“I CAN’T BELIEVE I JUST SAID THAT”: DISCURSIVE FILTERS AND BEGINNING MATHEMATICS TEACHERS’ ENACTMENT OF AMBITIOUS PRACTICE

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ABSTRACT

After being introduced to “ambitious” teaching methods during teacher education, only some beginning teachers attempt to enact those methods, as others gravitate toward the conventional practices that overwhelm K-12 settings. To learn more about *why,* I conducted a multiple-case study of four beginning secondary mathematics teachers who graduated from three cohorts of one teacher education program and went on to teach in two different schools. Through longitudinal interviews and classroom observations, I examined the teachers’ enactment of ambitious practice through the lenses of their discursive teaching identities (critical pedagogical discourses) and perceptions of messages about teaching circulating within their institutional settings (contextual discourses). Findings revealed that the extent to which teachers’ critical pedagogical discourses acted as resources for filtering out contextual pressures to teach in conventional ways helped to explain their enactment of ambitious practice. Among other implications, these results suggest that teachers’ discourse development should be a more explicit focus of teacher education.

CHAPTER 1: INTRODUCTION

A focus on "reform" has dominated the field of mathematics education in the U.S. for over thirty years, tracing at least as far back as the National Council of Teachers of Mathematics' (NCTM) release of the *Curriculum and Evaluation Standards for School Mathematics* in 1989. While reform efforts in the field have varied in terms of both scope and focus, many have operated under the assumption that the goals for mathematics learning in K-12 schools need to change (e.g., Association for Mathematics Teacher Educators [AMTE], 2017; Common Core State Standards Initiative, 2010; Kilpatrick, Swafford, & Findell, 2001; NCTM, 1989; 1991; 1995; 2000; 2014; 2018). Proposing visions for new and more ambitious goals for mathematics learning, proponents of reform have recommended the affordance of opportunities for students to, among other things, develop conceptual understanding and procedural fluency, engage in reasoning, sense-making, and argumentation, and cultivate a productive disposition toward the discipline (e.g., Kilpatrick et al., 2001; NCTM, 1989; 2000; 2014).

It is widely acknowledged that conventional approaches to teaching mathematics, which are generally teacher centered and focused on the replication of facts, skills, and procedures, do not afford the opportunities necessary for supporting students in meeting the ambitious learning goals for which the field has called. Consequently, a primary goal of mathematics education research has been to identify instructional practices that can. While the field has seemed to converge on those practices, researchers have used a number of terms to describe them, including "high-leverage" (e.g., Ball, 2002), "high-quality" (e.g., Munter, 2014), and "ambitious" (e.g., Lampert & Graziani, 2009). Here, I follow Lampert and Graziani in using the term "ambitious teaching" to refer to the field's vision for mathematics instruction.

Drawing on a definition offered by Kazemi, Franke, and Lampert (2009), ambitious teaching requires that “teachers teach in response to what students do as they engage in problem solving performances, all while holding students accountable to learning goals that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions” (p. 11). With respect to ambitious mathematics teaching in particular, a set of core practices is considered foundational to such an approach, including posing cognitively-demanding tasks that can be solved in multiple ways (Stein, Smith, Henningsen, & Silver, 2000), providing opportunities for students to reason about and discuss those tasks with peers (NCTM, 2000), and facilitating whole-class discussions about students’ solutions in order to highlight key mathematical concepts (Stein, Engle, Smith, & Hughes, 2008). Others, however, have argued for the expansion of definitions of ambitious mathematics teaching to include instructional practices that more explicitly attend to issues of equity (e.g., Jackson & Cobb, 2010). Such practices include, for example, launching mathematical tasks to ensure accessibility (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013) and addressing issues of status among students (e.g., Cohen, 1994).

Whether or not it has been intentional, the responsibility for supporting a pedagogical transition away from the use of conventional instructional practices toward the enactment of ambitious ones has rested, in large part, on teacher education. While principles and practices for ambitious teaching are now foregrounded in national guidelines for mathematics teacher education (e.g., AMTE, 2017), mathematics teachers’ instructional practice continues to look like it did at the time of NCTM’s (1989) first call for reform: teacher centered and focused on the replication of facts, skills, and procedures (Hiebert, 2013; Hiebert et al., 2005; Resnick, 2015).

Researchers have offered various explanations for why conventional teaching persists, even after teachers have been introduced to alternative methods (e.g., ambitious teaching) during teacher education. For example, Lortie (1975) suggested that beginning teachers tend to teach in ways that reflect their conceptions of teaching, which, he argued, are rooted in extensive observations of their own K-12 teachers ("apprenticeships of observation"). As the work of practicing teachers often reflects conventional ideals, the educational biographies and, in turn, the instructional practices that beginning teachers develop are typically conventional in nature. Zeichner and Tabachnick (1981) described—but also raised questions about—another explanation for the persistence of conventional teaching, which asserts that any "progressive" or "liberalizing" effects of teachers' experiences in colleges and universities are only later "washed out" by the dominant norms and values of K-12 settings (pp. 7–8). As a final example, Kennedy (1999) suggested that while teachers may take up the ideas promoted during teacher education, it is unlikely that they will know how to enact those ideas once they begin teaching ("a problem of enactment").

On a more hopeful note, there is empirical evidence suggesting that beginning teachers can and do enact ideals promoted in teacher education (e.g., Grossman et al., 2000; Horn, Nolen, Ward, & Campbell, 2008; Levin, Hammer, & Coffey, 2009; Thompson, Windschitl, & Braaten, 2013; Yurekli, Stein, Correnti, & Kisa, 2020). Even in settings in which conventional teaching prevails, beginning teachers have demonstrated potential to enact ambitious practices that go against the grain (e.g., Thompson et al., 2013). When considered against the backdrop of the literature discussed in the previous paragraph, these results raise an important question: If it is possible for beginning teachers to enact ambitious teaching within an institutional climate that is overwhelmed by conventional norms and practices, why is it that only some beginning teachers, when introduced to ambitious visions for teaching during teacher education, strive to enact those visions in their own classrooms?

How one attempts to answer this question, of course, depends on the theoretical perspective with which it is approached (Russ, Sherin, & Sherin, 2016). While cognitive perspectives on teacher learning were prominent throughout the 1980s and 1990s (e.g., Clark & Peterson, 1986; Fennema & Franke, 1992; Kagan, 1992; Leinhardt & Greeno, 1986; Shulman, 1986), sociocultural and situative perspectives on the matter began to emerge around the turn of the century (e.g., Borko et al., 2000; Franke & Kazemi, 2001; Grossman, Smagorinsky, & Valencia, 1999; Grossman et al., 2000). When it comes to explaining teacher learning, cognitive-oriented research has focused on individual-level constructs such as teacher knowledge and beliefs, whereas research conducted from sociocultural and situative standpoints has highlighted the roles of broader social, cultural, and historical contexts in learning-to-teach processes.

As a bridge between individuals and broader contexts, scholarship rooted in sociocultural and situative perspectives underscores *identity* as an important analytical construct for understanding the development of teachers' practice. Sfard and Prusak (2005) defined identity as a collection of "reifying, significant, endorsable stories" (p. 14) that are told about a person, perhaps to themselves, which are drawn from broader discourses that are "floating around" in the world (p. 18). These discourses, according to Sfard and Prusak, are narratives communicated by individuals and through aspects of institutional settings. As an individual participates in different learning communities, they may appropriate certain discourses to describe who they currently are or who they aspire to be. Appropriated discourses, as Sfard and Prusak suggested, sometimes become "critical" elements of one's identity, which means, when compared to other discourses, they are more consequential for one's actions. If discourses are critical in nature, then one might experience some sort of tension or conflict if they were unable to act in ways that aligned with those discourses.

To extend Sfard and Prusak's (2005) ideas to beginning teachers in particular, it is first important to acknowledge that beginning teacher learning often spans two institutional settings: universities and K-12 schools. Because the socially- and culturally-constructed roles, meanings, and values central to those settings typically vary in significant ways, beginning teacher learning can be framed as occurring in two different "figured worlds" (Holland, Lachicotte, Skinner, and Cain, 1998). As what it means to productively participate in university and K-12 contexts differs between them, it follows that the discourses circulating within the two worlds also vary. Thus, as beginning teachers interact with and participate in university and school contexts during teacher education, they construct teaching identities by drawing on different discourses available within those contexts. And further, because the discourses that teachers take up as their own will vary across individuals, it is likely that teachers' actions—or instructional practices—will also differ.

A number of researchers concerned with beginning teachers' adoption of ambitious instructional practices have focused on the role of identity construction in that process (e.g., Horn et al., 2008; Nolen, Horn, Ward, & Childers, 2011; Thompson et al., 2013). For example, in a longitudinal study of eight beginning secondary mathematics (4) and social studies (4) teachers, Horn and colleagues (2008) found that as teachers encountered different instructional practices during their university and field placement experiences, they made decisions about which practices to appropriate through the lens of their teaching identities. Similarly, in Thompson and colleagues' (2013) study of 26 beginning science teachers from two cohorts of one teacher education program, the authors found that teachers' discursive identities—or "critical pedagogical discourses"—acted as "filters" through which teachers considered whether to appropriate certain instructional practices. Describing critical pedagogical discourses as "personal theories about 'what counts' as productive teaching and learning…threads of internalized dialogue that constitute teachers' narratives about their current and future teaching selves" (pp. 578-579), Thompson and her colleagues found that teachers' critical pedagogical discourses explained whether teachers were susceptible to contextual pressures to teach in conventional ways and, thus, the extent to which they enacted ambitious practice.

Purpose

In tandem, the work of Horn, Thompson, and their colleagues (2008; 2013) suggests that to better understand variation in beginning teachers' uptake of ambitious practice and, more broadly, the persistence of conventional teaching in the U.S., it is necessary to examine the construction of beginning teachers' discursive identities, their early practice, and relations between the two—which was the primary motivation of this study. Because this study builds most explicitly on Thompson and colleagues' (2013) work, in the paragraph that follows, I reflect more on specific contributions of the authors' study and explain how my research is intended to extend those contributions.

First, Thompson and her colleagues (2013) found that the nature of beginning teachers' discursive identities (*critical pedagogical discourses*) was consequential for their enactment of ambitious practice. However, the *types* of discourses that comprise teachers' critical pedagogical discourses warrants further examination, as the authors broadly described critical pedagogical discourses as more and less focused on student learning. Second, results pointed to variation in teachers' susceptibility to external pressures to teach in conventional ways (*contextual discourses*). While Thompson and colleagues attended to differences in the contextual discourses that beginning teachers perceived, they broadly characterized those discourses as "instructionally-conservative" (p. 579), noting that how teachers acted in response to those discourses varied across individuals. It is reasonable to assume, however, that the nature of instructionally-conservative contextual discourses and the sources from which they originate may be more or less consequential for teachers' practice, which suggests there is more to be learned. Finally, the authors' study showed that beginning teachers' critical pedagogical discourses interacted with contextual discourses in different ways. For example, some teachers filtered out or resisted contextual pressures to teach in conservative ways while others were more susceptible to them. As interactions between critical pedagogical and contextual discourses were consequential for teachers' enactment of ambitious practice, investigating the nature of those interactions and how, exactly, they influence teachers' practice is worthy of further investigation.

Through a multiple-case study of four beginning secondary mathematics teachers who graduated from the same teacher education program (in which ambitious practice was promoted) across three consecutive years and went on to teach in two different schools, I attempted to pursue the lines of inquiry described in the previous paragraph. In particular, through longitudinal interviews with and classroom observations of the four teachers, I sought to learn more about variation in teachers' uptake of ambitious practice through the lenses of critical pedagogical and contextual discourses. The three research questions that guided this investigation are provided below, followed by an overview of the study design and findings and the structure of the remaining chapters.

Research Questions

1. How do the critical pedagogical discourses of four beginning secondary mathematics teachers who graduated from the same teacher education program across three consecutive years compare?

2. What contextual discourses do the four teachers perceive, and how do such discourses vary within and across two different school settings?
3. How do teachers' critical pedagogical discourses interact with contextual discourses, and how do such interactions help to explain their enactment of ambitious practice?

Overview of the Study

Following case-study design principles (e.g., Creswell, 2013), I conducted a multiple-case study to answer the research questions listed in the previous section. In my initial sample, I included two first-year mathematics teachers from one cohort of a teacher education program who were teaching in different schools, with the intention of spending an entire school year in their classrooms to investigate, most explicitly, the role of context in shaping beginning teachers' practice. After observing those teachers for one semester, I recruited one student teacher and one second-year teacher, who were from different cohorts of that same program and were teaching in the same school as one of the first-year teachers, as including them would result in a stronger understanding of how characteristics of individuals matter for teaching. Thus, through year- and semester-long interviews and classroom observations of two first-year teachers and one student teacher and one second-year teacher, respectively, I sought to investigate how the four individuals' discursive teaching identities (critical pedagogical discourses), the messages about teaching circulating within their schools (contextual discourses), and interactions between them helped to explain teachers' early practice.

Within- and cross-case analyses revealed that teachers' enactment of ambitious practice followed three patterns: a lack of appropriation (the second-year teacher), compartmentalization of discrete ambitious practices alongside conventional ones (the two first-year teachers), and integration of multiple ambitious practices (the student teacher). Ultimately, these patterns of enactment were explained by interactions between teachers' critical pedagogical discourses and the contextual discourses that they perceived. In particular, the extent to which different dimensions of teachers' critical pedagogical discourses acted as resources for filtering out contextual discourses that pressured teachers to teach in conventional ways helped to explain their enactment of ambitious practice.

From a broad perspective, while these results suggest that teacher education needs to do a better job of preparing beginning teachers to enact ambitious practice in an educational climate overwhelmed with forces that will surely work against our [teacher educators'] efforts, they also give reason to believe that programs can graduate teachers who will act as agents of change. The role that teachers' critical pedagogical discourses seemed to have in their efforts to effect change—by enacting ambitious practice—highlight the importance of attending to the development of such discourses, as they can function as discursive filters for contextual discourses and practices that maintain the status quo. Thus, more research needs to be conducted to further investigate the nature of critical pedagogical discourses, how they develop, and their role in teacher learning in both university and K-12 settings.

In the chapters that follow, I discuss my research in more detail. Given the study's focus on beginning mathematics teachers' enactment of ambitious practice, Chapter 2 begins with an overview of ambitious mathematics teaching, followed by a review of extant research focused on beginning teacher learning. Then, because this study was motivated by the work of Thompson and colleagues (2013) in its motivation and design, I revisit that study, focusing specifically on its theoretical implications.

In Chapter 3, I provide an overview of my research methods, beginning with a positionality statement to orient readers to the ways in which my personal experiences may have shaped this work. Then, after providing an overview of relevant contexts and introducing the four teachers who participated in this study, I describe my processes for data collection and analysis.

In Chapter 4, I discuss the cases of the four teachers separately. Each discussion begins with a general overview that highlights a case's key themes and is followed by multiple sections in which I reflect on the teacher's: critical pedagogical discourse, perceptions of contextual discourses and constraints, enactment of ambitious practice, and relations between those things. Given the idiosyncrasies of each case, however, the structures of the case discussions vary slightly across teachers.

In Chapter 5, I present the results of cross-case analyses. This discussion, which is structured according to the three research questions that guided this study, highlights themes that were prevalent across teachers' critical pedagogical discourses (research question 1), both similarities and differences in the contextual discourses that teachers perceived (research question 2), and interactions between critical pedagogical and contextual discourses, as well as how those interactions were consequential for teachers' enactment of ambitious practice (research question 3).

In Chapter 6, I discuss main findings, along with their implications for future research, teacher education, and K-12 schooling, reflect on the limitations of my work, and conclude the study.

CHAPTER 2: LITERATURE REVIEW

This chapter begins with a review of literature on ambitious mathematics teaching to provide an overview of current goals for mathematics teacher learning. Next, given this study's focus on beginning teachers, I synthesize extant literature on beginning teacher learning to demonstrate how this study fits within that literature. The chapter concludes with a description of the empirical study on which this research was intended to build, highlighting its theoretical implications.

Ambitious Mathematics Teaching

As mentioned in Chapter 1, calls for more ambitious goals for mathematics learning (e.g., learning mathematics "with understanding," Carpenter & Lehrer, 1999) have been coupled with attempts to define ambitious teaching practices that, when enacted, can support students in meeting those goals (e.g., Kazemi et al., 2009; Lampert & Graziani, 2009). In this section, I review research on ambitious mathematics teaching, focusing specifically on principles and practices to which the four teachers who participated in this study were likely introduced during their teacher education program (discussed more in Chapter 3). Though it does not capture the entirety of ambitious mathematics teaching, a particular lesson structure that is often promoted is a three-phase lesson in which teachers *launch* a mathematical task, provide opportunities for students to *explore* those tasks in small groups, and facilitate a whole-class discussion to *summarize* students' solution strategies. Below, I employ those three phases as categories for summarizing key aspects of ambitious mathematics teaching.

Launching Mathematical Tasks

An element foundational to ambitious mathematics teaching, and the first phase of a typical mathematics lesson, is the selection and implementation of cognitively-demanding mathematical tasks, which can, potentially, afford opportunities for students to explore mathematical concepts and explain their reasoning (Silver & Stein, 1996; Smith & Stein, 1998; Stein et al., 2000). Distinct from conventional, or low-level, tasks that promote the memorization of facts or application of previously-learned procedures, tasks of high-cognitive demand, or high-level tasks, are often said to have “low floors” (or “thresholds”) and “high ceilings,” (Boaler, 2016), which means that they can be accessed through multiple points of entry and solved using a number of strategies.

In order for opportunities potentially afforded by teachers’ use of high-level tasks to be realized, researchers have argued the importance of dedicating time to launching those tasks (e.g., Jackson et al., 2013; Jackson, Shahan, Gibbons, & Cobb, 2011; see also AMTE, 2017; Smith & Stein, 2018). According to Jackson and colleagues (2013; 2011), a primary purpose of launching tasks is to ensure that all students can access them. This requires that teachers, without revealing potential solution strategies, establish a common language for describing both aspects of the context in which the task is situated and its key mathematical features. Further, as it is possible, and arguably likely, that many students have only minimal experience in solving high-level tasks, launching tasks also requires that teachers be explicit about the norms and expectations for working on those tasks. Being explicit about participation norms and expectations, however, is not necessarily specific to the launch phase of a lesson, as teachers should strive to clearly communicate participation norms and expectations throughout the explore and summary phases as well (Jackson & Cobb, 2010).

Providing Opportunities for Students to Explore in Pairs or Small Groups

In regards to the second phase of a typical mathematics lesson, researchers have argued the importance of providing opportunities for students to explore and struggle through mathematical tasks, as doing so can support them in meeting more ambitious learning goals such as developing conceptual understanding (e.g., Hiebert & Grouws, 2007; Kilpatrick et al., 2001; NCTM, 2000; 2014). This means that rather than guiding students through deriving correct solutions, teachers should attempt to facilitate students' exploration of the concepts, structures, and relationships underlying mathematical tasks, usually as students work collaboratively in pairs or small groups (e.g., Cohen, Lotan, Scarloss, & Arellano, 1999). Resulting, potentially, in a shift in the role of the teacher from simply a deliverer of knowledge to more of a facilitator (Munter, 2014), the practice of providing opportunities for exploration and struggle may allow for the redistribution of mathematical authority so that rather than residing solely in the teacher, authority is shared with students (Gresalfi & Cobb, 2006).

As the mere provision of such opportunities does not necessarily result in desired learning outcomes, other teaching practices, according to mathematics education literature, are instrumental in ensuring that students' exploration of and struggle through mathematical tasks is productive. One such practice is that of anticipating, which requires teachers to solve mathematical tasks in as many ways as possible before posing them to students in order to envision a range of potential solution strategies (Stein et al., 2008). Anticipating students' solution strategies is intended to support teachers in being responsive to student thinking and struggle (NCTM, 2014). Anticipating is also important for the third phase of a typical mathematics lesson (the summary phase), as it can help teachers prepare for facilitating a whole-class discussion centered around students' solution strategies.

Another practice for ensuring productive exploration and struggle, monitoring, requires teachers to circulate the room and visit with pairs or small groups of students as they work on mathematical tasks (Smith & Stein, 2018; Stein et al., 2008). While one purpose of monitoring is for the teacher to select particular strategies to highlight in the subsequent whole-class discussion, monitoring is typically done in concert with other practices, including posing purposeful questions and eliciting and using evidence of student thinking (NCTM, 2014). According to NCTM, such practices should be employed to assess and advance students' current understanding and inform both in-the-moment and future instructional decisions.

Focusing specifically on the practice of posing purposeful questions, the nature of teachers' questions is consequential for the learning opportunities afforded to students. For example, some teacher questions merely require students to recall information or procedures (Boaler & Brodie, 2004), which can sometimes result in the teacher funneling students toward a desired conclusion (Herbel-Eisenmann & Breyfogle, 2005). In contrast, other, more open-ended teacher questions that, for example, prompt students to explain, justify, or consider relations between mathematical ideas are more likely to support students in thinking conceptually.

As teachers engage in the practice of monitoring, they should also ask questions to facilitate discussion among students who are working together. To facilitate student-to-student discussion, a teacher may, for example, pose questions that orient students to others' thinking. Examples of questions that accomplish this include those that prompt students to repeat in their own words what another student has said or to consider the details of a particular student's strategy (e.g., Kazemi & Cunard, 2016). Ideally, such questions would contribute to facilitating discussion among students that focuses not only on *how* students solved the problem but also *why* their methods work (e.g., Kazemi & Stipek, 2001).

Facilitating Whole-Class Discussions to Summarize Students' Solution Strategies

An ambitious practice foundational to the third phase of a typical mathematics lesson—the summary—is the facilitation of a whole-class discussion centered around students' solution strategies. To do this, teachers should first select strategies for students to share at the front of the room and then facilitate a discussion of those strategies in a way that supports students in developing an understanding of the key mathematical ideas, concepts, and/or procedures underlying the task at hand (Stein et al., 2008; Stigler & Hiebert, 1999). Mathematics education research has pointed to a number of principles and practices that are important for facilitating such discussions. First, Smith and Stein (2018) argued the importance of sequencing students' solution strategies in a way that supports students in meeting the lesson's goals. Depending on the nature of students' strategies, teachers may, for example, choose to sequence strategies by beginning with a presentation of a strategy associated with a common misconception in order to address confusion before moving forward, or by ordering strategies so that they are presented in an increasingly abstract manner, which may help to make clearer for students the mathematical connections between them.

Second, teachers should strive to facilitate whole-class discussions that are led primarily by students (Hufferd-Ackles, Fuson, & Sherin, 2004). This may require the teacher to establish, with students, norms and expectations for participating in such discussions. For example, it should be clear that students are expected to explain and justify their strategies as they present and to ask questions pertaining to strategies presented by others. Facilitating a student-led, whole-class discussion also necessitates that teachers refrain from engaging in the conventional "IRE" pattern of talk in which the teacher *initiates* a question to which students *respond*, and then *evaluates* whether students' responses are correct (Mehan, 1979). Alternatively, teachers should work to cultivate a "mathematical discourse community" (Lampert, 1990) by inviting students to initiate discussion with others, and asking questions to generate student-to-student discussion.

A third aspect of whole-class discussions to which teachers practicing ambitious teaching should attend is the focus or nature of those discussions. To distinguish between types of discussions, researchers have described two types of orientations (Thompson, Philipp, Thompson, & Boyd, 1994) or discourse (Cobb, Stephan, McClain, & Gravemeijer, 2001): calculational and conceptual. Rather than orienting whole-class discussions to focus solely on procedures used or steps followed (calculational discourse), teachers should organize discussions around both problem-solving processes and the mathematical concepts underlying them (conceptual discourse). To do this, teachers might engage in questioning that prompts students to explain why they took a particular approach to solving a task (McClain & Cobb, 1998), to ask others about their strategies, or to consider connections between the strategies and mathematical representations presented (NCTM, 2014; Smith & Stein, 2018).

While not necessarily specific to the third phase of a typical mathematics lesson, positioning students as competent—in ways that extend beyond speed and accuracy—is a final practice foundational to ambitious mathematics teaching (Boaler & Staples, 2008; Horn, 2012). While the ways in which students are positioned depends, in part, on the nature of the mathematical tasks that teachers pose (Gresalfi, Martin, Hand, & Greeno, 2009), teachers can position students as competent by publicly praising their ideas (Boaler & Staples, 2008), valorizing "informal" strategies (Turner et al., 2012), or normalizing making mistakes and being confused (Gresalfi et al., 2009; Horn, 2012). Positioning students as competent in such ways can also contribute to alleviating issues of status by expanding notions of what it means to be good at mathematics. For example, teachers can work to elevate students' status by publicly affirming less "traditional" forms of participation, such as asking good questions or making astute observations (Horn, 2012).

Summary

To summarize, ambitious mathematics teaching, as it is defined here, typically occurs throughout three phases of a lesson in which teachers launch a high-level mathematical task, provide opportunities for students to collaboratively explore the task in pairs or small groups, and facilitate a whole-class discussion to summarize students' solution strategies. To ensure the effective enactment of such practices, teachers should employ them in concert with other practices including (but not limited to): anticipating solution strategies prior to posing the task; being explicit about participation norms and expectations; monitoring students as they work on the task while asking questions to elicit, assess, and advance student thinking; selecting and sequencing the presentation of students' solution strategies; facilitating a student-led, conceptually-oriented discussion of those strategies and the mathematical connections between them; and positioning students as competent in ways that extend beyond speed and accuracy.

As mentioned in Chapter 1, the enactment of ambitious mathematics teaching has been established as a national goal for teacher learning (AMTE, 2017). Perhaps not surprisingly, teacher education has come to bear a great deal of responsibility for supporting teachers in meeting this goal. Darling-Hammond (2010) has argued, for example, that "nations that have steeply improved their students' achievement…attribute much of their success to their focused investments in teacher preparation and development" (p. 194). Given, at the very least, our nation's financial investment in teacher education—despite having minimal evidence for its effectiveness (Cochran-Smith & Zeichner, 2005)—it is necessarily worthwhile to investigate how our current efforts might be improved to better support teachers' enactment of ambitious practice. Arguably, this requires studying the experiences of beginning teachers. To reflect on what is currently known about beginning teacher learning, I review relevant literature in the section that follows.

Beginning Teacher Learning

Beginning teacher learning is unique in that it is typically occurs within two institutional settings: universities (or colleges) and K-12 schools. For most traditional, university-based teacher education programs, prospective teachers experience a prolonged period of coursework, usually over a course of three to four years, followed by a student teaching internship conducted under the mentorship of a practicing teacher. As a result of pressure to incorporate more opportunities for teachers to conduct work in the field (Barnett, 1975; Cochran-Smith & Villegas, 2016; Moore, 1979), more modern versions include fieldwork components that span the length of the program, requiring teachers to spend more time in classrooms. Regardless of program structure, however, many prospective teachers move between university and school settings as they formally prepare for teaching.

The "Two-Worlds" Problem

Based on learning-to-teach literature, it is well established that a stark contrast often exists between the visions for learning and teaching promoted in university and school settings. In general, many teacher education programs are driven by so-called progressive or reform agendas that encourage teachers to take up constructivist perspectives on learning and corresponding teaching practices (e.g., ambitious practices). These reform agendas are, in many ways, at odds with the traditional landscape of schools, in which conventional, teacher-centered practices—along with curriculum and assessment accountability pressures—are prevalent. Given this conflict between the norms, values, and practices central to the "two worlds" (Feiman-

Nemser & Buchmann, 1985), beginning teachers are pulled in different directions as they move between university and school settings during formal education.

This particular disjuncture and its implications for beginning teacher learning have been a focus of research for decades. While some researchers have focused on potential ways to build bridges between universities and schools (e.g., Anagnostopoulos, Smith, & Basmadjian, 2007; Zeichner, 2010), others were concerned with making sense of how beginning teachers experience the two-worlds problem and what happens as a result (e.g., Borko & Mayfield, 1995; Borko et al., 2000; Braaten, 2019; Ensor, 2001; Grossman et al., 1999; Hebard, 2016; Horn et al., 2008; Kennedy, 1999; Lortie, 1975; Ritchie & Wilson, 1993; Smagorinsky, Cook, Moore, Jackson, & Fry, 2004; Smith & Avetisian, 2011; Thompson et al., 2013; Ward, Nolen, & Horn, 2011; Zeichner & Tabachnick, 1981; 1985). Regarding the latter body of work, results have shown that characteristics of individuals, aspects of institutional settings, and, in some cases, relations between them influence whether and to what extent beginning teachers enact the practices promoted in teacher education. To highlight some of the factors that are important for understanding beginning teacher learning, I synthesize relevant literature in the sections that follow.

Characteristics of Individuals

In learning-to-teach literature, it is well established that beginning teachers' *conceptions of teaching* shape how they interact with and participate in university and school contexts (e.g., Horn et al., 2008; Nolen et al., 2009) and, thus, whether and to what extent they adopt the practices promoted in teacher education programs (e.g., Borko & Putnam, 1996, Lortie 1975; Zeichner & Tabachnick, 1981). However, conceptions of teaching is sometimes used as a catch-all term for describing teachers' perspectives on myriad matters of teaching, including knowledge and subject matter, what constitutes high-quality teaching, expectations for students, and purposes of schooling (e.g., Parajes, 1992; Skott, 2001; Thompson, 1992). Therefore, in an effort to more clearly illustrate how different aspects of individuals' conceptions of teaching might help to explain teachers' practice, I review selective literature in the paragraphs that follow.

While not necessarily focused on beginning teachers in particular, research has suggested that teachers' conceptions of knowledge and subject matter have implications for what they do in the classroom (Horn, 2007; Kang, 2008; Kang & Wallace, 2005; Schoenfeld, 1988; Tanase & Wang, 2010; Thompson, 1992). For example, based on her study of 23 beginning science teachers, Kang (2008) argued that teachers' beliefs about the nature of knowledge and knowing—or *personal epistemologies*—shape teachers' instructional goals and subsequent teaching actions. In particular, Kang found that teachers who conceptualized learning as "answering one's [own] questions" and described science as an "evolving body of knowledge and inquiry" were most likely to establish inquiry-oriented learning goals and enact ambitious science teaching practices (p. 484).

Similarly, Horn (2007) found in her study of two high school mathematics departments that teachers' views of mathematics as a discipline related to the learning opportunities they planned to afford in their classrooms. For example, teachers who viewed mathematics as a series of connected ideas—rather than a static body of knowledge—were willing to adapt the designated curriculum in order to offer students more rigorous opportunities to learn. In tandem, the results of Kang's (2008) and Horn's (2007) studies suggest that the ways in which teachers talk about learning and content help to explain why they establish particular goals for learning and attempt to enact certain instructional practices.

Although teachers' practice is not always reflective of their conceptions of good teaching (e.g., Barkatsas & Malone, 2005; Law, Wong, & Lee, 2012; Skott, 2001), there is empirical evidence suggesting that such conceptions may help to explain why teachers enact particular practices (e.g., Horn et al., 2008; Munter & Correnti, 2017; Nespor, 1987; Nolen et al., 2009; Parajes, 1992; Philipp, 2007; Richardson, 1994; Stodolsky & Grossman, 2000; Thompson, 1992; Thompson et al., 2013; Wilhelm, 2014). For example, studies have shown that teachers' beliefs about or visions of "good teaching" act as filters through which teachers decide to take up or reject the pedagogical tools they encounter (e.g., Horn et al., 2008; Nespor, 1987; Nolen et al., 2009; Parajes, 1992; Thompson et al., 2013). Similarly, the results of studies by Munter and Correnti (2017) and Wilhelm (2014) revealed relations between mathematics teachers' (future) enactment of high-quality instructional practices and the extent to which those teachers' visions for instruction reflected definitions of high-quality teaching established by the mathematics education research community.

As the previous paragraph suggests, how researchers have conceptualized teachers' conceptions of good teaching has varied across studies. For example, some researchers have studied teachers' instructional practice through the framework of teacher beliefs (e.g., Nespor, 1987; Pajares, 1992; Philipp, 2007; Richardson, 1994; Stodolsky & Grossman, 2000). More recently, others have relied on the notion of teachers' "instructional visions" to characterize such conceptions (Munter, 2014). Drawing from Hammerness's (2001) construct of "teacher vision," Munter and Wilhelm (in press) defined instructional vision as "the discourse that teachers or others employ to characterize the kind of 'ideal classroom' practice to which they aspire but have not yet necessarily mastered" (p. 2). As a way to operationalize teachers' instructional visions, Munter (2014) offered a set of interview prompts and rubrics, which delineate trajectories of teachers' visions with respect to three dimensions of practice: role of the teacher, classroom discourse, and mathematical tasks. Each trajectory maps pathways of increasing sophistication, with the most sophisticated level aligning with much of what was summarized about "ambitious mathematics teaching" in a previous section of this chapter.

In addition to teachers' personal epistemologies and instructional visions, prior research has shown that teachers' views of students and their capabilities shape their practice (e.g., Brown, 1986; Jackson, 2009; Jackson et al., 2017; McLaughlin & Talbert, 1993; Skott, 2001; Wilhelm, 2014; Wilhelm, Munter, & Jackson, 2017; Windschitl, Thompson, & Braaten, 2011). In particular, results suggest that teachers' enactment of ambitious practices may depend on whether they consider their students as capable of engaging with the learning opportunities afforded by such practices (Wilhelm, 2014) or whether they locate the responsibility for undesirable learning outcomes within students, rather than within instructional or institutional opportunities (Wilhelm et al., 2017; Windschitl et al., 2011). For example, in Windschitl and colleagues' (2011) study of 11 first-year secondary science teachers who were introduced to ambitious practice during teacher education, the authors found that teachers who attributed instructional challenges to problems with students (e.g., "My students don't get it" p. 1322) were inclined to employ teaching practices grounded in a transmissive model of instruction, meaning that teachers' practices were generally unresponsive to students' ideas.

To operationalize teachers' views of students, Jackson and colleagues (2017) offered a framework for assessing the productiveness of those views. Based on Snow and Benford's (1988) notion of "problem framing," Jackson and colleagues' (2017) framework provides a way to characterize the extent to which teachers' framing of both problems of student learning ("diagnostic framing") and solutions for addressing such problems ("prognostic framing") is productive. Asserting that teachers' diagnostic and prognostic framings can be productive, unproductive, or a mix of both, the authors offered a description for each category of framing. Productive framings—associated with a view of students as capable of engaging in rigorous mathematics—attribute students' struggle in mathematics to limitations of instructional and/or schooling opportunities, and depict instructional adjustments that would maintain rigorous learning goals for students. Unproductive framings—associated with a view of students as incapable of engaging in rigorous mathematics—attribute students' struggle in mathematics to inherent traits of students (or deficits in their families and communities), and depict instructional adjustments that would reduce the rigor of learning goals for students.

Finally, although less studied, prior research has suggested that teachers' awareness of the political nature of teaching may have implications for the practices they aspire to enact in their classrooms (e.g., Bartolomé, 2004; Ginsburg & Newman, 1985; Gutiérrez, 2013; 2015). For example, because teachers are typically "winners" of the school system (Gutiérrez, 2015), it is likely that many of them see broader political and economic inequities as "natural or unproblematic" (Ginsburg & Newman, 1985, p. 49), which provides them little motivation to consider how teaching might act as a tool for contributing to or addressing those inequities. In contrast, teachers who demonstrate what Bartolomé (2004) called "political clarity" may be more likely to take up instructional practices that can support them in acting as agents of change. Bartolomé (2004) defined political clarity as:

> [an] ever-deepening consciousness of the sociopolitical and economic realities that shape [teachers'] lives and their capacity to transform such material and symbolic conditions…[an] understanding of the possible linkages between macro-level political,

> economic, and social variables and subordinated groups' academic performance in the micro-level classroom. (p. 98)

Therefore, the extent to which teachers can articulate how "macro-level political, economic, and social variables" contribute to the inequities that play out in schools—and whether they acknowledge teaching as a tool for addressing those inequities—may help to explain teachers' enactment of instructional practices that can disrupt the status quo (or not).

To summarize, the research reviewed in this section highlights individual-level characteristics that may be important for understanding variation in beginning teachers' enactment of ambitious practice. Those factors include teachers': personal epistemologies (e.g., Kang, 2008); instructional visions (Munter, 2014); views of students and their capabilities (Jackson et al., 2017); and political clarity (Bartolomé, 2004). As mentioned earlier, other research has shown that aspects of the institutional settings in which beginning teachers learn to teach are also consequential for their practice. To reflect on potential influences of those settings, relevant research is reviewed in the following section.

Aspects of Institutional Settings

In regards to university settings in particular, research has suggested that the structure, coherence, and focus of teacher education programs may have important implications for beginning teacher learning (e.g., Ball & Forzani, 2009; Borko et al., 1992; Eisenhart et al., 1993; Grossman & McDonald, 2008; Hebard, 2016; Jansen, Gallivan, & Miller, 2018; Tatto, 1996; 1998). First, researchers have proposed a number of changes regarding the general structure of teacher education programs, ranging from extending the study of teaching methods beyond one 12-week course (e.g., Eisenhart et al., 1993) to increasing the amount of time beginning teachers spend in schools. However, as such changes would do little to address the disjuncture between university and school settings previously discussed, others have offered suggestions for building bridges between the two contexts (e.g., Zeichner, 2010).

The results of other studies have pointed to program coherence as consequential for what beginning teachers learn (e.g., Tatto, 1996; 1998). For example, as Tatto (1998) found that students in more coherent programs were more likely to share the perspectives of faculty members (Tatto, 1998), she argued the importance of establishing program coherence in terms of "shared understandings among faculty and in the manner in which opportunities to learn have been arranged…to achieve a common goal" (Tatto, 1996, p. 176). Tatto's argument for program coherence is somewhat similar to one provided more recently by Jansen and colleagues (2018), which was based on the results of their study of 81 beginning mathematics teachers who experienced the same teacher education program. Jansen and her colleagues found that while the program's instructors had established an "intended vision" for teachers to develop, teachers' visions post-graduation were aligned with the program's intended vision to varying extents. Consequently, the authors proposed establishing a vision that spans multiple courses to increase the likelihood that teachers adopt a program's intended vision.

Finally, as one might imagine, the focus of teacher education programs—in terms of the learning opportunities afforded—have direct implications for what teachers enrolled in those programs learn. Noting that teacher education programs tend to focus most explicitly on effecting cognitive change, some researchers have argued for a shift in focus to allow for the provision of more opportunities for beginning teachers to carry out the work of teaching (Ball & Cohen, 1999; Ball & Forzani, 2009; Grossman, Hammerness, & McDonald, 2009; Grossman & McDonald, 2008; Lampert, 2010). For example, arguing that teacher education has focused primarily on pedagogies of reflection and investigation, Grossman and colleagues (2009) suggested that teacher education expand its focus to include pedagogies of enactment or "the clinical aspects of practice and experiment" (p. 274).

In another line of work, researchers have proposed leveraging the tensions that beginning teachers perceive between university and school settings to facilitate learning (e.g., Horn et al., 2008; Hebard, 2016; Johnson & Barnes, 2018; Nolen et al., 2011; Smagorinsky et al., 2004; Ward et al., 2011). Hebard (2016), for example, offered this argument based on the results of a comparative case study of two teacher education programs, which showed that beginning literacy teachers who were afforded opportunities to reflect on contradictions between university and school settings were more likely to appropriate the ideas and practices promoted in their program. Similarly, Ward and colleagues' (2011) study showed that beginning teachers' perceptions of tensions or "frictions" between university and school settings supported their enactment of ambitious practice. In particular, the authors found that beginning teachers sometimes recontextualized ambitious practices to meet goals specific to K-12 classrooms.

Turning to influences of aspects of school settings in particular, research has pointed to a number of contextual factors that have implications for beginning teacher learning. First, studies have shown that individuals who oversee and facilitate the work of beginning teachers, including university supervisors and mentor teachers, are consequential for the development of teachers' practice (or lack thereof). In some cases, researchers have found that beginning teachers' work with university supervisors and school mentors did little to support teachers' uptake of new practices (e.g., Allen, 2009; Borko & Mayfield, 1995; Bullough & Draper, 2004; Valencia, Martin, Place, & Grossman, 2009). However, other studies suggested that collaborative interactions between teachers and mentors, which focused on improving the learning experiences of K-12 students, afforded opportunities for teachers to experiment with and refine their use of ambitious practices (e.g., Braaten, 2019; Thompson et al., 2013).

Other research has highlighted the influences of subcultures and institutional structures within schools on beginning teachers' practice (e.g., Allen, 2009; Arzi & White, 2007; Eisenhart et al., 1993; Lacey, 1977; Steele, 2001; Zeichner & Tabachnick, 1985). Zeichner and Tabachnick's (1985) three categories of "control"—personal, bureaucratic, and technical—are particularly helpful for characterizing institutional constraints that teachers may face as they attempt to enact unconventional practices. Zeichner and Tabachnick's categories of control refer to the sources from which particular constraints emanate, including individuals who monitor teachers' work (personal), school rules and social relations and hierarchies (bureaucratic), and aspects of an institution's physical structure, such as designated instructional time and curriculum materials (technical). While constraints imposed by each type of control may pose challenges for beginning teachers, Zeichner and Tabachnick found in their longitudinal study of four beginning teachers, that technical controls, including curriculum standards and expectations for pacing, were most influential for beginning teachers' practice.

Finally, other studies have shown that students may influence beginning teachers' enactment of ambitious practices (e.g., Brown, 1986; Friedrichsen, Chval, & Teuscher, 2007; Skott, 2001; Spradbery, 1976), as those practices often contradict the norms that were central to students' previous mathematics classes. For example, in studies by both Brown (1986) and Skott (2001), the authors found that students' reactions to teachers' enactment of ambitious practices influenced teachers' future use of those practices. In particular, because teachers perceived that their use of unconventional practices was not aligned with students' expectations (Brown, 1986) or instrumental in building students' confidence (Skott, 2011), the teachers engaged in practices that were different from those they had intended to use.

To summarize, the research reviewed in this section highlights contextual factors of both university and school settings that may be important for understanding variation in beginning teachers' enactment of the ideals promoted in teacher education. Regarding factors central to university settings, researchers have argued that program structure (e.g., Borko & Mayfield, 1995), coherence (e.g., Tatto, 1996), and focus (e.g., Ball & Forzani, 2009) are consequential for beginning teacher learning. In regards to school settings, research has shown that relationships between teachers, school mentors, and university supervisors (e.g., Valencia et al., 2009), institutional subcultures and structures (e.g., Zeichner & Tabachnick, 1985), and students' expectations for teaching (e.g., Brown, 1986) also have a role in explaining beginning teachers' practice.

Bridging the Individual to the Institutional

The research reviewed in the two previous sections highlights individual and institutional factors that help to explain why some beginning teachers enact ideas and practices promoted in teacher education to a greater extent than others. Some researchers have argued, however, that to more fully understand the process of learning to teach—in all of its complexities—requires attending more explicitly to relations between characteristics of individuals and aspects of the institutional settings in which they learn and work (e.g., Borko et al., 2000; Grossman et al., 1999; Horn et al., 2008; Peressini, Borko, Romagnano, Knuth, & Willis, 2004; Putnam & Borko, 2000; Thompson et al., 2013). Because sociocultural and situative theories of learning lend themselves to making sense of those relations (Cobb & Bowers, 1999; Greeno, 1997; Greeno, Collins, & Resnick, 1996; Lave & Wenger, 1991; Rogoff, 1994; Wenger, 1998), such theories have been gaining traction in research on beginning teacher learning.

Although researchers operating from sociocultural and situative standpoints have relied on an number of theoretical and analytical frameworks to investigate matters of beginning teacher learning, *identity* seems to be a recurring construct. Sfard and Prusak (2005) defined identity as a collection of "reifying, significant, and endorsable stories" told about a person that are products of "collective storytelling" (p. 14). One's identity, then, is constructed by drawing on socially and historically established narratives that one encounters as they move through the world. Others researchers have conceptualized such narratives as originating from different Discourses—socially constructed "ways of being certain kinds of people" (Gee, 2001, p. 110)—or figured worlds, in which "particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p. 52).

While the narratives that constitute a person's identity shape the ways in which they interact with and, in turn, come to participate in different Discourses or figured worlds, for example, those interactions may also influence one's appropriation (or not) of particular narratives, illustrating the co-constituting nature of identity construction and learning. According to Sfard and Prusak (2005), narratives that one appropriates into their identity sometimes become "critical" elements of their sense of self, meaning that they are most consequential for one's actions. To be clear, the meaning of critical, here, refers to a narrative's essential nature and, thus, is distinct from meanings associated with Critical Theory and *critique*.

These ideas are mirrored in research on beginning teacher learning, as various studies have illustrated how teachers' identity construction and teachers' (uptake of) practices are intertwined (Peressini et al., 2004). A study by Nolen and colleagues (2009), for example, highlighted the ways in which teachers' identities act as filters through which learning occurs. The authors found that beginning mathematics and social studies teachers adopted or rejected pedagogical tools based on whether they identified with those tools in terms of their perceived usefulness or feasibility. But because teachers' identities are in constant flux as they move between university and school settings, teachers' filters were subject to change as they experienced what Nolen and colleagues referred to as "productive friction," which was caused by interactions between aspects of teachers' current identities and the "norms and images of teaching" projected within each setting (p. 274). As teachers negotiated frictions through interactions with other social actors in university and school settings, they sometimes filtered in new ideas and, thus, were more likely to experiment with those ideas in practice.

Nolen and colleagues' (2009) notion of productive friction is similar to the identification and negotiation processes described by Horn and colleagues (2008). In their study of eight beginning teachers, Horn and her colleagues found that as teachers considered teaching practices promoted in coursework through the lens of their current teaching identities, they filtered in practices that were seemingly at odds with their preexisting identities through processes of identification and negotiation:

> Through identification, interns trust in the value of practices that may not be immediately seen as consonant with their emergent teaching identities. This trust, placed in a friend or seemingly competent instructor, allows interns to affiliate with and learn about a practice that may actually seem counterintuitive to them. With negotiation, interns modify their teaching identities to incorporate new images of good teaching practice that they have encountered in [university settings]. (p. 71)

Based on Horn and colleagues' descriptions of these processes, the extent to which teachers filter in "new images of good teaching practice" may depend on whether their adoption of those images occurs through processes of identification or negotiation. For example, while identification may involve a temporary adoption of an idea alongside preexisting ones, negotiation requires a modification of one's current identity.

Arguably, constructs introduced by Thompson and colleagues (2013) more clearly illustrate relations between beginning teachers' identities and the projected "images" that Nolen and Horn and colleagues (2009; 2008) described. Thompson and colleagues' (2013) study of 26 beginning science teachers pointed to relations between teachers' discursive identities—*critical pedagogical discourses*—and teachers' perceptions of messages about teaching circulating within various learning-to-teach settings—*contextual discourses*. Drawing from Sfard and Prusak's (2005) notion of "critical" narratives discussed earlier, Thompson and colleagues (2013) defined critical pedagogical discourses as "personal theories about 'what counts' as productive teaching and learning…threads of internalized dialogue that constitute teachers' narratives about their current and future teaching selves" (pp. 578-579). Critical pedagogical discourses, the authors explained, are constructed as teachers encounter and draw on broader narratives—or contextual discourses—about teaching. More specifically, Thompson and colleagues described contextual discourses as "perceptions of messages about teaching and learning communicated by actors in social situations and institutional environments or through policy statements" (p. 579). In their study, the authors found that interactions between critical pedagogical and contextual discourses shaped teachers' enactment of the ambitious teaching practices promoted in their teacher education program. For example, teachers' whose critical pedagogical discourses were aligned with contextual discourses central to the teacher education program were more likely to resist contextual pressures to teach in conventional ways and most successful in enacting ambitious practice.

As mentioned in Chapter 1, this research builds on the work of Thompson and colleagues (2013) with respect to motivation and design. Drawing on the authors' constructs of critical pedagogical and contextual discourses, I sought to further investigate: the types of discourses that constitute the critical pedagogical discourses of beginning mathematics teachers, differences in contextual discourses that teachers may encounter within and between school settings, interactions between critical pedagogical and contextual discourses, and relations between those interactions and teachers' practice. In contrast to Thompson and colleagues' sociocultural framing of their study, this work is grounded in a *situative* perspective on learning (Greeno, 1997). I make this distinction based on definitions of the two perspectives provided by Russ and colleagues (2016). In particular, rather than studying "broader activity systems by placing individuals within the larger cultural and historical context" (sociocultural), I claim to have focused more on "the in-the-moment interactions of people with one another and their immediate world and environment" (situative) (p. 403). This is because while the contextual discourses that teachers perceived may have stemmed from "historical, institutional, and sociocultural forces" (Gee, 2001, p. 100), from the teachers' perspectives, contextual discourses are more so messages communicated by individuals in and aspects of their current school environments.

CHAPTER 3: METHODS

As discussed in the previous chapter, teachers' discursive identities, or *critical pedagogical discourses*, are consequential for their enactment of ambitious practice, as discourses that are critical elements of one's identity are most consequential for one's current and future actions. For beginning teachers, the construction of their critical pedagogical discourses occurs as they interact with different settings in which particular messages about teaching—*contextual discourses*—are communicated by various aspects of those settings, including other social actors, norms, values, and practices. As noted earlier, a primary purpose of this study was to examine the practices of beginning mathematics teachers through the lenses of critical pedagogical and contextual discourses to learn more about why teachers attempt to enact ambitious practice to varying extents. To do this, I designed and implemented a multiple-case study, as researchers have argued that understanding relations between teachers' discourses and practice requires close-up, qualitative work (e.g., Munter & Correnti, 2017).

A multiple-case study is an approach to qualitative research through which a researcher attempts to understand a phenomenon by exploring "multiple-bounded systems (cases) over time, through detailed, in-depth data collection involving *multiple sources of information* (e.g., observations, interviews, audio-visual material, and documents and reports) and reports a case *description* and case-based themes" (Creswell, 2013, p. 97; emphasis in original). A case, according to Merriam and Tisdell (2016), is characterized by the unit of analysis "around which there are boundaries" (p. 38). A primary purpose of studying multiple cases is to construct an explanation of the phenomena of interest that fits each of the cases, which, arguably, enhances the generalizability of the results (Yin, 2014; Miles et al., 2014).

As mentioned earlier, this particular study was organized around four cases of beginning secondary mathematics teachers who graduated from one teacher education program across three consecutive years and went on to teach in two different schools. Those cases included Sara, a second-year teacher, Emma and Joe, two first-year teachers, and Bri, a student teacher (all names are pseudonyms). This sample—four beginning teachers with varying experience working between two school settings—was intentional, as it afforded the opportunity to investigate the role of both individual- and contextual-level variables in explaining the phenomenon of interest: beginning teachers' enactment of ambitious practice.

Before describing those cases in more detail, I provide a positionality statement in which I reflect on how my personal experiences may have shaped this research, and then describe relevant contexts, including the teachers' teacher education program and the two school settings. Following a description of the four teachers, I close out the chapter by describing processes of data collection and analysis.

Positionality Statement

This research was shaped by my experiences as a white woman, former prospective and practicing secondary (Grades 6–12) mathematics teacher, and methods course intern and instructor. First, my interest in beginning teachers was sparked by my own experiences as a prospective and practicing teacher. After graduating from a teacher education program that promoted ambitious teaching practices, I was looking forward to enacting those practices in my own classroom, only to find later that my efforts would be hindered by a lack of support and standardized testing policies. I soon after left the profession, wondering whether my experiences as a beginning teacher were typical.

It is important to note, then, that I was, of course, rooting for these teachers, hoping not only that they had endorsed the ambitious teaching principles and practices promoted in their methods courses, but also that they would experience success in enacting them. Being aware of these biases, however, I consistently reflected on them as I conducted the study and was sure to keep the study's primary goals in mind.

My experiences as a methods course intern and instructor had implications for my work as well, as it was during these experiences that I met and got to know three of the four teachers who participated in this study. I met Joe and Emma, a white man and a white woman, respectively, in the spring of 2017 during their second of three secondary mathematics methods courses. As an intern for the course, I interacted with Joe and Emma during class in the context of small-group discussions, and because we got along well, we sometimes chatted after class had ended. As a research assistant, I also had a professional, working relationship with Emma, as during that same spring, she served as an undergraduate research assistant on a project on which I was working with my advisor. The project focused on establishing research-practice partnerships between our university and school districts across the state.

I met Bri, a white woman, in the fall of 2017 during her first of three secondary mathematics methods courses. As an intern for the course, I primarily interacted with Bri during small-group discussions. Then, in the fall of 2018, I was the instructor of record for Bri's third secondary mathematics methods course, prior to her spring semester of student teaching. During this time, Bri and I grew closer, as we saw each other twice a week throughout the semester.

Unlike the others, I did not meet Sara at the university. I met Sara, a white woman, during the winter of 2018 at the high school where she worked, after my advisor (who had been her student teaching supervisor during the spring of 2017) had put me in touch with her via email. Because Sara had graduated from the same teacher education program as the others and was working in the school where Emma was experiencing her first year of teaching and Bri was conducting her student teaching internship, I asked her to participate in the study so that I could learn about the experiences of three—rather than two—teachers in different years of their careers working within the same institutional setting.

Given my relationships with the four teachers, along with my K-12 teaching experiences and affiliation with the university, issues of social desirability were likely at play during the execution of this study and, thus, may have limited my access to teachers' discourses and instructional practices. For example, because I endorsed the perspectives promoted in the university's methods courses, teachers may have felt pressured to say that they, too, valued those perspectives. Further, it is possible that pressures were stronger for Bri, given that I, a white woman, had encouraged her and other students in my methods course to interrogate sources of (racial) inequity in schools. The issue of social desirability extends to the teachers' practice as well, as their actions during observations may have been influenced by my presence. Second, teachers' talk and practices may have been influenced by my affiliation with the university. In particular, to maintain social affiliations with the university, teachers may have talked and acted in ways that did not necessarily reflect (all of) their values. It is also important to note that as an affiliate to the university, I approached and conducted this research from a place of privilege. Consequently, I was consistent in attempting to honor the teachers' experiences.

Overview of Relevant Contexts and Cases

Teacher Education Program

The teacher education program from which the four teachers graduated comprised three phases of study, each of which included coursework and field experience requirements. During

Phase I (Freshman year to Sophomore year), teachers were required to complete general education and mathematics-content coursework and conduct 36 hours of fieldwork between classroom (16 hours) and community settings (20 hours). In Phase II (Junior year to fall of Senior year), teachers were required to participate in the program's three secondary mathematics methods courses and work as an intern, for at least 24 hours total, in a local school district under the mentorship of a practicing secondary mathematics teacher. While the nature of the internship work in which teachers engaged likely varied across individuals, mentors were responsible for supporting teachers in working with individual and small groups of students and teaching components of, if not entire, mathematics lessons. In Phase III (spring of Senior year), teachers worked full-time (up to 40 hours per week) as a student teaching intern under the mentorship of a secondary mathematics teacher.

Three of the four teachers, Bri, Emma, and Sara, conducted their student teaching internships with the same teacher they had worked with during the fall of the previous semester. While the other teacher, Joe, spent the spring and fall of his senior year working in the same high school, he worked with two different mentor teachers during that time. Student teaching experiences were intended to be more intensive than the Phase II internship, as student teachers were expected to "take over" the classroom by planning and implementing lessons and supporting and assessing student learning on a daily basis. Both mentors and university supervisors were responsible for supporting interns in developing and implementing such practices. In most cases, however, mentors likely provided a majority of such support, as supervisors were required to observe student teachers on only four occasions during the semester-long student teaching internship.

Given the focus of this study, it is important to note that student teachers' mentors, as well as other practicing teachers and administrators who worked at their internship sites, did not necessarily value or attempt to enact the ambitious practices promoted by methods course instructors at the university (described more in the next section). This means, then, that student teachers likely perceived differences in the norms, values, and practices central to the contexts in which they were learning to teach (Feiman-Nemser & Buchmann's [1985] "two-worlds" problem). Although the university's teacher education program did have designated procedures and structures for supporting student teachers during their internship experiences (e.g., a bi-weekly student teaching seminar), helping teachers make sense of and navigate perceived differences between the university and K-12 settings was not an explicit focus of those supports.

Methods Courses

As mentioned earlier, a key characteristic of this study is that the four teacher participants graduated from the same teacher education program across three consecutive years. Given that mathematics education faculty did not collaboratively design the three-course methods sequence with an intended vision for mathematics teaching and learning in mind (cf. Jansen et al., 2018), it is likely that course instructors introduced and emphasized ambitious teaching principles and practices to varying extents, which means there were likely differences in the contextual discourses that the four teachers perceived at the university. However, it is also reasonable to assume that there were consistencies across the sequences because instructors shared course materials, taught between sequences, and were practicing researchers in mathematics education who likely highlighted ideas and practices espoused by the broader field.

To characterize potential similarities and differences between the ideas and practices introduced across methods course sequences, I administered a two-question survey to the six course instructors (A–F), myself included (F), each of whom taught at least one of the three methods courses across the three years (nine courses total). Two of the six instructors, A and C, taught between two of the sequences, and one of those two, instructor A, taught twice within the same sequence. Using three-letter combinations to represent the courses that each instructor taught within a given sequence (e.g., combination ABA indicates that instructor A taught the first and third methods courses and instructor B taught the second methods course), Table 1 provides an overview of the sequences that the four teachers experienced, along with pseudonyms of their mentor teachers and university supervisors.

Table 1
Overview of Teachers' Methods Course Sequences, Mentors, and Supervisors

Teacher	Years	Instructor Sequence	Mentor Teacher	University Supervisor
Sara	2016–2017	ABA	Ms. G	Instructor C
Joe, Emma	2017–2018	CAD	Ms. H, Mr. I	Mr. J
Bri	2018–2019	CEF	Mr. I	Instructor C

To restate, the purpose of surveying course instructors was to gain insight into overlaps and distinctions between the three methods course sequences in terms of the ideas, theories, frameworks, teaching practices, and resources (referred to hereafter as "pedagogical tools," Grossman et al., 1999) that were introduced. Investigating similarities and differences across the sequences was particularly important for understanding the contextual discourses—messages about mathematics teaching and learning—that the four teachers may have encountered during their methods course experiences. The two-question survey (see Appendix A) was administered via email, and questions therein prompted instructors to describe intended learning opportunities and outcomes, as well as other messages about mathematics teaching and learning that they may have tried to communicate to their students at the time. Instructors were also encouraged to look back at course materials while responding to the survey.

Based on the results of the survey, there was significant overlap in the pedagogical tools introduced across the three methods course sequences. While there may have been variation in the extent to which pedagogical tools were emphasized, instructors within each of the three sequences said they had hoped to support their students (prospective mathematics teachers) in:

- Conceptualizing mathematics as a series of connected ideas
- Extending ideas about mathematical proficiency beyond speed and accuracy (e.g., developing conceptual understanding)
- Conceiving the role of the teacher as one who facilitates learning rather than one who demonstrates how to solve problems
- Reflecting on the affordances of providing opportunities for students to struggle through and discuss high-level mathematical tasks
- Practicing anticipating and making sense of student thinking, with the idea that student thinking can be used to guide instruction
- Developing asset-oriented views of students

Recall, most of the ideas listed above were discussed in Chapter 2 to clarify the contextual discourses to which teachers may have had access in their methods courses.

In addition to the previously listed consistencies, there were also differences in the pedagogical tools introduced across the methods course sequences that are worth noting. First, in two of the three sequences (Emma's and Joe's sequence and Bri's sequence), Smith and Stein's (2018) framework from *Five Practices for Orchestrating Productive Mathematics Discussions* was introduced as an instructional model that might act as an alternative to more conventional approaches to teaching mathematics. Whereas only one instructor in Emma's and Joe's sequence promoted this framework, two instructors in Bri's sequence highlighted Smith and Stein's five practices. Second, while instructors within each of the sequences introduced issues related to equity in mathematics education, the nature of those issues varied both across and within sequences. For Sara—the second-year teacher—only one instructor reported discussing issues of equity in their course, explaining that they spent some time discussing how there may be bias inherent in standardized test questions. For Joe and Emma, three instructors reported attending to issues of equity in their courses, but the nature of those issues varied across individuals. To specify, the instructors reported: introducing social justice as a goal for teaching mathematics, suggesting that teachers may consider working to be agents of change (Instructor C); discussing how algebra operates as a gatekeeper to opportunities for some students, particularly students of color (Instructor A); and emphasizing the importance of relationship building (Instructor D). Last, for Bri, two of her instructors reported focusing on equity in their courses by: emphasizing relationship building, investigating historical and political aspects of the schools in which prospective teachers were conducting fieldwork, and introducing ideas related to teaching mathematics for social justice (Instructor C); and discussing strategies for engaging in creative insubordination to challenge practices and policies that operate to oppress students, particularly those who have been historically marginalized in mathematics (Instructor F—me).

Teachers and School Settings

All four of the secondary mathematics teachers identified racially as white and were in their early 20s at the time of the study. Three of the four teachers, Bri, Emma, and Sara, identified as women, and the fourth teacher, Joe, identified as a man. As mentioned earlier, three teachers, Bri, Emma, and Sara, were teaching at one high school, Schenley, which is located in a large, non-metropolitan area in the Midwest. According to publicly available district data, Schenley served a student population that was both racially and economically diverse (47% of students were students of color; about 53% of students received free-and-reduced lunch). Additionally, Schenley employed a block scheduling system, which means that classes were about 90 minutes long. The other teacher, Joe, was teaching at Beechwood, a small rural school district (Grades K–12 were housed within one building) that was approximately 40 miles from the district in which the other three teachers were working. In comparison to Schenley, Beechwood served a student population that was less racially and economically diverse (93% of students were white; 32% of students received free-and-reduced lunch). Beechwood followed a more traditional scheduling system, so classes lasted for about 50 minutes. Table 2 provides an overview the four cases, which are described in more detail in the paragraphs that follow.

Table 2
Overview of Teachers and School Settings

Teacher	Year Teaching	School	Courses Taught
Sara	2	Schenley	Algebra Functions, Geometry
Joe	1	Beechwood	Algebra I and II, Geometry, Pre-Calculus, Applied Mathematics, Agriculture Mathematics
Emma	1	Schenley	Algebra I and II
Bri	0	Schenley	Geometry in Construction, Geometry

Schenley teachers: Bri, Emma, and Sara. At the time of the study, Bri was student teaching at Schenley and had conducted fieldwork there during the previous semester under the mentorship of Mr. I (her host teacher) and a special education teacher who co-taught with him. During her student teaching internship, Bri was responsible for teaching one section of Geometry in Construction—a course intended to support students in learning geometry content through working on construction tasks—and three sections of Geometry. Over the course of the semester, Bri was observed four times by her university supervisor, who happened to be one of her former methods course instructors (Instructor C, see Table 1). Emma, who had conducted fieldwork and student taught at Schenley one year earlier (spring of 2018)—also under the mentorship of Mr.

I—was in her first year of full-time teaching. She was teaching three sections of Algebra II and two sections of Algebra I. Sara, who had conducted fieldwork and student taught under the mentorship of Ms. G at another high school in the district and was supervised by Instructor C two years prior, was in the second semester of her second year as a full-time teacher. She was teaching one section of Geometry and four sections of Algebra Functions—a transitional course between Algebra I and Geometry intended to support students who had received a failing grade in at least one semester of Algebra I. One of Sara's Algebra Functions classes was co-taught with the special education teacher who was also working with Bri at the time.

Given that Bri and Sara were teaching different sections of the same course (Geometry), they met every other day, along with the other Geometry teachers, during Professional Learning Team (PLT) meetings. PLT meetings were intended to provide opportunities for individuals teaching the same course to, among other things, share resources, discuss course standards, and track student data and plan interventions. Emma attended PLT meetings every other day as well but met with fellow Algebra I and Algebra II teachers. In addition to PLT meetings, the three teachers also attended mathematics department meetings once a month. Additionally, all full-time teachers at Schenley (which excludes Bri, the student teacher) were observed and evaluated multiple times throughout the school year by the mathematics department chair and/or a school principal.

Beechwood teacher: Joe. Joe was in his first year of teaching. As mentioned earlier, Beechwood was a small rural district where Grades K-12 were housed within one building. Given that Joe was the only high school mathematics teacher, he was teaching six different courses: one section of Algebra I, one section of Algebra II, two sections of Geometry, one section of Applied Mathematics, one section of Pre-Calculus, and one section of Agriculture

Mathematics, which was co-taught with and typically led by a practical arts teacher. According to Beechwood's school website, the Applied and Agriculture Mathematics courses were focused on making connections between mathematics and the "real world" (e.g., using mathematics to model compounding interest) and agricultural occupations (e.g., farming), respectively. Joe attended monthly staff meetings with all teachers in the building and, on occasion, met informally with the middle school mathematics teacher. Additionally, Joe was observed and evaluated multiple times during the school year by the principal and/or district superintendent.

Data Collection Processes

For all participants, a majority of data collection occurred during the spring semester of 2019 (January–May). For each of the two first-year teachers, data were also collected immediately before and throughout the fall of 2018 (July–December). Methods used to collect data for each of the cases reflected an ethnographic approach to research, as sources of data included field notes of classroom observations, formal interviews, debriefing interviews, and artifacts. Once transcribed, observation and interview data for each teacher were stored in digital folders. Artifacts (e.g., handouts for students) were also stored separately but in tangible folders. An overview of data collected for each teacher is provided in Table 3.

Table 3
Overview of Data Collected

Teacher	Number of Classes Observed	Number of Days Observed	Total Observation Time	Number of Formal Interviews	Number of Debriefing Interviews	Total Interview Time
Sara	10	9	15:00:00	2	7	5:26:22
Joe	19	10	15:12:00	3	8	10:00:37
Emma	10	10	13:10:00	3	7	6:52:24
Bri	10	9	14:05:00	2	9	5:11:16

Field notes

For each teacher, I observed at least 10 lessons and recorded descriptive field notes during each observation (Emerson, Fretz, & Shaw, 2011). Field notes were either handwritten or typed in Microsoft Word, but handwritten field notes were transcribed following each observation. During early observations, I attended to teachers' instructional practice, noting specifically when teachers enacted ambitious practices to which they may have been introduced in their methods courses (see previous section: Methods Courses), and provided details of teachers' interactions with students, including, for example, the nature and patterns of talk during small-group discussions. Over time, as I conducted more interviews, I sometimes changed or narrowed the focus of observations, attending more explicitly to teachers' enactment of certain practices (Savin-Baden & Major, 2012). For example, if a teacher noted in a debriefing interview that they were working to improve a particular form of practice, I would focus more closely on how the teacher used that practice during an observation.

During classroom observations, I assumed the roles of observer and participant (Savin-Baden & Major, 2012) but prioritized my role as an observer. To avoid interfering with instruction, my participation in the classroom was limited to supporting students during individual or small-group work time per the request of the teacher or students. When relevant, I provided accounts of my own participation in the classroom, noting the details of my interactions with students. During these interactions, I moved back and forth between participant and observer, as I recorded details of what the teacher was doing when possible (see Clandinin & Connelly, 2000, p. 105).

Formal Interviews

Formal, semi-structured interviews were conducted with each teacher in person or, in the case of two interviews, over the phone. The duration of each formal interview was between one and two hours. All formal interviews were audio recorded and transcribed. Each teacher was formally interviewed at least twice, once before the beginning of the spring semester of 2019 (December or January) and once at the end (May). I also interviewed the two first-year teachers (Emma and Joe) before their official start dates in the fall of 2018, providing a total of three formal interviews for each of them.

Formal interview questions were designed to elicit teachers' personal theories about productive mathematics teaching and learning—*critical pedagogical discourses*—and teachers' perceptions of expectations for teaching within their school settings—*contextual discourses* (Thompson et al., 2013). Questions prompted teachers to describe their: visions of high-quality mathematics instruction (Munter, 2014), perceptions of others' instructional visions, and differences between them (Jansen et al., 2018); views of students' mathematical capabilities (Jackson et al., 2017); and interactions with colleagues, administrators, and others in various school and district contexts (Cobb, Jackson, Henrick, Smith, & the MIST Team, 2018; Thompson et al., 2013) (see Appendices B–D).

Debriefing Interviews

Following classroom observations, I conducted debriefing interviews with teachers that lasted between 5 and 45 minutes. Because teachers were not always available to debrief, I was not able to conduct interviews following each of the 10 observations. Debriefing interviews were typically audio recorded (I took notes during two unrecorded interviews) and later summarized. Summaries of debriefing interviews included field notes that corresponded with those interviews, and were organized according to topics discussed during debriefing conversations. Summaries also included interview excerpts that were transcribed verbatim, as well as analytical and observational notes in which I described emergent themes and recurring patterns.

Debriefing interviews were informal in nature (Merriam & Tisdell, 2016), beginning with open-ended questions, followed by questions specific to teachers' instructional practice and their school settings. Debriefing interview questions prompted teachers to describe: reactions to a lesson, rationales for practices used, instructional goals and perceived obstacles to meeting those goals, and perceived expectations for teaching specific to their school setting (Thompson et al., 2013) (see Appendix E). Generally, these interviews were opportunities for me to listen to in-the-moment insights that teachers might have had. However, if during debriefing interviews, teachers solicited feedback or support from me, I did offer ideas. For example, if a teacher asked for support in designing a high-level mathematical task because they found available curriculum resources to be insufficient, I honored their request by sharing my ideas.

When reporting findings, data from field notes, formal interview transcripts, and debriefing interview summaries are referenced using the following format to indicate the data source and date on which the data was collected: Data source, YYMMDD. For example, a reference to a formal interview conducted on April 4, 2019 will appear as follows: Formal interview, 190404.

Data Analysis

Data analysis for this multiple-case study occurred in two phases: within-case analysis and cross-case analysis (Merriam & Tisdell, 2016; Miles, Huberman, & Saldaña, 2014). In the first phase, I analyzed both interview and observation data for the four cases separately, attending to each teacher's critical pedagogical discourse, perceptions of contextual discourses, instructional practice, and relations between them. After developing cases for each teacher, I conducted a cross-case analysis to identify patterns and themes that spanned the four cases (Merriam & Tisdell, 2016). In the sections that follow, I describe within-case analyses by describing how data were coded to characterize critical pedagogical discourses, contextual discourses, instructional practices, and relations between them, and then discuss cross-case analysis processes.

Within-Case Analyses

Analysis of teachers' critical pedagogical discourses. For each teacher, I characterized their critical pedagogical discourse by coding transcripts of all formal interviews and debriefing interview summaries in two cycles using the NVivo qualitative software program. Processes followed for the two cycles of analysis are discussed in the two sections that follow.

Cycle one. Beginning with the transcript of each teacher's first formal interview, I generated "Descriptive" and "In Vivo" codes (Miles et al., 2014; Saldaña, 2016) to categorize the ways in which teachers talked about pedagogical ideas and practices—both in general (e.g., their visions for instruction; Munter, 2014) and, if possible, in relation to their own instruction (Thompson et al., 2013). Descriptive codes, which are intended to summarize the main topic of an excerpt of data, were generated and applied to excerpts in which teachers talked about instructional practices (e.g., *teaching for conceptual understanding*) or challenges (e.g., *behavior*). Because in some interview excerpts, teachers described practices in unique ways, I generated In Vivo codes that were based on the teachers' own language. For example, if in an interview excerpt, a teacher said that they wanted to use "collaborative learning" strategies more often, that particular phrase was used to code that excerpt. While it was common for Descriptive codes to be applicable across teachers' interviews (e.g., mathematical tasks), In Vivo codes were generally particular to individuals (Clandinin & Connelly, 2000).

While I coded teachers' talk about any pedagogical idea or practice, only those that teachers identified as goals for their teaching or described when articulating their visions for instruction became candidates for "threads" of their critical pedagogical discourses. For example, if in excerpts from the first formal interview transcript a teacher named posing "open-ended tasks" as a goal for their instruction and described "giving notes" as something they wanted to refrain from doing, both of those excerpts were coded (using the codes *mathematical tasks* and *giving notes,* respectively), but only the content of the former was identified as a thread of dialogue that may constitute that teacher's critical pedagogical discourse. Coded excerpts in which teachers described practices that they did not aspire to use were not considered as candidates for threads of teachers' critical pedagogical discourses, given Thompson and colleagues' (2013) definition of the construct: "personal theories about 'what counts' as *productive* [emphasis added] teaching and learning" (pp. 578–579).

After coding a teacher's first formal interview transcript, I coded the debriefing interview summaries and remaining formal interview transcripts in chronological order by applying the Descriptive and In Vivo codes applied to the first formal interview transcript (if possible) and generating new ones if necessary. Once all transcripts and summaries were coded, I coded them a second time by applying the complete list of codes. Following this process, I began the second round of coding to identify themes, or threads, in a teacher's talk across interview transcripts and summaries.

Cycle two. To identify threads of a teacher's critical pedagogical discourse, I attended to both the frequency with which each code was applied, and identified broader themes that spanned coded segments. First, for codes that could be traced across both formal interview transcripts and debriefing interview summaries, I identified the content of coded excerpts as candidates for threads of the teacher's critical pedagogical discourse. If a code was applied to excerpts of formal interview transcripts but not to debriefing interview summaries, then the content of those coded excerpts did not count as a candidate for a thread. This was because the teacher did not draw on ideas reflected in that content when reasoning about their own practice.

After candidates for threads of a teacher's critical pedagogical discourse were identified, I considered whether broader themes in that teacher's talk consistently spanned coded excerpts. When themes emerged, I coded those excerpts a second time by applying a new "Pattern" code to account for the broader theme (Miles et al., 2014). For example, if a teacher consistently described two instructional practices as means for ensuring student engagement, I coded relevant excerpts by applying the Pattern code, *teaching as a goal for student engagement*, to account for this broader theme. After Pattern codes were identified and applied across interview transcripts and summaries, the content of excerpts to which those codes were applied were deemed threads of a teacher's critical pedagogical discourse (see Appendix F for list of codes for each teacher).

Analysis of teachers' perceived contextual discourses. To characterize teachers' perceived contextual discourses, I followed coding processes similar to those described in the previous section in that I coded all formal interview transcripts and debriefing interview summaries in NVivo but in only one cycle (Miles et al., 2014).

Because contextual discourses were conceptualized as teachers' self-reported perceptions of broader messages about mathematics teaching and learning communicated in their schools, different segments of interview data—from both formal interview transcripts and debriefing interview summaries—were coded for this round of analysis. Examples of such segments included instances in which teachers discussed their perceptions of: others' visions for instruction; expectations communicated by colleagues, mentors, and administrators; issues addressed in meetings with other teachers (e.g., PLT meetings); curriculum and learning standards; school policies and procedures; and constraints and obstacles to meeting instructional goals.

First, I generated codes to categorize a teacher's perceptions of contextual discourses or (anticipated) constraints in the transcript of their first formal interview. When coding relevant excerpts, I applied both Descriptive and In Vivo codes to categorize sources of contextual discourses (i.e., who or what, from the teacher's perspective, communicated that discourse), as well as the content of those discourses. After generating codes for the first formal interview transcript, I coded the debriefing interview summaries and remaining formal interview transcripts in chronological order. When possible, I applied codes generated when analyzing the first formal interview transcript to subsequent summaries and transcripts but also generated new codes when necessary. Once all interview summaries and transcripts were coded, I coded them a second time by applying the complete list of codes.

To illustrate these coding processes, if in an excerpt of a formal interview transcript a teacher explained that when their principal observes them, the principal looks to see how many students were engaged, I coded the excerpt using the Descriptive codes, *principal* (to account for the source of the contextual discourse), and *student engagement* (to account for the content of that discourse). When a teacher did not name the source of a contextual discourse, only the teacher's description of the nature of that discourse was coded (see Appendix G for list of codes for each teacher).

Analysis of teachers' instructional practice. For each teacher, I characterized their instructional practice by coding field notes in NVivo in two cycles (Miles et al., 2014). In the first cycle, I generated codes to characterize a teacher's instructional practices and routines.

Then, in the second cycle, I coded field notes a second time to make qualitative distinctions among a teacher's practices and routines. Processes followed for each cycle of coding are described in more detail in the two sections that follow.

Cycle one. Beginning with field notes collected during each teacher's first observation, I generated and applied Descriptive codes to categorize field notes excerpts in which I described teachers' instructional practices and routines. Descriptive codes were generated by drawing on ambitious practices described in Chapter 2, or by naming common routines. For example, Descriptive codes from the literature reviewed in Chapter 2 included *launch* (Jackson et al., 2013) and *mathematical task* (Smith & Stein, 1998), while Descriptive codes generated to name common routines included *warm-up* and *structured note-taking*. Codes generated from field notes collected during the first observation were then applied to field notes collected during subsequent observations, and new codes were generated and applied when necessary.

Cycle two. During the second round of coding, I generated and applied new codes to further characterize a teacher's practice within field notes segments that had already been coded. For example, for segments of field notes that had been categorized using the Descriptive code, *mathematical task*, I generated and applied codes from Smith and Stein's (1998) Task Analysis Guide to characterize the cognitive demand of the tasks that a teacher posed to students during observations. Since some of those tasks were posed via handouts, I coded those as well. As another example, for field notes segments in which classroom discourse was described (e.g., while a teacher interacted with students in small-group settings), I coded the nature of the dialogue (e.g., calculational, conceptual, or both), as well as evident patterns of talk (e.g., talk turns alternating between teacher and students) (see Appendix H for list of codes for each teacher).

Analysis of relations between discourses and practices. To consider how teachers' practice related to interactions between critical pedagogical and contextual discourses, I first drew on analyses of observation data to construct narratives that characterized a teacher's mathematics teaching. Then, following methods employed by other researchers (e.g., Braaten, 2019; Ward et al., 2011), I identified discourse interactions by comparing a teacher's critical pedagogical discourse to the contextual discourses they reported, attending specifically to whether or not teachers reported feelings of *tension*. For example, if in an interview excerpt a teacher reported feeling pressure to teach in a way that conflicted with their vision for teaching, I characterized that excerpt as a tension. Interactions were then analyzed in relation to the instructional narratives written for each teacher in order to consider how such tensions may have been consequential for a teacher's practice.

Cross-Case Analyses

After conducting the within-case analyses described in the previous section, I reported findings of those analyses by writing narratives for each individual case, which are shared in Chapter 4. The four case narratives were then compared to identify themes and patterns that spanned the cases and, ultimately, to answer the three research questions that guided this study. In particular, I compared critical pedagogical discourse threads, self-reported contextual discourses, discourse interactions, teaching patterns, and relations between those things to identify cross-cutting themes. Based on some of those themes, I drew on literature reviewed in Chapter 2 to make qualitative distinctions between the four cases. For example, because initial cross-case comparisons showed that critical pedagogical discourses varied in terms of teachers' descriptions of learning, teaching, students, and broader (political) contexts, I drew on frameworks offered by Kang and Wallace (2005), Munter (2014), Jackson and colleagues

(2017), and Bartolomé (2004), respectively, to help describe the differences I noticed. First, applying Kang and Wallace's (2005) framework for epistemological beliefs or "personal epistemology" (Kang, 2008), I returned to coded interview excerpts to identify differences in the ways in which teachers' critical pedagogical discourses reflected their teaching goals and views of students in relation to mathematics. Then, following similar processes when applying the other frameworks, I attended to differences in how teachers' critical pedagogical discourses reflected their instructional visions (Munter, 2014), views of students and their mathematical capabilities (Jackson et al., 2017), and relations between students' schooling experiences and broader sociopolitical and economic contexts (Bartolomé, 2004).

Additionally, since cross-case analyses revealed differences in teachers' enactment of ambitious practice, I drew on the teaching patterns identified by Thompson and colleagues (2013) to further characterize those differences: lack of appropriation, compartmentalization, and integration. The results of cross-case analyses described in this section are reported in Chapter 5.

CHAPTER 4: FINDINGS OF WITHIN-CASE ANALYSES

In this chapter, I discuss each of the four cases, Joe, Emma, Sara, and Bri, separately. The cases are presented in such a way to account for similarities and differences between them. The case of Joe, one of the first-year teachers, is presented first because he worked in a school setting in which the others did not. In the interest of comparing the two first-year teachers, Emma's case is presented after Joe's. Then, for the sake of comparing cases situated within the same school setting, Emma's case is followed by those of Sara, the second-year teacher, and Bri, the student teacher. The presentation of each teacher's case begins with a description of its overarching themes and is followed by a discussion of (relations between) the teacher's critical pedagogical discourse, perceptions of contextual discourses, and instructional practice. However, given the idiosyncrasies of each case, there are slight differences in the ways in which the discussions are structured.

The Case of Joe

Joe's case provides an example of a beginning mathematics teacher who consistently promoted yet struggled to enact ambitious practice because he was susceptible to contextual pressures to teach in conventional ways and lacked sufficient resources. Consequently, Joe's practice was reflective of his critical pedagogical discourse to only a small extent and, therefore, was largely driven by the contextual discourses he perceived. In the sections that follow, I describe Joe's case in more detail by first providing an overview of his mathematics teaching. Then, after discussing four prominent threads of Joe's critical pedagogical discourse, I reflect on the contextual discourses (and constraints) he reported, and how those discourses helped to explain gaps between his envisioned and enacted practice.

Joe's Mathematics Teaching

In general, a few things were evident from observations of Joe's teaching. First was Joe's consistent effort to build relationships with students, as each day he dedicated time to talking with them about their interests and lives outside of school, as well as his own experiences. Second was that Joe's primary approach to teaching was conventional in nature, as he typically demonstrated to students how to solve low-level mathematical tasks and then provided opportunities for students to practice solving similar tasks on their own. Important to note, however, is that Joe sometimes enacted ambitious practice by posing high-level tasks that prompted students to make connections between mathematical concepts, asking students to consider why the procedures they were using to solve problems worked, and encouraging students to share different strategies and answers to problems. In the sections that follow, I discuss Joe's teaching in more detail by describing instructional practices and routines that were foundational to his teaching.

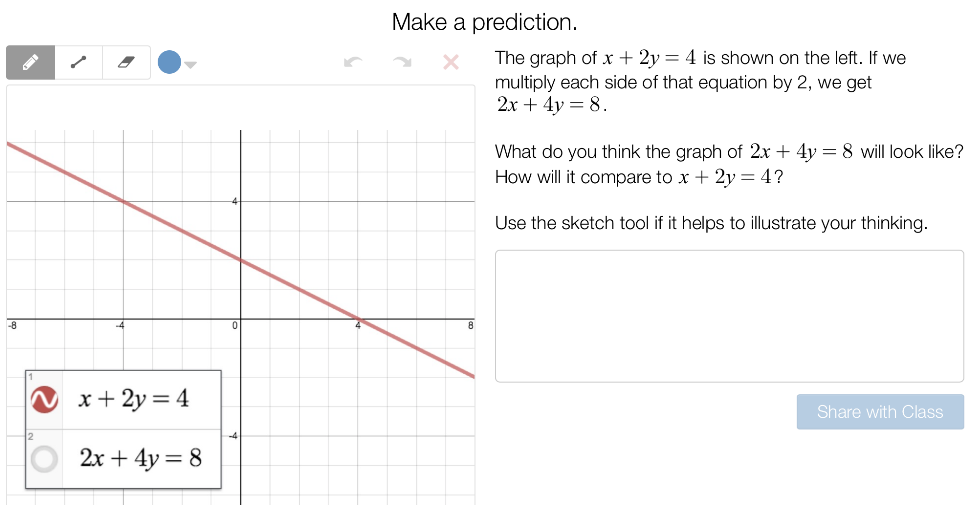
Pre-lesson conversations. Joe often began class by initiating informal conversations with students. During those conversations, which usually lasted around five minutes, Joe asked students how their days or weekends had been, and shared "jokes of the day" or silly personal stories. Students regularly laughed at Joe's jokes and stories and sometimes requested to hear ones he had told them before. Like Joe, students willingly shared stories of their own and talked about their lives outside of school or what was going on in other classes. Conveying what seemed like a great sense of comfort, Joe's students often addressed him by his last name—"Hey, Anderson" (Field notes, 190502)—and expressed sarcasm in conversations with him: "Anderson, I didn't do my math homework this weekend. Are you proud of me?" (Field notes, 190122).

Warm-up tasks. Warm-up tasks, which Joe projected on the SMART Board after pre-lesson conversations, were always focused on mathematical topics discussed in previous lessons and typically consisted of one to a few low-level mathematical tasks (Smith & Stein, 1998). Examples of warm-up tasks that Joe posed to students included: "Solve $x^2 + 8x + 12 = 0$ using the quadratic formula" (Field notes, 190308); and "Simplify $(4c^3)^2$" (Field notes, 190110).

Desmos activities. On three occasions, Joe prompted students to "get out [their] Chromebooks" to work on a Desmos activity—Desmos is an online application that provides free access to an advanced graphing calculator. When introducing Desmos activities, Joe did not explain the activities beyond describing broad foci and goals (e.g., "[This is] a review of quadratics." Field notes, 181210) nor did he clarify expectations for working on them beyond encouraging students to work together.

Joe retrieved Desmos activities from an online bank of resources where he searched for tasks by typing the topic he planned to teach on a given day into the search bar (e.g., "direct variation" Field notes, 190502). The Desmos activities that Joe assigned were quite long—one consisted of 22 sub-tasks—and could be characterized as "Procedures with Connections" tasks (see Smith & Stein, 1998) as they prompted students to explain their reasoning and make connections between different mathematical representations (e.g., equations and graphs). As an example, consider Figure 1 in which a sub-task from a Desmos activity that Joe posed to his Algebra I class is presented (Field notes, 181011). For this particular sub-task, students were prompted to consider the graph of the equation $x + 2y = 4$, predict how its graph would change after multiplying the equation by two, and describe how the new graph (of $2x + 4y = 8$) might compare to the original. Because this task requires making connections between a procedure (multiplying an equation of a linear function by 2) and a graphical representation, "Procedures with Connections" is an appropriate characterization.

Figure 1
Desmos Sub-Task Example



Note-taking sessions. To begin note-taking sessions, which either followed a warm-up task or Desmos activity or compromised entire lessons, Joe specified what the day's topic was (e.g., "Today, we're going to continue learning how to factor quadratics." Field notes, 190116). After introducing the day's topic, Joe demonstrated procedures for solving problems related to that topic and then asked students to try solving similar problems. During demonstrations, Joe solicited ideas about steps for solving the problem (e.g., "How do we cancel the exponents?" Field notes, 181210) and invited questions from students, who seemed to be comfortable asking them (e.g., "Why did you [Joe] write positive and negative (+/-)?" Field notes, 190110). On some occasions, Joe also attended to mathematical concepts by explaining (or asking students to explain) connections between mathematical representations (Field notes, 181011, 181210, 190110, 190308) or why certain procedures worked (Field notes, 190122, 190403). However, on only one occasion were such concepts the central focus of a note-taking session (Field notes, 190110).

Individual work time. Students typically worked on tasks alone. Despite the prevalence of off-topic conversations during individual-work time, students always worked on the tasks that Joe posed, sometimes asking each other questions when they were unsure of how to move forward (e.g., "How do you factor again?" Field notes, 181210). While Joe sometimes redirected students during individual work time, he more often allowed students to engage in informal conversations, occasionally joining in himself.

Monitoring. While students worked on warm-up tasks, Desmos activities, or practice problems, Joe circulated the room, monitoring students' progress. As Joe monitored, students would sometimes ask him for help or, if they had finished solving the task, whether their answers were correct. As he visited with students, Joe sometimes reminded them to get started (e.g., "Why don't you go ahead and open a new tab there?" Field notes, 190502), asked questions (e.g., "What does it mean to expand?" Field notes, 190122), made suggestions (e.g., "If you do it that way, you're gonna get a fraction." Field notes, 181011), or told students whether their answers were correct. After Joe visited with students, they sometimes stopped working until Joe called the class together to discuss correct solutions. Once Joe had written solutions on the SMART Board, it was typical for students to copy his work in their notes.

Whole-class discussions. Whole-class discussions always followed students' work on warm-up tasks, Desmos activities, or practice problems, and reflected a pattern of talk that alternated between teacher and student. During whole-class discussions of Desmos activities, Joe projected sub-tasks from the activity on the SMART Board, and then led a discussion that focused on "what [students] were supposed to take away" from it (Field notes, 181011). For example, in the discussion of the Desmos sub-task shared earlier (see Figure 1), Joe shared a list of students' predictions—about how the graph of the equation $x + 2y = 4$ would change after multiplying it by two—on the SMART Board. Referencing the list of student predictions, Joe noted that some students suggested that the slope of the line would change (which is incorrect), and asked one student who made that prediction if he would share his rationale. The student explained his prediction by asserting that the "slope is in front of the *x*" and, therefore, would change in value if it were multiplied by two. In response, Joe explained that the entire equation—and "not just the slope"—would be multiplied by two, noting that the values of the constant term and coefficient of the *y* term would also change. Then, Joe clarified a key idea, or "takeaway," from the activity: "If we multiply both sides of the equation, we get the same equation, but it just looks different" (Field notes, 181011). After emphasizing a task's intended takeaways, Joe typically transitioned into leading a formal note-taking session.

During whole-class discussions that followed students' work on warm-up tasks or practice problems, Joe either called on students to describe their strategies, or explained his own strategy step by step. When Joe took the lead on explaining how to solve a problem, he often solicited ideas from students about next steps (Stein et al., 1996). Though less frequently, Joe sometimes shared common mistakes he noticed when monitoring and asked students whether they had found alternative strategies (e.g., solving for *x* instead of *y* when solving a system of equations using substitution, Field notes, 181011) or answers (e.g., 25% and ¼, Field notes 190403).

Joe's Critical Pedagogical Discourse: Four Prominent Threads

As a reminder, teachers' critical pedagogical discourses are threads of appropriated dialogue through which teachers articulate their personal theories about what constitutes high-quality teaching and learning (Thompson et al., 2013). And while teachers' critical pedagogical discourses are consequential for their instructional decision-making, their actions may not always align with the values and practices reflected in their discourse. In my analysis, I found that Joe's critical pedagogical discourse included four prominent threads of dialogue, which means that he maintained ideas central to those four threads as important for mathematics teaching and learning throughout the course of the study. Those four threads are discussed in the four sections that follow.

Conceptual understanding as a goal for student learning: Thread one. Through one prominent thread of Joe's critical pedagogical discourse, he argued the importance of establishing *conceptual understanding as a goal for student learning*. Throughout interviews, Joe was persistent in asserting that good mathematics instruction necessitates teaching for conceptual understanding, and often reflected on whether he had been successful in providing opportunities for students to develop it. In Joe's rationales for why conceptual understanding mattered to him, he argued that students should not "mindlessly" carry out procedures but develop an understanding of *why* those procedures work. From Joe's perspective, this required attending to ideas underlying mathematical procedures and solutions. To illustrate this thread of Joe's critical pedagogical discourse, consider the following exchange from our second formal interview in December of 2018 in which I asked him to explain why teaching for conceptual understanding was important:

C: You said you would look for teaching for conceptual understanding. Can you say a little bit about that? Why is that important, and how would you know that's happening?

J: Yeah so, you know, not just limiting the context in the classroom to just solving particular problems with a procedure, but actually exploring some of the intricacies of the math going on.

C: What do you mean by intricacies?

J: Good question [*laughs*].

C: You can give an example or something.

J: Okay. So not only telling [students]- I don't know. Think absolute values, I guess. Not only telling them how to solve an absolute value problem but making sure they can relate the solution back to what the absolute value does. Like, how does the solution being -5 and 5 represent the meaning of absolute value as a distance? So not only can they solve the problem, but they recognize that there's this correlation with distance. And how does this connect to the property of distance? How does the answer being 3 and -6 actually relate to distance? (Formal interview, 181229)

In this exchange, Joe suggested that in addition to solving problems by applying procedures, students should also be able to make sense of mathematical concepts related to those procedures. To provide an example, Joe suggested that if students can solve absolute value problems, they should also be able to recognize that the solutions to those problems "relate to distance."

While not the focus of this section, it is worthwhile to note Joe's depiction of *how* students might learn to solve and understand the concepts that underlie absolute value problems. In this particular instance, Joe portrayed the teacher's role as one of "telling" students how to solve the problem. Though it is unclear whether Joe considered the practice of telling instrumental in supporting students in developing conceptual understanding, telling is in contrast to another thread of Joe's discourse, *student learning requires providing opportunities for thinking, reasoning, and struggle,* which is discussed in the next section.

Student learning requires providing opportunities for thinking, reasoning, and struggle: Thread two. Through a second prominent thread of his discourse, Joe argued that *student learning requires opportunities for students to think, reason, and struggle* through mathematics, and identified the provision of such opportunities as an important goal for his practice. According to Joe, instrumental in his efforts to provide such opportunities was leading a particular kind of lesson that he described as "task-based" (Formal interview, 190527).

From Joe's perspective, the importance of leading "task-based" lessons (and the subsequent opportunities afforded) was an important takeaway from teacher education. Illustrated by the following excerpt from our first informal interview in August of 2018 (which proceeded Joe's official start date), he suggested that his own ideas about the mathematical activity in which students should have opportunities to engage—and, thus, the types of tasks they should be working on—changed after the start of his teacher education program:

> Before I entered college, I would've said that students should always know what they should be doing on a problem [and] what steps they need to take. But now I believe that students should be able to struggle through a problem, figure it out for themselves, really get to think about it, make their own conclusions and realize for themselves what they need to do. Because that will help them remember it better…So I guess my view of learning switched from kind of like the basic drilling into more of like, "Well no, students should really be thinking hard." They should not just be repeating the same steps with their pencil, drilling it into their heads. But they should be drilling, you know, the thoughts and, like, what mathematical thinking is. Like, you know, "Let's try this. Oh, it

> doesn't work. But what ideas can I learn from this experience?" And like, "Oh, maybe this strategy would work." Just being able to test things out for themselves. (Formal interview, 180803)

In reflecting on how his ideas changed over time, Joe made distinctions between the demands of different types of mathematical activity: that which involves students knowing "what steps they need to take" and "repeating the same steps," and that which involves engagement in mathematical practices, such as struggling through a problem, testing out strategies, and drawing conclusions. In the excerpt above, Joe privileged the latter type of activity for reasons related to student learning, as he asserted that students are more likely to "remember" ideas if provided opportunities to first struggle and think through them. Recognizing that such opportunities would arise only from lessons in which certain types of mathematical tasks were posed, Joe hoped to enact such tasks during his first year of teaching.

When prompted to describe tasks that might afford the opportunities he envisioned, Joe named multiple defining characteristics. Across a number of interviews, he suggested that an ideal mathematical task would: be "open-ended" (Formal interview, 180803); require reasoning that extends beyond applying a previously learned algorithm (Formal interviews, 180803; 190527); "foster pattern recognition" (Formal interview, 190527); "have multiple levels to it" (Formal interview, 181229); and allow for "multiple solutions" (Formal interviews, 180803; 181229; 190527) that "connect with each other" (Formal interview, 180803). In addition to their potential to support students in thinking, reasoning, and struggling through mathematics, Joe argued that lessons centered around the tasks he envisioned were important because they are accessible to a range of students, regardless of their current capabilities, and ensure student engagement as, Joe suggested, they "can't be solved immediately." To illustrate this further, consider the following excerpt from our final formal interview in May of 2019 in which Joe described why a particular "task-based" lesson was ideal:

> There's a lot of different levels that you can access the task. So at the bare minimum, I could just go to a student and be like, "Okay, see what happens at 9." And then, you know, other students can be making tables, connecting it to like- A really cool pattern emerges with powers of 2 and modular arithmetic, and you can go really advanced with that. But you can also still ask students at the most basic level like, "Hey, just see what happens at 7." And they can be like, "Oh, at 7? Well, the third person [wins]. And you're like, "Okay." (Formal interview, 190527)

Here, Joe argued the benefits of posing a particular task by pointing to both its accessibility and affordance of opportunities for exploration (e.g., making tables, recognizing patterns).

From the outset of the study, however, Joe anticipated that it would be challenging to regularly engage students in task-based lessons (described more later in a discussion of the *contextual discourses* Joe perceived). Consequently, Joe aspired to implement only one task-based lesson per month in each of his classes. Because of the challenges Joe anticipated, he envisioned that students would be working on tasks that varied in notable ways from those he considered ideal. For example, consider the following interview excerpt from our first formal interview in which Joe described tasks that, from his perspective, were more feasible to pose:

> I would like to still not do the basic kind of drills where it's like, "Hey, now find the angle of this. Now find the angle of this." But still do, not really open- obviously I'd like to do open-ended problems. But more realistically, I'll probably do difficult problems that [students] can still work together to solve [and] kind of [incorporate] multiple things. One of my favorite things to do is have a problem that's relevant to the unit but requires

> you to use past knowledge to solve it. Because it really makes mathematics seem connected rather than, "We're learning topic A, topic B, and now we're gonna learn topic C." But [I'd like to] have a problem that has a little bit of everything in it…A really basic example is instead of having all of the side lengths [of a shape] given to you, you have to do some algebra. So if this side length is x, and this side is $2x+7$, what's the area? I don't know if that's even a real problem you can solve, but you get what I mean. (Formal interview, 180803)

Here, Joe suggested that "more realistically," he would pose "difficult" problems that require students "to use past knowledge to solve it," allowing them to recognize mathematics as a series of connected, rather than isolated, ideas. However, the demands of the types of tasks that Joe described in the excerpt above are, in many ways, different from those of the tasks he described as ideal. For example, consider the "difficult" task that Joe described in the segment above. By "difficult" task, Joe seemed to mean one that requires students to draw on previously learned procedures and carry out multiple steps (e.g., first recall an area formula and then "do some algebra"). Such opportunities, however, would not necessarily support students in thinking, reasoning, and struggling in the ways he envisioned. That Joe did not recognize this tension is interesting and suggests, perhaps, that aspects of his critical pedagogical discourse may have been at odds with one another.

Student learning requires providing opportunities for discussion: Thread three. In a third thread of his critical pedagogical discourse, Joe emphasized the importance of *providing opportunities for discussion,* describing his use of this practice as another goal for his teaching. In Joe's rationales for why providing opportunities for students to discuss mathematics mattered to him, he suggested that such opportunities could lead to multiple outcomes including student engagement, learning, and enjoyment.

In recalling his own experiences as a student in the classroom of one of his former mathematics teachers, Joe expressed appreciation for opportunities for discussion as, based on his experiences, they allowed for “more engaging” lessons:

> [O]n the instruction side, she definitely made it more engaging. She tried to let students always talk, and she did the- kinda what I want to do with small groups. Or, you know, students could talk with one another. And then she would reconvene and have representatives [from each of the groups] talk…(Formal interview, 180803)

Here, Joe’s rationale for valuing opportunities for discussion is rooted in engagement, as he described the practice of “let[ting] students talk” as a means for “engaging” students. While not reflected in the excerpt above, across other interviews Joe envisioned that student engagement would ideally involve students “sharing ideas,” “constantly thinking,” and “doing more than taking notes.”

In other instances, Joe suggested that providing opportunities for discussion mattered for others reasons, such as student learning and enjoyment. From Joe’s perspective, these ideas were also important takeaways from his teacher education program, which he described in the following excerpt from our first formal interview:

> Before [methods courses], I was like, “Oh yeah, student discussion. Have one student help another student.” But that’s like the most basic possible thing you could do. There’s a lot more you could do with it. You can do some really fun activities and have students like, you know, even enjoy the work occasionally. And you can have students not only learn the mathematics but learn the skills of arguing civilly [and] discussing with others,

> which I guess are technically a part of mathematics—being able to argue appropriately, being able to see the flow of an argument and critique it… Standard 3 of Common Core! That's what I'm thinking of. (Formal interview, 180803)

In this quote, Joe explained what, to him, seemed like obvious reasons for providing students opportunities to talk: at a very "basic" level, those opportunities could allow for student-to-student support. Beyond that, however, Joe argued that opportunities for discussion can lead to students learning and enjoying mathematics. Also illustrated in the excerpt above is Joe's framing of student discussion as a valuable outcome in and of itself, as "arguing civilly" and "discussing with others," he suggested, "are technically a part of mathematics."

That opportunities for discussion can lead to student learning was something Joe maintained throughout the course of the study. In the following excerpt from our midyear formal interview in December of 2018, Joe explained why providing opportunities for students to talk was something he hoped to accomplish:

> I think it's one of the ways that makes them start thinking about the content, and it makes them aware of what they know and what they don't know. I found that when doing just basic practice problems, a lot of students, if they don't understand it, are just gonna stop and wait for an explanation on their own. And that kind of hurts the knowledge of it because, you know, in math you're gonna get the best results if you think about it yourself. And then, you know, you're getting refinement of that idea rather than just waiting to hear someone else's idea and trying to force your mind to accept that idea. So I think the student discourse is forcing them to start thinking about the idea and then that helps them understand the content better. (Formal interview, 181229)

Here, Joe framed discussion as instrumental in students' construction of knowledge, noting potential limitations of "wait[ing] for an explanation" without having the opportunity to think through and talk about problems first. His assertion, then, seems to be that requiring students to talk not only shifts authority to students but also "forces" them to think, which, ultimately, leads to learning.

However, often absent from Joe's talk about student discussions and why he valued them was the focus of those discussions: what students would be talking about. Although Joe, when prompted, described what he hoped student discussions would entail, he more frequently attended to patterns—rather than the nature—of student talk. For example, consider the following excerpt in which Joe reflected on a discussion that occurred in his Algebra I class:

> Like when Gene was saying, "Okay, well the slope is in front of the *x*," and, you know, I asked a question about that. And then I had Lou respond to that. I would like to like cut out the middle man. Like have Lou, who is already thinking in his head, be comfortable directly responding after Gene finishes his statement. (Debriefing interview, 181011)

Rather than reflecting on the nature of the discussion, Joe expressed concerned with patterns of talk, noting how he played the role of "middle man" in a discussion that, ideally, would have occurred between students. While not reflected in this particular excerpt, "cut[ting] out the middle man" (i.e., the teacher) in discussions was important to Joe because, from his perspective, it allowed him to shift authority to students: "student opinions [should be] valued as part of the learning process…I feel like a lot of times, [students] just wait for me to be like, 'Yep! That's right'" (Formal interview, 190527).

Important to note is that Joe was, at times, concerned with the focus or nature of discussions in his classroom (recall that *conceptual understanding as a goal for student learning* was a thread of his critical pedagogical discourse). But perhaps because Joe often struggled in his attempts to facilitate discussions between students (discussed more in a later section), he was more likely to attend to whether students were talking to each other than what, exactly, they were talking about.

The importance of building relationships with students: Thread four. In the fourth and final thread of his critical pedagogical discourse, Joe emphasized the importance of *building relationships* with his students. Consistently among the reasons Joe provided for caring about building relationships was so that students felt like they had an adult at school whom they could "trust" and "confide in." In the excerpt below from our first formal interview, Joe expressed this sentiment:

> I think it's a pretty big responsibility that you build a relationship with your students. I mean, I love- I could talk about math all day. But, you know, especially since I started student teaching, I realized just how much of a difference being an adult to some students makes. Like, making sure they have at least one good relationship at school…I mean, you're not gonna get along with all of your students. But making sure that students have at least one adult in the building that they can rely on. (Formal interview, 180803)

In this excerpt, Joe reflected on how his student teaching experiences made him realize that being a mathematics teacher requires doing more than "talk[ing] about math all day," noting that it is also important for teachers to build relationships with their students so that students "have at least one good relationship at school."

Joe often distinguished between the practice of building relationships and "the pedagogy side" of his job, sometimes framing the former as more important than the latter. Consider, for example, the following excerpt from our final formal interview in which Joe described his responsibility to teach mathematics in relation to his responsibility to care for his students:

J: A lot of students are just incredibly stressed out, and they have so many problems in their life. I want students to take my class seriously, I just don't want to- A lot of students don't need the extra panic caused with stuff in their life. I have a fair group of students that comes to talk to me about things that I wish they weren't having to talk to someone about considering—if you catch my drift. So it's like, you know, I want to not only- Obviously I care about math, but also- I mean, I got into education not only because I care about math but because I care about students. So, you know, the dual role of the teacher. You're teaching math, but you're also taking care of people that sometimes aren't getting taken care of other places.

C: What is more important to you?

J: For me, it's taking care of the other stuff. Most of the time it doesn't come up, so I'm defaulting to the math. But for me, the other part of the job is more important. (Formal interview, 190527)

In this exchange, Joe shared that he "got into education" because he cared about both mathematics and students. However, as seen here, Joe prioritized "taking care" of his students over teaching mathematics, asserting that the latter may sometimes be an added source of stress for students. Joe acknowledged the likelihood that students have a lot to worry about outside of school (on occasion he discussed the community's struggle with poverty), and therefore, in addition to being their mathematics teacher, he aspired to be a source of support for them.

Also reflected in this thread of Joe's discourse was the idea that building relationships with students had "practical benefits." For example, in an account of his own experiences as a K-12 student, Joe noted that because one of his former mathematics teachers "took the time to build a relationship" with him, he was more inclined to "pay attention in her class." As illustrated in the following excerpt from our final formal interview, Joe extended this logic to his own classroom, suggesting that building relationships is also useful for ensuring student compliance:

> I think I've admitted that the thing I care the most about is making sure students are, like, not going crazy. You know what I mean? I find that very important. But then there's benefits just practically as a teacher. If you have good rapport with your students, and you're saying, "Hey, would you please work on this right now," then they're like, "Okay." And you actually get less of those questions where it's like- the honesty with your students. You get less questions of like, "Why are we doing this," I find. Because, you know, like, "Okay, Mr. Anderson is saying that I should do this. So whatever, I'm just going to trust him on this one." And then they can work through the problems…So I mean, I think there's practical benefits as well as that- I mean students are a priority for me anyway. (Formal interview, 190527)

In this excerpt, Joe maintained that students' well-being was of the utmost importance to him and mattered more than any practical benefits of relationship building. However, as mentioned earlier, the practical benefits that Joe described included student compliance, as he suggested that "good rapport" can lead to student buy in and even decrease the frequency with which students ask questions like, "Why are we doing this?"

To summarize, the four threads of dialogue that comprised Joe's critical pedagogical discourse included: (1) *Conceptual understanding as a goal for student learning*; (2) *Student* *learning requires providing opportunities for thinking, reasoning, and struggle*; (3) *Student learning requires providing opportunities for discussion;* and (4) *The importance of building relationships with students*. This means that in describing who he was—or who he wanted to be—as a mathematics teacher, Joe consistently invoked ideas central to those four threads of dialogue. As threads of Joe's critical pedagogical discourse, one might expect to see ideas from those threads reflected in his practice (e.g., teaching for conceptual understanding). However, teachers are not always capable of making decisions that align with their personal values, as their practice may also be driven by perceived expectations for their teaching—*contextual discourses*—and other constraints imposed by broader contexts. In the section that follows, I describe the contextual discourses and constraints that Joe perceived, and reflect on the ways in which they, along with threads of his critical pedagogical discourse, helped to explain aspects of his practice.

Joe's Perceptions of Contextual Discourses (and Constraints)

Recall, Joe's case is one of a beginning mathematics teacher for whom gaps existed between his critical pedagogical discourse and practice. This means that Joe acted in ways that aligned with threads of his critical pedagogical discourses to only some extent. When Joe struggled to enact his vision for instruction, he sometimes expressed feelings of tension. Most often, those tensions resulted in Joe's reliance on conventional practices that he did not aspire to use. Consequently, Joe explained his use of conventional practices by pointing to contextual constraints, including the demands imposed by "having six preps" to prepare for and a lack of curriculum and professional resources, and by drawing on perceived expectations for his teaching (contextual discourses) communicated by the state standards for learning, administrators, and students. In the sections that follow, I describe those contextual constraints and discourses in more detail and illustrate how they, along with Joe's critical pedagogical discourse, helped to explain his practice.

The demands of "having six preps." As the only high school mathematics teacher at Beechwood, Joe was responsible for teaching six different mathematics courses. In our first formal interview, Joe expressed excitement about teaching six courses, suggesting that the heavy course load would afford him the opportunity to explore which courses he most enjoyed teaching. But over time, Joe began to argue that "having six preps" imposed time constraints that inhibited his enactment of ambitious practices, including facilitating student discussions and providing opportunities for students to think, reason, and struggle through mathematics. Consequently, Joe most often relied on conventional methods that he described as "familiar" and "reliable." In the sections that follow, I reflect on how Joe's perceptions of the demands of having six preps seemed to influence aspects of his practice.

Inhibiting the provision of opportunities for discussion. First, recall that through one prominent thread of his critical pedagogical discourse, Joe framed the provision of opportunities for discussion as a means to, among other things, support learning. Joe often expressed a desire to facilitate discussions among students, noting that he hoped to play the role of the "middle man" in those discussions less frequently. Although he sometimes attempted to facilitate student discussions by encouraging students to "talk to each other," the predominant pattern of talk in his classes was one that alternated between Joe and his students. And while students were almost never silent during lessons, they only occasionally talked to each other about mathematics.

Among other factors, Joe suggested that the time constraints imposed by having six preps could explain his infrequent attempts to facilitate discussion among students. More specifically, Joe explained how planning for six different classes kept him from engaging in the practice of anticipating (described in Chapter 2), which, he suggested, was necessary for facilitating the kind of discussions he envisioned. As an example, consider the following excerpt from our final formal interview in which Joe reflected on what he perceived as limitations of his practice:

> J: I think I really only used [GeoGebra and Desmos] superficially. I was hoping to do more discussions based on them, but I kind of just ended up using them superficially. I was like, "Look what happens with the linear function as the *m* value changes." Stuff like that.
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> C: Why do you think that you only used them superficially? What makes you say that?
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> J: I don't know, I think it comes back to time again. Because I didn't want to- I didn't feel like I had the opportunity to work through the whole activity myself before giving it to students. So I didn't feel comfortable having a big discussion about it. (Formal interview, 190527)

In this exchange, Joe described his use of technology (i.e., GeoGebra and Desmos) as "superficial" in part because he rarely coupled it with the practice of facilitating discussions. Joe attributed the absence of this practice to lacking the planning time required to "work through the whole activity" to anticipate what the focus of discussions might be.

According to Joe, he spent most of his planning time deciding what the topic of each lesson should be, leaving little, if any, time to plan for discussions. Consequently, Joe suggested that most of his lessons were "average" and "not really the best for students." During one midyear debriefing interview, Joe argued that if he had more time, both before and in class, he would be able to plan for and facilitate discussions and, thus, implement ambitious practices more often:

J: Sometimes I try to lead discussions or at least do activities that could promote discussion like you saw with Algebra II. But I kind of ran short of time based on what I wanted to get done that day. So we didn't get into as much as I wanted. But with stuff like that, I think if I had more time I would be more equipped to deal with certain student answers. So, one thing- to do a task really well…you kind of [need to] know what students are thinking…So like today, I would have to say, "Hey, they got this number," and have to think on the spot, "Where did that number come from?" Whereas if you got more time to spend with that task, you're able to think, "Oh, okay. I see what they have, and I know what that solution is already." If that makes sense.

C: So are you saying that with the amount of time you have to plan for each lesson, you don't really- that doesn't really allow you to anticipate student solutions?

J: Right...It's like I give- I have enough time to work out what the correct solution is already, so I know when I'm talking with them what the answers are to each of the problems. But I don't have enough time to think about what all the wrong- or different answers would be to the problem. (Debriefing interview, 181210)

In this exchange, Joe asserted that he could "get into" discussions more if he had additional class time to spend on those discussions. He also suggested that facilitating discussions more often would necessitate that he be "equipped to deal with certain student answers." In other words, Joe felt that if he had fewer classes to prepare for, he would have more time to spend "think[ing] about…different answers" to the problems he posed to students and, as a result, would be more prepared to facilitate the discussions he envisioned.

Relying on conventional practice. Recall that through another prominent thread of his critical pedagogical discourse, Joe asserted that students should have opportunities to think, reason, and struggle through mathematics. But Joe only occasionally afforded those opportunities to students, as most of his lessons were centered around note-taking sessions in which he employed conventional practices. As discussed earlier, Joe's note-taking sessions usually began with him reminding students of the previous day's topic (e.g., "Today, we're going to continue learning how to factor quadratics." Field notes, 190116) or introducing a new one. Then, after demonstrating how to solve a problem, Joe often posed a number of low-level tasks, or practice problems, for students to try solving on their own. From Joe's perspective, he relied on conventional practices like giving notes and leading demonstrations in part because he lacked the time required to plan ambitious, "task-based" lessons.

While Joe expressed feelings of tension when reflecting on his use of conventional practices, he did not express disappointment with lessons in which he utilized such practices—even though they rarely afforded the kinds of opportunities he envisioned. In the following excerpt from a midyear debriefing interview in December of 2018, Joe reacted to a question about whether he felt as though he had to make instructional sacrifices due to the time constraints imposed by having six classes to prepare for:

J: It's not that I'm disappointed by my lessons. But I know that they could be better if I had enough time. Not if I had enough time, but, like, you know- it's one of those things like- When I was student teaching, right? I was only teaching Geometry. With the same amount of time, I'm giving average lessons. But with the amount of time I was spending student teaching, I could make a really awesome, engaging lesson for my students. But now I have to create, in the same amount of time, like six, you know, not bad but average [lessons]. Obviously they're not bad if I'm getting teacher of the month and fours and fives on my [evaluation] scores and stuff from my administrators. So I'm not doing bad. I guess everyone is their own worst critic.

C: Can you describe why you think your current lessons are average and if you had more time, what those lessons might look like if you improved them to make them more engaging? I guess I'll break that down. What makes you think that the lessons that you're currently leading are average?

J: I rely a lot more on note-taking than I would like. But other than that it's just when I work at the board…I rely a lot more on that than I'd like. When I- I don't know. I like to give myself credit. When I do do note-taking, I think I try to be more engaging about it and make sure I'm calling on people and still even, as they're taking notes, exploring why. So like, when I did notes on the Triangle Sum Theorem, I wasn't just telling them, "[The angles] add up to 180. Let's move on." But I had them take notes on why [the angles] add up to 180 and was connecting it to parallel lines and stuff. So it's like I go in depth with my notes, I just don't actually have much- I just actually give notes. And that's the primary thing we do that day in that class. (Debriefing interview, 181210)

Despite suggesting that he "relied a lot more on" giving notes and "work[ing] at the board" than he had hoped, Joe justified his use of those practices by explaining that he coordinated them with ambitious ones such as teaching for conceptual understanding (e.g., "I wasn't just telling them…I had them take notes one why…"). Joe also justified his use of conventional practices by drawing on messages of approval communicated by his administrators. For example, Joe defended his practice by noting that he had been awarded "teacher of the month" and received positive evaluation scores following observations. Thus, in light of his attention to students' conceptual understanding and (perceived) approval from administration, Joe justified his use of conventional practices that were not reflected in his critical pedagogical discourse.

Joe's practice of posing low-level problems for students to solve following a demonstration also contrasts with the thread of his critical pedagogical discourse through which he suggested that student learning requires opportunities to think, reason, and struggle. In addition to time constraints, Joe explained his use of this practice by admitting that he was often "tempted to do rote and drill stuff because it's an easy option to plan for" (Debriefing interview, 190116). In a debriefing interview that took place toward the end of the spring semester, Joe elaborated on this more, describing how he regularly employed direct instruction—an "I do, We do, You do" approach—in his classes:

> My main instructional approach is kind of, as you say, it's where I do the problem first, then kind of collaboratively we do the problem, and then [students] do the problem at the end. Like the I do, We do, You do scenario. That's mostly what I've been using just because- I don't know. I think I'm kind of viewing it as- It's kind of an easy formula to apply to things. Like when I'm stuck, and I don't have a good idea about how to explain something, it's good to go to old reliable. So I show them what to do, we work through it together, then they do it. It's easy to see if they're getting the material down or not. So I think mostly just because it's easy and like I said, kind of a lack of planning time. (Debriefing interview, 190403)

Given that Joe had not aspired to employ an "I do, We do, You do" approach to instruction, and because it contrasted with the thread of his critical pedagogical discourse through which he promoted opportunities for struggle, I asked him to reflect on his use "I do, We do, You do" and whether it was a source of contention for him:

> C: Based on what you've seen with your students, how do you feel about doing [I do, We do, You do] on a regular basis? Do you feel conflicted, or is it something that's actually working for you and that you'll probably continue to do?
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> J: I think for the most part I feel pretty fine about it if I'm honest. It's working decently so far. One thing I'm feeling [is that] I need to get better at spacing out the amount of planning I put into each class. I put a lot of effort into first hour and fifth hour, and fourth hour occasionally. The other ones I'm just like, "Okay, here's what we're gonna do today." And I think that's part of things. The classes that I don't put as much thought into planning, they do the fallback of I do, We do, You do or just practice like, "Here's some problems. Practice them, and I'll be around to answer any questions."
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> C: So when you say that it seems to be- What makes you say that it's working? What are you seeing that's making you feel like, "This is fine"?
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> J: Not a significant decrease in test scores. There's been almost no change in test scores and stuff like that. Like compared to- The method of instruction hasn't changed test scores and quiz scores at all. The quizzes and tests I give, the scores, the averages are pretty much the same. (Debriefing interview, 190403)

Despite the disconnect between his vision and practice, Joe justified—and said he "fe[lt] fine" about—his conventional approach to instruction. However, in this case, Joe's justification for relying on a practice he did not aspire to use was rooted in student achievement: "There's been almost no change in test scores..." By test scores, here, Joe was referring to student performance on classroom assessments, which, based on my observations of the mathematical tasks that Joe posed and examination of one artifact of a unit assessment he designed, were focused primarily on solving low-level problems.

Joe provided a similar justification for his use of an “I do, We do, You do” approach to instruction during a midyear debriefing interview. Asserting that “it’s one of those things [where] if it ain’t broke, don’t fix it,” Joe explained that he had little motivation to be “inventive” if “the same amount of learning [would] occur by doing something that doesn’t take a lot planning” (Debriefing interview, 190122).

That Joe justified his use of conventional practices is interesting, especially given his consistent assertion that learning requires thinking, reasoning, and struggling through mathematics—because opportunities for students to “figure things out for themselves” (Formal interview, 180803) are not typically afforded through an instructional approach in which problem-solving strategies are demonstrated by the teacher. Considering Joe’s talk about student achievement discussed earlier, it may be that students’ relatively strong performance on classroom assessments began to challenge his assumptions about what learning requires. That is, perhaps Joe considered posing tasks that provide opportunities for thinking, reasoning, and struggle and implementing an “I do, We do, You do” approach to instruction as two means to the same end: learning how to solve different types of problems. And, because planning for the latter was easier and required less time—a resource that was scarce for Joe—he could justify using it.

To further rationalize why students’ performance on classroom assessments validated his approach to teaching, Joe drew on contextual discourses communicated by his principal and superintendent. When describing his perceptions of their expectations for his teaching, Joe consistently suggested that, for the most part, his principal and superintendent encouraged him to maintain his current instructional approach because they were “seeing results with [his students’] scores” on classroom assessments (Debriefing interview, 181011). As an example, consider the following excerpt from our final formal interview in which Joe described feedback he once received from his principal:

> One time when he was reviewing me, he was like, “Oh, your class seems pretty chaotic.” And then he looked through- I was grading, and he looked through some of the tests, and he was like, “Oh, these students actually know what they’re talking about though.” So it was kind of- For him, he kind of realized, “Okay. Different approaches-” So now when we talk, he’s kind of like, “Well, I don’t really understand, but it seems to be working.” So, I don’t know. He can’t really give good feedback, but he’s open because he sees results, even though it’s not at all like what he would be doing in this situation. So he’s kind of like, “Okay, just do whatever” [*laughs*]. (Formal interview, 190527)

Here, Joe explained that, from his principal’s perspective, his class was “pretty chaotic” (which Joe later suggested was due to students talking). This instance highlights an interaction between a thread of Joe’s critical pedagogical discourse—that students need to talk in order to learn—and a contextual discourse communicated by his principal—that classes are not usually as “chaotic” as Joe’s. However, based on Joe’s account, his principal expressed little concern for the chaos he initially perceived because Joe’s students were performing well on his classroom assessments. Consequently, for Joe, this perceived message of approval—which was rooted in student achievement—validated “whatever” he was doing.

To summarize, Joe’s perceptions of time constraints imposed by having six classes to prepare for influenced his practice in multiple ways. First, because Joe felt that he lacked the time required to plan for good discussions (by working through the problems himself and anticipating student responses), he did not facilitate discussions among students as often as he had envisioned. Second, Joe suggested that time constraints, among other things, could explain his use of conventional practices that were not necessarily aligned with his critical pedagogical discourse, such as giving notes and engaging in direct instruction. Eventually, however, Joe began to justify his use of conventional practices by noting the ways in which he *was* acting on his values (e.g., by coordinating giving notes with teaching for conceptual understanding) and drawing on perceived messages of approval communicated by administration: contextual discourses that emphasized the importance of student achievement (regardless of the instructional approach that gave rise to it).

Expectations of coverage communicated by state learning standards and course textbooks. Over the course of the study, Joe was consistent in suggesting that one of his main responsibilities as a teacher was to support students in meeting the state learning standards. But Joe perceived disconnects between his own expectations for mathematics learning and teaching and his interpretations of the expectations communicated by the state learning standards. For example, while Joe envisioned providing students opportunities to think, reason, and struggle through mathematics and develop conceptual understanding, he struggled to enact those aspects of his vision in part because of his adherence to expectations of coverage communicated by the state standards. In the sections that follow, I describe Joe's perception of his responsibility to teach the state standards and then discuss how that perception—a contextual discourse—influenced his efforts to afford the kinds of learning opportunities he envisioned.

Teaching the state standards: A contractual responsibility. According to Joe, administrators and other teachers in his district did not explicitly emphasize the importance of teaching the state learning standards. Joe even suggested that, for the most part, he could "teach what [he] want[ed]" (Debriefing interview, 190502). Rather, Joe described his responsibility to support students in meeting state standards as a contractual one—an expectation implicit in taking a job as a mathematics teacher in the state:

J: I think a lot about, "Why are we talking about this?" I don't ever say this to my students, but like, "Next chapter, we have to teach polynomial long division. Cool? Okay. I guess we'll do that. Why are we doing that? No real reason." But, and that's sometimes what I answer students when they ask, "Why are we covering this?" I'm like, "Well, it's one of the standards, you know. [The state's] decided that it's important that we talk about this." And they're like, "Okay" [*laughs*].

C: So what made you take that on as a responsibility?

J: I feel like it's literally the job description for a teacher in [the state] is to teach the [state] standards.

C: Is that something that's emphasized in your district?

J: Not particularly. I feel like if I really wanted to, I could do something else. But I mean, that's what we should be doing I feel like. I don't know. I don't really have any good reason just other than that's what- [*laughs*]. (Formal interview, 190527)

Although Joe questioned the relevance of some state standards (discussed more later), he maintained a responsibility for teaching them because to do so, he suggested, is "literally the job description for a teacher in [the state]."

Joe relied on publicly available state resources, as well as his course textbooks (which he perceived to be well aligned with the state standards), to decide what to "cover" in each of his classes and for how long. In a midyear debriefing interview, I asked Joe to describe his routine for determining the focus of his instruction and assessments within a given unit. Here is what he said:

> Let me use this textbook as an example. So I'll open up to- so recently in my Algebra II, they took their test—it was more of a quiz—on Friday. And it was about matrices. And what I would've done, is that I know from August Joe—shout out to that guy's work—is- So this was a good unit because the system of equations and inequalities chapter is almost exactly aligned with the standards, except I didn't need to cover Cramer's Rule because that was in the Pre-Calc section in the [state] learning standards. So I was able to just follow what they had here [in the book]. And I didn't use- Linear programming wasn't in the standards if past me is to be believed. So I have systems of equations- And sometimes, most of the time, I will just look at the chapters. I'm just looking at [the systems of equations chapter], and I know what goes with that. I know that's talking about graphing, elimination, and substitution…Most of the time I can tell you everything that's in a section just based on the title. So I'm just looking at the titles, and I know everything that's going to go in there according to that title. So, you know, if it's systems of equations with three variables, I know [the book is] going to want me to teach how to solve that by hand and on a calculator. So I'll use the chapter titles and the order of them to get everything that's related in the unit. And that also gives me the freedom to then go as deep as I need to with it. So with multiplying matrices, "Well, do [students] need to know that by hand? Or do they need to know"- You know, "How deep am I going with that?" And then if I have extra time, I can cover the book stuff. For the matrices unit, I didn't have extra time, so I didn't cover Cramer's Rule because [the state] told me not to. (Debriefing interview, 181210)

In this excerpt, Joe explained how he used the state standards and his course texts to decide what to teach in his classes, noting what he perceived to be alignments between the two. However, the content of the state standards seemed to take precedence over that of the course texts, as Joe suggested it was not necessary to "cover" topics in the textbooks that were not included in the state standards.

Influences on Joe's practice. Because Joe relied on state standards and course texts to decide what to teach and assess in his classes, those resources—and Joe's perceptions of expectations communicated by them—directly influenced his practice. Recall that through threads of his critical pedagogical discourse, Joe argued the importance of providing opportunities for students to think, reason, and struggle through mathematics and develop conceptual understanding. Based on observations of his teaching, however, Joe acted in ways that aligned with those threads only occasionally, as his instruction typically involved a teacher demonstration followed by individual work time during which students solved low-level practice problems on their own.

Based on Joe's talk about his practice, opportunities for thinking, reasoning, and struggle were often absent from his lessons because he prioritized covering the learning standards deemed important by the state. As an example, consider the following excerpt from our final formal interview in which Joe reflected on what he perceived to be obstacles to enacting tasks that afforded the opportunities he envisioned:

> I feel like I know a lot of high-quality tasks, I just don't understand connections to the standards within them. So like, "Okay, I know this great task. But I'm also supposed to be teaching factoring polynomials. Where's the connection between them?" Like, "How

> can I get a great task that also addresses *this*?" And if I can't find one, then it's like, "Okay, I'll just do something else then." (Formal interview, 190527)

While Joe suggested that he could find "high-quality tasks" if he wanted to, he explained that he posed those tasks only occasionally because connections between the mathematical foci of high-level tasks and the state standards were often unclear to him. Thus, because Joe felt bound by contract to support students in meeting the state standards, he sometimes sacrificed affording the types of opportunities he said he valued. Worth noting is that this constraint seems largely self-imposed, as, according to Joe, individuals within his school did not explicitly communicate expectations for teaching the state standards. Regardless, Joe's reliance on the state standards for guidance illustrates an example of how a lack of resources (that contain high-level tasks) may influence a beginning teachers' practice.

Joe also suggested that it was difficult to attend to students' conceptual understanding in some classes because, in comparison to others, they required coverage of a greater number of standards. For example, Joe claimed that it was easier to do "conceptual activities" in Algebra II and Geometry than in Algebra I because there were so many standards to "cover" in Algebra I. In one debriefing interview, Joe questioned how he could be expected to "get through" all of the Algebra I standards if he also had to "be conceptual" (Debriefing interview, 190308). Additionally, because Joe perceived that many of the state standards were procedural in nature, he was able to justify a focus on procedures in his instruction and classroom assessments. For example, in that same debriefing interview, Joe explained why the questions he included on an Algebra I unit test for quadratics prompted students to recall and apply procedures without requiring any explanation:

> I just look at the standards for what is incorporated into a unit and then make a test based on them. So one of the state standards was the discriminant, so I put a question on the discriminant in…When it came to quadratics, [the standards] were pretty procedural in nature. (Debriefing interview, 190308)

Thus, because Joe relied on the state standards documents to guide his instructional and assessment practices, the foci of such practices often reflected the foci of the state learning standards.

Despite his adherence to contextual discourses communicated by the state learning standards, Joe challenged the importance of some of the standards as, from his perspective, meeting them did not necessarily require "doing mathematics." For example, in our final formal interview, Joe reflected on how the mathematics that is typically valued in schools differs from his own perspectives on what doing mathematics entails:

> J: In Algebra…the logic [required] is kind of surface level. In Geometry, it's a bit more apparent. But in the Algebra classes…it's mostly like, "Okay, well I need to get *x* by itself." Algebra—I just thought of this—is [learning] increasingly convoluted ways to solve for *x*. It's like, "Okay, we've got addition, multiplication, and division. What if we threw in exponents to solve for *x*? What if we threw in square roots to solve for *x*?" And then, a lot of it is almost remembering steps and then following through on them...I don't really understand how you can turn graphing a logarithm into an intellectual logic exercise.
>
> C: So would you say that that kind of stuff is not doing mathematics?

J: I mean, personally I would. I understand it's a standard, but I don't really understand the benefit of it. I think [students] should know what a logarithm is and the general shape [of its graph], but-

C: So can you describe something that's different from that that you would consider to be doing mathematics?

J: Hm. Yeah, I guess you can ask students- Maybe just like motivating a need for things rather than doing mathematics for *this* example. Like rather than just graphing a logarithm, you create a problem with an exponent. I did this with my Algebra II. Like, "Okay, we've got x to the [power of] $0.5t$ equals 2. Wouldn't it be nice if we could solve this problem?" So I'm like- then my to students—this is like one of the better things I did this year—to my students I said, "Okay, try to solve for x." And then obviously they're all coming to dead ends, and they're like, "Okay, tell us how to solve for x now." Like, "Alright, this handy thing called logarithms were invented for just this purpose"…But it's like, what is the purpose of graphing all of these functions? In my college-level math [courses], I didn't really graph all that much. I only graphed basically to "cheat" on problems where it's like, "Oh, this would be really easy if I just graphed it on a calculator," rather than thinking about it. So, "Oh, the answer's 2." I don't know. I feel like the things I like about math aren't always apparent in actual practice. I mean, I enjoy most of the stuff we do in schools, but I'm kind of a biased source I guess.

(Formal interview, 190527)

In this exchange, Joe criticized the Algebra standards deemed important by the state, arguing that students merely have to "remember steps" to meet some of them. According to Joe, recalling and applying procedures was not necessarily an example of "doing mathematics." Instead, Joe explained that doing mathematics involves "motivating a need" for the procedures students are ultimately expected to learn. While Joe recalled an instance in which he attempted to motivate a need for introducing logarithms to students (and suggested that it was "one of the better things [he] did this year"), he distinguished such instances from "most of the stuff we do in schools." Joe's talk, here, suggests at least two things: that he considered the work of mathematicians as distinct from what students are typically asked to do in mathematics classes; and/or that he considered the regular enactment of his vision for teaching unfeasible.

To summarize, Joe perceived a responsibility to teach his state's learning standards for mathematics even though the importance of teaching those standards was not necessarily emphasized by others in his district. Rather, Joe described this responsibility as an unspoken expectation for any mathematics teacher working in the state. Consequently, Joe relied on state standards documents and his course textbooks as resources for deciding what to teach and assess in his classes. Despite questioning the relevance and rigor of some of the state standards, Joe's adherence to his interpretation of the expectations communicated by the standards outweighed his desire to provide students opportunities to think, reason, and struggle and develop conceptual understanding of mathematics, which were two prominent threads of his critical pedagogical discourse. Consequently, the focus of Joe's instructional and assessment practices often reflected what Joe perceived to be the focus of the state learning standards and/or the course textbooks to which he had access.

A lack of curriculum and professional resources. In addition to time, Joe lacked other resources that, if available, may have supported him in more regularly acting in ways that aligned with his critical pedagogical discourse. In particular, Joe's course textbooks and the professional expertise of both colleagues and mentors did not necessarily support him in enacting ambitious practice and, thus, may help to explain Joe's reliance on conventional practices. Details of how available curriculum materials and sources of expertise influenced Joe's practice are discussed in the sections that follow.

A lack of high-quality, standards-aligned curriculum materials. When reflecting on his teaching in debriefing interviews, Joe described the challenges of finding curriculum materials that were both aligned with the state learning standards and contained readily available mathematical tasks that could afford opportunities for students to think, reason, and struggle and develop conceptual understanding of mathematics. As an example, consider the following interview excerpt in which Joe described how he used his Geometry course text as a resource for teaching the Vertical Angles Theorem:

J: Most of the students knew something about vertical angles, and some of them could even tell me that they were congruent ahead of time. So I just had students- I said, "Okay, well draw on your piece of paper a line intersecting another line, and call them angles 1, 2, 3, and 4, and tell me as much as you can about the relationships between the angles." So, we got to that these add to 180 [degrees] because they're on a line, and these add to 180 [degrees]. And then some students said these are equal, and these are equal. And then I said, "Why are those two equal?" And then they explained the reasoning. And that's when I stepped in and was like, "Yeah, so your reasoning- Let me write down the steps in your reasoning." And that's how we introduced proofs as steps of reasoning basically.

C: Where did you come up with that idea?

J: Like I said, the book tells me the concepts, and then I just ask, "What kind of activity goes with that concept?"…This is one of those that- Divine inspiration mostly. I was just like, "Hey, what if I ask them what those angles are?" Because I knew the goal was to show that these angles are congruent. So I was just like, "Why don't I have students explain to me why they think these angles are congruent?" (Debriefing interview, 181210)

In this exchange, Joe explained that he used his course text only to decide which concepts to teach, which was the Vertical Angles Theorem in this particular case. Because from Joe's perspective, his course texts lacked high-level mathematical tasks, he had to search for or design such tasks on his own if he wanted to provide opportunities for his students to reason about and develop understanding of concepts central to the mathematics he was teaching (e.g., why the measures of vertical angles are always congruent). But, as Joe alluded to here, generating ideas for high-level tasks was not necessarily easy: "This is one of those that- Divine inspiration mostly." Thus, the difficulty of finding and designing such tasks, coupled with a lack of planning time, helped to explain why Joe only occasionally afforded the types of learning opportunities he envisioned.

Mentors with expertise outside of mathematics education. As discussed earlier, Joe experienced a sense of autonomy in making decisions about both what and how to teach in his classes. Based on descriptions of his relationships and interactions with colleagues and mentors, the autonomy that Joe sensed may have been related to how he was positioned—both by himself and by others—in relation to his colleagues and mentors. From Joe's perspective, he was the person in his district most qualified to teach secondary mathematics. This sentiment was reinforced through messages of approval communicated by both his colleagues and mentors.

Further, because Joe recognized that his colleagues and mentors lacked expertise in mathematics education specifically, he rarely sought their support and took up only some of the feedback they offered. While being positioned as an expert likely contributed to the autonomy that Joe sensed, it also meant that he lacked colleagues and mentors with expertise similar to his own, which may have inhibited the enactment of his instructional vision.

Joe's colleagues. Joe insisted that he could seek support from just about anyone in his district but suggested that none of those people had the expertise necessary to support him in achieving the goals he had set for himself. Recall, for example, that Joe co-taught a class called Agriculture Mathematics with another teacher. However, Joe said that he rarely sought support from his co-teacher because mathematics was not his "area of expertise," and he consistently assured Joe that "whatever [he was] doing [was] probably fine" (Debriefing interview, 181210).

In addition to his co-teacher, Joe sometimes interacted with the middle school mathematics teacher. But because she was a long-term substitute teacher who had not formally studied mathematics education—and had a vision for mathematics teaching that was seemingly different from Joe's (e.g., "[she would say] students should be quiet, working by themselves, and ask if they have questions," Debriefing interview, 181011)—he rarely sought instructional support from her either.

Still, Joe occasionally met with the middle school mathematics teacher to discuss course content. As an example, consider the following interview excerpt from our final formal interview in which Joe described his conversations her:

> [S]he's really good about asking me like, "What do they need to know for your class?" I think that's one of the advantages of a smaller school. If you know the only class that'll be going into 9th grade, you can just ask, "Hey, do I need to cover this that much in my

8th grade class?" Like, "Well, kind of treat that like it's brand new," or something like, "No, I'm expecting that they know how to solve that." (Formal interview, 190527)

Evident from Joe's account of his interactions with the middle school mathematics teacher is a focus on what to teach and when—an emphasis on coverage and pacing—rather than how. Thus, because Joe perceived that his colleagues could not provide feedback useful for improving his mathematics teaching, he did not see them as potential sources of support for enacting his vision.

Joe's mentors. As discussed earlier, Joe invoked contextual discourses that were communicated by his principal and superintendent to justify his reliance on conventional practices that he did not envision using (e.g., "Obviously [my lessons are] not bad if I'm getting teacher of the month and fours and fives on my [evaluation]." Debriefing interview, 181210). In addition to communicating their approval of his practice, the nature of the feedback that Joe's principal and superintendent provided—which was likely related to their expertise—may also have contributed to the infrequency with which Joe attempted to enact ambitious practice. As an example, consider the following excerpt from our midyear formal interview in which Joe reflected on feedback he received from his principal and superintendent:

J: So like…their main feedback is that I connect really well with the students. And their areas of improvement were to just continue trying out different techniques: "Don't make every class so much of the same. Try to kind of vary it up a little bit."

C: What were some things they said they'd like to see you switch up sometimes?

J: Yeah they said- One thing they said was…when we're doing examples, instead of just having [students] solve them by themselves in their notes, maybe I have them use a whiteboard. And then we can share and compare answers as a whole class.

So just things to make it more engaging for students…They always say like, "You don't do a bad job engaging the students because you connect with them, but here are some tricks so that students might be more interested, more engaged"…Which is like- I prefer. I think I'm pretty good at teaching the math side of things, so I prefer their feedback to be in that kind of area.

C: Like general strategies for engaging students?

J: Yeah, and especially because obviously it's such a small school, I'm not gonna get feedback- I mean this is why I like these [interviews] so much because I don't get feedback that's tailored to math education specifically…Because I'd rather them give me feedback on things that they're actually qualified to talk about, rather than trying to be like, "You should try teaching quadratics like this." Like, "No offense, but your background is elementary ed." (Formal interview, 181229)

In this exchange, Joe suggested that the feedback he received from both his principal and superintendent was focused primarily on strategies for making his lessons more engaging (e.g., having students write on a whiteboard rather than writing in their notes). In other instances, Joe described their feedback as a series of recommendations to add "dressing" to his lessons, which, from his perspective, would merely result in surface-level changes (Formal interview, 190527). Positioning himself as the district expert in teaching mathematics, Joe suggested that their feedback could provide little support for improving his practice beyond employing strategies that could better foster student engagement. Additionally, worth noting is Joe's acknowledgement of our debriefing interviews as a source of support: "I don't get feedback that's tailored to math education specifically." Although the purpose of the debriefing interviews was not to provide feedback, as mentioned in Chapter 3, I offered ideas if teachers solicited feedback or support from me. Because I did not do this often, however, it is likely that Joe was framing the nature of our conversations as a source of support, as my interview questions often prompted him to reflect on his vision and goals for teaching.

To summarize, Joe lacked resources that, if available, may have supported him in more regularly acting on the threads of his critical pedagogical discourse. First, without high-quality curriculum materials that were aligned with state learning standards, Joe had to search for and/or design high-level tasks that could afford the opportunities he envisioned providing for his students. Because Joe's planning time was already spread thin, he struggled to plan for using high-level tasks in his instruction. Second, Joe's colleagues and mentors did not support his enactment of ambitious practice, as, from his perspective, they lacked expertise in mathematics education. Consequently, Joe most often relied on conventional practices that were not reflected in his critical pedagogical discourse (e.g., giving notes) because he found them familiar and easy to use. Additionally, Joe's use of conventional practice was likely reinforced by his perceptions of contextual discourses communicated by his colleagues and mentors, as they expressed approval of his practice and focused primarily on coverage and student engagement (respectively) rather than on instructional improvement.

Students' (and the community's) expectations for mathematics teaching. The last contextual discourse that was consequential for Joe's practice was communicated by his students and, more broadly, the small, rural community in which he taught. First, Joe perceived that in his students' previous schooling experiences, they had not encountered the type of teaching and, thus, the kinds of learning opportunities that he envisioned. Consequently, Joe explained, he sometimes employed conventional practices that, from his perspective, would appease his students. For example, when reflecting on his teaching in debriefing interviews, Joe explained that per his students' requests, he introduced standard procedures (Debriefing interview, 181210) and "gave more worksheets than [he] was planning to" because it seemed to make students feel "more comfortable" (Debriefing interview, 190403).

In addition to explicitly asking for standard procedures and worksheets of practice problems, Joe suggested that students communicated their expectations by resisting his attempts to enact ambitious practices. Joe's claim is supported by observation data, as I documented incidents of student resistance on a number of occasions. For example, when Joe attempted to pose a high-level task from the front of the room, I noted that some students verbally rejected it (e.g., "Yeah, that's never gonna happen." Field notes, 190116). Based on another observation, when Joe attempted to facilitate student discussion by requesting that students talk to each other about the task they were working on or a question he had just posed, students sometimes ignored his request and, instead, opted to work on their own or directly responded to the questions Joe had asked (e.g., Field notes, 190110).

From Joe's perspective, students' reluctance to engage in discussions with peers may have been because they lacked confidence in mathematics (e.g., abstaining from mathematical discussions with peers was a way to avoid the potential embarrassment of being wrong, Formal interview, 190527) or because students at Beechwood did not necessarily understand the "culture of debating." In regards to the latter, consider the following excerpt from a debriefing interview in which Joe reflected on why his discussions were primarily teacher-led:

J: I think a lot of the time, especially in this community, it's very close knit. And often times, they don't even want to correct their peers unless the peer is asking for help. I've even had- Sometimes they're like, "Hey, teacher, will you say this because I don't want to seem like I'm contradicting someone else." Which I think is a really- That's not something I've experienced with students before.

C: So what you're saying is, if a student- let's say Gene was wrong. And Lou wants to respond, he'd rather- it seems that he'd rather have you do it because he doesn't want to publicly correct his peer?

J: Yeah. He'd rather have me correct his peer so he doesn't look like he's being rude. Even though they joke around with each other a lot, they actually do really care. Because these kids- I mean, thinking about it in this community, they've been with each other since kindergarten—the same exact group of peers. So, you know, think about how closely they know each other. They don't want to legitimately upset one of their peers.

C: But so you'd like to get, you'd like to work toward them being able to do that kind of thing, but maybe-

J: Yeah, yeah. I think I just have to explicitly mention like, "Hey, you really shouldn't consider it- When we're having a discussion in class, it's fine to pose ideas and bounce off of each other." I just don't think that they understand the culture of debating and that it's fine to disagree with someone. And that when you get disagreed with, that's also fine. (Debriefing interview, 181011)

In this exchange, Joe attributed the absence of student discussion from his lessons to his students' expectations for their participation. In particular, Joe suggested that students may not want to "correct their peers" during a discussion because doing so might "upset" students or, perhaps, convey a sense of disrespect. These participation norms, Joe suggested, may have been a function of the small size and "close knit" nature of the community.

Important to note, however, is that Joe did not consistently attribute students' struggle to engage in the opportunities he sought to afford them to resistance on their part. In some cases, Joe considered whether limitations of his instruction (e.g., a failure to make clear expectations for participation) contributed to a lesson's shortcomings. Still, he more regularly pointed to student expectations and also behaviors (e.g., students "being off-task" or "not doing anything") to explain the struggles they experienced.

As mentioned at the beginning of this section, in addition to contextual discourses communicated by his students, Joe perceived contextual discourses communicated by the broader community in which he worked, and suggested that such discourses had implications for his teaching. In particular, Joe explained that because the student population was predominantly white and Beechwood was located in a "right-wing" community, he had to abandon teaching mathematics for social justice—an idea he said he had learned about during teacher education—as a potential pursuit. Further, Joe explained that while he would have liked to explore issues of social justice and "diversity" in his classes, he did not feel that his students would find such issues relevant. To illustrate this, consider the following excerpt from a debriefing interview conducted in October of 2018.

C: [Since beginning teaching at Beechwood], is there anything you feel like you've had to give up?

J: Yeah, actually. A lot of questions, especially ones related to social justice, I kind of gave up—especially given the community we're in. It's very right-wing, as rural communities tend to be. So [social justice] is not something I want to address, especially when most people are like…You've seen my classes. There wasn't a single student of color in my classes. There is- I have one seventh hour.

That's all of my classes though. I have one student of color, and he's half Mexican. So that's one of the things I've kind of abandoned. Questions of diversity for this group of students, I think they're important. But they aren't relevant because [students] don't see [diversity]. You know, they go into town, even in [another nearby town] where I live, there's a much larger Hispanic population than here, but it's still almost 80% white people.

C: So you don't think at any point moving forward that you'll be able to talk about social justice issues in the context of your math classes with this student population?

J: It's just one of those things where I would like to. I'm just not sure how to do it or make it relevant because they don't have, they don't see other people that look different from them. It's not something that they think about or that's relevant to them. I just don't know how to approach it. I'm all for talking about student diversity, but what if there's no diversity to speak of? (Debriefing interview, 181011)

According to this excerpt, Joe felt as though he could not take up questions of social justice or diversity in his classes because, from his perspective, investigating such questions was not necessarily relevant for a predominantly white group of students who lived in a conservative community. While Joe attributed his decision to abandon the idea of teaching mathematics for social justice to contextual discourses he perceived (e.g., "It's not something they think about or that's relevant to them."), Joe's decision might also be explained by, at least, two other things. First, Joe's decision may have been driven by his own ideas about issues of social justice and diversity (and who should be thinking and talking about them), as he seemed to think that “questions of diversity,” for example, were not relevant for his students. Second, it is possible that Joe was unsure of how to enact (Kennedy, 1999) or recontextualize (Ensor, 2001) teaching mathematics for social justice in a predominantly white, conservative community.

In two interviews that followed the one from which the last excerpt was pulled, I revisited Joe’s argument that he could not teach mathematics for social justice at Beechwood, questioning whether, after some time had passed, he still thought this to be the case. In those interviews, Joe maintained his initial argument, expressing concern that his students would not be receptive to investigating social justice issues because such issues were not relevant to them (Formal interviews, 181229; 190527). Worth noting, however, is that in our midyear formal interview, Joe reported posing a mathematical task focused on minimum wage for workers in the U.S., which is necessarily an issue of social (i.e., economic) justice. Joe described this task during a segment of the interview in which were discussing whether (his perceptions of) his students’ and colleagues’ conservative views had implications for his practice:

J: I’ve kind of avoided talking about [political issues] with students. But I’ve talked to a few students about topics. Minimum wage was a big one we talked about because it was more relevant to math, which is interesting. A lot of them are used to the environment of Beechwood. So we- I mentioned how…I was trying to relate [minimum wage] to ratios. So I was like, “What’s an apartment cost in [Beechwood] versus California the state? Shouldn’t the minimum wage in California be much higher because it’s much more expensive to live in?”

C: What did they think?

J: Most of them were in agreement with that, which is kind of what I left it at. I didn’t want to seem like I was pushing a certain viewpoint.

C: Did you do a lesson around that, or was it just something that you informally talked about?

J: I did a lesson around that in my Pre-Calc and Applied [Mathematics] classes.

C: Well, I know that you said in past interviews that social justice was sort of something that you felt like you aligned with in your teacher education program but had to abandon because you weren't sure how to cover it in Beechwood. But that sounds social justice oriented.

J: I mean, that was actually kind of a product of our last interview. Because this is something I did on the last few days of school after they had taken their final. So I was like, "We have free time. Let's talk about something that's interesting." They wanted to watch a movie, but you know [*laughs*]. [I was] like, "Here, let's look at some math problems in the real world."

C: That's cool. Do you remember the task that you posed, or did you just say, "Look at the"- Did you use the example you just described?

J: Yeah, I used the same example.

C: So were you like, "Is this fair?" Was that your question or-

J: Yeah, basically. And in the end they had to come up with a reason in support or if they were opposed [to Beechwood and California having different minimum wages]…I think the most popular answer was that—I mean this would be impossible to implement—but they wanted [to base minimum wage] on the average livable wage. Because most of their concerns were like- They'll have a father who owns an axe shop, and they're like, "Well, if my father had to pay his employees [a certain minimum wage], he wouldn't turn a profit," which might be true in Beechwood…So they were trying to balance average income versus what the minimum wage should be, which is a very interesting idea.

C: Yeah, so can you explain that a little more? Can you go back to the example of "If my father has to pay employees…"

J: Yeah, yeah. So they're like, "Obviously Walmart would have enough money to pay a higher minimum wage to their employees." But one of them brought up their father's business, which is kind of like selling feed for animals. And they were like, "If he had to pay his employees that much, then they wouldn't actually be able to turn a profit in their business, in which case they wouldn't have jobs." So in their minds, they were like, "Isn't $7.25 better than $0?" I'm like, "Yeah, that's true—$7.25 is better than $0." (Formal interview, 181229)

In this excerpt, Joe explained that, for the most part, he avoided talking about political issues with his students because, from his perspective, both students and colleagues had political views that differed from his own. Joe noted, however, that he had discussed political issues with some students, as he recalled an instance in which he posed a task that required students to consider whether the minimum wage for workers in Beechwood and the state of California should be the same. A few things about Joe's talk here warrant discussion. First is that Joe expressed concern regarding his students' perception of his political stance—recall that Joe said he "didn't want to seem like [he] was pushing a certain view point." This suggests that Joe may have considered it inappropriate for teachers to share their political views with students. Second is that Joe explained that he posed this particular task "on the last few days of school after [students] had taken their final," which suggests that Joe may have posed the minimum wage task only because he and his students had some "free time." That is, perhaps Joe was hesitant to dedicate class time to working on such tasks because he was more concerned with "covering" the state learning standards. Last, it is important to note that Joe did not initially characterize the minimum wage task as one focused on an issue of social justice—I did. However, Joe suggested that the minimum wage task was "a product" of our conversation in a previous interview (in which we discussed his decision to abandon teaching mathematics for social justice as a possible pursuit), which speaks to the potential influence of contextual discourses that *I* communicated.

It is also worth noting that based on Joe's account of how his students engaged in the minimum wage task, the nature of students' reasoning was very much a function of their experiences as individuals living in Beechwood. Recall, for example, Joe said that when working on the task, students drew on personal experiences and circumstances (e.g., that their families owned small businesses) when making their arguments. This supports the validity of Joe's concerns with posing such tasks, as the socially oriented goals of the minimum wage task may depend on the context (i.e., the community) in which the task is posed.

To summarize, Joe consistently described how his perceptions of students' expectations and community norms and values influenced his practice. For example, because students at Beechwood seemed comfortable doing worksheets, Joe gave more worksheets than he had initially planned. Additionally, because he perceived that his students lacked confidence in sharing their ideas out loud and had expectations for how they should interact with peers (e.g., not challenge each other), Joe led discussions more often than he had hoped. Last, because from Joe's perspective, his students—who were predominantly white and lived in a conservative town—would not be receptive to exploring issues of social justice and diversity, he decided to abandon the idea of teaching mathematics for social justice, even though he said it was something he would have liked to have done. Thus, the contextual discourses communicated by students and the broader community helped to explain the practices that Joe employed (or not).

Summary of Joe's Case

To summarize, Joe's practice occasionally reflected his critical pedagogical discourse as he sometimes: posed high-level tasks, attended to conceptual understanding, and attempted to facilitate discussion among students. However, given a lack of resources and his susceptibility to perceived contextual discourses that promoted conventional teaching, Joe more often relied on conventional practices that were "familiar" and "reliable" (e.g., "I do, We do, You do"). Further, it seems that Joe appropriated those contextual discourses as his own, as he often justified his use of conventional practices by referencing messages communicated by the state learning standards, colleagues and administrators, and students.

The Case of Emma

Emma's case is one of a beginning mathematics teacher who promoted ambitious practices but struggled to enact them because she was susceptible to contextual discourses that pressured her to teach in conventional ways and positioned her as a novice in relation to other teachers. Although Emma recognized that the contextual discourses she perceived were, in some ways, in conflict with her values, she did not express frustration when describing those tensions. Consequently, Emma appropriated contextual discourses alongside threads of her critical pedagogical discourse, and drew on those discourses to explain and justify her use of conventional practices.

In the sections that follow, I first provide an overview of Emma's teaching. Then, I discuss three threads of Emma's critical pedagogical discourse. Following this discussion, I describe the contextual discourses that Emma perceived, illustrate how she appropriated them, and reflect on how they helped to explain her practice.

Emma's Mathematics Teaching

Observations of Emma's teaching revealed a few things about her instructional practice. First, Emma was committed to covering the content required for each of her classes and enacting the tasks that she and her Professional Learning Team (PLT) had planned to use. Second, Emma occasionally enacted ambitious practices, as she sometimes provided opportunities for students to make sense of mathematical ideas on their own or discuss them with peers, and posed tasks that required students to explain their reasoning, make predictions, draw conclusions based on patterns, and/or consider connections between mathematical representations. However, such efforts were undermined by Emma's tendency to: guide students through tasks by asking leading questions (Stein et al., 1996) or telling them what to do in order to make progress; and leading structured note-taking sessions in which she demonstrated how to solve particular types of problems and explained intended takeaways. In the sections that follow, I discuss Emma's teaching in more detail by describing practices and routines that were foundational to her instruction.

Pre-lesson routine. Before starting class, Emma often wished good day to her students by saying, for example, "Happy Wednesday" as they walked in the door and again before introducing the first activity of the day. During Emma's first class of the school day, she sometimes thanked students for "quietly listening" to the announcements (Field notes, 190118). Students' desks were arranged in either pairs or groups of four, and Emma sometimes assigned students to seats by writing their names on desks with a dry erase marker.

Warm-up tasks. To begin class, Emma typically posed what she referred to as "warm-up" tasks. Such tasks usually prompted students to solve problems—most of which were low-level—that were topically related to the activity or quiz that followed them. For example, before posing an activity focused on "modeling percent change" by writing exponential equations, Emma posed a warm-up task that prompted students to solve problems that required them to make conversions between percentages and decimals (Field notes, 190118).

"Investigations." On four occasions, I observed Emma engage students in what she called "investigations." Emma's investigations were high-level tasks that her PLT had designed recently or during a previous school year. The four investigations that I observed Emma pose were "Procedures with Connections" tasks (Smith & Stein, 1998) because they required students to: consider relations between equations and the situations they represented (Field notes, 190508; 190510), or predict and test (using Desmos in one case) how changing a function's equation would transform its graph (Field notes, 190109; 190201).

To introduce investigations, Emma briefly explained to students both what they would be investigating (e.g., "You're going to be investigating Euler's number—the number *e*." Field notes, 190510) and how (e.g., "You should be looking at the pattern and describing what you see." Field notes, 190109). Once students had started working, Emma would circulate the room, asking questions to help students progress through the activity and sometimes, if her questioning was unhelpful, telling students what to do next. For example, on one occasion, because many students were struggling to make progress, Emma stopped the class from working on an investigation and went over the activity on the SMART Board at the front of the room. Following investigations, Emma led either note-taking sessions or whole-class discussions that focused on what students should have found after working through the activity.

Structured note-taking sessions. To begin note-taking sessions, which followed either warm-up tasks or investigations, Emma distributed a structured notes worksheet for students to fill out while she led the session. The notes worksheet, which, according to Emma, was a tool commonly used by other members of her PLT, had a space for writing in an essential question related to the day's topic (e.g., "What is an absolute value function?" Field notes 190109), a column on the left side of the paper for jottings, and a section at the bottom where students could write a summary of the notes. The body of the notes worksheets had spaces designated for writing definitions of mathematical terms (e.g., the meaning of parts of exponential functions, Field notes, 190508), working through example problems, and, sometimes, listing "steps to remember."

After distributing notes worksheets, Emma instructed students to write down the essential question in its designated space. Then, Emma guided students through filling out the remaining sections of the notes by writing out definitions of mathematical terms and/or working through example problems on the SMART Board. She also occasionally made connections to prior work students had done by referencing findings of prior investigations. As Emma worked through notes worksheets, she usually solicited input from students by asking them questions. To illustrate this, consider the excerpt below from the field notes I collected on February 1, 2019 while Emma led a note-taking session focused on transformations of rational functions of the form $y = \frac{a}{x-h} + k$:

> Emma begins guiding students through filling out the notes on transformations of rational functions. To make connections to prior work on transformations, Emma asks students questions like: "What did *a* do for absolute value [functions]? What did *h* do?"
>
> Students respond to Emma's questions using informal language (e.g., "*h* goes left to right"). While Emma verbally affirms their responses, she writes down formal language for describing transformations while filling in the notes (e.g., *h* relates to "horizontal translations").
>
> As she copies the notes on the SMART board, Emma continues to make connections to prior work: "For functions up until this point, our domain has been negative infinity to infinity—so any *x* will work. But we found that the function in the investigation was undefined at one value of *x*. What was that value?"
>
> "Zero," a student toward the back of the room suggests.
>
> Emma affirms her answer and begins filling in a table of *x* and *y* values for the function $y = \frac{1}{x}$, telling students to copy down the values that she writes in the table. Next, Emma tells students that they need to graph the asymptotes of the function on a coordinate plane provided on the notes worksheet. She asks students if anyone remembers what an asymptote is, and when no one offers an idea, Emma reminds them that an asymptote is

> "a line that the graph never crosses." She then writes the definition in the left-hand column of the notes sheet. To display the graph's horizontal asymptote, Emma sketches a horizontal line at $y = 0$ and labels it "H.A." Then she explains to students that the equation for the horizontal asymptote will always be $y = k$. She repeats the same process for graphing and writing the equation of the vertical asymptote.

First, it is important to note the mathematical focus of this note-taking session. Even though Emma sometimes posed conceptually-oriented questions that were intended to support students in making connections between equations, tables, and graphs, she never asked students to explain or justify their answers to those questions. Also worth noting is the pattern of talk evident in this excerpt: Emma posed questions and students responded to them.

Small-group and partner work. After completing notes worksheets, Emma usually posed practice problems for students to work on that were related to example problems in the notes. For example, following the note-taking session just described, Emma wrote an equation of a rational function on the SMART Board and asked students to describe what the graph of that function would look like and to identify the equations of the function's horizontal and vertical asymptotes.

While students were working on tasks, Emma encouraged them to discuss those tasks with a partner or other group members. Although a majority of students talked to each other during this time, a few students often chose to work alone, sometimes while wearing headphones. And others who were working in groups still regularly requested help from Emma by raising their hands when they were unsure of how to move forward or wanted her feedback on whether their work was correct.

Monitoring. While students worked on warm-up tasks, investigations, or practice problems, Emma circulated the room, monitoring students' progress. As Emma monitored, students frequently requested her help by raising their hands and waiting for her to arrive at their desks. Or, if they had finished a task, students would raise their hands to ask Emma whether their answers were correct. As she visited with individual students, Emma often asked questions to help students make progress (e.g., "How does *h* move the graph?" Field notes, 190201). In other cases, Emma reminded students to get started (e.g., "Why haven't we started this one? Is it too easy? Maybe you need a harder one." Field notes, 190410), made suggestions about what to do next (e.g., "You need to take the opposite of the sign on the inside [of the parentheses]." Field notes 190513), or affirmed their answers as correct (e.g., "Beautiful!" Field notes, 190201).

Quizzes. On four occasions, Emma administered quizzes after students had worked through some practice problems to assess whether they could solve the types of problems that were the recent focus of her instruction. Based on the classroom artifacts I collected, the foci of the quizzes that Emma administered included solving low-level problems (e.g., on complex and synthetic division, Field notes, 190109; 190201) and ones that required students to make connections between mathematical representations of functions (Field notes, 190118).

Whole-class discussions. Emma led whole-class discussions during note-taking sessions (discussed earlier) or after students had worked on warm-up tasks, investigations, or practice problems. Those discussions always reflected a pattern of talk that alternated between teacher and student, although Emma occasionally made attempts to orient students to others' contributions by asking whether they agreed with something a student had said.

During whole-class discussions of warm-up tasks or practice problems, Emma typically demonstrated how to solve problems on the SMART Board while asking questions to solicit ideas from students about next steps (e.g., "How many times does x go into $5x^2$?" Field notes, 190201). For investigations, whole-class discussions were different in that they tended to focus more on concepts than procedures. To illustrate an example, consider the following excerpt from field notes I collected on January 1, 2019 when Emma was leading a whole-class discussion of an investigative (Desmos) task for which students were asked to explore transformations of absolute value functions of the form $y = a|x - h| + k$:

> Emma addresses the activity's questions one by one on the SMART Board, calling on students to offer answers to those questions. In some instances, a few students sitting near the middle of the classroom chorally respond to the questions Emma poses.
>
> Emma asks the class, "What happens when we change *a*?"
>
> Some students chorally respond, "It gets smaller."
>
> Emma asks what they mean by "get smaller."
>
> One student says it means that the graph "gets skinnier."
>
> Agreeing with this student's assertion, Emma refers to the transformation as a "vertical stretch" and writes the term on the SMART Board under a question that reads: "How does '*a*' effect the graph of the absolute value function? Be specific and use vocabulary from previous units."
>
> Next, Emma asks, "What happens when *a* is between 0 and 1?"
>
> A few students chorally respond, "It gets bigger."
>
> Emma affirms their response, noting that this particular transformation is called a "vertical compression." As Emma writes the term on the SMART Board, she explains that *a* "can tell you whether the graph vertically stretches or compresses."
>
> Emma asks next, "What if *a* is negative?"

One student responds, “It flips.”

Responding directly to the student, Emma asks, “What do we call that flip?”

She answers her, “A reflection.”

Moving quickly onto the next question, Emma asks, “What does *h* do?”

Some students chorally respond, “Horizontal shift.”

After affirming this response, Emma asks students to turn to someone nearby and explain what *k* does. Then, she tells students that she’s going to call on someone who “hasn’t talked yet” to share their explanation.

Following students’ exchanges with the person next to them, Emma calls on a student who claims that “*k* moves up and down the y-axis.”

After agreeing with his answer, Emma describes the transformation as a “vertical shift.”

Then, Emma alerts students they have run out of time and will continue discussing this activity tomorrow.

Note that in this field notes segment, the questions that Emma asked students—and, thus, students’ responses to those questions—were conceptual in nature, as they pertained to relations between equations and graphs of absolute value functions. Also important to note, however, is that questions of *why* such relations existed were not central to this (or other) conceptually-oriented discussions.

Emma’s Critical Pedagogical Discourse: Three Prominent Threads

As a reminder, teachers’ critical pedagogical discourses are not static and, in fact, are “in flux as they transition across learning-to-teach contexts” (Thompson et al., 2013, p. 579) because such contexts often communicate competing messages about what is important for teaching and learning. In my analysis of Emma’s case, I found that her critical pedagogical discourse included three prominent threads of dialogue. However, Emma was unable to draw on those threads of discourse to filter out contextual discourses that pressured her to teach in conventional ways. Consequently, Emma appropriated contextual discourses as her own, which helped to explain why her practice reflected her critical pedagogical discourse to only some extent.

In the following sections, I first discuss the three threads of Emma's critical pedagogical discourse, and reflect on her attempts to act in ways that aligned with those threads. Then, I discuss Emma's appropriation of contextual discourses to illustrate how those discourses acted as the main drivers of her practice.

Student engagement as a goal for teaching: Thread one. Through one prominent thread of her critical pedagogical discourse, Emma described *student engagement as an important goal* for her teaching. During our first formal interview in July of 2018, just prior to beginning her first year of teaching, Emma envisioned that in an ideal mathematics classroom, students would be "interacting" and "having productive conversations" with each other. Further, Emma suggested that it was the teacher's responsibility to "engag[e] all the kids" in a lesson. For students who were not engaged (e.g., "kinda just sitting there") or seemed "bored," Emma proposed that in response, the teacher should "pose a question that gets [students] interested" or "give them something that challenges their thoughts and makes them think differently" (Formal interview, 180731).

That student engagement was an important goal for her teaching was something Emma maintained throughout the course of the study. In our midyear formal interview in January of 2019, however, the ways in which Emma described student engagement had changed somewhat. In particular, when Emma explained what student engagement would look like in an ideal mathematics classroom, she portrayed the role of students as more passive than she had in our first formal interview:

> I think [I would look for] student engagement, which would look like students actively listening. If questions are posed, students answer them. If students are asked to have a discussion with each other, they actually talk to each other about the math. I think that students are doing what they need to be doing to learn and gain an understanding of whatever the topic is. I would also hopefully look for that the teacher is not doing all of the talking and the thinking—that the students are doing at least half of the talking and thinking, if not most of the thinking in the classroom. (Formal interview, 190102)

Recall, in her first formal interview, Emma envisioned that students would be “interacting” and “having productive conversations with each other” (Formal interview, 180731). But here, Emma explained that student engagement could also look like students “actively listening.” Also worth noting is that in this particular excerpt, Emma depicted a classroom in which students would be doing what the teacher expects of them. In particular, she suggested that students should answer questions “if questions are posed,” talk to each other if they “are asked to have a discussion,” and, ultimately, do “what they need to be doing to learn.” Seemingly new to this thread of Emma’s critical pedagogical discourse is a more explicit focus on student compliance, which is an idea central to contextual discourses that Emma perceived about student behavior (discussed more in a later section). Last, Emma also suggested that students should be “doing at least half of the talking and thinking.” The sentiment that students—and not the teacher—should doing most of “the talking and thinking” is an idea that Emma promoted through another thread of her critical pedagogical discourse, which is discussed in the section that follows.

The importance of shifting mathematical authority from teacher to students: Thread two. Another idea central to Emma's critical pedagogical discourse was that good mathematics teaching requires shifting mathematical authority from the teacher to students. From Emma's perspective, a good mathematics teacher was one who does not "take over" students' thinking by telling them how to approach or work through problems. In an ideal classroom, she suggested, students would be "thinking through" mathematics on their own or with each other "instead of constantly referring to the teacher as the only source of knowledge in the classroom" (Formal interview, 180731).

Based on Emma's descriptions of her own experiences as a K-12 student, she initially thought that mathematical authority resided primarily within the teacher. Thus, the idea that students should have mathematical authority was relatively new to Emma, as, she claimed, it was something she had learned about during her teacher education program:

> A lot of my [K-12] math classes were very lecture-based. It was lecture, take notes, and then you work on homework. I thought that was fine because it worked for me. I was good at taking notes…I was able to learn material that way. But after taking [methods] courses and after student teaching, it's not like- You can't lecture every single class period. That's boring! It's boring for the teacher, and it's boring for the kids. And for some of the kids, it's harder for them to learn if they don't get in there and actually try solving an interesting problem. They won't automatically be engaged in the material. I was automatically engaged because that's just the way I was, but not everybody's like that. And not all my students are gonna be walking into the classroom super hyped up to learn math every single day. Everybody has off days. Everybody has days like, "I don't want to do math today." Before the program, I don't think I would've thought about those

> things. I would’ve been like, “Well, why don’t you wanna take notes every day? It’s easy. We get through the most material. We can move really fast.” But it’s not interesting all the time. It’s not engaging. And it doesn’t give you a chance to really grapple with mathematics. You just kind of absorb the information and then spit back whatever your teacher tells you. You don’t get a chance to actually think it through and think about why it works or how it works. (Formal interview, 180731)

In this quote, Emma explained that before the program, she would not have problematized conventional practices (e.g., lecturing and giving notes) because, as a K-12 student, she was able to “learn material that way,” and such practices ensured content coverage and allowed for moving at a fast pace. But, according to Emma, experiences in her teacher education program challenged her views of conventional practices. For example, arguing that note-taking was “not engaging,” “boring,” and merely required students to “absorb information and then spit back whatever [the] teacher tells [them],” Emma suggested that students should have opportunities to “really grapple with mathematics” and think about why or how things work. Therefore, because Emma perceived that conventional practices did not lend themselves to supporting student engagement (thread one) and shifting mathematical authority to students (thread two), she did not envision making regular use of those practices in her instruction.

Throughout the course of the study, Emma maintained her ideas about the importance of shifting mathematical authority from teacher to students. In describing how she might accomplish the shift in authority she envisioned, Emma promoted the use of three instructional practices: posing particular kinds of mathematical tasks (e.g., “investigative tasks”); asking questions of students while they worked on problems (rather than telling them what to do); and providing opportunities for students to discuss mathematics in small groups (independent of the teacher). In regards to mathematical tasks, Emma argued that ideal tasks would be “open-ended” and have: “multiple pathways to solutions”; “a low floor and high ceiling”—meaning that “all kids in the classroom can do the problem” but some would be able to take the task “really, really far”—; and “an explanation element” that requires students to “justify their thinking” (Formal interviews, 180731; 190102). To further illustrate how Emma characterized high-quality tasks, consider the following excerpt from our final formal interview in May of 2019:

> [In an ideal classroom, students would be working on] problems that make them think on their own—problems where there’s room for productive struggle. So if you’re introducing a new topic, for example, maybe something that’s like an investigation where they’re doing something with some stuff that they maybe don’t fully understand yet, but then we do a lot of stuff with it and kind of arrive to this conclusion that we can all describe what we saw. Something like that. And I mean in a truly ideal world, we would take math and do cool stuff with it like solve problems that are actually interesting and relevant. So looking at things that are happening in our world, coming up with ways to describe them with numbers, and figuring out ways to create solutions to problems. Ideally. I think that’s very idealistic. (Formal interview, 190522)

While Emma described ideal tasks as ones that require students to “think on their own,” her description of how those tasks required this of students was somewhat vague. For example, Emma explained that, ideally, tasks would require students to work through ideas that they “don’t fully understand yet,” and after “do[ing] a lot of stuff with [those ideas],” students would “arrive at a conclusion” about what they had investigated. Thus, while Emma suggested that posing a particular type of task would allow for a shift in mathematical authority to students, her explanation for how such tasks would help her accomplish this was relatively unclear. Also worth noting with respect to this excerpt is that Emma explained that "in a truly ideal world," she and her students would use mathematics to make sense of and address problems in the world, which, as she suggested elsewhere in the interview, would "help kids see the value in math." However, Emma strongly emphasized the ideal nature of the scenario she depicted, perhaps to imply that using mathematics to do such things in school was not necessarily feasible.

Emma named questioning as a second practice that could support her in shifting mathematical authority to students. For example, Emma explained that, ideally, she would be able to "just batter students with questions until they come up with an answer," because doing so would require students to "do more of the thinking" and make their own decisions about how to approach a problem (Formal interview, 190102). Beyond students "com[ing] up with an answer" on their own, however, Emma did not explain what "batter[ing] students with questions" would accomplish. Thus, what Emma envisioned as the end goal of her attempts to shift authority to students was not entirely clear.

The third practice that Emma described as instrumental for shifting authority to students was providing opportunities for students to discuss mathematics in small groups. Because the importance of discussion, more broadly, was the focus of another thread of Emma's critical pedagogical discourse, I describe the ways in which she talked about providing opportunities for discussion in another section.

Over the course of the study, Emma often reflected on the ways in which her attempts to shift authority to students were unsuccessful. Because such attempts were, arguably, an attempt to act on this thread of her critical pedagogical discourse, I discuss Emma's talk about her struggles with shifting authority in the paragraphs that follow.

First, Emma (and I) noticed that as she monitored students' progress during lessons, students often waited for her to visit them at their desks before getting started on a task. Even after Emma had visited with students, they sometimes waited for her to return again before moving forward (Field notes, 190201; Debriefing interview, 190201). Consequently, Emma explained that for the sake of making progress, she had to tell students what to do more often than she would have liked.

From Emma's perspective, students' tendency to rely on her support was likely attributable to her own struggles to support students in building the confidence necessary for solving problems without her guidance:

> I don't think I did enough this year to build up confidence and autonomy in all my students so that they feel comfortable trying things without me. I know I didn't do that because some of them won't feel comfortable doing a problem unless I'm standing there. And I don't have to do anything. I'm just there. And that makes them feel secure enough to try a problem. So I have not done enough to create confidence in them that they can do things on their own. (Formal interview, 190522)

In this excerpt from our final formal interview in May of 2019, Emma claimed that it was not only her guidance that students relied on but also her mere presence, as, she noted, students seemed more "comfortable" and "secure" when solving problems if she was standing nearby.

In that same interview, Emma reflected on her struggles to shift mathematical authority to students in other instances. While, as illustrated by the last interview excerpt, Emma named students' lack of confidence as an obstacle to shifting authority to them, she also acknowledged that her own actions may have reinforced students' tendency to rely on her support:

> It's difficult for me to watch a kid struggle. It's difficult to stand there and watch them kind of flounder. And keeping myself from jumping in to assist is really hard. And I think sometimes I do it, but I don't do it consistently—the stepping back and letting them figure it out. I don't do that enough, I think, for them to feel like they can do things on their own. And that is something that I really need to improve on because I think doing that, like just taking a step back sometimes, and letting them figure it out because they can, would help build confidence. I know that's something that I have a hard time with. And I think part of the reason is that I'm still trying to figure out when to make the call of when I need to step in. Kind of seeing like, "How far can I push this kid before they have a full-blown meltdown?"…And I think it's definitely fear of those full-blown meltdowns that have kind of kept me from maybe pushing some kids. Because some kids I have pushed, and they've had meltdowns. And that's not fun. (Formal interview, 190522)

In this excerpt, Emma explained that it was difficult for her to "watch [students] struggle" and refrain from "jumping in to assist" them. Reflecting on the ways in which providing assistance may have inhibited students from developing confidence, Emma suggested that she could more consistently "step back" and let students figure things out on their own. Toward the end of the excerpt, however, Emma attributed her tendency to "step in" to the ways in which students sometimes reacted to her attempts to shift authority to them. In particular, Emma said that students sometimes had "full-blown meltdowns" in response to her "pushing" them. Such reactions, she suggested, may have reinforced her use of other practices that, from her perspective, were less likely to result in "meltdowns" (e.g., telling students what to do next). In addition to pointing to student behavior to describe the challenges she faced, Emma's talk, here, suggests that students may have communicated expectations for Emma's teaching (e.g., that she would tell them what to do).

To summarize, while Emma maintained her view that shifting mathematical authority to students was important because it required students to "do more of the thinking," she struggled in her attempts to enact that view. According to Emma, this was for, at least, three reasons: (1) she did not sufficiently support students in developing the confidence required to solve problems on their own; (2) she found it difficult to refrain from providing guidance when students struggled; and (3) she feared students' reactions (e.g., "meltdowns") to her attempts to support them in doing things on their own. From Emma's perspective, then, her efforts to shift mathematical authority to students were often unsuccessful because of limitations of her own practice and resistance on the part of students.

While such things may have contributed to the challenges that Emma faced, her attempts to shift authority to students were likely undermined by her regular use of conventional practices, such as giving structured notes and demonstrating to students how to solve problems. And although Emma recognized that such practices would not support her in shifting authority to students, she justified using them for other reasons. Those justifications were often rooted in contextual discourses that Emma perceived within Schenley, which I describe more in a later section.

Student learning requires providing opportunities for discussion: Thread three. A third thread of Emma's critical pedagogical discourse focused on *providing opportunities for students to discuss mathematics* both in small- and whole-group settings. In general, Emma emphasized the importance of providing opportunities for students to discuss mathematics because, from her perspective, learning mathematics necessitates talking through ideas. With respect to small-group discussions in particular, Emma suggested that sharing ideas in small-group settings was “lower risk” for students when compared to making contributions to whole-group discussions (Formal interview, 190522). Further, because small-group discussions allowed students to talk independently of the teacher, Emma considered the provision of opportunities for small-group discussion as a means for shifting authority to students. To further illustrate this thread of Emma’s critical pedagogical discourse, consider the following excerpt from our first formal interview in July of 2018. In this excerpt, Emma reflected on why providing opportunities for discussion mattered to her:

> I think that learning is such a social thing. And with math like- I feel like traditionally it’s not been very social. But we learn so much better when we can talk to each other and think through our ideas and pose ideas that we’re not quite sure of, and we don’t know if it’s gonna work out. And then, when you’re talking with a classmate as opposed to with the teacher, the power is more equal. Because when you’re talking to a teacher, the teacher is automatically in a position of power over their students, which can be intimidating for some kids. But if they have the chance to have discussions with a peer or someone they like and trust and can relate to, then I think they’re more likely to maybe steer in directions that they might not with the teacher. They might feel like it’s a little scary to go there with the teacher because the teacher is sometimes this whole, like, figure. Whereas their classmates are their friends, and they can ask them questions about things that they might think are kinda silly, or that they’re not sure of the answer to, and feel okay about it. (Formal interview, 180731)

In this quote, Emma problematized perspectives on the “not-very-social” nature of learning mathematics by arguing that talking through ideas is how learning happens. Then, in describing student-teacher power dynamics, Emma suggested that allowing for small-group discussions provides opportunities for students to take mathematical risks and pose questions without feeling pressured or "intimidated" by the teacher. As alluded to earlier, this illustrates relations between Emma's focus on student discussion and the thread of her discourse through which she promoted shifting mathematical authority from the teacher to students. That is, Emma framed the practice of providing opportunities for small-group discussions as means for supporting students in doing mathematics independently of the teacher.

That opportunities for discussion can cultivate student learning and autonomy was something Emma maintained throughout the course of the study. By the end of school year in May of 2019, Emma's rationales for the importance of student discussion were similar to those that she had articulated in July of 2018:

> [In an ideal mathematics classroom], you'd see students talking to each other about math and discussing what they're doing and asking each other questions and not necessarily just turning to the teacher to be the expert. And I think groupwork is really great for building kids' understanding. So if they are participating in groupwork, having it be something that genuinely needs to be groupwork so that it's actually groupwork instead of kids silently passing a paper back and forth, which can happen. So if they are doing group tasks, making sure there's something that actually necessitates being in a group so everybody has a role to play and something to contribute. And they understand what that role is, and they're actually doing it. And ideally, if the kids are engaging in groupwork and talking to each other, the teacher is mostly there to facilitate and make sure that groupwork is happening smoothly, which means that you're there if the kids are absolutely stuck and can't get anywhere together…Classroom discussion I think ideally

> would be almost entirely the students. And, I know I kind of said this earlier, but facilitated by the teacher. So students asking other students questions and then answering those questions. And I also feel like the dialogue should always be friendly. (Formal interview, 190522)

Again, Emma described providing opportunities for small-group discussions as instrumental in "building [students'] understanding" and sharing authority with them. Additionally, Emma suggested that organizing students for small-group work would not necessarily cultivate student learning; it was also important, she argued, that students worked on group-worthy mathematical tasks that required all students in the group to contribute in some way. Last, this excerpt provides another example of how the student authority and student discussion threads of Emma's critical pedagogical discourse were related. For example, notice that at the end of the excerpt, Emma explained that in an ideal mathematics classroom, students would be the main drivers of discussions, while the teacher would assume the role of facilitator.

Worth noting here is that Emma rarely focused on *what* students would be talking about when reflecting on her lessons. Although when prompted, Emma explained that, ideally, students would be "challenging each other's reasoning" (Formal interview, 180731) and "asking each other questions" in a "friendly" manner (Formal interview, 190522), she more often attended to whether students were talking to each other at all. This is not necessarily surprising, however, given that through this particular thread of her discourse, Emma did not explain how, exactly, opportunities for discussion would support student learning.

Emma's instructional practice often reflected the thread of her critical pedagogical discourse focused on discussion, as in each lesson I observed, she provided opportunities for students to discuss mathematics in both small- and whole-group settings. However, whole-group discussions were always led by Emma and, consequently, followed a pattern of talk that alternated between the teacher and students. Further, Emma often struggled to facilitate the small-group discussions she envisioned, as some students often worked alone, even when sitting in small groups. From Emma's perspective, this was likely due to limitations of her own practice (e.g., not doing enough to cultivate a culture in which students willingly talk to each other) and challenges posed by student behavior. Regarding the latter, Emma explained that students sometimes "refused to work in groups" (Debriefing interview, 190109), and argued that they were incapable of having productive ("on-topic") conversations with peers (Debriefing interview, 190118), which made it difficult to facilitate small-group discussions.

To summarize, while Emma maintained the importance of providing opportunities for student discussion through this thread of her critical pedagogical discourse—and, to some extent, was able to act on this thread—she struggled to fully enact her vision for student discussion. From Emma's perspective, this was due to limitations of her own practice and issues of student behavior. While Emma's analysis may have been accurate, the challenges she faced in facilitating the types of discussions she envisioned (and her subsequent actions) may have been influenced by her use of conventional practices, which likely undermined her attempts to get students to talk about mathematics independently of the teacher.

Emma's Perceptions of Contextual Discourses (and Constraints)

Before describing the contextual discourses that Emma perceived, I first recall specific details about her case. While Emma had appropriated threads of discourse that were aligned with ambitious practice (e.g., shifting mathematical authority to students and providing opportunities for discussion), she was still susceptible to contextual discourses that promoted conventional teaching. Further, while Emma noted tensions between the kind of teaching she envisioned and perceived pressures to teach in conventional ways, she did not express feelings of frustration when describing those tensions. As a result, Emma appropriated contextual discourses alongside threads of her critical pedagogical discourse and drew on those discourses to explain and justify her use of conventional practices.

In the sections that follow, I illustrate the ways in which Emma appropriated contextual discourses, explain, when possible, how those discourses interacted with threads of her critical pedagogical discourse, and discuss how discourse interactions helped to explain her practice.

Emphasis on student behavior and engagement. Emma described contextual discourses that communicated messages about issues of student behavior and engagement at Schenley. Worth recalling, here, is that in regards to Schenley's student population, students of color made up 47%, and students who received free-and-reduced lunch made up 53%. Based on interviews with Emma, contextual discourses pertaining to issues of student behavior and engagement were communicated through Schenley's disciplinary policies and practices, as well as by her colleagues and mentors.

First, regarding Schenley's disciplinary policies, Emma described two commonly used practices: "writing referrals" to document incidents of misbehavior and disengagement, and conducting hallway "sweeps." According to the hallway sweeps policy, adults were to "sweep" students to the office if they were late for class without a pass or found wandering the hallways. Then, after checking in with an adult in the office, students were escorted back to class. In the following excerpt from our first formal interview in July of 2018, Emma explained, based on her student teaching experiences, why Schenley had those policies in place:

> My impression as to why [those policies exist] is based upon the demographics of the kids and maybe the small amount of control that there is over some of the home lives of

> the students. And so school is a controlled, very stable environment, or at least tries to be a stable environment for the kids. Um, because that population of kids needs it, or at least it seems like they need it…[Compared to the other high schools in this town,] the student population at Schenley is more diverse both ethnically and in terms of socioeconomic status…It's interesting listening to students talk about things that they've experienced. I mean, there are students I had in my classroom who had been shot and stuff, or one of their cousins has been shot. And their parents aren't super stable necessarily. Not all of the kids, um, just some of them. And so Schenley is more controlled…(Formal interview, 180731)

In this interview excerpt, Emma suggested that Schenley's hallway sweeps and referral policies existed due to student demographics and a lack of stability in students' home lives. Emma agreed with (her perception of) why those policies were in place, arguing that because Schenley's students were diverse in terms of "ethnicity" and "socioeconomic status" and, in some cases, had "[un]stable" parents, students needed a school environment that was "controlled." Given that Emma attributed the (negative) circumstances that some of her students may have faced to individual characteristics and students' home lives—rather than to injustices perpetuated by social and economic structures—her discourse, here, is rooted in deficit narratives.

In addition to disciplinary policies, contextual discourses about student behavior and engagement were also communicated by teachers and other individuals overseeing Emma's instruction, including her principal and department chair. For example, Emma said that one of the classroom "indicators" on which she was evaluated was "student engagement." Therefore, when her principal or department chair dropped in to observe her, Emma expected them to focus on things like "how many students were engaged in the lesson" or "how many kids weren't on their phones or asleep" (Formal interview, 190522). To further illustrate Emma's perception of these contextual discourses, consider the following excerpt from our midyear formal interview in which she described the focus of classroom observations conducted by her principal and department chair:

> [My department chair] mentioned that there were a couple students in that class that pretty continuously use profanity—just like speaking out of turn or that just are kind of disruptive. We talked about a couple strategies I could use, like some grouping strategies, which would have worked if the disruptive kids weren't trying to fight each other. So they can't sit next to each other. Yeah I think mostly- When they've come in, I've asked them mostly to focus on my classroom management and the questions I ask students…I have not asked them to focus on my, like, instructional approach so much. (Formal interview, 190102)

According to this excerpt, Emma's department chair gave her feedback that was focused on issues of student behavior and engagement, and offered ideas for addressing them (e.g., grouping strategies). Further, Emma had not necessarily filtered out contextual discourses about student behavior and engagement, as she explained that she more often wanted feedback on her "classroom management" practices than on her "instructional approach." This is not necessarily surprising, however, given that through the first thread of her critical pedagogical discourse, Emma described student engagement as a goal for her teaching. Additionally, recall that Emma's talk about student engagement had changed over time, as midway through the study she suggested that in an ideal classroom, students would be doing things that the teacher had asked them to do. This change in Emma's critical pedagogical discourse might be explained by her appropriation of contextual discourses that were focused on controlling student behavior.

Emma's appropriation of these contextual discourses is also illustrated by the focus of her reflections during debriefing interviews. In particular, Emma consistently considered whether and how students were engaged, and often reflected on the effectiveness of the strategies she employed in her attempts to engage them. For example, Emma explained that she sometimes grouped students in particular ways in order to "break up groups that were incredibly disruptive" (Debriefing interview, 190508) or to keep students from "constantly chatting about [off-topic] things" (Debriefing interview, 190410).

Additionally, Emma often expressed frustration with her attempts to engage students in class, as, from her perspective, they were often unsuccessful. By the end of the study, Emma lamented that her failed attempts to engage students had caused her to "lose patience" (Debriefing interview, 190410). While she considered that her issues with student engagement may have been a symptom of her own actions (e.g., "not holding kids accountable," Formal interview, 190102), she asserted that some students "just don't like doing things" and that there was little she could to do address that (Debriefing interview, 190109). Emma also argued that disciplinary policies could be stricter, as, based on her experiences, they were largely unsuccessful in effecting change in students' behavior (Formal interview, 190522). To further illustrate this, consider the following excerpt from our final formal interview in which Emma reflected on why she experienced difficulty fostering student engagement:

> I think sometimes if students don't understand something right away, they get frustrated and then they check out of the process. And they don't want to be re-engaged in it because it's hard for them. And they don't want to do it because it's hard…I think sometimes they just feel like they would rather not. And so they would rather just sit there and—well maybe not all class—but they would rather just sit there and be on their

> phones. And if I've already written a referral for you being on your phone seven times, it's an issue. At that point—I'm thinking of a couple students in particular—at that point, I don't know what other tools I have to address their behavior. (Formal interview, 190522)

In this excerpt, Emma attributed students' disengagement to their tendency to "check out" when something is "hard for them" and to their general disinterest in classroom activity. After describing the disciplinary policy of writing referrals as ineffective, Emma suggested that she lacked the tools necessary to re-engage students. Emma's talk, here, illustrates her appropriation of contextual discourses about student behavior and engagement, as she framed the actions of students as obstacles to enacting aspects of her vision for instruction.

In summary, that Emma consistently talked about student behavior and engagement when reflecting on her practice—and seemingly without reservation—suggests that she was susceptible to and, ultimately, appropriated contextual discourses that focused on such issues. Her appropriation of those contextual discourses might be explained by two things. First, because student engagement was a focus of one thread of Emma's critical pedagogical discourse, it could be that the contextual discourses discussed in this section only reinforced her preexisting ideas about mathematics learning and teaching (and students). Second, although managing issues of behavior was not a focus of Emma's critical pedagogical discourse (meaning that she did not describe herself as a teacher who, for example, manages behavior well), threads of her discourse (e.g., shifting mathematical authority to students) may not have been defined enough to act on, which may help to explain why student behavior and engagement were among Emma's primary concerns.

Other teachers' use of conventional practices. Recall that mathematics teachers at Schenley were members of content-specific Professional Learning Teams (PLTs), so Emma spent time with the Algebra I and Algebra II teachers every other day during Algebra PLT meetings. According to Emma, the primary purpose of PLT meetings was to provide regular opportunities for teachers who were teaching the same classes to "plan and collaborate" (Formal interview, 190522). While Emma described the Algebra PLT as more structured than the Geometry PLT, she insisted that she had autonomy in teaching as she saw fit. This is interesting, however, because Emma also expressed feeling pressure to use the activities for which her PLT had planned (e.g., giving notes).

For example, midway through the study, Emma noted that she had not enacted some ambitious practices as often as she had initially envisioned and, instead, frequently employed conventional ones (e.g., "I've done way more notes that I've ever wanted to do." Formal interview, 190102). While Emma suggested that this was partially because her PLT more regularly planned for giving notes than posing investigative tasks, she did not express frustration with the pressure she felt:

E: I do feel like I have to give the notes because everybody else is giving them. I think I feel that pressure partially because it's my first year teaching. I've never taught Algebra II or Algebra I before. And I don't want to not get through the notes because of some deficiency in my teaching, which is probably the case because it's my first year teaching. I would love to do more investigative stuff. It is a lot more time consuming, and the structure and planning part is a lot more than you have to do with notes. Notes are pretty easy to make and to give. Investigations, not so much. And there's a lot of variables with investigations that you really, you can't necessarily plan for because you just aren't sure of some things…

C: So do you think that- Are there other obstacles aside from the note-taking structure that you think inhibit you from doing more investigative stuff? You said- Is it that it's hard to even plan those tasks?

E: Um, I mean it is hard to plan those tasks. And I think it's partially because the PLT is so organized, and I don't necessarily want to disrupt that. Not that they're not receptive to disruption. I think that's just an insecurity I feel because I feel like I don't know enough to disrupt something that has been working relatively okay…especially if it means more work for everybody because they all work really, really hard already. So I think that partially just my insecurity in myself as a teacher kind of keeps that from happening. (Formal interview, 190102)

Based on this exchange, it seems that because Emma perceived herself as a novice in relation to other members of her PLT, she was able to justify *not* engaging in practices that, for example, would allow for shifting mathematical authority to students like she had envisioned. In particular, from Emma's perspective, planning for and implementing "investigative" tasks would create more work for her PLT and pose challenges that, as a novice teacher, she could "not necessarily plan for." Consequently, conventional practices like giving notes were safer for Emma, as they could *ensure coverage of the designated content* (a contextual discourse discussed more later) and would not disrupt the order of the PLT.

Toward the end of the study, Emma more strongly justified her use of giving notes, as well as other conventional practices regularly used by members of her PLT (e.g., direct instruction). While she still expressed skepticism that such practices could support students in "thinking on their own," Emma had developed new arguments for the merits of leading structured note-taking sessions and employing direct instruction. First, Emma promoted such tools for their "efficiency," suggesting that they helped her ensure that "everybody [got] the same information." Second, because giving notes and direct instruction were "low risk" activities, she argued that it was easier to "sell" them to students: "It's pretty easy to get kids to do notes" (Debriefing interview, 190502).

At the end of the school year, Emma suggested that her views on giving notes and enacting direct instruction had changed, distinguishing her current perspectives on such practices from those promoted in her methods courses (e.g., "We were encouraged to stay away from direct instruction." Formal interview, 190522). Further, she attributed her new outlook on the merits of conventional practices, in part, to the work she had done with her PLT. To illustrate this, consider the following excerpt from our final formal interview in which Emma reflected on these ideas:

> I think part of [the changes in my views] comes from discussions I've had with my PLT this year and the way they teach. Because their approach to teaching is probably more direct instruction than indirect instruction. Especially for introducing new topics, it's more direct instruction. I think towards the latter part of this year, we have done more stuff with indirect instruction and investigations. And I think both ways were successful. I think students learned both ways. Something that I did notice about indirect instruction versus direct instruction is that when you do an investigation, kids are forced to think. And if you're doing direct instruction, especially notes, they're not always thinking. I think that is one big downside to direct instruction. And kids know how to do it because they've been in school for a while…And so, it's a very easy thing for you to ask of them.

> Like, they'll gladly do it because they're just sitting there copying stuff down. But I don't think I've seen a difference in student achievement either way…

Evident in this quote are traces of Emma's critical pedagogical discourse thread through which she articulated the importance of shifting mathematical authority from teacher to students. For example, Emma suggested that "doing direct instruction [or] notes" does not necessarily require student thinking, which she characterized as a "downside." However, Emma drew on discourses pertaining to engagement and students' expectations for teaching to rationalize her use of conventional practices. For example, note that Emma described giving notes as a tool for both engaging and appeasing students: "Like, they'll gladly do it because they're just sitting there copying stuff down." And, as she noted at the end of the excerpt, she could point to the results of student assessments or "achievement" outcomes to further justify her actions.

To summarize, while Emma envisioned shifting mathematical authority to students, she often engaged in conventional practices, which, based on Emma's accounts, were regularly used by members of her PLT. While Emma recognized that her enactment of such practices would not support her efforts to shift mathematical authority to students, she rarely expressed feelings of conflict when acknowledging this. Instead, Emma justified her use of conventional practices by drawing on threads of her critical pedagogical discourse (e.g., a focus on student engagement), as well contextual discourses she perceived. For example, Emma suggested that giving notes could support her in covering content (a contextual discourse described in more detail next), controlling student behavior (emphasized by individuals who observed her teaching), and meeting students' expectations for their participation in class.

Responsibility to cover content and stay on pace. As alluded to in the previous section, Emma perceived contextual discourses that communicated the importance of covering the content designated for a particular course and staying on pace with the other members of her PLT. While Emma sometimes explained that these contextual discourses pressured her to teach in ways she had not necessarily envisioned, she ultimately appropriated those discourses as her own. This is illustrated by Emma's talk in debriefing interviews, as she consistently considered whether she and her students had "[gotten] through" everything she had planned for the day, and contemplated the implications of content coverage (or lack thereof) for upcoming lessons. Because Emma appropriated contextual discourses focused on content coverage and pacing, those discourses, to some extent, guided her practice.

Emma's recurring suggestion that she sometimes had to forgo doing what may have been best for students for the sake of covering content further illustrates her appropriation of contextual discourses focused on content coverage and pacing. For example, Emma explained that, from her perspective, many students lacked the number sense necessary to be successful in Algebra I and II and needed additional support for developing "foundational skills." However, Emma expressed feeling like there was little she could do to support students in developing those skills because of her responsibility to cover designated content:

> Finding ways to build in chances for students to increase their number sense while also getting to standards isn't something I have found the time to do yet this year. I don't think you were here, but we practiced, we just did fraction stuff a couple cycles ago—just like adding, subtracting, multiplying, and dividing fractions—and it was so rough. And I felt bad because a lot of them felt bad because they didn't remember how to do it. But they haven't done it since elementary school and their understanding then probably wasn't super solid…But I feel like I haven't had the time to help them have a solid number sense and get to the stuff we need to cover. (Debriefing interview, 190201)

Here, Emma suggested that she could not necessarily spend time supporting students in developing number sense because she also had standards to "get to." Even though she perceived that students "felt bad" about struggling to solve problems that they likely solved in elementary school, Emma prioritized "get[ting] to the stuff [they needed] to cover," which, arguably, illustrates the critical nature of this particular contextual discourse.

Similarly, while Emma often suggested that it would be beneficial for student learning to pose particular kinds of tasks, she explained that it was difficult to pose those tasks and also ensure coverage of the designated content. This is illustrated in the following excerpt from our final formal interview in May of 2019 in which Emma reflected on her struggles to enact certain aspects of her vision for teaching:

E: I don't think I have come up with a really good application problem this year that is actually interesting. I think the closest I've gotten was doing something with climate change. But I don't think that that was enough. And I don't know if I did enough with it to make it do anything…And I think the reason why I haven't done that is time. Even not doing things like [application problems], there's content we haven't covered this year in both my classes. So, yeah. I think time. And also they're hard to come up with. Like, it can be hard to think of a really cool way to apply like polynomial long division. I don't even know if you can.

C: So when you say time- Because you brought up stuff that you didn't cover, do you mean it's hard to fit those [tasks] in given requirements for content?

E: Yes. I do. And I think we spend a lot of time learning how to do something and then we need to move onto the next thing. And so we don't have time to, I guess, spend time learning how to apply that thing we just learned, which sucks because

> it would stick more and make more sense if we were able to do things with everything we learn instead of just learning the process of doing it. (Formal interview, 190522)

In this excerpt, Emma suggested that in addition to struggling to generate ideas for “application problems” (perhaps due to a lack of particular resources), she posed such problems less often than she would have liked because she lacked the time needed to do so. She added that despite posing a small number of “application problems” throughout the school year, there was still “content [she hadn’t] covered” in both her Algebra I and Algebra II classes. At the end of the excerpt, Emma’s concern with content coverage seems to take precedence over her desire to pose particular types of tasks, as she suggested that “[she and others] don’t have time to spend [on] learning how to apply [things],” even though doing so would, from Emma’s perspective, benefit student learning.

Emma’s appropriation of contextual discourses about content coverage and staying on pace also seemed to influence the rate at which she moved through the activities she had planned for. In field notes from one of my classroom observations, I noted that it seemed like Emma was in a bit of a rush, as she was “moving and talking pretty quickly” while passing out a notes worksheet after collecting a quiz from students (Field notes, 190410). Assuming that her students, to some extent, had sensed her urgency, I noted in my field notes that one student had asked Emma if she would “please chill out” as she walked by him while handing out papers for the next activity.

In our debriefing interview following this particular observation, I asked Emma how she felt about her lesson that day. In response, she suggested that the quiz she administered was “not well received” by students. Emma attributed students’ frustration with the quiz to the fast pace at which they were moving through the current unit, noting that she administered the quiz only one class after introducing the unit. However, because Emma perceived that she was “a day behind” the other teachers in her PLT, she suggested that the pace at which she was moving was necessary.

In response, I asked Emma whether she “[felt] pressure” to catch up to the other teachers in her PLT. She said that while “no one was being mean about [where she was with respect to content],” she decided to teach at a faster pace so that she and her students could “get to everything.” To rationalize this decision, Emma explained that she needed to “cover radical functions” in her Algebra II classes because radical functions were included in the learning standards that would be assessed on the end-of-course test. Thus, it is possible that in addition to her PLT, the pressure Emma perceived to cover content and stay on pace may have been communicated by matters of accountability (e.g., state testing).

To summarize, Emma appropriated contextual discourses about covering content and staying on pace. Emma’s appropriation of such discourses is demonstrated, in part, by the ways in which she prioritized covering content and staying on pace over other goals. For example, while Emma was able to articulate the ways in which focusing on content coverage may have come at the expense of student learning, she rarely, if ever, problematized content coverage as a priority. Thus, because Emma’s critical pedagogical discourse did not act as a resource for filtering out these contextual discourses, perceived messages about content coverage and staying on pace were consequential for her practice.

Summary of Emma’s Case

As demonstrated in the discussion of this case, Emma’s teaching occasionally reflected threads of her critical pedagogical discourse through which she promoted ambitious practice, as she sometimes: posed “investigative” tasks that required students to explain their reasoning, organized students in small groups to facilitate student discussion, and posed questions to advance students’ thinking rather than telling them what they should do next. But Emma more often employed conventional practices (e.g., giving notes) that, from the outset of the study, she had not aspired to use. Eventually, Emma began to justify her use of conventional practices by arguing that such practices could ensure student compliance and content coverage, illustrating her susceptibility to—and appropriation of—contextual discourses that she perceived within Schenley. While Emma’s appropriation of such discourses was likely due, in part, to the nature of her critical pedagogical discourse, it may have also been influenced by (her perception of) her status as a novice teacher.

The Case of Sara

Sara's case is one of a beginning mathematics teacher whose critical pedagogical discourse minimally reflected ambitious practice and was well-aligned with the contextual discourses she perceived within Schenley. While there were notable differences between contextual discourses and threads of Sara's critical pedagogical discourse, such differences did not cause tension for her. Consequently, Sara consistently acted in ways that reflected her critical pedagogical discourse.

In addition to this general overview, there are two important points to make about Sara's case. First, although Sara's critical pedagogical discourse and perceptions of contextual discourses did not interact in ways that posed challenges for her teaching, she sometimes drew on her critical pedagogical discourse to challenge contextual discourses that communicated messages related to student behavior. Second, one thread of Sara's critical pedagogical discourse—teaching for conceptual understanding—emerged over the course of the study. Further, ideas central to this thread reflected both ideas promoted in her methods courses and contextual discourses she perceived within Schenley.

In the sections that follow, I first provide an overview of Sara's teaching, highlighting the instructional practices and routines that were most frequently observed. Then, after introducing five prominent threads of Sara's critical pedagogical discourse and describing the contextual discourses she perceived, I reflect on the ways in which those discourses interacted, and discuss the implications of those interactions for Sara's actions in the classroom.

Sara's Mathematics Teaching

Before describing the details of Sara's mathematics teaching, at least four things are worth noting about her instructional practice. First is Sara's dedication to building relationships with her students, as she consistently demonstrated knowledge about students and their lives outside of school, and shared with them her own experiences and interests. Second is that Sara's primary approach to teaching was conventional in nature, as she regularly demonstrated to students how to solve low-level mathematical tasks (Smith & Stein, 1998). Third is that following demonstrations, Sara typically engaged students in what she called "collaborative learning" activities during which students practiced solving problems similar to those that Sara had just shown them how to solve. Fourth is that Sara often introduced strategies or tricks that students might use to recall definitions and procedures. These themes, along with other details about Sara's teaching, are further described in the sections that follow, which are organized according to the instructional practices and routines that were prevalent across observations.

Warm-up tasks. As students entered Sara's classroom, she was usually preparing for class but always acknowledged and chatted with students while doing so (e.g., To a student who walks into class wearing a Saint Louis Cardinals shirt, Sara [a Cubs fan] says jokingly, "Cardinals? Ew. That's gross." Field notes, 190201). Before sitting down at their desks, which were arranged in either pairs or groups of four, students usually picked up a warm-up worksheet from a table at the front of the room. While waiting for class to begin, students began working on the warm-up task and/or casually chatted with peers. During this time, it was typical for Sara to join students' conversations. Sometimes, when students had not picked up a worksheet on their way into class, Sara joked with them about how they should be familiar with the warm-up routine by now (e.g., To one group of students, Sara says with a smile, "And none of you have a warm-up. It's February 1st. Come on now." Field notes, 190201).

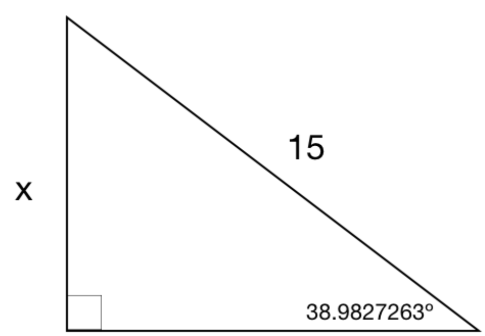
Sara's warm-up tasks typically prompted students to solve low-level problems and were topically related to a previous or upcoming activity. For example, before introducing an activity focused on identifying, naming, and sketching geometric figures (e.g., lines, line segments, and rays), Sara asked students to work on a warm-up task that required them to use "proper geometry notation" to make "true statements" about a diagram of intersecting lines (Field notes, 190109). While students occasionally talked to each other while working on warm-up tasks, they usually worked alone.

Structured note-taking sessions. Following discussions of warm-up tasks (discussed more later), Sara led students through structured note-taking sessions. While she typically led note-taking sessions from the front of the room on the SMART Board, on a few occasions, Sara instructed students to copy notes from a Microsoft PowerPoint presentation or from definitions that were posted around the room on sheets of paper. Some of the notes worksheets that Sara distributed were structured similarly to those used by Emma, as they provided spaces for writing down an essential question, marginalia, definitions of mathematical terms, example problems, and a summary. In other cases, notes worksheets followed a different, less rigid structure. For example, on one occasion, Sara distributed a notes worksheet that provided a blank table in which students recorded definitions of and drew diagrams to represent mathematical terms.

After distributing notes worksheets, Sara guided students through filling them out on the SMART Board, often while soliciting student input by posing questions to the entire class (Stein et al., 1996). To illustrate a typical note-taking session, consider the following excerpt from the field notes I collected on February 1, 2019. For context, it is important to note that in classes preceding this note-taking session, Sara (and, according to her, other members of the Geometry PLT) had engaged students in an activity sequence focused on right triangle trigonometry. For this sequence, a primary goal was to teach students how to calculate side length ratios of right triangles (opposite side/hypotenuse [O/H]; adjacent side/hypotenuse [A/H]; and opposite side/adjacent side [O/A]) *before* introducing the formal names for those ratios (i.e., sine, cosine, and tangent) and the trigonometric function on a calculator. Leading up to the note-taking session described below, students had been taught how to solve right triangle trigonometry problems using only two worksheets: one that listed eight "reference triangles," and another that provided a trigonometric table (on which the "sine," "cosine," and "tangent" ratios had not been named).

Prior to beginning the session, Sara distributed a notes worksheet and asked students to solve the first example problem. This problem, seen in Figure 2, required students to calculate the length of a side of a right triangle using their trigonometric table worksheet.

Figure 2
First Example Problem from Notes Worksheet



Because the measure of the reference angle in this problem was not included on the students' trigonometric table, they were unable to find the value of the ratio associated with it (O/H) and, therefore, could not use the table to solve the problem. To begin the session, Sara asked students why they thought they may have been struggling to solve the first example problem:

> Sara asks, "So why do you think we can't solve the first problem with the [trigonometric table]?"
>
> A few students say out loud, "because of the decimal."

Sara responds, “The angle is a decimal. Right!” Then, Sara tells students to hold their trigonometric table sheets up in the air, say “nice knowin’ ya,” crumple them up, and throw them into the recycle bin.

Students follow her instructions and begin crumpling up and disposing of their trigonometric table sheets.

After students return to their seats, Sara tells them that “instead of the [trigonometric table],” they will now need to “use calculators to solve these types of problems.”

After making sure that everyone has a calculator, Sara guides students through putting their calculators “into degree mode.” While doing this, Sara warns students that they’ll “get the wrong answers” if their calculators are not in degree mode.

Next, Sara asks the class to remind her of the three ratios they’ve been using to solve “similar [right] triangle problems.”

In response, students begin shouting out ratios (e.g., “O-H!”).

On the SMART Board, Sara writes the three ratios that students name (i.e., O/H, A/H, and O/A) into a 1 by 3 table provided on the notes worksheet under the question, “What are the 3 ratios we use during trigonometry that you are familiar with?”

Then, Sara reveals that “there are actually names for the ratios [they’ve] been using,” which are “sine, cosine, and tangent.”

While writing down those terms on the SMART Board, Sara jokes about how some people say “sin” and “coss,” and tells students to be sure to pronounce the “ine” at the end.

Then, Sara introduces the acronym for remembering the three trigonometric ratios, SOH-CAH-TOA, which is written in large font at the bottom of the notes worksheet. To represent the three trigonometric ratios within the acronym, Sara draws slashes between the last two letters of each three-letter string on the SMART Board: SO/H CA/H TO/A. Next, Sara moves to stand directly in front of the SMART Board and begins shouting pairs of sides to see whether students can correctly identify which ratio they'll need to use: "O-H! What are we gonna use?"

In response, some students shout "sine," while others shout "sin." Laughing, Sara affirms their answers.

After repeating the same process for the other two ratios, Sara calls students' attention back to the right triangle problem on the notes worksheet that they had attempted (unsuccessfully) to solve earlier, and begins guiding them through it.

First, Sara tells students to identify the given ratio and then to find the trigonometric function associated with it. Next, "Like [they] did before," Sara says, "[they'll] need to set up a proportion, write the trig function over 1, and cross multiply."

While explaining how to set up the proportion, Sara writes it on the board:

$$\frac{x}{11} = \frac{sin\,(38.9827263)}{1}$$

Then, Sara asks students to try solving for *x* using their calculators.

As students begin solving, one student expresses frustration that the trigonometric table worksheet they had just learned how to use are no longer relevant: "So the yellow paper was just pointless."

"It wasn't pointless to me," Sara says in response. To rationalize teaching students to use the trigonometric tables to solve right triangle trigonometry problems, Sara says, "You gotta learn the hard way before you learn the easy way."

Two things are worth noting about this field notes excerpt. First is the low-level cognitive demands of the mathematical activity in which students were engaged. Note, for example, that after Sara introduced trigonometric functions to students, she asked them (collectively) to practice identifying relevant functions based on the ratio provided, and demonstrated how to set up and solve practice problems step by step. Second is that the discussion reflects an initiate-respond-evaluate (IRE) pattern of talk, as Sara consistently posed questions to which students responded, and then immediately evaluated those responses.

"Collaborative learning" activities. Following structured note-taking sessions, Sara usually posed practice problems for students to work on that were related to example problems in the notes. While Sara sometimes distributed worksheets of practice problems for students to work on at their desks, she also engaged students in solving practice problems through "collaborative learning" activities. According to Sara, she had learned about some of those activities from "a book of strategies" she received at an Advancement Via Individual Determination (AVID) professional development session, and occasionally referenced that book when planning to have students "practice" but also "be engaged" (Formal interview, 190102). A list of collaborative learning activities that Sara named, along with brief descriptions of how Sara used them, is provided below:

- Math trails: In small groups, students solved one of several problems listed on sheets of paper posted around the room. After arriving at an answer for one of the problems, students looked for that answer, which, if correct, was posted somewhere else in the room along with another problem that they would solve next. If students could not find their answer on any of the sheets of paper, it meant that they had solved the most recent problem incorrectly and had to revise their work. (Field notes, 190412)

- Math stations: Similar to math trails, small groups of students solved one of several problems listed on sheets of paper posted around the room. After groups had been given time to solve their designated problem, they would move to either the left or right to solve the next problem in the sequence. (Field notes, 190219)
- Pass and play: Students worked through problems which were listed on individual sheets of paper organized in a pile on a desk. After selecting one of the papers and solving the problem on it, students checked their answers, which were posted on the back of each sheet. If students had arrived at the correct answer, they "passed" their sheet back to the pile, selected a new one, and then solved another problem. (Field notes, 190319)
- Rally robin: In groups, students took turns solving problems on a worksheet. If a student was struggling, they were allowed to ask the other members of their group for guidance. After one student had solved a problem, they passed the worksheet along to the next person, who was to check the student's work and answer. If the person conducting a review disagreed with the student's work, the group would discuss the problem until they arrived at a conclusion. (Field notes, 190109)
- Sage and scribe: In pairs, students took turns being the "sage"— who explained, step by step, how to solve a problem to the "scribe"—who solved the problem following the sage's instructions. (Field notes, 190109; 190201)
- Quiz-quiz-trade: Individual students were given flash cards with vocabulary words or problems written on the front and definitions or answers written on the

> back. Then, students circulated the room, finding partners to "quiz." After students in each pair had asked and answered a question, they found another partner and repeated the process. (Field notes, 190109)

Important to note is that regardless of the "collaborative learning" activity in which students were engaged, they were always working to solve low-level problems that involved recalling and applying mathematical definitions and procedures. During this time—and depending on the nature of the activity—Sara sometimes asked students if they wanted to listen to music while they worked, and, if they did, she solicited their requests for "school appropriate" songs (Field notes, 190109).

Monitoring. While students worked on warm-up tasks, practice problems, or collaborative learning activities, Sara circulated the room, monitoring students' progress. As she visited with individual students, she often asked questions to help students make progress (e.g., "How do you get that b-squared to the other side?" Field notes, 190319) or told students what to do next (e.g., "You have to solve for the side that both triangles share before you can solve for *x*." Field notes, 190201). When Sara realized that students were not working, she redirected them in a caring or playful manner (e.g., "Can you put your head up for me, hon?" Field notes, 190109; "Alliyah, doesn't sound like you're workin' girl!" Field notes, 190118; "Ryan, you alive?" Field notes, 190412).

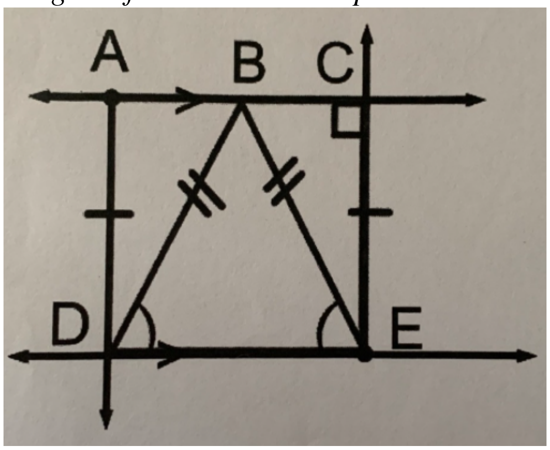
Quizzes. On three occasions, I observed Sara allot time to administer quizzes on a learning management system and to provide opportunities for students to retake quizzes in order to "get their grade up" (Field notes, 190219). During those observations, Sara first provided opportunities for students to practice solving the type of problems that would be on the quiz she would eventually administer.

Whole-class discussions. Sara led whole-class discussions during note-taking sessions (discussed earlier) or after students had worked on warm-up tasks or practice problems. During those discussions, which always reflected a pattern of talk that alternated between teacher and students, Sara explained procedures for solving problems while demonstrating how to carry them out on the SMART Board. While Sara demonstrated how to use a procedure, she often solicited ideas from the class or individual students about what she might do next (e.g., “Now, how do I show that [these lines are] perpendicular?” Field notes, 190123).

Sara consistently tried to make whole-class discussions fun by joking around with students, making references that they might know and laughing at herself if she poorly explained something. Further, while providing explanations, Sara regularly checked in with students to see whether they understood the work they had done together thus far (e.g., “We good up until this point?” Field notes, 190412) and would validate their feelings if they expressed confusion or frustration (e.g., “I understand this is frustrating, especially if you don’t understand it. So you gotta stay with me because it’s not gonna get any easier.” Field notes, 190118). During this time, Sara also shared strategies that students might use to recall vocabulary terms or procedures. As two examples, Sara sketched a sting ray on the SMART Board with an arrow at the end of its tail to “help [students] remember how to represent a ray” using geometric notation (Field notes, 190109), and suggested thinking of linear pairs of angles as “people sitting on a bench” (Field notes, 190123).

To illustrate a typical whole-class discussion, consider the following excerpt from field notes I collected on January 23, 2019. During this part of the observation, Sara was at the front of the room at the SMART Board leading students through a discussion of a warm-up task that focused on using geometric notation to describe and label diagrams of geometric figures. For the problem discussed in the excerpt below, students were prompted to "make four true statements about the diagram" in Figure 3 "using proper geometry notation":

Figure 3
Diagram for First Warm-Up Task



To begin the discussion, Sara asks students, "Who had a hard time with this? Be honest."

A few students raise their hands.

Sara reminds them that "this work is hard" and "takes a lot of practice" because learning to use geometric notation is like "learning a new language."

For the first warm-up problem for which students had to "make four true statements about the diagram," Sara chooses one relationship in the diagram to begin with and traces over the "carrots" on lines AC and DE. As she traces them, she asks students, "Does anyone know what these carrots mean?"

A few students yell out, "Parallel!"

Sara affirms their responses and writes $\overleftrightarrow{AC} \parallel \overleftrightarrow{DE}$ on the SMART Board as one true statement that students might make about the diagram. While writing out the statement, Sara tells students that she "[doesn't] have a trick for remembering the difference between carrots and tick marks" and suggests that "it's just engrained into her brain at this point."

Moving on, Sara asks students about other relationships they noticed in the diagram.

Noticing that some students have their phones out, Sara says, “Keep your phones down for me please.”

Referring to the congruence markings for line segments DB and EB, Sara asks, “What do these tick marks mean?”

Some students shout out, “Congruent!”

After validating their response, Sara writes $\overline{\mathrm{DB}} \cong \overline{\mathrm{EB}}$ on the board while explaining notational differences between representing lines and line segments.

Then, referring to the right angle in the diagram, Sara asks students, “What does this box mean?”

One student says, “It’s a right angle.”

Affirming this, Sara tells the class that the right angle indicates that line AC and ray CE are perpendicular. Then, using conventional notation, Sara writes a statement to indicate the relationship.

Next, Sara then traces over “the swoosh marks” that represent angles BDE and BED, and asks students what they represent.

A few students say, “Congruent.”

After confirming their answers, Sara asks students to help her name the two angles, reminding them that they’ll need three letters to do so. While tracing angle BDE in the diagram, Sara asks, “How can I name this one?”

One student shouts out, “BDE!”

“Good,” Sara says as she writes $\angle BDE$ on the board. “How about this one,” she asks next tracing angle BED.

A few students shout “BED,” and Sara writes down the rest of the congruence statement. A few things are worth noting about this excerpt. First, Sara checked in with students to see how they were feeling and validated any sense of struggle they may have experienced. Second, the discussion described in this excerpt follows an IRE pattern of talk and is calculational in nature. Finally, Sara’s tendency to introduce strategies for recalling definitions or procedures is reflected here, as she regrettably told students that she “[didn’t] have a trick for remembering the difference between carrots and tick marks [congruence markings].”

Closing out lessons. As the end of the class period approached, Sara typically acknowledged this by letting students know that time was about to run out. On some occasions, Sara thanked students for the work they had done that day (e.g., “I’ll just let you guys hang out because you were awesome today. Thanks for doing everything I asked.” Field notes, 190123) and spent the remaining class time interacting with them. To illustrate this, consider the following excerpt from field notes I recorded at the end of class on April 12, 2019:

> Sara notices that one student of color is sitting with her hands on her cheeks at her desk. Leaning over another desk directly across from her, Sara mimics the student’s position with a smile on her face. Then, Sara says, jokingly, that she’s going to go to [a local grocery store where the student works] and sit like that at the end of the conveyor belt while the student works the checkout line. The student laughs in response, and both she and Sara continue on with the joke. (Field notes, 190412)

In addition to her playfulness, this field notes excerpt illustrates an example of Sara’s efforts to establish and maintain positive relationships with her students by leveraging her knowledge of students’ lives outside of school.

Sara's Critical Pedagogical Discourse: Five Prominent Threads

In my analysis of Sara's case, I found that her critical pedagogical discourse included five prominent threads. While those threads interacted with contextual discourses that Sara perceived within Schenley in different ways, they were, for the most part, well aligned with contextual discourses. In the following section, I introduce the five threads of Sara's critical pedagogical discourse. Then, I describe the contextual discourses that Sara perceived and reflect on the ways in which they interacted with her critical pedagogical discourse.

Student engagement as a goal for teaching: Thread one. Through the first prominent thread of Sara's critical pedagogical discourse, she consistently described *student engagement as an important goal* for her teaching. In an ideal mathematics classroom, Sara envisioned that students would be doing more than taking notes and, when possible, engaged in "collaborative learning" activities during which students could "work together" and "coach each other" through solving problems. "Collaborative learning" activities, or the mere notion of students working on something together, Sara explained, was something that she learned about during her teacher education program. However, Sara asserted that lessons could consist not only of collaborative learning activities because teachers had to "teach at the front of the room" before affording opportunities for students to "collaborate" (Formal interview, 190102). But Sara lamented that it was difficult "to keep students engaged" while teaching at the front of the room because it required them to "do a lot of listening" (Debriefing interview, 190123).

Important to note is that while Sara had a vision for how students, ideally, would be engaged in a mathematics classroom, of particular importance to her was that students would complete class work, regardless of the nature of that work or the means by which it was accomplished. That is, more important to Sara than *what* students were engaged in was whether they were engaged at all: “If some students can just put an ear bud in and get going, they’re more willing to take notes” (Debriefing interview, 190123). To illustrate this further, consider the following excerpt from a debriefing interview in which Sara reflected on whether and how her students were engaged in the lesson that day:

> Most of them did the [math] stations. Carter was like, “I’m not getting around and doing that.” I’m like, “Okay, then take pictures. I don’t care as long as you’re doing math.” And he’s like, “No, I don’t want to.” And I’m like, “Oh, well here’s a study guide then. You can work on that. You have to be doing something to prepare yourself for the test.” (Debriefing interview, 190219)

Despite having preferences for what student engagement would look like in her classroom (e.g., “a lot of collaborative learning,” Formal interview, 190102), Sara was willing to compromise her vision “as long as [students were] doing math.” This was likely attributable to at least three things. First, as alluded to earlier, student engagement was a primary goal for Sara’s teaching. So while she envisioned that students would be engaged in a particular way, perhaps that they were at engaged at all was more important to her. Second, because Sara valued her relationships with students, she may have given them autonomy in deciding how they would complete the work she assigned to avoid compromising her relationships with them. Last, because Sara perceived contextual discourses about importance of student engagement (discussed more later), it is also possible that in ensuring that students were, at least, “doing something,” she was adhering to the expectations communicated by those discourses.

The importance of teacher explanations: Thread two. Through a second thread of Sara’s critical pedagogical discourse, she described *teacher explanations* as an important element of instruction. In particular, Sara argued that teachers should be sure to use “correct vocab terms," especially when introducing tricks for recalling procedures (e.g., "the butterfly method [is actually a method for] cross multiplying," Formal interview, 190102). Worth noting is that Sara did not problematize the idea of students using tricks to solve problems (e.g., drawing a "rainbow" to help them distribute when multiplying polynomials). Rather, she argued the importance of introducing and repeating the "actual content terms" (e.g., "distribute") in concert with tricks. Ultimately, providing quality explanations was something Sara expressed interest in improving with regards to her own practice:

> I'm always trying to improve everything I do. So even- Sometimes when I'm teaching something, I almost feel like my words aren't even coming out like I want them to. So just the most basic things, like getting better at being precise and using good language. (Formal interview, 190102)

Throughout the course of the study, Sara consistently reflected on the quality of her explanations and whether those explanations seemed to "hit with everyone" (Debriefing interview, 190201). When Sara was under the impression that students had struggled to understand her explanations, she often attributed their struggle to limitations of the ways she had explained things. For example, in reflecting on a lesson during which Sara introduced steps for partitioning a line segment, she expressed concern regarding her students' understanding of the steps because, from her perspective, she was "not very good with [her] words" (Debriefing interview, 190412).

At the end of the study, Sara suggested that she had made improvements in explaining things to her students. She attributed such improvements, in part, to the knowledge she had gained about students over time, suggesting that because she had developed a better understanding of "how students' brains work," she could more consistently provide explanations that students would “get.” However, Sara insisted that she could continue to work on this aspect of her practice because, from her perspective, students still occasionally struggled to understand her explanations:

> I’m not very good with words. So the nice thing is is that my students tend to be pretty honest with me. So if I’m like, “Is that making sense in your guy’s brains,” sometimes they’re like, “No, you need to try again.” So that helps because I like when they tell me those things. It doesn’t offend me at all. It’s like, “Alright, I need to redo this.” So sometimes it’s my words and just getting better at the way I say things. (Formal interview, 190522)

In this excerpt from our final formal interview in May of 2019, Sara suggested that when students struggled to understand things, it was likely due, in part, to limitations of explanations she had provided. Further, Sara explained that she sometimes checked in with students to see whether her explanations made sense to them. If students voiced that they had not understood her explanations, Sara said she considered other ways to explain things, which suggests that she was willing to adapt her instruction based on student feedback.

The importance of building relationships with students: Thread three. Through a third prominent thread of her critical pedagogical discourse, Sara argued the *importance of building relationships* with her students. Noting that she had “always liked kids,” Sara said that “getting to hang out with high schoolers” on a regular basis was motivation for becoming a teacher (Formal interview, 190102). In addition to the personal enjoyment Sara gained from spending time with students, she suggested that building relationships was also important for students, as getting to know them, she thought, would help them to feel more comfortable and confident in class. To give an example, consider the following excerpt from our final formal interview in May of 2019 in which Sara explained why building relationships with students was important to her:

> Just to make them feel welcome, important, and wanted. Like, when I talk to a girl that works at McDonald's, I'll just be like, "Hey girl, where's my coffee today," or like, "Bring me summin'." I think it just makes them feel comfortable. And some students don't want to go home after they leave school. So comforting them and getting to know them. Getting to know them helps comfort them and makes them feel comfortable to take risks and, you know, get to know a new person this day or try something new. (Formal interview, 190522)

According to this excerpt, Sara made efforts to get to know her students on a personal level, including, perhaps, where they worked (e.g., McDonald's). Further, Sara explained that gaining knowledge about students' lives outside of school, and leveraging such knowledge in interactions with them (e.g., "Hey girl, where's my coffee today?"), helped her to ensure that students felt comfortable in her classroom. Feeling comfortable, Sara suggested, could potentially support students in taking risks and trying new things. Further, while it is not entirely clear from this excerpt, Sara may have considered establishing relationships with students important because of students' circumstances at home.

For Sara, that she built relationships with her "Algebra Functions kids" was especially important because, as those students had previously failed at least one semester of Algebra I, she assumed that they lacked confidence in mathematics:

> So especially with [Algebra] Functions [students]…you're in Functions because you failed at least a semester of Algebra I. So they can come in thinking, "Oh, this is math class. This is totally gonna suck. I hate math. I'm not good at it." Whereas if they get to

> know me, and they know that I'm pretty patient and that I'm willing to help them whenever they need it, they will do much more, and they will take more risks… I have some kids that have never studied for a test before, but now that they are comfortable and confident with math. [They're] like, "Oh, I wanna get an A on this test." And you've never heard that out of some of the kids. Granted it's not the hardest class in the world, but for these kids, math has been their hardest subject always. So if you get the buy-in, get them to at least do more and more every day, eventually you'll have students where it's like, "I'm gonna study for this test." I'm like, "You've never studied before!" And I had one kid in my [first class] who got an A on his area and volume 3D test that he took, and he's like, "I never got an A on a math test before in my life. That's crazy." And he's like, "Will you email my mom and tell her that?" You know, so just helping them build the confidence too. It doesn't work for everyone, but building relationships does help make them want to try and do better. (Formal interview, 190522)

As this excerpt reveals, Sara was under the impression that students in her Algebra Functions classes had negative experiences in former mathematics classes, and viewed *building relationships* as a way to counter that history. Sara suggested that she could leverage her relationships with students to ensure that they experienced success, which, from her perspective, would help instill a sense of confidence that Algebra Functions students commonly lacked.

Also evident from this excerpt is Sara's framing of building relationships as a means for ensuring student "buy-in" or engagement (an idea central to the first thread of her critical pedagogical discourse). As demonstrated by the previous quote, Sara often suggested that she could leverage her relationships with students to get them to "do math" (e.g., Debriefing interview, 190219). Sara's actions reflected this thread of her critical pedagogical discourse, as when students voiced that they did not want to work on an activity, she sometimes asked them if they would be willing to "do it for [her]" (e.g., Field notes, 190118). Likely underlying Sara's requests for students to do something "for [her]" is an assumption that students will be more willing to do what a teacher asks them if they like that teacher. Therefore, perhaps Sara thought that if she could build positive relationships with her students—and got them to like her—she could leverage those relationships to help meet an important goal for her teaching: student engagement.

The importance of preparing students for the future: Thread four. Through a fourth thread of Sara's critical pedagogical discourse, she espoused a responsibility to *prepare her students for the future*. For example, Sara argued that learning mathematics in school was important in part because students could apply mathematics in their lives after school when "putting a down payment on a car or house," "figuring out how much interest is going to cost," or calculating a tip at a restaurant (Formal interview, 190522). However, Sara often explained that while it was important to her that students learn mathematics, she prioritized supporting students in developing other skills that, from her perspective, would be more useful for navigating life after high school. According to Sara's talk about preparing students for the future, there are at least two assumptions underlying this thread of her critical pedagogical discourse: (1) that most of her students would not continue studying mathematics after graduating high school; and (2) that her students lacked skills for productively participating in professional workspaces.

To illustrate how these ideas were reflected in Sara's critical pedagogical discourse, consider the following excerpt from our first formal interview in January of 2019 in which Sara described what she perceived to be her biggest responsibilities as a teacher:

> For me, it's not necessarily always about how much math [students] learn by the time they come out of your class. Obviously you want them to do well with math, but leading by example and teaching students other things like organization and how to behave well across like- I think you were here the day we talked about code-switching. Just preparing them for the future. Not just educating them to do math forever, because most of them will not...But trying your best to prepare them for what's to come outside of school. Because I know, especially with the Algebra Functions kids, their history in math hasn't been that great. So I like to instill a little bit of confidence in them just because, usually, they haven't been good at math. But it's more than just the math usually. I don't know. Just behaviors and how to treat a person and "how would you do this in the real world" kind of thing. Like those kids, they're not gonna go get a math major when they leave my class. A lot of them won't. Some of them might but…(Formal interview, 190102)

Here, Sara argued the importance of preparing students—especially ones in her Algebra Functions class—for "what's to come outside of school." Again, this may have been because from Sara's perspective, it was unlikely that many of her students would continue to study mathematics after high school. Thus, she asserted that it was equally if not more important to teach students things like "organization" and "how to behave well."

Also in the previous quote, Sara cited an example of how she worked to prepare her students for the future by referencing an incident I had observed during which she talked to a few students of color about "code-switching." When asked to elaborate on this incident and why teaching students to code-switch was important to her, Sara said:

> So when they start cursing all the time, I try- It's not like I can necessarily control it. I try my best to, but I can't control what comes out of their mouths. But just having that

> conversation like, “In the future, when you have a job, you can’t just be like, ‘F! B!’ [You can’t] just start yelling curse words”—because that’s what they do. Their language is crazy. So I just talk about what it means to code-switch. And…I’ve had that conversation a couple times with them. But they think that code-switching is just turning on and off the F-bomb, which isn’t true either. It’s also with their slang and how they do it. So I just try to make them aware of that because I truly think that most of them- Like, if they got a job at Walmart or something, they wouldn’t be ringing someone up at the cash register dropping the F-bomb…But in school I try to explain to them that it’s a professional environment. (Formal interview, 190102)

In addition to “yelling curse words,” Sara suggested that students’ use of “slang” would likely pose problems for them once they “had a job.” Thus, so that students were prepared to act accordingly in a “professional environment,” Sara encouraged them to practice code-switching in school, which demonstrates an arguably keen awareness of the racialized nature of her students’ experiences. Worth noting, however, is that while Sara likely talked to her students of color about code-switching because she genuinely cared and wanted the best for them, she attributed the need for students to code-switch to deficits in their speech rather than to broader societal issues (e.g., racism).

Sara maintained the importance of doing more than teaching mathematics throughout the course of the study and consistently prioritized preparing students for life after high school. As noted earlier, in addition to the (perceived) unlikelihood of her students pursuing mathematics-related careers, Sara assumed a responsibility to prepare students for the future because, from her perspective, they lacked some of the skills necessary for productively navigating workspaces. To illustrate this, consider the following exchange from our final formal interview in May of 2019:

S: I think [it's my responsibility] to do my best to prepare [students] for life outside of high school. So in [Algebra] Functions…I have had a lot of juniors and seniors. So when they come to me, and they're like, "Okay, I'm making my schedule for next year. What do I put on there for math? I have to have my third credit." I try and kind of always ask them like, "Well, what are you gonna do after high school? Because if you're not gonna do anything math related, I don't know that Algebra II is your best choice after Geometry just because you've had such a difficult time with math." So just preparing them for what they need to know. That's why we talk a lot about respecting one another in the classroom…Not just being like, "Okay, today we're gonna teach you how to do this." So teaching them how to be civil with adults and other peers and getting them ready to be outside of high school.

C: How do you think those things will help them once they leave?

S: So their relationships skills. It's just like when you have a job, especially if you go into anything customer service related or anything, you have to be able to talk to people. And you have to, especially when you're frustrated, you have to be able to still hold a civil and serious conversation with someone. So we're goofy all the time in my class, but also, I've pulled kids out in the hallway and yelled at them or talked to them about why this behavior is unacceptable…because you can't do that to your peers. You can't do that to an adult, especially your boss. So I think it's just kind of saving them- Because they're learning like, "Oh, yeah, maybe I shouldn't tell someone to F off because that's inappropriate, and that's something that could get me into a lot of trouble in the future." So I think it's important just

> to get them ready for like what- Obviously I'm not teaching them how to do their future job but teaching them how to deal with the people and their surroundings in their future job. (Formal interview, 190522)

First, reflected in this excerpt is the thread of Sara's critical pedagogical discourse focused on building relationships with students. For example, note that rather than making assumptions, Sara expressed an interest in learning about her students' plans after high school (e.g., "Well, what are you gonna do after high school?"). For students whose plans were not contingent on taking Algebra II, for example, Sara recommended following an easier course trajectory and focusing more on "what they need to know." From Sara's perspective, this included, among other things, learning to navigate professional workspaces. Again, worth noting is that while Sara's actions were likely motivated by a desire to act in the best interest of her students (e.g., by implicitly problematizing cultural course-taking norms), one might also argue that underlying her decision to prioritize the teaching of social skills over teaching mathematics is an assumption about *who* mathematics is for. That is, Sara's sentiment regarding the importance of preparing students for the future (especially those who end up in the Algebra Functions course) may be, to an extent, rooted in deficit discourses, as seemingly underlying that sentiment is the assumption that mathematics may just not be for some students.

Conceptual understanding as a goal for student learning: Thread five. The final thread of Sara's critical pedagogical discourse, through which she described *conceptual understanding as a goal for student learning*, emerged over the course of the study. While Sara never mentioned conceptual understanding in our first formal interview, she sometimes considered whether her students seemed to "conceptually understand" what she was teaching (e.g., Debriefing interview, 190201). For example, while reflecting on one of her Geometry lessons, Sara suggested that she "didn't do a good job teaching volume" because she struggled to "make connections" and support students in understanding "where things come from" (Formal interview, 190522).

In our final formal interview at the end of the semester, Sara named "getting kids to conceptually understand things" as a goal for her future teaching (Formal interview, 190522). This had become important to her, Sara said, because supporting students in developing conceptual understanding seemed to help them make sense of why procedures "are the way that they are," which provided "more of a grasp [for them to] hold onto." To further illustrate this thread of Sara's critical pedagogical discourse, consider the following interview excerpt from that same interview in which she described what it meant to conceptually understand something:

> If we're talking about a circle equation, you have to complete the square with both the *x* and the *y* variable…If you work backwards, you can kind of see like, "Oh, that's why this number's here. That's why this number's here. That's why you have to put them on this side." (Formal interview, 190522).

Worth noting about Sara's example of conceptual understanding in the excerpt above is that it does not illustrate a connection between a procedure and a mathematical concept. Rather, Sara described relations between components of an equation and a mathematical procedure without attending to concepts.

Also worth noting about this thread of Sara's critical pedagogical discourse is how she described the development of conceptual understanding. In particular, Sara suggested that in order to develop conceptual understanding, it may be important to, first, learn how to carry out procedures:

> I think you kind of have to understand just how to do it, even if it is just mimicking or memorizing how to do something, before you get into [conceptual understanding]…If you asked me to do most of the Geometry problems before I started teaching, I could remember how to do most of them. But as far as getting in front of the classroom and telling [students], teaching them why or trying to explain why something happened, I needed to know more of the conceptual stuff than I did know. So I think it helped that I knew how to do it before I figured out everything. (Formal interview, 190522)

Reflecting on the development of her own conceptual understanding of Geometry content, Sara explained that it was helpful that she had first learned “how to do” certain types of problems. Thus, Sara proposed that, in general, one might have to learn how to “mimic” or “memorize” procedures before developing an understanding of why those procedures work.

Based on excerpts from our final formal interview, at least two things might explain the emergence of this thread of Sara’s critical pedagogical discourse. First, Sara recalled that teaching for conceptual understanding was an idea promoted in her university methods courses. Even though she suggested that teaching for conceptual understanding “can’t happen all the time” (Formal interview, 190522), the emergence of this particular thread may represent residue from her experiences in those courses (Grossman et al., 2000). Second, this change in Sara’s critical pedagogical discourse may also be a result of working with colleagues in her PLT who focused on conceptual understanding in their own practice (and through their discourse). For example, Sara explained that she could approach Eric, a fellow member of her PLT, and ask, “How come we do this like this,” suggesting that he could “explain everything” to her. This implies that Eric may have communicated contextual discourses that Sara had begun to appropriate herself.

Sara's Perceptions of Contextual Discourses (and Constraints)

Recall that Sara's case is one of a beginning mathematics teacher whose critical pedagogical discourse was, for the most part, aligned with contextual discourses that she perceived within Schenley. This means that Sara's efforts to enact her instructional vision were often successful in part because the contextual discourses she perceived did not pressure her to teach in a way that differed from her vision. Important to note, however, is that while Sara's critical pedagogical discourse was well aligned with the contextual discourses she perceived, at times, her critical pedagogical discourse acted as a resource for challenging some of the messages she encountered, particularly ones about student behavior.

In the sections that follow, I describe the contextual discourses that Sara perceived and reflect on how those discourses interacted with her critical pedagogical discourse. And, when possible, I discuss how those interactions helped to explain Sara's practice.

Emphasis on student engagement. According to Sara, the importance of student engagement was communicated by her principal, who observed and evaluated her teaching multiple times throughout the school year. Noting that "cognitively engaging students in the content" was a criterion on which she was evaluated, Sara suggested that when conducting an observation, her principal attended to how many students Sara visited with, how often, and "who was doing what [Sara wanted] them to do" (Debriefing interview, 190219). In providing examples of feedback she had received following observations, Sara said that her principal had encouraged her to "make a game so that kids might be more motivated" and complemented the "rock, paper, scissors strategy" that Sara used to assign students to small groups (Debriefing interview, 190123).

Because the contextual discourses communicated by Sara's principal were well aligned with the thread of Sara's critical pedagogical discourse focused on student engagement, together, those discourses likely reinforced aspects of her practice. As mentioned earlier, because "keeping students engaged" was an important goal for her instruction (Formal interview, 190102), Sara consistently made use of a range of "collaborative learning" activities (some from her AVID book of strategies) to ensure that students were, at least, doing something related to mathematics. And, as Sara reported that her principal often provided positive feedback about student engagement following observations, it is likely that the contextual discourses Sara perceived only supported her use of those activities.

To illustrate how those contextual discourses reinforced aspects of her practice, consider the excerpt below from a debriefing interview in March of 2019 in which Sara shared feedback that she had received from her principal following an observation. To provide some context, in the following quote, Sara described her principal's reaction to a self-made activity called "family dinner time," in which students were required to work on and talk about problems in small groups without being distracted by their cell phones:

> As soon as [the principal] walked in, I was like, "Okay, it's family dinner time! Get with a table!" And I usually let them pick [their groups]. And she [the principal] asked me in our [debriefing] meeting, "What is family dinner?" And I was like, "When they have to put their phones away and respect each other at the table because it's dinner time." And she was like, "Oh my god!" (Debriefing interview, 190319)

According to this excerpt, Sara made use of an activity ("family dinner time") in part because she assumed that her principal would appreciate it. And, based on Sara's account of their debriefing conversation, her assumption was valid, as the principal expressed approval of Sara's use of the "family dinner time" activity. Important to note is that (Sara's perception of) the principal's positive feedback was focused not on *what* mathematical work Sara had asked students to engage in but on *how* she had asked them to engage (e.g., by working in small groups without looking at their cell phones). Emphasizing the importance of student engagement, the message communicated by the principal's feedback aligns with Sara's "as long as [students are] doing math" sentiment. And because Sara viewed "collaborative learning" activities as means by which she could foster student engagement, contextual discourses communicated by her principal likely reinforced Sara's use of those activities.

Sara's use of collaborative learning activities may also have been reinforced by other contextual discourses, which, according to Sara, were communicated by members of the Geometry PLT. For example, Sara explained that in Geometry PLT meetings, she was often reminded of AVID strategies that she could use for upcoming lessons:

> Had I not been in the Geometry PLT, I probably would've struggled a lot more my first year teaching it. Just getting ahold of the content and how to teach it, and even my creativity and a lot of the AVID strategies I'm just reminded of [during PLT meetings]. So it's nice to have five brains in a room, rather than just one, sparking creativity. Because I probably would've been a little lost without them. (Formal interview, 190102)

In this quote, Sara suggested that working with the Geometry PLT, among other things, helped to cultivate her "creativity" and prompted her to recall "AVID strategies" she might make use of in her classes. Therefore, because Sara perceived that PLT members communicated messages of approval regarding the use of AVID strategies, it is likely that those messages reinforced Sara's primary method for engaging students: "collaborative learning" activities.

Emphasis on student behavior. Like Emma, Sara perceived contextual discourses about issues of student behavior issues at Schenley. According to Sara, contextual discourses about problems of student behavior (and engagement) were communicated by the disciplinary policies that Schenley had in place, as well as by other adults in the building. While Sara argued that the disciplinary policies at Schenley were necessary for managing student behavior—and should, perhaps, be stricter than they were—she sometimes criticized how administrators and other teachers enacted those policies, suggesting that their responses only contributed to more management problems.

To an extent, Sara had appropriated perceived contextual discourses about student behavior at Schenley because she often described the ways in which student behavior (e.g., cutting class, being disruptive, using cell phones) posed challenges for teaching and learning at Schenley. For example, across interviews, Sara lamented about the ways in which some students were “blatantly defiant” or “on their phones” during class (Formal interview, 190522), attributing such challenges to at least two things: (1) that students often struggled to see the relevance of what they were learning in class; and (2) that some students would rather opt out of class than try and fail. Reflected in such explanations are ideas from the thread of Sara’s critical pedagogical discourse focused on preparing students for the future. In particular, because from Sara’s perspective, mathematics was not relevant for some of her students, especially those who lacked confidence in mathematics, she argued that it was more beneficial to support those students in developing social skills.

Although Sara felt that she could leverage her relationships with students to manage the bulk of behavior challenges, she viewed Schenley’s disciplinary policies (e.g., hallway sweeps and writing referrals) as necessary and, as mentioned earlier, suggested that they could be stricter. This was in part because Sara perceived that students faced minimal consequences for their actions, which, she argued, only exacerbated behavior issues. To illustrate this, consider the following excerpt from our first formal interview in January of 2019 in which Sara reflected on Schenley's disciplinary policies:

> Students complain a lot about like, "Schenley sucks. Schenley is like a jail," because of all the rules. But I think that- I mean, the rules are there for a reason. But for some of them- I think we're kinda too easy on them. Some of them just need to be disciplined more…(Formal interview, 190102)

As noted earlier, Sara described contextual discourses about student behavior that were communicated by other adults in the building, including administrators and other teachers. While Sara often described student behavior as an important challenge, she expressed disapproval of how some administrators and teachers approached that challenge, arguing that their (over)emphasis on and responses to behavior issues only contributed to the problem. For example, Sara was skeptical about her principal's behaviorally focused feedback and suggested that there may have been more important things to focus on when observing her:

> So most of the time, [the feedback I receive from my principal] is all behavioral, which is frustrating because…a lot of them are difficult students. So there were two kids in the back of the classroom, and we were reviewing for a test, which- It's hard to get them to motivate themselves to review. But I got a low [evaluation] score because one kid was working on his review packet while eating Hot Cheetos. But I was like, "But he's doing math"…I don't know if it's because I'm more laid back than most teachers, but the comments on some of it, it's like, "You're not making me a better teacher by making me

> become more strict." If he was doing math then what should I have to worry about? (Formal interview, 190102)

Based on Sara's account here, she received negative feedback from her principal for allowing a student to eat during class. Because this student was still "doing math"—a sentiment that Sara expressed through the thread of her critical pedagogical discourse focused on student engagement—she questioned the usefulness of the principal's feedback. Illustrated by her skepticism of contextual discourses communicated by her principal (i.e., that being "more strict" in her classroom would make her a better teacher) is an instance of Sara drawing on her critical pedagogical discourse to challenge and, perhaps, filter out perceived messages that were incompatible with her values.

To further demonstrate how Sara challenged contextual discourses about student behavior, it is important to consider how she talked about her colleagues' responses to behavior issues. Most often, Sara suggested that behavior issues may have been more manageable if her colleagues had established positive relationships with students. In the following excerpt from a debriefing interview in April of 2019, Sara reflected on the ways in which she differed from her colleagues when enacting disciplinary policies:

> So instead of, "Do you have a pass? You have to come with me." It [should be] like, "Oh hey, you have a pass with you? Oh, no? Alright. Let's go check into the office, and I'll take you right back to class." There's a very different- You see the older math men, like angry people. They have a much harder time with the kids and [the hallway sweeping policy] and being defiant just because they're mean to [students]. (Debriefing interview, 190412)

Arguing the importance of tone, for example, Sara described more and less productive ways to address issues of student behavior. Further, in noting that "the older math men" were "mean" to students when enacting disciplinary policies, Sara suggested that their actions likely exacerbated the behavior issues that they were trying to manage.

Therefore, while Sara had, to an extent, appropriated contextual discourses about student behavior, those discourses did not necessarily overpower other threads of her critical pedagogical discourse. That is, Sara agreed that student behavior was an issue at Schenley but drew on threads of her critical pedagogical discourse to argue that other issues (e.g., student engagement and building relationships) were more important.

Autonomy. Finally, Sara perceived contextual discourses that communicated messages of autonomy, meaning that, from her perspective, she could teach in any way she saw fit. Such messages, according to Sara, were communicated by her mentors at Schenley, including the department chair and her principal:

> Our department chair and our principal and everyone who hires us, they trust that we're doing a really, really good job. And so I think that they're hoping, when they come into my classroom, students are struggling with things but understanding things and working through things because that's how learning works. But I don't feel as if I have the expectation that I have to do [certain things]. Like, [I don't feel like] I have to do Cornell style notes, or I have to do only AVID collaborative learning strategies. I don't feel like I'm forced into anything. I feel like I have a lot of freedom. And I think that they trust- We get observed a lot by our principal and department chair and stuff, so they see that I sort of know what I'm doing, that I'm confident with the material and

> whatever. But there's a lot of trust, and I feel like I do have a lot of freedom. (Formal interview, 190522)

Based on this excerpt from our final formal interview, Sara suggested that individuals who hired her and oversaw her teaching trusted her (and other teachers) to do a "good job." While Sara explained that those individuals did express some expectations for what students would be doing (e.g., that students would be "struggling," "understanding," and "working through" things), from her perspective, they did not necessarily prefer a particular "learning strategy" or instructional approach.

Sara suggested that messages of autonomy were also communicated by members of her Geometry PLT, as she explained that she "[didn't] feel forced to do everything they [did]" (Formal interview, 190522). The only expectation, Sara explained, was to "deliver the same content":

> S: I think we all have our own different personality when it comes to teaching. But we deliver the same content in different ways. So if I'm sick out of nowhere one day, and I said, "Hey, I have this lesson made on this. If you have [first period] off, can you teach it for me," I would trust that Eric would deliver the same type of material. The students would understand it [either way]...There is a lot more togetherness [this year] even though we all might not be practicing in the same way, or we might not be delivering the notes in the same way. But it could be the same exact notes sheet. It's just that the deliverance is different.
>
> C: So if I see you giving Cornell notes, is it likely that those Cornell notes are being given in other classrooms too?

S: No. Or if I do, I'll be like, "Hey, I made these Cornell notes on graphing a circle." And I'll send it to all of them, and then they can kind of edit it how they want or change it. Ashley and I, our pacing and schedule this year is more- We've taught lessons in the same order, same pace. And so a lot of times, I'll be like, "Hey, I was gonna make a sage and scribe for this. Here you go." And she'll use that. And she might send me Cornell notes that she made, and I'll look at that and change it if I want to. (Formal interview, 190522)

In this excerpt, Sara suggested that the Geometry teachers had their own "personalit[ies] when it [came] to teaching" but argued that such differences were not necessarily consequential for student learning. In other words, from Sara's perspective, what was more important than *how* teachers "delivered content" was that they delivered it at all. Also evident in Sara's talk about autonomy is an assumption that the Geometry teachers would deliver content at a similar, if not at the same, pace. Notice that while Sara expressed feeling free to make decisions about the learning activities she used in her classes (e.g., Cornell notes), she also described teaching lessons "in the same order" and at the "same pace" as one of the other Geometry teachers. This may mean, then, that Sara was susceptible to or, perhaps, had appropriated contextual discourses that communicated messages about the importance of content coverage and staying on pace with other teachers.

A couple of things are worth noting in regards to Sara's perception of contextual discourses about autonomy. First is that such discourses, which, according to Sara, were communicated by both her mentors and fellow PLT members, potentially reinforced aspects of her practice. That is, because Sara perceived that individuals at Schenley did not prefer a particular instructional approach and valued, at least some of the time, the same learning strategies that she valued, she did not feel pressure to adapt her practice in any particular way. Second, Sara may have perceived contextual discourses that communicated autonomy because her practices were likely similar to those used by other members of her PLT. For example, based on Sara's accounts of her interactions with PLT members, the Geometry teachers sometimes shared materials (e.g., AVID activities) for upcoming lessons, which suggests that they were teaching in ways that were at least somewhat similar. Consequently, it is likely that Sara stayed on pace with other teachers by default, which may, to some extent, explain the sense of autonomy that she perceived.

Summary of Sara's Case

To summarize, Sara's instructional practice consistently reflected threads of her critical pedagogical discourse, as across the lessons I observed, she regularly: explained mathematical terms and procedures from the front of the room; engaged students in "collaborative learning" activities after demonstrating how to solve particular types of problems; exhibited and leveraged knowledge about students and their lives outside of school; and supported students in developing skills for navigating life after school (e.g., code-switching). That the fifth thread of Sara's critical pedagogical discourse was emergent (and likely still developing) might explain why I never observed Sara attend to conceptual understanding in her instruction. Additionally, because the contextual discourses that Sara perceived were, for the most part, aligned with threads of her critical pedagogical discourse, it is likely that contextual discourses circulating within Schenley reinforced aspects of Sara's practice to at least some extent. Recall, however, that while Sara had appropriated contextual discourses about student behavior, she drew on threads of her critical pedagogical discourse (focused on student engagement and building relationships) to problematize the ways in which her mentors and colleagues (over)emphasized and addressed behavior issues.

The Case of Bri

Bri's case is one of a beginning mathematics teacher whose critical pedagogical discourse—which aligned with perspectives and practices promoted in her methods courses—acted as a resource for filtering out contextual discourses that communicated messages about mathematics teaching that were in conflict with her own ideas. While threads of Bri's critical pedagogical discourse were often reflected in her instruction, at times, she enacted conventional practices that she had not aspired to use. In particular, Bri sometimes led note-taking sessions and demonstrated to students how to solve problems at the front of the room, which were practices commonly used by other mathematics teachers at Schenley. However, by supplementing her use of those practices with ambitious ones, Bri was able to reconfigure conventional practices to make them fit within her vision.

In the sections that follow, I first provide an overview of Bri's mathematics teaching. Then, after introducing three prominent threads of Bri's critical pedagogical discourse and describing the contextual discourses that she perceived, I reflect on interactions between critical pedagogical and contextual discourses and discuss how those interactions helped to explain her teaching.

Bri's Mathematics Teaching

A few things were evident from observations of Bri's teaching. First was Bri's consistent effort to position students as smart and mathematically capable, as she often highlighted students' strategies at the front of the room and provided public affirmations of their contributions. Second was Bri's use of practices that, from her perspective, constituted an "inquiry-based" approach to teaching mathematics. Third was Bri's tendency to supplement her use of conventional practices with ambitious ones that aligned with her vision for mathematics teaching and learning. More details regarding Bri's mathematics instruction are discussed in the sections that follow and are organized according to the instructional practices and routines that were observed most frequently.

Warm-up tasks. As students entered the classroom, Bri greeted them and then often called their attention to the SMART Board at the front of the room where lists of student names (for small-group work) and materials needed for class were projected. Once students had found their seats and class had officially started, Bri usually introduced the day's warm-up task.

The nature of warm-up tasks varied across observations. In most cases, warm-up tasks were low-level problems that were topically related to previous or upcoming activities. For example, prior to a task that required students to create and solve their own right triangle trigonometry problems based on measurements they had calculated themselves, Bri posed the following warm-up task: "Katie, who is five feet tall, is standing in a parking lot looking up at the top of a building with an angle of elevation of 72 degrees. If the building is 467 feet tall, how far is Katie standing from the building?" (Field notes, 190213). In some cases, Bri's warm-up tasks could be characterized as "Procedures with Connections" tasks (Smith & Stein, 1998), of which the foci were always conceptually related to the activities that followed. For example, prior to an activity that required students to find the shortest path for walking from one pizza shop to another, Bri posed a warm-up task focused on establishing "why the Pythagorean Theorem is true for all right triangles" (Debriefing interview, 190416). For this task, groups of students were given a set of triangles and asked to "draw squares off of" the triangles' three legs so that each leg of each triangle was also a side of one of three squares. Then, students were instructed to calculate the areas of the squares they had constructed, record their findings in a table projected on the SMART Board at the front of the room, and consider what they notice about the areas listed in the table.

Launch-explore-summarize lessons. On five occasions, I observed Bri lead what she referred to as “inquiry-based” lessons (discussed more later), which typically followed a “launch-explore-summarize” activity structure (Madsen-Nason & Lappan, 1987). To begin those lessons, Bri launched high-level mathematical tasks by leading whole-class discussions focused on the tasks’ contextual and mathematical features (Jackson et al., 2013). For example, when launching a task described earlier—one that required students to find the shortest walking path from one pizza shop to another—Bri dedicated time to discussing the task with students before they began working on it. Details of this launch are provided in the following excerpt from field notes I collected during my observation:

> To begin the launch of the pizza shop task, Bri asks students whether they’ve ever been to Sal’s [a pizza shop downtown]. In response, a couple of students groan, one even suggesting that “Sal’s is gross.” Bri and students spend a little time joking around about and debating whether “Sal’s is gross.” Bri eventually moves the conversation forward by admitting that she likes the pizza at Sal’s. She then says that because Sal’s is often busy, she sometimes she decides to walk to Moira’s [another local pizza shop] because she doesn’t feel like waiting.
>
> Next, Bri projects a map from Google Maps on the SMART Board that shows the locations of the two pizza shops, and points out to students where those shops are on the map. Then, to model one possible path she might take to walk from Sal’s to Moira’s, Bri extends a vertical line from Sal’s and then, from that line’s endpoint, draws a horizontal line to Moira’s. Suggesting to students that the path she drew is not the shortest one, Bri

> asks students if they could determine the path from Sal's to Moira's that would require her to walk the shortest distance possible.
>
> After posing the task's main question, Bri tells students that she's "mathematized the situation for them" by plotting the locations of Sal's and Moira's as points on a coordinate plane, and distributes half-sheets of paper with two points plotted on a coordinate plane. Before students begin working, there are some questions of whether they can assume that Bri can "walk through buildings" on her way from one shop to the other. Bri tells students that for this particular problem, they should assume that she can. In response, a few students problematize the assumption, arguing that it would be impossible for her to do that. Eventually, students accept the circumstances and begin solving the problem. (Field notes, 190416)

Following Bri's efforts to launch high-level tasks, she typically provided opportunities for students to work on solving those tasks in small groups, although some students often chose to work on their own. While students worked on tasks, Bri circulated the room and monitored their progress. As she visited with small groups or individual students, Bri usually asked questions to gain insight into students' solution strategies (e.g., "How did you find those distances?") or to help them move forward (e.g., "Is there a way I could find *this* distance?") (Field notes, 190416).

After Bri had given students time to work on tasks, she called their attention to the front of the room. Then, Bri initiated whole-class discussions that focused on the solution strategies that students had used to solve to task. During those discussions, Bri highlighted certain strategies to support students in reflecting on the mathematical concepts underlying the task. To illustrate an example of this, consider the following excerpt from the field notes I recorded during a whole-class discussion of the pizza shop task described earlier, of which an important mathematical goal, Bri suggested, was for students to recognize the connection between two procedures (the Pythagorean Theorem and the distance formula):

> Bri calls students' attention to the front of the room and says, "Okay, I think we agree that *this* is the shortest distance," drawing a diagonal line connecting the points on the coordinate plane that represent the locations of Sal's and Moira's.
>
> Then, Bri asks one student to share how he calculated the distance represented by the diagonal line she drew (which he had drawn as well): "Clay, can you explain what you did?"
>
> From his seat, Clay responds by saying that he "drew a triangle." Bri then invites him to draw his triangle on the SMART Board so that the rest of the class could see his representation.
>
> Using the diagonal line that Bri drew as the hypotenuse of his triangle, Clay extends a horizontal line from Sal's, then, from that line's endpoint, extends a vertical line to Moira's, creating a right triangle.
>
> After Clay had finished drawing his triangle, one student says, "But my triangle looks different than that!"
>
> Responding to her, Bri says, "We'll definitely get there in a minute." Then Bri asks Clay more questions about his solution strategy: "So what did you do, Clay?"
>
> Clay says, "I used the Pythaogrean Theorem."
>
> Bri then asks, "How did you find [the length of] these sides," pointing to the legs of the triangle he drew.
>
> He says in response, "I counted."

After affirming counting as a valid strategy for determining the length of the triangle's legs, Bri revisits the student who, earlier, announced that she had drew her triangle in a way that was different from how Clay had drawn his: "What did you say earlier, Aliyah?"

"Mine is up," Aliyah says, explaining that her triangle had a different orientation.

"Did you get the same thing," Bri asks while drawing the triangle Aliyah described.

Aliyah says that her answer is the same as Clay's.

After explaining to the class that Clay and Aliyah used different strategies but arrived at the same answer, Bri poses the question: "Why does that work?"

Due to a lack of response from students perhaps, Bri asks a variation of the question: "Why are they getting the same answer if their triangles are different?"

One student suggests that it's because "the distances up and over will still be the same."

Bri agrees with his suggestion and shows students why this is true by tracing the horizontal and vertical lines that constitute the legs of Clay's and Aliyah's triangles.

Moving on to another student's solution, Bri asks, "Lou, can you tell us what you did?"

Lou says that he counted (like Clay) but "kept making mistakes."

In response, Bri tells the class that because the legs of the triangle are "so long," it's easy to make mistakes when finding the lengths of those legs by counting. Then, Bri asks the class, "Is there a better way to find those distances?"

Bri calls on Jada to share how she found the lengths of the legs of the triangle.

From her seat, Jada explains that she found the lengths by calculating the absolute values of the differences between both the *x* and *y* values of the coordinates for Sal's and Moira's: $|x_2 - x_1|$ and $|y_2 - y_1|$.

Then, Bri asks Jada why she decided to find the absolute value of those distances.

> In response, Jada explains that she did this “because the line between the two points is a distance—and distance is always positive.”
>
> Bri then poses a question to the whole class about why Jada is “able to do that” (i.e., find the absolute value of those differences to find the lengths of the sides of the triangle).
>
> Lou—the student who, earlier, said he counted to find the side lengths and “kept making mistakes” while doing so—says, “[Jada] was trying to find a procedure for finding [the lengths] rather than counting.”
>
> Bri repeats Lou’s contribution and then, next to the horizontal and vertical lines constituting the legs of the triangle, represents their lengths using Jada’s notation: $|x_2 - x_1|$ and $|y_2 - y_1|$.
>
> Then Bri introduces the distance formula as a procedure for calculating the distance between any two points on a coordinate plane.
>
> While writing out the distance formula under the triangle on the SMART Board, $y = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, Bri makes connections to “what Jada did” by explaining that the “subtraction of the coordinates in the formula represents how Jada found the absolute value of the differences between the *x* and *y* values of the coordinate points.”
>
> Then, Bri points out that “the distance formula is basically just the Pythagorean Theorem” and notes that for the pizza shop problem, “rather than being given the lengths of the triangle’s sides, [students] needed to find them by using a strategy like Jada’s.”

In this excerpt, Bri invited three students to share (different) strategies for solving the task and asked questions about those strategies to both the individuals who were sharing and the whole class (e.g., “Why is Jada able to do that?”). Then, Bri introduced the distance formula by highlighting connections between it and the strategies that students had used to solve the problem.

Note-taking sessions. On three occasions, Bri led note-taking sessions from the front of the room. During two of those sessions, Bri distributed structured notes worksheets that were similar to those that Emma and Sara used, which provided spaces for writing down an essential question, marginalia, definitions, example problems, and a summary. In the third case, Bri distributed blank sheets of paper to students and instructed them to copy the notes that she was about to write on the SMART Board.

During note-taking sessions, Bri defined mathematical terms and sometimes demonstrated to students how to solve particular types of problems. It was also common for Bri to ask students whether they already knew definitions of any of the terms she was introducing (e.g., "Does anyone know what the reflexive property is?" Field notes, 190307) and to explain why procedures worked before asking students to practice using them on their own. As an example of a note-taking session, consider the following excerpt from field notes I collected during an observation in April of 2019:

> On the SMART Board, Bri projects an image of a quadrilateral circumscribed within a circle and tells students that the opposite angles of that quadrilateral are supplementary (i.e., the measures of those angles add up to 180 degrees). Then, while occasionally soliciting ideas from students by asking questions (e.g., Bri asks, "What does *this* have to be," pointing to one of the angles of the quadrilateral), Bri constructs a proof to show that the opposite angles of any quadrilateral circumscribed within a circle are supplementary. After explaining the proof, she tells students that "it's okay if [what they just did] was weird or confusing" and explains that the only reason she showed them the proof is

> because it's important to her that they "understand why the formula [they're going to be using] works."
>
> Next, Bri asks students to now, "knowing what they know about the opposite angles of a quadrilateral circumscribed in a circle," solve for missing angles in a quadrilateral labeled *x* and *y*, which are across from angles that measure 88 and 108 degrees, respectively. On a blank sheet of paper that Bri had distributed earlier, students work on their own to solve the problem for a few minutes while Bri circulates the room. Then, Bri asks students if they would share their answers to the problem and explain how they got them. Bri calls on two students to share, who provide answers of 92 degrees and 72 degrees and give similar explanations for how they found those answers (i.e., by subtracting the measures of the given angles from 180 degrees).
>
> After students finish sharing, Bri asks students if they have any questions. They do not, so she moves on to the next part of the session and tells students to copy down the notes that she is about to write down. Bri writes "parts of a circle" at the top of the SMART Board and then draws a circle. Then, one by one, Bri constructs different features of circles, asking students whether they know what each one is called (e.g., After drawing a chord, Bri asks students, "Does anyone know what this is called?"). After a feature is named, Bri explains the characteristics of that feature (e.g., a chord is line segment whose end points are on a circle's circumference). (Field notes, 190404)

During this note-taking session, Bri proved why opposite angles of quadrilaterals circumscribed within a circle are supplementary in order to, as she said, help students "understand why the formula [they were] going to be using works." After explaining the proof to students, Bri provided an opportunity for students to solve a problem that required them to apply the procedure central to the proof (i.e., if x and y are measures of opposite angles of a quadrilateral circumscribed in a circle, then $x + y = 180^{o}$), and then asked two students to share and explain how they got their answers.

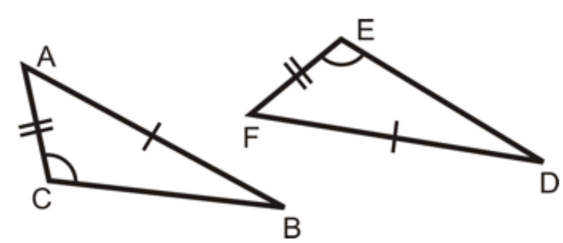
Monitoring. While students worked on tasks, Bri circulated the room to monitor students' progress. As Bri visited with small groups or individual students, she often asked questions to help students make progress (e.g., "What do I know about these two triangles?" Field notes, 190305) or, per students' request, pointed out where they had made a mistake (e.g., "Here, you used *d* [the diameter] to find the area [of the sector], but for this problem, we need to use *r* [the radius]." Field notes, 190404). When students asked Bri whether their answers were correct, she sometimes asked them to "walk [her] through" their work first (Field notes, 190117; 190129; 190314). If Bri noticed while monitoring that some students were not working, she redirected them in a caring or playful manner much like Sara did (e.g., "If I hear you say one more thing that isn't about this problem, we're gonna have a problem," Bri said with a smile on her face. Field notes, 190314).

Quizzes. On two occasions, I observed Bri administer a quiz after providing students an opportunity to practice solving problems that were similar to those that would be on the quiz. According to Bri, she administered quizzes after students had practiced because this was a routine that many of the mathematics teachers at Schenley, including her host teacher, engaged in (recall, I observed both Emma and Sara do this as well). However, as I will discuss more later, Bri expressed interest in "phasing out" the practice of reviewing before giving an assessment because, she suggested, "it doesn't really allow you to see what students actually understand" (Debriefing interview, 190117).

Whole-class discussions. In addition to the summary phase of Bri's launch-explore-summarize lessons, she also led whole-class discussions during note-taking sessions (discussed earlier) after students had worked on a warm-up task or at the beginning of class to revisit ideas or unresolved issues from a previous lesson. During these discussions, which typically reflected a pattern of talk that alternated between teacher and student, Bri clarified the topic of interest and then asked questions about that topic. Bri's questions typically prompted students to share or rationalize next steps for solving a problem (e.g., "Why is it important to take away the five feet?" Field notes, 190213) or justify any claims they made (e.g., "What do you mean because of the [side-side-side triangle congruence theorem]?" Field notes, 190307). In response to students' contributions, including questions they asked, Bri often provided affirmations (e.g., "That's a good way of thinking about that, Sam." Field notes, 190404). Bri also publicly acknowledged when she made mistakes, suggesting that any confusion students expressed was likely due to her instruction. Below, I provide an excerpt from field notes I collected during an observation in March of 2019 when Bri was leading a whole-class discussion about a question that one student had raised during the previous class about how to decide which triangle congruence postulate to use to prove congruence between two triangles:

> Bri begins the discussion by calling students' attention to a diagram (shown in Figure 4) of two triangles projected on the SMART Board at the front of the room:

Figure 4
Triangle Congruence Task



Then, Bri reminds students of an unresolved discussion from the previous class, which started with one student asking why the side-angle-side (SAS) triangle congruence postulate could not be used to prove that the two triangles in the diagram are congruent (the information provided in the diagram is actually insufficient for proving the triangles congruent). Bri then asks that student to restate his question.

Once he does, Bri describes his question as “a good one” and explains that while SAS cannot be used to prove the triangles congruent, he may have thought that SAS was appropriate in this case because of an insufficient explanation that Bri had provided during the previous class. Bri explains that although she told students that “the order of the triangle’s [components]” matters for choosing the appropriate triangle congruence postulate, she neglected to tell them that the “location” of each triangle’s components in relation to one another also matters.

After explaining this, Bri revisits the student’s question again, and asks the class, “So why can’t we use SAS?”

In response, one student suggests that “there are too many [components]” in between the given angle and the given side across from it (i.e., that the provided information does not warrant applying the SAS postulate because the given angle is not between the two given sides). This same student then explains that the given information, two adjacent sides and an angle (SSA), cannot be used to prove triangles congruent.

Bri affirms his answer and asks students if they have any questions. Student indicate that they do not, so Bri then tells students that they are now “going to practice applying the triangle congruence postulates.” (Field notes, 190305)

In this excerpt, Bri revisited an unresolved question posed by a student during the previous class about why the SAS postulate could not be used to prove the two triangles (in Figure 4) congruent. After positioning this student's question as a "good one," Bri suggested that she was unclear in providing an explanation for how to determine which postulate to apply and when. Then, she explained the importance of attending to both the "order" and "location" of triangle components. Finally, by posing the student's question to the whole class, Bri provided an opportunity for students to justify why the SAS postulate could not be used in this particular case.

Bri's Critical Pedagogical Discourse: Three Prominent Threads

In my analysis, I found that Bri's critical pedagogical discourse included three prominent threads. Because the ideas central to those threads did not align with some of the contextual discourses that Bri perceived within Schenley, her critical pedagogical discourse acted as a resource for filtering out those contextual discourses. In the following sections, I introduce the three threads of Bri's critical pedagogical discourse. Then, after describing interactions between Bri's critical pedagogical discourse and the contextual discourses she perceived, I reflect on the ways in which those interactions helped to explain aspects of her practice.

Conceptual understanding as a goal for student learning: Thread one. Like Joe and Sara, Bri argued that *conceptual understanding was an important goal for student learning* through one thread of her critical pedagogical discourse. Across interviews, Bri was consistent in asserting that students should have opportunities to develop understanding of the mathematical concepts they were learning about in class. For example, when reflecting on a lesson she had implemented during her field work experiences at Schenley in the Fall of 2018, Bri explained that in addition to learning how to solve problems using right triangle trigonometry, she wanted to support students in understanding that trigonometric ratios of right triangles were actually measurements of angles in those triangles (Formal interview, 181218). For Bri, setting conceptual learning goals was important because, from her perspective, knowing mathematics involves more than knowing how and when to apply procedures—it requires understanding the "meaning behind" them:

> [It's important] that you understand [mathematics] beyond just performing an operation. You understand the meaning behind what you're doing. If you're really doing mathematics, you know why the tools you're using work, and you're thinking about it critically. Math is problem solving. It's working through problems that don't have a set answer. It's really doing math and understanding it through working with it, not just memorizing how to do things. (Formal interview, 190530)

Noting that her current conceptualization of mathematics was relatively new, Bri claimed that she had adopted this view during her teacher education program. Most influential in changing her views, she suggested, were instances in which she recognized the procedural nature of her own understanding:

> I'm convinced that the way that we're learning to teach math now is better and will make kids understand it better. Like, saying that something is procedural and being able to see that, "Oh, it is [procedural], and I have been studying to be a math teacher for my entire life, and I don't understand math at all." That's kind of the main thing that convinced me. Like, "Oh, I have to do better than that." Because I don't want that to happen to one of my students—for them to just realize like, "Oh, I don't know anything."…Because I will admit, I don't know anything. Like, anything I've learned, I've retaught myself or thought about more. (Formal interview, 181218)

In this excerpt, Bri recalled coming to realize that her understanding of mathematics was limited in a significant way, despite having studied “to be a math teacher for [her] entire life.” To avoid fostering a similar false sense of knowledge or confidence in her students, Bri insisted that she had to “do better” for them by teaching mathematics for understanding.

To support students in developing conceptual understanding, Bri argued that students need access to learning opportunities afforded through an “inquiry-based” approach to teaching—an idea central to the thread of her critical pedagogical discussed next. However, as I discuss in more detail later, Bri suggested that, while not ideal, teachers could also support students in developing conceptual understanding by demonstrating to students why procedures work.

Student learning requires “inquiry-based” teaching: Thread two. Through a second thread of her critical pedagogical discourse, Bri emphasized *the role of an “inquiry-based” model of instruction in supporting student learning*. Often contrasting it with conventional “direct instruction,” Bri’s talk about inquiry-based mathematics teaching was similar to the language that Smith and Stein (2018) used to describe their Five Practices framework. This is not necessarily surprising given that Smith and Stein’s framework was promoted in Bri’s methods course sequence. As an example of her use of such language, when describing an ideal mathematics classroom, Bri suggested that the teacher would be doing the following things: “launching” mathematical tasks that are “accessible,” “connect to things [students] have seen or done,” and have “no clear solution strategy”; “anticipating” students’ solution strategies for those tasks; “walking around the room” while students discussed tasks in small groups; “picking out [solution strategies] to have students share themselves” during a whole-group discussion; and finally, "making connections between [those] strategies" (Formal interviews, 181218; 190530; Debriefing interview, 190129).

From Bri's perspective, an inquiry-based approach to teaching mathematics was ideal because it could provide opportunities for students to "discover" mathematical relationships, "figure out" how to solve problems on their own, and "work through ideas," even if students are "wrong." Bri also suggested that beyond discovering ideas that the teacher had planned for, an inquiry-based approach to instruction could support students in pursuing genuine paths of inquiry, as she argued the importance of students being provided opportunities to "branch out on new topics" and explore things they are interested in. Such opportunities, Bri suggested, would support students in "caring about," "making meaning of," and "remembering" mathematics (Formal interview, 181218; Debriefing interviews, 190307; 190404). In other words, Bri thought that inquiry-based teaching could cultivate interest, sense-making, and, ultimately, learning in ways that conventional practice (e.g., direct instruction) could not.

Bri explained that she had not always valued an inquiry-based approach to teaching because she had first learned about it during her teacher education program:

> I went into this program not even thinking that students would be able to discover anything on their own. I thought that teaching was telling…that teachers fill kids' heads with information…I thought learning was the teacher telling you something and you remembering that—not using what you already know or figuring things out on your own. I didn't think teaching math like that was possible at all because I had never seen anything like that. (Formal interview, 181218)

Here, Bri described at least two changes in her ideas about mathematics teaching and learning that, from her perspective, resulted from experiences in her teacher education program. The first is her reconceptualization of teaching and the role of the teacher: a teacher is not one who merely “tells” or “fill[s] kids’ heads with information.” The second is a shift in her conception of a student’s role in the classroom from one who gets told and must remember things to one who “discovers” and “figures things out on their own.”

Bri maintained the importance of inquiry-based teaching throughout the course of the study, suggesting by the end that she “[couldn’t] even imagine teaching a math class in any other way” (Formal interview, 190530). Bri’s strong commitment to following an inquiry-based model of instruction was likely attributable, in part, to her commitment to students (an idea central to another thread discussed in the following section). That is, because Bri felt that only some students benefited from conventional approaches to instruction, she saw inquiry-based teaching (along with teaching for conceptual understanding) as a means to ensure that *all* of her students experienced success in mathematics, especially those who had, historically, been “forgotten about”:

> I think you’re leaving behind most of your students if you do it that way—the traditional way. I mean, there will be students who always sit in the back of the classroom and don’t say anything because they don’t think that they have anything worth saying. A lot of kids just get looked over if math classes are taught traditionally because they’re not the ones that are raising their hands and are immediately answering questions. If students are working in groups on a problem, it really just stems from what they think. And if problems have low thresholds to be accessible to all students, they can feel—no matter what their level of mathematical ability is—they can feel like they’re engaging with the math. But if [the teaching is] traditional, [students are] just told to do something. And if

> they can't do it, it's like a you-don't-know-how-to-do-it-so-we're-moving-on kind of situation. (Formal interview, 190530)

Here, Bri explained how from her perspective "traditional" teaching practices contributed to the marginalization of some students, particularly those who had not been supported in developing positive mathematics identities and, thus, may be reluctant to participate in ways that are traditionally valued in mathematics classrooms (e.g., by "raising their hands and immediately answering questions"). Framing inquiry-based instruction as a way to better support those students, Bri suggested that posing mathematical tasks with "low thresholds," for example, could extend opportunities for learning and experiencing success to a wider range of students.

The importance of supporting students in experiencing success and seeing themselves as smart: Thread three. Through a third thread of her critical pedagogical discourse, Bri professed a commitment to *supporting students in experiencing success and seeing themselves as smart*, especially those who, historically, had not been afforded opportunities to do so. Bri suggested that "a lot of power" is associated with mathematics because being seen as and seeing themselves as "good at it" is consequential for whether students are constructed as or see themselves as mathematically capable. Therefore, as a teacher of mathematics, Bri felt that she could "have a part in changing all that" by "showing [all students] that they were smart" (Formal interview, 181218).

Bri explained that supporting students in experiencing success and seeing themselves as smart was important for her work as a teacher in part because common institutional structures and practices, such as tracking and the labeling of students based on their performance in mathematics, influenced the extent to which students had positive experiences—both in classrooms and school more broadly:

> I think that school for a lot people is a really negative experience. Like, they hate school because- I think it's in large part because of the tracking that happens with students. And some students are labeled as not smart. Yeah, I think that schools have a bad habit of labeling kids and making them feel less than they are. And I think that that's a really big problem. And I think that's in large part because schools like to say, "This kid's gifted, and this one isn't." Or, "This kid's smart, and this one isn't." And that kind of goes back to why I think math is important. Because I think a lot of schools use math as the reason that kids are smart or that kids are gifted. A lot of times, a kid that's in an honors math course is also in the other honors courses. (Formal interview, 181218)

In this exchange from our first formal interview in December of 2018, Bri described the ways in which tracking and the labeling of students seemed to influence the opportunities they were afforded in school. In particular, Bri suggested that the labels to which students were assigned related to their opportunities to develop positive academic identities, take upper-level courses, and actually enjoy school. While not necessarily reflected in this excerpt, from Bri's perspective, all students—regardless of how they had been labeled or tracked—were entitled to such opportunities.

In addition to following an inquiry-based approach to teaching, Bri described other ways in which she could support students in experiencing success and seeing themselves as smart. First, Bri argued that building trusting relationships with her students was important for, among other reasons, supporting their learning:

> B: [It's my job] to support all of my students, not just academically, but also with the things that they're going through in their lives. To be somebody that they know they can trust and who cares about them beyond just the math that they're

> learning. That I value what they want and who they are and what they're interested in. Um, yeah being, just being there for students in more ways than just teaching them.
>
> C: Why do you think that's important for your work as an educator?
>
> B: Well one- I think that the relationships that you build with students are what make them trust you enough to want to learn from you. I think that it means a lot more, learning something means a lot more when you know it's coming from somebody who cares about you and wants what's best for you. (Formal interview, 190530)

Here, Bri suggested that expressing care for and interest in students' lives could contribute to establishing relationships of trust with them, which could, in turn, cultivate students' interest in learning.

In a similar vein, Bri felt that if she wanted to support students in seeing themselves as smart, it was important for her to communicate to them that they already are, regardless of their experiences in previous mathematics classes (Formal interview, 181218). To illustrate this further, consider Bri's description of the kind of teacher she aspired to be in our final formal interview in May of 2019:

> Really [I'd just like to be] a teacher who believes that their students can do something—like really believes in all of their abilities and is just trying to highlight that. [I want to be] a teacher who is showing my students that they already are capable—not teaching them how to be. Not like giving them information, but them using what they already have. (Formal interview, 190530)

Evident from this excerpt are Bri's productive views of her students and their mathematical capabilities (Jackson et al., 2017). Suggesting that students' smartness was not contingent on the "information" they learned from her, Bri argued that all of her students were *already* capable and possessed valuable knowledge, some of which students may have been unaware. Consequently, Bri felt responsible for supporting her students in realizing this, which she felt she could accomplish by "highlighting all of their abilities" and providing opportunities for students to "use what they already have"(perhaps by posing mathematical tasks with "low thresholds").

To summarize, the three threads of dialogue that comprised Bri's critical pedagogical discourse included: (1) *Conceptual understanding as a goal for student learning*; (2) *Student learning requires "inquiry-based" teaching*; and (3) *The importance of supporting students in experiencing success and seeing themselves as smart*. These three threads of discourse acted as resources that supported Bri in resisting pressures—communicated through the contextual discourses she perceived—to act in ways that did not align with her vision for mathematics teaching and learning. While Bri did, at times, rely on practices that she did not envision using (e.g., showing students how to solve a problem using a procedure before giving them an opportunity to "discover" that procedure on their own), she often reconfigured those practices in ways that made them fit within her vision (e.g., by proving why the procedures she was introducing worked).

In the section that follows, I describe the contextual discourses that Bri perceived and, when possible, provide examples of the ways in which she drew on her critical pedagogical discourse to filter them out. Then, I reflect on the ways in which Bri's critical pedagogical discourse—and how it interacted with perceived contextual discourses—helped to explain aspects of her practice.

Bri's Perceptions of Contextual Discourses (and Constraints)

Recall that both Bri and Sara were members of the Geometry PLT at Schenley, so they met with other Geometry teachers every other day. According to Bri, one of the purposes of PLT meetings was to provide opportunities for teachers to plan for activities they might use in upcoming lessons. For example, in describing what PLT meetings were typically like, Bri said that teachers often discussed which learning opportunities—or learning activities—"would work" for teaching particular content: "Every day it's like, 'I want to do a rally coach with this activity. Do you guys think that would work, or do you think it would be better if I did [a math trail]?'" (Formal interview, 190530).

Bri claimed that she did not feel pressure from her PLT (or her host teacher, who was also a PLT member) to teach in a particular way (which, Bri suggested, was likely not the case for Algebra teachers [e.g., Emma]). Based on interviews with Bri, however, her colleagues communicated contextual discourses about teaching mathematics that she filtered out by drawing on threads of her critical pedagogical discourse. Those contextual discourses—and examples that help to illustrate how Bri filtered them out—are discussed in the sections that follow.

Conflicting conceptions of "inquiry-based" teaching. Bri felt that her colleagues valued inquiry-based teaching practices like she did, but perceived that their ideas about goals for and features of inquiry-based lessons were, in some ways, different from her own. For example, Bri suggested that when her colleagues claimed to have planned inquiry-based lessons, she did not always agree with their characterization of those lessons as inquiry-based. Because Bri perceived such differences, she sometimes filtered out messages that other teachers communicated about inquiry-based teaching by, for example, being critical of—and ultimately rejecting—some of the feedback they offered her. To illustrate this, consider the following excerpt from our first formal interview in December of 2018:

B: I've only been to a few of their [PLT meetings], but they talk about the importance of things being cognitively-demanding…Like, their ideas are the same [as mine], but I think the execution doesn't come off the way that they want it to. I think what my host teacher thinks he's doing is inquiry, is letting students discover things, when really it's leading them but in a different way. It's packets instead of notes. It's weird talking to the teachers at Schenley and then seeing what they're doing. I think they think they're doing [inquiry-based instruction]. And that's why I was so excited to be there. Because from talking to the teachers I've been placed with, or even other teachers who plan with them [in PLT meetings], it sounds like they're gonna do what we've been learning how to do [in methods courses]. But then when I've tried to do what we've been learning how to do, I was critiqued because I didn't tell the students enough. So it's weird. I don't think that I'll be actively discouraged every day from trying new things or from teaching the way that we've been taught to teach. But I don't necessarily think that they'll understand that that's what I'm doing.

C: So you thought you all were on the same page, and then you tried something and got feedback about not telling students enough. Can you say, can you describe that a little bit?

B: Okay, so I did a lesson that was a- It was supposed to introduce the trig unit. So we were talking about trig ratios, and we gave students four triangles with different angle measures for one specific angle. And one of the angles was what we were comparing across the triangles. And we gave them a ruler, and asked them to try to come up with a way to measure the angles and [rank them] in order

> from smallest to biggest without using degrees. And my host teacher before the lesson said, “You might want to introduce what adjacent, hypotenuse, and opposite are before you start doing that.” And I was like, “Oh, I really don’t want to because I want them to look at side lengths on their own. And he kept saying, “The objective of the lesson is to get students to understand SOH-CAH-TOA.” And I’m like, “Okay. I’m, we’re gonna get there.” And then we did it, and after we went through it, and students spent a lot of time discovering these ratios and actually came up with the ratios on their own as ways to measure the angles, after the lesson, my host teacher’s immediate feedback was, “You should’ve just told them what [the ratios] were at first. I think this was great what you did, and I like it. It looked like it went fine. I just think it would’ve been a lot better if you just told them at first about the ratios.” [He wasn’t talking] just about [naming] the sides but [telling students] what [the ratios] were called. But if I had told them that- That’s kind of what they were looking for. So I don’t really think he saw the benefit in them figuring it out or discovering it. (Formal interview, 181218)

This excerpt reveals a tension between Bri’s critical pedagogical discourse and contextual discourses communicated by her host teacher. In particular, because for this lesson, Bri had hoped that students would invent trigonometric ratios as a way to measure the angles they were given, she was critical of her host teacher’s suggestion to introduce and name the trigonometric ratios at the beginning—rather than at the end—of the lesson. Thus, Bri’s critical pedagogical discourse acted as a filter, here, as she recognized that taking up her host teacher’s suggestion (i.e., that she should first “tell” students about trigonometric ratios) would defeat a primary purpose of her lesson.

Deficit narratives about students and their capabilities. Bri perceived differences in the ways in which she and her colleagues viewed Schenley students and their capabilities, as well as in their ideas about the supports students may need in order to be successful. Recall that for Bri, an important instructional goal was to extend opportunities to learn (with understanding) and experience success to all of her students, especially those who, historically, had not had regular access to such opportunities. And, because Bri framed teaching for conceptual understanding and inquiry-based instruction as a means for achieving that goal, she planned to enact those practices as often as possible. From her perspective, however, members of her PLT did not share her views, as they often argued that enacting such practices on a regular basis was not feasible given their perceptions of the mathematical capabilities of most students at Schenley. This is illustrated in the following excerpt from a debriefing interview in April of 2019 in which Bri described her fellow PLT members' reactions to her plan to construct and explain a proof for why opposite angles of quadrilaterals inscribed in circles were always congruent:

> I talked about it with my PLT yesterday, and they were like, "[Students are] not gonna get it, but sure. You can try." And then this morning I asked [my host teacher] because I was second guessing whether or not I should even talk about it, and he was like, "Give it numbers. Don't prove it broadly. Show them it for one [case]." But then I'm like, "That kind of takes away from it as a proof for all cases." (Debriefing interview, 190404)

By suggesting that students were "not gonna get it," or that they would struggle to understand a generalized proof, members of Bri's PLT (including her host teacher) communicated unproductive assumptions about what students were mathematically capable of. Because such assumptions were in conflict with Bri's critical pedagogical discourse, namely her view of all students as capable of learning mathematics with understanding, she ignored—and filtered out—

the reactions of PLT members, deciding to stick to her original plan of explaining to students the proof she had planned for (see previously discussed excerpt from Field notes, 190404).

Bri's critical pedagogical discourse regularly acted as a resource for filtering out contextual discourses that communicated unproductive views of students. To provide another example of her critical pedagogical discourse at work, consider the following excerpt from the same debriefing interview in which Bri expressed frustration with herself immediately after referring to a group of students as "low":

B: I can't believe I just said that [low students]. I'm actually mad at myself.

C: No [*laughs*], it's-

B: But it's like- I don't think I would've said that a few months ago. It just makes me mad.

C: Oh, really? Why?

B: Because I don't think that way.

C: Is that how other people talk about students?

B: Yeah, that they're low. That Schenley gets the low kids. And that made me really mad when I first started. But I don't know. I guess the way I would want to say it, and the way I do think about it—I just used the language that everyone else uses—is that [students] have less of a math background at Schenley. Like, they don't have the foundational stuff. They're starting at- not lower, but with less than other kids.

C: Why do you think that is?

B: I don't know. I think it has a lot to do with a group of students that are being disserved… (Debriefing interview, 190404)

In this exchange, Bri filtered out a contextual discourse about students that she otherwise may have been susceptible to if not for the nature of her critical pedagogical discourse. In particular, because Bri described herself as a teacher who saw all students as mathematically capable, she expressed frustration with herself when she described students using “low” as an adjective. Further, Bri suggested that her use of such language was likely attributable, in part, to contextual discourses about students that she often encountered (e.g., “That Schenley gets the low kids.”). Perhaps to resist the appropriation of such discourses, Bri considered another—and perhaps, from her perspective, more productive—way to explain what she meant by “low” students: students with “less of a math background.”

Other teachers’ use of conventional practices. According to Bri, her colleagues privileged conventional approaches to teaching that involved, for example, supporting students in memorizing a series of steps for solving a particular type of problem and providing opportunities for students to practice solving those problems on their own or in small groups. In addition to their (unproductive) views of students’ mathematical capabilities (e.g., that Schenley students would not “get” a conceptual explanation of something), Bri suggested that her colleagues privileged conventional practices for other reasons:

> There’s a big emphasis on practice because practice is what’s gonna get test scores up, and all the teachers at Schenley are concerned about their test scores, and because they’ve been on a watchlist for [the state] for their Algebra I students being so far behind the other high schools in [the district]. So [Schenley’s] vision is like practice, practice, practice…Because I’ve heard so many times like, “Our kids are just coming in so far behind. We’re just trying to catch them up.” And I think that if you haven’t been exposed to this new way—or not even- I guess new, I don’t know—way of teaching math, the

> only way you know how to make it better is practice, practice, practice. Like, show them kinds of problems that are gonna be on those state assessments because that's how you take up math scores, and that's how [students will] succeed…And I get why [teachers] care about test scores because that's what administration and all the people at [the district level] care about too. It's like, "Schenley kids are just so far behind." So I get wanting to get those up…And I think that there are really good math teachers at Schenley. I just think that the way that they're teaching is leaving behind the kids that they're trying to bring forward.…And what they wanna see is higher test scores, and I don't know how meaningful that is. (Formal interview, 190530)

Because Bri was skeptical that higher test scores were a "meaningful" representation of student learning, she wondered whether her colleagues at Schenley were "leaving behind the kids they were trying to bring forward" by emphasizing practice. Bri suspected that one reason Schenley teachers emphasized the importance of providing opportunities for students to practice was that it was the "only way [they knew] how" to support students in "catch[ing] up" to other students in the district—a goal that, from Bri's perspective, may have been emphasized by leaders at the district level.

Because of the contextual pressure to provide opportunities for students to practice solving problems, Bri, on occasion, enacted conventional practice. For example, Bri occasionally led note-taking sessions during which she introduced a series of steps that students could use to solve particular problems. When she did this, however, Bri expressed feelings of conflict because, from her perspective, she had not supported her students in really understanding the ideas central to the problems they were solving—even though, subsequently, students did well on assessments that focused on their ability to apply the steps that Bri had introduced. Such feelings of conflict are reflected in the following excerpt from notes I took during a debriefing interview with Bri in January of 2019:

> Bri explained that she had been trying to support students in solving multi-step right triangle trigonometry problems in ways that "made sense to them" and "without giving them steps to follow." But because students "weren't making progress fast enough" and were becoming "frustrated" with Bri's reluctance to introduce a step-by-step procedure, Bri said that both her host teacher and support teacher encouraged her to provide a list of steps for students to follow when solving such problems (e.g., label, find the ratio, set up the proportion, cross multiply). Bri complied, she said, and demonstrated to students how to solve such problems by following a series of steps, later providing opportunities for students to practice applying the steps on their own. Bri noted that on an assessment that shortly followed students' work on practice problems, a majority of students performed well. However, Bri expressed frustration with students' success because, from her perspective, the positive assessment results did not suggest that students understood the "meaning behind" the steps they were following or the key mathematical ideas underlying those problems. (Debriefing interview, 190117)

First, illustrated by the notes above is that multiple contextual discourses validated the practice of "giving students steps to follow." While it was Bri's mentors who encouraged her to teach in a particular way, pressure to give students steps to follow may have also been communicated by a focus on "pace" (e.g., students "weren't making progress fast enough") and students' expectations for mathematics teaching (e.g., an expectation for being told how to solve problems). Also illustrated in the notes is an instance of Bri's critical pedagogical discourse acting as a resource for filtering out messages communicated by students' performance on procedurally focused assessments. Because Bri valued conceptual understanding as a goal for student learning, students' relatively strong performance on an assessment of their fluency with a procedure that had been demonstrated to them did not count as evidence for the development of their conceptual understanding.

Responsibility to cover content and stay on pace. Throughout the study, Bri described perceived contextual discourses about the importance of "staying on pace" with members of her Geometry PLT. According to Bri, the importance of staying on pace—or "covering" Geometry content at the same rate—was communicated by other teachers during PLT meetings:

> Every day coming into PLT, they ask where you're at. And if I was a day or a topic behind, they'd immediately be like, "Well you should- You can cover these two things in one day so you can catch up." [People] say like, "Come to Schenley. Teachers can do whatever they want." But you have your PLT that you're supposed to- It's the teachers' expectation that they're supposed to be in line with each other…Like, "It should take us eight days to do [these three things]." (Formal interview, 190530)

While Bri perceived contextual discourses about teacher autonomy at Schenley (e.g., "Come to Schenley. Teachers can do whatever they want."), she questioned the validity of those discourses, noting that her own autonomy was suppressed by the pressure to "be in line" with the members of her PLT.

Acknowledging that there were, at least, some benefits to teachers covering content at a similar pace, Bri suggested that "it was helpful to know where the other teachers [were] at" with respect to content (Formal interview, 190530) and that failure to cover designated content might pose challenges for students "later on down the road" (Debriefing interview, 190307). Bri argued, however, that prioritizing staying on pace to cover content over providing opportunities for students to learn and actually do mathematics was not in students' best interest, as it could be detrimental to their learning. Bri's sentiment, here, suggests that her critical pedagogical discourse was in conflict with contextual discourses that communicated messages about the importance of content coverage. To illustrate this further, consider the following excerpt from a debriefing interview in April of 2019:

> You can cover a lot of content and kids will remember it maybe to take your last test but they're not, they don't care about it or have any meaning for it if they don't get to see it applied or discovered on their own. They won't care about it. Because why would they? They'll remember it for the test and then it's meaningless to them if you don't give it meaning. So I think pushing through content the way we've been doing isn't really worthwhile. (Debriefing interview, 190404)

Here, Bri suggested that "pushing through content" often came at the expense of supporting students in developing "meaning" for or "discover[ing]" mathematical ideas and procedures, which, for Bri, were important goals for student learning. Therefore, because she did not consider covering content as a means for cultivating student learning and interest, Bri, to at least some extent, resisted contextual pressures to prioritize content coverage.

Because Bri was critical of contextual discourses that emphasized the importance of covering content, her actions were often driven by threads of her critical pedagogical discourse—which means that her teaching practices (e.g., implementing inquiry-based lessons and teaching for conceptual understanding) regularly reflected ideas central to her discursive teaching identity. Consequently, however, Bri often "fell behind" the other members of her PLT because she covered content at a slower rate. Based on Bri's accounts of her interactions with PLT members, when she had fallen behind them, they often encouraged her to try to "catch up." Bri even reported that one teacher told her that "it's just not realistic" to cover designated content while enacting ambitious practices on a regular basis. Thus, in addition to contextual discourses about content coverage, PLT members may have also communicated messages about the (un)feasibility of ambitious teaching.

Given the pressure Bri was under to keep pace with the other teachers, she sometimes made use of conventional practices that did not align with the ideas central to her critical pedagogical discourse. Important to note, however, is that Bri did not completely attribute her use of conventional practices to contextual pressures, as she pointed to other contributing factors including a lack of resources (like Joe and Emma had) and limitations of her own practice (e.g., being unprepared to respond to student thinking that she had not anticipated). Still, when Bri enacted practices that, from her perspective, would not support students in developing conceptual understanding, discovering mathematical ideas and procedures, or seeing themselves as smart, she expressed feelings of conflict, lamenting that she was only perpetuating what she was trying to disrupt. Consequently, when Bri made use of conventional practices, she reconfigured those practices by using them in strategic ways (e.g., giving notes as a way to "formalize" what students had discovered during an inquiry-based lesson, Debriefing interview, 190305) or coordinating them with ambitious ones:

> If I couldn't give a task where students were able to find something on their own, like a theorem or something, I could show them where it was coming from. Or when we talked about partitioning, I knew that I had to just give them the steps because that's what the PLT wanted us to do. So I showed them what it meant to be finding the rise and the run and how you're partitioning the segment, like how you're cutting it. I'd try to at least emphasize conceptual understanding, but I was still sometimes teaching the way that I

> don't like to. Like still ending up with giving them practice or showing them how to do it basically. (Debriefing interview, 190530)

According to Bri, she sometimes "show[ed] students how to do [problems]" because members of her PLT had encouraged her to do so. However, because demonstrating to students how to solve problems did not align with Bri's instructional vision, when she provided students with steps to follow when solving a problem, she also tried to support students in understanding the meaning behind those steps.

Worth recalling, here, is that while Bri noted that students sometimes performed well on assessments after she had demonstrated how to solve problems and provided opportunities for students to practice on their own, she argued that the results of those assessments merely proved that students had learned how to apply a procedure (Debriefing interview, 190307). Therefore, that students performed well on procedurally focused classroom assessments did not reinforce Bri's use of conventional practices that were not reflected in her critical pedagogical discourse. To claim that her students had really learned, Bri needed evidence that students had developed conceptual understanding. And, when she found such evidence, it only validated her decisions to spend time teaching in ways that mattered to her, even if it meant falling behind the members of her PLT. To illustrate this, consider the following excerpt from our final formal interview in May of 2019 in which Bri rationalized her decision to pose high-level mathematical tasks at the expense of "falling behind the PLT":

> [My argument] is that my students are talking and understanding things more than [theirs] do. Like the other- This is one thing that made me feel good. After we did the distance formula and Pythagorean Theorem [lesson]…I don't know if you remember the kid Kahlil, but he was really obnoxious during the whole lesson because that's just his

> personality. He's just a really funny kid. The next day, he was in Sara's classroom for [study hall], and he was helping one kid with his Geometry homework. [And that kid] wanted to go up to the vending machines but couldn't until Sara had checked off that he had done the work. And Sara told me that [this student] was solving for distance in a coordinate plane and was like, "Ugh, what's the distance formula," looking around [at the lists of procedures posted] in Sara's room. And Kahlil was like, "Well, just use the Pythagorean Theorem." And [the other kid] was like, "What?" And Kahlil was like, "Dude, they're the same! This is the *x* minus the *x*! This is the *y* minus the *y*! You square them both!" And [when Sara told me that], she was like, "That was really cool that he actually understood where it came from." (Formal interview, 190530)

In this excerpt, Bri described an instance in which Sara (from this study) told her that Kahlil (one of Bri's students) helped one of Sara's students with his geometry homework. According to Bri's account of what Sara told her, Kahlil helped his peer by pointing out the relation between the distance formula and Pythagorean Theorem. Because, for Bri, Sara's story provided evidence that Kahlil understood connections between mathematical ideas in ways that, perhaps, other students did not, Bri rationalized her decision to fall behind members of her PLT for the sake of dedicating more instructional time to working on tasks that supported the development of students' conceptual understanding.

Emphasis on student behavior and engagement. Like Emma and Sara, Bri described problems of student behavior and engagement when reflecting on her teaching. But unlike Emma and Sara, Bri attributed the challenges she faced to the norms and disciplinary policies that Schenley had in place, suggesting that student behavior and engagement issues were merely symptoms of those structures. For example, in a debriefing interview in March of 2019, Bri expressed frustration that her lesson—in which she had posed a high-level ("Doing Mathematics," Smith & Stein, [1998]) task—did not go as well as she had hoped. In reflecting on why, Bri suggested that students may have disengaged with the task because her actions (e.g., refraining from telling students how to solve the problem) did not align with students' expectations for her teaching. However, Bri suspected that students' expectations were a product of their past experiences in mathematics classrooms at Schenley (Debriefing interview, 190307).

To illustrate this further, consider the following excerpt from our final formal interview in May of 2019 in which Bri explained why she sometimes struggled to enact the ambitious practices she envisioned:

> If it wasn't already encouraged for kids to share their solutions and want to, feel confident enough to not just do the same thing the teacher does, it's kind of hard to get those kind of conversations [going] if students are apprehensive about what they're saying…[Schenley students] were very used to taking notes. [The norm is] like: "I'm gonna introduce the topic, maybe we'll do a little warm-up activity or a refresher from something you looked at in middle school math, and then you're gonna take notes on it. I'm gonna show you exactly how to do what I want you to do, and then you're gonna do a worksheet or a sage and scribe, or a rally coach. You're gonna do something where you're practicing exactly what I told you to." And there's a lot of emphasis on "you have a notes sheet that you can use and keep using." Like, "Refer to your notes because you can use notes on quizzes and tests." They're just so used to that because…the semester before I got there, their whole math lives, that's all they've been doing. (Formal interview, 190530)

In this excerpt, Bri attributed her struggles to engage students in discussion and other aspects of inquiry-based instruction to students' experiences in mathematics classrooms prior to working with her. Bri suggested that because, over time, students had grown increasingly accustomed to following a routine of solving practice problems after their teacher had given notes and demonstrated how to solve such problems, she was not surprised when students were "apprehensive" about participating in other ways (e.g., by sharing their own ideas).

In regards to Schenley's disciplinary policies (e.g., writing referrals to document incidents of misbehavior and disengagement and carrying out hallways sweeps), Bri perceived contextual discourses similar to those communicated by Emma and Sara. However, Bri was skeptical that enacting stricter policies would mitigate issues of behavior and engagement:

> It feels weird talking about this because I don't know how to help Schenley. But what's happening right now is not working…I don't even think it's a policy that needs to change. I think it's like, if kids are avoiding your classroom, like actively avoiding it, it's something that the teacher should be trying to fix. [Teachers should make] their classroom somewhere where kids don't, where kids want to be or aren't dreading going to. And I know that that's hard and that's a lot to put on teachers, but yeah. I don't know. I had a student who skipped every day before I started teaching. So I think you can do it. (Formal interview, 190530)

This excerpt provides evidence that Bri's critical pedagogical discourse may, in some ways, have been in conflict with contextual discourses about student behavior and engagement. Note that rather than advocating for the modification of disciplinary policies to more sufficiently address behavior and engagement issues, Bri argued that teachers should be asking themselves why students are "avoid[ing] their classroom[s]"in the first place. Further, while Bri acknowledged that shifting responsibility from the school- to teacher-level may be a bit unfair, Bri suggested that teachers could do more to make their classes places to which students look forward to going. Thus, rather than taking up contextual discourses that blamed students for issues of behavior and engagement, Bri drew on her critical pedagogical discourse to provide alternative explanations for such issues.

Summary of Bri's Case

To summarize, the three threads of dialogue that comprised Bri's critical pedagogical discourse included: (1) *Conceptual understanding as a goal for student learning*; (2) *Student learning requires "inquiry-based" teaching*; and (3) *The importance of supporting students in experiencing success and seeing themselves as smart*. Over the course of the study, those threads acted as resources for filtering out contextual discourses that were in conflict with Bri's critical pedagogical discourse, supporting her enactment of ambitious practice. While Bri did, at times, rely on conventional practices that she did not envision using, she often reconfigured those practices in ways that made them fit within her instructional vision, or coordinated them with other ones that, from her perspective, were of high quality.

CHAPTER 5: FINDINGS OF CROSS-CASE ANALYSES

The following chapter details the results of cross-case analyses of the four cases: Joe, Emma, Sara, and Bri. The chapter includes three sections, which are organized by the research questions that guided this study. In the first section, I discuss differences in teachers' critical pedagogical discourses (research question 1). Then, after reflecting on differences in the contextual discourses that teachers reported both between and within the two school settings (question 2), I reflect on how critical pedagogical and contextual discourses—and interactions between them—had implications for teachers' practice (question 3).

Teachers' Critical Pedagogical Discourses

As a reminder, the first research question that guided this study was: How do the critical pedagogical discourses of four beginning secondary mathematics teachers who graduated from the same teacher education program across three consecutive years compare? One purpose of investigating this question was to attend to nuances in the discourses that comprised teachers' critical pedagogical discourses. To attend to such nuances, I characterized *threads* of teachers' critical pedagogical discourses by identifying patterns in their talk about practice—both envisioned and enacted—across formal interview transcripts and debriefing interview summaries. Table 4 provides an overview of the threads identified for each teacher (which were described in detail in Chapter 4), where shaded cells represent commonalities across individuals.

The results reported in Table 4 showed that at least one thread of each teacher's critical pedagogical discourse reflected the ambitious teaching principles and practices (referred to hereafter as "ambitious practice") promoted in their methods courses, which were listed in Chapter 3 (e.g., teaching for conceptual understanding, the role of the teacher as facilitator, the importance of discussion). Alignment between threads of teachers' critical pedagogical discourses and ambitious practice was also reported by the teachers themselves, as each individual attributed their teaching goals and values to teacher education experiences to at least some extent. While the purpose of this study was not to examine the impact of teacher education, these co-occurring results suggest that teacher education influenced developments in teachers' critical pedagogical discourses. However, as teachers' critical pedagogical discourses aligned with ambitious practice in different ways, these findings also suggest that residue from teacher education (as articulated through teachers' talk) may vary across individuals (Jansen et al., 2018). For example, results of my analyses, along with teachers' self-reports, showed that the threads of dialogue that teachers appropriated included a focus on the importance of: providing opportunities for struggle (Joe), shifting mathematical authority to students (Emma), facilitating "collaborative learning," (Sara) and employing "inquiry-based" teaching (Bri).

Table 4
Overview of Teachers' Critical Pedagogical Discourses

	Critical Pedagogical Discourses				
	Thread 1	Thread 2	Thread 3	Thread 4	Thread 5
Joe	Conceptual understanding as a goal for student learning	Student learning requires providing opportunities for thinking, reasoning, and struggle	Student learning requires providing opportunities for discussion	The importance of building relationships with students	
Emma	Student engagement as a goal for teaching	The importance of shifting mathematical agency from teacher to students	Student learning requires providing opportunities for discussion		
Sara	Student engagement as a goal for teaching	The importance of teacher explanations	The importance of building relationships with students	The importance of preparing students for the future	Conceptual understanding as a goal for student learning
Bri	Conceptual understanding as a goal for student learning	Student learning requires inquiry-based teaching	The importance of supporting students in experiencing success and seeing themselves as smart		

In addition to general differences in alignment between teachers' critical pedagogical discourses and ambitious practice, results of cross-case analyses revealed two main findings. First, teachers' talk differed in terms of the extent to which the ideas that teachers promoted were at epistemological odds at each other, which suggests that *how* teachers appropriated new discourses—in terms of replacing or merely joining preexisting ones—varied across individuals. Second, a closer look at differences across teachers' critical pedagogical discourses revealed variations with respect to four *dimensions*. In particular, teachers' critical pedagogical discourses differed in terms of their talk about *what* they were teaching (personal epistemology), *how* to teach (instructional vision), *who* they teach (views of students), and *why* they teach (political clarity), all of which varied in alignment with ambitious practice. These findings are explained further in the two sections that follow.

Differences in Appropriation of Contextual Discourses about Ambitious Practice

Based on both analyses of teachers' critical pedagogical discourses and self-reports of their ideas about mathematics teaching and learning prior to teacher education, it seems that how teachers appropriated contextual discourses about ambitious practice differed across individuals. While all teachers suggested that their views had changed, it seemed that only Bri *modified* preexisting discourses in order to appropriate "new" contextual discourses that were, in some ways, at odds with the old. In contrast, the contextual discourses about ambitious practice that Joe, Emma, and Sara appropriated were merely *integrated* with preexisting ones. To illustrate these differences, I draw on examples from each of the four cases.

Throughout the study, Bri, unlike the others, consistently made distinctions between old and new ideas by articulating how some of them were at odds with each other. Thus, to take up new discourses, Bri recognized that she had to let go of preexisting ones. For example, to take up new contextual discourses through which good mathematics teaching was framed as requiring the provision of opportunities for students to "discover" things on their own, Bri explained that she had to disassociate teaching with the practice of "telling": "I went into this program not even thinking that students would be able to discover anything on their own. I thought that teaching was telling—that teachers fill kids' heads with information." That Bri may have modified her preexisting critical pedagogical discourse is also illustrated by the fact that threads of her discourse consistently fell under the umbrella of what she called "inquiry-based" teaching. Further, when Bri made use of practices that, from her perspective, did not fit under that same umbrella (e.g., demonstrating to students how to solve problems), she recognized that using such practices likely undermined her efforts to accomplish other, more ambitious goals (e.g., providing opportunities for student to invent their own strategies). To make up for this, Bri explained how she reconfigured conventional practices that were commonly used by other teachers in her school to make them fit within her vision for instruction (e.g., by explaining to students *why* the procedures she was introducing worked) (Braaten, 2019; Ensor, 2001; Ward et al., 2011).

In contrast to Bri, Joe, Emma, and Sara seemed to take up contextual discourses about ambitious practice without having to let go of preexisting ideas that focused on conventional teaching. For example, while Joe and Emma promoted practices they said they had learned about during teacher education (e.g., providing opportunities for students to *struggle* through and *explore* mathematics), they consistently justified their use of conventional ones (e.g., demonstrating to students how to solve problems and giving notes). And even though both Joe and Emma could articulate the ways in which some of the practices they espoused and enacted were at odds with each other, such contradictions posed tensions that diminished over time as Joe and Emma encountered contextual discourses that reinforced conventional practice. This suggests that, unlike Bri, Joe and Emma did not take issue with ambitious and conventional teaching practices living together.

While similar in some ways, Sara's case was a bit different from those of Joe and Emma, as Sara's critical pedagogical discourse reflected ideas promoted in methods courses to the smallest extent. Recall, for example, that unlike the others, Sara consistently described a good mathematics teacher as one who provides quality explanations and rarely, if ever, made distinctions between different types of mathematical tasks and the (potential) learning opportunities afforded by them. Further, Sara did not express feelings of conflict or tension when reflecting on her teaching, which is not necessarily surprising given that the ideas and practices she espoused and enacted could be characterized under the umbrella of conventional teaching. Worth noting, however, is that Sara eventually began drawing on contextual discourses that communicated the importance of teaching for conceptual understanding, which, she explained, was a practice promoted both in her teacher education program and by other mathematics teachers in her school. But, because Sara suggested that in order to develop conceptual understanding one may first have to learn how to "mimic" or "memorize" procedures, it is likely that appropriating contextual discourses about conceptual understanding did not require modification of her preexisting critical pedagogical discourse. That is, since Sara did not sense that teaching for conceptual understanding was at odds with other practices she valued, contextual discourses about conceptual understanding easily fit alongside current threads of her critical pedagogical discourse.

These differences suggest that the processes by which the four teachers appropriated new contextual discourses may have varied across the four teachers, as those processes seemed to

Table 5
Differences in Teachers' Critical Pedagogical Discourses Along Four Dimensions

	Joe	Emma	Sara	Bri
Personal Epistemology	Coverage was a primary goal Emphasized process and results; prioritized results Framed students as consumers	Engagement and coverage were primary goals Emphasized process and results; prioritized results Framed students as consumers	Engagement and coverage were primary goals Emphasized results Framed students as consumers	Conceptual understanding was a primary goal Emphasized process and results; described process as equally, if not more, important than results Framed students as contributors
Instructional Vision	Described teacher as facilitator Envisioned providing opportunities for discussion that was both calculational and conceptual in nature; provided rationales rooted in authority and learning Envisioned posing high-level tasks; provided rationales rooted in learning, access, and engagement; framed regular enactment of high-level tasks as unfeasible	Described teacher as facilitator Envisioned providing opportunities for discussion that was both calculational and conceptual in nature; provided rationales rooted in authority and learning Envisioned posing high-level tasks; provided rationales rooted in access, authority, engagement, learning, and utility; framed regular enactment of high-level tasks as unfeasible	Described teacher as monitor Envisioned providing opportunities for discussion that was calculational in nature; provided rationales rooted in engagement and authority Envisioned engaging students in "collaborative learning" activities; provided rationales rooted in authority and engagement	Described teacher as more knowledgeable other Envisioned providing opportunities for discussion that was both calculational and conceptual in nature; provided rationales rooted in authority and learning; argued that small-group discussion must precede whole-group discussion Envisioned posing high-level tasks; provided rationales rooted in access, authority, learning, and relevance
Views of Students	Emphasized the importance of relationship building; provided rationales rooted in care and engagement Framed students' capabilities in both productive and unproductive ways	Did not consistently emphasize the importance of relationship building Framed students' capabilities in both productive and unproductive ways	Emphasized the importance of relationship building; provided rationales rooted in care and engagement Framed students' capabilities in both productive and unproductive ways	Emphasized the importance of relationship building; provided rationales rooted in care, trust, and learning Framed students' capabilities in productive ways
Political clarity	Demonstrated awareness of students' social and economic realities; framed teaching as a tool to transform those realities but not those of his students	Demonstrated awareness of students' social and economic realities Invoked deficit discourses	Demonstrated awareness of students' social and economic realities; framed teaching as a tool to transform those realities Invoked deficit discourses	Demonstrated awareness of instructional and institutional inequities; framed teaching as a tool to address those inequities Rejected deficit discourses

vary in terms of what they demanded of teachers (e.g., trying out new ideas or completely disrupting old ones). Not surprisingly, perhaps, results also suggested that the outcomes of appropriation processes—the focus of teachers' critical pedagogical discourse threads—varied across individuals. Such differences are discussed further in the section that follows.

Differences in Dimensions of Teachers' Critical Pedagogical Discourses

As mentioned earlier, results of cross-case analyses showed that teachers' critical pedagogical discourses are multidimensional, as teachers' talk about mathematics (what they teach), instruction (how they teach), students (who they teach), and the role broader social contexts (why they teach) varied across individuals. As mentioned in Chapter 3, to characterize differences in teachers' critical pedagogical discourses along these four dimensions, I drew on constructs from the literature reviewed in Chapter 2: personal epistemology (Kang, 2008; Kang & Wallace, 2005); instructional vision (Munter, 2014); views of students (Jackson et al., 2017); and political clarity (Bartolomé, 2004). To be clear, I applied these constructs only in an effort to further characterize differences across teachers' critical pedagogical discourses. Thus, I do not make claims about, for example, teachers' personal epistemologies (Kang, 2008) or views of students (Jackson et al., 2017). Rather, I point to the ways in which teachers' *discourses* around epistemology and students varied across individuals. Findings of these analyses are reported in Table 5 and discussed further in the sections that follow, which are organized according to the four constructs mentioned earlier.

Teachers' personal epistemologies. The first dimension along which teachers' critical pedagogical discourses varied was what Kang (2008) described as personal epistemology: teachers' beliefs about knowledge and knowing. Drawing on Kang's argument that teachers' description of *goals* for their teaching can serve as a "critical window" into their personal epistemologies (p. 481), I focused on differences in the goals that were reflected in teachers' critical pedagogical discourses. When conducting those analyses, I focused not only on whether the goals that teachers consistently espoused aligned with ambitious practice but also on the extent to which those goals were critical—or consequential for—teachers' actions. Findings showed that while some teachers alluded to goals that aligned with ambitious practice on a regular basis, whether teachers *prioritized* those goals varied across individuals.

For example, Joe, Sara, and Bri identified conceptual understanding as an important goal for their teaching, which is among the goals specified in definitions of ambitious teaching (e.g., Lampert et al., 2013). Based on their talk about conceptual understanding, however, there was variation in the extent to which teaching for conceptual understanding was *critical* for those individuals. For example, Bri was consistent in framing conceptual understanding as a *primary* goal for student learning, as she often criticized lessons and assessments for which conceptual understanding was not an explicit focus. Recall, for example, Bri's arguments that teaching students how to carry out procedures without attending to underlying ideas would foster a false sense of knowledge or confidence, and that students' ability to carry out procedures did not provide sufficient evidence for their learning. In contrast, for Joe and Sara, the goal of supporting students in developing conceptual understanding was *secondary* to other goals, such as ensuring that students meet the state standards—even if those standards were primarily procedural in nature—(Joe) and complete designated assignments (Sara). That this goal was secondary for Joe and Sara is illustrated by Joe's justification of his procedurally focused lessons and Sara's argument that teaching for conceptual understanding "can't happen all the time," respectively.

A second example of differences in teachers' talk about goals is illustrated by comparisons of Emma's and Sara's focus on *engagement* and Bri's focus on *learning*. For Emma and Sara, that students were engaged was a primary goal for their teaching. In Emma's case, this is demonstrated by her argument that while conventional practices (e.g., giving notes) do not require students to do much thinking, such practices can ensure both engagement (e.g., "It's pretty easy to get kids to do notes.") and content coverage (i.e., that she "got through" everything). In Sara's case, that engagement was a primary goal for her is illustrated by her consistent concern with not *what* students were engaged in but whether they were engaged at all: "I don't care, as long as [they're] doing math."

In contrast, Bri's primary goals were focused more on learning. In addition to supporting students in developing conceptual understanding, Bri consistently argued the importance of providing opportunities for students to "discover" or invent mathematical procedures and experience success. That these were primary goals for Bri is illustrated by the feelings of tension she expressed after enacting practices that, from her perspective, did support the realization of such goals (e.g., introducing steps that students might follow to solve a particular type of problem). Further, Bri consistently prioritized the provision of opportunities for students to make mathematical discoveries over other goals that, according to her, were not associated with student learning:

> You can cover a lot of content and kids will remember it maybe to take your last test but they're not, they don't care about it or have any meaning for it if they don't get to see it applied or discovered on their own. They won't care about it. Because why would they? They'll remember it for the test, and then it's meaningless to them if you don't give it meaning. So I think pushing through content the way we've been doing isn't really worthwhile.

Note that in this excerpt, Bri suggested that more important than covering or "pushing through" content was how, exactly, students were afforded opportunities to learn that content. In particular, Bri argued that providing opportunities for students to apply or discover ideas helped students to develop investment in and construct meaning for those ideas, whereas merely introducing the content would not afford such opportunities.

In addition to teachers' talk about their goals, Kang (2008) argued that teachers' personal epistemologies may vary with respect to a relational aspect: how teachers describe relationships between students and mathematics. According to Kang and Wallace (2005), teachers' whose personal epistemologies are "more sophisticated" would emphasize "a process of meaning making more than the result" because doing so would "bring students closer to [mathematics] through engaging them in doing [mathematics] for themselves" (p. 143). An emphasis on process rather than results aligns with literature on ambitious mathematics teaching, as student sense-making and discourse, for example, are framed as important outcomes in and of themselves (e.g., NCTM, 2014). Because Bri, when compared to the other three teachers, more consistently emphasized the importance of processes, the epistemological dimension of her critical pedagogical discourse was most "sophisticated," or most aligned with epistemologies associated with ambitious practice. To illustrate this, I draw on examples from each of the four cases.

First, Sara, unlike Joe, Emma, and Bri, did not suggest that student learning requires teachers to provide opportunities for students to explore and figure things out on their own. Rather, Sara was more concerned with whether students had completed the work she had assigned, regardless of the means (i.e., the nature of the activity) through which they completed it. This focus on *completion* highlights the results-oriented nature of the epistemological dimension of Sara's critical pedagogical discourse.

That Joe, Emma, and Bri argued the importance of *process* (e.g., students struggling through problems) suggests that the epistemological dimensions of their critical pedagogical discourses were less results-oriented than Sara's. However, Bri's critical pedagogical discourse was, arguably, more focused on process than Joe's and Emma's. That is, while each of the three teachers claimed that providing opportunities for students to explore mathematics was important, only Bri insisted that providing such opportunities was *necessary*, suggesting that she considered meaning-making processes to be equally, if not more important, than the results of those processes. In contrast, Joe and Emma considered the provision of such opportunities to be *optional*, as both teachers argued—by pointing to the results of classroom assessments—that students could learn just the same through conventional approaches to instruction (e.g., demonstrating to students how to solve problems). That Joe and Emma were more concerned with student learning outcomes than the processes by which such outcomes were achieved suggests that, when compared to Bri's, the epistemological dimensions of their critical pedagogical discourses were more oriented toward results.

Also worth noting is that only Bri was consistent in suggesting that students had knowledge and experiences that they could leverage to do mathematics in school. Recall that Bri said: "[I want to be] a teacher who is showing my students that they already are capable—not teaching them how to be. Not like giving them information—them using what they already have." And while the other three teachers did not explicitly describe students as mere consumers of mathematics, they rarely, if ever, reflected on students' contributions, showing more concern for whether students: understood a task's intended takeaway (Joe), got through everything they needed to get through (Emma), and did the work that was asked of them (Sara).

To summarize, cross-case analyses revealed differences in how teachers talked about goals for their teaching and described students in relation to mathematics, which highlights differences in the epistemological dimensions of their critical pedagogical discourses. Regarding teachers' talk about goals, for Bri, supporting students in developing conceptual understanding was a primary goal, whereas for Joe and Sara, it was secondary to other goals including ensuring that students meet state standards (Joe) and complete designated assignments (Sara). Additionally, while engagement was a primary goal for Sara and Emma, Bri was more concerned with learning. In terms of how teachers described students in relation to mathematics, Bri consistently connected students to mathematics, as she emphasized the importance of meaning-making processes and suggested that students could leverage preexisting knowledge and personal experiences to do mathematics. In contrast, Joe, Emma, and Sara were more concerned with results than processes and tended to talk in ways that positioned students as consumers of—rather than contributors to—mathematical knowledge.

Teachers' instructional visions. The second dimension along which teachers' critical pedagogical discourses varied was their instructional visions, which, as a reminder, are "the discourse[s] that teachers or others employ to characterize the kind of 'ideal classroom' practice to which they aspire but have not yet necessarily mastered" (Munter & Wilhelm, in press, p. 2). In particular, when articulating their ideas about what an ideal mathematics classroom would look like with respect to three sub-dimensions of practice—role of the teacher, classroom discourse, and mathematical tasks (Munter, 2014)—teachers described different *forms* and *functions* of the practices they described (Saxe et al., 1999). Further, teachers not only envisioned different practices (*forms*) but sometimes articulated different rationales (*functions*) for why, from their perspectives, such practices were important.

With respect to visions for ambitious practice described in the literature (i.e., Munter, 2014), Bri's critical pedagogical discourse was the most aligned, whereas Sara's was the least aligned. While Joe's and Emma's critical pedagogical discourses were more aligned than Sara's, they were not reflective of visions for ambitious practice to the extent that Bri's was. Further, while Joe and Emma promoted ambitious practices, unlike Bri, they did not necessarily describe those practices as components of a holistic approach to instruction (e.g., "inquiry-based" teaching). To illustrate these differences, I compare teachers' critical pedagogical discourses to the three sub-dimensions of ambitious practice specified earlier: role of the teacher, classroom discourse, and mathematical tasks.

Role of the teacher. Sara's vision for role of the teacher was least aligned with ambitious practice, as she described the teacher as a *monitor*: one who, after demonstrating to students how to solve a problem, circulates the room to support students as they work together to practice what the teacher has just demonstrated.

The two first-year teachers, Joe and Emma, depicted the role of the teacher in similar ways that, when compared to Sara's vision, were more reflective of ambitious practice. In particular, both Joe and Emma consistently described the role of the teacher as a *facilitator*: one who provides opportunities for students to "figure things out for themselves" (Joe) and does not "take over" students' thinking, intervening only to pose questions that will guide students along a productive path (Emma). Worth noting, however, is that while both teachers described an ideal mathematics teacher as one who shifts mathematical authority to students, they did not imagine that students would pursue lines of inquiry that would not necessarily lead to conclusions or goals for which the teacher had planned.

In contrast, Bri's vision for role of the teacher was most aligned with ambitious practice, as she consistently described the role of the teacher as a *more knowledgeable other*. For example, Bri argued that an ideal mathematics teacher would enact "inquiry-based" teaching, which, from her perspective, involved launching mathematical tasks, providing opportunities for students to work on and discuss those tasks in small groups, and facilitating discussions of students' solution strategies—and connections between them—during whole-group discussions. Further, when compared to Joe and Emma, Bri described students' mathematical authority in a different way. In particular, unlike Joe and Emma, Bri imagined that students would be provided the opportunity to pursue ideas they were interested in, even if it resulted in them "branch[ing] off" onto something new.

Classroom discourse. While all four teachers argued the importance of providing opportunities for student-to-student discussion in the context of small groups, their rationales for why the provision of such opportunities was important aligned with visions for ambitious practice to varying extents. Beginning with Sara, her rationale for providing such opportunities was to ensure engagement and shift authority to students, as she suggested that while students worked in small groups, they could "coach each other through" solving problems. However, Sara's characterization of student authority was somewhat limited, as she depicted students only as "mediators of the teacher's instruction" (Munter, 2014, p. 629).

Also worth noting is the nature of discourse that Sara described. Sara's portrayal of students "coach[ing] each other" suggests that the discourse in which she envisioned students engaging was primarily calculational in nature (Cobb et al., 2001; Thompson et al., 1994).

In contrast to Sara, the two first-year teachers, Joe and Emma, rationalized the importance of providing opportunities for student discussion for reasons related to student learning and authority, and envisioned discourse that was both calculational and conceptual in nature. First, Joe and Emma argued that student learning was contingent on the provision of opportunities for discussion. More specifically, the two teachers suggested that allowing students to talk through their ideas would help students "learn" (Emma) and "understand the content" (Joe). Second, Joe and Emma framed the provision of opportunities for discussion as a means for shifting mathematical authority to students. When compared to Sara's, Joe's and Emma's talk about authority is more reflective of ambitious practice, as rather than depicting students as coaching others through replicating procedures introduced by the teacher (Sara), they envisioned students "arguing" (Joe) and "taking risks" (Emma) independently of the teacher.

Bri's rationales for and descriptions of opportunities for student discussions were somewhat similar to those articulated by Joe and Emma. In particular, Bri provided rationales for the provision of opportunities for discussion that were rooted in learning and authority, and envisioned classroom discourse that was both calculational and conceptual in nature. For example, Bri suggested that allowing students time to work on and discuss mathematical tasks in small groups would support them in "discovering" and "making meaning of" ideas and procedures on their own. However, Bri described a function of student-to-student discussion that the others had not, making her vision for classroom discourse most reflective of ambitious practice. In particular, Bri framed providing opportunities for students to discuss mathematics in small groups as an essential element of what she called "inquiry-based" instruction. That is, Bri consistently depicted small-group work as occurring *before* whole-class discussions, during which, she explained, students—whom the teacher had purposefully selected—would present the ideas and strategies they had talked about with peers in small groups. Further, Bri suggested that as students presented their work, the teacher should facilitate a whole-class discussion that would help students "mak[e] connections between their strategies." This suggests that, when compared to Joe's and Emma's, Bri's vision for instruction was more holistic.

Mathematical tasks. As with the role of the teacher and classroom discourse, Sara's vision with respect to mathematical tasks was the least aligned with the tasks foundational to ambitious practice (i.e., tasks of high cognitive demand, Stein et al., 1996). In contrast, the visions of the other three teachers, Joe, Emma, and Bri, were more reflective of ambitious practice albeit in slightly different ways. Additionally, when compared to Bri, Joe and Emma considered the enactment of the tasks they envisioned less feasible (e.g., Nolen et al., 2009). Such differences are explained further in the paragraphs that follow.

Beginning with Sara, she explained that in an ideal mathematics classroom, she would want to see students working on "collaborative learning" activities but did not necessarily describe those activities as being of lesser or higher quality. That is, more important to Sara than the nature of the activities that students would be working on was that they would be working on them "collaboratively." Further, Sara suggested that collaborative learning activities serve the functions of ensuring engagement and, as mentioned earlier, providing opportunities for students to "coach each other through" solving problems following a teacher demonstration. Therefore, Sara's rationales for promoting such activities were rooted in engagement and authority (e.g., giving students the opportunity to explain how to solve a problem to their peers).

Joe's descriptions of ideal mathematical tasks were more aligned with ambitious practice, as he suggested that such tasks should be "open-ended," provide opportunities for "struggle," "foster pattern recognition," and "allow for multiple solutions." Joe also expressed an interest in posing tasks that could support students in exploring issues of social justice and diversity, but, as mentioned in the discussion of his case, he consistently argued that such tasks were not relevant for the student population at his school. In rationalizing why the tasks he envisioned were important, Joe argued that they serve, at least, three functions: (1) providing opportunities for struggle to help students "remember" key ideas; (2) affording access to a range of students regardless of their current mathematical capabilities; and (3) ensuring student engagement. Thus, Joe's rationales were rooted in ideas of learning (i.e., "remember[ing]" ideas), access, and engagement. As mentioned earlier, while Joe consistently argued the importance of posing the tasks he envisioned for the reasons just described, he did not consider the regular enactment of such tasks to be feasible due to time restrictions and the demands of covering state learning standards.

Emma envisioned posing tasks that were similar to the ones that Joe described, as she explained that, ideally, mathematical tasks would be "open-ended," provide opportunities for students to "think on their own," and have "a low floor and high ceiling." However, Emma also suggested that "in a truly ideal world," students would be provided opportunities to solve problems that are "interesting" and "relevant." More specifically, Emma depicted a classroom in which students would be using mathematics to make sense of and come up with solutions to problems in the world. In addition to shifting mathematical authority from the teacher to students, Emma argued that such tasks were important because they were accessible, more "engaging" than lectures, required students to "explain" and "justify" their reasoning, and could help students "see the value in math." Emma's rationales, then, were rooted in authority, access, engagement, learning, and utility. However, like Joe, Emma did not consider the regular enactment of such tasks to be feasible given the demands of content coverage.

Finally, Bri's vision for what students would be working on in an ideal mathematics classroom was not much different from those of Joe and Emma. In particular, Bri promoted tasks that “connect to things that [students] have seen or done” and have “low thresholds,” “no clear solution strategy” and, in some cases, multiple answers. In arguing why such tasks were important, Bri pointed to their potential to support students in accessing and “caring about” mathematics, “figuring things out on [their] own,” and developing conceptual understanding. Thus, Bri’s rationales were rooted in access, relevance, authority, and learning. When compared to Joe and Emma, Bri was less skeptical about the feasibility of posing such tasks, as she noted in our final interview, she “[couldn’t] even imagine teaching a math class in any other way.”

To summarize, the four teachers’ critical pedagogical discourses varied in terms of their instructional visions (Munter, 2014). In particular, the instructional vision dimensions of teachers’ critical pedagogical discourses varied with respect to the practices (*forms*) that teachers named, as well as their rationales (*functions*) for why those practices are important. Overall, Sara’s instructional vision was the least reflective of ambitious practice, while Bri’s was the most reflective. While Joe’s and Emma’s instructional visions were more aligned than Sara’s, they were not reflective of ambitious practice to the extent that Bri’s was, as Bri envisioned a more holistic approach to instruction (i.e., “inquiry-based” instruction). Further, unlike Bri, Joe and Emma argued that enacting aspects of their instructional vision (e.g., posing high-level mathematical tasks) on a regular basis was unfeasible.

Teachers’ views of students. A third dimension along which teachers’ critical pedagogical discourses varied was views of students. In particular, findings pointed to differences in teachers’ discourses around students and building relationships with them, as well as in how they framed students’ mathematical capabilities (Jackson et al., 2017).

First, while all four teachers expressed care for their students, Joe, Sara, and Bri were most consistent in arguing the importance of building relationships. For all three teachers, demonstrating to students that they cared about them (and not just about what students learned in their classes) was a priority. Of those three, Joe's and Sara's discourses were most similar, as they both described getting to work with students as a motivating factor for becoming a teacher, and named similar *functions* of relationship building. For example, Joe and Sara suggested that building relationships with students was important for ensuring that students felt comfortable in class (Sara) and as though they had a source of support in school (Joe), which may have been, in part, because both teachers perceived that students experienced stress in their lives outside of school. Additionally, both teachers described practical affordances of relationship building, suggesting that they could leverage their relationships with students to ensure engagement and compliance. In particular, Joe described having "good rapport" with students as a means for getting them to work on things (even if the relevance of such work was unclear to students), and according to Sara, if students enjoyed being in her class, they would be more willing to "do [math] for [her]."

In contrast, Bri's talk about the importance of building relationships with students often coincided with her talk about providing opportunities for students to experience success and develop positive mathematics and academic identities. But unlike Joe and Sara, who suggested that they could build and leverage their relationships with students in order to ensure compliance, Bri framed relationship building as a way to cultivate students' trust in her, which, from her perspective, would not only support students in learning mathematics but also make learning more meaningful for them:

> …I think that the relationships that you build with students are what make them trust you enough to want to learn from you. I think that it means a lot more, like learning

> something means a lot more when you know it's coming from somebody who cares about you and wants what's best for you.

Important to note is that Bri's sentiment in the excerpt above is not dissimilar from Sara's talk about the importance of building with students in her Algebra Functions classes—who had failed at least one semester of Algebra I. In particular, Sara suggested that building relationships with students in her Algebra Functions classes was critical because they had, historically, endured negative experiences in mathematics. And, from Sara's perspective, if she could build relationships with those students, she could better support them in taking risks, studying more, and, ultimately, experiencing success which, in turn, would build their confidence in mathematics.

However, the ways in which Sara and Bri described means for ensuring student success differed in notable ways. In particular, Bri argued that high-level tasks were instrumental in ensuring the success that low-level tasks may have denied students in the past, whereas, for Sara, it was most important that students experienced success at all, regardless of the nature (or quality) of the activity through which it was achieved. Therefore, for Bri, it was not only important that students were successful but that they were successful in *rigorous* mathematics. Such differences might be explained by the differences in their instructional visions discussed earlier, further highlighting the multidimensional nature of critical pedagogical discourses.

The four teachers also differed in terms of how they framed students' mathematical capabilities. In particular, the ways in which teachers described students' struggle ranged from more to less productive (Jackson et al., 2017). First, in explaining why, from their perspectives, students sometimes did not learn as expected, all four teachers considered potential limitations of their instructional practice, including a lack of explicitness about expectations for participation

(Joe), a struggle to support students in building confidence (Emma), the provision of unclear or confusing explanations (Sara), and an unpreparedness to respond to students' thinking on the spot (Bri). Also worth noting is that all four teachers sometimes framed students' struggle as a symptom of students' experiences in previous mathematics classrooms.

Of the four teachers, however, Bri was most consistent in locating responsibility for students' struggle in mathematics to limitations of instructional—and institutional—opportunities. For example, recall that Bri: framed her use of conventional practices (e.g., asking students to replicate teacher-demonstrated procedures) as contributing to students' lack of success and confidence (a problem she was trying to disrupt); explicitly rejected using ability labels that others in her school used to describe students (once even correcting herself after accidentally referring to a group of students as "low"); and suggested that her school's emphasis on practice (for the sake of getting test scores up) was essentially detrimental to student learning. Bri also framed issues of student behavior in a productive manner, arguing, for example, that if students were cutting class to avoid a teacher's classroom, then that teacher should be working to make their classroom a place to which students look forward to going.

The other three teachers, Joe, Emma, and Sara, wavered between attributing students' struggle to instructional or institutional limitations (productive) and faults of students and their families (unproductive). For example, while Joe sometimes framed students' resistance to participate in discussion as a function of their past experiences in more conventional mathematics classrooms, in other cases he explained that students' "off-task" behavior made it difficult to accomplish all he had hoped to achieve within a given lesson. Similarly, although Emma and Sara occasionally considered how shortcomings on their part may have been responsible for undesirable outcomes, they also pointed to students' behavior to explain some of the challenges they faced. In particular, when explaining a lack of student engagement, Emma alluded to a lack of student interest in classroom activity (e.g., some students "just don't like doing things") and suggested there was little she could to do address their disinterest, and Sara argued that some students would rather opt out than try and fail. Further, Emma and Sara consistently located the responsibility for behavior issues within students and their families. Rather than considering ways in which they might adjust their instruction to address behavior issues, both teachers suggested that their students needed *more* structure and discipline.

To summarize, the four teachers' critical pedagogical discourses varied in terms of their talk about students and their relationships with them, as well as in the extent to which their framings of students' mathematical capabilities were productive. Through threads of their critical pedagogical discourses, Joe, Sara, and Bri, argued the importance of building relationships with their students. However, there were differences in the three teachers' rationales (*functions*) for why building relationships mattered to them. While Joe and Sara said that building relationships was important because they cared deeply about their students and their well-being, they also suggested that they could leverage their relationships with students to ensure engagement or compliance. In contrast, Bri did not frame her relationships with students as leverage for getting them to do things for her. Rather, Bri described relationship building as serving the primary function of establishing relationships of trust, which, in turn, could support students' learning and success.

In terms of teachers' framings of students' mathematical capabilities, all four teachers provided productive explanations for students' struggle in mathematics, as each of them, on at least one occasion, attributed learning-related challenges to limitations of their own instruction and students' experiences in previous mathematics classrooms. However, Joe, Emma, and Sara also located responsibility for the challenges they faced within students without considering how such issues may have been a symptom of their instruction or institutional constraints. In contrast, Bri consistently attributed students' struggle (and behavior issues) to instructional and institutional limitations and explicitly rejected using labels to describe her students' abilities.

Teachers' political clarity. The last dimension along which teachers' critical pedagogical discourses varied was political clarity—the extent to which teachers acknowledged teaching as a political act (Bartolomé, 2004; Gutiérrez, 2013). As a reminder, Bartolomé (2004) defined political clarity as:

> [an] ever-deepening consciousness of the sociopolitical and economic realities that shape [teachers'] lives and their capacity to transform such material and symbolic conditions…[an] understanding of the possible linkages between macro-level political, economic, and social variables and subordinated groups' academic performance in the micro-level classroom. (p. 98)

The ways in which teachers' critical pedagogical discourses reflected the "consciousness" and "understanding" that Bartolomé described varied across individuals. To illustrate those differences, I reflect on particular instances in which each individual alluded to relations between sociopolitical and economic issues and matters of teaching, beginning with Joe, as his case was the first presented in Chapter 4, followed by Emma, Sara, and finally, Bri.

Based on threads of his critical pedagogical discourse, Joe exhibited at least some political clarity. First, reflected in Joe's critical pedagogical discourse was an awareness of the "economic realities" of his students. Recall that Joe worked in a school that was located in a small, rural town in the Midwest, and from his perspective, members of the community and, thus, some of his students and their families, struggled with poverty. These circumstances, Joe suggested, likely had implications for how students experienced class:

> A lot of students are just incredibly stressed out, and they have so many problems in their life. I want students to take my class seriously, I just don't want to like- a lot of students don't need the extra like panic caused with stuff in their life. I have a fair group of students that comes to talk to me about things that I wish they weren't having to talk to someone about considering—if you catch my drift. So it's like, you know, I want to not only- obviously I care about math, but also- I mean, I got into education not only because I care about math but because I care about students. So, you know, the dual role of the teacher. You're teaching math, but you're also taking care of people that sometimes aren't getting taken care of [in] other places.

Given his perception of students' struggle with issues they experienced outside of school, Joe prioritized taking care of them, arguing that teaching mathematics content should not always be his primary focus.

Arguably, Joe was cognizant of the political nature of his teaching because, in some instances, Joe framed his teaching as a tool through which he might work to raise awareness of and address issues of "social justice" and "diversity." Because almost all of his students were white, however, Joe did not see the relevance of investigating issues of social justice and diversity in his classes, which suggests a few things. First, Joe's reference to his students' race when explaining why they would not find questions of social justice and diversity to be relevant suggests that his conception of "diversity" may be limited to *racial* diversity. Second, underlying Joe's suggestion that investigating issues of social justice and diversity is not *for* white students might be the assumption that white people (who live in a town where the population is predominantly white) do not necessarily have a responsibility to understand and work to address issues of *racial* injustice. Third, given Joe's perception of Beechwood as a conservative, or "right-wing" community, it is possible that he worried about the potential backlash he would face, from both students and parents perhaps, for raising questions about social justice and diversity in his classroom. Therefore, while it is reasonable to suggest that Joe's critical pedagogical discourse reflected political clarity to at least some extent, his talk about political, economic, and social issues may not have been sufficiently refined to act on (discussed more in relation to research question 3).

While Emma's critical pedagogical discourse reflected an awareness of the sociopolitical and economic realities that shaped her students' lives, she drew on deficit discourses when explaining the implications of those realities (e.g., Bertrand, Perez, & Rogers, 2015; Valencia & Black, 2002). For example, when describing issues of student behavior at Schenley, Emma suggested that the disciplinary policies and practices in place at her school were necessary and, in fact, could be stricter. This was in part because, from her perspective, some of the parents and "home lives" of Schenley students were "[un]stable." Thus, rather than considering the ways in which her school's practices and policies might be better designed to meet the needs of her students—which are necessarily a function of broader economic and social circumstances—Emma attributed issues of behavior to deficits of students and their families. And although Emma's assumptions about students' lives may have been based on interactions she had with them (recall that she said, "…there are students I had in my classroom who like had been shot and stuff…"), she did not necessarily consider connections between the (negative) circumstances of students' lives and the injustices perpetuated by broader economic and social structures (Bartolomé, 2004). That Emma did not consider such connections and invoked deficit discourses when describing students' schooling experiences suggests that her critical pedagogical discourse was limited in terms of the extent to which it reflected political clarity.

Sara's critical pedagogical discourse reflected political clarity to at least some extent, as she demonstrated an awareness of the racialized nature of students' experiences. However, like Emma's, Sara's talk about students was sometimes rooted in deficit discourses. For example, based on how her students talked in class (e.g., using "slang" and "curse words"), Sara expressed concern that students would struggle to successfully navigate professional workspaces in the future. Assuming responsibility to prepare her students for navigating those spaces, Sara attempted to support them in developing skills that would be useful for subverting hegemonic norms and expectations (e.g., code-switching). However, Sara's talk about code-switching was rooted, to some extent, in deficit discourses. Rather than problematizing the oppressive nature of typical workplace norms and expectations, Sara attributed the need for students to code-switch to deficits in their speech.

Also, Sara was not critical of the disciplinary policies and practices that her school had in place and, like Emma, suggested that they could be stricter. Recall that Sara said:

> Students complain a lot about like, "Schenley sucks. Schenley is like a jail," because of all the rules. But I think that- I mean, the rules are there for a reason. But for some of them- I think we're kinda too easy on them. Some of them just need to be disciplined more…

Thus, rather than considering the validity of students' experiences (i.e., that Schenley feels like "a jail") and how policies and practices might be better designed to meet students' needs, Sara argued that students merely needed more discipline, which highlights the deficit-oriented nature of her discourse.

Finally, Bri's critical pedagogical discourse reflected political clarity in multiple ways. First, Bri connected the experiences of her students, both in mathematics and school more broadly, to normative instructional practices and institutional structures. Beginning with the former, Bri made connections between a school-wide emphasis on practice (for the sake of getting students' test scores up) and students' limited success. In particular, Bri argued that teachers' enactment of conventional practice was detrimental to students' learning, as it merely prepared students to do well on standardized tests. And further, Bri suggested that the pervasiveness of conventional practice within schools (in general) was likely a reason that students were considered "behind" in the first place.

Bri also framed issues of behavior as a symptom of teachers' practice. That is, rather than blaming students (and their families) for the disciplinary issues that were prevalent within Schenley, Bri suggested that teachers could do a better job of being responsive to students by trying to make their classrooms places to which students look forward to going. However, Bri extended this responsibility beyond teachers, implying that there were other, perhaps systemic, changes that needed to happen in order for students to want to go to class.

Regarding the implications of institutional structures, Bri described the ways in which students' schooling experiences were shaped by the tracking and labeling of students. In particular, Bri argued that tracking and labeling contribute to the construction of students as smart or not and, thus, result in the success of some and marginalization of others. Further, Bri asserted that there is power in mathematics, suggesting that the extent to which students are constructed as smart in mathematics has implications for their access to other advanced courses, and, consequently, the construction of their academic identities more broadly.

Last, Bri's political clarity was reflected in her explicit rejection of the use of deficit discourses to describe students. To push back on deficit discourses, Bri consistently framed students' backgrounds and experiences as assets on which students might draw to do mathematics, rejected contextual discourses that communicated negative assumptions about students' capabilities, and problematized when others drew on ability labels (e.g., "low") to describe students. Regarding the last of these, Bri argued that the labels to which students are often assigned are merely a symptom of the ways in which the school, and the system more broadly, is "disserving" them.

Another important aspect of Bri's political clarity is reflected in her framing of her instruction as a tool through which she might work to address the inequities perpetuated by instructional and institutional practices. Recall, for example, that Bri consistently described the enactment of "inquiry-based" teaching as a way to foster learning in ways that conventional practice could not and as a means by which she could support students in experiencing success they had been denied in the past.

Also worth noting here is the overlap in the *views of students* (Jackson et al., 2017) and *political clarity* (Bartolomé, 2004) dimensions of teachers' critical pedagogical discourses, as both dimensions seem to reflect where teachers locate the responsibility for undesirable outcomes. While further exploring potential overlaps in those dimensions extends beyond the scope of this study, I distinguish between them here by arguing that the demonstration of political clarity requires acknowledgement—and criticism—of the role of the status quo (in both school and society more broadly) in shaping students' schooling experiences.

To summarize, the four teachers' critical pedagogical discourses were different in terms of the ways in which they reflected political clarity. Joe's discourse reflected an awareness of the economic realities of his students, although he did not consider his teaching as a means by which he could work to transform those realities. While Emma's discourse demonstrated an awareness that students' schooling experiences were shaped by social and economic circumstances, she consistently drew on deficit discourses when describing students' experiences, locating responsibility for behavior issues within students and their families. Sara also drew on deficit discourses to explain behavior issues but demonstrated an awareness of the racialized nature of students' experiences. Further, Sara recognized the role of her instruction in shaping students' experiences, both present and future, as she sought to support students in developing skills that, from her perspective, would support them in navigating life after high school. Finally, Bri's discourse reflected an understanding of how students' experiences in both mathematics and school more broadly were shaped by instructional and institutional norms. In particular, Bri pointed to the tracking and labeling of students when explaining some students' (lack of) success. Additionally, Bri's talk about her own instruction suggested that she considered the enactment of ambitious practice (e.g., "inquiry-based" instruction) as a means by which she might work to disrupt the inequities perpetuated by the system.

Summary. The results reported in this section illustrate the multidimensional nature of teachers' critical pedagogical discourses (see Table 5 for an overview). That is, teachers' critical pedagogical discourses varied in terms of the extent to which they reflected ideas about ambitious practice as it relates to epistemology (e.g., Kang, 2008), instructional vision (Munter, 2014), views of students (e.g., Jackson et al., 2017) and political clarity (Bartolomé, 2004). Overall, Bri's critical pedagogical discourse was most aligned with ambitious teaching practices and principles, whereas Sara's was the least aligned. As I will discuss more in relation to the third research question, such differences in teachers' critical pedagogical discourses had implications for how teachers responded to the contextual discourses they perceived within their schools. However, in the section that follows, I discuss findings for the second research question, reflecting on how the contextual discourses that teachers perceived varied across individuals.

Teachers' Perceived Contextual Discourses

The second research question that guided this study was: What contextual discourses do the four teachers perceive, and how do those discourses vary within and across two different school settings? One purpose of investigating this question was to gain insight into contextual discourses (and constraints) that teachers may face as they begin working in different school settings. To characterize contextual discourses, I identified instances in interviews in which teachers described their perceptions of: messages communicated about matters related to mathematics teaching and learning within their schools; and contextual constraints that may have inhibited their practice. Table 6 provides an overview of the contextual discourses and constraints that teachers often reported. Because teachers occasionally alluded to other contextual discourses and constraints that, from their perspective, were less consequential for their practice, those are also reported in Table 6, with shaded cells representing commonalities across individuals.

Table 6

Overview of Teachers' Contextual Discourses and Constraints

	Sources and Nature of Contextual Discourses and Constraints					
Joe (Beechwood)	Lack of resources (e.g., time, support)	Responsibility to cover state learning standards	Autonomy	Student (and community) preferences for conventional practices		
Emma (Schenley)	Emphasis on student behavior and engagement	Other teachers' use of conventional practices	Responsibility to cover content and stay on pace	Autonomy	Student preferences for conventional practices	

Sara (Schenley)	Emphasis on student behavior and engagement	Autonomy				
Bri (Schenley)	Messages about “inquiry-based” teaching (e.g., unfeasibility)	Deficit narratives about students and their capabilities	Other teachers’ use of conventional practices	Responsibility to cover content and stay on pace	Emphasis on student behavior and engagement	Student preferences for conventional practices

The results reported in Table 6 show both similarities and differences in the contextual discourses perceived by the four teachers both between and within school settings. In the paragraphs that follow, I unpack these similarities and differences further by discussing comparisons of cases both *between* and *within* the two school settings.

Between-School Comparisons of Contextual Discourses (and Constraints)

Findings of between-school comparisons of teachers’ perceived contextual discourses (and constraints) pointed to both similarities and differences in those discourses. Regarding similarities, teachers in both schools framed students as a source of contextual discourses, and reported an emphasis on student engagement that was communicated by administration. Regarding the former, teachers explained that students seemed to prefer their use of conventional practice (e.g., giving notes) because it was what students were used to (Joe, Emma, and Bri), one teacher noting, even, that students explicitly expressed disapproval of unfamiliar activities (Joe). In regards to the latter, teachers between the two schools reported engagement as a focus of observations and evaluations (Joe, Emma, and Sara), suggesting that when mentors and administrators conducted classroom observations, they were attending to whether teachers had successfully engaged students throughout the lesson. That such contextual discourses spanned the two schools suggests that some (sources of) contextual discourses may be less context-specific than others.

In regards to differences, while teachers in both schools reported experiencing pressure to teach in conventional ways (Joe, Emma, and Bri), the sources from which those pressures originated differed between schools. At Beechwood (where Joe taught), presses for conventional practice stemmed primarily from resources and students, whereas at Schenley, prominent sources of contextual discourses that encouraged conventional practice were communicated by individuals (e.g., other teachers) and disciplinary policies and practices. To reflect on these differences further, I compare the contextual discourses reported by the two first-year teachers: Joe and Emma.

Differences in contextual discourses reported by Joe and Emma. As just mentioned, for Joe, *resources* (or a lack thereof) and *students* were the most prominent sources that imposed pressure to enact conventional practice. Beginning with resources, the lack of time that Joe perceived, along with his access to particular curriculum materials (i.e., publicly available state standards documents and course texts), promoted conventional teaching. Regarding a lack of time, recall that Joe was responsible for planning for six different classes, so he often described time as a scarce resource. Because, from Joe's perspective, planning ambitious lessons required time that was not available to him, he relied on practices that required less preparation (e.g., "I do, We do, You do"). The curriculum resources to which Joe had access also communicated contextual discourses that promoted conventional teaching. Due, in part, to a lack of support, Joe relied primarily on state standards resources and course texts to guide his instruction. For Joe, those resources communicated contextual discourses about the importance of content coverage, as he often described feeling pressure to "cover" state standards and content. Because Joe described conventional methods as more useful for covering content than ambitious ones, pressure to cover content spawned pressure to enact conventional practice.

Last, Joe (more consistently than Emma) reported feeling pressure from students, as well as the community more broadly, to teach in conventional ways. For example, Joe often reflected on his students' explicit rejection of his attempts to enact ambitious practice and argued that students communicated preferences for conventional teaching by, for example, asking him to introduce standard procedures and provide worksheets of practice problems. From Joe's perspective, students' preferences were likely a product of their previous experiences in other mathematics classrooms and the small size of the community (recall Joe's suggestion that students were likely reluctant to engage in argumentation because they did not want to embarrass their long-time peers).

Additionally, Joe perceived contextual discourses that communicated messages about the relevance of some ambitious practices for students at Beechwood. In particular, Joe assumed that teaching mathematics for social justice and taking up questions of "diversity" would not be received well by his students or by the community more broadly. From Joe's perspective, this was because his students and their families seemed to have political views that were conservative in nature. Further, because almost all of his students were white, Joe anticipated that they would not have interest in exploring issues of social—and, perhaps, racial—justice.

Also worth noting is that Joe did not describe his colleagues, or even his administrators, as sources of pressure to teach in conventional ways. Rather, Joe's colleagues and administrators communicated messages of approval, which, for Joe, evoked a sense of autonomy in making decisions about his teaching.

In contrast, for Emma, the most prominent sources of contextual discourses that promoted conventional teaching emanated from *colleagues* and, although more indirectly, *disciplinary policies and practices*. For example, Emma reported feeling pressure to teach in conventional ways because, from her perspective, members of her PLT privileged conventional practices (e.g., giving notes) over ambitious ones (e.g., posing “investigative” tasks). Therefore, unlike Joe, Emma encountered contextual discourses communicated by colleagues (and mentors) through which a particular approach to teaching was promoted.

Additionally, Emma described expectations for enacting the school’s disciplinary policies and practices, which, according to her, were intended to address issues of student engagement and behavior. While contextual discourses communicated by those policies and practices did not necessarily privilege conventional teaching, they emphasized the importance of establishing *control* over students, which is, arguably, easier to achieve through the enactment of conventional practices (e.g., leading structured note-taking sessions). Based on Emma’s self-reports of other contextual discourses, she perceived messages that pressured her to ensure that students complied with expectations for behavior (e.g., that students would report to class on time, comply with teachers’ requests, and refrain from engaging in “disruptive” behavior). From Emma’s point of view, the disciplinary policies and practices that the school had in place were “born out of necessity” because, when compared to other high schools in the district, Schenley’s student population was more diverse in terms of “ethnicity” and “socioeconomic status.” As this sentiment implies that the severity of control measures depends on the racial and economic diversity of the student population, the contextual discourses that Emma perceived were rooted in deficit narratives of students—even if she did not articulate this herself.

Summary. As illustrated by comparisons of Joe’s and Emma’s cases, contextual discourses varied between the two school settings. While both teachers experienced pressure to teach in conventional ways, for Joe, those pressures stemmed from (a lack of) resources and students, whereas for Emma, such pressures were imposed by other individuals and disciplinary policies and practices.

The nature of the two teachers' self-reported contextual discourses also varied in some ways. According to Joe, his students (and their families) seemed to have conservative political views, which, for him, communicated the message that teaching mathematics for social justice, for example, may not be well received by the community. In contrast, Emma perceived messages that communicated the importance of ensuring that students complied with school-wide expectations for behavior. Although not aware of it herself, those messages were rooted in deficit discourses that pathologized students of color and students of lower socioeconomic status, characterizing them as in need of strict disciplinary policies.

Within-School Comparisons of Contextual Discourses (and Constraints)

Within-school comparisons of the Schenley teachers' perceived contextual discourses (and constraints) revealed two main findings. First, based on the three teachers' descriptions of their PLTs, the Geometry and Algebra PLTs operated differently and, thus, communicated different expectations for teaching. Second, whether teachers reported particular contextual discourses seemed to depend on whether their critical pedagogical discourses acted as resources that supported them in identifying and describing contextual discourses. These two findings are discussed in more detail in the paragraphs that follow.

Recall that Sara and Bri were members of the Geometry PLT, while Emma was a member of the Algebra PLT. Both Sara and Bri reported a sense of autonomy in making decisions about the activities or tasks they would use in their classes. In contrast, Emma described the Algebra PLT as "organized," noting that Algebra teachers tended to implement the same activities and at a similar pace. While Emma insisted that members of her PLT would be receptive to ideas she had, she worried about disrupting the order of the PLT given her status as a novice teacher. These differences in expectations communicated by the two PLTs point to the existence of "subcultures" within schools, which can vary in terms of the constraints they impose on—or the autonomy they afford—beginning teachers (Zeichner & Tabachnick, 1985).

While there were similarities in the contextual discourses reported by the Schenley teachers, subsets of the three teachers perceived contextual discourses that the other(s) did not. As mentioned earlier, this was likely due, in part, to the nature of teachers' critical pedagogical discourses, as some of them had appropriated threads of dialogue that allowed them to identify and describe contextual discourses that others could not.

For example, while Emma and Bri suggested there was an expectation to cover content at a pace similar to the teachers in the PLT and, therefore, enact conventional practices, Sara never described her PLT as a source of pressure. However, this is not necessarily surprising, as Sara, unlike Emma and Bri, aspired to enact conventional practices that were either similar to or commonly used by other members of the PLT. Consequently, Sara may have stayed on pace with other teachers by default. In contrast, Emma and Bri envisioned enacting ambitious practices that other teachers used less often (e.g., providing opportunities for students to figure things out on their own). Because enacting those practices would likely result in falling behind the other teachers, Emma and Bri perceived pressure to employ conventional practices in an effort to stay on pace, recognizing that such practices would not support them in affording the opportunities they envisioned.

Another difference in contextual discourses reported by the Schenley teachers is illustrated by Bri's perceptions of messages about "inquiry-based" teaching and students' capabilities, as only she was attuned to the ways in which other teachers described such things.

For example, Bri questioned her host teacher's conception of inquiry-based teaching, as after observing her teach a lesson in which she had hoped students would invent their own strategies, he suggested to Bri that while launching a high-level task, she should have introduced the very strategies that she hoped students would invent. As another example, Bri pointed to the language that colleagues sometimes used to refer to students (e.g., "low") and described instances in which other teachers suggested that her students would be incapable of doing something she had planned for: "They're not gonna get it, but sure. You can try." That Bri perceived such contextual discourses—while the others did not—is likely attributable to the thread of her critical pedagogical discourse through which she argued the importance of supporting students in experiencing success and seeing themselves as smart.

Summary

The results reported in this section illustrate the ways in which teachers' self-reports of contextual discourses varied across individuals both between and within schools (see Table 6 for an overview). Regarding differences between schools, results showed that while both first-year teachers encountered contextual pressures to teach in conventional ways, the sources from which those pressures emanated most often varied. In particular, while Joe's students and the resources to which he had (or did not have) access consistently pressured him to enact conventional practices, the pressure that Emma experienced stemmed predominantly from other individuals in her school (e.g., members of her PLT). In regards to differences within Schenley, the contextual discourses that the three teachers perceived depended on the sub-contexts (e.g., PLTs) in which they worked and the extent to which their critical pedagogical discourses acted as resources for naming contextual discourses. More specifically, because Emma and Sara and Bri were members of different PLTs (Algebra and Geometry, respectively), the teachers' perceptions of teaching expectations depended on the norms central to the PLT in which they worked. Additionally, because Bri, for example, had appropriated discourses through which she promoted an "inquiry-based" approach to teaching, and Emma and Sara had not, only Bri was attuned to the ways in which other teachers communicated messages about that approach.

(Interactions Between) Critical Pedagogical and Contextual Discourses in Relation to Practice

The third and final research question that guided this study was: How do teachers' critical pedagogical discourses interact with contextual discourses, and how do such interactions help to explain their enactment of ambitious practice? The primary purpose of investigating this question was to understand how beginning teachers' practice might be guided by personal and contextual discourses and relations between them. Efforts to answer this question began with writing narratives (based on the results of coding field notes) to characterize each teacher's instructional practice. Then, to identify discourse interactions, I compared threads of teachers' critical pedagogical discourses to the contextual discourses they reported, attending specifically to whether or not teachers reported *tensions* between those discourses. Then, after considering discourse interactions in relation to each teacher's narrative, I conducted cross-case analyses to identify emergent themes and patterns.

Findings pointed to patterns in teachers' enactment of ambitious practice that reflected three patterns observed by Thompson and colleagues (2013): lack of appropriation, compartmentalization, and integration. These patterns of enactment were associated with different interactions between teachers' critical pedagogical and perceived contextual discourses. In particular, a lack of appropriation was associated with alignment between critical pedagogical and school contextual discourses (Sara), the compartmentalization of ambitious practice was associated with interactions that caused tensions that faded over time (Joe and Emma), and the integration of ambitious practices was associated with interactions that caused tensions which resulted in the filtering out of contextual pressures (Bri). To illustrate relations between discourse interactions and teachers' practice, I describe the three patterns of enactment in more detail and then consider them in relation to the three types of discourse interactions.

Patterns of Enactment

Sara's (the second year teacher's) case demonstrated a lack of appropriation of ambitious practice. Recall, Sara regularly guided students through structured note-taking sessions during which she defined vocabulary or demonstrated how to solve problems, and then provided opportunities for students to practice applying what she had introduced through "collaborative learning" activities.

The cases of the two first-year teachers, Joe and Emma, illustrated the compartmentalization of ambitious practice. Joe and Emma only occasionally attempted to enact ambitious practices and relied more consistently on conventional teaching methods (e.g., demonstrating to students how to solve problems). When the two teachers did attempt to enact an ambitious practice, such as posing a high-level task or facilitating conceptually oriented discourse, they often did so in isolation from other ambitious practices. For example, Joe posed conceptually oriented questions during teacher demonstrations, and Emma regularly explained—and proceduralized—the intended takeaways of "investigative" tasks through subsequent note-taking sessions.

Bri's case represented the integration of ambitious practice, as she tended to enact multiple forms of ambitious practice within a given lesson. When Bri posed high-level tasks, for example, she also provided opportunities for students to discuss those tasks with peers, and selected particular students to share their work during subsequent whole-class discussions. Arguably, then, Bri's enactment of ambitious practice was more holistic than that of Joe and Emma. But Bri was not always successful in integrating ambitious practice, as she sometimes relied on conventional teaching methods (e.g., giving notes). However, much like Joe and Emma, Bri sometimes coordinated conventional and ambitious practices. For example, before introducing and demonstrating to students how to carry out a procedure, Bri dedicated time to showing students "where [those procedures] come from." Thus, when Bri was unable to integrate ambitious practices, she enacted them alongside conventional ones.

Explaining Patterns of Enactment Through Discourse Interactions

Differences in teachers' enactment of ambitious practice can be explained through the lenses of critical pedagogical and contextual discourses and interactions between them. Beginning with Sara again, recall that her critical pedagogical discourse, while minimally reflective of ambitious practice, was well aligned with her self-reports of contextual discourses within her school. Given this alignment, Sara regularly acted in ways that aligned with her critical pedagogical discourse. For example, threads of dialogue through which Sara framed student engagement as a goal for teaching and emphasized the importance of using "correct vocab" when explaining things to students were often reflected in her practice. This was in part because Sara did not perceive contextual discourses that pressured her to change her actions. In fact, Sara's actions were likely reinforced by contextual discourses that she perceived, as they communicated messages that, for example, conveyed teacher autonomy and prioritized student engagement as an outcome.

Joe's critical pedagogical discourse, while well aligned with ambitious practice, was at odds with contextual discourses that he perceived within his school. Aware of these differences, Joe described *tensions* between his desire to enact ambitious practice and (his perceptions of) the demands of his course load (i.e., a lack of time), a responsibility to "cover" the state learning standards, and students' preferences for conventional teaching. But those tensions ultimately resulted in Joe's reliance on conventional teaching practices for at least two reasons: (1) his critical pedagogical discourse did not act as a sufficient resource for filtering out contextual pressures; and (2) he lacked resources and support for enacting his vision. Further, such tensions became less salient for Joe over time, as he began to justify his use of practices that were not aligned with his vision. For example, Joe justified his enactment of conventional practices (e.g., "I do, We do, You do") by arguing that his use of such practices: required little preparation, ensured coverage of the state learning standards, were approved by both administrators and students, and resulted in student learning. Therefore, Joe's teaching was, in large part, guided by contextual discourses, some of which he had appropriated as his own.

Emma's critical pedagogical discourse, which was somewhat aligned with ambitious practice, was also in conflict with her self-reports of contextual discourses within her school. Like Joe, Emma could articulate such tensions, as, for example, she suggested that giving notes—a normative practice within Schenley—required only minimal thinking on the part of students, and that ensuring content coverage sometimes came at the expense of student learning. These tensions resulted in Emma's reliance on conventional methods. This was because of at least two reasons: (1) her critical pedagogical discourse was an insufficient resource for filtering out contextual pressures; and (2) she was positioned—by herself and by other perhaps—as a novice teacher (recall that Emma was reluctant to disrupt the order of her PLT). As was the case for Joe, these tensions became less salient for Emma over time, as she began to justify her use of conventional methods by drawing on contextual discourses. For example, Emma justified giving notes by arguing that it required minimal planning, was a practice commonly used by fellow PLT members, ensured student compliance and content coverage, and resulted in student learning. Therefore, Emma, to an extent, had appropriated the contextual discourses she perceived which, in turn, guided her practice.

Bri's critical pedagogical discourse reflected ambitious practice to the greatest extent and was, in many ways, at odds with the contextual discourses she perceived within Schenley. Like Joe and Emma, Bri was able to articulate those tensions. First, Bri described how her vision for inquiry-based teaching was in conflict with messages communicated by other teachers. For example, Bri perceived (and was critical of) contextual discourses that framed the regular enactment of inquiry-based teaching as unfeasible. She also problematized other teachers' claims that they had enacted inquiry-based teaching because, from her perspective, those teachers had not engaged students in genuine inquiry but merely funneled students toward a desired outcome. Second, Bri reported tensions between her vision and a perceived school-wide emphasis on content coverage and practice. In particular, Bri sensed that prioritizing content coverage and opportunities for practice would, in some ways, be detrimental to student learning and success. Last, Bri perceived contextual discourses that communicated deficit narratives about students and their capabilities, which were in conflict with her consistent assertion that all students were capable of experiencing success.

At times, Bri was susceptible to contextual pressures, as she sometimes acted in ways that did not align with her critical pedagogical discourse. But, unlike Joe and Emma, Bri did not appropriate contexutal discourses as her own and, in some cases, filtered them out by drawing on her critical pedagogical discourse. For example, Bri rejected others' recommendations to provide students with prescribed steps for solving problems, and corrected herself when inadvertently drawing on static ability labels to describe students. Thus, because Bri's critical pedagogical discourse acted as a filter for contextual discourses, when compared to the others, she was less susceptible to contextual pressures to teach in conventional ways.

CHAPTER 6: DISCUSSION AND IMPLICATIONS

In this final chapter, I discuss the study's main findings—some of which were shared in Chapter 5—in more detail. Before beginning this discussion, I provide a brief overview of the study's motivation to help situate the results within a broader context. Then, after considering implications for future research, teacher education and K-12 schooling, I reflect on the limitations of my work and conclude the study.

Discussion

As discussed in Chapters 1 and 2, teacher education has come to bear a great deal of the responsibility for supporting a pedagogical transition away from the use of conventional practices toward the enactment of ambitious ones that can bring to fruition the mathematics learning opportunities for which the field has called for the last 30 years (e.g., AMTE, 2017; NCTM 1989; 2014). Despite our nation's financial investment in teacher education, there is minimal evidence of its effectiveness (Cochran-Smith & Zeichner, 2005), as mathematics teachers' instructional practice continues to look like it did at the time of earlier calls for reform: teacher centered and focused on the replication of facts, skills, and procedures (Hiebert, 2013; Hiebert et al., 2005; Resnick, 2015). But in light of success stories that have highlighted beginning teachers' potential to enact ambitious practice—even within a broader educational climate that is overwhelmed by conventional practice and accountability pressures (e.g., Thompson et al., 2013; Ward et al., 2011; Yurekli et al., 2020)— it seems that teacher education efforts may not be altogether futile.

While Bri's case gives reason to believe that teacher education programs can graduate teachers who will act in the best interest of students and their well-being, the other cases suggest that we still have a lot of work to do. To be clear, this argument is not an indictment of Joe's, Emma's, and Sara's teaching, as they surely have their students' best interest at heart, and, as my findings reveal, did plenty to create opportunities for students' learning. Rather, it is an indictment of the system in which those teachers are caught—a system that privileges test scores over learning, normalizes oppressive practices and policies, and perpetuates deficit narratives about children and their families. And until that system can be remedied through significant changes in policy, I argue that teacher educators have a responsibility to prepare teachers who will—through, at the very least, the enactment of ambitious practice—work to improve the experiences of students whom schools, ostensibly, are intended to serve.

The results of this study raise questions that provide starting points for conversations around how research, teacher education, K-12 schools can contribute to addressing the issues raised in the previous paragraph. First, that only Bri's critical pedagogical discourse acted as a discursive filter for contextual pressures raises questions of why. Results suggest that this may have been due to the threads of dialogue comprising the four dimensions of her critical pedagogical discourse: personal epistemology (Kang, 2008), instructional vision (Munter, 2014), views of students (Jackson et al., 2017), and political clarity (Bartolomé, 2004). For example, Bri's *instructional vision* may have acted as a resource through which she filtered out messages that associate the practice of telling with "inquiry-based" teaching. Likewise, the discursive filters of Bri's *personal epistemology* and *views of students* may have supported her in resisting pressures to prioritize content coverage over other goals for teaching (e.g., teaching for conceptual understanding) and problematizing negative assumptions about students' mathematical capabilities, respectively. As a final example, Bri's *political clarity* might help to explain why she rejected the use of ability labels to describe students, as, from her perspective, the labels to which students are so often assigned are merely a symptom of the ways in which the school system is "disserving" them.

However, it is likely that these dimensions did not operate as discursive filters in isolation from one another. For example, recall that Bri framed inquiry-based teaching (an aspect of her *instructional vision*) as a means through which to ensure—and communicate—that all students are capable of experiencing success in rigorous mathematics (a *view of students*). Thus, perhaps the extent to which teachers' critical pedagogical discourses act as resources for resisting contextual pressures to teach in conventional ways can also be explained by interrelations between the four dimensions.

The dimensions of teachers' critical pedagogical discourses may also help to explain why the feelings of tension expressed by Joe and Emma began to fade over time, ultimately resulting in their appropriation of contextual discourses and regular enactment of conventional practices. For example, it is possible that dimensions of Joe's and Emma's critical pedagogical discourses were *vulnerable* in ways that Bri's were not. Recall that while both Joe and Emma emphasized the importance of providing opportunities for students to struggle and think through mathematics—a focus on *process*—they were more concerned with the *results* of those processes, as both teachers argued the effectiveness of conventional practices (which did not afford the opportunities that they ostensibly valued) by pointing to students' relatively strong performance on assessments.

As mentioned earlier, it is also possible that relations between the dimensions of Joe's and Emma's critical pedagogical discourses can help to explain the teachers' susceptibility to pressures to teach in conventional ways. In Joe's case, one such relation may be that between his *instructional vision* and *political clarity*. In particular, Joe suggested that he had to abandon his vision for teaching mathematics for social justice (an aspect of his *instructional vision* that never developed) because his students—who were white and came from families with politically conservative points of view—would not see the relevance in investigating issues of social justice and diversity (*political clarity*). This relation may help to explain why Joe was susceptible to (his perceptions of) messages communicated by students—and the community more broadly—that his classroom was not a place to explore such issues.

As another example, consider the relation between the *personal epistemology* and *instructional vision* dimensions of Emma's critical pedagogical discourse. While Emma envisioned providing opportunities for students to reason about and think through mathematics on their own (*instructional vision*), a primary goal for her instruction was ensuring student engagement (*personal epistemology*). That Emma did not necessarily consider the provision of such opportunities as a means for meeting her primary goal may help to explain why she was susceptible to pressures to enact conventional practices that, from her perspective, were more likely to ensure student engagement (e.g., giving notes).

While Sara's critical pedagogical discourse was well aligned with the contextual discourses she perceived, she did, in some instances, draw on her critical pedagogical discourse—through which she emphasized the importance of building relationships with students—to problematize how others treated students when enacting disciplinary policies. But Sara's critical pedagogical discourse did not act as a sufficient resource for filtering out the messages communicated by her school's disciplinary policies: that Schenley students needed to be controlled. Based on discussions of the other three cases, this may have been due to limitations of isolated dimensions of Sara's critical pedagogical discourse (e.g., *political clarity*) or relations between them (e.g., *political clarity* and *views of students*).

A second question raised by the results of this study pertains to why Bri—a teacher who grew up in a small, conservative community and experienced conventional teaching as a K-12 student—appropriated discourses reflective of ambitious practice to the greatest extent. As discussed in Chapter 5, it is possible that the *processes* by which the four teachers appropriated such discourses varied across individuals. Because Bri's critical pedagogical discourse did not reflect epistemological contradictions like Joe's and Emma's did, it is possible that Bri, when appropriating new contextual discourses, had to abandon preexisting ideas, which were likely established during her apprenticeship of observation (Lortie, 1975). This suggests that Bri appropriated contextual discourses about ambitious practice through *negotiation* processes, which involve letting go of old ideas to make room for new ones that are, perhaps, contradictory to the old (Horn et al., 2008). In contrast, Joe, Emma, and Sara seemed only to appropriate new contextual discourses alongside preexisting ones, suggesting that, for them, appropriation processes did not require modifications (see, e.g., Ritchie & Wilson, 1993). Although this does not necessarily mean that Joe, Emma, and Sara took up new ideas through *identification* processes (by identifying with other individuals who promote those ideas themselves), these findings suggest that, for these three teachers, the appropriation of contextual discourses about ambitious practice was not as transformative—or, perhaps, as *critical*—as it was for Bri.

A third question, and the final one I raise in this discussion, relates to the influence of particular contextual discourses—or constraints—on beginning teachers' practice. In particular, although the two first-year teachers, Joe and Emma, were susceptible to contextual pressures to teach in conventional ways in part because of the nature of their critical pedagogical discourses, might either of them have been more successful in enacting ambitious practice had they been in the other's situation, encountering *different* contextual constraints? For example, if Emma had taught in Joe's school, where she would have been free from pressures communicated by other, more experienced mathematics teachers, might she have been more successful in enacting the types of mathematical tasks she envisioned? Similarly, had Joe been teaching in Emma's school, where his course load would have been lighter and less demanding, might he have been more successful in planning for and facilitating the types of discussions he imagined? While the results of this study cannot provide answers to such questions, they highlight the variation in beginning teachers' experience in terms of the contextual constraints they might be up against once they have classrooms of their own.

Implications

Implications for Research

The results of this study add nuance to Thompson and colleagues' (2013) construct of critical pedagogical discourses. Whereas others have equated the construct with instructional vision (e.g., Munter & Correnti, 2017), findings suggested that instructional vision may be one dimension among others that constitute one's critical pedagogical discourse. This study points to personal epistemology, views of students, and political clarity as likely candidates. However, future research is needed to determine the validity of those dimensions and the possibility of others. In addition to investigating whether and to what extent other teachers' critical pedagogical discourses are multidimensional, future research efforts might also focus on specifying differences within each of the four proposed dimensions, and investigating relations between them. While some researchers have offered tools for assessing such differences (e.g., Jackson et al., 2017; Munter, 2014), those tools are limited in terms of what they can capture. For example, Munter's (2014) assessment tool was helpful for characterizing differences in teachers' discourses around (or visions for) high-quality mathematics teaching practices, but the tool did not account for teachers' talk about the *feasibility* of enacting such practices (e.g., Nolen et al., 2009). Thus, more work needs to be conducted to understand the multidimensionality of teachers' critical pedagogical discourses, qualitative differences within those dimensions, and relations between them.

Second, as results showed that the processes by which teachers appropriated contextual discourses about ambitious practice may have varied across individuals, future research is needed to further investigate those processes and their implications for the development of teachers' critical pedagogical discourses. This may require conducting more work that is dedicated to understanding differences in appropriation processes (e.g., Horn et al., 2008), the kinds of experiences that give rise to them, and how teachers' preexisting critical pedagogical discourses might influence their responses to encountering new contextual discourses.

Third, as mentioned in the discussion, that the two first-year teachers—whose critical pedagogical discourses were relatively similar—were both susceptible to contextual pressures to teach in conventional ways despite having taught in different schools is interesting. Had each of the teachers worked in the other's school, would patterns in their early practice have been different, and if so, how? To answer such questions, more research is needed to investigate differences in the extent to which particular contextual discourses (and their sources) influence the practice of beginning teachers whose critical pedagogical discourses are similar.

Fourth, while results showed that Bri, the student teacher, was successful in resisting contextual pressures to teach in conventional ways and most consistent in her attempts to enact ambitious practice, more longitudinal research is needed to determine whether such efforts are sustainable over time. In particular, to understand the long-term effects of continued press for conventional practice on beginning teachers' practice, research might focus on whether, to what extent, and under what conditions critical pedagogical discourses that initially act as filters converge with—or are "washed out" by—conservative contextual discourses (Zeichner & Tabachnick, 1981). Relatedly, such research might also consider whether a threshold of appropriation exists over which if beginning teachers cross, they are no longer susceptible to particular contextual pressures.

Implications for Teacher Education

As this study highlights the persistence of the "two-worlds" problem (Feiman-Nemser & Buchmann, 1985), its results have implications for each of the two worlds: teacher education and K-12 schools. Beginning with teacher education, researchers have argued the need for structural changes to minimize conflicts between the two worlds. For example, Zeichner (2010) offered ideas for cultivating more collaborative relationships between teacher educators and practitioners by, for example, bringing the knowledge of K-12 teachers into campus-based classrooms and/or creating a hybridized role for teacher educators that requires participation in both university and school settings.

Because the structural changes such as those that Zeichner (2010) proposed require significant time and collaboration among multiple people and institutions, it is also important to consider how (groups of) teacher educators can support beginning teachers in navigating tensions between the two worlds to, ultimately, learn how to enact ambitious practice. In contrast to arguments that emphasize a focus on the enactment of ambitious practice (e.g., Ball & Forzani, 2009; Grossman et al., 2009), the results of this study suggest that a more explicit focus on discourses—or the development of discursive filters—may be promising. To better facilitate the appropriation of particular discourses (e.g., those that reflect ambitious teaching principles and practices), teacher educators may consider: establishing intended visions for their programs that are promoted consistently across courses (e.g., Jansen et al., 2018; Tatto, 1996, 1998); supporting beginning teachers in deconstructing and problematizing common contextual discourses that contradict ambitious ones (e.g., those that communicate deficit narratives about students) (e.g., Gutiérrez, 2013); treating discursive conflicts between university and school settings as objects of inquiry (e.g., Hebard, 2016); and reflecting on changes in self-narratives (e.g., de Freitas, 2008), perhaps over the duration of the program.

The results of this study also highlight the importance of preparing teachers to enact ambitious practices in *different* institutional contexts, as contextual discourses and constraints will likely vary across school settings. My argument is not for the individualization of teacher education programs but rather, for the responsiveness of such programs to the future plans of prospective teachers enrolled in those programs. For example, prospective teachers planning to work in under-resourced schools where a majority of community members are white and, perhaps, affiliate with conservative points of view may benefit from learning opportunities focused on adapting low-quality curriculum resources to increase the cognitive demand of mathematical tasks (e.g., Smith & Stein, 2018) and supporting white students in using mathematics to investigate issues of social—and *racial*—justice in a way that avoids unproductive backlash (e.g., Gutiérrez, 2016). In contrast, prospective teachers planning to work in large, well-resourced schools that serve racially and economically diverse student populations may benefit from learning opportunities focused on navigating collaborative spaces as a new teacher and problematizing dehumanizing disciplinary policies.

Implications for K-12 Schooling

As the results of this study showed that aspects of K-12 settings can inhibit beginning teachers' enactment of ambitious practices, which, if implemented effectively and equitably, can improve the learning experiences of students, they also have implications for K-12 schooling. Based on the contextual discourses reported by the teachers in this study, teachers' struggles to enact ambitious practices—and their reliance on conventional ones—may have been exacerbated by commonly used tools (e.g., AVID "collaborative learning" activities), accountability practices (e.g., a focus on preparing students to be successful on state tests), structures intended to support teacher collaboration (e.g., PLT meetings), and punitive disciplinary policies. While such things are likely enacted with the best of intentions (aside from the last perhaps), K-12 practitioners—especially district leaders—should consistently work to interrogate whether taken-for-granted norms, practices, and policies operate in the best interest of students and their well-being or whether such things help to protect the status quo and, thus, perpetuate inequities.

That teachers across the two school settings named students as a source of contextual discourses through which the use of conventional practices was encouraged highlights a final implication for K-12 schooling. In particular, students' explicit rejections of teachers' attempts to enact ambitious practices and verbalized preferences for conventional ones speaks to the influence of K-12 schools on students' perceptions of what mathematics is, what (and who) it is for, and how one successfully participates in it. As it is likely that such perceptions begin to develop early on, it is crucial for practitioners to reflect on the messages they send to their youngest students, approaching the problem from a K-12 perspective.

Limitations

Before concluding, I wish to acknowledge a number of this study's limitations. First, a common critique of case study research pertains to validity, as the researcher acts as the primary instrument of data collection and analysis (Merriam & Tisdell, 2016). To establish validity, Merriam and Tisdell recommended a number of strategies including reflexivity, triangulation, peer review, and member checks. While, due to time demands, I was unable to conduct member checks with teachers, I consistently reflected on my biases and frames of reference (see positionality statement in Chapter 3) while collecting data, conducting analysis, and writing the final report, triangulated data by cross-checking field notes and interviews, and engaged in discussions with both my advisor and a fellow graduate student throughout the course of the study to discuss data collection processes, tentative findings, and interpretations.

A second limitation—which is methodological in nature—pertains to my use of contextual discourse as an analytical construct. While an affordance of focusing on teachers' self-reports is that they reflect teachers' own perceptions of external pressures that may or may not be consequential for their practice, a limitation of self-reports is that they do not account for the potential effects of contextual forces of which teachers may be unaware.

A final limitation relates to the duration of the study, as I observed and interviewed four teachers over the course of one semester (Sara and Bri) and school year (Joe and Emma). Thus, it is important to acknowledge that the study's results represent only a limited view of the teachers' early-career experiences.

Conclusion

Despite any progress the field has made since NCTM's call for reform in 1989, the results of this study suggest that there is more work to be done. As researchers, teacher educators, and K-12 practitioners continue to work to improve the mathematics learning experiences of students in the U.S., it is my hope that these results will provide guidance for those stakeholders. However, not only must we continue our efforts to investigate, define, and support teachers in developing forms of practice that can address the inequities (re)produced by school mathematics, we must also interrogate how taken-for-granted norms, values, and practices within both university and school settings undermine those efforts.

REFERENCES

Allen, J. M. (2009). Valuing practice over theory: How beginning teachers re-orient their practice in the transition from the university to the workplace. *Teaching and Teacher Education*, *25*(5), 647–654.

Anagnostopoulos, D., Smith, E. R., & Basmadjian, K. G. (2007). Bridging the university-school divide: Horizontal expertise and the "two-worlds pitfall." *Journal of Teacher Education*, *58*(2), 138–152.

Arzi, R. J., & White, R. T. (2007). Change in teachers' knowledge of subject matter: A 17-year longitudinal study. *Science Education*, *92*, 221–251

Association of Mathematics Teacher Educators. (2017). *Standards for preparing teachers of mathematics*. Retrieved from http://amte.net/standards

Ball, D. L. (2002). What mathematics knowledge is needed for teaching mathematics? Remarks to the Secretary's Summit on Mathematics, U.S. Department of Education, Washington, D.C., February 6, 2003. Available at http://www.ed.gov/inits/mathscience/ball.html.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Towards a practice-based theory of professional education. In L. Darling- Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32). San Francisco: Jossey-Bass.

Ball, D., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education*, *60*(5), 497–511.

Barkatsas, A. T., & Malone, J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal*, *17*(2), 69–90.

Barnett, D. (1975). The emergence of new concepts for teacher education field experience. *Interchange*, *6*, 44–48.

Bartolomé, L. I. (2004). Critical pedagogy and teacher education: Radicalizing prospective teachers. *Teacher Education Quarterly*, *31*(1), 97–122.

Bertrand, M., Perez, W. Y., & Rogers, J. (2015). The covert mechanisms of education policy discourse: Unmasking policy insiders' discourses and discursive strategies in upholding or challenging racism and classism in education. *Education Policy Analysis Archives*, *23*, 1–35.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco, CA: Jossey-Bass.

Boaler, J., & Brodie, K. (2004). The importance, nature and impact of teacher questions. In D. E. McDougall & J. A. Ross (Eds.), *Proceedings of the 26th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 774–782).

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record, 110*(3), 608–645.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily?. *Journal for Research in Mathematics Education*, *23*(3), 194–222.

Borko, H., & Mayfield, V. (1995). The roles of the cooperating teacher and university supervisor in learning to teach. *Teaching and Teacher Education*, *11*(5), 501–518.

Borko, H., Peressini, D., Romagnano, L., Knuth, E. J., Willis-Yorker, C., Wooley, C.,…Masarik,

K. (2000). Teacher education does matter: A situative view of learning to teach secondary mathematics. *Educational Psychologist, 35*(3), 193–206.

Borko, H., & Putnam, R. (1996). Learning to teach. In D. Berliner & R. Calfee (Eds.)*, Handbook of educational psychology* (673–708). New York: Macmillan.

Braaten, M. (2019). Persistence of the two-worlds pitfall: Learning to teach within and across settings. *Science Education*, *103*(1), 61–91.

Brown, C.A. (1986). A study of the socialization to teaching of a beginning secondary mathematics teacher. In *Proceedings of the Tenth International Conference for the Psychology of Mathematics Education* (pp. 336–341). London: University of London Institute of Education.

Bullough, R. V., & Draper, R. J. (2004). Making sense of a failed triad: Mentors, university supervisors, and positioning theory. *Journal of Teacher Education*, *55*, 407–420.

Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding*. Mahwah, NJ: Lawrence Erlbaum Associates.

Clandinin, D. J., & Connelly, F. M. (2000). *Narrative inquiry: Experience and story in qualitative research*. San Francisco: Jossey-Bass.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 255-296). New York: Macmillan.

Cobb, P., & Bowers, J. S. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher, 28*(2), 4–15.

Cobb, P., Jackson, K., Henrick, E., Smith, T. M., & the MIST team. (2018). *Systems for* *instructional improvement: Creating coherence from the classroom to the district office*. Cambridge, MA: Harvard Education Press.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences*, *10*(1–2), 113–163.

Cochran-Smith, M., & Villegas, A. M. (2016). Research on teacher preparation: Charting the landscape of a sprawling field. In D. Gitomer & C. Bell (Eds.), *Handbook of research on teaching* (5th ed., pp. 439–538). Washington, DC: American Educational Research Association.

Cochran-Smith, M., & Zeichner, K. M. (Eds.). (2005). *Studying teacher education: The report of the AERA Panel on research and teacher education*. Mahwah, NJ: Erlbaum.

Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research*, *64*(1), 1–35.

Cohen, E. G., Lotan, R. A., Scarloss, B. A., & Arellano, A. R. (1999). Complex instruction: Equity in cooperative learning classrooms. *Theory Into Practice*, *38*(2), 80–86.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org

Creswell, J.W. (2013). *Qualitative inquiry & research design: Choosing among the five approaches*. Thousand Oaks, CA: Sage Publications, Inc.

Darling-Hammond, L. (2010). *The flat world and education*. New York, NY: Teacher College Press.

de Freitas, E. (2008). Troubling teaching identity: Preparing mathematics teachers to teach for diversity. *Teaching Education, 19*, 43–55.

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education*, *24*, 8–40.

Emerson, R., Fretz, R., & Shaw, L. (2011). *Writing ethnographic fieldnotes* (2nd ed.). Chicago: The University of Chicago Press

Ensor, P. (2001). From preservice mathematics teacher education to beginning teaching: A study in recontextualizing. *Journal for Research in Mathematics Education, 32,* 296–320.

Feiman-Nemser, S., & Buchmann, M. (1985). Pitfalls of experience in teacher preparation. *Teachers College Record*, *87*, 53–65.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 147–164). New York: Macmillan.

Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory Into Practice*, *40*(2), 102–109.

Friedrichsen, P., Chval, K. B., & Teuscher, D. (2007). Strategies and sources of support for beginning teachers of science and mathematics. *School Science and Mathematics*, *107*(5), 169–181.

Gee, J. P. (2001). Identity as an analytic lens for research in education. In W. G. Secada (Ed.), *Review of research in education*, Vol. 25 (pp. 99–125). Washington, DC: American Educational Research Association.

Ginsburg, M. B., & Newman, K. K. (1985). Social Inequalities, schooling, and teacher education. *Journal of Teacher Education*, *36*(2), 49–54.

Greeno, J. G. (1997). On claims that answer the wrong questions. *Educational Researcher,* *26*(1), 5–17.

Greeno, J. G., Collins, A. M., & Resnick, L. B. (1996). Cognition and learning. In D. Berliner and R. Calfee (Eds.), *Handbook of educational psychology* (pp. 15–46). Macmillan, New York.

Gresalfi , M. S., & Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies: An International Journal*, *1*(1), 49–57.

Gresalfi, M., Martin, T., Hand, V., & Greeno, J. (2009). Constructing competence: An analysis of student participation in the activity systems of mathematics classrooms. *Educational Studies in Mathematics*, *70*(1), 49–70.

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: Theory and Practice*, *15*, 273–289.

Grossman, P. L., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal, 45*(1), 184–205.

Grossman, P. L., Smagorinsky, P., & Valencia, S. (1999). Appropriating tools for teaching English: A theoretical framework for research on learning to teach. *American Journal of Education*, *108*(1), 1–29.

Grossman, P. L., Valencia, S., Evans, K., Thompson, C., Martin, S., & Place, N. (2000). Transitions into teaching: Learning to teach writing in teacher education and beyond. *Journal of Literacy Research, 32*, 631–662.

Gutiérrez, R. (2016). Strategies for creative insubordination in mathematics teaching. *Teaching for Excellence and Equity in Mathematics*, *7*(1), 52–60.

Gutiérrez, R. (2015). Nesting in Nepantla: The importance of maintaining tensions in our work.

In Joseph, N. M., Haynes, C. & Cobb, F. (Eds.), *Interrogating Whiteness and relinquishing power: White faculty's commitment to racial consciousness in STEM classrooms* (pp. 253–282). New York: Peter Lang.

Gutiérrez, R. (2013). Why (urban) mathematics teachers need political knowledge. *Journal of Urban Mathematics Education*, *6*(2), 7–19. Gutiérrez, R. (2012). Embracing Nepantla: Rethinking" knowledge" and its use in mathematics teaching. *Journal of Research in Mathematics Education*, *1*(1), 29–56.

Hebard, H. (2016). Finding possibility in pitfalls: The role of permeable methods pedagogy in preservice teacher learning. *Teachers College Record*, *118*(7), 1–42.

Herbel-Eisenmann, B., & Breyfogle, L.M. (2005). Questioning our patterns of questions. *Mathematics Teaching in the Middle School, 10*(9), 484–489.

Hiebert, J. (2013). The constantly underestimated challenge of improving mathematics instruction. In K. R. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 45–56). New York, NY: Springer.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Charlotte, NC: Information Age Publishing.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M.,…& Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis*, *27*(2), 111–132.

Holland, D., Lachicotte, W., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Boston, MA: Harvard University Press.

Horn, I. S. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in math teachers' conversations. *Journal of the Learning Sciences*, *16*(1), 37–79.

Horn, I. S. (2012). *Strength in numbers: Collaborative learning in secondary mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Horn, I. S., Nolen, S. B., Ward, C., & Campbell, S. S. (2008). Developing practices in multiple worlds: The role of identity in learning to teach. *Teacher Education Quarterly*, *35*(3), 61–72.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116.

Jackson, K. (2009). The social construction of youth and mathematics: The case of a fifth grade classroom. In D. B. Martin (Ed.), *Mathematics teaching, learning, and liberation in the lives of Black children* (pp. 175–199). New York, NY: Routledge.

Jackson, K., & Cobb, P. (2010, May). *Refining a vision of ambitious mathematics instruction to address issues of equity*. Paper presented at the annual meeting of the American Educational Research Association, Denver, CO.

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education*, *44*(4), 646–682.

Jackson, K., Shahan, E., Gibbons, L., & Cobb, P. (2011). Launching complex tasks. *Mathematics Teaching in the Middle School*, *18*(1), 24–29.

Jansen, A., Gallivan, H. R., & Miller, E. (2018). Early-career teachers' instructional visions for mathematics teaching: Impact of elementary teacher education. *Journal of Mathematics Teacher Education*, *23*, 183–207.

Johnson, L. L., & Barnes, M. E. (2018). Mediating the "two-worlds" pitfall through critical, project-based clinical experiences. *The New Educator, 14*(3), 270–291.

Kagan, D. M. (1992). Professional growth among preservice and beginning teachers. *Review of Educational Research*, *62*, 129–169.

Kang, N. H. (2008). Learning to teach science: Personal epistemologies, teaching goals, and practices of teaching. *Teaching and Teacher Education*, *24*(2), 478–498.

Kang, N. H., & Wallace, C. S. (2005). Secondary science teachers' use of laboratory activities: Linking epistemological beliefs, goals, and practices. *Science Education*, *89*, 140–165.

Kazemi, E., & Cunard, A. (2016). Orienting students to one another and to the mathematics during discussions. In S. Marx (Ed.), *Qualitative research in STEM education: Studies of equity, access, and innovation* (pp. 287–305). New York, NY: Routledge.

Kazemi, E., Franke, M. L., & Lampert, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell, & T. Burgess (Eds.) *Crossing divides: Proceedings* of the *32nd annual conference of the Mathematics Education Research Group of Australasia* (pp. 11–30). Palmerston, NZ: MERGA.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, *102*(1), 59–80.

Kennedy, M. M. (1999). The role of pre-service teacher education. In L. Darling Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 54–86). San Francisco, CA: Jossey-Bass.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

Lacey, C. (1977). *The socialization of teachers*. London: Methuen.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean?. *Journal of Teacher Education*, *61*(1–2), 21–34.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29–63.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H.,…& Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*(3), 226–243.

Lampert, M., & Graziani, F. (2009). Instructional activities as a tool for teachers' and teacher educators' learning. *The Elementary School Journal*, *109*(5), 491–509.

Lave, J., & Wenger, E. (1991). *Situated learning and legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Law, H. Y., Wong, N. Y., & Lee, N. Y. L. (2012). A study of espoused values in Hong Kong's mathematics classrooms. *ZDM Mathematics Education*, *44*(1), 45–57.

Leinhardt, G., & Greeno, J. G. (1986). The cognitive skill of teaching. *Journal of Educational Psychology, 78*(2), 75–95.

Levin, D. M., Hammer, D., & Coffey, J. E. (2009). Novice teachers' attention to student thinking. *Journal of Teacher Education*, *60*(2), 142–154.

Lortie, D.C. (1975). *Schoolteacher: A sociological study*. Chicago: University of Chicago Press.

Madsen-Nason, A., & Lappan, G. (1987). The middle grades mathematics project: Coaching as a strategy for changing teacher practice. In J. C. Bergeron, N. Herscovics, & C. Kieran (Eds.), *Proceedings of the 11th International Conference on the Psychology of Mathematics Education* (pp. 100–106). Montreal, Canada.

McClain, K., & Cobb, P. (1998). The role of imagery and discourse in supporting students' mathematical development. In M. Lampert & M. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 56–81). New York: Cambridge University Press.

McLaughlin, M. W., & Talbert, J. E. (1993). How the world of students and teachers challenges policy coherence. In S. H. Fuhrman (Ed.), *Designing coherent educational policy: Improving the system* (pp. 220–249). San Francisco, CA: Jossey-Bass.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Cambridge, MA: Harvard University Press.

Merriam, S. B. & Tisdell, E. J. (2016). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). Thousand Oaks, Califorinia: SAGE Publications, Inc.

Moore, C. (1979). National survey queries early clinical experiences. *ATE Newsletter, 12*, 3.

Munter, C. (2014). Developing visions of high-quality mathematics instruction. *Journal for Research in Mathematics Education*, *45*(5), 584–635.

Munter, C., & Correnti, R. (2017). Examining relations between mathematics teachers' instructional vision and knowledge and change in practice. *American Journal of Education*, *123*(2), 171–202.

Munter, C. & Wilhelm, A. G. (in press). Mathematics teachers' knowledge, networks, practice, and change in instructional visions. *Journal of Teacher Education*.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author

National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: Author.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies*, *19*(4), 317–328.

Nolen, S. B., Horn, I. S., Ward, C. J., & Childers, S. A.(2011). Novice teacher learning and motivation across contexts: Assessment tools as boundary objects. *Cognition and Instruction*, *29*(1), 88–122.

Nolen, S. B., Ward, C. J., Horn, I. S., Childers, S., Campbell, S. A., & Mahna, K. (2009). Motivation in pre-service teachers: The development of utility filters. In M. Wosnitza, S. A. Karabenick, A. Efklides, & P. Nenniger (Eds.). *Contemporary motivation research: From global to local perspectives (*pp. 265– 278). Ashland, OH: Hogrefe & Huber.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, *62*(3), 307–332.

Peressini, D., Borko, H., Romagnano, L., Knuth, E. J., & Willis, C. (2004). A conceptual framework for learning to teach secondary mathematics: A situative perspective. *Educational Studies in Mathematics, 56*(1), 67–96.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 257–315). Charlotte, NC: Information Age Publishing.

Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning?. *Educational Researcher*, *29*(1), 4–15.

Resnick, L. B. (2015). Talking to learn: The promise and challenge of dialogic teaching. In L. B. Resnick, C. Asterhan & S. Clarck (Eds.), *Socializing intelligence through academic talk and dialogue* (pp. 441–450). Washington, DC: American Educational Research Association.

Richardson, V. (1994). The consideration of teachers' beliefs. In V. Richardson (Ed.), *Teacher change and the staff development process: A case in reading instruction*. New York: Teachers College Press.

Ritchie, J. S., & Wilson, D. E. (1993). Dual apprenticeships: Subverting and supporting critical teaching. *English Education*, *25*(2), 67–83.

Rogoff, B. (1994). Developing understanding of the idea of communities. *Mind, Culture, and Society*, *1*(4), 209–229.

Russ, R., Sherin, B. L., & Sherin, M. G. (2016). What constitutes teacher learning? Teacher learning and the balance of expertise. In D. Gitomer & C. Bell (Eds.), *Handbook of research on teaching* (5th ed., pp. 391–438). Washington, DC: American Educational Research Association.

Saldaña, J. (2016). *The coding manual for qualitative researchers* (3rd ed.). Los Angeles: SAGE Publications.

Savin-Baden, M. & Major, C.H. (2012). *Qualitative research: The essential guide to theory and practice*. London: Routledge.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of 'well-taught' mathematics courses. *Educational Psychologist*, *23*, 145–166.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14–22.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4–14.

Silver, E. A., & Stein, M. K. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*(4), 476–521.

Skott, J. (2001). The Emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education*, *4*(1), 3–28.

Smagorinsky, P., Cook, L. S., Moore, C., Jackson, A. Y., & Fry, P. G. (2004). Tensions in learning to teach: Accommodation and the development of a teaching identity. *Journal of Teacher Education*, *55*(1), 8–24.

Smith, E. R., & Avetisian, V. (2011). Learning to teach with two mentors: Revisiting the "two-worlds pitfall" in student teaching. *The Teacher Educator*, *46*(4), 335–354.

Smith, M. S., & Stein, M. K. (2018). *Five practices for orchestrating productive mathematics discussions* (2nd ed.). Reston, VA: National Council of Teachers of Mathematics.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344–350.

Snow, D. A., & Benford, R. D. (1988). Ideology, frame resonance, and participant mobilization. In D. Klandermans, H. Kriesi, & S. G. Tarrow (Eds.), *International Social Movement Research* (Vol. 1, pp. 197–217). Greenwich, CT: Jai Press Inc.

Spradbery, J. (1976). Conservative pupils? Pupil resistance to a curriculum innovation in mathematics. In G. Whitty & M. Young (Eds.), *Explorations in the politics of school knowledge* (pp. 236–243). Driffield, England: Nafferton.

Steele, D. (2001). The interfacing of preservice and inservice experiences of reform-based teaching: A longitudinal study. *Journal of Mathematics Teacher Education*, *4*, 139–172.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Free Press.

Stodolsky, S. S., & Grossman, P. L. (2000). Changing students, changing teaching. *Teachers College Record*, *102*(1), 125–72.

Tanase, M., & Wang, J. (2010). Initial epistemological beliefs transformation in one teacher education classroom: Case study of four preservice teachers. *Teaching and Teacher Education*, *26*(6), 1238–1248.

Tatto, M. T. (1996). Examining values and beliefs about teaching diverse students: Understanding the challenges for teacher education. *Educational Evaluation and Policy Analysis*, *18*(2), 155–180.

Tatto, M. T. (1998). The influence of teacher education on teachers' beliefs about purposes of education, roles, and practice. *Journal of Teacher Education*, *49*(1), 66–77.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: a synthesis of the research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York: Macmillan.

Thompson, A. G., Philipp, R. A., Thompson, P. W., & Boyd, B. A. (1994). Calculational and conceptual orientations in teaching mathematics. In D. B. Aichele & A. Coxford (Eds.), *Professional development for teachers of mathematics*, 1994 yearbook of the National Council of Teachers of Mathematics (pp. 79–92). Reston, VA: National Council of Teachers of Mathematics.

Thompson, J., Windschitl, M., & Braaten, M. (2013). Developing a theory of ambitious early-career teacher practice. *American Educational Research Journal, 50*(3), 574–615.

Turner, E. E., Drake, C., McDuffie, A. R., Aguirre, J., Bartell, T. G., & Foote, M. Q. (2012). Promoting equity in mathematics teacher preparation: A framework for advancing teacher learning of children's multiple mathematics knowledge bases. *Journal of Mathematics Teacher Education, 15*, 67–82.

Valencia, R. R., & Black, M. S. (2002). "Mexican Americans don't value education!"—On the basis of the myth, mythmaking, and debunking. *Journal of Latinos and Education, 1*(2), 81–103.

Valencia, S. W., Martin, S. D., Place, N. A., & Grossman, P. (2009). Complex interactions in student teaching: Lost opportunities for learning. *Journal of Teacher Education*, *60*(3), 304–322.

Ward, C. J., Nolen, S. B., & Horn, I. S. (2011). Productive friction: How conflict in student teaching creates opportunities for learning at the boundary. *International Journal of Educational Research*, *50*(1), 14–20.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge, UK: Cambridge University Press.

Wilhelm, A. G. (2014). Mathematics teachers' enactment of cognitively demanding tasks: Investigating links to teachers' knowledge and conceptions. *Journal for Research in Mathematics Education, 45*(5), 636–674.

Wilhelm, A. G., Munter, C., & Jackson, K. (2017). Examining relations between teachers' explanations of sources of students' difficulty in mathematics and students' opportunities to learn. *The Elementary School Journal, 117*(3), 345–370.

Windschitl, M., Thompson, J., & Braaten, M. (2011). Ambitious pedagogy by novice teachers: Who benefits from tool-supported collaborative inquiry and why?. *Teachers College Record, 113*(7), 1311–1360.

Yin, R. (2014). *Case study research: Design and methods* (5th ed.). Thousand Oaks, CA: Sage Publications, Inc.

Yurekli, B., Stein, M. K., Correnti, R., & Kisa, Z. (2020). Teaching mathematics for conceptual understanding: Teachers' beliefs and practices and the role of constraints. *Journal for Research in Mathematics Education*, *51*(2), 23–-247.

Zeichner, K. (2010). Rethinking the connections between campus courses and field experiences in college-and university-based teacher education. *Journal of Teacher Education*, *61*(1–2), 89–99.

Zeichner, K. M., & Tabachnick, B. R. (1985). The development of teacher perspectives: Social strategies and institution control in the socialization of beginning teachers. *Journal of Education for Teaching*, *11*(1), 1–25.

Zeichner, K. M., & Tabachnick, B. R. (1981). Are the effects of university teacher education 'washed out' by school experience? *Journal of Teacher Education*, *32*(3), 7–11.

APPENDIX A: METHODS COURSE INSTRUCTOR SURVEY

Name:
Course:
Semester/Year:

1. What were some of the key opportunities you tried to afford students in your course, and what were the intended learning outcomes of those opportunities? In other words, what did you hope students would “take away” from those opportunities?

2. As a methods course instructor, what messages about mathematics teaching and learning did you try to communicate to your students?

APPENDIX B: FIRST FORMAL INTERVIEW PROTOCOL

1. First, I'd like to ask you some questions about your view of high-quality mathematics instruction. If you were to observe another mathematics teacher teach one or more lessons, what would you look for to decide whether the instruction was of high quality?
   a. Why do you think it is important to use/do _____ in a math classroom?
   b. Is there anything else you would look for? *If so*, why?

2. What are some of the things that you would expect to find the teacher actually doing in the classroom if the instruction were of high quality?

3. What kinds of problems or mathematical tasks would you expect to see the students working on if the instruction were of high quality?
   a. Can you please describe a _____ task that you would consider to be of high quality?

4. Can you please describe what classroom discussion would look and sound like if instruction were of high quality?
   a. Would you expect to see the entire class participating in a single discussion, or would students be talking primarily in small groups?

5. What would you look for to determine whether instruction is equitable?

6. Next, I'd like to ask you some questions about your perspectives on education. Could you please describe personal experiences or histories that led you to teaching?
   a. What do you consider to be your responsibilities as an educator?
   b. More broadly, what do you think schools should accomplish?

7. If you think about your responses to the questions I just asked about high-quality mathematics instruction and education more broadly, would you say those responses are different from what you would have said before starting your teacher education program? *If so*, in what ways?
   a. What do you think brought on those changes?
   b. *If not addressed*: How did your coursework experiences influence your ideas about teaching and learning mathematics?
   c. Were there any experiences that you found particularly challenging? *If so*, what were they, and why do you think they were particularly challenging?
   d. Do you feel that the program did a good job of preparing you to teach?
   e. Do you have ideas about other kinds of supports and/or opportunities that might have been helpful?

8. How, if at all, do your views of high-quality mathematics instruction differ from the ideas and approaches promoted in your teacher education program?

9. How, if at all, do your views of high-quality mathematics instruction differ from the ideas and approaches promoted in your school?

10. Next I'd like to ask you some questions about your teaching experiences. How is teaching going so far?
    a. Have you tried to apply anything you learned in coursework? *If so*, how did it go?
    b. Is there anything that you've had to abandon?

11. Next I'd like to ask you some questions about your student teaching experiences. Can you please describe the school in which your student teaching took place?

12. *If not addressed:* How do you think student teaching influenced your ideas about teaching and learning mathematics?
    a. Were there any experiences that you found particularly challenging? *If so*, what were they, and why do you think they were particularly challenging?

13. *If not addressed in response to the previous question:* How did your field placement experiences relate to your coursework experiences?

14. Did you try to apply any ideas you learned from coursework to your work in the field? *If so*, which ideas did you apply, and how did it go?
    a. Were there any ideas from coursework that you intentionally did not apply in the field? *If so*, what were they, and why did you not apply them?

15. *If not addressed in response to the previous question*: Do you feel like your field teacher was receptive to your ideas, and was s/he willing to let you try things out?

16. Do you feel like you learned things in your field placement that you didn't/couldn't have learned in your coursework experiences? *If so*, what were they?

17. Can you say a little about the school district in which you'll be working this fall?
    a. What were some reasons you decided to take this position?
    b. Was the choice to take a position at a non-charter, public school intentional?

18. What are some current goals that you have?

19. What are your biggest challenges?

20. When students don't learn as expected, what do you typically find to be the reasons?

21. Do you feel like you need to adjust your instruction in any way for particular groups of students?

22. Have you noticed any inequities?

23. Are there any students or groups of students you're particularly worried about?

24. Who are your main sources of support?

25. What feedback have you gotten so far about your teaching?
    a. What, if anything, have you done with that feedback?

APPENDIX C: MIDYEAR FORMAL INTERVIEW PROTOCOL

1. How is teaching going so far?
    a. Who or what are your main sources of support?
    b. *For each source of support*, what would you say have been your biggest takeaways?
    c. What feedback have you gotten so far about your teaching?
    d. What, if anything, have you done with that feedback?

2. What are some current goals that you have?

3. What are your biggest challenges?

4. When students don't learn as expected, what do you typically find to be the reason?

5. Do you feel like you need to adjust your instruction in any way for particular groups of students?

6. Have you noticed any inequities?

7. Are there any students or groups of students you're particularly worried about?

8. Next, I'd like to ask you some questions about your view of high-quality mathematics instruction. If you were to observe another mathematics teacher teach one or more lessons, what would you look for to decide whether the instruction was of high quality?
    a. Why do you think it is important to use/do _____ in a math classroom?
    b. Is there anything else you would look for? *If so*, why?

9. What are some of the things that you would expect to find the teacher actually doing in the classroom if the instruction were of high quality?

10. What kinds of problems or mathematical tasks would you expect to see the students working on if the instruction were of high quality?
    a. Can you please describe a _____ task that you would consider to be of high-quality?

11. Can you please describe what classroom discussion would look and sound like if instruction were of high quality?
    a. Would you expect to see the entire class participating in a single discussion, or would students be talking primarily in small groups?

12. What would you look for to determine whether instruction is equitable?

13. How, if at all, do your views of high-quality mathematics instruction differ from the ideas and approaches promoted in your teacher education program?

14. How, if at all, do your views of high-quality mathematics instruction differ from the ideas and approaches promoted in your school?

15. Since beginning teaching here, would you say that your views of high-quality mathematics instruction have changed at all?
    a. What do you think brought on those changes?

16. Now that you've spent some time teaching, do you feel that your teacher education program did a good job of preparing you to teach?
    a. Do you have ideas about other kinds of supports and/or opportunities that might have been helpful?

APPENDIX D: FINAL FORMAL INTERVIEW PROTOCOL

1. With this first set of questions, I'd like to ask you about your view of high-quality mathematics instruction. If you were to observe another mathematics teacher teach one or more lessons, what would you look for to decide whether the instruction was of high quality?
    a. Why do you think it is important to use/do _____ in a math classroom?
    b. Is there anything else you would look for? If so, why?

2. What are some of the things that you would expect to find the teacher actually doing in the classroom if the instruction were of high quality?

3. What kinds of problems or mathematical tasks would you expect to see the students working on if the instruction were of high quality?
    a. Can you please describe a _____ task that you would consider to be of high-quality?

4. Can you please describe what classroom discussion would look and sound like if instruction were of high quality?
    a. Would you expect to see the entire class participating in a single discussion, or would students be talking primarily in small groups?

5. What would you look for to determine whether instruction is equitable?

6. Is there anything that you just described that you yourself are not quite doing yet? *If so*, what do you think makes enacting those practices especially difficult?
    a. *If not addressed*, some of the practices you named are what some people describe as ambitious teaching practices. Do you think those practices are difficult to enact? *If so*, what do you see as reasons for that?

7. Have you always had this view about ideal mathematics teaching?
    a. *If not,* can you please describe events or situations that inspired changes in this view, and in what ways did these events or situations prompt changes in your image of ideal mathematics teaching?

8. What can you tell me about the view of good mathematics teaching that was promoted in your undergraduate courses? Also, what was/is your opinion of this view?

9. What can you tell me about the view of good mathematics teaching that is promoted here in your school (e.g., by your PLT, your principal, etc.)? In what ways is that view promoted (e.g., can you give me an example of something you've heard or seen?) Also, what are your opinions of those views?

10. Next, I'd like to ask you some questions about your views of mathematics education and education more broadly. First, what do you think it means to know and do mathematics?

11. How do you think people learn mathematics?

12. Do you think it's important that we teach mathematics in high school? *If so*, why?

13. Can you briefly describe why you want(ed) to become a teacher? Was that always the reason?

14. What do you consider to be your responsibilities as an educator?

15. What do you think it is that schools should accomplish?

16. Next I'd like to ask you some questions about your teaching experiences and experiences in your school this year. Looking back on this year, how would you say it went?

17. What were some of your biggest challenges?

18. When students didn't learn as expected, what did you typically find to be the reasons?

19. This year, did you notice anything that you perceived to be inequitable?
    a. When you think about your students' futures, does anything concern you?

20. You named ____ as goals you had this year. Do you feel like you've made progress in meeting those goals? *If so*, what do you think supported you in doing so? *If not* (as much as you'd have liked), what do you perceive to be obstacles that you faced?

21. In previous interviews, you consistently named ____________________________ as something that's important to you. Can you say a little bit about why that's important for your teaching?

22. How would you describe your instructional approach?
    a. Would you say that your instructional approach changed at all over the year? *If so*, in what ways?
    b. To what do you attribute those changes?

23. What kind of teacher would you say you are?
    a. Is that different from the kind of teacher you hope to become? *If so*, how?

24. Who or what acted as other sources of support for you this year?
    a. *For each source of support,* what would you say have been your biggest takeaways?
    b. What would you say ____ expects you to do in order to be a good math teacher?
    c. Do you agree with them? *If not*, in what ways do you disagree, and why?

25. *If not addressed*: In previous interviews, you described working with your PLT. From your understanding, what is the main purpose of PLTs?

a. Do you find the supports that the PLT provides to be helpful? *If so*, in what ways?
b. Do you have ideas about the ways in which the PLT could function in order to be more helpful for either you or your students?

26. Now that you've taught for some time, would you say that your teacher education program did a good job of preparing you to teach?
    a. Do you have ideas about what other supports or experiences might have been helpful (and why)?

27. Now that you've been a teacher for a little while, why do you want to keep being a teacher?

28. Can you say a little bit about how you think students feel about going to school here? Why do you think they feel that way, and what is your opinion of it?

APPENDIX E: DEBRIEFING INTERVIEW PROTOCOL

1. How do you feel about your lesson today?

2. What were your goals for this lesson?

3. Would you mind telling me about the lessons that came before this lesson and the lessons that will follow it?

4. In regards to your lesson, what did you try that seemed successful?
    a. Why would you call it successful?
    b. Why were you excited to see _____ happen?
    c. Is there something that you would have liked to have seen or done that didn't happen?

5. What did you hear students talking about today?

6. I noticed that _____. Can you tell me a bit about that?
    a. Why did you decide to _____?
    b. How do you think it went?

7. Do you have ideas about what you'd like to work on in the future?
    a. What kinds of supports do you think you might need in order to make that improvement?
    b. Last time I spoke with you, you said you were working on _____. How is that going?

8. At this point, how do you feel about your job here?
    a. Do you have any frustrations or feel like you are facing any challenges?
    b. Is there anything that you learned from your teacher education coursework that you've tried to apply in your teaching? *If so*, how did it go?
    c. Is there anything you've abandoned? *If so*, why?
    d. What or who would you say are your main sources of support?

9. At this point, who do you feel is the person overseeing your instruction?

10. Can you describe what ______ expects you to do to be an effective math teacher in your school?

11. Would you say that ______ expects you to teach math in a certain way?

12. How would you say that others' views of high-quality mathematics instruction compare to your own?

APPENDIX F: TABLES OF CODES FOR TEACHERS' CRITICAL PEDAGOGICAL DICOURSES

Table 7
Codes for Joe's Critical Pedagogical Discourse

Cycle One: Descriptive and "In Vivo" Codes	Cycle Two: Pattern Codes
Anticipating	Conceptual understanding as a goal for learning
Behavior	Learning requires thinking, reasoning, and struggle
"Desmos"	Learning requires opportunities for discussion
Direct instruction	Building relationships with students
Discussion	
Engagement	
"GeoGebra"	
"I do, We do, You do"	
Mathematical tasks	
Monitoring	
Relationships	
Small-group work	
"Social justice"	
"Students exploring, thinking, struggling"	
Teaching for conceptual understanding	
Whole-group discussion	

Table 8
Codes for Emma's Critical Pedagogical Discourse

Cycle One: Descriptive and "In Vivo" Codes	Cycle Two: Pattern Codes
Behavior	Student engagement as a goal for teaching
Direct instruction	Shifting authority from teacher to students
Discussion	Learning requires opportunities for discussion
Engagement	
"Indirect instruction"	
Mathematical tasks	
Monitoring	
"Not telling"	
Questioning	
Relationships	
Small-group work	
"Students thinking"	

Table 9
Codes for Sara's Critical Pedagogical Discourse

Cycle One: Descriptive and "In Vivo" Codes	Cycle Two: Pattern Codes
Behavior	Student engagement as a goal for teaching
"Collaborative learning" activities	Teachers' explanations
Discussion	Preparing students for the future
Engagement	Building relationships with students
"Life skills"	Conceptual understanding as a goal for learning
Monitoring	
Questioning	
Relationships	
Small-group work	
Teacher explanations	
Teaching for conceptual understanding	

Table 10
Codes for Bri's Critical Pedagogical Discourse

Cycle One: Descriptive and "In Vivo" Codes	Cycle Two: Pattern Codes
Anticipating	Conceptual understanding as a goal for learning
"Giving notes"	Learning requires "inquiry-based" teaching
"Inquiry-based teaching"	Students experience success and feel smart
"Launching" mathematical tasks	
Monitoring	
Relationships	
Small-group work	
"Showcasing students' ideas"	
"Students experiencing success"	
"Students discovering things"	
"Students seeing themselves as smart"	
Teaching for conceptual understanding	
Whole-group discussion	

APPENDIX G: TABLES OF CODES FOR TEACHERS' SELF-REPORTED CONTEXTUAL DISCOURSES

Table 11
Codes for Joe's Contextual Discourses

Sources	Content
Colleagues	Approval
Community	Autonomy
Principal	Conservative views
Resources	Engagement
Students	"Having six preps"
Superintendent	Preferences for conventional practices
	"Responsibility to teach state standards"

Table 12
Codes for Emma's Contextual Discourses

Sources	Content
Department chair	Autonomy
Disciplinary policies and practices	Behavior
PLT	"Covering" content
Principal	Engagement
Students	Giving notes as a norm
	Staying "on pace"

Table 13
Codes for Sara's Contextual Discourses

Sources	Content
Disciplinary policies and practices	Autonomy
PLT	Behavior
Principal	Engagement

Table 14
Codes for Bri's Contextual Discourses

Sources	Content
Disciplinary policies and practices	Behavior
PLT	"Covering" content
Students	Emphasis on practice
	Engagement
	Giving notes as a norm
	Feasibility of "inquiry-based" teaching
	Nature of "inquiry-based" teaching
	Staying "on pace"

Student labels and capabilities

APPENDIX H: TABLES OF CODES FOR TEACHERS' INSTRUCTIONAL PRACTICE

Table 15
Codes for Joe's Instructional Practice

Cycle One	Cycle Two
Desmos activity	Calculational discourse
Individual work time	Conceptual discourse
Mathematical task	Mathematical task- Level 1
Monitoring	Mathematical task- Level 2
Note-taking session	Mathematical task- Level 3
Pre-lesson conversation	Mathematical task- Level 4
Questioning	Sharing of multiple solutions/answers
Teacher demonstration	Teacher-student pattern of talk
Warm-up	
Whole-class discussion	

Table 16
Codes for Emma's Instructional Practice

Cycle One	Cycle Two
Investigation	Calculational discourse
Mathematical task	Conceptual discourse
Monitoring	Mathematical task- Level 1
Note-taking session	Mathematical task- Level 2
Small-group work	Mathematical task- Level 3
Questioning	Mathematical task- Level 4
Quiz	Teacher-student pattern of talk
Teacher demonstration	
Warm-up	
Whole-class discussion	

Table 17
Codes for Sara's Instructional Practice

Cycle One	Cycle Two
Collaborative learning activity	Calculational discourse
Mathematical task	Conceptual discourse
Monitoring	Mathematical task- Level 1
Note-taking session	Mathematical task- Level 2
Questioning	Mathematical task- Level 3
Quizzes	Mathematical task- Level 4
Small-group work	Students share strategies
Teacher demonstration	Teacher-student pattern of talk

Warm-up
Whole-class discussion

Table 18
Codes for Bri's Instructional Practice

Cycle One	**Cycle Two**
Launch	Calculational discourse
Mathematical task	Conceptual discourse
Monitoring	Mathematical task- Level 1
Note-taking session	Mathematical task- Level 2
Questioning	Mathematical task- Level 3
Quizzes	Mathematical task- Level 4
Small-group work	Students share strategies
Teacher demonstration	Teacher-student pattern of talk
Warm-up	
Whole-class discussion	

VITA

Cara Haines grew up in Greensburg, Pennsylvania, a suburb of the city of Pittsburgh. After graduating high school in 2006, she attended the University of Pittsburgh, where she earned a Bachelor of Science in Mathematics and a Master of Arts in Teaching Secondary Mathematics.

From 2012 to 2014, Cara worked as a secondary mathematics teacher at a public high school in Wilmington, North Carolina. In the fall of 2014, Cara returned to the University of Pittsburgh to begin a Ph.D. program and later transferred to the University of Missouri to continue working with her advisor. In the summer of 2020, Cara graduated from the University of Missouri, earning a Ph.D. in Learning, Teaching, and Curriculum with an emphasis in Mathematics Education.

Cara currently lives in Nashville, Tennessee, where she works as a postdoctoral researcher at Vanderbilt University.