Theory of Relativistic Reference Frames for High-Precision Astrometric Space Missions

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Recent modern space missions deliver invaluable information about origin of our universe, physical processes in the vicinity of black holes and other exotic astrophysical objects, stellar dynamics of our galaxy, etc. On the other hand, space astrometric missions make it possible to determine with unparalleled precision distances to stars and cosmological objects as well as their physical characteristics and positions on the celestial sphere. Permanently growing accuracy of space astronomical observations and the urgent need for adequate data processing algorithms require corresponding development of an adequate theory of reference frames along with unambiguous description of propagation of light rays from a source of light to observer. Such a theory must be based on the Einstein's general relativity and account for numerous relativistic effects both in the solar system and outside of its boundary. The main features of the relativistic theory of reference frames are presented in this work. A hierarchy of the frames is described starting from the perturbed cosmological Friedmann-Robertson-Walker metric and going to the observer's frame through the intermediate barycentric and geocentric frames in the solar system. Microarcsecond astrometry and effects of propagation of light rays in time-dependent gravitational fields are discussed as well.

1 Introduction

The role of high quality reference frames in astronomy has been recognized early by both theorists and observers. Astrometric and navigation data rely on observations referred to a frame, either local or global. The choice of the reference frame may be driven by instrumental considerations or be based upon deeper theoretical grounds. This paper deals with the latter subject.

The original approaches to construct reference frames in astronomy were completely based on the concepts of Newtonian gravity and Euclidean absolute space and time [1]. Modern astrometry, however, is operating at the angular resolution already exceeding 1 milliarcsecond (see, for example, [2]). At this level the primary gravitational theory must be the General Relativity Theory (GRT) with a corresponding replacement of the Euclidean space and time by the four-dimensional Riemannian space-time manifold. In other words, the theoretical basis of modern astrometry must be entirely relativistic. Recognition of this fact is rapidly spreading in the astrometric community especially after the successful completion of the HIPPARCOS mission [4], materialization of the International Coordinate Reference

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Frame in the sky, development of technologically new ideas in space astrometry, and the adoption by the XXIVth General Assembly (GA) of the International Astronomical Union (IAU) of a new relativistic framework for the reference frames in the Solar system [3].

In what follows we shall describe in more detail the initial theoretical motivations that stimulated the development of modern relativistic framework for the reference frames in the solar system and new ideas driving the development of the theory beyond 2000. Then, in section 2, we give a description of the IAU-1991 reference frame framework and compare it in section 3 with the present day formulation adopted by the XXIVth GA of the IAU-2000. A possible way of matching the IAU-2000 framework with cosmological model of the universe is considered in section 4. The basic theoretical ideas for approaching microarcsecond accuracy in relativistic space astrometry is outlined in section 5.

1.1 Initial Motivations

Transition from the Newtonian concepts towards more profound theoretical relativistic approach for the construction reference frames in modern astrometry and celestial mechanics can be traced back to the period 1975-1992 and is associated with the breakthrough in the solution of two major problems: development of a self-consistent framework for derivation of the post-Newtonian equations of translational and rotational motion of self-gravitating extended bodies in the solar system [5, 6] and development of the higher-order relativistic celestial mechanics of binary pulsars [7, 8, 9] discovered by Hulse and Taylor [10]. The main concern of theorists working on relativistic celestial mechanics of the solar system bodies was related to the problem of unambiguous interpretation of astronomical measurements and separation of small coordinate perturbations from the real physical relativistic effects. Later on the question regarding the best choice for the gauge conditions imposed on the metric tensor arose, for a number of gauges were used in calculations by various groups and there were arguments about their advantages and disadvantages. But, probably the most serious was the problem of construction of the local geocentric (and planetocentric) reference frame in the post-Newtonian approximation(s). Apparently there existed principal difficulties in solving these problems. This was because the simplest approximation of the Earth's gravitational field by the Schwarzschild solution could not be considered as accurate enough due to the noticeable contribution to the metric from the Earth's rotation and oblateness as well as the existence of tidal forces from the Moon, Sun, and other planets. It was also recognized [5] that the well-known procedure of construction of the Fermi normal coordinates [11] can not be applied due to the ambiguity in choosing the background space-time manifold and deviation of the Earth's center-of-mass world line from geodesic motion. Lack of a rational approach to the problem led to the fact that a simple (Euclidean-like) spatial translation of the origin of the barycentric reference frame to the Earth's geocenter was used in order to construct a geocentric reference frame at the post-Newtonain approximation. However, though such a procedure is allowed in GRT because of the coordinate freedom, the Euclidean translation to the geocenter produces large relativistic effects having pure coordinate origin and, hence, unobservable. In addition, in such a frame the geometric shape of the Earth moving along the elliptic orbit undergoes the Lorentz and gravitational contractions which must be compensated by spurious internal stresses in order to prevent the appearance of unphysical deformations. Similar problems were also met in developing general relativistic celestial mechanics of binary pulsars [12].

1.2 Motivations in the New Millennium

Recent technological developments make it necessary to extend the domain of applicability of the relativistic theory of reference frames outside the boundaries of the solar system. New generation of astrometric satellites which include FAME [13], SIM [14], and GAIA (recently adopted as a cornerstone mission of ESA [15]) requires an absolutely new approach for unambiguous interpretation of astrometric data obtained from the on-board optical interferometers. FAME's resolution is about 100 microarcseconds for stars having 10-th stellar magnitude. The resolution of GAIA for the same kind of stars is expected to be already 100 times better. At this level of accuracy the problem of propagation of light rays must be based on the recently developed post-Minkowskian "Lorentz-covariant" approximation [16] that allows us to integrate the equations of light propagation without artificial assumptions about the motion of light-ray-deflecting bodies. Besides, the treatment of the parallax, aberration, and proper motion of celestial objects becomes much more involved requiring better theoretical definitions of reference frames on a curved space-time manifold.

Additional motivation for improving relativistic theory of reference frames is related to the problem of calculation of the incoming signals of the space interferometric gravitational wave detectors like LISA [17] which will consist of three satellites flying in space separated by distances of order 1.5×10^6 km. Detection of gravitational waves can be done only under the condition that all coordinate-dependent phenomena are completely understood and subtracted from the signal. On the other hand, LISA will fly in the near-zone of the Sun which is considered as a source of low-frequency gravitational waves produced by oscillations of its interior (known as g-modes) [18]. Motion of the satellites carrying lasers and mirrors as well as propagation of light rays along the baseline of LISA in the field of such g-modes must be carefully studied in order to provide a correct interpretation of observations.

Many other relativistic experiments for testing GRT and alternative theories of gravity also demand advanced theory of relativistic reference frames (see, for example, [19, 20, 21]).

1.3 Historical Remarks

Historically it was de Sitter [22, 23] who worked out a relativistic approach to build reference frames in GRT. He succeeded in the derivation of relativistic equations of motion of the solar system bodies and discovered the main post-Newtonian effects including gravitomagnetic perihelion precession of a planetary orbit due to the angular momentum of the Sun as well as the geodetic precession ("de Sitter-Fokker effect") that has been verified with the precision of about 1% [24, 25]. These effects are essential in the present-day definition of dynamical or kinematical rotation of a reference frame [6]. Later on Lense and Thirring gave a more general treatment of the gravitomagnetic dragging of a satellite orbiting around a massive rotating body [26] but hardly believed that the effect can be measured in practice.

It was Ginzburg [27] who first realized that the Lense-Thirring effect can indeed be measured using artificial satellites and considered the problem of separation of coordinate and physical effects. But only recently the experimental verification of the Lense-Thirring effect has come about [19].

Ginzburg's paper and the success of the soviet space program motivated Brumberg[28] to develop a post-Newtonian Hill-Brown theory of motion of the Moon in the solar barycentric coordinate system. Baierlein [29] extended Brumberg's approach accounting for the eccentricity of Earth's orbit.

A different original approach to the problem of motion of the Moon and the construction of reference frames in GRT was suggested by Mashhoon and Theiss in a series of papers (see, for example, [30] and references therein). Instead of making use of the post-Newtonian approximations (PNA) they developed a post-Schwarzschild treatment of gravitomagnetic effects in the three-body problem. It allowed them to discover that the validity of PNA is restricted in time and the geodetic and gravitomagnetic precessions are parts of the more general phenomena involving a long-term relativistic nutation ("Mashhoon-Theiss effect"). The same authors also introduced a precise definition of the local geocentric frame with the Earth considered as a massive monopole particle [30, 31].

A more general approach to the problem of construction of reference frames in GRT was initiated in [5] (see also [32]), where the matched asymptotic expansion technique and a decomposition of gravitational fields of bodies in the (Newtonian) multipoles were employed for this purpose. Our approach has been further used for the development of the extended Brumberg-Kopeikin (BK) formalism for building relativistic astronomical frames. This formalism relies upon Einstein's equations, which are solved for the construction of local geocentric and global barycentric reference systems, and matching technique, which is used for setting up relativistic transformations between these two systems [6, 33].

T. Damour, M. Soffel and C. Xu (DSX) have extended the Brumberg-Kopeikin theory by applying the post-Newtonian definitions of the ("Blanchet-Damour") gravitational multipole moments [34]. Elements of the BK formalism were introduced in resolutions of the GA of the IAU in 1991 [35]. The complete BK-DSX theory is presently accepted by the XXIVth GA of the IAU-2000 as a basic framework for setting up relativistic time scales and astronomical reference frames in the solar system.

2 The IAU-1991 Reference Systems Framework

Official transition of the astronomical community from Newtonian positions to relativistic concepts began in 1991 when a few recommendations (resolution A4) were adopted by the GA of the IAU.

In the first recommendation, the metric tensor in space-time coordinates $(ct, \mathbf{x})$ centered at the barycenter of an ensemble of masses is recommended to be written in the form

$$g_{00} = -1 + \frac{2U(t,\mathbf{x})}{c^2} + O(c^{-4}) \ , \tag{1}$$

$$g_{0i} = O(c^{-3}) \ , \tag{2}$$

$$g_{ij} = \delta_{ij} \left[1 + \frac{2U(t,\mathbf{x})}{c^2}\right] + O(c^{-4}) \ , \tag{3}$$

where c is the speed of light in vacuum, U is the sum of the gravitational potentials of the ensemble of masses, and of a tidal potential generated by bodies external to the ensemble, the latter vanishing at the barycenter. This recommendation recognizes that space-time cannot be described by a single coordinate system. The recommended form of the metric tensor can be used not only to describe the barycentric celestial reference system (BCRS) of the whole solar system, but also to define the geocentric celestial reference system (GCRS) centered on the center of mass of the Earth with a suitable function $\hat{U}$, now depending upon geocentric coordinates. In analogy to the GCRS a corresponding celestial reference system can be constructed for any other body of the Solar system.

In the second recommendation, the origin and orientation of the spatial coordinate grids for the solar system (BCRS) and for the Earth (GCRS) are defined. The third recommendation defines TCB (Barycentric Coordinate Time) and TCG (Geocentric Coordinate Time) – the time coordinates of the BCRS and

GCRS, respectively. The relationship between TCB and TCG is given by a full 4-dimensional transformation

$$TCB - TCG = c^{-2} \left[\int_{t_0}^{t} \left(\frac{v_E^2}{2} + \overline{U}(t, \mathbf{x}_E) \right) dt + v_E^i r_E^i \right] + O(c^{-4}), \tag{4}$$

where $(x_E^i) \equiv \mathbf{x}_E(t)$ and (v_E^i) are the barycentric coordinate position and velocity of the geocenter, $r_E^i = x^i - x_E^i$ with $\mathbf{x}$ being the barycentric position of the observer, and $\overline{U}(t, \mathbf{x}_E)$ is the Newtonian potential of all solar system bodies evaluated at the geocenter apart from that of the Earth.

In August 2000, a new resolution B1 on reference frames and time scales in the solar system was adopted by the XXIVth General Assembly of the IAU [3]. The resolution is based on the first post-Newtonian approximation of GRT and completely abandons the Newtonian point of view on space and time [b].

3 The IAU-2000 Reference Systems Framework

3.1 Conventions for the Barycentric Celestial Reference System

BCRS is defined mathematically in terms of a metric tensor which reads

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + O(c^{-5}) \ , \tag{5}$$

$$g_{0i} = -\frac{4}{c^3} w_i + O(c^{-5}) \ , \tag{6}$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2}{c^2} w \right) + O(c^{-4}) \ . \tag{7}$$

Here, the post-Newtonian gravitational potential w generalizes the usual Newtonian potential U and w^i is the vector potential related with gravitomagnetic effects.

This form of the barycentric metric tensor implies that the barycentric spatial coordinates x^i satisfy the harmonic gauge condition. The main arguments in favor of the harmonic gauge are: (1) tremendous work on GRT has been done with the harmonic gauge that was found to be a useful and simplifying gauge for all kinds of applications, and (2) in contrast to the standard post-Newtonian (PN) gauge (see, for example, [38]) the harmonic gauge can be defined to higher PN-orders, and in fact for the exact Einstein theory of gravity.

Assuming space-time to be asymptotically flat (no gravitational fields exist at infinity) in the standard harmonic gauge the post-Newtonian field equations of

[b] How the resoltuion B1 was prepared can be traced in the materials of the corresponding working groups of the IAU [36, 37]

GRT are solved by

$$w(t,\mathbf{x}) = G\int d^3x'\,\frac{\sigma(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \frac{G}{2c^2}\frac{\partial^2}{\partial t^2}\int d^3x'\,\sigma(t,\mathbf{x}')|\mathbf{x}-\mathbf{x}'|\,, \tag{8}$$

$$w^i(t,\mathbf{x}) = G\int d^3x'\,\frac{\sigma^i(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,, \tag{9}$$

where $\sigma(t,\mathbf{x}) = c^{-2}(T^{00}+T^{ss})$, $\sigma^i(t,\mathbf{x}) = c^{-1}T^{0i}$, and $T^{\mu\nu} = T^{\mu\nu}(t,x^i)$ are the components of the stress-energy tensor in the barycentric coordinate system, $T^{ss} = T^{11}+T^{22}+T^{33}$.

3.2 Conventions for the Geocentric Celestial Reference System

GCRS is defined in terms of the geocentric metric tensor

$$G_{00} = -1 + \frac{2W}{c^2} - \frac{2W^2}{c^4} + O(c^{-5}), \tag{10}$$

$$G_{0a} = -\frac{4}{c^3}W_a, \tag{11}$$

$$G_{ab} = \delta_{ab}\left(1+\frac{2}{c^2}W\right) + O(c^{-4})\,. \tag{12}$$

Here $W = W(T,\mathbf{X})$ is the post-Newtonian gravitational potential in the geocentric system and $W^a(T,\mathbf{X})$ is the corresponding vector potential. These geocentric potentials should be split into two parts: potentials W_E and W_E^a arising from the gravitational action of the Earth and external parts $W_{\rm ext}$ and $W_{\rm ext}^a$ due to tidal and kinematic effects. The external parts are assumed to vanish at the geocenter and admit an expansion into positive powers of $\mathbf{X}$. Explicitly,

$$W(T,\mathbf{X}) = W_E(T,\mathbf{X}) + W_{\rm kin}(T,\mathbf{X}) + W_{\rm tidal}(T,\mathbf{X})\,, \tag{13}$$

$$W^a(T,\mathbf{X}) = W_E^a(T,\mathbf{X}) + W_{\rm kin}^a(T,\mathbf{X}) + W_{\rm tidal}^a(T,\mathbf{X})\,. \tag{14}$$

The Earth's potentials W_E and W_E^a are defined in the same way as w_E and w_E^i but with quantities calculated in the GCRS. $W_{\rm kin}$ and $W_{\rm kin}^a$ are kinematic contributions that are linear in X^a

$$W_{\rm kin} = Q_b X^b, \qquad W_{\rm kin}^a = \frac{1}{4}\,c^2\varepsilon_{abc}(\Omega^b - \Omega^b_{\rm prec})\,X^c\,, \tag{15}$$

where Q_b characterizes the deviation of the actual worldline of the origin of the GCRS from geodesic motion in the external gravitational field (for more details see [5, 6, 34])

$$Q_b = \partial_b w^{\rm ext}(\mathbf{x}_E) - a_E^b + O(c^{-2})\,. \tag{16}$$

Here $a^i_E = dv^i_E/dt$ is the barycentric acceleration of the origin of the GCRS (geocenter). The function $\Omega^a_{\rm prec}$ describes the relativistic precession of dynamically nonrotating spatial axes with respect to remote objects:

$$\Omega^i_{\rm prec} = \frac{1}{c^2}\,\varepsilon_{ijk}\,\left(-\frac{3}{2}\,v^j_E\,\partial_k w^{\rm ext}(\mathbf{x}_E) + 2\,\partial_k w^j_{\rm ext}(\mathbf{x}_E) - \frac{1}{2}\,v^j_E\,Q^k\right). \qquad (17)$$

The three terms on the right-hand side of this equation represent the geodetic, Lense-Thirring, and Thomas precessions, respectively. One can prove that $\Omega^a_{\rm iner}$ is dominated by geodetic precession amounting to $\sim 2''$ per century plus short-periodic terms usually called geodetic nutation. One sees that for $\Omega^a = \Omega^a_{\rm prec}$ the vector potential $W^a_{\rm kin}$ vanishes. This implies that dynamical equations of motion of a test body, e.g., a satellite orbiting around the Earth, do not contain the Coriolis and centrifugal terms, i.e., the local geocentric spatial coordinates X^a are *dynamically non-rotating.* For practical reasons, however, the use of *kinematically non-rotating* geocentric coordinates defined by $\Omega^a = 0$ is recommended.

Potentials $W^{\rm tidal}$ and $W^{\rm tidal}_a$ are generalizations of the Newtonian tidal potential. We also note that the local gravitational potentials W_E and W^a_E of the Earth are related to the barycentric gravitational potentials w_E and w^i_E by the relativistic transformations [3].

3.3 Transformations between the reference systems

The coordinate transformations between the BCRS and GCRS are written as

$$\begin{aligned} T = t &- \frac{1}{c^2}\left[A(t) + v^i_E\,r^i_E\right] \qquad (18)\\ &+\frac{1}{c^4}\left[B(t) + B^i(t)\,r^i_E + B^{ij}(t)\,r^i_E\,r^j_E + C(t,\mathbf{x})\right] + O(c^{-5}), \end{aligned}$$

$$X^i = r^i_E + \frac{1}{c^2}\left[\frac{1}{2}v^i_E v^j_E r^j_E + w_{\rm ext}(\mathbf{x}_E) r^i_E + r^i_E a^j_E r^j_E - \frac{1}{2}a^i_E r^2_E\right] + O(c^{-4}), \qquad (19)$$

where functions $A(t), B(t), B^i(t), B^{ij}(t), C(t,\mathbf{x})$ can be found in [3]. Let us also remark that the harmonic gauge condition does not fix the function $C(t,\mathbf{x})$ uniquely. However, we prefer to fix it in the time transformation for practical reasons.

4 Matching the IAU-2000 Framework with Cosmological Reference Frame

The rapidly growing accuracy of astronomical measurements makes it necessary to take into account some important cosmological effects for an adequate interpretation of optical and radio observations of cosmological lenses, anisotropy in cosmic microwave background radiation, etc. For this reason the matching of the IAU-2000 framework for reference systems in the solar system with the cosmological reference frame becomes vitally important. In this section we outline the main ideas in this matching for the spatially flat Friedmann-Robertson-Walker (FRW) cosmological model.

First, we consider the perturbation $h_{\alpha\beta}$ of the gravitational field as

$$g_{\alpha\beta} = a^2(\tau)(\eta_{\alpha\beta} + h_{\alpha\beta}) \;, \qquad \gamma^{\alpha\beta} = -h^{\alpha\beta} + \frac{1}{2}\eta^{\alpha\beta} h \;, \qquad h = \eta^{\alpha\beta} h_{\alpha\beta} \;, \tag{20}$$

and impose the quasi-harmonic cosmological gauge conditions [c]

$$\partial_\beta \gamma^{\alpha\beta} = -2H\left(\gamma^{\alpha 0} - \frac{1}{2}\eta^{\alpha 0}\gamma\right) \;, \qquad H(\tau) = \frac{\dot{a}}{a} \;, \tag{21}$$

which eliminate almost all first order derivatives of the metric perturbation. Then, the linearized Einstein equations read

$$\begin{aligned} \Box\gamma^{\alpha\beta} - 2H\,\partial_\tau \gamma^{\alpha\beta} + 2(2\dot{H} + H^2)h^{\alpha\beta} + 4(\dot{H} - 2H^2)\eta^{0(\alpha}h^{\beta)0} \\ - 2(\dot{H} - H^2)\eta^{\alpha\beta}h^{00} = 16\pi a^4 \delta T^{\alpha\beta} \;, \end{aligned} \tag{22}$$

where a dot over "the Hubble parameter" H denotes the time derivative and $\delta T^{\alpha\beta}$ is the tensor of energy-momentum of perturbing source ("solar system").

For the particular case of matter-dominated background FRW cosmological model the reduced linearized Einstein equations (22) read

$$\Box_g w - 8H^2 w = -4\pi a^4(\tau)\;\delta(T^{00} + T^{kk}) \;, \tag{23}$$

$$\Box_g(\gamma + \frac{1}{2}w) = -4\pi a^4(\tau)\;\delta(T^{00} - \frac{7}{2}T) \;, \tag{24}$$

$$\Box_g w^i - 5H^2 w^i = -4\pi a^4(\tau)\;\delta T^{0i} \;, \tag{25}$$

$$\Box_g w^{ij} = -4\pi a^4(\tau)\;\delta T^{<ij>} \;, \tag{26}$$

where $w = -\left(\gamma^{00} + \gamma^{kk}\right)/4$, $w^i = -\gamma^{0i}/4$, $w^{ij} = -\gamma^{<ij>}/4$ [d], and $H(\tau) = 2/\tau$. For the differential operator $\Box_g$ one has

$$\Box_g \equiv \Delta - \frac{\partial^2}{\partial\tau^2} - \frac{4}{\tau}\frac{\partial}{\partial\tau} \;, \tag{27}$$

so that the solution of the inhomogeneous equation $\Box_g F(\tau, \mathbf{x}) = -4\pi a^4(\tau)\delta T(\tau, \mathbf{x})$, can be found making use of the replacement [41]

$$F(\tau, \mathbf{x}) = \frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\frac{\Psi(\tau, \mathbf{x})}{\tau}\right) \;. \tag{28}$$

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[c] These conditions were also independently discovered in the paper [39] for the case of de Sitter space-time.

[d] The square brackets around spatial indices denote symmetric and trace-free tensor [40]

transforming the equation in question into

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\frac{\Box_\eta\Psi(\tau,\mathbf{x})}{\tau}\right) = -4\pi a^4(\tau)\delta T(\tau,\mathbf{x}) \ , \qquad \Box_\eta = \Delta - \partial_\tau^2 \ , \tag{29}$$

which has a simple particular solution

$$\Psi(\tau,\mathbf{x}) = \tau\int\frac{d^3\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|}\int_1^{\tau'} sa^4(s)\delta T(s,\mathbf{x}')ds - \int d^3\mathbf{x}'\int_1^{\tau'} sa^4(s)\delta T(s,\mathbf{x}')ds \ , \tag{30}$$

where $\tau' = \tau - |\mathbf{x}-\mathbf{x}'|$ is a retarded time in a flat space-time.

Matching this solution with that defined in the BCRS of the IAU-2000 framework as flat-space retarded potentials is achieved after the replacement $\tau = 1+H_R t$ and expanding all quantities depending on τ in the neighbourhood of the present epoch $\tau = 1$ along with making use of the tensor transformation law for metric tensor. Results of the matching will be described elsewhere.

5 Relativistic Microarcsecond Astrometry

The IAU-2000 framework for reference frames requires new advanced theory of astrometric data analysis. The key issue in this theory is solution of the problem of propagation of light rays with account for as many relativistic effects as required for making unambigious interpretation of the data. Previous approaches for calculating light ray propagation in the framework of relativistic astrometry were based on making use either metric tensor of exact solutions of GRT (Schwarzschild, Kerr, etc.), or metric tensor of the post-Newtonian approximation (PNA), or metric tensor of the plane weak quadrupolar gravitational wave. All these approaches have difficulties and/or inconsistencies in describing light propagation at the microarcsecond threshold. The question arises how to avoid these difficulties and what is the proper formalism to deal with the problem. Treatment of this problem has been given recently in the framework of the "Lorentz-covariant" theory of light propagation [42–43]. and the formalism developed has an unrestricted ability to make calculations at any desired order of approximation with respect to the small parameter v_a/c as well as many other advantages making it a powerful tool for theoretical predicitons of various relativistic effects being detectable by modern astrometric technique (see, for example, [44–46]).

6 Acknowledgments

I appreciate the help of Bahram Mashhoon (UMC) in giving valuable references and for taking part in numerous discussions. Interesting conversations with Juan Ramirez (Universidad Complutense de Madrid) and Mikhail Sazhin (Sternberg

Astronomical Institute) clarified some important issues for establishing the proper framework for the cosmological reference frames.

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