

# Testing Relativistic Effect of Propagation of Gravity by Very-Long Baseline Interferometry

Sergei M. Kopeikin

*Department of Physics and Astronomy, 322 Physics Building, University of Missouri-Columbia, Columbia, MO 65211*

kopeikins@missouri.edu

## ABSTRACT

It is shown that the finite speed of gravity affects very-long baseline interferometric observations of quasars during the time of their line-of-sight close angular encounter with Jupiter. The next such event will take place in 2002, September 8. The present *Letter* suggests a new experimental test of general relativity in which the effect of propagation of gravity can be directly measured by very-long baseline interferometry as an excess time delay in addition to the logarithmic Shapiro time delay (Shapiro, I. I., 1964, Phys. Rev. Lett., **13**, 789).

*Subject headings:* gravitation – relativity – techniques: interferometric – (galaxies:) quasars: individual (QSO J0842+1835)

## 1. Introduction

Experimental verifications of basic principles underlying Einstein's general relativity theory are important for fundamental physics. Numerous tests of general relativity in the solar system (Will 1993) and in the binary pulsar PSR 1913+16 (Taylor 1994) confirm its validity up to the precision of 1% or slightly better.

It is worth emphasizing that all of the Solar system tests of general relativity have relied upon the Schwarzschild solution and could say nothing about effects of retardation associated with the propagation of gravity. Einstein's theory of general relativity predicts that if the second time derivative of the quadrupole moment of a gravitating system (e.g., a binary pulsar) is not zero, the system emits gravitational waves that travel outward at the speed of light. Indirect evidence for the existence of gravitational waves consists of observations of the inspiraling orbits of binary pulsars (Taylor 1994). Observations of binary pulsar inspiral for PSR 1913+16 agree at the level of  $\sim 1\%$  with predictions based on the emission of energy by the binary in the form of (quadrupole) gravitational radiation (Kopeikin 1985; Schäfer 1985; Damour 1987). This also provides an indirect evidence that gravity waves must travel at the speed of light (Damour 1987; Will 1993). However, no one has yet *directly* detected gravitational waves, let alone measured their speed.

It is the purpose of this *Letter* to point out that observing propagation of light through the gravitational field of the solar system can serve as a tool for measuring effects associated with the finite speed of propagation of gravity. This is based on our previous papers (Kopeikin 1997; Kopeikin et al. 1999; Kopeikin & Schäfer 1999), where we have developed a post-Minkowskian approach for solving the problem of propagation of light through time-dependent gravitational fields in the geometric optics approximation. This approach is based on making use of the retarded *Liénard-Wiechert*-type solutions of the linearized Einstein equations and allows one to find a smooth analytic representation of light-ray trajectory for arbitrary locations of the source of light and observer without imposing any restrictions on the motion of the light-ray deflecting bodies. We find that electromagnetic signals interact with gravitating bodies only through the retarded gravitational field of the bodies. That is, if a light-ray deflecting body moves with respect to a chosen coordinate system the temporal variation in its gravity field must take time to reach the electromagnetic signal in order to perturb its trajectory. This observation constitutes the main idea of the proposed VLBI test of the propagation of gravity elaborated in the following sections of the present *Letter*.

In 1964, I. I. Shapiro (1964) suggested that the gravitational deflection of light by the Sun — one of the three classical effects of general relativity analyzed by Einstein — could be measured more accurately at radio wavelengths with interferometric techniques than at visible wavelengths with the available optical techniques. His idea led to stringent experimental limitations on the parameter  $\gamma$  of the parametrized post-Newtonian formalism (Shapiro 1967), thereby strongly restricting the number of viable theories of gravity. Our post-Minkowskian approach gives in the case of static spherically-symmetric field the same result as predicted by Shapiro. Furthermore, we are able to calculate additional corrections to the Shapiro time delay related to the non-stationarity of the gravitational field of the solar system and associate them with the finite speed of propagation of gravity. The largest contribution to the non-stationarity of gravitational field of the solar system comes about via the orbital motions of the most massive planets — Jupiter and Saturn. Therefore, it is reasonable to undertake an attempt to detect the effect of propagation of gravity by observing very accurately the deflection of light rays from a background source of light (quasar) caused by the motion of Jupiter or Saturn. This is the essence of our proposed VLBI test of general relativity discussed in the present paper.

In section 2 we consider the basic formula for relativistic time delay in time-dependent gravitational fields. Section 3 outlines the basic principles of measurement of the effect of propagation of gravity in radio interferometric experiments. Section 4 is dedicated to the description of the proposed VLBI experiment and gives numerical estimates for the Shapiro time delay in the field of Jupiter and for the effect of propagation of gravity. Finally, in section 5 we discuss the proposed VLBI experiment.

## 2. Relativistic time delay in time-dependent gravitational fields

Let us assume that the gravitational field is generated only by the Solar system's bodies and there is a global four-dimensional coordinate system  $(t, \mathbf{x})$  with origin at the barycenter of the Solar system. The total time of propagation of an electromagnetic signal from the point  $(t_0, \mathbf{x}_0)$  (quasar) to the point  $(t, \mathbf{x})$  (observer) is given by the expression

$$t - t_0 = \frac{1}{c} |\mathbf{x} - \mathbf{x}_0| + \Delta(t, t_0). \quad (1)$$

Here,  $|\mathbf{x} - \mathbf{x}_0|$  is the usual Euclidean distance between the points of emission,  $\mathbf{x}_0$ , and observation,  $\mathbf{x}$ , of the photon, and  $\Delta(t, t_0)$  is the relativistic time delay produced by the gravitational field of the moving gravitating bodies of the Solar system. The function  $\Delta(t, t_0)$  is given by (Kopeikin & Schäfer 1999)

$$\Delta(t, t_0) = \frac{2G}{c^2} \sum_{a=1}^N m_a \int_{s_0}^s \frac{(1 - c^{-1} \mathbf{k} \cdot \mathbf{v}_a(\zeta))^2}{\sqrt{1 - c^{-2} v_a^2(\zeta)}} \frac{d\zeta}{ct^* + \mathbf{k} \cdot \mathbf{x}_a(\zeta) - c\zeta}, \quad (2)$$

where  $m_a$  is the mass of the  $a$ th body,  $t^*$  is the time of the closest approach of electromagnetic signal to the barycenter of the Solar system<sup>1</sup>,  $\mathbf{x}_a(t)$  are coordinates of the  $a$ th body,  $\mathbf{v}_a(t) = d\mathbf{x}_a(t)/dt$  is the (non-constant) velocity of the  $a$ th light-ray deflecting body,  $\mathbf{k}$  is the unit vector from the point of emission to the point of observation,  $s$  is a retarded time obtained by solving the gravitational null cone equation for the time of observation of photon  $t$ , and  $s_0$  is found by solving the same equation written down for the time of emission of the photon  $t_0$

$$s + \frac{1}{c} |\mathbf{x} - \mathbf{x}_a(s)| = t, \quad s_0 + \frac{1}{c} |\mathbf{x}_0 - \mathbf{x}_a(s_0)| = t_0. \quad (3)$$

It is remarkable that formula (2) does not impose any restriction on the motion of bodies since we did not use any explicit solutions of their equations of motion while calculating the light ray propagation. The formula (2) can be further simplified by performing integration by parts which yields<sup>2</sup>

$$\begin{aligned} \Delta(t, t_0) = & - \sum_{a=1}^N \frac{2Gm_a}{c^3} \left\{ \ln \left[ \frac{r_a(s) - \mathbf{k} \cdot \mathbf{r}_a(s)}{r_{0a}(s_0) - \mathbf{k} \cdot \mathbf{r}_{0a}(s_0)} \right] - \frac{\mathbf{k} \cdot \mathbf{v}_a(s)}{c} \ln \left[ r_a(s) - \mathbf{k} \cdot \mathbf{r}_a(s) \right] + \right. \\ & \left. \frac{\mathbf{k} \cdot \mathbf{v}_{a0}(s_0)}{c} \ln \left[ r_{0a}(s_0) - \mathbf{k} \cdot \mathbf{r}_{0a}(s_0) \right] - \int_{s_0}^s \ln \left[ ct^* + \mathbf{k} \cdot \mathbf{x}_a(\zeta) - c\zeta \right] \frac{\mathbf{k} \cdot \mathbf{a}_a(\zeta)}{c} d\zeta \right\}, \end{aligned} \quad (4)$$

where  $\mathbf{r}_a = \mathbf{x} - \mathbf{x}_a(s)$ ,  $\mathbf{r}_{0a} = \mathbf{x}_0 - \mathbf{x}_a(s_0)$ ,  $r_a = |\mathbf{r}_a|$ ,  $r_{0a} = |\mathbf{r}_{0a}|$ ,  $\mathbf{v}_a = \dot{\mathbf{x}}_a(s)$ ,  $\mathbf{v}_{a0} = \dot{\mathbf{x}}_a(s_0)$ ,  $\mathbf{a}_a = \dot{\mathbf{v}}_a(s)$ , and the retarded times  $s$  and  $s_0$  are calculated from the gravitational null-cone equations (3).

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<sup>1</sup>The time  $t^*$  is used in calculations as a mathematical tool only.

<sup>2</sup>After integration we omitted for simplicity terms of order  $v_a^2/c^2$ .

Our expression (4) vastly extends the region of applicability of Shapiro’s work (Shapiro 1967) for it is valid for the case of arbitrary-moving bodies whereas the calculations by Shapiro had been done only for the case of static Schwarzschild field. Earlier work on improving the Schwarzschild approximation in the case of light passing through the gravitational field of moving bodies should be noted (Hellings 1986; Klioner 1991; Klioner & Kopeikin 1992; Sovers, Fanselow & Jacobs 1998); however, our expression (4) is much more general and completely solves the problem. We emphasize that solution (4) is valid everywhere both inside and outside of the Solar system at arbitrary distances including its near, intermediate, and wave zones, and, for this reason, represents a smooth analytic solution of the equations of light-ray geodesics for arbitrarily located source of light, observer, and the system of the light-ray deflecting bodies. However, the most important observation is that our formula (4) allows one to single out the retardation effects uniquely associated with the finite speed of propagation of gravity and indicates that for the correct calculation of the relativistic time delay positions of the gravitating bodies must be taken at the retarded times  $s_0$  and  $s$  corresponding to the instants of emission  $t_0$  and observation  $t$  of the electromagnetic signal

### 3. Principles of Measurement of the Propagation of Gravity by VLBI

Very Long Baseline Interferometry (VLBI) has recently reached a precision in measurements of differential phase delay of order  $10^{-12}$  second (ps), which makes it presently the most accurate technique for measuring relativistic time delay (Beasley & Conway 1995; Honma, Kawaguchi & Sasao 2000). In order to discuss how to measure the relativistic effect of propagation gravity by VLBI one transforms the formula (4) to the form used in VLBI analysis and called the differential VLBI time delay (Walter & Sovers 2000). In doing this conversion, we note that formula (4) is also valid in scalar-tensor theories after replacement of the universal gravitational constant  $G \rightarrow G(1 + \gamma)/2$ , where  $\gamma$  is one of the parameters of the PPN formalism (Will 1993).

Let us assume now that there are two earth-based VLBI stations and the plane front of electromagnetic waves from a quasar propagates towards the Earth. Taking two rays from the wave front and subtracting equation (1) for the first light ray from that for the second ray yields

$$t_2 - t_1 = \frac{1}{c}|\mathbf{x}_2 - \mathbf{x}_0| - \frac{1}{c}|\mathbf{x}_1 - \mathbf{x}_0| + \Delta(t_1, t_2), \quad (5)$$

where  $\Delta(t_1, t_2) \equiv \Delta(t_2, t_0) - \Delta(t_1, t_0)$ . Here  $t_1$  and  $t_2$  are coordinate times of arrival of the electromagnetic signal from the source of electromagnetic waves (quasar) to the first and second stations, respectively, and  $\mathbf{x}_1$  and  $\mathbf{x}_2$  are spatial coordinates of the first and second VLBI stations with respect to the barycentric frame of the solar system that is chosen as the primary non-rotating reference frame (Walter & Sovers 2000).

The difference  $\Delta(t_1, t_2)$  is obtained from the main formula (4) after long and tedious calculations and analysis of residual terms. It can be proved (Kopeikin & Schäfer 1999) that with accuracy

better than 1 ps,

$$\Delta(t_1, t_2) = (1 + \gamma) \sum_{a=1}^N \frac{Gm_a}{c^3} \left( 1 + \frac{\mathbf{K} \cdot \mathbf{v}_a(s_1)}{c} \right) \ln \frac{r_{1a}(s_1) + \mathbf{K} \cdot \mathbf{r}_{1a}(s_1)}{r_{2a}(s_2) + \mathbf{K} \cdot \mathbf{r}_{2a}(s_2)}, \quad (6)$$

where  $\mathbf{K}$  is the unit vector from the barycenter of the solar system to the quasar;  $\mathbf{v}_a(s_1)$  is the velocity of the  $a$ th gravitating body at time  $s_1$ ,  $r_{1a} = |\mathbf{r}_{1a}|$ ,  $r_{2a} = |\mathbf{r}_{2a}|$ ,  $\mathbf{r}_{1a}(s_1) = \mathbf{x}_1(t_1) - \mathbf{x}_a(s_1)$  and  $\mathbf{r}_{2a}(s_2) = \mathbf{x}_2(t_2) - \mathbf{x}_a(s_2)$ ; moreover, the retarded times  $s_1$  and  $s_2$  are calculated according to

$$s_1 = t_1 - \frac{1}{c} |\mathbf{x}_1(t_1) - \mathbf{x}_a(s_1)|, \quad s_2 = t_2 - \frac{1}{c} |\mathbf{x}_2(t_2) - \mathbf{x}_a(s_2)|. \quad (7)$$

The effect of propagation of gravity appears in equation (6) as a displacement of the light-ray deflecting bodies from their present to retarded positions. We note that velocities of the gravitating bodies  $\mathbf{v}_a$  are small with respect to the speed of gravity and the time taken by the light ray to reach the observer after passing by the gravitating body is much smaller than its orbital period. Thus, one can expand the retarded positions  $\mathbf{x}_a(s_i)$  ( $i = 1, 2$ ) of the bodies in Taylor series around their present positions  $\mathbf{x}_a(t_i)$  taken at the arrival times  $t_i$  ( $i = 1, 2$ ). Neglecting all terms quadratic with respect to velocities of the gravitating bodies and/or proportional to their accelerations one gets

$$\Delta(t_1, t_2) = (1 + \gamma) \sum_{a=1}^N \frac{Gm_a}{c^3} \left[ \left( 1 + \frac{\mathbf{K} \cdot \mathbf{v}_a}{c} \right) \ln \frac{r_{1a} + \mathbf{K} \cdot \mathbf{r}_{1a}}{r_{2a} + \mathbf{K} \cdot \mathbf{r}_{2a}} - \frac{\mathbf{B} \cdot \mathbf{v}_a + (\mathbf{K} \cdot \mathbf{v}_a)(\mathbf{N}_{1a} \cdot \mathbf{B})}{c(r_{1a} + \mathbf{K} \cdot \mathbf{r}_{1a})} \right], \quad (8)$$

where  $\mathbf{B} = \mathbf{x}_2(t_1) - \mathbf{x}_1(t_1)$  is the baseline between the two VLBI stations,  $\mathbf{r}_{ia} = \mathbf{x}_i(t_i) - \mathbf{x}_a(t_i)$  is the difference of coordinates of the  $i$ th VLBI station and  $a$ th gravitating body taken at the time of arrival of the radio signal to the  $i$ th station,  $\mathbf{N}_{1a} = \mathbf{r}_{1a}/r_{1a}$ ,  $\mathbf{v}_a \equiv \mathbf{v}_a(t_1)$  and all quantities in the second term on the right side of equation (8) are also evaluated at time  $t_1$ .

The logarithmic term on the right side of equation (8) was first discovered by Shapiro (1967). However, the finite speed of propagation of gravity leads to the appearance of an additional (second) term on the right side of equation (8) as well. Had the speed of gravity been set equal to infinity as it is done in Newtonian gravitation, the second term on the right side of equation (8) would be identically equal to zero. Experimental confirmation of the existence of such a term would directly prove that gravity propagates with a finite speed.

#### 4. The Proposed Radio Interferometric Experiment

In 2002, on September 8 Jupiter will pass at the angular distance of about 3.7 arcminutes from the quasar QSO J0842+1835, making an ideal celestial configuration for measuring the speed of propagation of gravity by using the differential VLBI technique. During the passage of Jupiter near the quasar line of sight, the time-dependent impact parameter  $|\xi|$  of the light ray from the quasar with respect to Jupiter will be always small as compared with the Earth-Jupiter distance,

$r_{\oplus J}$ , which is approximately equal to 6 astronomical units. Hence, it is convenient to introduce the unit vector  $\mathbf{n}$  along the direction of the impact parameter according to the definition  $\boldsymbol{\xi} = |\boldsymbol{\xi}| \mathbf{n}$  such that

$$\mathbf{N}_{1J} = -\cos \theta \mathbf{K} + \sin \theta \mathbf{n}, \quad (9)$$

where the subscript  $J$  refers to Jupiter and  $\theta \simeq |\boldsymbol{\xi}|/r_{\oplus J}$  is the (small) angle between the undisturbed geometric positions of the quasar and Jupiter. The baseline  $\mathbf{B}$  during the time of the experiment will be much smaller than  $|\boldsymbol{\xi}|$ ; therefore, the following expansions are valid

$$\mathbf{N}_{1J} = -\left(1 - \frac{\theta^2}{2}\right) \mathbf{K} + \theta \mathbf{n} + O(\theta^3), \quad (10)$$

$$r_{2a} + \mathbf{K} \cdot \mathbf{r}_{2a} = r_{1a} + \mathbf{K} \cdot \mathbf{r}_{1a} + \mathbf{N}_{1a} \cdot \mathbf{B} + \mathbf{K} \cdot \mathbf{B} + O(\mathbf{B}^2). \quad (11)$$

Other than Jupiter, the Earth and the Sun also contribute significantly to the gravitational time delay and must be included in the data processing algorithm in order to extract the effect of propagation of gravity unambiguously and to measure its speed. Accounting for this fact and making use of the expansions (10)–(11), we can recast formula (8) into the following convenient form

$$\Delta(t_1, t_2) = \Delta_{\oplus} + \Delta_{\odot} - (1 + \gamma) \left[ \frac{2GM_J \mathbf{n} \cdot \mathbf{B}}{c^3 r_{1J} \theta} + (1 + \delta) \frac{2GM_J \mathbf{B} \cdot \mathbf{v}_J - (\mathbf{K} \cdot \mathbf{v}_J)(\mathbf{K} \cdot \mathbf{B})}{c^4 r_{1J} \theta^2} \right], \quad (12)$$

where we have introduced a new phenomenological parameter  $\delta$  parameterizing the effect of propagation of gravity in data processing algorithms. Thus, we emphasize that there are two relativistic parameters to be measured in the VLBI experiment in order to test the validity of general relativity theory — the PPN parameter  $\gamma$  and the gravity propagation parameter  $\delta$ . The best experimental measurement of the parameter  $\gamma$  is due to Lebach et al. (1995), who obtained  $\gamma = 0.9996 \pm 0.0017$  in excellent agreement with general relativity. The primary goal of the new experimental test of general relativity proposed in the present *Letter* is to set *direct observational* limits on the parameter  $\delta$  which will measure the effect of retardation in propagation of gravity by the moving Jupiter. According to the Einstein theory of relativity one must expect that the numerical value of the parameter  $\delta$  must be equal to zero.

We emphasize that our consideration is fully based on the Einstein theory of general relativity in which the speed of propagation of gravity is equal to the speed of light in vacuum. Will (1971) analyzed the case when the two speeds are not equal and showed that the resulting theory would have a non-zero value of the PPN parameter  $\alpha_2 = (c/c_g)^2 - 1$ , where  $c_g$  is the value of the propagation speed of gravity in the rest frame of the universe.<sup>3</sup> This parameter has been strongly bounded by consideration of various “preferred frame effects”. The present best limit (Nordtvedt 1987) is  $|\alpha_2| < 4 \times 10^{-7}$ . The experiment proposed in the present *Letter* is not designed to compete

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<sup>3</sup>We would like to emphasize that our parameter  $\delta$  is just a fitting parameter and has no any relation to the PPN parameter  $\alpha_2$  as we work in the framework of general relativity where  $c_g = c$ .

with this tight bound, rather its main purpose is to measure the retardation effect (parameter  $\delta$ ) associated with finite speed of propagation of gravity which was never observed before.

The first term on the right side of equation (12) describes the Shapiro time delay due to the gravitational field of the Earth. It can reach 21 ps for a baseline of  $B \simeq 6000$  km. The second term on the right hand side of equation (12) describes the Shapiro time delay due to the Sun. It can vary (for  $B \simeq 6000$  km) from  $17 \times 10^4$  ps for the light ray grazing the Sun's limb to only 17 ps when the direction to the source of light is opposite to the Sun (Klioner 1991). The third term on the right side of equation (12) is the standard Shapiro time delay due to the static gravitational field of Jupiter. Finally, the fourth term on the right side of equation (12) is the time delay caused by the effect of propagation of gravity. Numerical estimates of the Shapiro time delay  $\Delta_J$  and the gravity propagation time delay  $\Delta_{JP}$  caused by Jupiter can be obtained directly from equation (12). One finds for  $\gamma = 1$  and  $\delta = 0$ ,

$$\Delta_J \simeq \alpha_\odot \left( \frac{R_\odot}{R_J} \right) \left( \frac{M_J}{M_\odot} \right) \left( \frac{\theta_J}{\theta} \right) \left( \frac{R_\oplus}{c} \right), \quad \Delta_{JP} \simeq \left( \frac{\Delta_J}{\theta} \right) \left( \frac{v_J}{c} \right), \quad (13)$$

where  $\alpha_\odot = 1.75''$  is the relativistic deflection of light by the Sun for the case of light grazing the Sun's limb,  $R_\odot$  is the radius of the Sun,  $R_J$  is the radius of Jupiter,  $R_\oplus$  is the radius of the Earth,  $\theta_J = 0.27'$  is the visible angular radius of Jupiter at the distance of  $r_{\oplus J} = 6$  AU,  $\theta = 3.7'$  is the minimum angular distance from Jupiter to the quasar in 2002, on September 8, and  $v_J$  is the orbital speed of Jupiter.

It is worth emphasizing that  $\Delta_J \sim \theta^{-1}$ , while the gravity propagation time delay  $\Delta_{JP} \sim \theta^{-2}$ . For this reason, the effect of propagation of gravity is not strongly suppressed by the presence in  $\Delta_{JP}$  of the small factor  $v_J/c = 4.5 \times 10^{-5}$ , and, it turns out to be measurable using VLBI techniques. Indeed, using the numerical values for the parameters in equation (13) for the case of the Jupiter-quasar "encounter" in 2002, September 8, we obtain ( $B \simeq R_\oplus$ )

$$\alpha_J \simeq 1.26 \text{ mas}, \quad \Delta_J \simeq 137 \text{ ps}, \quad \Delta_{JP} \simeq 6 \text{ ps}, \quad (14)$$

so that  $\Delta_{JP}/\Delta_J \simeq 0.04$ . Hence, the correction due to the propagation of gravity would be  $\sim 4\%$  of the Shapiro time delay. With the present differential VLBI accuracy it appears that the new effect ( $\Delta_{JP} \simeq 6$  ps) is measurable with careful observations. Indeed, one can achieve a phase-referencing VLBI accuracy of a few degrees of phase (averaged over a 10-hour observation) and can detect relative astrometric position changes of less than 0.010 mas ( $\sim 1$  ps) with the existing VLBA and Effelsberg radio antennas (E. Fomalont, private communication).

Formula (13) also explains why it is not so effective to observe the effect of the propagation of gravity in the field of the Sun. It turns out that for the case of the Sun the angle  $\theta$  can not be made as small as in the case of Jupiter, because the Sun is closer to the Earth and, in addition, its radius is ten times larger than that of Jupiter. This makes the effect of the propagation of gravity in the field of the Sun  $\leq 10$  ps for the light ray grazing the Sun's limb.

## 5. Discussion of the Proposed VLBI Experiment

The effect of propagation of gravity appears in addition to the logarithmic Shapiro time delay as an excess delay of  $\sim 4\%$ . The Shapiro time delay  $\Delta_J$  caused by the static part of the gravitational field of Jupiter was first measured in 1991 (Treuhaft and Lowe 1991). Klioner (1991) used the post-Newtonian expression for the metric tensor under the assumption that the planets of the solar system move uniformly along straight lines. He obtained the same result for  $\Delta_{JP}$  formally; however, due to the specific assumptions used for the derivation of the near field post-Newtonian metric, he was not able to interpret it properly as the effect caused by the finite speed of propagation of gravity. This is because there was no proof that the solution of the light geodesic equations obtained on the basis of the near field post-Newtonian metric can be smoothly matched with the far field metric describing emission gravitational waves by the orbital motion of planets in the Solar system. The present *Letter* provides evidence that this near field post-Newtonian metric directly depends on the speed of propagation of gravity in general relativity.

Other authors (Hellings 1986; Klioner 1991; Sovers, Fanselow & Jacobs 1998) have also attempted to derive more exact expressions for the Shapiro effect in the case of time-dependent gravitational field of the Solar system. These authors used the post-Newtonian metric for the calculation of the Shapiro effect with positions of bodies taken at the time of the closest approach  $t_a^*$  of a radio signal to the body

$$t_a^* = t - \frac{1}{c} \mathbf{k} \cdot (\mathbf{x} - \mathbf{x}_a) . \quad (15)$$

The instant of time  $t_a^*$  is numerically close to the retarded time  $s$  calculated as the solution of the gravitational null cone equation (3) and, for this reason, makes a good approximation for the calculation of the Shapiro time delay. However, in the general case when the impact parameter is large,  $t_a^*$  and  $s$  can have very different numerical values. Indeed, the physical meanings of the time of the closest approach  $t_a^*$  and the retarded time  $s$  are crucially different. As it is seen from its definition (15), the time  $t_a^*$  is calculated only approximately using the finite speed of propagation of *light*, while the retarded time  $s$  emerges in our more exact post-Minkowskian analytic calculations (4) due to the finite speed of propagation of *gravity*. It is because of this significant difference that the VLBI experiment in 2002, September 8 will probe the non-trivial effect of the propagation of gravity.

It is also useful to see how the effect of the propagation of gravity can be explained qualitatively without doing lengthy calculations. For this purpose, we note that equation (6) indicates that the Shapiro time delay is actually a function of the retarded time  $\Delta_J(s)$ . Expanding this function in Taylor series around the time of observation  $t$ , one finds

$$\begin{aligned} \Delta_J(s) &= \Delta_J(t) + \dot{\Delta}_J(t)(s - t) + \dots \\ &\simeq \Delta_J(t) + \Delta_J(t) \left( \frac{\dot{\theta}}{\theta} \right) \left( \frac{r_{\oplus J}}{c} \right) + \dots = \Delta_J(t) + \Delta_{JP}(t) + \dots , \end{aligned} \quad (16)$$

where  $\dot{\Delta}_J(t) \simeq -(\dot{\theta}/\theta)\Delta_J$  using equation (13),  $s - t \simeq -r_{\oplus J}/c$  using equation (3), and  $\Delta_{JP}$  is given

by the approximate expression (13).

One can see that the delay  $\Delta_{JP}$  due to the finite speed of propagation of gravity can become comparable with the standard (instantaneous) prediction for the Shapiro time delay  $\Delta_J(t)$  in case  $\theta \simeq v_J/c$ . This can never happen for Jupiter and the other solar system bodies, but can be interesting for VLBI experiments if one were able to find a binary star close to the line of sight of a quasar. Orbital motion of the binary star would cause periodic modulation of the Shapiro delay in the time of propagation of light from the quasar to the observer, which can be used for the determination of the effect of propagation of gravity outside the solar system (Kopeikin et al. 1999; Kopeikin & Gwinn 2000). It is worth noting that the effect of propagation of gravity can also be studied in the timing of binary pulsars with nearly edgewise orbits and by the doppler tracking of spacecraft in deep space.

We are indebted to Prof. B. Mashhoon who participated in numerous scientific discussions, carefully read the manuscript and made many essential suggestions for its improvement. We are grateful to Dr. S.M. Kudryavtsev for calculating the date of the Jupiter's encounter with QSO J0842+1835. Thanks are also due to Dr. E. Fomalont, Dr. L. Petrov, Prof. K. Nordtvedt, Prof. J. Burns, Prof. C. R. Gwinn, and Prof. O. Sovers for interesting and stimulating discussions and suggestions.

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