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Shapiro Effect as a Possible Cause of the Low-Frequency Pulsar Timing Noise in Globular Clusters

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A prolonged timing of millisecond pulsars has revealed low-frequency uncorrelated (infrared) noise, presumably of astrophysical origin, in the pulse arrival time (PAT) residuals for some of them. Currently available pulsar timing methods allow the statistical parameters of this noise to be reliably measured by decomposing the PAT residual function into orthogonal Fourier harmonics. In most cases, pulsars in globular clusters show a low-frequency modulation of their rotational phase and spin rate. The relativistic time delay of the pulsar signal in the curved space time of randomly distributed and moving globular cluster stars (the Shapiro effect) is suggested as a possible cause of this modulation. Extremely important (from an astrophysical point of view) information about the structure of the globular cluster core, which is inaccessible to study by other observational methods, could be obtained by analyzing the spectral parameters of the low-frequency noise caused by the Shapiro effect and attributable to the random passages of stars near the line of sight to the pulsar. Given the smallness of the aberration corrections that arise from the nonstationarity of the gravitational field of the randomly distributed ensemble of stars under consideration, a formula is derived for the Shapiro effect for a pulsar in a globular cluster. The derived formula is used to calculate the autocorrelation function of the low-frequency pulsar noise, the slope of its power spectrum, and the behavior of the σ_z statistic that characterizes the spectral properties of this noise in the form of a time function. The Shapiro effect under discussion is shown to manifest itself for large impact parameters as a low-frequency noise of the pulsar spin rate with a spectral index of $n = -1.8$ that depends weakly on the specific model distribution of stars in the globular cluster. For small impact parameters, the spectral index of the noise is $n = -1.5$. © 2006 Pleiades Publishing Inc.

Key words: pulsars, neutron stars, and black holes, Shapiro effect, globular clusters.

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INTRODUCTION

Explaining the astrophysical origin of the low frequency uncorrelated pulsar timing noise is an extremely important, but challenging problem of modern pulsar astronomy. It is well known that millisecond and binary pulsars can be used as the most stable natural frequency standards (Davis et al. 1985; Rawley et al. 1987; Kaspi et al. 1994; Petit and Tavella 1996; Ilyasov et al. 1998; Kopeikin 1999). However, obtaining an unbiased and statistically significant estimate of their stability depends considerably on the accurate determination of the parameters of the random low-frequency noise that is detected in the pulse arrival time (PAT) residuals observed on long time scales. In most cases, the low-frequency (infrared) noise can be described by an additive set of components each of which is described by a simple model of the autocovariance function with a power-law spectrum, $S(f) = A_n f^{-n}$, where the index n takes on integer values from unity and higher (Kopeikin 1997). Noise of different physical natures corresponds to different spectral indices. For this reason, studying the spectral parameters of the infrared noise is an important problem of observational astrophysics. It enriches significantly the metrological studies of the pulsar time scale and encompasses many divisions of modern astrophysics, the most important of which are cosmology and gravitational wave astronomy (Kopeikin 1997a; Lorimer 2001).

A self-consistent approach to estimating the stability of the rotational and orbital parameters of pulsars and pulsar noise models has been developed by many authors (Groth 1975; Cordes 1980; Bertotti et al. 1983; Blandford et al. 1984; Taylor 1991; Kopeikin 1997a, 1997b, 1999; Ilyasov et al. 1998; Kopeikin and Potapov 2004). They solved the main theoretical questions of the analytical observational data processing technique in the time domain. The data processing in the time domain is more informative than that in the frequency domain, since both the stationary and nonstationary noise components are taken into account in the former case, while an adequate data analysis in the frequency domain and obtaining unbiased estimates of the measured parameters of the time series are possible only for a stationary random process.

It should be noted that the accuracy of determining the pulsar PATs is currently very high and approaches ~ 50 ns on a time scale of several years (Bailes 2003). Therefore, we need a high-precision model for processing the PATs that are affected by many factors, such as the Earth's orbital and rotational motions, the proper motion of the pulsar, the gravitational potentials of the Solar system and other gravitating objects at the point of observation and along the pulse propagation path, and the spatial distribution of the interplanetary and interstellar media. The classical and relativistic effects related to the orbital motion of the pulsar around the barycenter of the binary and the proper motion of the latter in space, the propagation of the radio emission from the pulsar in the gravitational field and atmosphere of its companion, and the precession of the pulsar spin axis, which causes the pulsar pulse shape to change with time, are added for pulsars in binary systems.

Based on the relativistic theory of astronomical reference frames and time scales (Kopeikin 1988, 1989a, 1989b; Brumberg and Kopeikin 1989, 1990), whose improved and extended version was taken by the General Assembly of IAU-2000 as the basis for relativistic astronomical algorithms and was described in detail in review papers (Soffel et al. 2003; Kopeikin and Vlasov 2004), Doroshenko and Kopeikin (1990, 1995) developed a pulsar timing algorithm that includes the above effects, to within 10 ns, and that is suitable for processing the observations of both single and binary pulsars. This theory was effectively used to systematically develop the TIMAPR software package (Doroshenko 1997), which is designed for pulsar data processing (Larchenkova and Doroshenko 1995). The independent TEMPO code was developed at the Princeton University, USA (Taylor and Weisberg 1989). Both codes are widely used by Russian and foreign scientists at various radio observatories worldwide (Lorimer 2001).

The standard procedure for estimating the pulsar parameters on fairly short time scales is based on the assumption that white noise dominates in the PAT residuals. However, a fairly long monitoring reveals components of the correlated infrared noise of astrophysical origin almost in all pulsars, whose spectrum differs from the white noise spectrum (Cordes and Downs 1985; D'Alessandro et al. 2001). The spectrum of this correlated (infrared) noise diverges at zero frequency. This infrared catastrophe compels the researches to reconsider the standard methods of spectral analysis and forces them to resort to various kinds of regularization procedures that allow the divergence to be avoided when modeling the noise spectrum (Kopeikin and Potapov 2004).

One of the most important problems in modern pulsar timing is to separate out the infrared noise in the PAT residual spectra and to determine its amplitude and spectral index. Although this is a challenging problem, its solution can provide substantial information about the physical processes inside neutron stars and in the interstellar medium in the path of pulsar pulse propagation and help to detect low-frequency gravitational waves and other, no less interesting gravitational effects. The most suitable objects for solving this problem are millisecond pulsars with a very high spin stability and, as a result, a low level of intrinsic rotational noise (Guinot and Petit 1991). Millisecond pulsars were discovered in various regions of our Galaxy. However, the population of millisecond pulsars in globular star clusters is currently most representative. The first millisecond pulsar was discovered in 1987 in the core of the globular cluster M28 (Lyne et al. 1987). A systematic search for pulsars both in M28 and in other globular star clusters began after this discovery. Twenty four globular clusters in which more than 80 pulsars have been discovered are known to date (Freire 2004). The globular clusters 47 Tuc (NGC 104) and M15 (NGC 7078), which contain, respectively, 22 and 8 pulsars (Taylor et al. 1993; Camilo et al. 2000; Freire 2004), and the globular cluster Terzan 5, in which 26 pulsars were discovered (Ransom et al. 2005), are the record-holders in the number of discovered pulsars.

If the inherent causes of the pulsar spin instability are ignored, then three physical causes of the low frequency timing noise produced by external effects can be suggested. First, the stochastic Shapiro effect, i.e., the random spread in the total time it takes for the pulsar radio signal to pass through the fluctuating gravitational field of the globular cluster stars that are distributed and move randomly in a certain vicinity of the line of sight to the pulsar; second, the stochastic gravitational perturbations in the pulsar velocity and acceleration produced by close passages of globular cluster stars near the pulsar itself (Joshi and Rasio

1997; Rodin 2000); and, third, the random fluctuations of the interstellar medium (Smirnova and Shishov 2001). In this paper, we concentrate on detailed calculations and analysis of the spectrum of the pulsar noise produced by the stochastic Shapiro effect. The amplitude of this noise depends significantly on the radial star density distribution inside the cluster, which is unknown and model-dependent in most cases. From general considerations, we may expect the noise amplitude to be comparable to the gravitational radius of a typical cluster star, i.e., $\sim 10\ \mu$s. However, this value may be an order of magnitude higher, since the noise from the Shapiro effect has a cumulative property and is directly proportional to the number of stars on the line of sight to the pulsar that produce this noise. There is no doubt that the increase in the noise amplitude correlates with the characteristic time scale of its manifestation.

The random process produced by star passages should be considered as a special case of gravitational lensing. When applied to microarcsecond astrometry, this case was considered by Sazhin et al. (1998, 2001). In this paper, we calculate the noise spectrum, as applied to pulsar observations. Knowledge of the theoretical power spectrum of the stochastic Shapiro effect for a pulsar in a globular cluster will allow us, in the case of its direct measurement, to obtain very important astrophysical information about the structure of the globular cluster core, which is inaccessible to other observational methods, and to analytically extend the Salpeter mass function for globular clusters toward the low-mass stars comparable in mass to Jupiter. Observation of the stochastic Shapiro effect will also allow the mass of the dark matter that is possibly concentrated near the globular cluster cores to be estimated.

Kopeikin and Schafer (1999) were the first to derive an exact formula for the Shapiro effect produced by a moving massive body and showed that the positions of the light-deflecting gravitating bodies (stars) should be taken not at the pulsar pulse arrival time to the observer, but at the corresponding delayed time due to the finite speed of propagation of the gravitational perturbation (Kopeikin 2001). This relativistic effect has received convincing experimental confirmation through high-precision VLBI measurements of the deflection of light from a quasar by the gravitational field of a moving Jupiter (Famalont and Kopeikin 2003). We took into account this effect when deriving an expression for the total relativistic delay time of the pulsar signal in the gravitational field of arbitrarily moving globular cluster stars.

This paper is structured as follows. In the next section, we briefly consider the pulsar timing model and the procedure for estimating its parameters in the presence of low-frequency noise and introduce the concept of σ_z statistic, which serves as a quantitative measure of pulsar instability. Subsequently, we write out a formula for the cumulative Shapiro effect for a pulsar in a globular cluster and an expression for the σ_z statistic in the case where the physically significant noise is attributable to the Shapiro effect from random passages of stars near the line of sight. In conclusion, we use the σ_z statistic to estimate the noise spectrum and discuss prospects for numerically analyzing this effect.

PULSAR TIMING, ESTIMATION OF PULSAR PARAMETERS, AND THE σ_z STATISTIC

Let the observations begin at time t_0. The rotational phase of a pulsar is specified by a time polynomial:

$$N(t) = \nu_p \Im + \frac{1}{2}\dot{\nu}_p \Im^2 + \frac{1}{6}\ddot{\nu}_p \Im^3 + \frac{1}{24}\dddot{\nu}_p \Im^4 + \nu_p \phi_2(\Im) + O(\Im^5) \tag{1}$$

where $\Im = \Im(t)$ is the pulse emission time in the proper time scale of the pulsar; t is the pulse arrival time in the barycentric time scale of the Solar system; ν_p, $\dot{\nu}_p$, $\ddot{\nu}_p$ and $\dddot{\nu}_p$ are the pulsar spin rate and its derivatives taken at time $\Im = 0$; and $\phi_2(\Im)$ is the intrinsic noise of the pulsar rotational phase, spin rate, and its time derivatives.

Note that the pulsar proper time $\Im$ is not directly observable. The barycentric time, t, which is related to the proper time of the observer (atomic time) by a relativistic transformation (Kopeikin 1989b; Brumberg and Kopeikin 1990; Kopeikin and Vlasov 2004), is a measurable quantity. The relationship between the pulsar proper time $\Im$ and the barycentric time t is established by solving the equations for the light geodesics that describe the propagation of the radio pulse from the pulsar to the observer (Kopeikin 1990; Doroshenko and Kopeikin 1990). This relationship allows the observed pulsar rotational phase $N(t)$ to be expressed as a function of the barycentric time t (Doroshenko and Kopeikin 1990; Kopeikin 1999):

$$N(t) = N_0 + \nu t + \frac{1}{2}\dot{\nu} t^2 + \frac{1}{6}\ddot{\nu} t^3 + \nu\varepsilon(t), \tag{2}$$

where N_0 is the initial rotational phase of the pulsar; ν, $\dot{\nu}$ and $\ddot{\nu}$ are the barycentric pulsar spin rate and its derivatives at the initial epoch of observations T; and $\varepsilon(t)$ is the total, physically significant additive noise of the pulsar rotational phase, with the intrinsic noise of the pulsar $\phi_2(t)$ being one of its components. We emphasize that the noise $\varepsilon(t)$ is produced by both external and internal factors. By the external factors we mean all those factors that affect the propagation of the pulsar radio pulse in a random way. These also include the noise from the passages of globular cluster stars, which will be considered in detail in the next sections. The internal factors responsible for the pulsar noise are related to the pulsar spin mechanism and are not considered here. Lyne and Graham-Smith (2004) gave a comprehensive overview of the causes of the pulsar spin instability.

The PAT residual $r(t)$ is the difference between the observed pulsar rotational phase, N_{obs}, and its theoretical value, $N(t, \theta)$, predicted by the timing model and specified by Eq. (2) divided by the pulsar rotation rate, ν:

$$r(t, \theta) = \frac{N_{obs} - N(t, \theta)}{\nu},$$

where $\theta = \{\theta_a, a = 1, 2, ..., ..., k\}$ denotes a set k of measurable parameters ($k = 4$ in the model represented by Eq. (2)).

Let us introduce the quantities $\beta_a \equiv \theta^*_a - \hat{\theta}_a$ that are the corrections to the unknown true values of the parameters $\hat{\theta}_a$ and the timing model fits

$$\psi_a(t) = \left[\frac{\partial N}{\partial \theta_a}\right]_{\theta=\theta^*},$$

where θ^*_a are the least-squares estimates of the parameters (Kopeikin 1999). The parameters and fits are given in the table.

Assuming that m equally spaced and comparable (in accuracy) pulse arrival times are measured for N complete rotations of the pulsar around its axis, we have mN residuals: $r_i \equiv r(t_i)$, where $i = 1, 2, ..., mN$. The least-squares method yields the parameters $\beta_a(\tau)$ (Bard 1974):

$$\beta_a(\tau) = \sum_{b=1}^{4}\sum_{i=1}^{mN} L^{-1}_{ab}\psi_b(t_i)\varepsilon(t_i), \qquad a = 1, 2, 3, 4,$$

where the information matrix

$$L_{ab}(\tau) = \sum_{i=1}^{mN} \psi_a(t_i)\psi_b(t_i),$$

τ is the total observing time. The correlation matrix of the parameters is defined as $M_{ab} \equiv \langle\beta_a\beta_b\rangle$, where the angular brackets denote an ensemble average.

The behavior of the PAT residuals can be described best using the so-called σ_z statistic. It is defined as the weighted root-mean-square value of the coefficients of the cubic terms in the time polynomial fitted to the observed pulsar phase divided into the segments that correspond to equal time intervals τ (Matsakis et al. 1997). The σ_z statistic is formally defined (Matsakis et al. 1997) as

$$\sigma_z(\tau) = \frac{\tau^2}{2\sqrt{5}}\sqrt{M_{44}},$$

where M_{44} is a diagonal element of the correlation matrix M_{ab}. This diagonal element is defined by

$$M_{44}(\tau) = \sum_{c=1}^{4}\sum_{d=1}^{4} L^{-1}_{4c}L^{-1}_{4d} \times \left[\sum_{i=1}^{mN}\sum_{j=1}^{mN} \psi_c(t_i)\psi_d(t_i)\Re(t_i, t_j)\right] \tag{3}$$

where $\Re(t_i, t_j) = \langle\varepsilon(t_i)\varepsilon(t_j)\rangle$ is the autocorrelation function of the random process $\varepsilon(t)$. For a stationary noise process, the autocorrelation function depends not on the individual times t_i and t_j , but only on their difference τ:

$$\Re(t, \tau) = \langle\varepsilon(t+\tau)\varepsilon(t)\rangle \equiv \Re(\tau),$$

where $t = t_j$, $\tau = t_i - t_j$.

List of basic functions and parameters used to fit the parameters in the pulsar timing model specified by Eq.(2)

Parameter	Fit
$\beta_1 = \delta N_0/\nu$	$\psi_1(t) = 1$
$\beta_2 = \delta\nu/\nu$	$\psi_2(t) = t$
$\beta_3 = \delta\dot{\nu}/\nu$	$\psi_3(t) = t^2$
$\beta_4 = \delta\ddot{\nu}/\nu$	$\psi_4(t) = t^3$

THE SHAPIRO EFFECT FOR A PULSAR IN A GLOBULAR CLUSTER

The relativistic time delay of an electromagnetic signal in a static, spherically symmetric gravitational field of a point mass is called the Shapiro effect (Shapiro 1964). A generalization of the formula for the Shapiro effect for the propagation of light in a variable gravitational field of an arbitrarily moving body was found by Kopeikin and Schafer (1999) and experimentally confirmed by Fomalont and Kopeikin (2003). In this paper, we use the Kopeikin-Schafer formula.

Formula for the Generalized Shapiro Effect

The expression for the travel time of an electromagnetic signal between fixed points, $(t_0, \vec{x}_0)$ and $(t, \vec{x})$, in the gravitational field of an arbitrarily moving body (star) derived by Kopeikin and Schafer (1999) is

$$t - t_0 = |\vec{x} - \vec{x}_0| + \Delta(t, t_0),$$

where $|\vec{x} - \vec{x}_0|$ is the coordinate distance in the background Euclidean space between the two points, $\vec{x}_0$ and $\vec{x}$; and $\Delta(t, t_0)$ is the relativistic time delay attributable to the gravitational field of the moving bodies.

For a pulsar in a globular cluster, the Shapiro delay produced by the gravitational field of the moving cluster stars can be written as (Kopeikin and Schafer 1999)

$$\Delta(t, t_0) = -\sum_{a=1}^{N} \frac{2GM_a}{c^3} \ln \frac{r_a - (\vec{k}_0 \vec{r}_a)}{r_{0a} - (\vec{k}_0 \vec{r}_{0a})}, \tag{4}$$

$$\vec{r}_a = \vec{x}(t) - \vec{x}_a(s),$$

$$\vec{r}_{0a} = \vec{x}_0(t_0) - \vec{x}_a(s_0),$$

where we discarded the terms that are proportional to the velocity of the bodies and that appear in the amplitude of the logarithmic function, because they are small. The coordinates of the gravitating bodies are calculated at the delayed times s and s_0, which are defined by the equations for isotropic gravitational field characteristics:

$$t = s + |\vec{x}(t) - \vec{x}_a(s)|, \tag{5}$$

$$t_0 = s_0 + |\vec{x}_0(t_0) - \vec{x}_a(s_0)|.$$

Here, t_0 is the photon emission time, $\vec{x}$ specifies the observer's position relative to the globular cluster barycenter (GCB) at the observation time of the pulsar radio pulse, $\vec{x}_a$ specifies the position of star a relative to the GCB, $\vec{x}_0$ specifies the pulsar position relative to the GCB at the radio pulse emission time, and the unit vector $\vec{k}_0$ specifies the direction of the rectilinear, gravitationally unperturbed radio pulse motion from the pulsar to the observer and is defined by

$$\vec{k}_0 = \frac{\vec{x} - \vec{x}_0(t_0)}{|\vec{x} - \vec{x}_0(t_0)|}.$$

Let us expand r_a, $\vec{r}_a$, and $\vec{k}_0$ in a Taylor power series of $x_a/R \ll 1$, where $R = |\vec{x}(t)|$ is the distance from the observer to the GCB, much larger than the cluster size:

$$\vec{k}_0 = \vec{K} - \vec{K} \times (\vec{\xi} \times \vec{K}) + O(\vec{\xi}^2),$$

$$\vec{r}_a = R - R(\vec{K}\vec{\xi}_a) + \frac{1}{2}R(\vec{\xi}_a \times \vec{K})^2 + O(\vec{\xi}_a^3),$$

where $\vec{K} = \vec{x}(t)/|\vec{x}(t)|$ is a unit vector, $\vec{\xi} = \frac{\vec{x}_0(t_0)}{|\vec{x}(t)|}$, and $\vec{\xi}_a = \frac{\vec{x}_a(s)}{|\vec{x}(t)|}$.

Let T be a fixed start time of observations. Let us expand $\vec{x}(t)$ and $\vec{x}_a(s)$ in a Taylor power series of $(t-T)$, provided that the following conditions are satisfied: $|\vec{x}(T)| \gg |\vec{v}(T)(t-T)| \gg |\dot{\vec{v}}(T)(t-T)^2|$. Retaining only the linear (in velocities) terms and using the following notation: $R_0 = |\vec{x}(T)|$, $\vec{K}_0 = \vec{R}_0/R_0$ is a unit vector, $\vec{v}$ is the observer's velocity, $\vec{v}_a$ is the velocity of star a, and $\vec{v}_0$ is the pulsar velocity, we then obtain

$$\vec{x}(t) = \vec{x}(T) + \vec{v}(T)(t-T) + \ldots,$$

$$\vec{x}_a(s) = \vec{x}_a(S) + \vec{v}_a(s-S) + \ldots,$$

$$R = R_0 + (\vec{K}_0\vec{v}(T))(t-T) + \ldots,$$

$$\vec{K} = \vec{K}_0 + \left[\frac{\vec{v}(T) - \vec{K}_0(\vec{K}_0\vec{v}(T))}{R_0}\right](t-T) + \ldots,$$

where S is the delayed time related to the start time of observations T by the gravitational cone equations (5).

Using the equations for gravitational field characteristics (5), we obtain the following expression for the numerator of the fraction in the argument of the logarithm in Eq. (4) after algebraic transformations:

$$\vec{r}_a - (\vec{k}\vec{r}_a) = 1 + 2\frac{\vec{d}_a\vec{v}_a(S)}{\vec{d}_a\vec{x}_a(S)}(t-T) + \ldots,$$

where $\vec{d}_a = (\vec{K}_0 \times \frac{\vec{x}_a}{R_0}) \times \vec{K}_0$ is the impact parameter of the radio pulse trajectory with respect to the GCB.

Acting in a similar way, we expand $\vec{r}_{0a}$ and r_{0a} in a Taylor series, retaining only the linear (in velocity) terms, and obtain an expression for the denominator of the fraction in the argument of the logarithm in Eq. (4) after simple transformations. Finally, once all terms have been reduced to the same time T_0, which defines the pulsar pulse emission time corresponding to the start of observations T, we obtain the ultimate expression for the random process produced by the relativistic delay during the propagation of a radio pulse in a variable gravitational field of moving globular cluster stars:

$$\tilde{\varepsilon}(t) \equiv \Delta(t, t_0) - \Delta(T, T_0) \quad (6)$$

$$= \sum_{a=1}^{N} \frac{2GM_a}{c^3} \left[\ln \left\{ 1 + 2(t-T) \frac{\vec{d}_a(T_0)\vec{v}_a(T_0)}{\vec{d}_a(T_0)\vec{x}_a(T_0)} \right\} - \ln \left\{ 1 + \frac{(-\vec{q} - \vec{K}_0)\vec{V}_{0a}(T_0)}{Q_{0a} + \vec{K}_0 \vec{Q}_{0a}} (t-T) \right\} \right],$$

where $\vec{V}_{0a} = \vec{v}_0(T_0) - \vec{v}_a(T_0)$, $\vec{Q}_{0a} = \vec{x}_a(T_0) - \vec{x}_0(T_0)$, $Q_{0a} = (\vec{Q}_{0a}\vec{Q}_{0a})^{1/2}$, $\vec{q} = \vec{Q}_{0a}/Q_{0a}$ is a unit vector.

Autocorrelation Function of the Shapiro Effect

Let us pass to a new model of the random process $\varepsilon(t)$ defined by the formula

$$\varepsilon(t) = \tilde{\varepsilon}(t) - \langle \tilde{\varepsilon}(t) \rangle,$$

where the angular brackets denote a statistical ensemble average, and we assume that the mean value is $\langle \varepsilon(t) \rangle = 0$. The autocorrelation function in Eq. (3) can then be written as

$$\Re(t, \tau) = \int dm_a d\vec{x}_a d\vec{v}_a f(m_a, \vec{x}_a, \vec{v}_a) \varepsilon(t, m_a, \vec{x}_a, \vec{v}_a) \varepsilon(t+\tau, m_a, \vec{x}_a, \vec{v}_a), \quad (7)$$

where we assume that the statistical ensemble of stars is defined by uncorrelated parameters, so the distribution function can be fitted by the product of three statistically independent distribution functions:

$$f(m_a, \vec{x}_a, \vec{v}_a) = A f(m_a) f(\vec{x}_a) f(\vec{v}_a);$$

we find the normalization numerical coefficient A from the condition

$$A = \int dm_a d\vec{x}_a d\vec{v}_a f(m_a, \vec{x}_a, \vec{v}_a) = 1.$$

No integration limits are specified in Eq. (7), but we assume that they are known (see below) and specify the range of statistical ensemble parameters.

A globular cluster is characterized by two quantities: the cluster core radius r_c, which is defined as the distance at which the surface brightness is half its central value, and the tidal radius r_t, at which the surface brightness is zero. Given the mass of the Galaxy M_G, the cluster mass M_c, and the Galactocentric distance of the cluster center R_G, the tidal radius can be calculated as follows: $r_t^3 = \frac{M_c}{2M_G} R_G^3$. Equation (7) is integrated in the following limits: $m_a = [0.1M_\odot - 10M_\odot]$, $|\vec{v}_a| = [0 - 4\sigma]$ and $|\vec{x}_a| = [0 - r_t]$, where σ is the stellar velocity dispersion of the cluster, and $M_\odot$ is the solar mass.

We use the Salpeter function $f(m_a) \sim m_a^{-2.35}$ as the mass function of globular cluster stars. Let us consider two model density distributions of a globular cluster: the model of an isothermal sphere with a core and the King model. In spherical coordinates, the density distribution for the model of an isothermal sphere with a core is (Spitzer 1990)

$$f(r_a) = \frac{\rho_0 r_c^2}{r_c^2 + r_a^2}, \tag{8}$$

where ρ is the core density of the globular cluster. The velocity distribution in the model of an isothermal sphere has a Gaussian (Maxwellian) profile,

$$f(v_a) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{v_{ax}^2 + v_{ay}^2 + v_{az}^2}{2\sigma^2}\right]. \tag{9}$$

It is the Maxwellian distribution that is typical of unrelaxed systems, which the globular clusters in our Galaxy are. However, the model of an isothermal sphere is not devoid of shortcomings. The main shortcoming is the assumption of its infinite radius in geometric space and velocity space, which is not physically justified. This shortcoming can be eliminated by "truncating" the argument of the velocity distribution function (9) by the characteristic star escape velocity v_e. This allows the isothermal sphere to be made finite in velocity space while preserving the isotropic Maxwellian distribution. From physical considerations in the velocity range $v_a > v_e$, the distribution function $f(v_a)$ must be close to zero and can be fitted by a truncated Maxwellian function:

$$f(v_a) = \frac{e^{-v_a^2/\sigma^2} - e^{-v_e^2/\sigma^2}}{1 - e^{-v_e^2/\sigma^2}},$$

where the normalization was chosen in such a way that $f(v_a) = 1$ at $v_a = 0$. The models of star clusters based on the truncated Maxwellian distribution were calculated by King (1966) and are called the King models. The density distribution in the King models is also bounded in space by the tidal radius r_t and is defined as (King 1966)

$$f(r_a) = \rho_0 \frac{(1+\Gamma^2)\arccos\sqrt{\frac{1+(r_a/r_c)^2}{1+\Gamma^2}} - \sqrt{\Gamma^2 - (r_a/r_c)^2}\sqrt{1+(r_a/r_c)^2}}{[1+(r_a/r_c)^2]^{3/2}\left[(1+\Gamma^2)\arccos\sqrt{\frac{1}{1+\Gamma^2}} - \Gamma\right]}, \tag{10}$$

where $\Gamma \equiv r_t/r_c$. For the subsequent calculations, we use the globular cluster 47 Tucanae, in which the largest number of pulsars have been discovered to date. This cluster has the following parameters: $r_c = 0.52$ pc, $\rho_0 = 6 \times 10^4 M_\odot/\mathrm{pc}^3$, $r_t = 60.3$ pc, and $\sigma = 10$ km s^{-1}; the distance to the cluster center is $R_0 = 4.1$ kpc (Harris 1996). To determine the slope of the power spectrum for the noise process under study and the behavior of the σ_z statistic, we must calculate the integral in Eq. (7). Analysis indicates that this integral cannot be calculated analytically in general form and the effect under study must be simulated numerically, which is done here. The random process attributable to the Shapiro effect can be estimated analytically if we restrict our analysis to star passages far from the line of sight to the pulsar. The random process under study can be separated into long- and short-period parts. The long-period part of the process corresponds to the case where the impact parameter da of the pulsar pulse trajectory is large for the cluster stars (analytical case). The short-period part of the process corresponds to the case of small impact parameters.

Large Impact Parameters

Let us make the simplifying assumption that the distributions of globular cluster stars in space, velocity, and mass are uncorrelated. This simplifies significantly the form of the distribution function in Eq. (7):

$$f(m_a,\vec{x}_a,\vec{v}_a,m_b,\vec{x}_b,\vec{v}_b)=\delta(m_a-m_b)\delta(\vec{x}_a-\vec{x}_b)\delta(\vec{v}_a-\vec{v}_b)f(m_a,\vec{x}_a,\vec{v}_a),(a\neq b) \tag{11}$$

where $\delta(y)$ is the Dirac delta function. The assumption about large impact parameters of the pulsar pulse trajectory relative to the globular cluster stars allows the expression for the random process under study (6) to be expanded in a Taylor power series of $(t-T)$:

$$\tilde{\varepsilon}(t)=\alpha(t-T)+\beta(t-T)^2+\gamma(t-T)^3+\ldots, \tag{12}$$

where T is the initial epoch of observations and the coefficients α, β and γ are defined by

$$\alpha=\sum_{a=1}^{N}\frac{2GM_a}{c^3}\left[\frac{2\vec{d}_a\vec{v}_a}{\vec{d}_a\vec{x}_a}-\frac{(-\vec{q}-\vec{K}_0)\vec{V}_{0a}}{Q_{0a}+\vec{K}_0\vec{Q}_{0a}}\right],$$

$$\beta=\sum_{a=1}^{N}\frac{2GM_a}{c^3}\frac{1}{2}\times\left\{\left[\frac{(-\vec{q}-\vec{K}_0)\vec{V}_{0a}}{Q_{0a}+\vec{K}_0\vec{Q}_{0a}}\right]^2-\left(\frac{2\vec{d}_a\vec{v}_a}{\vec{d}_a\vec{x}_a}\right)^2\right\},$$

$$\gamma=\sum_{a=1}^{N}\frac{2GM_a}{c^3}\frac{1}{3}\times\left\{\left(\frac{2\vec{d}_a\vec{v}_a}{\vec{d}_a\vec{x}_a}\right)^3-\left[\frac{(-\vec{q}-\vec{K}_0)\vec{V}_{0a}}{Q_{0a}+\vec{K}_0\vec{Q}_{0a}}\right]^3\right\}.$$

In what follows, it will suffice to restrict the analysis to only the first term in expansion (12), which is linear in time $(t-T)$. Let us write the autocorrelation function for it using the expression for the distribution function (11):

$$\Re(t_1,t_2)=\int dmd\vec{x}d\vec{v}\alpha^2(m,\vec{x},\vec{v})\times f(m,\vec{x},\vec{v})(t_1-T)(t_2-T)= \tag{13}$$
$$\int dmd\vec{x}d\vec{v}\alpha^2(m,\vec{x},\vec{v})f(m,\vec{x},\vec{v})\times[t_1t_2-T(t_1+t_2)+T^2].$$

Let us designate $\tau=|t_2-t_1|$ and $t_+=(t_1+t_2)$. These designations allow the autocorrelation function (13) to be represented as

$$\Re(t_1,t_2)=\int dmd\vec{x}d\vec{v}\alpha^2(m,\vec{x},\vec{v})f(m,\vec{x},\vec{v})\times\left(t_+^2-\frac{\tau^2}{4}-2t_+T+T^2\right). \tag{14}$$

In this formula, the terms proportional to t_+^2, t_+T, T^2 constitute the nonstationary part of the noise. The terms proportional to t_+ contribute only to the initial rotational phase of the pulsar, while t_+^2, t_+T, T^2 are equal to the products of the fits. According to the theorem that was proved by Kopeikin (1999), these products of the fits in the structure of the autocorrelation function do not contribute to the *PAT residuals, which depend only on the stationary part of the noise.* Thus, only the stationary component $\frac{\tau^2}{4}$ in Eq. (14) contributes to the PAT residuals. For this reason, the random process attributable to the regular part of the Shapiro effect for pulsars in globular clusters will manifest itself mainly as the noise of the pulsar spin rate.

Numerical Model for the Pulsar Noise Produced by the Shapiro Effect

As we noted above, knowledge of the autocorrelation function (7) allows us to determine the slope of the power spectrum for the noise process and the behavior of the σ_z statistic. We calculated the triple integral (7) numerically. We constructed two numerical models for the density distribution of the globular cluster 47 Tuc described by Eqs. (8) and (10).We also considered two extreme cases: small and large impact parameters.

Large impact parameters. We calculated the autocorrelation function in two ways: (1) in a spherical coordinate system with the origin at the GCB and (2) in a Cartesian coordinate system. We equated the volumes of the integration space in both coordinate systems and took into account the analytic singularities of the integrand by choosing the appropriate path of integration. Indeed, it follows from the form of the expression for the Shapiro delay (4) attributable to the gravitational field of the moving cluster stars that a star on the line of sight produces an in.nite signal delay. For a numerical model, this necessitates cutting out a cylinder of minimum radius r_{cl} in the direction of the unit vector $\vec{k}_0$. Given the radius of the cutout cylinder, we can easily estimate the cluster mass, which is excluded from the analysis. For example, for $r_{cl} \sim 1$AU, it is $\sim 10^{-4} M_\odot$.

The relative accuracy of calculating the integral was determined by the standard method and is 0.9% for the King model and 1% an isothermal sphere with a core. The autocorrelation function for the King model and the model of an isothermal sphere with a pulsar at the cluster center is plotted in Figs. 1 and 2, respectively.

Analysis of the plots of the autocorrelation function indicates that it is well fitted by a quadratic polynomial for both the King model and the model of an isothermal sphere. For example, $\Re = 7 \times 10^{-23} + 1.16 \times 10^{-20} t + 7 \times 10^{-23} t^2$ for the King model ($\tau = 3$ yr) and $\Re = 5 \times 10^{-24} + 6.12 \times 10^{-22} t + 4 \times 10^{-24} t^2$ for the model of an isothermal sphere with a core ($\tau = 3$ yr). As would be expected, our numerical result matches the analytic prediction made in the previous subsection and confirms the validity of the numerical integration model.

A stochastic process can be described both in the time domain by a time-dependent quantity $h(t)$ and in the frequency domain by the amplitude of the process $H(f)$ as a function of the Fourier frequency f. These two quantities are related by the Fourier transform:

$$H(f) = \int_{-\infty}^{\infty} H(f) e^{2\pi i f t} dt$$

$$h(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(f) e^{-2\pi i f t} dt \qquad (15)$$

According to the Parseval theorem, the total power of the signal will be the same when calculated in both the time and frequency domains:

$$P \equiv \int_{-\infty}^{\infty} |h(t)|^2 dt = \int_{-\infty}^{\infty} |H(f)|^2 df.$$

If the function $h(t)$ is real, then we can define the onesided spectral power density as

$$P(f) = 2|H(f)|^2.$$

The total power will then be calculated as an integral of $P(f)$ in the Fourier frequency limits from 0 to ∞.

To analyze the noise process, we numerically determined the spectral power density $P(f)$ and the spectral slope n using the fast Fourier transform, whose algorithm has been well developed (Elliott and Rao 1982). Our calculations show that the slopes of the power spectrum are $n = -(1.78 \pm 0.04)$ for the King model and $n = -(1.76 \pm 0.04)$ for the model of an isothermal sphere, which are equal within the computational error limits. In Fig. 3, $\log P$ is plotted against log f for the King model.

Small impact parameters. The random process attributable to star passages with small impact parameters should be considered as a special case of gravitational lensing. In our case, the characteristic impact parameter is equal to the Einstein-Chwolson radius. For a pulsar at the center of a globular star cluster, the Einstein-Chwolson radius is de.ned by (Zakharov and Sazhin 1998)

$$R_E = \sqrt{\frac{4GM_a D_{ds} D_d}{c^2 D_s}},$$

where D_{ds} is the distance from the pulsar to the gravitating body (star), D_d is the distance from the observer to the gravitating body, and D_s is the distance from the observer to the pulsar. For the globular cluster 47 Tuc and a gravitating mass of the order of the solar mass $M_\odot$, the characteristic Einstein-Chwolson radius is $\sim$ 1 AU. Therefore, in the direction of the pulsar specified by the unit vector $\vec{k}_0$, we cut out a cylindrical volume with a polar radius equal to the Einstein-Chwolson radius. We calculated integral (7) in a cylindrical coordinate system with a coordinate grid whose density increased with decreasing polar radius, since the stars with the smallest impact parameters make a larger contribution to the noise process.

The autocorrelation function of the noise process produced by the Shapiro effect is plotted in Fig. 4 for impact parameters smaller than 1 AU in the model of an isothermal sphere for three observing times τ: 1, 3, and 5 yr. The derived time dependences of the autocorrelation function cannot be fitted by a quadratic time polynomial, as in the case of large impact parameters, because the gravitational field acts on the radio pulse produced by close star passages near the line of sight for a short time. The autocorrelation function in Fig. 4 is nearly logarithmic and, in all probability, can be interpreted as a flicker noise (Kopeikin 1997b, 1999). The spectral index of the pulsar noise for small impact parameters is $n = -(1.55 \pm 0.03)$. Figure 5 shows the slope of the power spectrum for the model of an isothermal sphere for impact parameters smaller than 1 AU and an observing time of $\tau = 5$ yr.

Dependence of the noise on cluster core radius. In conclusion, let us analyze the dependence of the behavior of the autocorrelation function for the stochastic Shapiro effect on the globular cluster core radius. The numerical model described in the previous section was constructed for the globular cluster 47 Tuc whose core radius is $r_c = 0.52$ pc. We modeled the behavior of the autocorrelation function for three different cluster core radii, $r_c = 0.08$, 0.1, and 0.52 pc. The results of our calculations are shown in Fig. 6. It is easy to see that in all three cases, the shape of the time dependence of the autocorrelation function is the same, and, hence, the slope of the power spectrum does not depend on the cluster core radius used in our calculations.

CONCLUSIONS

The rotational phase modulation of a pulsar in a globular star cluster may be attributable to the fluctuations in the relativistic time delay of the pulsar signal caused by the gravitational field variations due to the motion of the cluster stars. The effect of the low-frequency timing noise produced by this process can be studied by accurately determining the behavior of the PAT residuals in the time and/or frequency domain. Here, we derived an analytical expression for the Shapiro delay of the radio pulse from a pulsar in a globular cluster by applying small aberration corrections, of the order of (v/c), where v is the characteristic velocity of the stars in the cluster and c is the speed of light.

Assuming that the interactions between the cluster stars are uncorrelated for large impact parameters, the random process produced by the stochastic Shapiro effect manifests itself mainly as the noise of the pulsar spin rate. For small impact parameters, we numerically simulated the stochastic Shapiro effect by assuming that the mass distribution of the cluster stars was specified by the Salpeter function, the velocity distribution of the cluster stars was Maxwellian, and the globular cluster density was described either by the model of an isothermal sphere or by the King model.

Our numerical analysis showed that the autocorrelation function of the noise process produced by the Shapiro effect attributable to the gravitational field of the moving cluster stars could be fitted by a quadratic time polynomial for both the model of an isothermal sphere and the King model at large impact parameters. The slope of the power spectrum is $n \approx -1.8$ and depends weakly on the model density distribution of the globular star cluster. For small impact parameters, the functional time dependence of the autocorrelation function for the stochastic Shapiro effect is nearly logarithmic and the slope of the power spectrum decreases and is $n \approx -1.5$.

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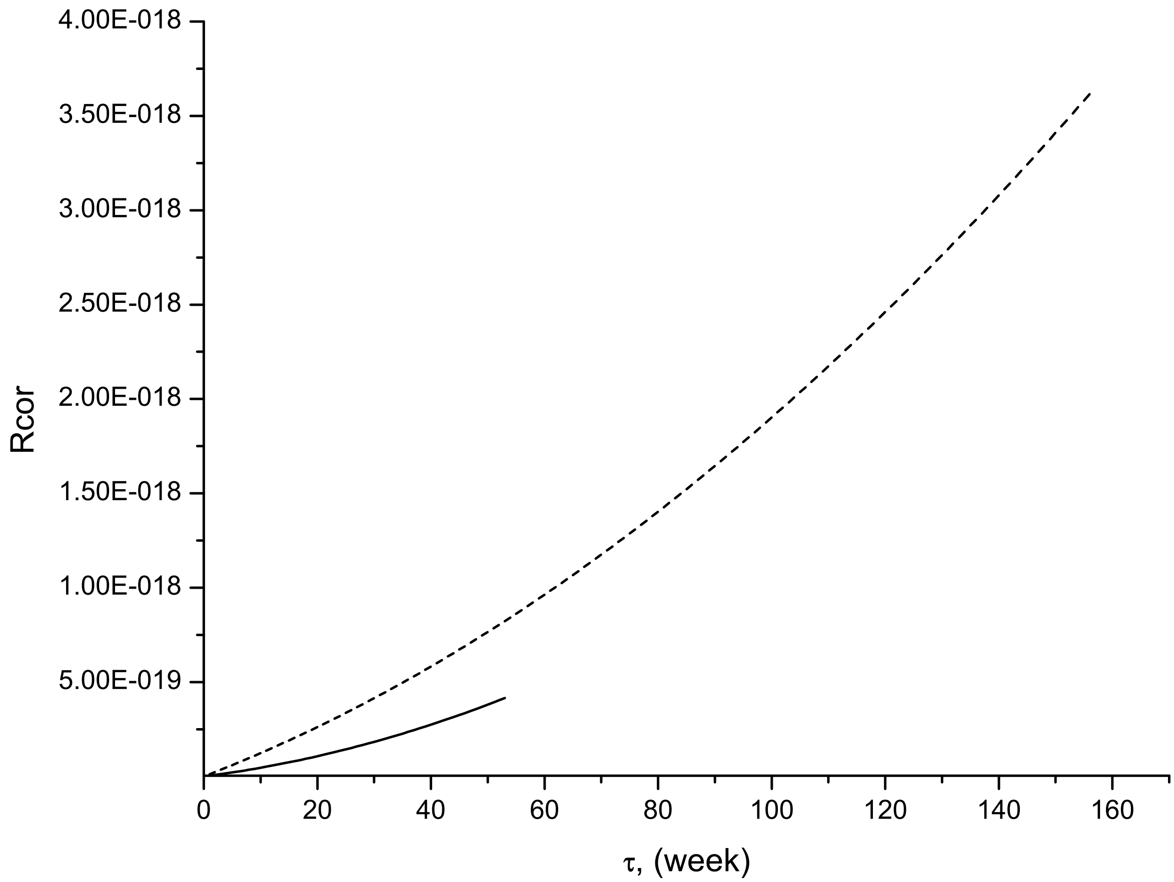

Fig. 1: Autocorrelation function of the stochastic Shapiro effect for a King model with a pulsar at the center of a globular cluster for observing times of 1 (solid line) and 3 yr (dotted line).

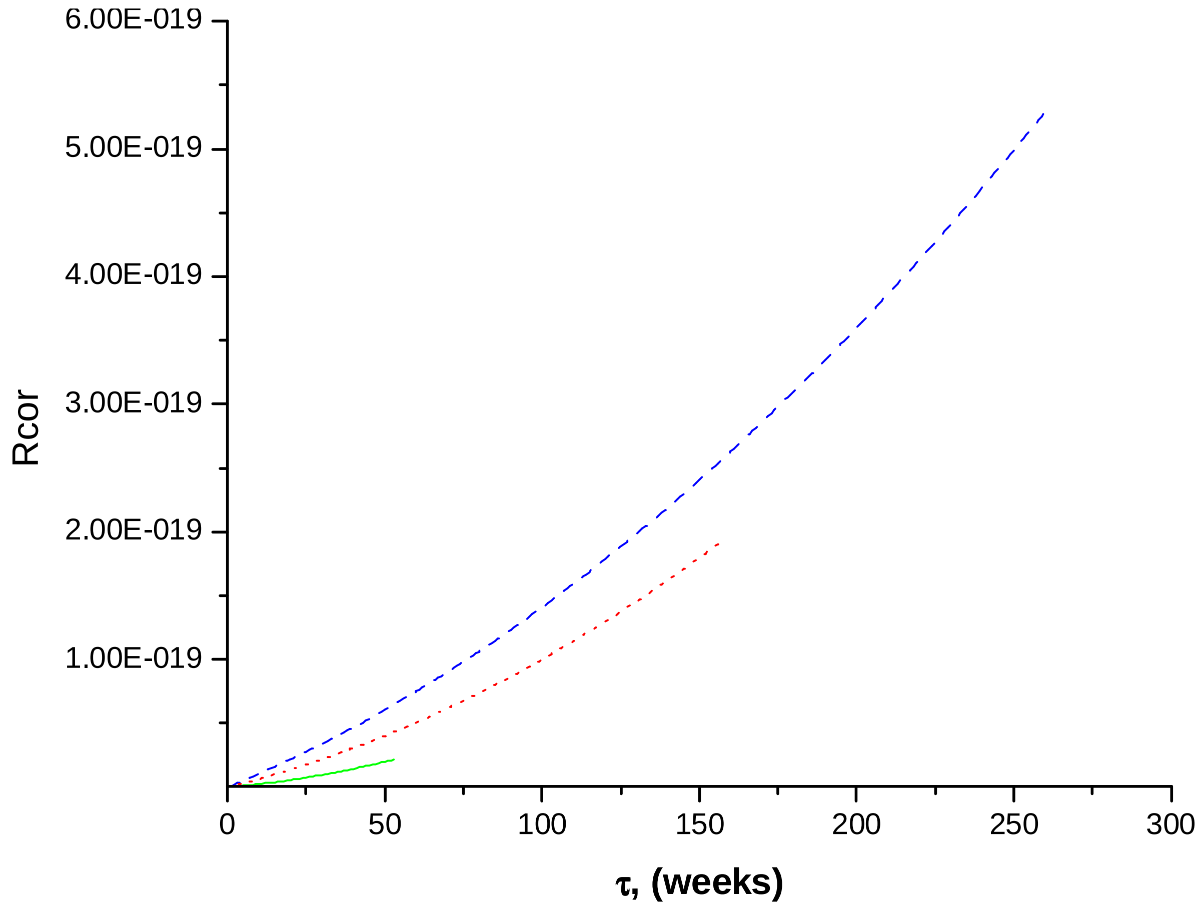

Fig. 2: Autocorrelation function of the stochastic Shapiro effect for the model of an isothermal sphere with a pulsar at the cluster center for observing times of 1 (solid line), 3 (dotted line), and 5 yr (dashed line)

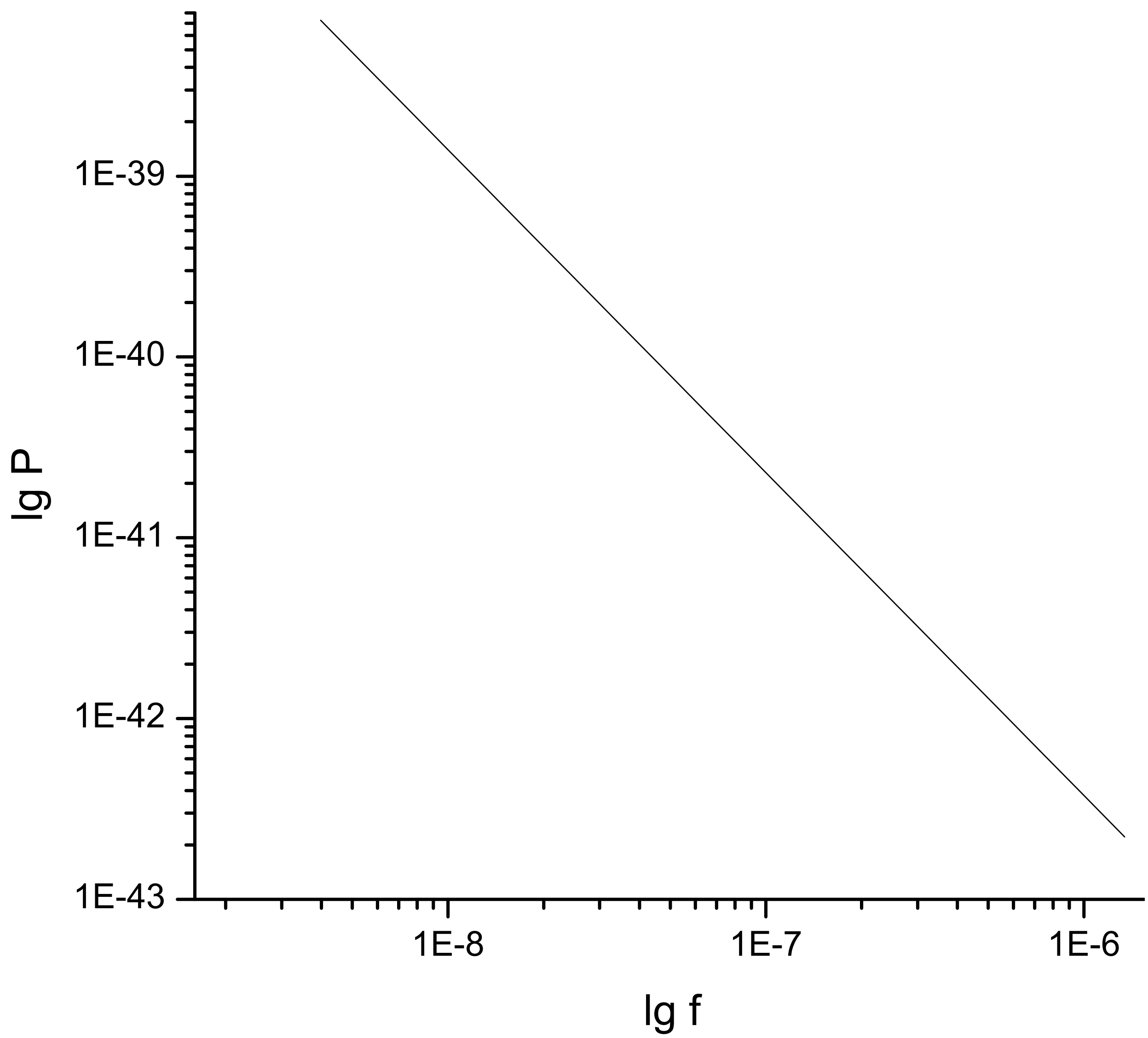

Fig. 3: Logarithm of the spectral power of the pulsar noise $\log P$ versus logarithm of the Fourier frequency $\log f$ for the King model.

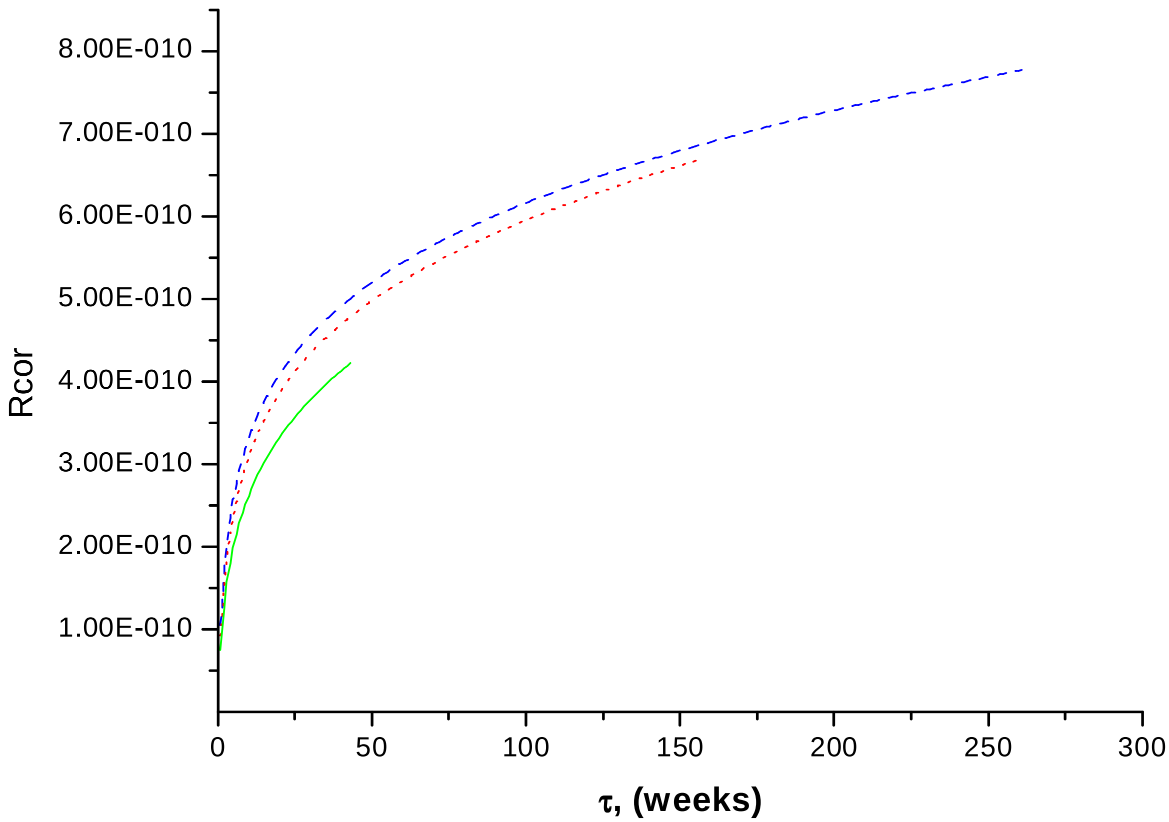

Fig. 4: Autocorrelation function of the stochastic Shapiro effect for impact parameters smaller than 1 AU in the model of an isothermal sphere with a pulsar at the center of a globular cluster for observing times of 1 (solid line), 3 (dotted line), and 5 yr (dashed line).

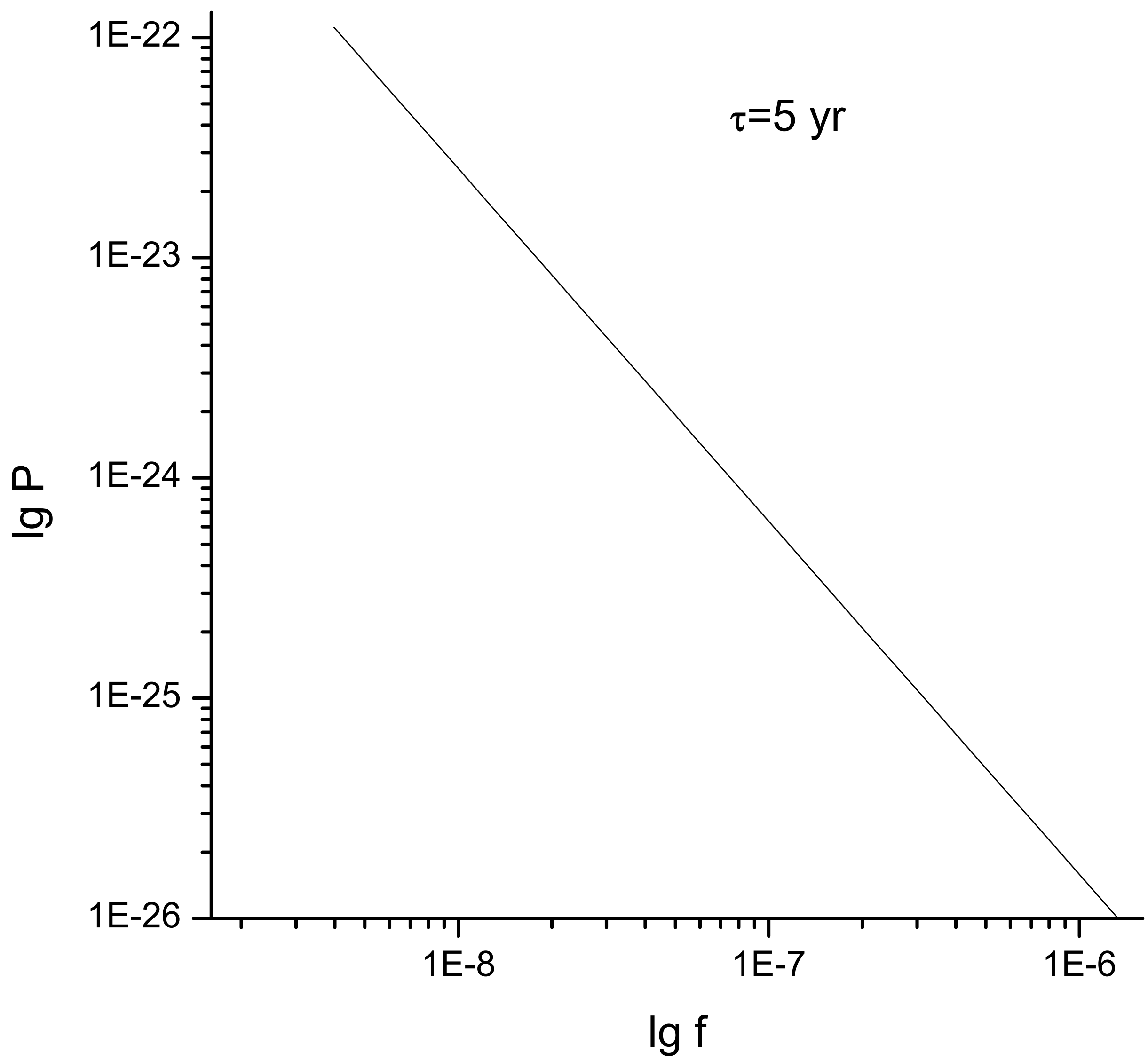

Fig. 5: Slope of the power spectrum for the stochastic Shapiro effect for impact parameters smaller than 1 AU ($\tau = 5$ yr)**.**

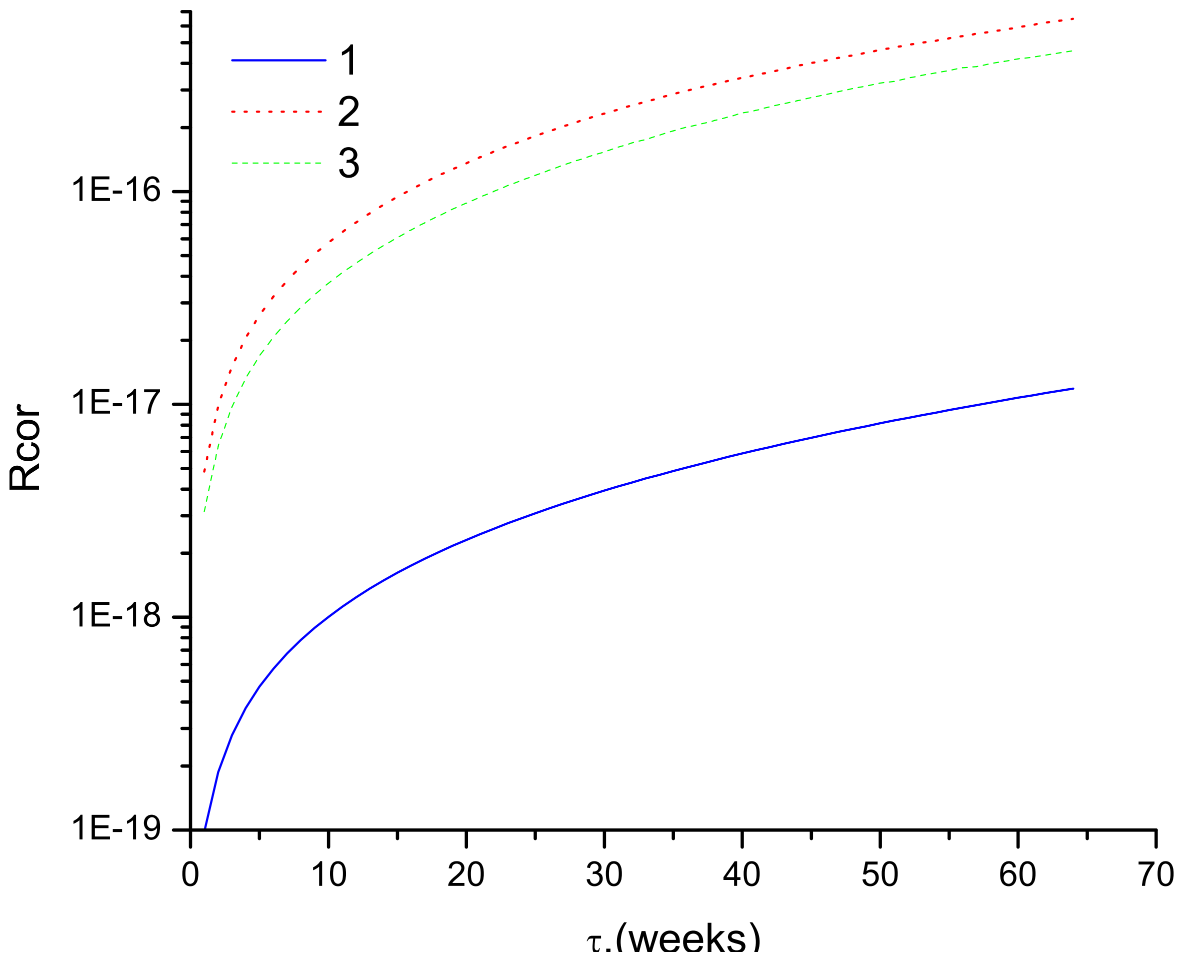

Fig. 6: Autocorrelation function of the stochastic Shapiro effect for the model of an isothermal sphere with various globular cluster core radii, 0.52 (1), 0.08 (2), and 0.1 pc (3), and with a pulsar at the cluster center. The observing time is $\tau = 1.5$ yr.