Existence and construction of real-valued equiangular tight frames

Daniel Redmond

Dr. Peter Casazza, Dissertation Supervisor

ABSTRACT

This paper presents results on real-valued equiangular tight frames (ETFs) and related topics. Some geometric theorems are developed, and aspects of frame theory are used to gain insight into ETFs. We develop a projection method for analyzing equiangular tight frames that leads to new existence results, and that establishes a link between the geometry of the ETF and the spectrum of the associated Gramian and signature matrices. A new lower bound on the number of frame vectors improves on the best known necessary conditions for existence. We recover the Holmes-Paulsen criterion two different ways, along with additional necessary conditions. We also show that ETFs can be rotated to match a standard position, and that this corresponds to a binary tree structure (partial ordering) of embedded sub-spheres of decreasing dimension. This leads to a new canonical form and an enumerative algorithm to algebraically construct or prove the non-existence of equiangular tight frames.