A free boundary gas dynamic model as a two-body field theory problem

Michael Thomas Heitzman

Professor Carmen Chicone, Dissertation Supervisor

ABSTRACT

Motivated by the two-body problem in the classical field theories of electrodynamics and gravitation, in which finite propagation speeds lead to radiation reaction and runaway solutions, we develop a free boundary problem in gas dynamics to explore the motion of sources in a medium whose dynamics are governed by hyperbolic, wave-like equations arising from physical conservation laws. In our linearized acoustic model, the fields can be eliminated to yield functional differential equations for the motion of the sources—delay equations with an infinite dimensional state space. Expansion and truncation gives rise to runaway solutions, just as in the classical field theories. We illustrate a scheme for eliminating runaway solutions by reducing to a finite dimensional, globally attracting, invariant manifold on which effective equations of motion for the sources can be obtained. The effective equations of motion approximate the asymptotic behavior of solutions in the full space as they approach the manifold. We also treat the full nonlinear free boundary problem and show that unique classical solutions exist locally, for initial fields close enough to their constant steady state.