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Effect of electrical bias on spin transport across a magnetic domain wall

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We present a theory of the current-voltage characteristics of a magnetic domain wall between two highly spin-polarized materials, which takes into account the effect of the electrical bias on the spin-flip probability of an electron crossing the wall. We show that increasing the voltage reduces the spin-flip rate, and is therefore equivalent to reducing the width of the domain wall. As an application, we show that this effect widens the temperature window in which the operation of a unipolar spin diode is nearly ideal. © *2004 American Institute of Physics*.
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INTRODUCTION

The discovery of the giant magnetoresistance effect[1] and the rapid growth in the number of its industrial applications have raised the hope that a similar breakthrough, perhaps of even broader consequence, may result from the combination of established semiconductor technologies with a precise control of the spin degree of freedom. As part of a growing effort in what has been called "semiconductor spintronics"[2,3] several spin-based devices have been designed and discussed during the past few years: we mention, for example, the Datta-Das[4] spin transistors, the bipolar spin diodes of Žutić *et al.*[5] and transistors of Flatté *et al.*,[6] and, lastly, the unipolar spin diode and transistor of Flatté and Vignale.[7,8] All these devices, while still largely theoretical, are actively pursued in the lab, since they might eventually prove useful for computer operation such as nonvolatile memory and reprogrammable logic.

At the heart of many of the above-mentioned devices is a magnetic junction (or magnetic domain wall), i.e., a region of inhomogenous magnetization connecting two regions of different homogeneous magnetizations. In this paper we extend the conventional theory of spin transport across such a junction to include the effect of the electric field in the inhomogeneous region between two highly spin-polarized materials. Our work is motivated, in part, by recent insights on the role of electric field on the efficiency of spin injection across a magnetic interface[9] and, more specifically, by the recent discussion of the unipolar spin diode in Refs. 7 and 8. A simple model for this device is two ferromagnetic conducting slabs, denoted F_1 and F_2, with oppositely aligned magnetizations, connected by a domain wall of width d. The direction of the exchange field $\vec{B}(x)$ within the domain wall rotates linearly through an angle π in the z-x plane, i.e.,

$$\vec{B}(x) = B_0[\cos\,\theta(x)\hat{x} + \sin\,\theta(x)\hat{z}], \tag{1}$$

where $-\pi/2 < \theta < \pi/2$ and $0 < x < d$. We distinguish between the component of the current due to "up-spin" electrons $J_\uparrow$ and that due to "down-spin" electrons $J_\downarrow$, and accordingly define the charge current $J_q = J_\uparrow + J_\downarrow$ and spin current $J_s = J_\uparrow - J_\downarrow$, where "up" points in the positive x direction. If the domain wall is sufficiently sharp (i.e., more precisely, if d is much smaller than $\hbar/\sqrt{2m^*\Delta}$, where m^* is the effective mass of the electrons and Δ is the magnitude of the exchange splitting between the up- and down-spin bands) then the spin of an electron crossing the junction is essentially conserved. Under these conditions a unipolar device (where the charge carriers on both sides have the same polarity) is analogous to a classical p-n diode, with up and down spins corresponding to electrons and holes, and the oppositely aligned magnetic regions playing the role of the p-type and n-type materials.[7] A bipolar device (where the charge carriers on different sides have opposite polarity) can also be analyzed in this context under conditions of forward bias. A key assumption, particularly in the analysis of the unipolar spin diode, is that the applied bias voltage drops almost entirely across the junction, whose resistance is therefore supposed to be much higher than that of the rest of the structure. Indeed, recent experimental work has confirmed that highly resistive and well localized domain walls can be realized at nanoconstrictions in GaAs.[10,11] Coherent spin transport across highly resistive vertical tunnel junctions[12–14] may also be analyzed based on models such as we present here.

An important deviation from ideality, namely, the possible occurrence of spin-flip processes in the junction, was examined in detail in Ref. 8. Such spin-flip processes are responsible for the appearance of a lower critical temperature below which minority-spin injection is no longer operative and direct tunneling between the majority-spin bands perverts the operation of the diode. However, the analysis of Ref. 8 did not account for the electric field that is present in the domain wall region when an external bias is applied. From the high-resistivity assumption we know that this field is significant, and from the work of Yu and Flatté[9] we know that even a modest electric field, in a semiconductor, can have a large and favorable effect on the efficiency of minority-spin injection. These considerations motivate us to refine the analysis of Ref. 8 to include the effect of the electric field on the spin-flip rate. The outcome of the improved analysis is both interesting and reassuring: on one hand, it shows that the electric field greatly favors minority-spin injection, thus widening the temperature window in which the spin-diode exhibits an "ideal" behavior; on the other hand it confirms the essential validity of the original treatment of Ref. 7.

THEORY

We now review some of the essential aspects of the analysis from which the results above are obtained. In pursuing the natural analogy between *p-n* diodes and unipolar spin diodes, a number of assumptions are required, which closely correspond to those introduced by Shockley for an ideal diode:[15] (1) within the diode, the voltage drop occurs mainly across the domain wall junction, (2) the Boltzmann approximation for transport is applicable, (3) the minority carrier density is small compared to that of majority carriers, and (4) there is no "recombination current" in the domain wall.

With these assumptions in mind, we can begin a reconstruction of the *I-V* characteristics by considering the action of a single electron incident on the domain wall. There are four possibilities [from the four possible combinations of reflection (r) or transmission (t), with spin flipped from its original orientation (sf) or not flipped (nf)], the probabilities of which will be denoted: $r_{\rm sf}, r_{\rm nf}, t_{\rm sf}, t_{\rm nf}$. Throughout our analysis, we will consider this set of coefficients to be the controlling quantities in the behavior of the spin diode, as they form the basis for all subsequent calculations. When a voltage V is applied to the diode, we can think of the regions F_1 and F_2 as two majority spin reservoirs of opposite alignment at quasichemical potentials $\mu_1=0$ and $\mu_2=eV$, respectively, which, it has been observed, are not appreciably altered by the presence of current. Then the majority- and minority-spin currents in these regions, due to electrons with energies in the range $(E, E+dE)$, are described component wise by (see Ref. 8),

$$
\begin{aligned}
j_{1\downarrow}(E) &= -[1-r_{\rm nf}(E)]f_{1\downarrow}(E) + t_{\rm sf}(E)f_{2\uparrow}(E),\\
j_{1\uparrow}(E) &= r_{\rm sf}(E)f_{1\downarrow}(E) + t_{\rm nf}(E)f_{2\uparrow}(E),\\
j_{2\uparrow}(E) &= [1-r_{\rm nf}(E)]f_{2\uparrow}(E) - t_{\rm sf}(E)f_{1\downarrow}(E),\\
j_{2\downarrow}(E) &= -r_{\rm sf}(E)f_{2\uparrow}(E) - t_{\rm nf}(E)f_{1\downarrow}(E),
\end{aligned}
\tag{2}
$$

where the functions $f_{n\sigma}(E)$ are the equilibrium distributions of the carriers of σ-spin orientation in region F_n, with $n=1$ or 2. To make use of these formulae, we observe that Boltzmann statistics implies $f_{1\downarrow}=f_{2\uparrow}e^{-eV/kT}$, and that, as will later be demonstrated in the general calculation, the coefficient $r_{\rm sf}$ is very small at all energies. We then integrate over all energies to obtain the total current in each region, and impose continuity conditions at $x=0$ and $x=d$ to get

$$
\frac{J_s(-d/2)}{J_s(d/2)} = \frac{\overline{t_-} + \overline{t_+}e^{-eV/kT}}{\overline{t_+} + \overline{t_-}e^{-eV/kT}}
\tag{3}
$$

where $\overline{t_\pm}=\overline{t_{\rm nf}}\pm\overline{t_{\rm sf}}$, and the two terms in the sum are population-averaged transmission coefficients given by

$$
\overline{t_{\rm nf(sf)}} = \frac{\int_0^\infty t_{\rm nf(sf)}(E)e^{-E/kT}dE}{\int_0^\infty e^{-E/kT}dE}.
\tag{4}
$$

Together with the standard drift-diffusion theory and other observations noted in Ref. 7, the continuity condition yields the following expressions for the charge current and spin currents near the domain wall as functions of voltage and temperature:

$$
\begin{aligned}
\frac{J_q}{J_0} &= \sinh\left(\frac{eV}{kT}\right)\left[1+\frac{\overline{t_{\rm sf}}}{t_{\rm nf}}\tanh^2\left(\frac{eV}{2kT}\right)\right],\\
\frac{J_s}{J_0} &= 2\sinh^2\left(\frac{eV}{2kT}\right)\left[1\pm\frac{\overline{t_{\rm sf}}}{t_{\rm nf}}\tanh\left(\frac{eV}{2kT}\right)\right],
\end{aligned}
\tag{5}
$$

where the upper sign holds in F_2, the lower sign in F_1, and $J_0 \equiv 2eDn_<^{(0)}/L_s$, D being the diffusion constant, $n_<^{(0)}$ the equilibrium value of the minority spin density, and L_s the spin diffusion length. Clearly the *I-V* characteristics of the diode depend critically on the value of the $\overline{t_{\rm nf}}/\overline{t_{\rm sf}}$, which will hereafter be referred to as the "key ratio."

In order to calculate the reflection/transmission probabilities, we must solve the Schrödinger equation for the electron wave function in the domain wall

$$
\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{\Delta}{2}\begin{pmatrix}\sin\,\theta(x) & \cos\,\theta(x)\\ \cos\,\theta(x) & -\sin\,\theta(x)\end{pmatrix} + V(x)\right]\begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow\end{pmatrix} = E\begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow\end{pmatrix},
\tag{6}
$$

where $V(x)=-e\mathcal{E}x$ is the term associated with the electric field $\mathcal{E}$ that is created by the potential applied across the domain wall. The presence of this term prevents us from finding a purely analytical solution, and a numerical solution is therefore computed. Imposing the appropriate matching conditions at the domain wall interfaces at $x=0$ and d, the transmission/reflection probabilities are obtained.

RESULTS

When bias produces an electric field such that $e\mathcal{E}d$ is of the same order as the spin splitting in this region, the values of these probabilities change according to whether the field accelerates or impedes the motion of incident electrons through the wall. To assist in observing these effects, we define the dimensionless parameters $\overline{\Delta}=\Delta/(\hbar^2/2md^2)$ which measures the relative size of the domain wall barrier, and $\epsilon = e\mathcal{E}d/(\hbar^2/2md^2)$, which measures the relative strength of the electric field. Values for $\overline{\Delta}$ in the range 0.1–0.5 will be considered to describe a thin domain wall, 1–5 an intermediate size one, and 10–50 a thick one. We note for the wall in Ref. 10 $\overline{\Delta}\sim 70$ which is within an order of magnitude of the intermediate size range. Figure 1(a) shows the four coefficients as a function of electron energy for $\overline{\Delta}=2.25$ and zero electric field. The essential trends can be easily discerned: at energies less than Δ, $r_{\rm nf}$ is approximately unity as expected,

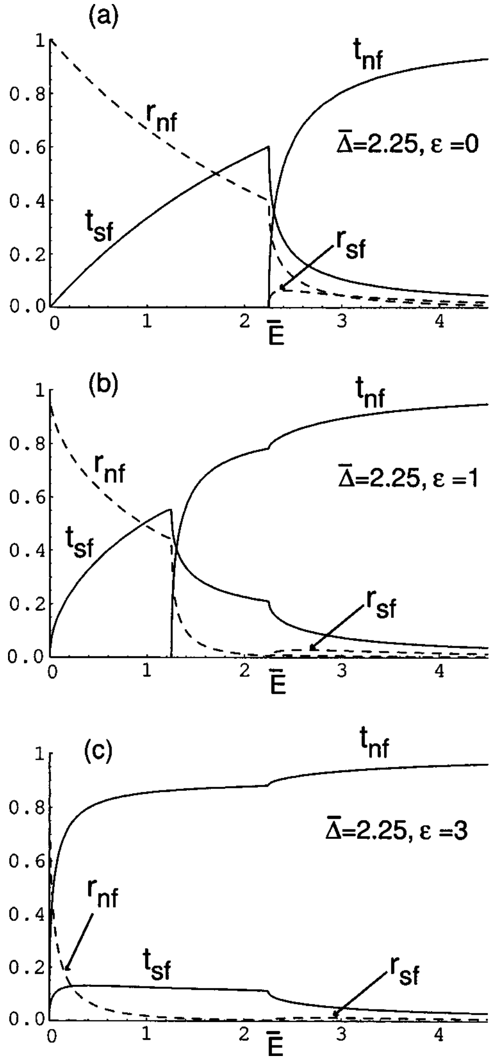

FIG. 1. The reflection and transmission coefficients for an intermediate size domain wall ($\bar{\Delta}=2.25$) vs incident electron energy [$\bar{E}\equiv E/(\hbar^2/2md^2)$] for three values of the electric field: (a) zero field, (b) an electric field interaction about half the size of the splitting ($\epsilon=1$), and (c) electric field exceeding the splitting ($\epsilon=3$). The labels of the various curves are shown in (a).

since the barrier dwarfs the energy of the incident electron. As the energy increases r_{nf} begins to drop and t_{sf} rises at the same rate, since it is now possible for the electron to cross the barrier if spin alignment is reversed. At the splitting energy threshold, the electron has sufficient energy to traverse the domain wall while maintaining spin orientation; t_{nf} increases rapidly while r_{nf} and t_{sf} plummet. We note finally that r_{sf} remains approximately zero uniformly over all energies, as previously announced.

The introduction of an electric field due to current flow has the effect of splitting the relevant energy thresholds [Figs. 1(b) and 1(c)], and the size of the domain wall will determine whether this shift is consequential. The minimum energy required for transmission without spin flip is reduced to $\bar{\Delta}-\epsilon$. The trends follow in a very similar fashion, with the transmission/reflection coefficients in the energy range $(0,\bar{\Delta}-\varepsilon)$ reaching approximately the same values as their zero-field counterparts in the range $(0,\bar{\Delta})$, but doing so more rapidly in the narrower energy interval, while the coefficients for energy larger than $\bar{\Delta}-\epsilon$ tend to move more gradually toward the same limits ($t_{nf}\rightarrow 1$ and $r_{nf}\rightarrow 0$ as the electron energy E grows). Of course electrons of smaller energy can now be transmitted through the reduced barrier, thus t_{nf} jumps at this earlier energy threshold, and again at the original barrier energy $\bar{\Delta}$ just slightly. As $\bar{\epsilon}$ exceeds $\bar{\Delta}$, the transmission probability is significant at almost all nonzero energies; t_{nf} continues to increase uniformly while all other coefficients are suppressed. This will occur almost immediately for small values of $\bar{\Delta}$. For large values of $\bar{\Delta}$, however, one would have to go to $\epsilon\gg\bar{\Delta}$ in order to have a substantial level of minority spin injection: but at this point the resistance of the junction would be too small to support such a large electric field. Hence the influence of the electric field is profound for thin domain walls and essentially negligible for thick ones.

These observations account for the main aspects of behavior of the key ratio as a function of electric field (see Fig. 2). Physically, values of the key ratio greater than unity signify the dominance of minority-spin injection. Again, for $\bar{\Delta}\lesssim 0.5$, the key ratio is tremendously amplified by the electric field, since in this limit $\bar{t}_{sf}$ goes to zero, and minority-spin injection is guaranteed for almost any temperature low enough not to disturb spin polarization in the conductors, but high enough to produce an ample supply of carriers above the exchange barrier (this range is typically given by $0.1\Delta/k<T<0.9\Delta/k$). The key ratio depends linearly on temperature for any value of $\bar{\Delta}$ and ϵ, thus for larger, intermediate barrier sizes, there will be a cutoff temperature below which majority spin transmission prevails since most of the system's electrons lack sufficient thermal energy to trans-

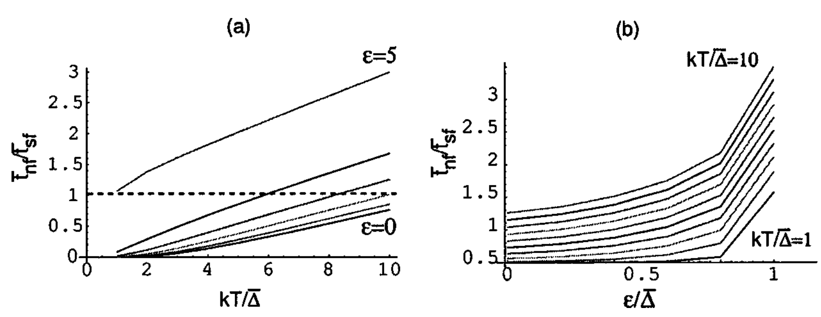

FIG. 2. (a) Key ratio vs $kT/\bar{\Delta}$ for several values of the electric field parameter $\epsilon=0,1,\ldots,5$ from bottom up at $\bar{\Delta}=5$. The dashed line represents the threshold for minority-spin injection. (b) Key ratio vs dimensionless electric field $\epsilon/\bar{\Delta}$ for $kT/\bar{\Delta}=1-10$ from bottom up.

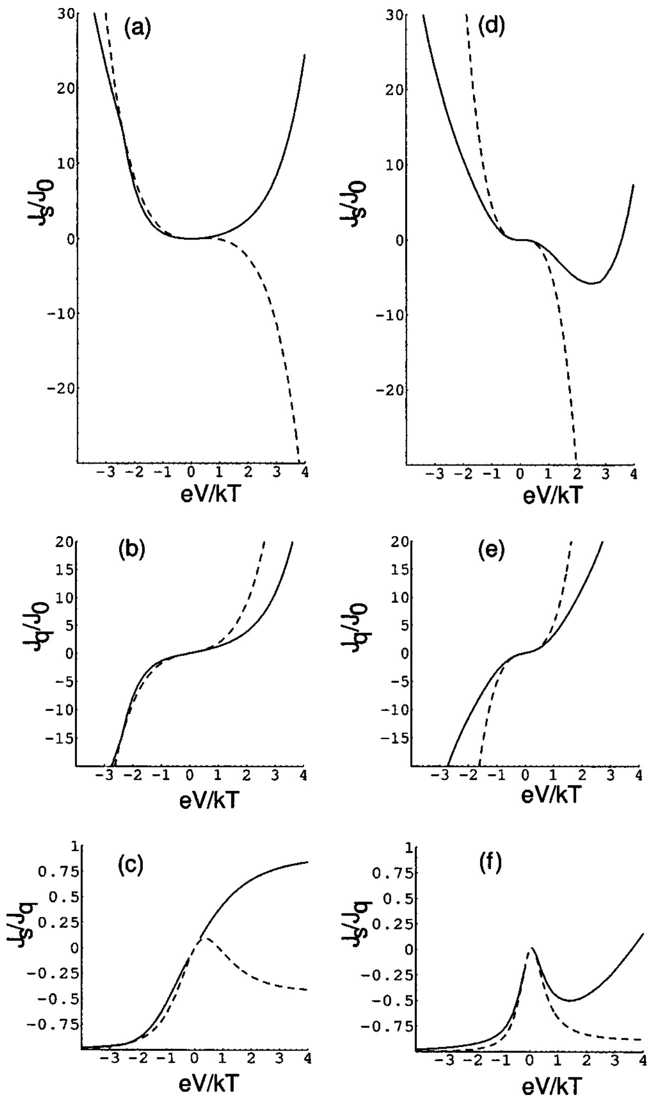

FIG. 3. Plots of the spin current J_s, the charge current J_q, and their ratio J_s/J_q vs bias voltage for $\bar{\Delta}=2.5$ and two different values of the temperature: (a)–(c) $kT/\bar{\Delta}=0.5$ and (d)–(f) $kT/\bar{\Delta}=0.2$. The dashed lines show the results obtained by treating the key ratio $\overline{t_{\mathrm{nf}}}/\overline{t_{\mathrm{sf}}}$ as a constant equal to its zero-field value, while the solid line is the result obtained with the voltage-dependent key ratio.

mit through the wall without spin flip. Our previous observations of effective barrier reduction due to forward bias imply that this cutoff temperature will shift downward generally. Indeed, Fig. 2 depicts the behavior of the key ratio over a feasible temperature range for $\bar{\Delta}=5$. The zero-field curve falls wholly under the minority-spin injection threshold $\overline{t_{\mathrm{nf}}}/\overline{t_{\mathrm{sf}}}=1$, for this barrier size, while those for finite values of ϵ exceed it at increasingly lower temperatures: for $\epsilon=5$ the key ratio lies completely above unity. The decaying exponential under the integral in the expression for $\overline{t_{\mathrm{nf}}}$ suggests that, for a given value of $\bar{\Delta}$, $\overline{t_{\mathrm{nf}}}/\overline{t_{\mathrm{sf}}}$ will rise most rapidly when the spin splitting and the bias voltage have comparable magnitude. This behavior is clearly seen in the exponential increase of the key ratio vs electric field in Fig. 2(b), otherwise the ratio is approximately linear with voltage for any barrier size. Thus we expect that the temperature window of device operation, bounded by the requirements for sufficient carrier energy and maintenance of ferromagnetism, will expand downward for intermediate barrier sizes, or equivalently, that larger barriers can be accommodated for a fixed temperature while still preserving minority-spin injection.

We are now ready to discuss the behavior of the *I-V* characteristics, calculated according to Eq. (5). Clearly when the key ratio is very large, say >5, the contribution of the second term in the square brackets of Eq. (5) is completely negligible. In this case the spin current J_s reduces to a strictly even function of voltage, the ratio of the spin to the charge current J_s/J_q (which serves as a measure of spin polarization) is odd-in-voltage, and they are both nonlinear. Fig. 3(a)–3(c) shows an example of the behavior of the spin current, the charge current, and their ratio, within the first region F_1. The dashed lines show the results obtained from Eq. (5) when the value of key ratio is set to the zero-field value. The domain is again of intermediate thickness ($\bar{\Delta}=2.5$), but at temperature $T=0.5\Delta/k$ the zero-field value is clearly quite small and J_s has a large odd-in-voltage component. When the voltage dependence of the key ratio is included, its rapidly increasing behavior, previously noted, leads to a quite different curve, which is shown by the solid line. This is clearly much closer to the "ideal" behavior of the spin current, described in Ref. 7.

In conclusion we have shown that the electric field can assist in maintaining the spin polarization of carriers traversing a magnetic domain wall, and consequently the ideal *I-V* characteristics of the spin diode should be more easily attainable than expected.

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