

Public Abstract

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Graduation Term:FS 2009

Department:Mathematics

Degree:PhD

Title:Existence and Construction of Real-Valued Equiangular Tight Frames

Equiangular frames are important for coding and communications theory, and many other applications. Very little is known about when equiangular frames exist or how to construct them. This paper presents results on real-valued equiangular tight frames (ETFs) and related topics.

Some geometric theorems are developed, and aspects of frame theory are used to gain insight into ETFs. We develop a projection method for analyzing equiangular tight frames that leads to new existence results, and that establishes a link between the geometry of the ETF and the spectrum of the associated Gramian and signature matrices. A new lower bound on the number of frame vectors improves on the best known necessary conditions for existence. We recover the Holmes-Paulsen criterion two different ways, along with additional necessary conditions. We also show that ETFs can be rotated to match a standard position, and that this corresponds to a binary tree structure (partial ordering) of embedded sub-spheres of decreasing dimension. This leads to a new canonical form and an enumerative algorithm to algebraically construct or prove the non-existence of equiangular tight frames.

This research advances pure mathematics in the area, as well as provides for numerous applications for which equiangular tight frames would be ideal. Examples include information transmission in noisy conditions and where retransmission is too expensive, such as for deep space probes.