Longitudinal Shear in Composite Beams

by

R. Paul Johnson

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By R. Paul Johnson*

Synopsis

In the vicinity of the shear connectors, the slab of a composite beam is subjected to a severe combination of longitudinal shear and transverse bending moment. Existing design methods for the design of the transverse reinforcement for this region are compared with the results of tests, and a new ultimate strength design method is proposed.

Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{tt}$, $A_{bt}$</td>
<td>Areas of top and bottom transverse reinforcement per unit length of beam.</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Cross-sectional area of steel joist at section of maximum positive bending moment.</td>
</tr>
<tr>
<td>$B$</td>
<td>Transverse spacing of steel joists.</td>
</tr>
<tr>
<td>$D_a$</td>
<td>Lever arm of composite beam at flexural failure.</td>
</tr>
<tr>
<td>$f_y$, $f_{gy}$</td>
<td>Yield strengths of reinforcing steel and of joist steel.</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of positive moment region of continuous beam, or span of simply-supported beam.</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Length of shear surface at shear connectors (as defined in CP 117: Part 1)</td>
</tr>
<tr>
<td>$m'$</td>
<td>Ultimate negative moment of resistance, per unit length of slab.</td>
</tr>
<tr>
<td>$p$</td>
<td>Proportion of transverse reinforcement, $= (A_{tt} + A_{bt})/t$.</td>
</tr>
<tr>
<td>$p_b$</td>
<td>Proportion of bottom transverse reinforcement, $= A_{bt}/t$.</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of concrete slab.</td>
</tr>
<tr>
<td>$v$</td>
<td>Mean ultimate longitudinal shear stress on a vertical cross-section of the slab.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Shape coefficient, $= tL/BD_a$</td>
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</tbody>
</table>

Suffixes:

- $d$: Design or calculated value
- $u$: Maximum value reached in a test to failure.

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Introduction

The concrete surrounding the shear connectors in a composite beam is subjected to a complex state of triaxial stress, due to the combined effects of longitudinal bending of the beam, transverse bending of the slab, the loads imposed by the connectors, and longitudinal shear in the slab acting as a flange of the beam. It is not practicable to base the design of this region on an analysis of these stresses; the problem is too complex and our knowledge of stress-strain relationships for reinforced concrete under triaxial stress is inadequate. A better approach is to develop a method from the results of tests to failure. This is the object of the present paper.

Two classes of composite beam may be distinguished:

1. **Beams for buildings**, in which design is not influenced by problems of dynamic or repeated loading, and in which positive (sagging) transverse bending of the slab in the vicinity of the shear connectors does not occur. Ultimate-strength design methods are appropriate. Superimposed loadings lie between two extremes: (a) a uniform distribution over the whole area of the slab, and (b) point or line loading on the centre-line of the beam (i.e., over the rolled steel member or “joist”).

2. **Beams for bridges**, which are subjected to repeated loading, so that design is normally based on the elastic theory and on the results of fatigue tests. Positive transverse bending of the slab can occur near the connectors, and the slab must carry concentrated loads applied anywhere. Slabs are therefore more heavily reinforced than is usual in buildings, and additional reinforcement to prevent local failure of the concrete near to the connectors is rarely required. The author is not aware of any such failures in tests on bridge beams.

This paper is mainly concerned with beams for buildings, as more test data is available and there is a greater need for an ultimate-strength design method for longitudinal shear.

Existing design methods

Composite beam design consists essentially of designing the slab to span between the joists, choosing a joist size adequate for the longitudinal bending and vertical shear, and then designing the shear connectors. Finally the longitudinal shear stress is checked, and the slab in the vicinity of the connectors is strengthened, if necessary, by the addition of transverse reinforcement. Thus the designer needs rules that relate the amount of this reinforcement to the longitudinal shear stress.
The rules used in Britain are given in CP 117: Part 1\(^{(1)}\). They were adapted from the ultimate-strength design method for beams in combined bending and shear of ACI 318-63\(^{(2)}\), and have never been related to the results of tests on composite beams in which transverse bending was present, so far as the author is aware.

In the United States, composite beams for buildings are usually designed in accordance with Section 1.11 of the AISC Specification \(^{(3)}\) and ACI 318-63, neither of which specifies any transverse shear reinforcement. It is therefore of interest to compare British and American designs with the results of tests.

A generalized presentation of the two methods can be made for structures consisting of a one-way composite floor system with steel joists of span \(L\), sectional area \(A_g\), and yield strength \(f_y\), joined by shear connectors to a continuous unhaunched concrete slab of thickness \(t\), reinforced with steel of yield strength \(f_y\). The slab is designed for a uniformly-distributed ultimate load \(w\) per unit area, and is continuous in the transverse direction over a number of similar parallel joists at spacing \(B\).

The British code specifies the bottom transverse reinforcement per unit length (\(A_{bt}\) in Fig. 1) in terms of \(L_s\), the "length of the shear surface at the shear connectors", which is to be taken as the lesser of the connector perimeter ABCD (Fig. 1) and twice the slab thickness. Tests in which transverse bending is present \(^{(4)}\) show that longitudinal cracks form along BE and CF, and that the concrete is likely to fail in shear-compression in regions AB and CD. Thus failure is not influenced by the length of BC. It is now concluded that \(L_s\) should always be taken as \(2t\). If the shear connectors extend into the zone of longitudinal compression, as they should, the perimeter ABCD is always less critical for shear failure than the lengths AE and FD.

CP 117 also requires that all transverse shear reinforcement shall be placed at the bottom of the slab. There is evidence from tests \(^{(5)}\) that top transverse steel can transmit shear even when the slab is at flexural failure, and that shear strength depends more on the total amount of transverse steel than on its position. It will now be assumed that the contribution of the steel is independent of its position in the slab. This makes it possible to compare design methods with tests in terms of two parameters: the yield strength of the transverse reinforcement per unit area of slab, given by

\[
pf_y = (A_{tt} + A_{bt}) \frac{f_y}{t}
\]

and \(v_u\), the mean ultimate longitudinal shear stress on a cross section such as AE (Fig. 1). It is assumed that at flexural failure of the composite beam, the neutral axis lies in the slab, and that the length EF is much smaller than the effective breadth of the slab. The total longitudinal shear is then \(A_g f_y\). It is transferred to the slab...
through an area of length $L/2$ and breadth $2t$, so that

$$v_u = A g f_{gy} / Lt$$  \hspace{1cm} (2)

It is shown in Appendix A that CP 117: Part 1 requires that

$$p f_y < v_u (0.5 + 0.876/\lambda)$$  \hspace{1cm} (3)

and

$$p f_y < v_u (1 + 0.876/\lambda) - 3.13 (f'_c)^{1/2},$$  \hspace{1cm} (4)

and that a slab designed for flexure in accordance with ACI 318:63 will have

$$p f_y < 1.16 v_u / \lambda,$$  \hspace{1cm} (5)

where

$$\lambda = tL/BD_a$$  \hspace{1cm} (6)

and $D_a$ is the lever arm for the composite beam at flexural failure. The non-dimen-
sional group $\lambda$ has been defined as the shape coefficient for a composite beam. It
is the ratio of the slenderness of the beam $(L/D_a)$ to that of the slab $(B/t)$. Its value
for composite beams for buildings almost always lies between 0.7 and 1.4.

The minimum values of $p f_y$ given by Eqs. (3), (4), and (5) are plotted in Fig. 2
for these values of $\lambda$ and for $f'_c = 3000$ and $6000$ lb/in$^2$ (20.7 and 41.4 N/mm$^2$). The
lines at $p f_y \approx 80$ lb/in$^2$ indicate typical minimum quantities of reinforcement permitted
in slabs. It is seen that CP 117 requires more reinforcement than does ACI 318,
except at low values of $\lambda$, where the agreement is remarkably close.

The other extreme type of loading is now considered: point or line loads on the
longitudinal centre-line of the beam. The curves for CP 117 are still applicable, for
they relate to the calculated longitudinal shear stress, $v_u$, but it appears that the ACI
and AISC specifications would be satisfied if the slab had only sufficient flexural rein-
forcement to carry its own weight. For most continuous slabs this would give a value
of $p f_y$ less than $100$ lb/in$^2$ (0.7 N/mm$^2$). If the composite beam supported columns
at its third points, $v_u$ might well exceed $400$ lb/in$^2$ (2.8 N/mm$^2$) in the shear spans.
Tests results will now be studied to find out if such a design would be safe.

**Tests on positive moment regions of composite beams**

Very few of the available results are from tests that were designed to check the
adequacy of the transverse reinforcement, and those of Kemp and Kipps still
seem to be the only ones in which the specimens were designed for simultaneous fail-
ure of the slab in transverse bending and the beam in longitudinal bending. Failures
that were obviously due to excessive longitudinal shear occurred only in the tests of
Toprac and Eyre, but it can be assumed that the ultimate shear strength of every
beam that failed in some other way was not less than the maximum value of $v_u$
reached in the test.
Mean longitudinal shear stresses at maximum load have been calculated from all available results for beams which had less transverse reinforcement than that required by the existing design methods, and are plotted against $p_f^y$ in Fig. 3. The calculations were in accordance with ultimate-strength design methods, using measured strengths of steel and concrete. The value of $v_u$ at the design ultimate moment, $M_{d'}$, was first calculated, and then scaled up or down by the ratio of the observed ultimate moment to $M_{d'}$ except where $M_u/M_d < 0.9$ due to inadequate shear connection. For these beams, $v_u$ was calculated from the ultimate shear strength of the connectors provided. The longitudinal shear was assumed to be uniformly distributed between cross-sections of maximum and zero moment (or over the length of the shear span, where appropriate), whether the shear-connector spacing was uniform or not.

The numbering of the points in Fig. 3 is related in Table 1 to the publication from which the data was taken and the original number of the beam; concrete strengths (assuming that $f_{c} = 0.8u$) and type of loading are also given. Further details of the beams without transverse bending in which there was a shear or tension failure in the slab are given in Appendix B.

The CP 117 design curves from Fig. 2 are also shown on Fig. 3. The test results suggest that the requirements for transverse reinforcement could be reduced. It appears that a beam with $p_f^y = 100\, \text{lb/in}^2$ would probably fail before $v_u$ reached 400\, \text{lb/in}^2$, so that the ACI-AISC design method may be unsafe for beams carrying heavy point loads.

The design of transverse reinforcement

In his study of the subject, Kemp (5) deduced from tests on plain concrete under compound stress that the concrete in the vicinity of the connectors could be assumed to fail at a transverse compressive stress of $0.56\, f_{c}^t$ combined with a longitudinal shear stress of $0.2\, f_{c}^t$. The present author developed a design method from this approach, assuming further that concrete cracked by transverse bending could resist no longitudinal shear and that bottom transverse steel of area $A$ could resist shear $A_f^y$ by dowel action. These assumptions imply that the shear strength of the concrete is zero when the transverse moment is zero, so that the method gives too much shear reinforcement when the design transverse moment is low.

The reinforcement provided in the beams of series TB (Nos. 28 to 35) ranged from 30 to 70 per cent of that required by this method. All of these beams
reached their design ultimate load, showing that the method is also too conservative when severe transverse bending is present. A new approach to the problem is now given.

**Beams without transverse bending**

A design method for conditions of zero transverse bending can be deduced from recent work by Hofbeck, Ibrahim, and Mattock (9). Thirty-eight ‘push-off’ specimens of the type shown in Fig. 4 were loaded to failure; 23 of these were cracked along the shear plane before the start of the test. These had a mean shear strength \( v_u \) about 250 lb/in\(^2\) (1.7 N/mm\(^2\)) less than that of a similar uncracked specimen. It was found that for the cracked specimens the relationship between \( v_u \) and \( p f_y \) (defined as in the present paper) was almost independent of the strength, size, and spacing of the reinforcement bars crossing the shear plane, and of the strength of the concrete, provided that \( f'_c \) was not less than 2500 lb/in\(^2\) (17.2 N/mm\(^2\)) and \( v_u \) did not exceed 700 lb/in\(^2\) (4.8 N/mm\(^2\)). This relationship, given in Figs. 3 to 6 of Ref. 9, is plotted in Figs. 2 and 3.

Figure 3 shows that the only specimens not subjected to transverse bending that were weaker in shear than predicted by this curve were Nos. 13 and 14, which were of the unusual design shown in Fig. 5. It is concluded that the curve provides the basis for a design method, except that account should be taken of concrete strength, as is usual in design methods for shear in reinforced concrete, since the concrete strengths in Ref. 9 ranged only from 2390 to 4510 lb/in\(^2\) (16.5 to 31.0 N/mm\(^2\)).

The proposed method is given (in lb/in\(^2\) units) by

\[
v_u = 3(f'_c)^{1/2} + 0.8p f_y, \tag{7}
\]

where \( p f_y < 80 \text{ lb/in}^2 \), and half of the reinforcement is placed near each face of the slab. This is plotted on Figs. 2 and 3, and is seen to be everywhere on the safe side of the results of the push-off tests.

**Beams with transverse bending**

Of the 12 beams for which results are given in Fig. 3, only Nos. 30 and 31 are reported as having failed in longitudinal shear; but the design transverse moment, \( m_{cl} \), was not reached in any of the five specimens of Kemp’s series CS (Nos. 36 to 40). His analysis of the results (5) shows clearly that longitudinal shear stress can reduce transverse flexural strength, and that premature failure of the slab in transverse bending leads to premature failure of the beam of which the slab is a flange. ‘Shear-flexure’ failures of this kind occurred in beams 38 and 39.
Ratios of observed to calculated ultimate moments in transverse negative bending of the slab and in longitudinal positive bending of the beam are given for these four beams in lines 2 and 3 of Table 2. The next three lines give the amount of transverse reinforcement (all placed near the top surface of the slab), the longitudinal shear stress \( v_d \) given by Eq. (7), and the ratio of the shear stress at maximum load \( v_u \) to \( v_d \).

The differences between the results from the two pairs of beams are thought to be chiefly due to the different types of reinforcement used. GK 60 deformed bars were used in specimens 28 to 35 and 8, 10, and 12 gauge steel wire in specimens 36 to 40. Both materials had a yield strength of about 60,000 lb/in\(^2\) (414 N/mm\(^2\)), but the stress in the deformed bars increased by over ten per cent as the strain was increased from 0.005 to 0.015, whereas that in the plain bars remained constant.

The ratio \( m'_u/m'_d \) is a better indication of the adequacy of the transverse reinforcement than the ratio \( v_u/v_d \). The results for beams 38 and 39 imply that reinforcement in accordance with Eq. (7) is inadequate under conditions of severe transverse bending when the steel has a flat yield plateau and a low bond strength. But in these specimens the slabs were only 1 3/4 in. (44 mm) thick, and the reinforcement was not typical of that used in practice.

More weight should be given to results 28 to 35 where the slabs were 2 1/2 in. (63 mm) thick. In these tests, point loads were applied to the slabs quite close to the edges of the joist. In specimens 30 and 31, the only two to fail in shear, these caused vertical shear stresses in the critical regions that exceeded 100 lb/in\(^2\), which is unusually high for a slab. In spite of this, \( M_u/M_d \) exceeded 1.0 for all these beams, and the lowest value of \( m'_u/m'_d \) was 0.95. This shows that when high-yield deformed bars are used, the reinforcement provided to resist flexure usually provides sufficient resistance to longitudinal shear, even when no bottom transverse reinforcement is provided.

**Bottom transverse reinforcement**

In absence of transverse bending, bottom transverse reinforcement is necessary to prevent longitudinal splitting of the slab along the lines of connectors. In effect, CP 117 requires that the amount of this reinforcement \( p_b \) or \( A_{bt}/t \) shall not be less than about 0.4 per cent of the slab area, which corresponds to \( p_f \leq 144 \text{ lb/in}^2 \) (1 N/mm\(^2\)) when mild steel reinforcement is used. In 1959, Adekola concluded that 0.4 per cent of top transverse reinforcement was sufficient to prevent longitudinal splitting, and the author understands that the CP 117 rule was based on this result.
A less conservative rule can be deduced from the tests of Hofbeck, Ibrahim, and Mattock\(^{(9)}\), in which half of the steel was placed near each face of the slab. Putting \( p_b = p/2 \) in Eq. (7), and in lb/in\(^2\) units:

\[
p_b f_y = 0.625 \nu - 1.9 (f'_c)^{1/2}
\]

with

\[
p_b f_y \leq 40
\]

This equation can be shown to be on the safe side when compared with the results of all the tests listed in Table 1 except Nos. 13 and 14, where the bottom steel was inadequately anchored (Fig. 5).

The results suggest that \( p_b \) could be reduced when transverse bending is present, since this opposes the tensile forces that cause splitting. Most floor slabs are designed for distributed load, but loading patterns that cause less transverse moment than is assumed in design often occur in service. It is therefore recommended that in all cases the bottom transverse reinforcement should be not less than that given by Eq. (8).
Conclusion

The following requirements for minimum transverse reinforcement for the slabs of composite beams not subjected to fatigue loading or transverse positive bending have been deduced from the results of tests on beams and shear tests on reinforced slabs. The results (equations 7 and 8) are re-stated in a form convenient for design.

The amount of top transverse reinforcement \( p_t \) (\( = \frac{A_{tt}}{t} \)) should not be less than that required to resist the transverse bending moment, and in addition

\[
 p_{tf} < 0.625v_u - 1.9(f_c')^{1/2}
\]

and

\[
 p_{tf} < 40
\]

The amount of bottom transverse reinforcement \( p_b \) (\( = \frac{A_{bt}}{t} \)) should satisfy

\[
 p_{bf} < 0.625v_u - 1.9(f_c')^{1/2}
\]

and

\[
 p_{bf} < 40
\]

These equations are in lb/in\(^2\) units. The terms \( 1.9(f_c')^{1/2} \) and 40 should be replaced by \( 0.14(u)^{1/2} \) and \( 0.276 \) when N/mm\(^2\) units are used and the concrete is specified in terms of its cube strength, \( u \).

In floor structures designed for uniformly distributed loading, the necessary top reinforcement for flexure will usually exceed that given by Eq. (9). Taking account of this, the total amount of transverse reinforcement given by this method can be expressed in terms of \( f_c' \) and the shape coefficient \( \lambda \) (which usually lies between 0.7 and 1.4), and is given in Fig. 2 for distributed load on the slab and for zero transverse bending. The Figure shows that the method requires slightly less reinforcement than the existing method of CP 117: Part 1 for distributed load, and much less when there is no transverse bending. Most designs in practice lie between these limits. The proposed method has been shown (Fig. 3 and Table 1) to be safe when compared with the results of tests to failure.

Acknowledgements

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References


2. ACI 318, Building code requirements for reinforced concrete. American Concrete Institute, Detroit, 1963.


Appendix A

Derivation of Equations 3, 4, and 5

For reinforced concrete floor slabs at flexural failure, the lever arm \( a \) normally lies between 0.72t and 0.81t, so that the error in assuming \( a = 0.76t \) is small. If the slab is designed by the yield-line theory, the designer is free to choose the ratio of negative to positive moments \( (m'/m) \); but there are advantages in making the slab relatively strong in negative bending, and the use of the 'elastic' value for a continuous slab, \( m' = wB^2/12 \), has been advocated \((5,6)\). With these assumptions, \( m' = wB^2/12 = \phi A_{tt}f_l a = 0.76\phi f_y t^2 \) \( (A1) \)

where \( \phi \) is the 'capacity reduction factor' of ACI 318, and \( p_t \) is the proportion of top transverse reinforcement.

For flexure of the composite beam,
\[
wBL^2/8 = A_{f_D} = v_L t D \text{ from Eq. (2).} \quad (A2)
\]

Eliminating \( w \) from \( (A1) \) and \( (A2) \) and putting \( tL/BD = \lambda \),
\[
p_f^y = 0.876 v_u/\lambda \phi \quad (A3)
\]

Clause 7(g) of CP 117: Part 1 requires that
\[
Q = 2.8L_s (u_w)^{1/2} + 2 A_{bt}f_y , \text{ where } Q = 2v_t \text{ in the present notation. Putting } L_s = 2t, \quad u_w = 1.25f_t'\text{c}', \quad \text{and } p_b = A_{bt}/t, \text{ this becomes}
\[
v_u > 3.13(f_y')^{1/2} + p_{fy} \quad (A4)
\]

A further requirement is that \( A_{bt} < Q/4f_y' \), which becomes
\[
v_u > 2p_{fy} \quad (A5)
\]

Putting \( p = p_t + p_b \), and \( \phi = 1.0 \) (since \( \phi \) is not used in CP 117), we have from \( (A3) \), \( (A4) \), and \( (A5) \):
\[
p_f^y < v_u (0.5 + 0.876/\lambda) \quad (3)
\]

and
\[
p_f^y < v_u (1 + 0.876/\lambda) - 3.13(f_y')^{1/2} \quad (4)
\]

For slabs of the type considered here, Clause 904 of ACI 318–63 states that \( m \) and \( m' \) may be taken as \( wB^2/16 \) and \( wB^2/12 \) respectively, so that Eq. \( (A3) \) is applicable. Putting \( \phi = 0.9 \) in this equation,
\[
p_f^y = 0.974 v_u/\lambda \quad (A6)
\]

The amount of bottom transverse reinforcement appears to be determined by Clause 918(f) which requires that one-fourth of the positive moment reinforcement be continued over the support. This gives \( p_b = (0.25 \times 12/16)p_t = 0.19p_t \). Hence from \( (A6) \),
\[
p_f^y = 1.19 p_f^y = 1.16 v_u/\lambda \quad (5)
\]
Appendix B

Notes on shear and tension failures in the slabs of composite beams

Beams are numbered as in Fig. 3. The longitudinal splitting failure of beam 6 (illustrated in Ref. 12) is due to inadequate transverse reinforcement; but this type of failure did not occur in beams 5 and 7.

The reinforcement in beams 19 and 22 consisted of a layer of fabric at mid-depth of the slab. Longitudinal cracks were observed in all these beams, but fracture of the shear connectors occurred at maximum load in beams 19, 20 and 22.

A typical cross-section of beams 13 and 16 is shown in Fig. 5. In the tests, shear failures occurred on planes ABCD and EBCH, but design would be based on the shortest shear surface, FBCG. The points plotted in Fig. 3 relate to the mean shear stress on this surface and the full yield strength of the bottom transverse reinforcement, even though it appears to be inadequately anchored.

Specimen 45 was a small-scale beam with a mortar slab and weak shear connection. Longitudinal shear failure occurred in the slab at 68 per cent of the ultimate load for full interaction. This was probably influenced by the excessive deflexion of the connectors, for similar beams with stronger connection (Nos. 43 and 44) failed in flexure at much higher loads.
<table>
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<th>Beam No. (Fig. 3)</th>
<th>Ref. No.</th>
<th>Original Test No.</th>
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*Lightweight concrete

Types of loading:

nP  n point loads placed over the joist
DB  Load distributed along the joist
DS  Load distributed over the slab
EL  Line loads on the slab causing transverse bending

TABLE 1. Key to numbered points in Figure 3.
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<td>( m_u / m_d )</td>
<td>0.95</td>
<td>1.11</td>
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<td>( M_u / M_d )</td>
<td>1.05</td>
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<td>( p_f y ) lb./in(^2)</td>
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<td>470</td>
<td>197</td>
<td>252</td>
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<tr>
<td>( v_d ) lb./in(^2)</td>
<td>478</td>
<td>531</td>
<td>315</td>
<td>341</td>
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<td>( v_u / v_d )</td>
<td>1.11</td>
<td>0.99</td>
<td>1.08</td>
<td>1.05</td>
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**TABLE 2.** Beams that failed in longitudinal shear.
Figures

1. Part cross-section of composite beam.
2. Design methods for transverse reinforcement in composite beams.
3. Results of tests on composite beams.
4. Typical push-off specimen (Ref. 9).
5. Part cross-section of beams tested by Toprac and Eyre (Ref. 8).

Fig. 1. Part cross-section of composite beam
Proposed design rule; load distributed over slab (Eqs. 9 and 9)

\[ \lambda = 0.7 \]

\[ \lambda = 1.4 \]

Eq. (3)

Eq. (4)

\[ f'_c = 3000 \]

\[ f'_c = 6000 \]

Cracked push-off specimens (Ref. 9)

Proposed design rule; no transverse bending (Eq. 7).

Fig. 2. Design methods for transverse reinforcement in composite beams.
Fig. 3. Results of tests on composite beams

- Shear or tension failure in concrete slab
- Other types of failure
- Beams with transverse bending
- Cracked push-off specimens

CP 117 Method
Proposed rule (Eq. 7)
Fig. 4. Typical push-off specimen (Ref. 9)

Thickness of specimen, 5" (0.13m)

Fig. 5. Part cross-section of beams tested by Toprac and Eyre (Ref. 8)
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