A PRACTICAL APPROACH TO THE CYCLIC MULTI-BIN JOINT REPLENISHMENT PROBLEM

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by

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The undersigned, appointed by the dean of the Graduate School,

have examined the thesis entitled

A PRACTICAL APPROACH TO CYCLIC MULTI-BIN

JOINT REPLENISHMENT PROBLEM

Presented by Michael T. Canlas,

a candidate for the degree of Master of Science,

And hereby certify that, in their opinion, it is worthy of acceptance.

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Professor W. Jang

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Professor J. Noble

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Professor S. Lee

DEDICATIONS

Thank you to my wife and best friend Meghan for her love and support in completing my schooling while working and raising a family.

Thank you mom and pops for your *contributions* in helping survive all eight… maybe nine… maybe 10 or 11 years of college.

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ABSTRACT

Inventory management practices of supplied parts in manufacturing facilities is a performance critical topic that is often underestimated in importance. Commonly, supplied parts in manufacturing are ordered and inventoried in bulk to leverage lower prices, etc. The result is a manufacturing environment in which inventory is stored in two locations; *bulk inventory* and *production inventory*. This research presents a new policy aimed at addressing the joint replenishment problem, or the movement of multiple inventoried items from one location to another within a manufacturing facility. Physical constraints are considered as well as risk of production stoppage due to inventory shortages. In shortage situations, emergency replenishments are considered. Practical advantages of the proposed policy over previously developed policies will be highlighted. Finally, two solution approaches are developed and applied to sample data.

1. Introduction

1.1. Overview of manufacturing operations and inventory joint replenishment

Successful manufacturing facilities require careful planning and critical timely decision-making. Manufacturing facilities are commonly thought of as robust operations that efficiently transform a raw material into a finished good for the general customer. The reality is that within each manufacturing facility lays an intricate network of operations and relationships that must occur synchronously to ultimately be competitive and successful. Included in these internal operations are production, maintenance, purchasing/logistics, quality, engineering, and inventory management to name a few. All areas generally function independently but have the same central goal of serving and supporting production of the finished good.

While all the aforementioned areas within the general manufacturing facility offer substantial opportunity for research and development, the emphasis of this work will be in inventory management. Specific to this paper, inventory management can be described as *the strategic movement and management of all supplied parts within a manufacturing facility that contribute to the production of the final good*.

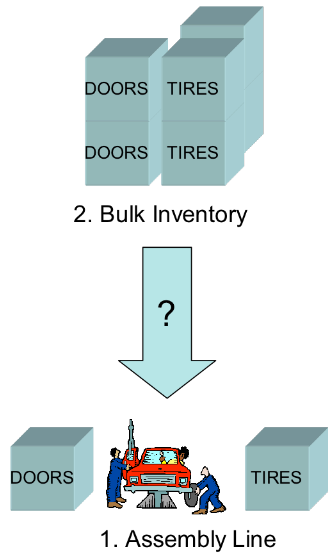

Figure 1. Facility level inventory management overview

As seen in Figure 1 there are two locations for parts inventory within the facility: (1) *Assembly Line* or *production inventory* where a small number of parts are stored, and (2) *Bulk Inventory* where a relatively large quantity is stored. Note that production inventory is assumed to be at the production line (i.e. point-of-use). Ideally, all inventories would be stored in a singular location. This hypothetical environment would avoid additional material handling between two inventory locations and nearly eliminate inventory micro-management. In reality, many small manufacturing facilities must buy their supplies in bulk to leverage cheaper costs. In other cases, larger parts must be stored in different areas due to safety or ergonomic guidelines. As well, many common parts might be used in multiple locations throughout a facility necessitating some sort of centralized bulk storage. Regardless of the reason, the manufacturing facility incurs the responsibility of managing additional inventory.

This *two-location inventory* layout is the very basic physical environment of this research. In the remainder of this chapter, an inventory management policy will be described, as well as the opportunities in further developing this policy. In the second chapter, previous work will be reviewed to give further background and understanding of the problem. It follows in the third chapter the problem will be outlined in greater detail, identifying key characteristics, etc. In chapter four, the general model will be developed. As well, approaches to solving the problem will be presented and applied to sample data. Finally, in chapter five, discussion of this topic will be concluded and potential for further research will be discussed.

1.2. Cyclic multi-bin approach to scheduling joint replenishment

When inventory is stored according to the aforementioned two-location framework, opportunities arise to design an efficient policy to move part inventories from bulk inventory to production inventory. A number of questions must be answered, including but not limited to: *When do you move inventory? How often? How much?* There are a number of policies in academia describing these models and answering these questions. Such models are generally described as some sort of *joint replenishment policy/problem* (i.e. JRP). *Joint* implies that more than one part is kept in inventory and therefore must be managed together. *Replenishment* describes the action of 'replenishing' inventory from bulk to production inventory. It is easy to deduce that such policies can become extremely complex in nature. Consider the inventory management of hundreds of individual parts. This chaos is faced in many manufacturing facilities today and is also the scenario that provides motivation for this work.

The problem to be further discussed in this thesis is already present in many facilities and will be referred to as the *cyclic multi-bin* JRP. See Figure 2. Consider multiple assembly stations in which doors and tires are being installed onto a car (1). Unfortunately, due to the physical size and quantity of the tires and doors, the bulk of the inventory must be stored some distance away from the assembly line in large bins (3). As bins of inventory are depleted at assembly (1), at some regular interval or cycle a material handler retrieves the empty bins from all the workstations (2), replenishes them at bulk inventory (3) then returns replenished bins to the assembly area.

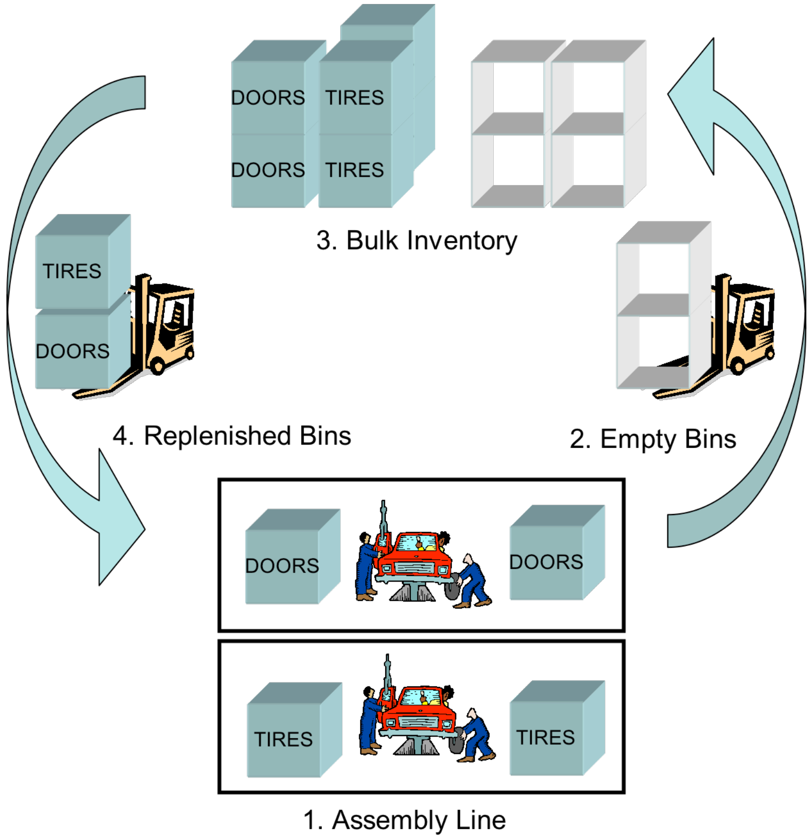

Figure 2. Cyclic multi-bin JRP

Although the above figure is basic, the intent is that the policy be applied to *all* parts that are stored in a two-inventory location environment.

There are several variables to take into consideration. In particular, number of bins in the system for any particular part, bin capacity, and standard cycle time length to name a few.

In practice, many of the aforementioned variables are pre-determined or highly inflexible. For example, bin capacity is constrained by the physical limitation of the material handler. Throughout an eight-hour workday, it is less feasible to expect a material handler to handle 50-pound bins of inventory all day versus 10-pound bins.

Beyond feasibility, this also can become a safety concern. Variables such as this become fixed. One variable that has considerably more flexibility is standard cycle time length. That is, how often a material handler cycles through each area, replenishing inventory. In general, standard cycle time length is a management decision that is often overlooked or underestimated in criticality. Additionally, the number of bins per part in the system has considerable flexibility. Again, this is generally a management decision.

This research will focus on a methodology for determining the ideal standard cycle time length by varying the numbers of bins per part in the system, given all other variables are fixed.

1.3. Opportunity for standard methodology

The motivating manufacturing facility is an industry leader in the design and manufacturing of electrical controls; circuit breakers. Specifically, the facility manufactures a number of drastically different product families of commercial and residential circuit breakers. This results in a number of different production lines producing various product families in differing volumes. In lower volume products, necessary supplied inventory is ordered and stocked in larger quantities to maintain low costs. As well, many parts are used in multiple products on different production lines. These facility characteristics necessitate the two-location inventory environment.

In the motivating facilities original state, any particular part was stored in a singular bin. Production would continue until any particular part was completely depleted. At this point, production would stop and wait while a material handler replenished inventory for this part. This was quickly identified as a substantial source of production downtime and instigated immediate action. The cyclic multi-bin policy was quickly adopted. However, system parameters were somewhat arbitrarily chosen. In an effort to make greater improvement, a simple *AB stratification* algorithm has been proposed. Later this will be described in greater detail.

The proposed actions of the manufacturing facility provide motivation for the development of standard decision-making tools within the inventory management function. This will ease the management burden while also moving towards the most efficient and beneficial practices in delivering inventory from bulk to production inventory.

2. Literature Review

2.1. Overview of relevant literature and terminology

The literature reviewed addresses previous research relevant to the background and development of the cyclic multi-bin JRP. All reviewed papers assume an environment in which inventory is moved from one location within a facility to another location within the same facility. While there are many commonalities between past work and this proposed research, opportunities for improvement will be ultimately highlighted.

Topics introduced in the literature review include but are not limited to: a practical discussion on issues related to inventory control, periodic versus continuous inventory review, stochastic versus deterministic demand, order discounting opportunities, and minimum order quantities. In general, costs considered in past research might include inventory/holding cost, major ordering costs, minor ordering costs, and shortage/backorder costs. The objective is to minimize costs, given an inventory system framework.

2.2. An innovative two-bin application for floor stock

Nicol (1989) presents an application of the two-bin inventory system in the electronics/missile controls industry. In its original state, as customer orders were received the storeroom would provide all necessary parts necessary for assembly one by one. This inventory management system was made necessary by strict quality assurance requirements and lot control.

A revised management system was developed to ease material handling and management costs of the low-value items. The result was 50% of the storerooms stockkeeping units were to be managed by a two-bin system. That is, half of the inventory originally delivered to production when a bill or material was presented would now be managed by the two-bin system. The described two-bin system is similar to the cyclic multi-bin JRP in that it provides production inventory availability and better controls material handling.

Nicol (1989) goes on to describe specifics of the program's setup. Of particular interest is the decision that each bin of inventory holds one month of parts. While this might sound like a physically massive amount of inventory, this particular stratum of inventory is comprised of nuts, screws, washers, etc. Further, 220 parts are to be managed by this system. The month-long cycle time benefits material handling costs of managing this many bins.

In this case study, the two-bin system's primary advantage is the continuous supply of inventory to production. Additionally, this system avoids the paperwork and use of a bill of material to obtain necessary parts for assembly. A case specific disadvantage relates to physical inventory audits. The studied facility is required to perform periodic physical inventory counts. In its original state, when all inventories were kept in the stockroom, this task was carried out relatively easily. However, with the addition of the two-bin system this task becomes more tedious as inventory is stored in multiple locations.

2.3. OR in inventory management: a review and critique

Silver (1981) provides an overview of operations research issues relevant to inventory management systems. Topics included, but not limited to, are policy descriptions, constraint considerations, potential problem objectives, relevant costs, and practical discussions.

Silver (1981) proposes three general questions that inventory management attempt to answer: (1) What is the inventory review interval? (2) When should replenishment occur? (3) What quantity should be replenished?

Several potential objectives for inventory management are presented. However, minimization of costs is generally observed in the vast majority of OR literature. Additionally, Silver (1981) presents very general constraint considerations as follows: (1) Supplier constraints (i.e. minimum/maximum order sizes, pack size restrictions, etc.), (2) Marketing constraints (i.e. minimum tolerable customer service levels), and (3) Internal constraints (i.e. physical limitations, budget, etc.)

Four relevant costs are identified as replenishment costs, carrying costs, costs of insufficient supply, and system control costs. The definitions of these costs are intuitive with the exception of system control costs. Silver (1981) describes this largely ignored cost as the expense of acquiring data necessary to execute decision rules.

Silver (1981) goes on to present a number of generally studied problems.

Perhaps the most interesting portion of the paper is the discussion of the gap between theoretical development and applications. For instance, often times instead of accurately modeling a given system, academia tends to focus on formulating the more mathematically interesting problem. The result might have academic credential, but is largely not applicable. As well, a number of points were made supporting a user-friendly inventory system that improves over the current system versus an abstract system that reaches optimality. In a practical environment, the user-friendly system is inherently easier to sell and implement.

2.4. Cyclic versus flexible approach to materials ordering

Sawik (2005) develops and analyzes two material ordering models aimed at the inventory management relationship between a supplier and manufacturer. A critical feature of the models is an assumed known production schedule.

The first model developed is the cyclic model, which consists of a stationary cycle time and order quantity for each delivered part. That is, the number of parts delivered is constant and the interval at which those parts are delivered is fixed (e.g. every hour, 20 of part a1 will be delivered, and every three hours 15 of part b2 will be delivered). In terms of practical use, this system is extremely easy to manage. Benefits of such a policy include ease of execution, and constant supply. At the same time, however, inventory/holding costs might become more significant.

Secondly, Sawik (2005) develops a flexible model. As the name implies, order intervals and quantity vary. With this policy, coordination is key.

In both presented cases, mixed-integer programs are formulated in a make-to-order environment. That is, product demand varies with time. However, it is assumed that production volumes are known and intuitively, a production schedule is known. The goal is to develop an optimal materials ordering schedule for a given set of data, for both models.

The two integer programs were applied to the same data set and results were evaluated. As intuition would suggest, the flexible model yielded the preferred results.

Two extensions of these models were also explored. First, a limit on inventory capacity was introduced. In any practical situation, there are physical limitations. Secondly, material order/delivery lead times were considered.

2.5. Periodic review policies for joint replenishment inventory systems

Viswanathan's (1997) major contribution is the development of a new policy to determining period length in a periodic review joint replenishment inventory environment.

Viswanathan (1997) policy applies to the periodic review (s, S) policy. In this policy, inventory levels are reviewed at some fixed interval of length t. At that time, if part i has an inventory level at or below s_i, then the inventory is replenished to an item specific level. The major contribution of this work is to determine the optimal period length.

Viswanathan (1997) references previous work in which the optimal period length was found for a single item environment. However, this paper extends this idea to the multi item, joint replenishment scenario.

The initial value is found based on the deterministic solution. This solution is based on the common run-out time on all items included in the system. Although not explicitly explained, it is assumed that this is the amount of time that it takes for all items (inventoried to capacity) to become depleted. Once the initial cost is evaluated, period length is incremented in steps of 0.03 in either direction until no improvement is observed.

This algorithm was applied to previously researched data sets and results were compared. Computational results demonstrate the advantages of the proposed approach over previous works.

2.6. Periodic can-order stochastic inventory systems with markovian discount opportunities

Zheng (1994) presents an inventory model of a single-item inventory system, with stochastic demand. The major contribution of this work is the demonstration that a particular policy shows optimal results when markovian discount opportunities are present.

The can-order model is commonly referenced by the notation (s, c, S). The framework of the model is as follows: When a particular item's inventory falls below a defined level, s, the item is replenished to a level, S. At the time, if other parts are below a level, c, those parts are also replenished to a given level S. The inventory levels have the relationship $s < c < S$. The advantage of this policy is the avoidance of incurring the major setup cost multiple times. It follows that the objective is to determine optimal inventory levels for s, c, and S. It is apparent that shortage costs must be considered in the instance that a particular item depletes its inventory and must wait some time to be replenished. Zheng (1994) defines this shortage cost as it relates to the number of products that must be backordered due to this absentee item.

A major contribution of the work is the inclusion of markovian discount opportunities. The policy indicates that when replenishment is carried out at discrete points in time, a discount occurs. In a practical scenario, this particular point in time might be associated with employees down time. That is, during normal work there is substantial loss when an employee has to leave their assignment/project to complete replenishment for another area. Conversely, when an employee is between assignments/projects this loss might not be as substantial. It follows that these markovian discount opportunities are stochastic.

Zheng (1994) begins formulations by defining three functions of inventory position: (1) expected time until next order is placed, (2) expected holding and shortage costs until next order is placed, and (3) the probability that the next order is triggered by a demand. Extensive proofs are demonstrated, ultimately indicating that the aforementioned (s, c, S) policy is ideal when markovian discount opportunities are present. Note that these results only apply to the basic item environment.

2.7. The stochastic joint replenishment problem

Ozkaya, Gurler, and Berk (2006) present a unique model and approach to the stochastic joint replenishment problem (SJRP) in a single location, multi-part environment.

In the approach developed by Ozkaya, Gurler, and Berk (2006), consider a continuous review, multi-item inventory system, subject to stochastic demands. Lead times are assumed to be item specific, but constant and backordering is considered. The major contribution of this paper is the way in which inventory replenishment occurs is a unique combination. First, orders might be triggered when a certain demand quantity, Q, is achieved for a specific part. For example, when 20 parts are used, replenishment is triggered. Secondly, inventory review and replenishment always occurs at a given time period after the last replenishment. For example, suppose that the given time period is two hours. Then parts are reviewed, and if necessary replenished two hours after the previous replenishment.

Ozkaya, Gurler, and Berk (2006) develop the cost function of their policy by first defining the following expressions: expected cycle length, order rate, and expected on-hand inventory and backorder levels.

Model analysis consists of numerical evaluations, and comparisons to previously researched policies. Optimization is achieved by an exhaustive search over a large solution space. Given that the evaluation of the policy is a comparison to past work, an extensive analysis is provided. In particular, attempts were made to determine what specific environment characteristics would lead one to favor one policy over another. However, it was concluded that the proposed policy does not have monotone dominance over other policies

2.8. Periodic versus can-order policies

The major contribution of this research by Atkins and Iyogun (1988) is the development of a new lower bound for a joint replenishment inventory problem, and the comparison of previously developed policies.

The lower bound and four policies were evaluated over a given set of data including 12 products. The lower bound and four policies are briefly described as follows:

Lower Bound – The numerical lower bound of the period length was found to be 0.65 time units.

Modified Periodic Heuristic – Using the period length from the lower bound, all replenishment must occur at multiples of this period length. Some parts are replenished every 0.65 time units, some every 1.30 time units, etc.

Period Policy – All products are reviewed/replenished EVERY period. All products are reviewed/replenished every 0.65 time units.

Can Order Policy – This policy is as described Zheng (1994) in section 2.6 of this paper.

Independent Policy – Each product is continuously reviewed and is replenished according to unique criteria. For each product, (s, S) is defined. That is, when a product reaches an inventory level s (or less) the product is replenished to a level S.

Although no proofs are demonstrated, computational results are shown in favor of the periodic policies over the more complex 'can-order' systems.

2.9. Iterative spreadsheet heuristic for solving JRP

Nilsson, Segerstedt, and van der Sluis (2007) develop a practical *spreadsheet tool*. The major contribution of this work is its movement towards bringing academic research and development in the JRP to a practical and easy to use tool.

Two critical assumptions that are made by the authors are that (1) replenishment orders are scheduled at (multiples of) strict intervals and that (2) at least one part must be replenished at each interval.

It is also assumed that this strict interval is known. It follows that the objective of this work is to identify at what multiple of this known interval should any particular part be replenished to minimize cost. For example, the heuristic might identify that *part 1* should be replenished every interval, while *part 2* should be replenished every third interval to minimize costs.

The cost structure of the objective function is represented by the time interval between inventory replenishment cycles as a function of *replenishment costs* and *inventory holding costs*. Within the replenishment costs are major costs, incurred once per interval, and minor costs that are dependent on the type and quantities of part(s) being replenished.

The intent of the heuristic is that by balancing replenishment and inventory holding costs, the cost function as a whole is minimized (i.e. the ratio equals one). This logic is deduced from previous work done with the *economic order quantity*.

The final result is an approach to solving a specific form of the JRP. Perhaps the biggest contribution is the inclusion of a spreadsheet driven two-step heuristic.

2.10. JRP with resource restrictions

Significant research has been done considering the cost structure, basic frameworks, and solution approaches to the JRP. Moon and Cha (2007) present work addressing more practical restrictions and constraints. A genetic algorithm is presented and compared against two previously developed approaches.

The authors site specific examples that have received little attention in previous JRP research. Specifically, minimal work has been done to address practical manufacturing constraints such as storage capacity, transportation capacity, or simply budget constraints.

The objective function used in this research consists of inventory holding costs and major and minor replenishment costs. The major contribution of this work is the addition of a budget constraint and how it is approached. This constraint is generally defined as a function of *demand rate, replenishment schedule,* and *unit cost*. As in other works, the decision variable is at what multiple of the replenishment interval is any particular part to be replenished.

Moon and Cha (2007) begin by modifying a previous developed solution methodology (i.e. *C-RAND*), such that it is applicable to the newly developed JRP with restrictions. Additionally, they present a new *genetic algorithm*. Both approaches are applied to a previously studied data set that was originally solved by a third approach, *Goyal's Algorithm*. For the particular data set, all three approaches yielded the same results.

The three methods were further applied to additional scenarios to better understand their performance characteristics. Based on these comprehensive computation experiments, it was found that the previously developed C-RAND approach performed better.

Moon and Cha (2007) presented a unique approach to solving the JRP with a single resource constraint. In computation experiments, previously developed methods appear to have performed better. However, as Moon and Cha point out, the fundamental benefit of their original genetic algorithm approach is the flexibility in that additional constraints can be *easily* added as necessary. This final thought leaves room for additional research in developing a JRP with multiple restrictions.

2.11. Summary of reviewed literature and relevance to proposed work

The purpose of this section is to briefly review portions of the policies previously researched and describe shortcomings of these policies relative to the cyclic multi-bin JRP.

Sawik (2005) compares and contrasts two distinct policies. The first rigidly controls review interval, replenishment scheduling, and replenishment quantity to fixed values while the second allows for complete flexibility. However, more popular in research is the (s, S) policy studied by Viswanathan (1997) and the can-order (s, c, S) policy studied by Zheng (1994). A key requirement for carrying out either policy is the requirement of continuous review. In both policies, replenishment for *part a* is triggered when inventory levels reach the value s, at which point they are replenished to maximum capacity, S. Additionally, in the (s, c, S) policy, if replenishment is already occurring for *part a* and *part b* has reach a inventory level of c such that $s<c<S$, then *part b* is also replenished. In these two policies, replenishment does not occur based on a time schedule, but is strictly triggered by inventory levels.

In the four policies presented by Sawik (2005), Viswanathan (1997), and Zheng (1994) inventory replenishment occurs either by time schedule, or when a given inventory level is reached. Ozkaya, Gurler, and Berk (2006) present a model that is a hybrid of both trains of thought. In their policy, replenishment occurs at regular intervals. Additionally, if inventory levels should drop to a given level as a result of increased production demand, replenishment will occur outside of the regular schedule.

Silver (1981) proposes three basic questions that a JRP policy must answer: (1) What is the inventory review interval? (2) When should replenishment occur? (3) What quantity should be replenished? In the policies reviewed for this research, these three questions posed by Silver (1981) can be answered. However, in applying these policies in practice, *ease of policy execution* must also be considered. Silver (1981) highlights the gap between academic development and practical applications for joint replenishment policies. For example, the (s, S) and (s, c, S) policies require *continuous review* for decision making (i.e. When must replenishment occur). That is the user must have a system in place that continuously reviews inventory levels. Such a system can be considerably expensive even for a small manufacturing facility. Again, while these policies might seem ideal in academia, implementing them might be deemed to costly or completely impractical. Additionally, many of the policies reviewed consider complex cost functions including major and minor setup costs. In many *heavy lifting* environments (e.g. punch-press operations, plating operations, etc.) these costs might be relevant, but in many cases there are no setup costs to consider.

It follows that there lies opportunities to develop a policy with practical and intuitive decision making tools and a simple cost function.

3. Environment and Problem Characteristics

3.1. Overview of environment and problem characteristics

As stated in section two, the motivating manufacturing facility is an industry leader in the design and manufacturing of electric controls equipment, circuit breakers. Market demand results in customized circuit breakers built to order. This single factor results in a large number of parts, each with vastly different production demand characteristics. At first glance, this fact might lead one to believe that a complex inventory management policy is necessary. However, this research proposes a simple and straightforward policy.

This development will focus on one production line in particular consisting of four assembly stations and utilizing a total of 32 supplied parts. A key characteristic of this line is that it functions as a *late customer adaptation* production line. That is, customized circuit breaker demand is met by *adapting* a basic circuit breaker to meet customer needs by adding various combinations of additional parts.

All parts necessary to produce the final product are supplied by outside manufacturers. In addition, a majority of the supplied parts are used in relatively small volumes. To maintain low price points parts must be ordered and stored in some bulk. As a result, inventory is stored in one of two locations; bulk inventory and production inventory. The material handler travel time between the two inventory locations is negligible. It follows that the physical environment and characteristics of the production line make it a prime candidate for the cyclic multi-bin JRP.

3.2. Current inventory replenishment operations

Both historic and current practices in the movement of inventory have not been systematic. In general, production would continue as long as possible until production inventory was completely depleted. The result is a production operator stopping the production line, calling a material handler to replenish production inventory, waiting, etc. Not only is this a nuisance, but it also decreases productivity. This is a key outcome that provided the motivation for this research.

3.3. Policy objective and decision variables

A major drawback of the current system is a lack in scheduling control. Section 3.2 describes an environment in which material handlers are *randomly* called to replenish inventory at a moments notice. The objective of this policy is to standardize the intervals or periods during which production inventory levels are reviewed and replenished. Specifically, *the objective is to maximize the standard length between inventory replenishment cycles*, subject to some constraints. A level of flexibility lies in the number of bins allowed in the policy. It follows that the number of bins per part is a decision variable.

The practical benefit of the stated objective function is that inventory replenishment decisions are vastly simplified, as these events will occur at fixed intervals. As well, as further discussed in 3.4, a more consistent flow of parts will be supplied to production.

3.4. Service level

The first constraint relates the material handlers performance to the level of service that is expected. One of the major benefits of the policy is in how it provides a constant flow of inventory to production.

In practical terms, if production had to stop daily for a conservative two minutes for replenishment, it might not seem like a major issue. However, consider that over the course of a year this *downtime* quickly accumulates to more than a full working shift (based on 250, 8 hour working shifts per year). Translating, this means that for a full day an entire production line of operators are paid to wait for inventory replenishment.

Based on this observation, it is absolutely necessary for any inventory management policy to ensure that production inventory is *mostly* available with some level of certainty.

3.5. Physical capacities

There is a maximum capacity constraint that a material handler is capable of replenishing in one cycle. Perhaps more important are rules and standards designed to minimize risk of personal injury. This is absolutely related to the quantity of bins that a material handler can replenish during a replenishment cycle.

In this research it is assumed that there is a known replenishment capacity. That is, a material handler is capable of replenishing a certain number of bins per cycle.

3.6. Review of problem characteristics

The focus of this research revolves around a production line with the key characteristic of having high variance in production inventory consumption. Additionally, each part's inventory consumption is assumed to be independent.

The objective of the cyclic multi-bin JRP is to maximize the standard length of time between material handler replenishment events. This will in turn minimize the usage of the material handler. To achieve this objective, the policy is given the flexibility to also adjust the number of bins allowed in the system. Two practical constraints will be considered. First and foremost is safety. Strict capacities are enforced on the quantity of bins that can be replenished in a single cycle. Secondly, as much as possible, continuous production must be maintained.

In section four, this model will be further developed. As well, two unique approaches will be described and ultimately applied to sample data collected from the motivating manufacturing facility.

4. Methodology

4.1. Methodology overview

The purpose of this section is to develop a mathematical model that describes the cyclic multi-bin JRP with the correct objective function and constraints. This will lay the framework to begin reviewing and developing techniques to efficiently solve the problem.

The following sections will begin with defining the variables necessary to construct the model. Next, the general model will be developed. Two approaches to solving the problem will be described. Finally, the methodologies will be applied to sample data from the motivating facility.

4.2. Policy and environment assumptions

The model being developed abides by the cyclic multi-bin JRP described in previous sections.

The environment consists of one production line made up of multiple *stations* (a negligible distance apart). Each station is capable of storing multiple bins for each part. It is possible for the same part to be required/stored on multiple stations.

It is assumed that part consumption and material handler capacity is known. Additionally, it is assumed that all bins are physically the same size. However, since different parts have different dimensions, the bins will not be able to hold the same number of every part.

4.3. Variable definitions

In constructing a mathematical model, it is necessary to define a number of variables to properly represent the system at hand. The following table outlines and describes given system and decision variable definitions:

Decision Variables

t	*Cycle length*; duration of time allowed between inventory replenishment cycles
x_i^j	Assigned number of bins for part i at station j

Given System Variables

c_i	Capacity of a bin in terms of part i
u_i^j	Hourly usage of part i at station j
u_i^j/c_i	Hourly usage of part i at station j in terms of bins
k^j	Capacity of station j in terms of bins
k	Capacity of a material handler in terms of bins per cycle
α	Material handler capacity estimator

Figure 3. Variable definitions

4.4. Mathematical model

Following is the complete deterministic cyclic multi-bin JRP.

$$\text{MAX } t \tag{1}$$

Subject To:

$$t \leq \frac{x_i^j - 1}{u_i}(c_i) \text{ for all } i \text{ and } j \tag{2}$$

$$\sum \left\lceil \frac{t \times u_i^j}{c_i} \right\rceil \leq k \tag{3}$$

$$\sum_j x_i^j \leq k^j \text{ for all } i \tag{4}$$

$$t > 0 \tag{5}$$

$$x_i^j = 2,3,4\ldots \text{ for all } i \text{ and } j \tag{6}$$

(1) describes the objective of the JRP, which is to maximize inventory replenishment cycle time. The first constraint, (2) *guarantees* that production inventory is always available thus negating any possibility of production downtime. (3) takes into account the physical constraint of the material handler and how many bins they are capable of replenishing during a cycle. Similarly, (4) takes into consideration the physical capacity of the workstation. (5) is a non-negativity constrain on time while (6) guarantees that at least two bins are in the system for any part i at station j.

The remainder of this subsection will be spent developing the specifics of the model.

As discussed in 3.3, the objective of the cyclic multi-bin JRP is to *maximize the standard period length between inventory replenishment cycles*, subject to some constraints. The formulation is (1).

The first constraint, (2), *guarantees* that production inventory is always available. The theoretical basis of (2) is that cyclic inventory usage cannot be so large that it leaves less than one bin of inventory available (i.e. to guarantee production inventory availability, the following must hold: $t \times u_i^j / c_i \leq x_i^j - 1$). In the following example, it is demonstrated that in a three-bin system in which periodic usage does not exceed two bins, inventory shortages will never seem to occur. Assume a single part follows a cyclic three-bin JRP with periodic bin usage of 1.9 bins. In the initial state, all three bins are filled to capacity. At the end of the first period, 1.9 bins are consumed with 1.1 remaining in inventory. After replenishment of one bin, 2.1 bins are available at the beginning of the second period. Again, at the end of the second period, 1.9 bins are consumed with 0.2 bins remain. Two bins are replenished leaving 2.2 bins to begin the third period. Note that an inventory shortage does not seem to occur. The following figure summarizes the inventory levels through the first five periods of this particular policy.

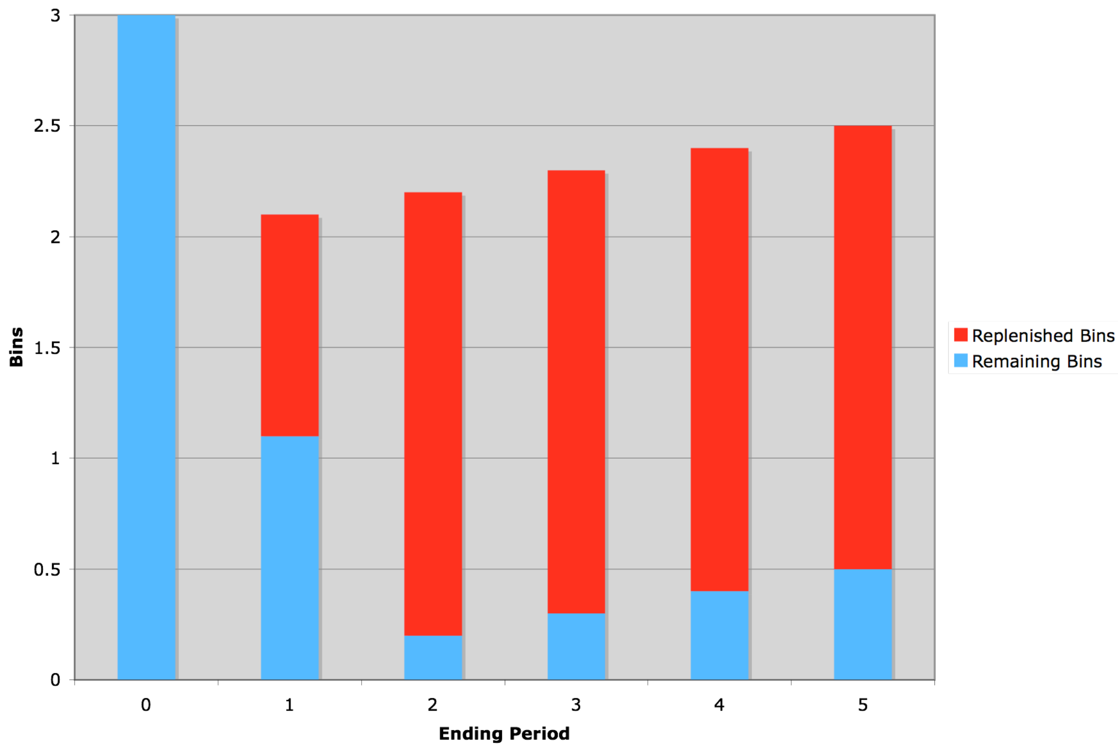

Figure 4. Example - bin replenishment schedule

Oppositely, in the following counter-example it is demonstrated that in a three-bin system in which periodic usage *exceeds* two bins, inventory shortages will occur by the end of the fourth period. In this JRP example, the number of bins in the system is three while periodic usage is 2.3 bins. In the initial state, all three bins are full. After 2.3 bins are consumed in the first period 0.7 bins remain. Two bins are replenished leaving 2.7 available at the beginning of the second period. At the end of the period, 0.4 bins are remaining. Again, two bins are replenished leaving 2.4 bins to start the third period. As demonstrated in the following figure, by period four, inventory shortages will occur.

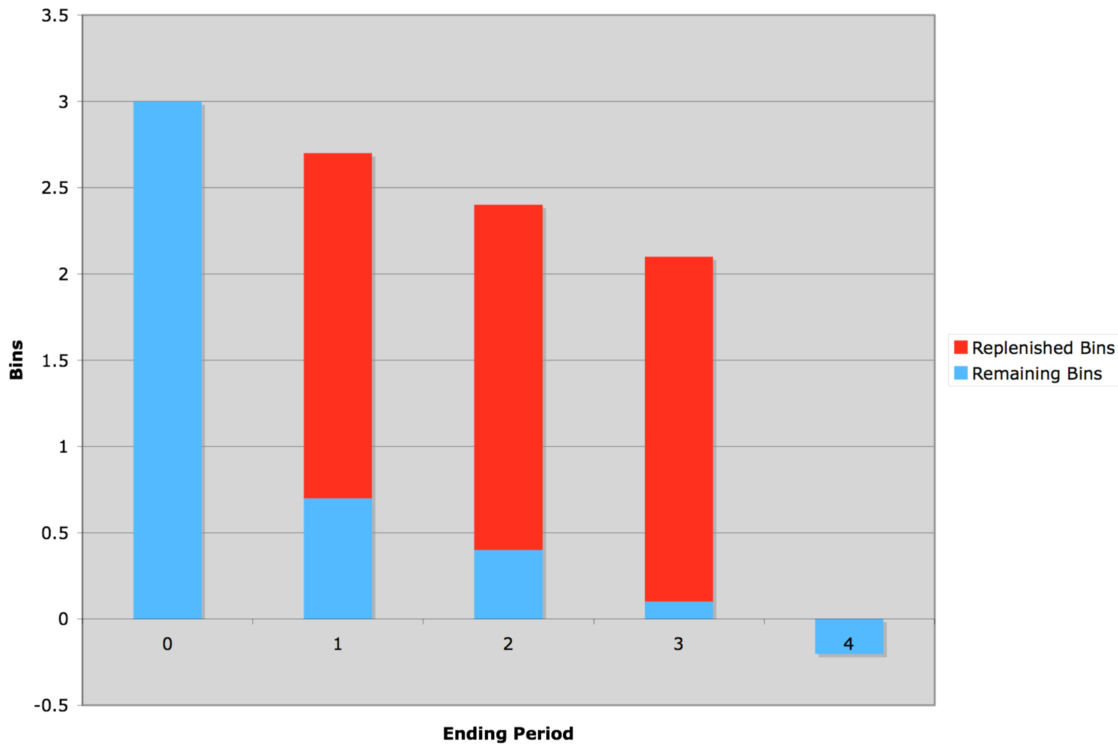

Figure 5. Counterexample – bin replenishment schedule

While these examples seem logical, the general case must be further discussed. Consider a scenario in which there are β full bins and one partial bin for part i at station j at the end of a cycle ($\beta = 0,\ 1,\ 2\ldots\ x_i^j - 1$). It follows that there are $x_i^j - \beta - 1$ empty bins waiting to be replenished. Note that these can be replenished assuming *other* constraints are met. At the beginning of the next cycle there will be $(\beta) + \left(x_i^j - \beta - 1\right) = x_i^j - 1$ full bins and one partial bin of inventory. During this next cycle, $t \times \frac{u_i^j}{c_i}$ bins of inventory are used. If $t \times \frac{u_i^j}{c_i} \le \left(x_i^j - 1\right) \Rightarrow$ (2) $t \le \frac{x_i^j - 1}{u_i^j}\left(c_i\right)$ is met, then part usage is less than or equal to inventory availability. No shortages will occur during the next cycle and there will be at least one partial bin available.

The next constraint, (3), takes into consideration the practical limitations of a material handler. Beyond obvious physical constraints there are often safety standards that must be abided by to help unsure employee well being. A major consideration in this constraint's development is the requirement that the material handler replenish *whole* bins of inventory. The formulation of the constraint is derived from the direct relationship between the number of bins to be replenished and periodic usage. That is periodic usage cannot exceed material handler capacity, (3) $\sum \left\lceil \frac{t \times u_i^j}{c_i} \right\rceil \leq k$.

Consider a previous example in which cyclic usage is known to be 1.9 bins/cycle in a three-bin system. The floor and ceiling of this periodic bin usage is one and two respectively. Looking again at Figure. 4 it is easy to see that the first time replenishment occurs, only one bin is replenished. However, in later cycles two bins are replenished. (3) accounts for the *worst-case* scenario by calculating the ceiling of periodic usage. The material handler will always have enough capacity to replenish the *worst-case* number of bins. This guarantees that the material handler will always have the capacity to replenish all empty bins and production inventory will always be available.

A practical physical constraint is also necessary at the workstation, (4). That is, there is a physical limit on the total number of bins that can be stored in production inventory, k^j.

Two additional constraints are required to guarantee the feasibility of any solution. First, a *non-negativity constraint* (5) is necessary for t. Secondly, a similar constraint is necessary for x_i^j to guarantee integer values and that *at least* two bins are in the systems for any part i at station j, (6).

4.5. Solution approaches

In this section, two solution approaches will be presented. The first approach necessitates an approximation of the material handler constraint, (3). Both approaches will be applied to a hypothetical situation and results will be compared and contrasted.

The following linear model can be solved in MS/Excel Solver. This model is similar to that developed in section 4.4 with the exception of the approximation of the material handler constraint. The original constraint consisted of a non-linear ceiling function, which poses some challenge when solving the model.

$$\text{MAX } t \tag{7}$$

Subject To:

$$t \le \frac{x_i^j - 1}{u_i}(c_i) \text{ for all } i \text{ and } j \tag{8}$$

$$\sum \frac{t \times u_i^j}{c_i} + \alpha \le k \tag{9}$$

$$\sum_j x_i^j \le k^j \text{ for all } i \tag{10}$$

$$t > 0 \tag{11}$$

$$x_i^j = 2,3,4\ldots \text{ for all } i \text{ and } j \tag{12}$$

The approximation of the material handler constraint provides practical benefit in approaching the problem. From 4.4, the material handler constraint considers the worst-case material handler capacity situation by calculating the ceiling, (3) $\sum\left\lceil \frac{t \times u_i^j}{c_i} \right\rceil \le k$.

Although this does guarantee production inventory availability, there is some difficulty in solving models containing a ceiling function, as it is non-linear.

Consider a more complex system in which multiple parts have known and unique x_i^j and u_i^j. In this system there are three parts that abide by the cyclic multi-bin JRP. *Part one* is known to have periodic usage of 2.5 bins per period with a total of four bins in the system. At the end of the first period of production 2.5 bins are consumed leaving 1.5 bins remaining in inventory while two bins are replenished. Thus, 3.5 bins are available at the beginning of the second period. At the end of the period, one bin will remain while three bins will be replenished. Note that at this point, *part one* inventory has returned to its initial state. It follows that this replenishment pattern will repeat. In the case of *part two*, it is known that 1.2 bins are consumed per period and that there are three bins in the system. Following similar logic as in *part one*, one bin will be required for replenishment in each of the first four period and two bins will need to be replenished in the fifth period. Again, at this point, *part two* has returned to its initial state of full inventory. Thus the pattern will continue to repeat itself. *Part three* has a known usage of 1.4 bins per period, with three bins in the system. Again, a pattern develops in which one bin is to be replenished in the first, second, and fourth while two bins are to be replenished in the third and fifth period. This pattern continues to repeat itself. While there is some interest in the individual patterns that have been observed thus far, the real interest is the implications that these patterns have on material handler capacity constraints. For example, one might observe the maximum required bin replenishment for each individual part and assume that capacity is always required to account for this maximum required replenishment. That is, *part one* requires at most three bins, *parts two* and *three* requires at most two bins each. It is deduced that a material handler must have capacity to handle seven bins. While this logic seems correct, what one might fail to realize is that these individual maximum required replenishment scenarios might not always occur during the same periods. In fact, in this particular example the maximum required replenishment scenario is not realized until the 10th period.

Given Policy Parameters		Replenishment per Period				
		Period 1	Period 2	Period 3	Period 4	Period 5
$x_1^1 = 4$	$\frac{t \times u_1^1}{c_1} = 2.5$	2	3	2	3	2
$x_2^1 = 5$	$\frac{t \times u_2^1}{c_2} = 1.2$	1	1	1	1	2
$x_3^1 = 3$	$\frac{t \times u_3^1}{c_3} = 1.4$	1	1	2	1	2
Req'd Replenishment Capacity		2+1+1=4	3+1+1=5	2+1+2=5	3+1+1=5	2+2+2=6
Max. Required Replenishment $\Rightarrow \sum \left\lceil \frac{t \times u_i^j}{c_i} \right\rceil = \lceil 2.5 \rceil + \lceil 1.2 \rceil + \lceil 1.5 \rceil = 7$						
Min. Required Replenishment $\Rightarrow \sum \left\lfloor \frac{t \times u_i^j}{c_i} \right\rfloor = \lfloor 2.5 \rfloor + \lfloor 1.2 \rfloor + \lfloor 1.5 \rfloor = 4$						

Figure 6. Theoretical VS actual material handler capacity requirements

This situation emphasizes the necessity of a constraint relaxation utilizing the constant α in place of the ceiling function. This results in the approximation, $\sum \frac{t \times u_i^j}{c_i} + \alpha \le k$. It might seem wise to account for the worst-case scenario of seven bins of material handler capacity, but this might be a gross overestimate as this may rarely occur. As well, situations might arise in which the ceiling function will *always* exceed material handler capacity. Considering only the best-case scenario of four bins per cycle will quickly lead to continuous part shortages requiring repetitive *emergency replenishment*. The use of α can provide a structured approach to resolving this dilemma.

In determining α, choosing smaller values might lead to shortage situations requiring emergency replenishment, while larger values might lead to an overestimate of required capacity.

Reconsider the information presented in Figure 6. Assume that actual material handler capacity is 5 bins per period and we wish to determine (1) if shortages can be expected and (2) if the material handler capacity constraint, $\sum \frac{t \times u_i^j}{c_i} + \alpha \le k$ is satisfied. Instead of using the floor or ceiling functions to estimate required replenishment capacity we will now use the estimation $\sum \frac{t \times u_i^j}{c_i} + (\alpha)$ where $\alpha = -0.3$. The result is that an estimated 4.2 bins will require replenishment per period. In this case, the constraint is satisfied as 4.2 replenished bins per cycle is less than the material handler capacity of five. In reality, this approximation of required replenishment underestimates actual requirements. Specifically, in the fifth period the material handler capacity is exceeded which would result in part shortage/emergency replenishment.

Similarly, completing the estimation when $\alpha = 1.0$ estimates that 8.1 bins will require replenishment. The constraint is *not* satisfied as $8.2 > 5$. The implication of this inequality is that the cycle time must be reduced, such that expected required replenishment does no exceed five. Interestingly, this approximation of required bin replenishment is larger than the maximum required replenishment. That is, in reality a material handler will never be required to replenish more than seven bins. The following table summarizes results.

<table>
<tr><td colspan="2" rowspan="2">Given Policy
Parameters</td><td colspan="5">Replenishment per Period</td></tr>
<tr><td>Period 1</td><td>Period 2</td><td>Period 3</td><td>Period 4</td><td>Period 5</td></tr>
<tr><td>$x_1^1 = 4$</td><td>$\frac{t \times u_1^1}{c_1} = 2.5$</td><td>2</td><td>3</td><td>2</td><td>3</td><td>2</td></tr>
<tr><td>$x_2^1 = 5$</td><td>$\frac{t \times u_2^1}{c_2} = 1.2$</td><td>1</td><td>1</td><td>1</td><td>1</td><td>2</td></tr>
<tr><td>$x_3^1 = 3$</td><td>$\frac{t \times u_3^1}{c_3} = 1.4$</td><td>1</td><td>1</td><td>2</td><td>1</td><td>2</td></tr>
<tr><td colspan="2">Req'd Replenishment
Capacity</td><td>2+1+1=4</td><td>3+1+1=5</td><td>2+1+2=5</td><td>3+1+1=5</td><td>2+2+2=6</td></tr>
<tr><td colspan="7">Req'd Replenishment (α = -0.3) $\Rightarrow$
$\sum \frac{t \times u_i^j}{c_i} + (-0.3) = (2.5 - 0.3) + (1.2 - 0.3) + (1.4 - 0.3)$
$= 4.2$</td></tr>
<tr><td colspan="7">Req'd Replenishment ($\alpha = 1.0$) $\Rightarrow$
$\sum \frac{t \times u_i^j}{c_i} + (1.0) = (2.5 + 1.0) + (1.2 + 1.0) + (1.4 + 1.0)$
$= 8.1$</td></tr>
</table>

Figure 7. Example of α

It follows that as α becomes smaller, the risk of incurring a part shortage increases for a given cycle length. If α is greater than or equal to one, estimates of required replenishment will be greater than or equal to maximum required replenishment.

For the purposes of this research, calculations will be carried out using varying values of α, and then results will be compared and contrasted.

To demonstrate solution approaches a simple example was constructed. The hypothetical scenario consists of two workstations, consuming a total of five unique parts. All parts follow the cyclic multi-bin JRP. Each of the two stations is capable of storing no more than 15 bins. The material handler can replenish at most nine bins per period. Although these physical constraints are known, it has not been determined how many bins should be in place for each part/station. Similarly, it has not been decided how often material handlers should review and replenish inventory. The following table summarizes given system parameters as well as an initial solution for x_i^j.

Material Handler Capacity (Bins/cycle, k)	Station j (Bins/station, k^j)	Part i	Known Usage u_i^j	Bin Capacity (Parts/bin, c_i)	Initial Solution (Bins/part, x_i^j)
9	a (15)	1	20	20	2
		2	20	25	2
		3	20	25	2
	b (15)	3	40	25	2
		4	40	40	2
		5	40	20	2

Figure 8. System parameters and initial solution

It follows that this scenario can be modeled and solved by the developed linear model. As well, it is arbitrarily assumed that $\alpha = 0.1$. In larger models this factor has considerable effect on the results. This will be demonstrated and further discussed in later sections.

The first solution approach will take advantage of readily available technology in determining a solution. In relatively small problems, the cyclic multi-bin JRP can be solved directly, in Microsoft Excel/Solver. All calculations in this research are carried out on an Apple iBook G4 using Microsoft Excel 2004 for Mac. As well, parameters are set to limit runtime to 600 seconds and 10000 iterations.

The following table summarizes the solution found by using the MS/Excel tool.

Station j (Bins/Station capacity k^j)	Part i	Known Part Usage u_i^j	Parts/Bin Capacity c_i	Bins per Part per Station x_i^j	Cycle Time t
a (15)	1	20	20	2	1 Hr.
	2	20	25	2	
	3	20	25	2	
b (15)	3	40	25	7	
	4	40	40	2	
	5	40	20	6	

Figure 9. MS/Excel Solver solution

As with most readily available technologies, as problems become larger other means for solving the problem must be developed. The second solution approach is a manual *greedy heuristic*. This approach does not require the use of the material handler constraint approximation but does remove the ceiling function. The following method details the execution of this heuristic:

i. Initiate: Set $x_i^j = 2$

ii. Verify that $\sum \frac{t \times u_i^j}{c_i} \leq k$ and $\sum_j x_i^j \leq k^j$ for all x_i^j. If these capacity constraints hold, continue to iv). If not continue to iii).

iii. If this is the initial iteration, no feasible solutions exist. If this is not the initial iteration, the final solution is from the last set of x_i^j's that satisfy $\sum \frac{t \times u_i^j}{c_i} \leq k$ and $\sum_j x_i^j \leq k^j$ where $t = \max\left[\frac{x_i^j - 1}{u_i}(c_i)\right]$.

iv. Calculate $\frac{x_i^j - 1}{u_i}(c_i)$ for all x_i^j. Set $t = \max\left[\frac{x_i^j - 1}{u_i}(c_i)\right]$.

v. Find $x_i^j{*} = \min\left[\frac{x_i^j - 1}{u_i}(c_i)\right]$ and increase $x_i^j{*}$ by one bin.

vi. Repeat ii).

Applying this greedy approach to the same data set as before, the following solution was found:

Station j (Bins/Station capacity k^j)	Part i	Known Part Usage u_i^j	Parts/Bin Capacity c_i	Bins per Part per Station x_i^j	Cycle Time t
a (15)	1	20	20	2	1 Hr.
	2	20	25	2	
	3	20	25	2	
b (15)	3	40	25	3	
	4	40	40	2	
	5	40	20	3	

Figure 10. Greedy heuristic solution

In this particular example, both approaches yield the objective function value of one hour. Interestingly, in two out of six cases the two approaches yield different values for the same respective x_i^j.

	Cycle Time t	MS/Excel Solver x_i^j	Greedy Heuristic x_i^j
x_1^a	1 Hour	2	2
x_2^a		2	2
x_3^a		2	2
x_3^b		3	3
x_4^b		5	2
x_5^b		7	3

Figure 11. Solver VS heuristic solution comparison

4.6. A Practical Application of the Linear Integer Program Approach and Greedy Heuristic Approach to the Cyclic Multi-Bin JRP

To this point, a model has been developed featuring a logical objective function with proven constraints aimed at fulfilling practical inventory management requirements. Two approaches were demonstrated on a small data set, greedy heuristic and MS/Excel solver. Both approaches yielded similar results. In this section, motivating manufacturing environment will be described. The original state of the motivating facility's inventory management practices will be described and quantified. This will be considered the *baseline*. For demonstration purposes, system parameters will be varied one at a time to demonstrate their affect. Following, *AB stratification* will be briefly discussed and quantified. Finally, the cyclic multi-bin JRP will be applied using the two techniques presented in this research. The outcomes of the various inventory management policies will be compared based on 1000 simulated replenishment cycles.

The motivating *manufacturing cell* consists of four small workstations in which variations of a circuit breaker component is assembled using various combination of 32 unique parts. The result is the ability to produce 52 unique final products aimed at meeting customer needs. The first station holds 17 parts in as many as 51 bins, the second holds five parts in as many as 15, the third holds four parts in as many as 12 bins, and the fourth station holds six parts in as many as 18 bins. Physically, the distance between the four workstations is negligible. As well, since bulk inventory for these 32 parts is stored a negligible distance away, the cyclic multi-bin JRP is applicable. It is also known that a material handler is capable of replenishing 14 bins of parts per cycle.

Production demand data was collected over a three-month period, June 2007 through August 2007. The *average hourly demand* was calculated over the period for each individual part. To maintain the integrity of the data, the average was taken across production that occurred during non-holiday Mondays, Tuesdays, Wednesdays, and Thursdays. During holidays, Fridays, and weekends, production scheduling is generally irregular potentially resulting in *non-normal* production behavior. Hourly demand and bin capacity is summarized in the appendix.

In the facility's original state, each part is stored in three bins. That is, a total of 96 bins are in the current system. Further, scheduled replenishment occurs every 15 minutes (0.25 hours). In the event that a part shortage occurs, emergency replenishment occurs. It is assumed that a material handler has unlimited capacity during emergency replenishment. In this baseline state, through 1000 replenishment cycles a material handler can expect to replenish 8.99 bins per scheduled replenishment, and 0.00 bins emergency replenished between scheduled replenishments. Additionally, the material handler will replenish at most 14 bins during scheduled replenishment and as many as 3 bins between scheduled replenishment. While this result seems desirable, as emergency replenishment risk appears to be relatively low, the baseline system parameters stipulate that the material handler must make replenishment cycles every 15 minutes (i.e. 32 cycles during a 8 hour working shift). It might be feasible for the material handler to visit less often but still achieve the same replenishment results. The following figure summarizes the affect of varying cycle time while all other system parameters remain the same.

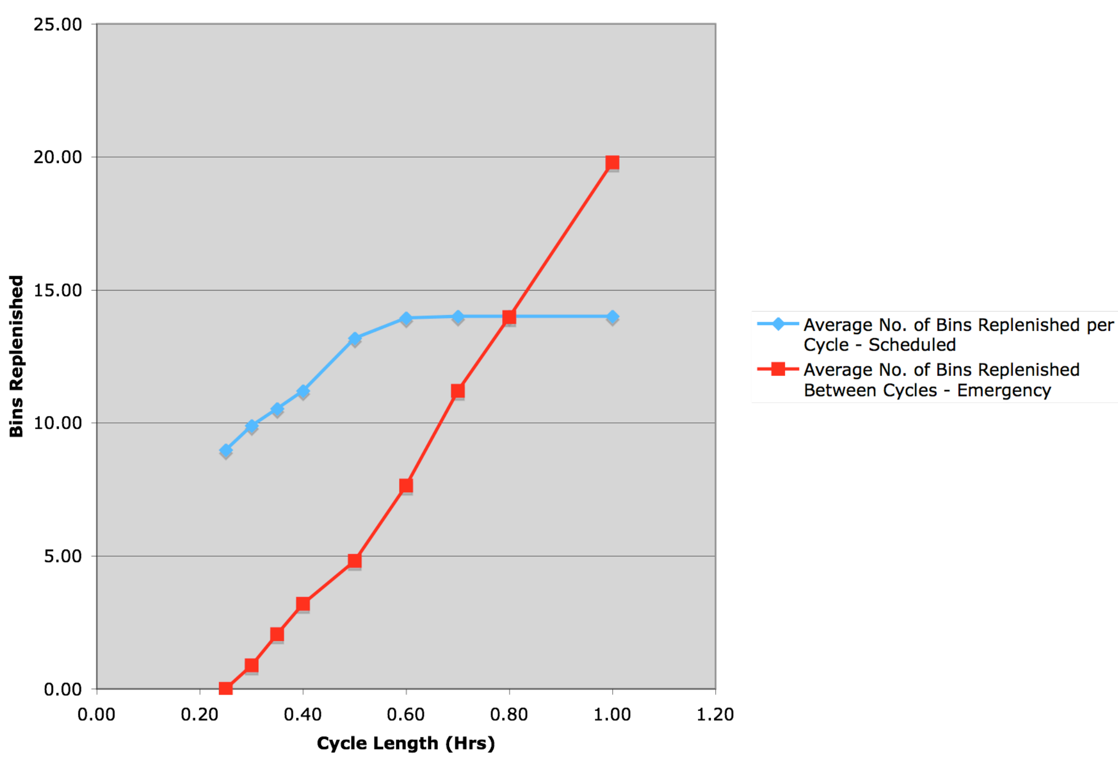

Figure 12. Bin replenishment VS cycle length

Interestingly, cycle length can be increased from 0.25 hours to 0.30 hours while still maintaining an *average number of bins emergency replenished between cycles* of less than one. Daily scheduled replenishment cycles can be reduced from 32 per working shift to less than 27 while still maintaining relatively low risk of emergency replenishment.

Although material handler capacity is fixed there might also be some interest, both academically and practically, in studying its affect on system performance. The following figure summarizes the affect of varying material handler capacity while all other parameters remain at baseline levels.

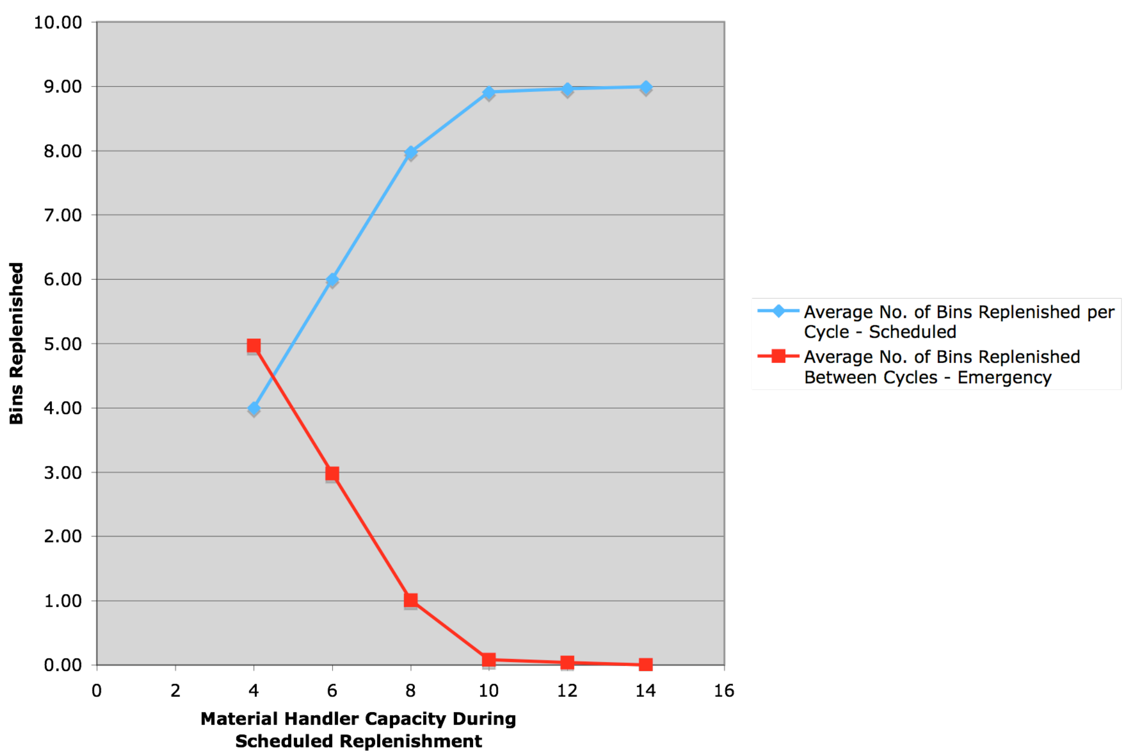

Figure 13. Material handler capacity VS bin replenishment

Looking at Figure 13, it might be reasonable to reduce material handler capacity to as low as 10 without affecting system performance. The practical benefit of this might be reducing safety risks or the possibility of reducing material handler resources.

The baseline system previously described was found through trial and error. As demonstrated in Figure 12, there is some possibility for improvement. A *quick and logical* solution to give some structure to developing this system is AB stratification. The following describes the algorithm:

i. Sort all 32 parts in descending order by *hourly bin usage*

ii. The first 16 parts listed will be assigned 3 bins each. The second 16 parts will be assigned 2 bins each.

iii. Identify the part that will be the first to be completely depleted. Based on this *constraining part* calculate how long it will take to deplete all but one bin of inventory. Set this value as the cycle time.

This is a logical approach to the problem. Higher volume parts, by bin usage, will have more bins in the system while lower volume parts will have fewer. As well, the part that is the quickest to be depleted will determine cycle length. The following summarizes the results of the AB Stratification compared to the baseline system.

	Baseline	AB Stratification
Cycle Length	0.25	0.30
Scheduled Replenishments per 8 Hr Day	32.00	26.67
Max Bins Replenished (Scheduled)	14.00	14.00
Max Bins Replenished (Emergency)	3.00	6.00
Average Bin Replenished (Scheduled)	8.99	9.90
Average Bins Replenished (Emergency)	0.00	0.89

Figure 14. Baseline VS AB stratification

The logical AB stratification approach appears to be an improvement over the baseline state of the system. Specifically, cycle length was increased to 0.30 hours without having a drastic effect on emergency replenishment between scheduled replenishments. However, there is still possible opportunity for improvement.

The greedy algorithm and MS/Excel solver tool are applied directly as described in 4.5. To demonstrate the effect of α, the MS/Excel Solver approach is applied at varying

α levels. Again, as α becomes small, the risk of incurring part shortage increases resulting in emergency replenishment. The following figure summarizes cycle length as a function of α.

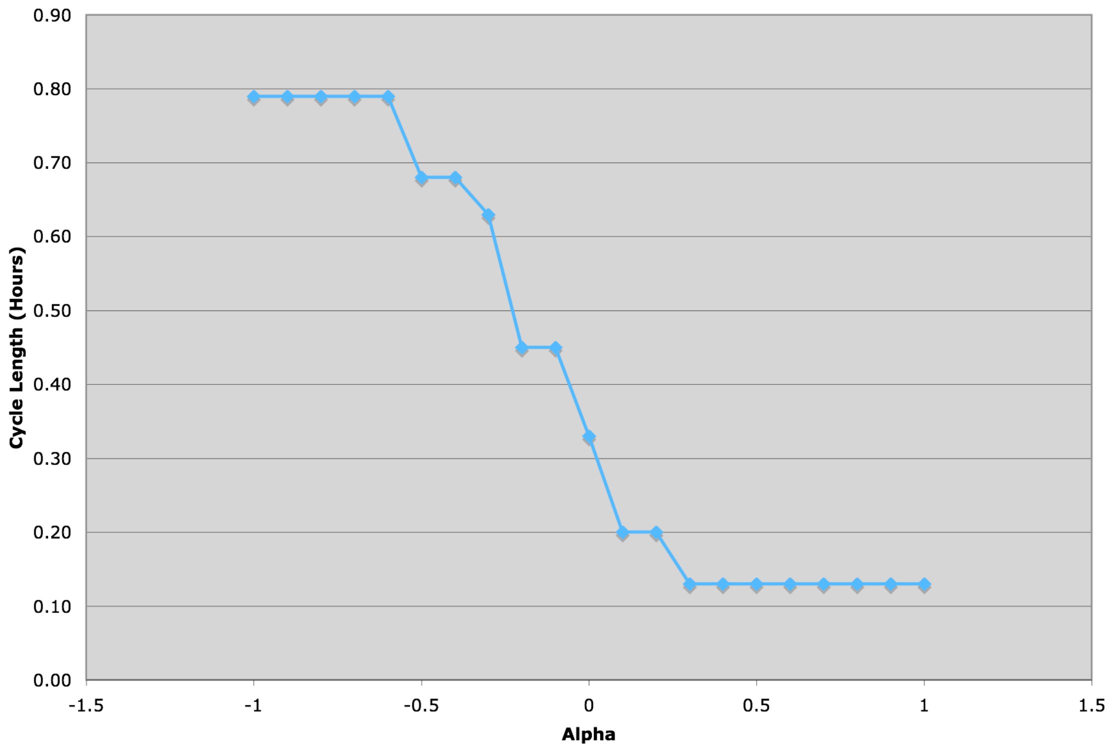

Figure 15. Cycle length VS α

Figure 16 summarizes the relationship between α and quantity of bins replenished.

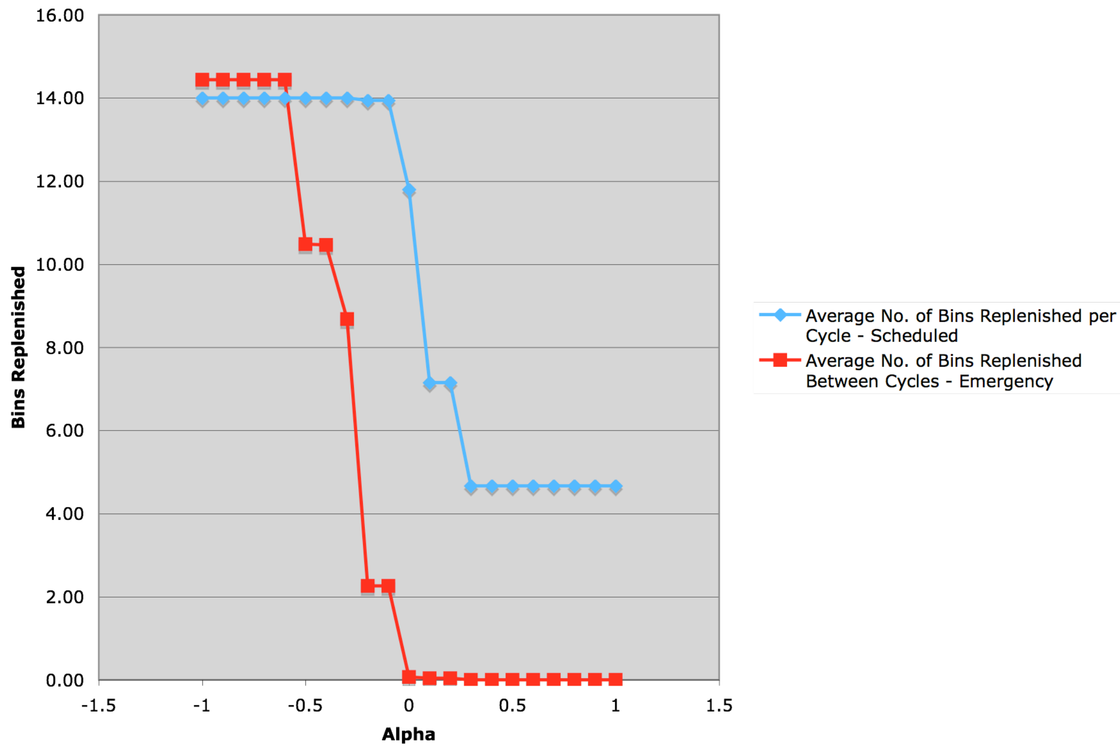

Figure 16. Bin replenishment VS α

Note from Figure 16 that bins emergency replenished between scheduled replenishments approaches zero when $\alpha \geq 0$. Of particular interest, is cycle length when $\alpha = 0$. Looking at Figure 15, when $\alpha = 0$ cycle length is greater than 0.30 hours. The following figure compares the baseline system, results of AB stratification, and the results of the MS/Excel Solver approach when $\alpha = 0$.

	Baseline	AB Stratification	$\alpha = 0$
Cycle Length	0.25	0.30	0.33
Scheduled Replenishments per 8 Hr Day	32.00	26.67	24.24
Max Bins Replenished (Scheduled)	14.00	14.00	14.00
Max Bins Replenished (Emergency)	3.00	6.00	4.00
Average Bin Replenished (Scheduled)	8.99	9.90	11.80
Average Bins Replenished (Emergency)	0.00	0.89	0.07

Figure 17. Baseline VS AB stratification VS MS/Excel Solver ($\alpha = 0$)

The last approach lengthened the cycle length to 0.33 hours while decreasing the average number of bins emergency replenished between scheduled visits to 0.07 bins.

The final approach to solving the cyclic multi-bin JRP is the use of the aforementioned greedy heuristic. The heuristic approach returned results after eight manual iterations. Figure 18 summarizes results of all approaches.

	Baseline	AB Stratification	$\alpha = 0$	Greedy
Cycle Length	0.25	0.30	0.33	0.74
Scheduled Replenishments per 8 Hr Day	32.00	26.67	24.24	10.81
Max Bins Replenished (Scheduled)	14.00	14.00	14.00	14.00
Max Bins Replenished (Emergency)	3.00	6.00	4.00	26.00
Average Bin Replenished (Scheduled)	8.99	9.90	11.80	13.74
Average Bins Replenished (Emergency)	0.00	0.89	0.07	12.91

Figure 18. Summary of results

Although the greedy heuristic returned the most desirable cycle length, it returned an unacceptably large number of average bins emergency replenished between scheduled visits.

Complete solutions are detailed in the appendix.

5. Conclusions & Directions for Future Research

Inventory management is a critical component in manufacturing. In particular, JRP policies must be oriented such that they *guarantee* inventory availability to the assembly line. While past research presents many interesting policies that can achieve this, few address practical concerns in policy management and implementation.

This paper presents a new policy, cyclic multi-bin JRP. Cyclic multi-bin JRP follows a periodically reviewed system designed to not only help ensure production inventory, but also ease implementation and management. Within this system there are some opportunities for improvement that inspired this work. Addressed in particular, is the determination of how often inventory is to be reviewed/replenished (i.e. cycle length). The decision variable in this case is the number of bins per part.

A mathematical model was developed with the practical objective of maximizing cycle length subject to material handler capacity constraints, workstation capacity constraints, as well as the *guarantee* of no inventory shortages during production. To address the complexity of a ceiling function and some practical concerns, a linear model was presented utilizing a material handler constraint approximation. The implications of the constraint approximation include incurring a calculated risk of experiencing emergency replenishments.

A logical approach, AB stratification, is described. Additionally, two more methodical approaches were demonstrated in solving this model. A linear model was solved with MS/Excel Solver utilizing the material handler constraint approximation. Similarly, a greedy heuristic was applied.

In the case presented, it was found that the MS/Excel Solver utilizing a material handler constraint approximation produced the most desirable results. Cycle length was substantially larger than the baseline system and AB stratification system, while still maintaining a negligible risk of emergency replenishment. The greedy heuristic did produce the largest cycle length, but also produced the highest risk of emergency replenishment.

While results were interesting, there are several possibilities for future work. Specific to this case, α was determined by evaluating the problem at varying values of α, then comparing. A substantial opportunity lies in developing a methodology for determining an exact value of α. Additionally, in this paper the greedy heuristic did not perform as intended. There lies an opportunity to improve on, or completely redevelop a heuristic. Similarly, the logical AB stratification approach showed some promise and might benefit from further development.

This paper has provided the basic framework for the unique cyclic multi-bin JRP. The practical benefits of this policy were described and include ease of implementation and decision-making. A logical approach was developed and it was also demonstrated that there is substantial opportunity to improve on this approach with a linear model utilizing constraint approximation. However, further opportunity lies in developing a methodology for determining the exact material handler constraint approximation.

6. Appendix

6.1. Collected data

Part No.	Bin Capacity (c_i)	Usage (u_i^j)	Part No.	Bin Capacity (c_i)	Usage (u_i^j)
1	35	61	17	25	29
2	8	61	18	18	40
3	25	10	19	18	15
4	25	11	20	18	54
5	25	1	21	18	11
6	25	18	22	18	58
7	25	1	23	12	61
8	25	4	24	100	40
9	25	10	25	100	19
10	25	14	26	13	19
11	25	5	27	12	19
12	25	5	28	60	3
13	25	17	29	60	6
14	25	23	30	60	21
15	25	8	31	60	19
16	25	21	32	60	10

6.2. Solution details

		Vary Cycle Length							
Cycle Length		0.30	0.35	0.40	0.50	0.60	0.70	0.80	1.00
Material Handler Capacity		14	14	14	14	14	14	14	14
Average Bin Replenished (Scheduled)		9.90	10.54	11.20	13.19	13.95	14.00	14.00	14.00
Average Bins Replenished (Emergency)		0.89	2.06	3.20	4.81	7.65	11.20	13.97	19.80
Bins per Part	**Part 1**	3	3	3	3	3	3	3	3
	Part 2	3	3	3	3	3	3	3	3
	Part 3	3	3	3	3	3	3	3	3
	Part 4	3	3	3	3	3	3	3	3
	Part 5	3	3	3	3	3	3	3	3
	Part 6	3	3	3	3	3	3	3	3
	Part 7	3	3	3	3	3	3	3	3
	Part 8	3	3	3	3	3	3	3	3
	Part 9	3	3	3	3	3	3	3	3
	Part 10	3	3	3	3	3	3	3	3
	Part 11	3	3	3	3	3	3	3	3
	Part 12	3	3	3	3	3	3	3	3
	Part 13	3	3	3	3	3	3	3	3
	Part 14	3	3	3	3	3	3	3	3
	Part 15	3	3	3	3	3	3	3	3
	Part 16	3	3	3	3	3	3	3	3
	Part 17	3	3	3	3	3	3	3	3
	Part 18	3	3	3	3	3	3	3	3
	Part 19	3	3	3	3	3	3	3	3
	Part 20	3	3	3	3	3	3	3	3
	Part 21	3	3	3	3	3	3	3	3
	Part 22	3	3	3	3	3	3	3	3
	Part 23	3	3	3	3	3	3	3	3
	Part 24	3	3	3	3	3	3	3	3
	Part 25	3	3	3	3	3	3	3	3
	Part 26	3	3	3	3	3	3	3	3
	Part 27	3	3	3	3	3	3	3	3
	Part 28	3	3	3	3	3	3	3	3
	Part 29	3	3	3	3	3	3	3	3
	Part 30	3	3	3	3	3	3	3	3
	Part 31	3	3	3	3	3	3	3	3
	Part 32	3	3	3	3	3	3	3	3

		Vary Cycle Length							
Cycle Length		0.30	0.35	0.40	0.50	0.60	0.70	0.80	1.00
Material Handler Capacity		14	14	14	14	14	14	14	14
Average Bin Replenished (Scheduled)		9.90	10.54	11.20	13.19	13.95	14.00	14.00	14.00
Average Bins Replenished (Emergency)		0.89	2.06	3.20	4.81	7.65	11.20	13.97	19.80
Bins per Part	**Part 1**	3	3	3	3	3	3	3	3
	Part 2	3	3	3	3	3	3	3	3
	Part 3	3	3	3	3	3	3	3	3
	Part 4	3	3	3	3	3	3	3	3
	Part 5	3	3	3	3	3	3	3	3
	Part 6	3	3	3	3	3	3	3	3
	Part 7	3	3	3	3	3	3	3	3
	Part 8	3	3	3	3	3	3	3	3
	Part 9	3	3	3	3	3	3	3	3
	Part 10	3	3	3	3	3	3	3	3
	Part 11	3	3	3	3	3	3	3	3
	Part 12	3	3	3	3	3	3	3	3
	Part 13	3	3	3	3	3	3	3	3
	Part 14	3	3	3	3	3	3	3	3
	Part 15	3	3	3	3	3	3	3	3
	Part 16	3	3	3	3	3	3	3	3
	Part 17	3	3	3	3	3	3	3	3
	Part 18	3	3	3	3	3	3	3	3
	Part 19	3	3	3	3	3	3	3	3
	Part 20	3	3	3	3	3	3	3	3
	Part 21	3	3	3	3	3	3	3	3
	Part 22	3	3	3	3	3	3	3	3
	Part 23	3	3	3	3	3	3	3	3
	Part 24	3	3	3	3	3	3	3	3
	Part 25	3	3	3	3	3	3	3	3
	Part 26	3	3	3	3	3	3	3	3
	Part 27	3	3	3	3	3	3	3	3
	Part 28	3	3	3	3	3	3	3	3
	Part 29	3	3	3	3	3	3	3	3
	Part 30	3	3	3	3	3	3	3	3
	Part 31	3	3	3	3	3	3	3	3
	Part 32	3	3	3	3	3	3	3	3

		Linear Model							
		-1	**-0.9**	**-0.8**	**-0.7**	**-0.6**	**-0.5**	**-0.4**	**-0.3**
Cycle Length		0.79	0.79	0.79	0.79	0.79	0.68	0.68	0.63
Material Handler Capacity		14	14	14	14	14	14	14	14
Average Bin Replenished (Scheduled)		14.00	14.00	14.00	14.00	14.00	14.00	14.00	14.00
Average Bins Replenished (Emergency)		14.44	14.44	14.44	14.44	14.44	10.48	10.47	8.68
Bins per Part	**Part 1**	5	5	5	5	5	12	11	11
	Part 2	10	10	10	10	10	9	9	9
	Part 3	2	2	2	2	2	2	2	2
	Part 4	2	2	2	2	2	2	2	2
	Part 5	2	2	2	2	2	2	2	2
	Part 6	2	2	2	2	2	2	2	2
	Part 7	2	2	2	2	2	2	2	2
	Part 8	2	2	2	2	2	2	2	2
	Part 9	2	2	2	2	2	2	2	2
	Part 10	2	2	2	2	2	2	2	2
	Part 11	2	2	2	2	2	2	2	2
	Part 12	2	2	2	2	2	2	2	2
	Part 13	2	2	2	2	2	2	2	2
	Part 14	2	2	2	2	2	2	2	2
	Part 15	2	2	2	2	2	2	2	2
	Part 16	2	2	2	2	2	2	2	2
	Part 17	8	8	8	8	8	2	2	2
	Part 18	3	3	3	3	3	3	3	3
	Part 19	2	2	2	2	2	2	2	2
	Part 20	4	4	4	4	4	4	4	3
	Part 21	2	2	2	2	2	2	2	2
	Part 22	4	4	4	4	4	4	4	4
	Part 23	5	5	5	5	5	6	6	6
	Part 24	2	2	2	2	2	2	2	2
	Part 25	2	2	2	2	2	2	2	2
	Part 26	3	3	3	3	3	2	2	2
	Part 27	8	8	8	8	8	8	8	2
	Part 28	2	2	2	2	2	2	2	2
	Part 29	2	2	2	2	2	2	2	2
	Part 30	2	2	2	2	2	2	2	2
	Part 31	2	2	2	2	2	2	2	2
	Part 32	2	2	2	2	2	2	2	2

		Linear Model							
		-0.2	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
Cycle Length		0.45	0.45	0.33	0.20	0.20	0.13	0.13	0.13
Material Handler Capacity		14	14	14	14	14	14	14	14
Average Bin Replenished (Scheduled)		13.94	13.94	11.80	7.15	7.15	4.67	4.67	4.67
Average Bins Replenished (Emergency)		2.26	2.26	0.07	0.04	0.04	0.00	0.00	0.00
Bins per Part	**Part 1**	2	2	2	2	2	2	2	2
	Part 2	19	19	19	19	19	2	2	2
	Part 3	2	2	2	2	2	2	2	2
	Part 4	2	2	2	2	2	2	2	2
	Part 5	2	2	2	2	2	2	2	2
	Part 6	2	2	2	2	2	2	2	2
	Part 7	2	2	2	2	2	2	2	2
	Part 8	2	2	2	2	2	2	2	2
	Part 9	2	2	2	2	2	2	2	2
	Part 10	2	2	2	2	2	2	2	2
	Part 11	2	2	2	2	2	2	2	2
	Part 12	2	2	2	2	2	2	2	2
	Part 13	2	2	2	2	2	2	2	2
	Part 14	2	2	2	2	2	2	2	2
	Part 15	2	2	2	2	2	2	2	2
	Part 16	2	2	2	2	2	2	2	2
	Part 17	2	2	2	2	2	2	2	2
	Part 18	2	2	2	2	2	2	2	2
	Part 19	2	2	2	2	2	2	2	2
	Part 20	5	4	2	2	2	2	2	2
	Part 21	2	2	2	2	2	2	2	2
	Part 22	3	4	7	2	2	2	2	2
	Part 23	6	6	6	2	2	2	2	2
	Part 24	2	2	2	2	2	2	2	2
	Part 25	2	2	2	2	2	2	2	2
	Part 26	2	2	2	2	2	2	2	2
	Part 27	2	2	2	2	2	2	2	2
	Part 28	2	2	2	2	2	2	2	2
	Part 29	2	2	2	2	2	2	2	2
	Part 30	2	2	2	2	2	2	2	2
	Part 31	2	2	2	2	2	2	2	2
	Part 32	2	2	2	2	2	2	2	2

		Linear Model					Greedy	AB Stratification
		0.6	**0.7**	**0.8**	**0.9**	**1**		
Cycle Length		0.13	0.13	0.13	0.13	0.13	0.74	0.30
Material Handler Capacity		14	14	14	14	14	14	14
Average Bin Replenished (Scheduled)		4.67	4.67	4.67	4.67	4.67	13.74	9.90
Average Bins Replenished (Emergency)		0.00	0.00	0.00	0.00	0.00	12.91	0.89
Bins per Part	**Part 1**	2	2	2	2	2	2	3
	Part 2	2	2	2	2	2	4	3
	Part 3	2	2	2	2	2	2	2
	Part 4	2	2	2	2	2	2	2
	Part 5	2	2	2	2	2	2	2
	Part 6	2	2	2	2	2	2	3
	Part 7	2	2	2	2	2	2	2
	Part 8	2	2	2	2	2	2	2
	Part 9	2	2	2	2	2	2	2
	Part 10	2	2	2	2	2	2	3
	Part 11	2	2	2	2	2	2	2
	Part 12	2	2	2	2	2	2	2
	Part 13	2	2	2	2	2	2	3
	Part 14	2	2	2	2	2	2	3
	Part 15	2	2	2	2	2	2	2
	Part 16	2	2	2	2	2	2	3
	Part 17	2	2	2	2	2	2	3
	Part 18	2	2	2	2	2	2	3
	Part 19	2	2	2	2	2	2	3
	Part 20	2	2	2	2	2	3	3
	Part 21	2	2	2	2	2	2	3
	Part 22	2	2	2	2	2	3	3
	Part 23	2	2	2	2	2	4	3
	Part 24	2	2	2	2	2	2	2
	Part 25	2	2	2	2	2	2	2
	Part 26	2	2	2	2	2	2	3
	Part 27	2	2	2	2	2	2	3
	Part 28	2	2	2	2	2	2	2
	Part 29	2	2	2	2	2	2	2
	Part 30	2	2	2	2	2	2	2
	Part 31	2	2	2	2	2	2	2
	Part 32	2	2	2	2	2	2	2

7. Bibliography

Atkins D, Iyogun P. Periodic versus ‘Can-Order’ Policies for Coordinated Multi-Item Inventory Systems. Management Science. Jun., 1988. Pp. 791-796.

Moon IK, Cha BC. The Joint Replenishment Problem with Resource Restriction. European Journal of Operational Research, 2006. Pp. 190-198

Nicol D. An Innovative Two-Bin Application For Floor Stock. Production and Inventory Management Journal. 1989. Pp. 27-29

Nilsson A, Segerstedt A, van der Sluis. A New Iterative Heuristic to Solve the Joint Replenishment Problem Using a Spreadsheet Technique. International Journal of Production Economics, 2007. Pp. 399-405

Ozkaya Banu Yuksel, Gurler Ulku, Berk Emre. The Stochastic Joint Replenishment Problem: A New Policy, Analysis, and Insights. Naval Research Logistics. 2006. Pp. 525-546

Sawik T. A Cyclic Versus Flexible Approach to Material Ordering in Make-To-Order Assembly. Mathematical and Computer Modeling. 2005 pp. 279-290

Silver E. Operations Research in Inventory Management: A Review and Critique. Operations Research, 1981. Pp. 628-645.

Viswanathan S. Periodic Review (s, S) Policies for Joint Replenishment Inventory Systems. Management Science. Oct., 1997. Pp. 1445-1454.

Zheng Yu-Sheng. Optimal Control Policy for Stochastic Inventory Systems with Markovian Discount Opportunities. Operations Research. (Jul. – Aug., 1994). Pp. 721-738.