EXAMINING SECONDARY TEACHERS'

PRACTICAL RATIONALITY OF MATHEMATICAL MODELING

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The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

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ABSTRACT

Research on mathematical modeling is growing rapidly in the field of mathematics education, and there are numerous benefits of engaging students in modeling activities. However, mathematical modeling is still marginalized in mathematics instruction. In order to understand the challenges and obstacles secondary teachers face to incorporate modeling into their classrooms, this study drew upon the theoretical perspective of practical rationality. It used a breaching experiment survey of 176 secondary teachers in Missouri and follow-up interviews with six purposefully selected survey respondents to examine secondary teachers' norms and professional obligations related to mathematical modeling. After data collection, I used descriptive statistical analyses to examine the norms and applied the professional obligation framework to analyze teacher obligations.

My findings confirmed four of the six hypothesized norms: Teachers tend to precisely identify which factors should be included in students' solutions, expect students to find a symbolic representation as their model, expect students to primarily work on politically neutral tasks, and are open to students doing model revisions. However, this study did not have robust enough evidence to confirm or disconfirm whether teachers tend to give students unambiguous directions about mathematical operations or components. Nor could it uncover whether students are expected to primarily engage in mathematical thinking rather than nonmathematical thinking during the modeling process.

The findings also revealed that teachers' preferred actions among the scenarios presented were influenced by their perceived professional obligations, including

commitments to individual students, the classroom's interpersonal dynamics, the practice of the mathematics discipline, and the institutions they work for. More specifically, disciplinary obligations featured most often in the survey responses of those teachers who complied with canonical actions. In contrast, those who selected noncanonical options (such as opening up the beginning phases of the modeling process) most often cited their obligations to individual student thinking to justify their survey responses.

The findings suggested that the mathematics education community needs to increase awareness about complex and often conflicting teacher obligations related to mathematical modeling. To support teachers in enacting modeling, the field might consider qualified professional development, viable modeling tasks and tools, and a flexible schedule as well as opportunities to build shared understandings of disciplinary practices and obligations to students' individual needs.

CHAPTER 1

Introduction

Mathematical modeling and its learning and teaching have become prominent topics in the past decade worldwide, as the realization of the importance of mathematics in science, technology, engineering, and everyday life has grown. Mathematical modeling is a process of "choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (Common Core State Standards Initiative [CCSSI], 2010, p. 72). This process comprises multiple stages of translation between real-world situations and mathematics in both directions, involving problem identification, assumption making, model formulation, mathematization, interpretation, validation, and revision (CCSSI, 2010; Garfunkel & Montgomery, 2016; Lesh & Lehrer, 2003). In these ways, mathematical modeling is a powerful way to use mathematics to understand real-world phenomena.

Various policy leaders and mathematics education scholars have advocated for teachers to implement modeling tasks in their classrooms. In the United States, for example, *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) addressed the importance of learning of modeling and suggested that "students at all levels should have opportunities to model a wide variety of phenomena mathematically in ways that are appropriate to their level" (p. 39). In 2009, *Focus in High School Mathematics: Reasoning and Sense Making*, the first of a series of documents centered on high school recommendations published by NCTM, regarded mathematical modeling as a "reasoning habit." The council called for modeling to be a fairly regular occurrence in school mathematics so that students could come to view real-world phenomena from a mathematical standpoint. Furthermore, modeling has been emphasized more than ever in the United States since the release of the *Common Core State Standards for Mathematics* (CCSSM) in 2010, which incorporated modeling as a “practice standard” for Grades K–12 and a “content standard” for high school mathematics curricula (CCSSI, 2010).

Research studies have shown a variety of benefits when students engage in mathematical modeling. For example, modeling helps students develop mathematical competency by building connections between real-world experiences and mathematical knowledge (e.g., Steen et al., 2007). Students can deepen their understanding of mathematical content and develop problem-solving abilities when working collaboratively on modeling tasks (e.g., Lesh & Doerr, 2003; Lesh & Lehrer, 2003). Modeling also affords a range of productive experiences as students can work at their own pace in modeling activities, and these activities allow multiple representations and different strategies (e.g., English, 2006). Furthermore, students can develop productive dispositions toward mathematics through seeing mathematics as useful and relatable (e.g., Lesh & Yoon, 2007) and develop a sense of mathematical agency and authority (e.g., Schoenfeld, 2013).

In spite of all the arguments in favor of modeling, the implementation of modeling tasks is still relatively rare in mathematics teaching practice in most countries, including the United States (Blum, 2015). This dearth aligns with my personal experience, where the secondary mathematics teachers I have interacted with have viewed mathematical modeling as potentially useful and relatable but challenging to teach. They seldom carried out modeling tasks in their classrooms. What lies at the root of these challenges for the implementation of mathematical modeling? What are the external factors at play? In what ways do teachers' day-to-day routines or views of their professional responsibilities align or not with modeling? And ultimately, how can we support teachers so that we can meet the promise of mathematical modeling in classroom practice?

Research has discussed that mathematical modeling presents teachers with a new way of teaching mathematics and demands changes in classroom practices (e.g., Cheng, 2013; Doerr, 2007; Kaiser, 2017). Early research approached this problem from the perspective of personal characteristics (i.e., teacher knowledge, teacher beliefs) and assumed that the obstacles and challenges directly resulted from teachers' lack of knowledge or productive beliefs (e.g., de Oliveira & Barbosa, 2010; Kaiser & Maaß, 2007). Specifically, de Oliveira and Barbosa (2010) took teacher knowledge into account to explain a secondary teacher's challenges of enacting modeling tasks, suggesting that the teacher's lack of knowledge about the mathematical connections to students' responses to the tasks prevented the teacher from moving forward. Kaiser and Maaß (2007) regarded teacher beliefs about mathematics as a fundamental reason for the low realization of modeling in mathematics classrooms. Teachers' beliefs about mathematics—such as believing that mathematics is "a collection of rules and formulas" or "an exact, formal and logical science"—restricted modeling in the classrooms (p. 107).

The above studies shed light on some of the challenges and obstacles of enacting modeling tasks. However, other researchers (e.g., Herbst & Chazan, 2003) have argued that the practice of teaching is not as simple as an expression of individual teacher knowledge, beliefs, or goals. One cannot ignore the forces from external circumstances, administrative requirements, and others that can inhibit or facilitate teaching practice. Teaching is influenced by the social and organizational environment or context as much as the individual characteristics of those who enact it (Herbst & Chazan, 2011, 2012). Mathematics teachers work under complex and sometimes competing obligations and norms. Their instruction reflects their “adaptations to conditions and constraints” in the context (Herbst & Chazan, 2011, p. 407).

Drawing on these ideas, some researchers pointed out that teachers’ practices are partly driven by their practical rationality that falls outside of personal characteristics (e.g., Herbst & Chazan, 2011; Nachlieli et al., 2009; Webel & Platt, 2015). *Practical rationality* is “the system that helps practitioners notice and justify (or else denounce) departures between actual actions and (implicit) norms” (Herbst & Chazan, 2011, p. 214). Practical rationality constitutes two central concepts: *norms* and *professional obligations*. *Norms* regulate what is normative in a situation (Bourdieu & Wacquant, 1992; Nachlieli et al., 2009). They are tacit expectations of the role professionals play in the professional situations they participate in. The influence of norms on teachers’ actions is often implicit; teachers act in a way that makes sense to them (Nachlieli et al., 2009). *Professional obligations*, on the other hand, are more explicit demands that characterize teachers’ professional position in relation to the expectations of its stakeholders (Herbst & Chazan, 2015). More specifically, teachers have an obligation to the discipline of mathematics, individual students, the collective group of students in their classroom, and the institution where they work (Herbst & Chazan, 2011, 2017). Teachers use their perceived obligations to determine what is possible in different instructional situations and give meaning and value to different instructional actions.

Understanding practical rationality can help explain why some recommendations for teaching change are perceived as viable by teachers, whereas others are seen as outrageous or daunting (e.g., Nachlieli et al., 2009). It can also explain why, under the same instructional recommendation, some teachers create authentic opportunities for mathematics learning for their students, whereas others implement the recommendation in ways that contradict the rationale behind it (e.g., Sobolewski-McMahon, 2017; Webel & Platt, 2015). In the case of mathematical modeling, for instance, a teacher's practical rationality might help explain why one teacher implements an open-ended modeling task while another breaks the task down into specific, teacher-directed steps. The first teacher may perceive obligations to provide rich sense-making opportunities while the other perceives obligations to use time efficiently and assure that students are not confused or frustrated.

In general, the implementation of modeling tasks is influenced by teachers' perceived norms and professional obligations, which should be taken seriously. As described before, a majority of studies have already examined mathematical modeling from the angle of teacher characteristics. Practical rationality, however, provides a new perspective for mathematics education researchers to understand the norms and professional obligations behind mathematical modeling and how teachers may be likely to enact modeling. Research on practical rationality is needed to expand our understanding of mathematical modeling and give a fuller answer for the low realization of modeling in classrooms. Hence, it is worth investigating teachers' perceived norms and examining the obligations they use to justify their teacher actions.

Research Questions

This dissertation aimed to understand secondary teachers' practical rationality with regard to mathematical modeling and classroom situations that are potentially related to mathematical modeling. I examined the following research questions:

1. What norms are perceived by secondary teachers in relation to mathematical modeling?
2. What professional obligations are perceived by secondary teachers in relation to mathematical modeling?

These two questions addressed the key elements of the theoretical framework of practical rationality, namely, the norms (the first research question, or RQ1) and professional obligations (the second research question, or RQ2) that secondary mathematics teachers perceive (or intend to enact) in relation to mathematical modeling.

Significance of the Study

Mathematical modeling has been regarded as an important practice that students should engage in because it can benefit their understanding of mathematics. However, for the benefit to be realized, the teacher must see the inclusion of mathematical modeling as probable and desirable. Teachers' actions might be constrained by the norms of instructional situations and the obligations from various stakeholders. My study reflected the importance of teachers' voices and has contributed to understanding teachers' perceived norms and professional obligations in relation to mathematical modeling.

This study's findings showed that mathematics teachers negotiated various types of obligations, among which disciplinary obligations hindered their efforts to open up modeling tasks and individual obligations often drove them to create more learning opportunities for individual students' success. These results can help teacher educators, curriculum developers, and policymakers recognize the complexity of enacting modeling in mathematics classrooms. It can further offer insights for education researchers and school leaders to find productive ways to work with teachers to enact modeling.

Dissertation Outline

In the following chapters, the study and its results and conclusions are presented. Chapter 2 lays out the background literature about mathematical modeling and the key findings of the related studies. In addition, Chapter 2 provides a detailed description of the study's theoretical framework—practical rationality—and evaluates other scholars' work on practical rationality in relation to the research questions being investigated. Chapter 3 describes the setting, participants, instrument design, and processes of data collection and analysis. Chapter 4 presents the findings of this study. Finally, Chapter 5 includes a summary of the findings as well as a discussion of the study's limitations and implications.

CHAPTER 2

Literature Review

This chapter is a review of the literature and is divided into two sections. The first section focuses on mathematical modeling. The review begins by defining mathematical modeling and distinguishing it from other similar terms with the word "modeling." Then, I compare model-eliciting activities to word problems. Following this is a discussion on how mathematical modeling relates to equity and social justice. Moreover, I summarize studies on instructional practices related to modeling and conclude with literature aimed at investigating the challenges of enacting modeling tasks.

The second section of this chapter focuses on theoretical perspectives, particularly the theoretical framework that guided this study. The section begins with a review of the literature concerning practical rationality. It is followed by a discussion of constructs essential to practical rationality, such as norms and professional obligations. I close by describing how the extant research concerning practical rationality informed my study's design.

Mathematical Modeling

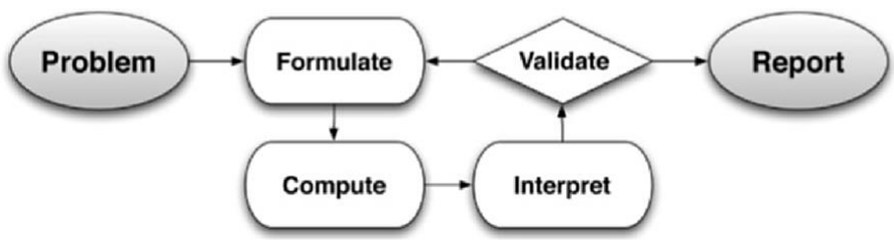
Mathematical modeling is often conceptualized as a multifaceted process of analyzing empirical situations with appropriate mathematics or statistics (e.g., Bliss & Libertini, 2016; Blum & Leiß, 2005; CCSSM, 2010). CCSSM identified the different stages in mathematical modeling (see Figure 1):

1. identifying variables in the situation and selecting those that represent essential features,

2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between variables,
3. analyzing and performing operations on these relationships to draw conclusions,
4. interpreting the results of the mathematics in terms of the original situation,
5. validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable; and
6. reporting on the conclusions and the reasoning behind them. (pp. 72–73)

Figure 1

The Mathematical Modeling Cycle from CCSSM

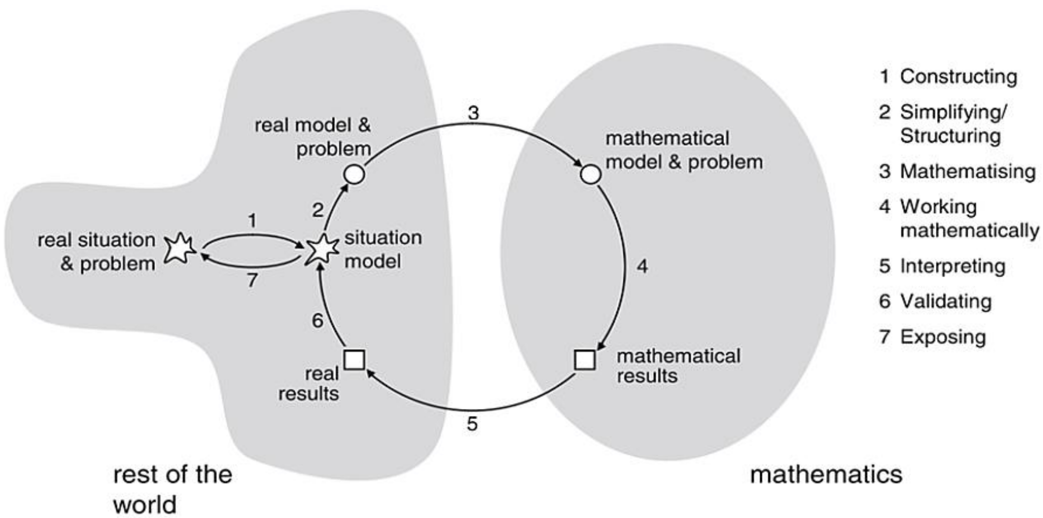


Note. From *Common Core State Standards for Mathematics*, by CCSSM, 2010, p. 72 (http://www.corestandards.org/Math/). Copyright 2010 by CCSSM.

While CCSSM highlighted the different stages of modeling, the conceptualization of mathematical modeling from Blum and Leiß (2005) emphasized the two important but separate realms of mathematical and nonmathematical entities (see Figure 2). First, students identify and understand the problem situation and develop a mental representation of the situation, called a situation model (Stage 1). Then, they structure and simplify the situation and come up with a real model that brings certain variables into play (Stage 2). In Stage 3, students transform the real model into a mathematical model and continue to work on it, which yields the mathematical results (Stage 4). These mathematical results are interpreted in the real-world context as real results (Stage 5). Students present their models if the results are validated (Stages 6 & 7); otherwise, they revise their models and go through the cycle a second time.

Figure 2

The Mathematical Modeling Cycle from Blum and Leiß

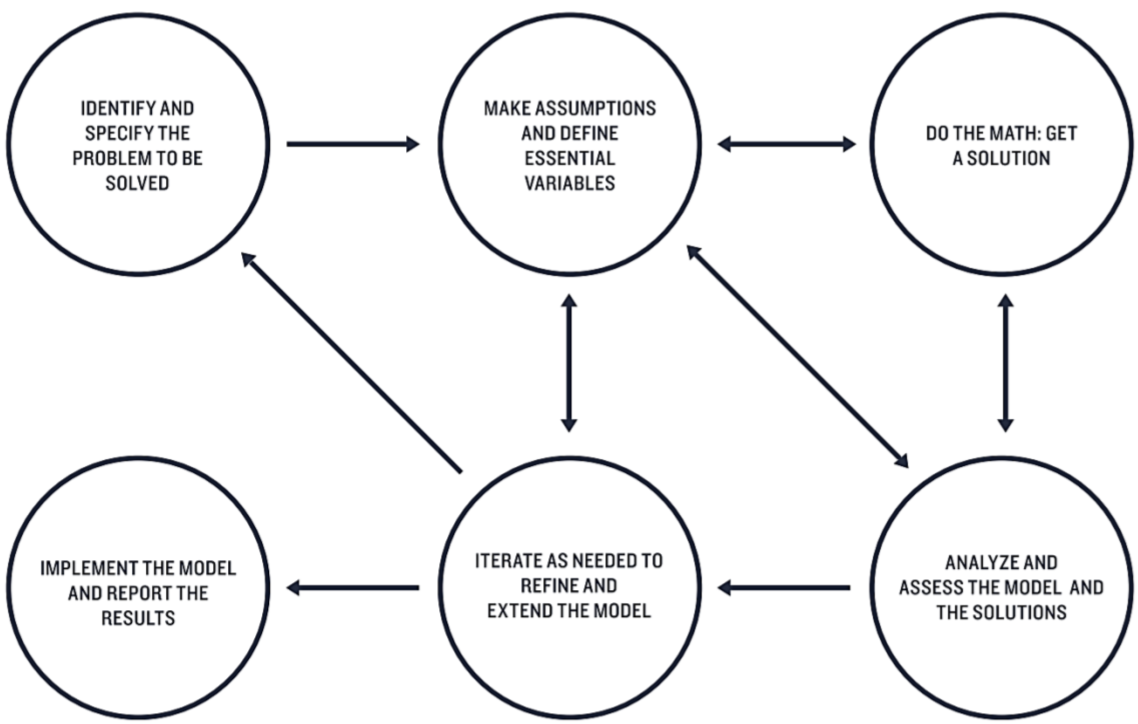


Note. From "'Filling up'–The problem of independence-preserving teacher interventions in lessons with demanding modelling tasks," by W. Blum and D. Leiß, 2005, in M. Bosch (Ed.), *Proceedings of the 5th Congress European Research Mathematics Education*, p. 1626. Copyright 2005 by the Universitat Ramon Llull.

In contrast to the conceptualization of modeling from CCSSM (2010) and Blum and Leiß (2005), the conceptualization from Bliss and Libertini (2016) emphasized that students could move back and forth between mathematics processes at any point and there is not a fixed route during the process. They pointed out the key components of the modeling process: identifying the problem, making assumptions and defining variables, doing the mathematics, analyzing and assessing the solution, iterating, implementing, and reporting the model (see Figure 3).

Figure 3

The Mathematical Modeling Process from Bliss and Libertini



Note. From "What is Mathematical Modeling?" by K. Bliss and J. Libertini, 2016, in S. Garfunkel & M. Montgomery (Eds.), *Guidelines for Assessment & Instruction in Mathematical Modeling Education (GAIMME)*, p. 13. Copyright 2016 by the Consortium for Mathematics and Its Applications (COMAP) and Society for Industrial and Applied Mathematics (SIAM).

Other Interpretations of Modeling

Since there are various uses of the terms model and modeling, I use this section to distinguish between the various meanings. *Mathematical modeling* is different from *modeling mathematics* (Cirillo et al., 2016). Mathematical modeling is a specific mathematical process to understand and analyze empirical situations, whereas modeling mathematics is a means of representing mathematical ideas. For example, the CCSSM mentions the process of *modeling mathematics* as follows:

> Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare strategies to solve arithmetic problems with these operations. (CCSSI, 2010, p. 13)

The models in the above excerpt are *concrete models*, which are typically physical objects that students can manipulate to understand mathematical concepts. Similar to *concrete models*, there are *visual models* or *area models* that help students develop reasoning for fractions or other abstract concepts. Adding a group of five items with four items could be represented by combining five strips with four strips. This strategy is *direct modeling* (Carpenter et al., 2015). In all these cases of modeling mathematics, something is used as a representation of a mathematical idea.

Another term is *teacher modeling,* through which teachers show students how to solve a mathematical problem by explaining each step for students to follow and imitate (Haston, 2007). During teacher modeling, all the actions and thinking are completed by teachers instead of students, which is different from mathematical modeling, where students actively engage in the process. Notice that these other interpretations of modeling are *not* what my study focuses on.

In this study, I conceive of mathematical modeling as a process of “choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions” based on the description from CCSSM (CCSSI, 2010, p. 72). In addition, I consider the transition of the nonmathematical and mathematical entities (Blum & Leiß, 2005) as fundamental in my study. Modeling activities, in my opinion, should engage students in connecting the nonmathematical and mathematical entities. If students work on procedural problems in mathematics (e.g., perform operations) without attending to the nonmathematical context, one cannot say that they are modeling since they are only practicing learned procedures from their textbooks. The reverse is also true. Students who merely engage in the nonmathematical part of the problem (e.g., chatting with peers on the out-of-school issue related to the situation of the problem while ignoring the mathematical part of the problem) are not participating in modeling because such an activity does not fulfill the goal of using mathematics or statistics to analyze situations.

In terms of the stages within the modeling cycle, I conceive of mathematical modeling as an iterative process involving various stages, such as problem identification, assumption making, model formulation, mathematization, interpretation, validation, and revision. I predominantly use the stages laid out by Blum and Leiß (2005), as depicted above in Figure 2. Mathematical modeling activities in school, however, do not always proceed through a full modeling cycle. It is understandable that mathematics teachers might focus on parts of the modeling process based on their learning goals for specific lessons.

Model-Eliciting Activities and Word Problems

A model-eliciting activity (MEA) is one way of implementation of mathematical modeling. MEAs are activities in which students go beyond short answers and, instead, are expected to express their ways of thinking and invent and test models (Lesh et al., 2000). Since this study focuses on mathematical modeling, this section describes the unique characteristics of MEAs and, in particular, clarifies the distinctions between MEAs and word problems.

The features of MEAs include (1) meaningful problem situations, (2) a basis for subsequent model exploration and application, (3) multiple interpretations and approaches, (4) mathematical communication, (5) documentation of end products, and (6) self-evaluation (Doerr & English, 2006). Somewhat related to MEAs but different in important ways, word problems or real-world problems have been a part of the mathematics curriculum for a long time. Drawing on some past literature (e.g., Pollak, 2012; Tran & Dougherty, 2014), I clarify the distinctions that separate MEAs from word problems in the next few paragraphs.

The problem context is relevant. Problem solving can be irrelevant to the real world. Pollak (2012) pointed out that even if a problem begins with a real-world context, problem solving that is separate from modeling "usually begins with the idealized real-world situation in mathematical terms, and ends with a mathematical result" (p. viii). Word problems in textbooks typically "'dress up' a purely mathematical problem in words referring to a segment of the real world" (Niss et al., 2007, p. 11). On the contrary, mathematical modeling usually "begins in the 'unedited' real world, requires problem formulating before problem solving, and once the problem is solved, moves back into the real world where the results are considered in their original context" (Pollack, 2012, p. viii). The context of modeling is essential since the context impacts the interpretation of the problem and the identification of the variables and mathematical relationships.

The process is the product. Unlike conventional word problems, reasoning and thinking processes are regarded as the product of modeling. They are not simply "postscripts that students give after 'the answer' has been produced" and "ARE the most important components of the responses that are needed" (capitalized in the original text, Lesh & Doerr, 2003, p. 3). On the other hand, most word problems contain precisely the information needed to solve the problem, afford particular solution strategies, and yield only one correct solution (Tran & Dougherty, 2014). They are often tasks requiring low cognitive demand (Smith & Stein, 2011) and do not encourage self-guided sense making. MEAs, on the other hand, are often tasks with high cognitive demand, which aim to engage students in high-level mathematical reasoning (Blum, 2015).

The interpretations and revisions are ongoing and necessary. An important characteristic of mathematical modeling is its evaluation and revision process. Unlike traditional word problems—that emphasize a search for correct procedures for a particular problem situation with the elimination of wrong procedures—students in modeling begin with the somewhat "wrong" ideas as they engage in modeling, which is considered as “a part of a continuum” toward improving and revising models, rather than part of a “failing solution path” (Zawojewski, 2010, pp. 239–240). As students go through the iterations of model validation and revision, they will continue to improve their models and develop a deeper understanding of the mathematical ideas involved. However, when people speak of word problems or problem solving, they usually refer to mathematization, which is part of modeling, but they often omit the evaluation and revision processes, which are also important components of modeling.

In sum, MEAs are distinguished from word problems by the relevance of the context, the process as the product, and the ongoing interpretations and revisions. Admittedly, the above distinctions are not the unique features of MEAs, and the distinctions are not limited to these three aspects. These distinctions between MEAs and word problems partly constitute the rationale behind the design of this study’s two tasks and the breaching experiment, which will be detailed in the Method chapter. Note that the two tasks in this study are neither MEAs nor word problems. They are merely an initial prompt for open-ended tasks and do not include a full plan for learning experiences.

Genres of Mathematical Modeling Within Educational Settings

The previous two sections distinguished between mathematical modeling and modeling mathematics and between MEAs and word problems. This section looks more closely at mathematical modeling by defining varying approaches.

There are two generic approaches to mathematical modeling within education (Julie, 2002). These two approaches, according to Julie (2002), are *modeling as content* and *modeling as vehicle. Modeling as content* refers to constructing mathematical models to better understand natural and social phenomena, while *modeling as vehicle* refers to modeling as the context for learning mathematical concepts and procedures.

Each approach has particular goals. The approach of *modeling as content* gives primacy to the modeling activities and has a goal of learning and applying modeling to solve real-world problems. Since modeling itself is the objective of this approach, the cognitive activities at each stage in the modeling cycle are considered foundational and need to be addressed. The studies by Doerr (2007) and Anhalt and Cortez (2016) employed this approach to introduce prospective teachers to some basic modeling ideas and techniques; they examined the evolution of prospective teachers' understanding of modeling.

In the *modeling as vehicle* approach, modeling serves other curricular needs or educational purposes. With this approach, educators use modeling activities to introduce new knowledge or apply learned knowledge (e.g., de Beer et al., 2015; Zbiek & Conner, 2006). The study by de Beer and colleagues (2015) investigated how to introduce new concepts (i.e., calculus-like topics) via modeling to primary school students, while Zbiek and Conner (2006) focused on deepening prospective teachers' understanding of centroid by engaging them in a modeling task.

These two genres describe how modeling has been employed within educational settings. They should not be viewed as necessarily antagonistic, although their respective goals are distinct. Teachers' priorities will be different depending on which approach they are pursuing in the classroom at a given time. In the Discussion and Conclusion chapter, I use the different modeling genres to interpret this study's findings.

Mathematical Modeling, Equity, and Social Justice

Some of the key attributes of mathematical modeling are closely related to efforts seeking to use mathematics for equity and social justice. Modeling tasks provide numerous opportunities to connect school mathematics and students' out-of-school experiences, which have been used to introduce culturally relevant contexts in mathematical activities (Aguirre et al., 2013). Mathematical modeling activities also have the potential to connect and build on students' funds of knowledge while honoring the cultural resources students bring to school (Aguirre et al., 2013; Gutierrez & Irving, 2012). Furthermore, studies have shown that appropriate implementations of modeling tasks can increase students' learning opportunities by appreciating their diverse approaches to a given situation and allowing each student to work at their own level (e.g., Wernet, Lawrence, & Gilbertson, 2016). Using modeling in this way aligns with NCTM's dedication to equity and access (NCTM, 2014).

Connections between modeling and social perspectives are especially evident in cases where the goal of modeling is to promote "a critical understanding of the surrounding world" (Kaiser & Sriraman, 2006, p. 304). Some studies (e.g., Barbosa, 2006; Gutstein, 2016) showed that teachers implemented the modeling-related tasks to develop students' social and political consciousness and motivate them to be actively involved in rectifying social inequities. In his study, Gutstein (2016) showed how he empowered African American and Latin@ students to use mathematics to understand social and racial issues relevant to their communities and prepare themselves to change the situation (Gutstein, 2016). Barbosa (2006) illustrated that a Brazilian teacher purposefully selected a piece of local news about a governmental decision to engage students in discussing the issue and thinking more critically about the role of mathematics in society.

Modeling offers opportunities to engage with ideas of social justice and equity, and some researchers have highlighted how modeling can help students think critically about real-world issues. Still, not all mathematics teachers are accustomed to mathematics problems that engage social justice issues or consider engaging students in culturally relevant tasks to be a learning goal. When it comes to modeling, there can be a discrepancy between the benefits of promoting students' critical thinking and the actual classroom situation. This study examines this potential discrepancy and attempts to understand the teachers' rationality underlying it.

Research on Enactment of Modeling Tasks

Although research on mathematical modeling has gained much attention, most related work has focused on three aspects. The first is the theorized modeling process (e.g., Borromeo-Ferri, 2006; Haines & Crouch, 2010; Zawojewski, 2010). Second, research has examined students' and pre- or in-service teachers' cognitive activities or learning involved in the modeling process (e.g., Anhalt & Cortez, 2016; Doerr & English, 2003, 2006; Zbiek & Conner, 2006). Third, research has focused on students' and teachers' perceptions of modeling (e.g., Gould, 2013).

Yet, some researchers have started to study the pedagogical dimension of modeling—for example, how teachers enact modeling in their classrooms (e.g., Fulton, 2017; Leiß, 2007). These existing studies have yielded somewhat conflicting findings. On the one hand, research has shown that the enactment of modeling is often teacher-centered and that teachers tend to scaffold a great deal of student activity (e.g., Leiß, 2007). Leiß (2007) compared two ways of teaching modeling: directive and operative-strategic. The directive approach is more teacher-centered, while the operative-strategic approach is more student-centered and emphasizes group work and strategically scaffolding instruction. Leiß found that strategic interventions were included marginally in German classrooms from the eighth to tenth grade. In fact, teachers often gave the students so much support that the students only had to find one step by themselves to overcome the difficulty. On the other hand, Fulton (2017) showed that four elementary teachers in the United States gave their students the authority to explore problems and make their own decisions. These teachers allowed students to introduce most of the mathematical ideas used to investigate the modeling tasks.

How can we understand these two varied types of teacher actions in relation to mathematical modeling? Are they both plausible within their own contexts? Both studies reported what happened in the classroom but did not explain what might lie beneath the teachers' actions. Looking beneath the actions to the teachers' perspectives and practical rationality may help researchers and educators understand these distinct findings. I argue that we should recount the practice of mathematical modeling with a primary focus on investigating teachers' practical rationality.

Challenges of Enacting Modeling Tasks

Scholars have elaborated on the challenges and obstacles of implementing modeling in their lessons. Some attempted to explain the challenges of enacting modeling by examining teachers' individual resources, such as knowledge (e.g., de Oliveira & Barbosa, 2010) and teacher beliefs (e.g., Kaiser & Maaß, 2007), while others related the obstacles to sociocultural influences (e.g., Blum & Niss, 1991; Niss, 1987). This section explores each approach in turn.

Approach 1: Teachers' individual resources. The individual resources can be divided into teacher qualifications (e.g., teacher knowledge) and teacher characteristics (e.g., teacher belief; Goe, 2007). de Oliveira and Barbosa (2010) took teacher knowledge into account when explaining a Brazilian secondary teacher's tensions while enacting modeling tasks. By analyzing the data from classroom observations and teacher interviews with a grounded theory method, they uncovered three types of tensions when the teacher tried to engage students in modeling. First, because students' development of modeling did not follow the progress that the teacher had planned, the teacher was uncertain of how to lead students and move forward. Second, when students responded with unanticipated ideas during the modeling process, the teacher was not sure how to respond. Third, the teacher was unsure of how to teach skills upon realizing that students did not have the prior mathematical skills he expected. de Oliveira and Barbosa attributed the above tensions to a lack of pedagogical knowledge.

Besides teacher knowledge, teacher beliefs are also used to explain the challenges of teaching modeling. Kaiser and Maaß (2007) regarded teacher beliefs about mathematics as an essential reason for the low realization of modeling in mainstream mathematics classrooms. Their teacher interviews found that different beliefs about mathematics influenced how teachers thought modeling activities should be implemented. Teachers with schematic mathematical beliefs (mathematics is "a collection of rules and formulas") restricted modeling to examples that enable easy mathematizations or lead directly to a formula. For teachers with formalistic beliefs (mathematics is “an exact, formal and logical science”), the context of the task played little to no role in modeling. Teachers with a process-oriented understanding of mathematics (mathematics is “a science which mainly consists of problem-solving processes”) reduced modeling to merely developing solutions (p. 107). However, it is unclear how Kaiser and Maaß made this connection between teachers’ beliefs about mathematics and teachers’ ways of enacting modeling since they did not provide any evidence related to teacher enactment or their enactment plan.

Approach 2: Sociocultural influences. Niss (1987) introduced three types of barriers to explain why, in practice, modeling was marginalized in mathematics education, and in my view, these barriers have remained to the present day. The first barrier is the *time-related barrier*. Modeling activities are time-consuming. They must find their place beside other mathematical activities because it is “rarely possible” to expand the total time allotted to mathematical studies (p. 502). Secondary teachers are not ready to make the change because increasing modeling activities might mean a “reduction of the purely mathematical syllabus,” and this reduction might be perceived as a “reduction in students’ access to insights into and experiences with the realm of mathematics” (p. 503). The teachers who hold these views would interpret an increase in modeling as a decrease in the quality of mathematics education without considering the compensating gains from modeling activities. The *student-related barrier* is the second type. While working on modeling tasks, students cannot be convinced of the power of mathematics to deal with real-world problems because they view the role of mathematics less relevant to “the core of the problems but only its outskirts” (p. 503). The third barrier is the *assessment-related barrier*. Nonmathematical phenomena and considerations are a necessary part of modeling, which makes the qualification of modeling difficult to assess with the traditional forms of evaluation that tend to focus solely on mathematical components. A satisfactory form of assessment has not yet been found or, at least, not implemented as the crucial assessment tool in many school contexts.

Building on research by Niss published in 1987, a few years later, Blum and Niss (1991) again reviewed the obstacles preventing modeling from playing an important role in mathematics classrooms. They considered additional obstacles, such as teachers' doubt about whether modeling belongs to mathematics instruction since some elements of modeling seem to distort the "clarity, aesthetical purity, context-free universality" of mathematics (pp. 53–54). Moreover, it seems easier for students to solve routine problems by following given procedures, whereas it is more demanding and less predictable for them to deal with modeling tasks. The demand required to enact modeling reduces the likelihood of teachers using modeling tasks. However, these scholars' explanations of the challenges did not build upon empirical studies. Both articles were theoretical pieces and did not present any data.

Summary

The research summarized so far in this chapter attempted to understand the challenges and obstacles of enacting modeling tasks through different approaches (e.g., individual knowledge, beliefs, sociocultural influences). However, without considering how environmental impacts (e.g., disciplinary, institutional) or teachers' own perspectives affect practice, the mathematics education community cannot fully understand the challenges of teaching modeling. Although some scholars (e.g., Blum & Niss,1991; Niss, 1987) have highlighted the environmental impacts, limited empirical evidence supports their claims. The present study builds upon some, but is not limited to, environmental impacts displayed in the existing literature. My study also contributes a new depth to the field's understanding of the practical rationality of mathematical modeling, with an emphasis on norms and professional obligations.

Theoretical Perspectives

I introduced practical rationality in the opening chapter, but in this section, I focus in further detail on the theoretical perspective that guided this study. I start with a description of practical rationality. Next, I illustrate several important constructs within practical rationality: norms and professional obligations. More specifically, I discuss four categories of professional obligations. Furthermore, I present the studies that embedded practical rationality and elaborate on how their designs and findings informed my study.

Practical Rationality

In mathematics educational research, efforts to understand mathematics instruction have often focused on individual teachers' qualifications and characteristics, such as different types of teacher knowledge (e.g., Hill et al., 2008; de Oliveira & Barbosa, 2010) and teacher beliefs (e.g., Kaiser & Maaß, 2007; Maass, 2011). This research paradigm conceptualizes teaching as individual expressions (Skott, 2009). However, a study conducted by Barkatsas and Malone (2005) challenged this paradigm, and their findings showed that teachers do not simply enact their beliefs but also are impacted by the norms and constraints that are attached to their roles. Several scholars have pointed out that teaching is a cultural activity (e.g., Hiebert, 2013; Stigler & Hiebert, 1999; Webel & Platt, 2015). They have acknowledged that teaching is not merely an act of making individual choices. Teachers need to adapt to a sophisticated system involving various agents (e.g., students, school administrators, parents, and society's expectations) and manage the complexity that involves interactions among roles, relationships, goals, and expectations (Herbst & Chazan, 2012; Lampert, 1985).

Building on the situated and sociocultural perspectives of learning and practice (e.g., Engestrom, 1992; Wenger, 1999), Herbst and colleagues (2011) employed the term practical rationality to lay the grounds for justifying mathematics teachers' actions and honoring their act of teaching. The construct of practical rationality is the theoretical framework guiding this study. *Practical rationality* refers to "the categories of perception and appreciation with which teachers talk about how they handle the multiple demands of their work and the dispositions that observers ascribe to teachers' action" (Herbst et al., 2011, p. 224). It draws the boundaries between what is acceptable and unacceptable and between desirable and undesirable actions for a mathematics teacher. The practical rationality of a teacher's action, as Herbst and Chazan (2012) mentioned, can be argued in terms of dispositions that "combine the norms of instructional activity and the obligations of the mathematics teaching profession" (p. 611). The two central concepts of practical rationality, norms and professional obligations, are illustrated next.

Norms

The term *norm* refers to the "normal or unmarked behavior that is tacitly expected in the setting" (Herbst & Chazan, 2011, p. 411). In terms of educational settings, instructional norms are the tacit expectations that teachers perceive about what subject matter instruction should be like. They usually regulate the interactions among teachers, mathematics, and students (Herbst & Chazan, 2003). Herbst and colleagues (2011) illustrated the notion of norms in the case of introducing a new theorem in high-school geometry: A teacher is expected to sanction or endorse those propositions that are to be remembered as theorems for later use. Although norms regulate what is expected to happen, individual teachers might perform differently under the same norm. Some might comply with the norm, while others might breach it. How teachers react to the norms of an instructional situation is the *rationality* that moves them to follow or breach the norms.

One characteristic of norms is that they are often tacit for the practitioners who carry out the action (Herbst & Chazan, 2011, 2015). The norms might make sense to the teachers, but teachers might not always realize that their actions arise from choices available to them. In certain instructional situations, the teacher might discard some of the choices. For example, if the norm of solving word problems is to find a particular answer, then the teacher might neglect the idea of discussing multiple ways to problem solving.

Examining norms requires either observation of the actual teaching practice or vicarious immersion in action, for example, by engaging teachers in tasks that include some representations (e.g., images, written cases, and video recordings) of teaching practice (Herbst & Kosko, 2014). Because of its tacit nature, an instructional norm becomes visible when it is violated. Hence, the technique of breaching experiments has been developed to elicit the norms as teachers confront an instructional situation that includes a breach of a norm. Breaching experiments will be further discussed in the Method chapter.

Professional Obligations

Researchers have used *professional obligations* to understand mathematics teachers' practical rationality (Herbst & Chazan, 2011). Herbst and Chazan (2017) indicated that a mathematics teacher is accountable to different stakeholders, involving *knowledge*, *organization*, *client*, and *society*. These four types of stakeholders correspond to four categories of professional obligations: *disciplinary obligation*, *institutional (schooling) obligation*, *individual obligation,* and *interpersonal obligation* (Table 1).

With regard to the mathematical *knowledge* that is expected to be taught and learned, mathematics teachers have a *disciplinary obligation* to teach a valid representation of mathematical knowledge, practices, and applications based on the discipline of mathematics. Regarding the *organization* perspective, mathematics teachers have *institutional (schooling) obligations* to their department (e.g., textbook choices, curriculum coverage), to the school (e.g., calendar, bell schedules), to the school district (e.g., assessment goals), and to their professional associations and unions (e.g., duration of the workday). From the *client* perspective, mathematics teachers have an *individual obligation* to each individual student who is seeking to gain skills, abilities, and dispositions, while teachers also have an *interpersonal obligation* to the classroom community from the *society* perspective. Although the content and extent of the obligations might vary across different environments, Herbst and Chazan (2011) assumed that all mathematics teachers have the above categories of obligations to varying degrees, though teachers' actions that comply with one obligation might *not* always comply with other obligations.

Table 1

Four Categories of Professional Obligations

Disciplinary obligation	Individual obligation
Obligation to teach a valid representation of mathematical knowledge, practices, and applications	Obligation to each individual student who is in the classroom to gain skills, abilities, and dispositions
Institutional obligations	**Interpersonal obligation**
Obligations to the department (e.g., curriculum coverage), school (e.g., bell schedules), school district (e.g., assessment goals), and professional associations and unions (e.g., duration of the workday)	Obligation to the classroom community

Research on Practical Rationality

Much of the past research has used breaching experiments to study teachers' practical rationality (e.g., Chazan et al., 2012; Erickson & Herbst, 2018; Nachlieli et al., 2009). A breaching experiment, as Nachlieli and colleagues (2009) described, consists of immersing the participants in a familiar instructional situation where some of the key characteristics have been altered. Teachers actively "relate to the story" and "construct various representations" of the scenarios via their practical rationality (p. 435). If the presented situation is not within teachers' expectations, teachers will project their own experiences onto the situation and react to a breach (a violation of a norm) by engaging in "repair strategies," which reveal what the teachers think should have happened or would be more natural to have happened (Herbst & Miyakawa, 2008, p. 473).

Using a breaching experiment, Nachlieli and colleagues (2009) examined a hypothesized norm concerning mathematical proofs, that is, "every statement must be followed immediately by a reason." They showed the teacher participants a video that included a breach scenario where a teacher allowed a student at the board to assume a statement to be true without giving reasons and encouraged him to continue developing the argument. The discussions from five focus group meetings revealed different perceptions of the participants, which were further aggregated into five categories regarding whether each teacher participant viewed the teacher's actions in the video as acceptable (or unacceptable), desirable (or undesirable), or usual (unusual) when doing proofs.

Studies have used interviews and observations of actual practice as ways of understanding the professional obligations of teaching. By examining the goals, instructional practices, and justifications of two secondary teachers, Webel and Platt (2015) studied how teachers' expressed goals of changing their practices were in conflict with their perceived obligations as mathematics teachers and how the conflicts helped explain teaching practices that were inconsistent with their goals. In their study, Webel and Platt interviewed the two teachers as a pair to elicit their goals, to have them evaluate past teaching in relation to their expressed goals, to invite them to articulate the strategies they planned to use to accomplish the goals of upcoming lessons, and to explain the changes in their goals throughout the year. Their findings showed that teachers' perceived obligations negatively impacted their desires to change their practices. For example, one teacher often directed students to use algebraic methods in accordance with a disciplinary obligation to teach efficient solution methods, rather than creating more opportunities for students to try their own strategies. Another teacher followed disciplinary and individual obligations to introduce standard vocabulary and avoid multiple terms for a single concept; she did not provide opportunities for students to develop names for ideas that made sense to them even though such an activity might align with a goal to help students with sense making.

Although most of the studies on professional obligations employ qualitative inquiry, some researchers have conducted surveys to examine the relations between professional obligations and teachers' actions, as well as the relations between obligations and teaching experience. Erickson and Herbst (2018) studied whether secondary teachers were willing to depart from the norms of teacher-led proofs to create discussion opportunities for students. The researchers asked 42 teachers to choose whether they would encourage discussions by showing them instructional scenarios developed through Lesson*Sketch* platform (https://www.lessonsketch.org/). Furthermore, the study asked teachers to choose from a list of premade justification statements with regard to the four categories of professional obligations. The results from a *t*-test analysis showed that teachers tended to choose to encourage discussions. For teachers who closed off discussions, the institutional and individual obligations were associated with their actions. Experienced geometry teachers cited the disciplinary obligation more often as their justification for allowing more open discussions, but novice geometry teachers used the disciplinary obligation to justify closing down discussions. However, the study merely asked the participants to select the obligation statements from the predesigned list, which might *not* have covered all the possible obligations or revealed the complex interactions between obligations that teachers might perceive.

Professional obligations are not necessarily stable and can evolve as teachers move along in their careers. For example, Bieda and colleagues (2015) investigated the shifts in intern teachers' reasoning about the practice of teaching during a year in which they took their final methods course and then did intern teaching. The researchers measured the intern teachers' evolving understanding of teaching by analyzing their arguments and focusing on how they warranted their claims. The results show that the intern teachers emphasized disciplinary and individual obligations. However, as these teachers moved into teaching, their justifications focused more on institutional and interpersonal obligations.

Teachers not only feel obligated to the institution but also feel that the institution is obligated to them. In her dissertation, Sobolewski-McMahon (2017) examined the influences of practical rationality on three middle school teachers' instructional decision making as they planned to enact two Common Core State Standards for Mathematical Practice: CCSS-MP1 (make sense of problems and persevere in solving them) and CCSS-MP3 (construct viable arguments and critique the reasoning of others). The study analyzed multiple sources of data: teachers' responses to the Mathematics Teaching Efficacy Beliefs Instrument, teachers' reflective journals after they watched videos of other teachers engaged in lessons intended to demonstrate the two standards within classrooms, teachers' video-recorded lessons, and teacher interviews in order to understand their decision making regarding the two standards. The study found that teachers took on the role of facilitator in students' learning and allowed students to make changes to their mathematics strategies. Although all four categories of obligations factored into their instructional decisions, the institutional obligations seemed the most influential. However, the interview protocol in the study might not be well suited to thoroughly eliciting teachers' practical rationality. It seems problematic that the researcher directly asked teachers, “What norms exist in your instructional system that enable you to enact CCSS-MP1 and CCSS-MP3?” Norms are often tacit (Herbst & Chazan, 2011), therefore the participants in the Sobolewski-McMahon study (2017) might not have been able to explicitly identify the norms acting upon their decision making.

These extant studies that have explored teachers’ practical rationality related to other mathematical practices, such as doing proofs and problem solving, have informed the present study’s investigation into practical rationality around mathematical modeling. The existing studies on the challenges of teaching modeling have only considered the role of individual teachers’ knowledge and beliefs. They have not focused on the role of external factors, such as norms and professional obligations. These studies cannot help us fully understand why teachers regard modeling as important but still marginalize it in their lessons. Research acknowledging the environmental impacts that affect teaching, such as research on teachers’ practical rationality, has not yet examined the role of professional obligations in the context of mathematical modeling. To go beyond the above studies, my study extended their work to understand the perceived norms and professional obligations of an important mathematical practice, mathematical modeling.

CHAPTER 3

Method

As described in prior chapters, this study examined the following two questions:

1. What norms are perceived by secondary teachers in relation to mathematical modeling? (RQ1)
2. What professional obligations are perceived by secondary teachers in relation to mathematical modeling? (RQ2)

This chapter presents the research design and methods for the study. I first describe the selection criteria for teacher participants and the design of the instruments. Then, I elaborate on the procedures for data collection, the techniques for data analysis, and the methods of establishing trustworthiness.

Participants and Setting

I recruited participants for this study through an online survey (see Appendix A), seeking more than 100 mathematics teachers at the secondary level who were interested in sharing their perspectives on instruction related to mathematical modeling. I sought at least 100 respondents to allow for variation and large enough numbers in the subgroups to potentially check for statistical differences. This study, however, did not aim for a representative nationwide sample for two reasons. First, it is challenging to get teachers' contact information from all the different states. Second, the study goal was to understand some teachers' practical rationality, rather than achieve a definitive record of all teachers' perspectives. Geographically, I sent out the survey to teachers encompassing rural, suburban, and urban areas in Missouri. As an incentive to respond, I offered a $20 e-gift card for each of the 50 teachers who completed the survey and another $20 gift card for each of the six interviewees. I concluded the survey data collection with 176 valid responses, including 88 responses in each subgroup.

My criteria for participant selection were to include a variety of mathematics teachers with respect to schools, years of teaching experience, and implementation of modeling-related tasks to get fruitful research findings. Including participants from different schools helped me understand the potential institutional differences in teachers' professional obligations. Teachers with different teaching experiences might react differently to different obligations (Erickson & Herbst, 2018). I also assumed that the different frequencies with which teachers used modeling tasks might impact their reactions to different obligations. Note that it was unnecessary to select participants who have enacted mathematical modeling tasks since my data collection centered on their perceptions of instructional situations related to modeling, rather than their enactments of full modeling itself.

Research Design and Instruments

In this study, I mainly employed a breaching experiment survey (Chazan et al., 2012; Nachlieli et al., 2009) and a follow-up interview to understand teachers' perceived norms and professional obligations concerning instructional situations related to mathematical modeling. An online survey allowed me to collect data from a relatively large group of teachers to examine the norms and obligations without the expense of classroom observations. Compared with a smaller sample of classroom observations or interviews, larger groups of teachers shed more light on what the norms are, since norms in this study were "socially expected patterns of action that an observer represents in a law-like proposition" (Herbst & Chazan, 2015, p. 274). Furthermore, a survey was more appropriate than classroom observations because norms and obligations arise from teachers' points of view rather than becoming evident in a given lesson. Obligations felt by the teachers can be invisible in practice, and what teachers perceive to be normal within secondary mathematics classrooms may not always match what they are doing in a given lesson. Moreover, a breaching experiment survey allowed me to script lessons beforehand to control what norms would be breached, helping to overcome the difficulties in capturing the emerging norms by detecting the regularities from real-time classroom observations. The premade scenarios were cartoon based, and participants were able to project their circumstances onto the scenarios and talk about them as if they were part of the participants' actual practice (Chazan & Herbst, 2012).

After completing the survey, three pairs of participants (n=6) were invited for a follow-up interview. They were selected in an attempt for me to further understand how the obligations supported or breached the hypothesized norms. Their perspectives helped me triangulate and potentially explain findings from the survey data. The next section describes both of this study's instruments, a scenario-based breaching experiment survey and a follow-up interview protocol, including the selection criteria for interviewees.

Breaching Experiment Survey

I designed the survey by developing six breaching scenarios within two fictional lessons—an exponential function lesson and a rational function lesson. The first lesson depicted a class working on a bacterial reproduction task, while the second lesson depicted a class working on a water usage task. See Appendix B for more details about the two tasks. I used two lessons instead of one because setting the breaching experiments in only one content area would have made it difficult to parse the norms related to the mathematical modeling practice from the norms related to teaching that specific content. For example, if the survey participants were to use symbolic representations heavily, and if they were to affirm this use as a norm, they might do so because they are accustomed to specifically teaching exponential functions symbolically rather than upholding a general norm that all modeling activities should yield symbolic equations. I chose exponential functions and rational functions as the survey's content areas because they are two important topics in algebra, and algebra is considered important for mathematical learning (Williams, 2011). These content areas often appear as the mathematical components of tasks in modeling research (e.g., Blum & Borromeo-Ferri, 2009; Chinnappan, 2010; Doerr, 2007; Mall & Risinger, 2014). In addition, there are more algebra teachers than teachers of other mathematics topics in Missouri. I assumed more teachers would have an algebra teaching experience; therefore, choosing algebra as the modeling content would help participants feel familiar with the content, thus reducing design flaws resulting from teachers' unfamiliarity with the content. Notice that my findings might be more relevant to algebraic modeling than to statistical or geometrical modeling tasks.

I used the depict tool in the Lesson*Sketch* platform (www.lessonsketch.org/) to create the scenarios for the breaching experiment. Lesson*Sketch* allows for the incorporation of multimedia (e.g., image, text) in which participants can view and answer questions about representations of teaching. I used Lesson*Sketch* storyboards rather than actual classroom videos for two main reasons. First, it can be challenging to find such videos because "norms are not usually breached" (Dimmel & Herbst, 2014, p. 395). Second, the illustrated fictional scenarios can minimize distractions from other parts of the classroom environment, such as school announcements, classroom decor, and classroom management. The survey was administered through Qualtrics (https://missouri.qualtrics.com/) because the Lesson*Sketch* platform will shut down part of its functions in December 2020.

As I worked to build the breaching experiment, I proceeded through four steps, which I describe in turn in the subsequent subsections:

1. hypothesizing the norms,
2. developing the breaching scenarios,
3. crafting the three options for each scenario, and
4. drafting the justification statements.

Step 1: Hypothesizing the Norms

I started by generating a set of hypothesized norms to describe what I believe to be teachers' typical behaviors as they enact modeling tasks. These hypothesized norms were based on prior studies of the enactment of modeling tasks (e.g., Leiß, 2007), nontraditional tasks (e.g., Herbst, 2003), and word problems (e.g., Chazan et al., 2012). I also hypothesized norms based on the second-year study that I conducted to understand prospective teachers' conception of modeling before commencing my dissertation work. Note that these predetermined hypothesized norms did *not* mean that I expected to confirm all these norms in the breaching experiment. I knew that it was possible that I would need to adjust, refine, or even disconfirm some of the norms based on the evidence from the participants' survey responses and interview data. But by formulating a hypothesized norm, I supposed that it was worth investigating.

The following subsections list this study's six hypothesized norms and the rationale for why I hypothesized them.

Hypothesized Norm 1 (Factor Specification). The teacher (rather than students) identifies precisely which information or factors should be included in the task.

Evidence That the Norm Exists. In a study of modeling activities, Leiß (2007) found that teachers often give students too much support. The students only need to figure out one step by themselves to overcome the challenges. Similarly, the teacher from my prior classroom observation tended to provide all the information and factors needed for solving the task. Students did not have to interact with the real data or decide what information to seek or include in their work. Thus, I hypothesized that it is typical for teachers to offer students a clear direction about the factors to consider when working with modeling tasks. Note that if a textbook supplied the relevant information, I still considered this situation adherence to this norm. I viewed the teacher as having the agency to enact or adapt the problem from the textbook (e.g., they could have altered the directions but instead chose to present the problem as written).

Hypothesized Norm 2 (Symbolic Representation). Students are expected to find a symbolic representation (e.g., equations) as their model.

Evidence That the Norm Exists. Studies have shown that algebra instruction in the United States predominantly focuses on symbolic manipulations (e.g., Kieran, 2007). In certain states' standards, modeling is explicitly associated with finding a symbolic representation for a situation. For example, the Missouri learning standards have listed that students in Algebra 1 are expected to "construct and compare linear, quadratic and exponential models" (Missouri Department of Elementary and Secondary Education, 2019a), and those in Algebra 2 are expected to "use functions to model real-world problems" (Missouri Department of Elementary and Secondary Education, 2019b). Prioritizing algebraic or symbolic representations in modeling activities seems to have its heritage in word problems. Students are often expected to write and solve an algebraic equation to solve algebraic word problems (Gerofsky, 1996). Consequently, it is reasonable to assume that a mathematics teacher might expect students to come up with a symbolic representation as their product of modeling.

Hypothesized Norm 3 (Mathematical Components). The teacher gives students unambiguous directions on what they are expected to use in terms of the mathematical operations and components.

Evidence That the Norm Exists. The mathematical operations or components tend to be explicit for students in textbook problems. An example from the textbook series *Precalculus: Graphical, Numerical, Algebraic* is "assuming the growth is exponential, when did the population of Austin surpass 800,000 persons" (e.g., Demana et al., 2008, p. 262). In this example, "the growth is exponential" is the type of mathematical function that should be used and was provided by the textbook. In other examples, the textbook or teacher explicitly reminds students of the standard form of the function to be used. According to Chazan and colleagues (2012), the purpose of problems like the above example is to have students practice an algorithm or a mathematical procedure recently presented in their mathematics lessons. It is reasonable to assume that a teacher might follow this norm of traditional textbook problems by explicitly providing students with the mathematical operations needed in the modeling activities or, for example, telling the students at the beginning that they should try to use what they have recently learned about exponential functions. Furthermore, research has found that over 50% of 263 teachers from Grades 7 through 12 completely agree that the purpose of modeling is for students to learn how to apply the mathematics that they are learning (Gould, 2013).

Note that Hypothesized Norm 1 (factor specification) is related to the information (e.g., "800,000" presented in the example from the previous paragraph) and factors (e.g., years, time), while Hypothesized Norm 3 (mathematical components) is related to the operations or mathematical functions (e.g., "exponential"). In some cases, factors, information, and operations are provided in the problem simultaneously. For example, in the problem "Detroit's population model is $p(t) = 1{,}203{,}368 \cdot 0.9858^t$," initial population, yearly growth rate, and time are the *factors* considered; 1,203,368 and 0.9858 is the *information* provided; and exponential function is the mathematical *operation* suggested in the problem (e.g., Demana, et al., 2008, p. 265). This example adheres to Hypothesized Norm 1 and Hypothesized Norm 3 at the same time.

Hypothesized Norm 4 (Mathematical Thinking Dominance). Students are expected to primarily (or exclusively) engage in mathematical thinking rather than nonmathematical thinking.

***Evidence That the Norm Exists*.** A typical expectation of a mathematics lesson is that students should work mathematically, where working mathematically is often conceived as procedural execution, abstraction, or deductive reasoning. Nonmathematical thinking, which is described in this study as thinking with knowledge from everyday life and other disciplines, would seem to distort the "clarity, aesthetical purity, [and] context-free universality" of the mathematics entity for some teachers (Blum & Niss, 1991, pp.

53–54). In addition, teachers might think that using nonmathematical knowledge and reasoning nonmathematically poses a risk that "[students might] ignore the mathematics and escape into reality" (Stillman, 2000, p. 336), reducing teachers' control over what is likely to happen in a lesson (de Oliveira & Barbosa, 2010) or diverting class time to something that is not directly related to the lesson's mathematical learning goals. Thus, I hypothesized that teachers allocate more time and energy to the mathematical side of modeling (e.g., formulate a model, calculate the mathematical results using the model, interpret the mathematical results in the real-world context) than other parts of modeling, such as interpreting the situation with one's realities, reasoning in a nonmathematical way, and validating results within the context.

Hypothesized Norm 5 (Model Revision Omitted). All students are expected to arrive at the same model, and model revision (beyond resolving discrepancies between different models) is not expected.

Evidence That the Norm Exists. Mathematical modeling can result in various models because students may take different approaches for the same task (Mathematical Association of America [MAA], 1972). In addition, different assumptions can lead to different models. Therefore, there is no such thing as the best model for a specific problem, as there always exists a better one or a different one that is equally appropriate. Because most of the problems in mathematics textbooks have specific procedures and one correct answer, enacting real-world problems can often be influenced by the norms of word problems and so involves computations and adherence to mathematical procedures. For example, in classroom observations I have conducted, once students arrived at a correct or reasonable answer, they were not required to revise or strive for a better solution nor to find a different but equally reasonable answer. Gould (2013) also found that 54.4% of the teachers surveyed perceived that, over half the time, modeling processes resulted in an exact answer or exact answers. Therefore, I hypothesized that modeling activities typically conclude once a completed model is determined or once a completed model is used to make a single prediction. Relatedly, I hypothesized that it is not customary for teachers to do model revision in their classrooms.

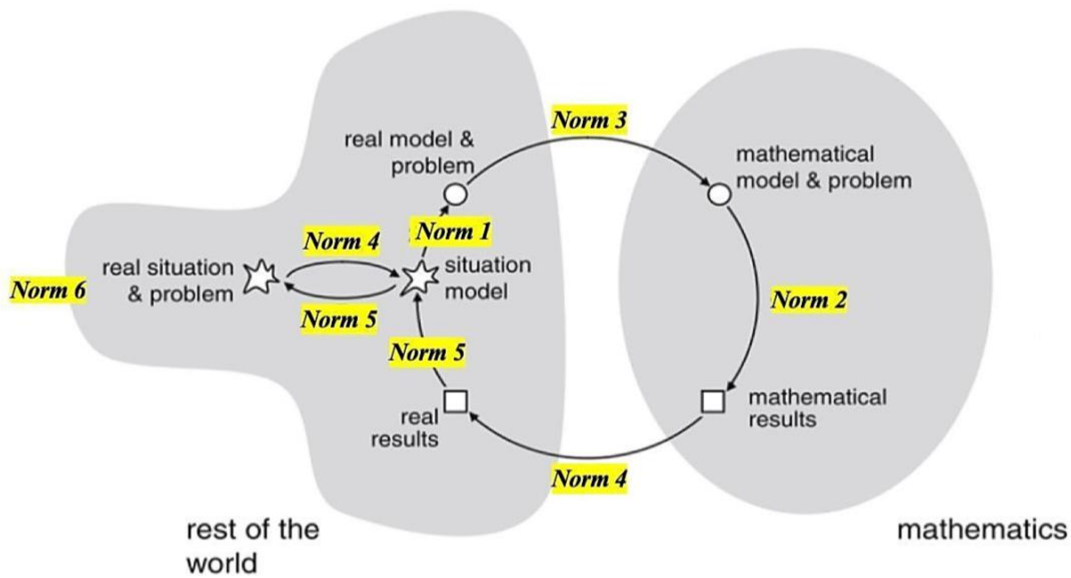
Hypothesized Norm 6 (Politically Neutral Contexts). Students are expected to primarily (or exclusively) work on politically neutral tasks instead of social-justice-oriented tasks.

Evidence That the Norm Exists. The use of modeling tasks connecting to social justice can engage students in "experienc[ing] mathematics as an analytical tool to make sense of, critique, and positively transform our world" (Aguirre et al., 2019, p. 8). Although some mathematics educators have dedicated a substantial amount of effort to promoting social justice and equity in schools, social-justice-oriented teaching is still seldom applied in mainstream mathematics classrooms in the United States. There are four potential explanations for this phenomenon (Gutstein, 2003). Based on Gutstein (2003), teachers might put themselves at risk by raising the inequality issues that can implicate power deconstruction and threaten authority. Second, the prevalence of high-stakes standardized tests in the United States, which do not tend to measure social-justice mathematics skills or orientations, impose pressure on mathematics teachers and schools, causing them to shy away from an equity pedagogy. Third, it is commonly "accepted" by the public that mathematics is neutral and context-free, so teachers may view it as beyond their purview to engage in social-justice mathematics teaching. Last, the field of mathematics education has traditionally and historically focused more on cognition than on sociocultural contexts (although this situation is changing), so teachers may not have the confidence or resources to enact social-justice activities in mathematics classrooms. Therefore, it is reasonable to assume that teachers might not attend to social justice and equity issues when implementing modeling tasks.

In sum, this study examined six hypothesized norms and teachers' perceived professional obligations related to mathematical modeling. These six hypothesized norms have their corresponding places within the cycle of mathematical modeling (see Figure 4). More specifically, Hypothesized Norm 6 (politically neutral contexts) is related to the *situation* of the problem that students engage in. Hypothesized Norm 4 (mathematical thinking dominance) is related to the *constructing* stage, when students identify and understand the problem, and the *interpreting* stage, when students interpret the mathematical results back to the real-world context. Hypothesized Norm 1 (factor specification) is related to the *simplifying* stage, where students prioritize certain factors to simplify the situation. Hypothesized Norm 3 (mathematical components) is related to the *mathematizing* stage, where students apply mathematical operations to creating certain types of mathematical products. Hypothesized Norm 2 (symbolic representation) is related to the *working mathematically* stage, where students spend their time on selecting and creating representations that describe relationships among the factors they consider. Hypothesized Norm 5 (model revision omitted) is related to the *validating* and *exposing* stages, where students present their models if the results are validated or revise the models.

Figure 4

Hypothesized Norms and the Corresponding Stages of the Modeling Cycle



Note. Adapted from "'Filling up'–The problem of independence-preserving teacher interventions in lessons with demanding modelling tasks," by W. Blum and D. Leiß, 2005, in M. Bosch (Ed.), *Proceedings of the 5th Congress European Research Mathematics Education*, p. 1626. Copyright 2005 by the Universitat Ramon Llull.

Step 2: Developing Breaching Scenarios

In order to test the hypothesized norms, I created a set of scenarios in which a fictional teacher undertakes actions that either adhere to or, more importantly, depart from each of the hypothesized norms. Table 2 summarizes the hypothesized norms and related scenarios. Each breaching scenario consisted of a sequence of classroom images to represent the instructional situation within a modeling activity. When starting the breaching experiment, the participants were notified that the scenarios were about mathematical modeling in the context of high school algebra, but they were not told that the scenarios included breaches of hypothesized norms.

Table 2

Description of the Breaching Scenarios

Hypothesized norm	Task	Summary of the corresponding breaching scenario
1. Factor specification: The teacher (rather than students) identifies precisely which factors should be included in the task.	Task 1: Bacterial reproduction	In the bacterial reproduction task, students came up with many factors to consider, such as the initial number of bacteria, time, temperature, bacterial types, and humidity. In a noncanonical option (NC), the teacher encouraged them to decide what they wanted to include rather than directly leading them to include specific factors in their model.
2. Symbolic representation: Students are expected to find a symbolic representation (e.g., equations) as their model.	Task 1: Bacterial reproduction	In the bacterial reproduction task, a group of students came up with a table to show their model of bacterial reproduction. In an NC, the teacher approved their use of a tabular representation without requiring the students to translate it into a function.
3. Mathematical components: The teacher gives students unambiguous directions on what they are expected to use in terms of the mathematical operations/components.	Task 1: Bacterial reproduction Task 2: Water usage	After launching the task, students wondered what mathematical operations or components were needed to solve the task. In an NC, the teacher maintained a productive ambiguity on the mathematical operations or components. She encouraged students to choose the operations or components that made sense to them rather than directing them to specific equations.

Hypothesized norm	Task	Summary of the corresponding breaching scenario
4. Mathematical thinking dominance: Students are expected to primarily (or exclusively) engage in mathematical thinking rather than nonmathematical thinking.	Task 2: Water usage	In an NC, instead of quickly moving forward to the stage of mathematization, the teacher encouraged students to share a variety of personal experiences and incorporate them into the model-building process (not merely as motivation or an introduction of the task).
5. Model revision omitted: All students are expected to arrive at the same model, and model revision is *not* expected (beyond resolving discrepancies between different models).	Task 1: Bacterial reproduction Task 2: Water usage	After most of the class got the "textbook-type" answer, a student brought up a new idea for their model. In an NC, the teacher prompted the whole class to continue revising their models and allotted much in-class time for a more complex model.
6. Politically neutral contexts: Students are expected to primarily (or exclusively) work on politically neutral tasks instead of social-justice-oriented tasks.	Task 2: Water usage	In an NC, to develop students' social and political consciousness, the teacher implemented a modeling task with a social justice emphasis rather than a task with a politically neutral context.

Since the study examined six hypothesized norms, it would be too long for a single survey to cover all corresponding scenarios. A long survey tends to have a low response rate or substantial respondent attrition. To avoid the potential negative effects of a lengthy survey, I split the survey respondents into two groups (see Table 3). Group 1 completed a survey that included Hypothesized Norms 1, 2, 3, and 5 (factor specification, symbolic representation, mathematical components, and model revision omitted, respectively), and Group 2 completed Hypothesized Norms 3, 4, 5, and 6 (mathematical components, mathematical thinking dominance, model revision omitted, and politically neutral context). Individual participants were randomly assigned to Group 1 or 2. The hypothesized norms were placed in an intuitive order, from set-up to working to look-back phases of modeling enactment (Otten & Soria, 2014). Hypothesized Norm 6 was last because it involved a potentially different context for a modeling problem, and I did not want to confuse respondents by presenting multiple different tasks at the start of the survey.

Note that for each group, I intentionally had two overlapping hypothesized norms (i.e., Hypothesized Norms 3, mathematical components, and 5, model revision omitted) for triangulation. I selected these two hypothesized norms for replication because they were unique to modeling activities while the rest were not. Hypothesized Norm 3 is unique to modeling because students can take multiple approaches and apply different operations to simulate the situation, while most problem solving in school mathematics targets one particular operation. Hypothesized Norm 5 is also unique to modeling since the fitness and delicateness of the model is a result of iterative model revisions, whereas problem solving usually aims for a certain solution. Other hypothesized norms, such as using different types of representations or nonmathematical thinking, are related to modeling, but they can be the norms of real-world problems and project-based learning as well.

Table 3

Distribution of Tasks and Hypothesized Norms in Two Groups

Group 1 Task 1: Bacterial reproduction	Group 2 Task 2: Water usage
Scenario 1: Hypothesized Norm 1 – factor specification	Scenario 1: Hypothesized Norm 4 – mathematical thinking dominance
Scenario 2: Hypothesized Norm 2 – symbolic representation	Scenario 2: Hypothesized Norm 3[c] – mathematical components
Scenario 3: Hypothesized Norm 3[a] – mathematical components	Scenario 3: Hypothesized Norm 5[d] – model revision omitted
Scenario 4: Hypothesized Norm 5[b] – model revision omitted	Scenario 4: Hypothesized Norm 6 – politically neutral contexts

[a, b, c, d] Hypothesized Norms 3 and 5 were explored with both groups for triangulation purposes.

Step 3: Crafting the Three Options for Each Scenario

Within the simulated instructional scenarios, the three options for how the teacher might proceed were designed so that one involved a (hypothesized) noncanonical teacher action and the other two involved canonical teacher actions.

The noncanonical teacher action (the noncanonical option [NC] in this study's survey) departed from the hypothesized norm in the situation, while the canonical teacher actions (Canonical Option 0 [C0] and Canonical Option 1 [C1] in this survey) were consistent with the hypothesized norm. Within the two canonical teacher actions, C1 followed the norms more strictly than C0 (see Table 4 for examples). One way to think about this is that C1 was hypothesized to be fully adhering to the hypothesized norm. On the other hand, C0 was closer to the boundary of the hypothesized norm (0 is akin to the boundary condition), but it was still within the hypothesized norm and so still canonical (hence the C in C0). On the other hand, NC was hypothesized as a breach. A complete list of options can be found in Appendix C.

I chose to have two canonical options and one NC rather than the same number of each because I wanted to recognize that there is a diversity of instructional options within the hypothesized norm space; a norm does not mean that all teachers handle situations exactly the same way. Therefore, I created two possibilities that were within the canonical space of the hypothesized norms. I did not, however, want to include two options for the breaching space because breaches are hypothesized to be abnormal and having two of them might cause one to normalize the another. That is, if two NCs were presented together, they might make one another look not so abnormal in comparison. Also, in the spirit of breaching experiments (Erickson & Herbst, 2018), each moment when a teacher chooses the breach option is intended to be a very meaningful event, so that option should be rarer than the canonical options.

To illustrate the range of possibilities on tasks, I include a written solution to both modeling tasks in Appendix D. The written solution for the bacterial reproduction task departs from all hypothesized norms whereas the written solution for the water usage task abides by all hypothesized norms.

Table 4

Description of Three Canonical and Noncanonical Options for Norms 1, 3, and 5

Hypothesized norm	Canonical option 1 (C1)	Canonical option 0 (C0)	Noncanonical option (NC)
Norm 1 (factor specification)	Ms. Lee: Good question. You can't consider everything, so for this problem, just focus on the initial number of bacteria, the reproduction rate, and the length of time.	Ms. Lee: Good question. You can't consider everything, so you can narrow it down. You don't have to consider factors like temperature or humidity.	Ms. Lee: Good question. That is something your group needs to think about. Choose whatever makes sense to you.
Norm 3 (math components)	Ms. Lee: You can use an exponential function. Remember, it has the form $y = a \cdot b^x$.	Ms. Lee: What did we learn yesterday? Emma: Exponential functions? Ms. Lee: Then try some exponential functions.	Ms. Lee: You should decide on the function. Use whatever type of function you think is appropriate.
Norm 5 (model revision omitted)	Ms. Lee: Group 2 brought out an interesting idea about bacteria dying. That was part of their answer. There are multiple good answers to these kinds of problems, so we are going to wrap up. We are going to start the next lesson and finish this chapter by this Friday. (The class spent the rest of time on the new lesson.)	Ms. Lee: Group 2 brought out an interesting idea about bacteria dying. Other groups, go ahead and revise your current model to try to incorporate bacteria dying. Group 2, you can finalize it and then turn in your work. (Other groups spent the rest of time revising their models.)	Ms. Lee: Group 2 brought out an interesting idea about bacteria dying. Besides bacteria dying, what else can we add to the current model we have? Let's think about a couple of other things and continue revising our model. (The whole class spent the rest of time revising the models.)

Note that the three options of the first five hypothesized norms can be viewed on a continuum, shifting from teacher-centered to student-centered mathematical authority. Mathematical authority is “the degree to which students are given opportunities to be involved in decision making and whether they have a say in establishing priorities in task completion, method, or pace of learning” (Gresalfi & Cobb, 2006, p. 51). In this study, the C1 options cohered around the notion of teacher-centered authority and direct guidance for the students, whereas the NC options cohered around sharing mathematical authority with students.

More specifically, for Scenario 1, C1 was the instance where the teacher determined which factors to consider, while NC was the instance where students were given an opportunity to choose the factors. For Scenario 2, C1 was the instance where the teacher told students which representation to use, while NC was the instance where students chose the representation that made sense to them. Likewise, in Scenario 3, the teacher provided the mathematical components to students in the C1 option and gave students opportunities to explore the mathematical components in the NC option. For Scenario 4, C1 was the instance where the teacher decided the validness of students’ types of thinking; NC was the instance where students clarified and justified both mathematical and nonmathematical thinking. For Scenario 5, C1 was the instance where the teacher determined the correctness of the solution and the completion of the problem, while NC was the instance where students were given the opportunity to further explore their work. C0 options, like C1, still involve teacher-centered authority but in a slightly less directive or restrictive manner. For Scenario 6, the C1, C0, and NC did not follow the continuum of authority. Instead, CI was the instance where the teacher presented a politically neutral task. For C0, the teacher introduced a real-world issue with an environmental concern. The NC included explicit references to the social justice issues surrounding the environmental concern.

Step 4: Drafting Justification Statements for the Choice of Option

In order to understand the justification of their choice for each breaching scenario, participants were asked to choose from a collection of eight justification statements (Table 5) after they selected the option. This collection of the justification statements was adapted from Erickson and Herbst's study (2018), where they tried to examine whether secondary teachers would create discussion opportunities for students through an online survey. In my study, participants could choose as many of these statements as they felt applicable. Then, they were asked to indicate how strongly they felt obligated for each of the justifications selected on a 3-point scale (i.e., did not consider, considered, strongly considered).

I originally considered having open-ended items to solicit teachers' justifications, but I modified the design to multiple-choice items. There were both pros and cons to this change: A potential advantage was that closed responses helped shorten the survey's length because participants only needed to select those that matched their justification without spending much time on writing. It also eased the process of coding justifications to some extent since each statement was created with a particular obligation in mind, whereas the open-ended responses might not have clearly alluded to a single obligation or any obligation at all.

The potential drawbacks of closed responses were that these statements could not cover all the possibilities participants had in mind and that the statements might have reminded participants of other obligations that they might not have considered when they picked the choice of action. To address the first drawback, I had an additional question for participants to potentially write about what else they had considered if they wished. For instance, they could have written about the option that best matched their teaching style. To check whether participants were interpreting the closed options differently than anticipated, I also used the follow-up interview to further probe into some of the obligation choices. In terms of the second drawback, I made it explicit in the question item for the participants to *only* choose what they considered at the moment to avoid over-selection.

Table 5

Examples of the Justification Statements

Obligation	Justification statement
Individual	• Because this action best facilitates individual student thinking (JS 1.1). • Because this action best supports individual emotional needs (JS 1.2).
Interpersonal	• Because this action best maintains a social environment that is conducive to learning (JS 2.1). • Because this action best supports interactions among students in the class (JS 2.2).
Disciplinary	• Because this action best represents the mathematical concepts and/or properties under consideration (JS 3.1). • Because this action best supports understanding of the nature of mathematical practice (JS 3.2).
Institutional	• Because this action best meets my department and/or school policies (e.g., curriculum coverage, pacing guide) (JS 4.1). • Because this action best matches the assessment goals (e.g., standardized state testing) (JS 4.2).

Note. Adapted from “Will teachers create opportunities for discussion when teaching proof in a geometry classroom?” by A. Erickson and P. Herbst, 2018, *International Journal of Science and Mathematics Education, 16*(1), p. 172. Copyright 2018 by the Ministry of Science and Technology, Taiwan.

There were three major stopping points for each scenario (Figure 5). First, the participants read the task and decided whether it was the type of task they would use with their students (Stopping Point 1). Then, participants viewed the first part of a classroom episode in which students worked on a modeling task. They were presented with several prompts (see Appendix E), including requests to choose one out of three options to respond to the instructional situation and select justification statements as their rationale for what they decided to do next (Stopping Point 2). The three options and the justification statements were placed in a random order when the participants took the survey to avoid a possible order effect. However, Appendix C represents the options in a fixed order, from NC to C0 to C1, for readers' convenience.

After answering those prompts, participants were presented with the actions of one of the fictional teachers, Ms. Johnson. Importantly, the fictional action that respondents saw was *different* from the action they chose at Stopping Point 2. If the participant selected one of the canonical options, then the survey logic would guide them to the NC. If the participant selected the NC, then the logic would guide them to C1. Participants who picked the NC were forced to view C1 rather than C0 to elicit their justifications between two extreme choices. All participants were then asked to rate the typicalness of the fictional teachers' actions (Stopping Point 3). By analyzing participants' selections as well as their evaluations of the fictional teacher's reaction to students depicted in the scenarios, I could infer whether participants perceived some teacher actions as breaches of typical instructional patterns. As a result, I could confirm, disconfirm, or refine the hypothesized norms around modeling (Herbst & Kosko, 2014). More detailed breaching scenarios can be found in Figure 5 and in Appendix F.

Once the teacher participants answered the questions regarding the scenarios, they were prompted to describe their conceptualization of mathematical modeling by answering the question "What does the term 'mathematical modeling' mean to you?" This question provided additional information about the teachers' general perspectives to potentially inform my interpretations. Next, they were asked to fill out their demographic information. I placed the demographic questions at the end of the survey to keep momentum while participants completed the survey's substantive parts and to reduce survey fatigue. The purpose of the demographic information section was to identify aspects of each participant's background that might be used to gain insights into how different characteristics (e.g., years of teaching, modeling experiences) influenced their perceived norms and professional obligations. In addition, demographic information, such as contact information, enabled me to recruit teachers for the follow-up interview described in the next section.

Figure 5

An Overview of the Breaching Experiment for Hypothesized Norm 3a – Mathematical Components

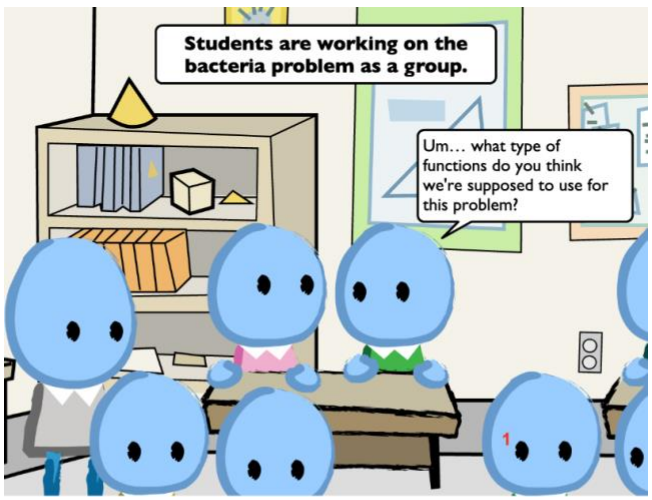
Stopping Point 1: View the modeling task

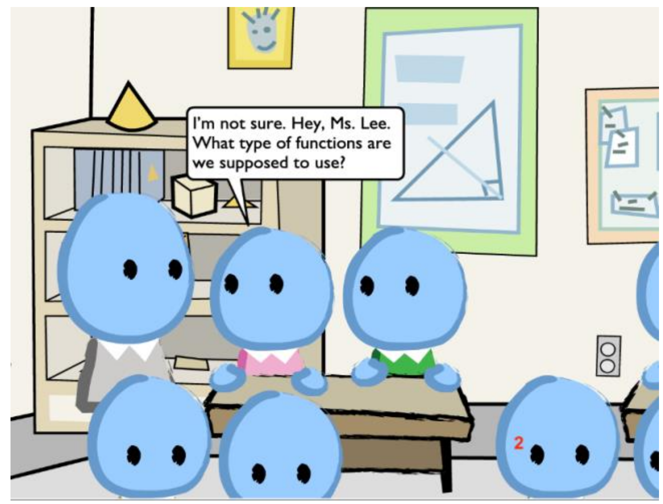
Bacterial Reproduction Task

"Droplets containing bacteria are released into the air when a person sneezes. You can get sick when you come into contact with these tiny droplets. If a person sneezes on his/her hand, how many individual bacteria are on his/her hand over time?"

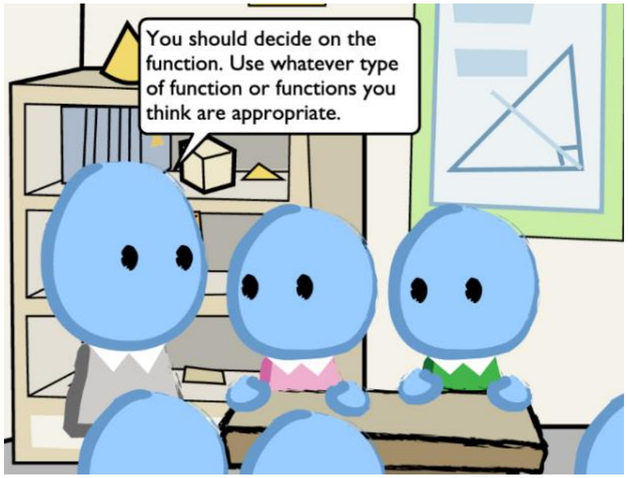
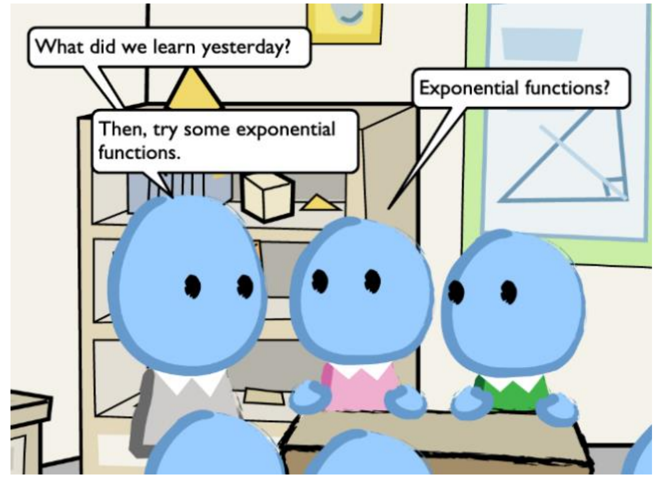
- This task has a real-world context and does not specify what students have to do. Would you use this type of task as written with your students? [Choosing from Yes/Maybe/No]. Explain your choice. [Open responses].
- If yes, how often would you use this type of task as written with your students? [Choosing from Daily/Weekly/Monthly/Semesterly].
  If maybe/no, would you use the task if you could change it? [Choosing from Yes/No] What kinds of changes would you make and why?

Stopping Point 2: View a classroom episode and then select one of the options to react to (and explain)

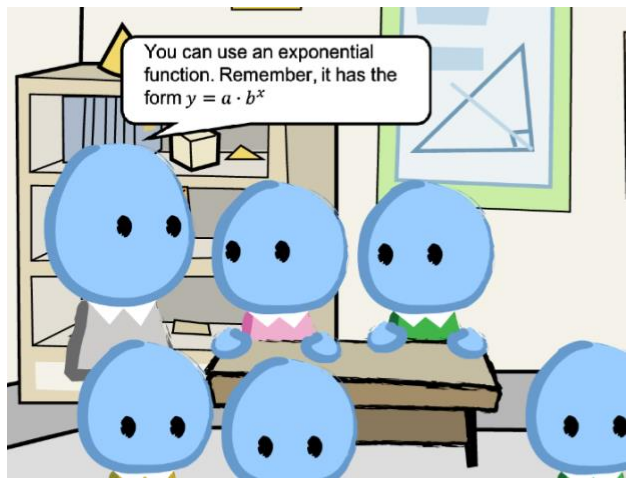




- Here is the scenario of a high-school lesson using the same task. Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.

Noncanonical option (NC)	**Canonical option 0 (C0)**
	

Canonical option 1 (C1)



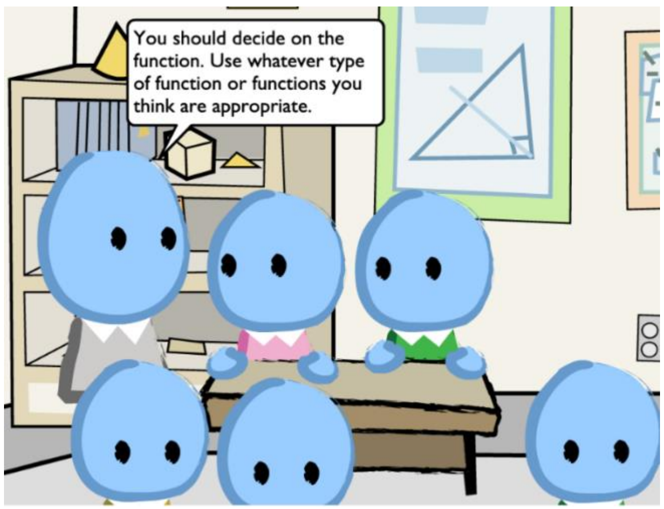
- Indicate the extent to which you considered these reasons as you selected your teacher action.
  Justification statements: JS 1.1—JS 4.2 [Choosing from Did not consider/Considered/Strongly considered].
- Are there other considerations when you selected the teacher action? If so, you may describe them here. [Open responses].
- Are there other ways you would respond to the students? And why? [Open responses].

[If the participants selected the NC, then the logic guided them to C1. If the participants selected one of the canonical options, then the logic guided them to the NC.]

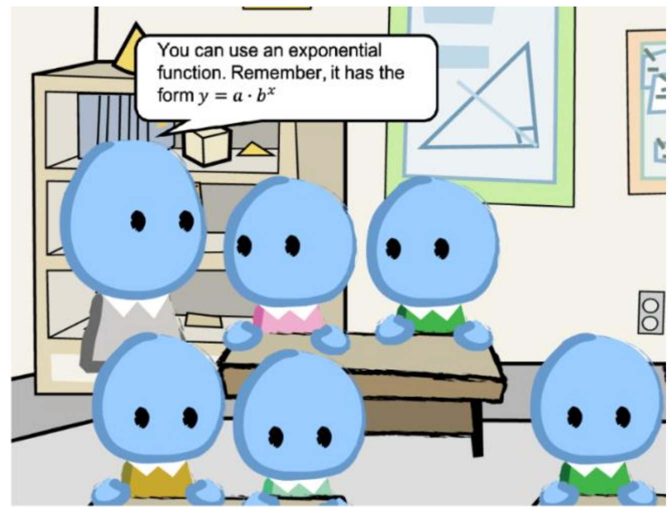
Stopping Point 3: Continue to watch the scenario and react to the actions of another teacher, Ms. Johnson

Noncanonical option (NC)	Canonical option 1 (C1)

Noncanonical option (NC)



Canonical option 1 (C1)



- For mathematics teachers in general, how typical would it be to respond to the students as Ms. Johnson did?
  [Choosing from Very typical/Somewhat typical/Somewhat rare/Very rare].

Interview

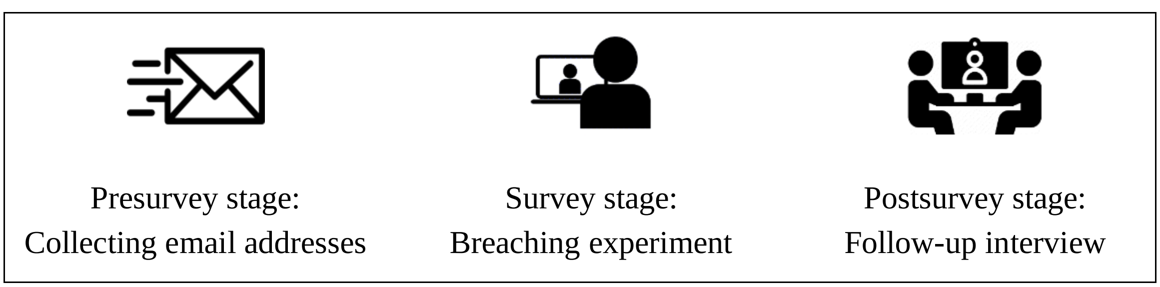
To gather more detailed information about the survey responses and to triangulate the data from the breaching experiments, I conducted semi-structured interviews with six participants after the breaching experiment. Sample interview questions were as follows: “You indicated that it is very typical (depends on what the teachers selected) for mathematics teachers in general to respond to the students. Can you talk more about it?” “What, if any, are the expectations from your department, school, or school district that impact your enactment of modeling?” See more questions in Appendix G. In the Data Collection section below, I describe in detail how the six interviewees were selected from the survey respondents.

Data Collection

The procedure of data collection (Figure 6) included collecting email addresses (presurvey stage), distributing the survey (survey stage), and interviewing a subgroup of participants from the survey responses (postsurvey stage).

Figure 6

Three Stages of Data Collection



Presurvey Stage

The data collection started with requesting email addresses from the Missouri Department of Elementary and Secondary Education and searching teachers' emails from the school and school district websites in Missouri. After attaining secondary mathematics teachers' information (2,781 email addresses), I sent out an invitation email containing the breaching experiment survey link on February 5, 2020. Participants were randomly assigned to one of two groups in which they completed one of two versions of the breaching experiment survey (see Table 3). A total of 323 teachers started the survey. I paused the survey response collection on February 21, 2020 (17 days in total) after I received 176 valid responses (54.5%), which constituted the usable data for the study.

Survey Stage

Each participant engaged in an approximately 20-minute breaching experiment administered through an online survey (i.e., Qualtrics). The survey followed a common structure, involving the study's introduction, four breaching scenarios (and three stopping points per scenario), the participants' conceptualizations of modeling, and demographic information (see Appendix E).

To start with, participants read a description of the research goal and decided whether they wanted to participate in the study. If they provided their consent, participants acclimated themselves to one modeling task and shared their ideas about the task—for example, their willingness to use the task in the classroom and the frequencies they used a similar type of task, if applicable. Later, participants proceeded through one lesson implementing the same task with four breaching scenarios, which were designed to elicit the participants' perceptions about what a normal classroom occurrence is and what a breach of the norms is. After viewing the scenario, participants answered questions about the scenario, such as the following: “Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.” “Indicate the extent to which you considered these reasons as you selected your teacher action.” “For mathematics teachers in general, how typical would it be to respond to the students as Ms. Johnson did?” (see Appendix E).

Each survey subgroup received 88 valid responses, making a total of 176 responses. Of the 176 respondents, 94 (53.4%) were mid-career teachers who had more than 5 years but less than 20 years of experience. Another 56 (31.8%) were in their late career and had at least 20 years of teaching experience, whereas 26 (14.8%) were early-career teachers with no more than 5 years of teaching experience (see Table 6). These respondents were from school districts of different sizes, with 50 (28.4%) from small school districts with less than 1,000 students and certified teachers and staff. Fifty-nine (33.5%) were from school districts with 1,001–5,000 students and certified teachers and staff, and 20 (11.4%) were from school districts of 5,001–10,000 students and certified teachers and staff. Another 14 (8.0%) were from school districts with 10,001–15,000 students and certified teachers and staff, and 33 (18.8%) were from larger school districts with more than 15,000 students and certified teachers and staff.

In terms of modeling teaching experience, of the 176 respondents, 20 (11.4%) had no experience enacting modeling tasks, 65 (36.9%) had at least 1 year but less than 5 years, 58 (33.0%) had at least 6 years but less than 20 years, and 20 (11.4%) had more than 20 years of experience enacting modeling tasks experience. The remaining 13

(7.4%) were not sure whether they had enacted modeling tasks in their classrooms (Table 6). As with career stage, school district size, and years of modeling teaching experience, the sample might not proportionally reflect the distribution of teachers in Missouri since the study did not control for how many teachers in each category volunteered.

Table 6

General Sample Population

Group	Career stage			School district size					Years of modeling teaching experience				
	Early	Middle	Late	<1,001	1,001–5,000	5,001–10,000	10,001–15,000	>15,000	0	1–5	6–20	>20	Not sure
Group 1 (*n*=88)	12	48	28	29	25	8	6	20	12	32	33	7	4
Group 2 (*n*=88)	14	46	28	21	34	12	8	13	8	33	25	13	9
Total	26	94	56	50	59	20	14	33	20	65	58	20	13

Note. The classification of the career stage is based on research by Christensen and Knezek (2017) in which they defined early-career teachers (1–5 years), mid-career teachers (6–20 years), and late-career teachers (>20 years).

Postsurvey Stage

After the breaching experiment, I selected six interview participants based on the emergent themes as I analyzed the survey data to gain a fuller picture of the participants' obligations and perceived norms. Based on my research questions, three types of participants were included in the follow-up interview:

1. For hypothesized norms that were confirmed: This type of participants chose C1 and indicated the alternative option (NC) as very rare. I interviewed these participants to help shed light on the justifications and obligations that were at play with regard to a modeling norm.
2. For hypothesized norms that I was unable to confirm or disconfirm: This type of participants chose the NC and indicated the alternative option (C1) as very typical. I interviewed these participants to help reveal information about why some teachers did not follow the hypothesized norm, specifically from the perspective of those who recognized that many teachers do follow the hypothesized norm.
3. For hypothesized norms that were disconfirmed: This type of participants chose the NC and indicated the alternative option (C1) as very rare. Interviewing these participants helped me understand the justifications and obligations that teachers perceive with regard to instructional actions that I might have wrongly hypothesized to be rare.

I used a maximum variation sampling strategy (Creswell, 2013) to select one participant per hypothesized norm, which allowed me to preserve multiple perspectives and shared patterns about their perceived obligations across different participants. Six participants (see Table 7) were selected and interviewed through one-on-one Zoom calls (https://www.zoom.us), each of which lasted for approximately one hour. The interviews were audio-recorded and transcribed automatically by an online program called Temi (https://www.temi.com/).

Table 8 shows the profiles of the six interviewees based on the information they provided in the survey and the interview. All interviewees identified their ethnicities as White. There was an even split between interviewees identifying their gender as male or female. Half of the interviewees were from Group 1, and the rest of them were from Group 2. Their teaching experiences varied between 2 and 31 years. Four of them indicated that they had experience enacting modeling tasks in the past. The interviewees' names used in the study were pseudonyms.

Table 7

Selection of Interviewees for Each Norm (n=6)

Hypothesized Norm	Pick C1 & indicate the alternative option NC is very rare	Pick NC & indicate the alternative option C1 is very typical	Pick NC & indicate the alternative option C1 is very rare
1. Factor specification (confirmed)[a]	1		
2. Symbolic representation (confirmed)	1		
3. Mathematical components (unable to confirm)		1	
4. Mathematical thinking dominance (unable to confirm)		1	
5. Model revision omitted (disconfirmed)			1
6. Politically neutral contexts (confirmed)	1		

Note. C1 = Canonical Option 1. NC = the noncanonical option.

[a] I further elaborate on which hypothesized norms were confirmed, disconfirmed, or unable to be confirmed in the Findings chapter.

Table 8

Background Information of the Interviewees (n=6)

Interviewee	**Ethnicity**	**Gender**	**Years of teaching experience**	**Years of teaching modeling experience**	**Group**
Lena	White	Female	22	20	1
Tyler	White	Male	12	8	1
Aaron	White	Male	2	0	1
Pate	White	Female	31	31	2
Ben	White	Male	13	0	2
Rebecca	White	Female	5	3	2

Note. The interviewees' names in this table were pseudonyms.

Data Analysis

Generally speaking, this study employed both qualitative and quantitative methods to analyze the breaching experiment survey and interview data. I describe the analyses for each of the overarching research questions in the following sections.

Analysis of Hypothesized Norms (RQ1)

The breaching experiment survey data served as the primary data source to investigate the participants' perceptions of the norms—that is, their responses about how they would react to the given scenarios and whether they thought other teachers would respond in a manner similar to Ms. Johnson (the fictional teacher). I used basic descriptive statistics for survey data, including percentage and central tendency (e.g., mode) of the multiple-choice items—for instance, "From the three different options below, indicate which action YOU would most likely do next." If the most-favored action was one of the hypothesized canonical teacher actions, then it potentially confirmed that it was a normative action in teachers' perceptions of modeling. If neither C0 nor C1 was the most-favored action, I looked at the participants' estimation of the typicalness of the comparison choice. If teachers predominantly selected the NC but indicated that C1 is very typical, this result could still provide evidence in support of the hypothesized norm. Norms are also about a community's *perception* of what is normal and not necessarily about what is truly normal. On the other hand, if they chose the NC and indicated that C1 is rare, this might disconfirm the hypothesized norm.

In this study, I did not use other advanced statistical analyses, such as the analysis of variance (ANOVA), to examine whether there were statistically significant differences among different norms or the potential differences across the subgroups in terms of teaching experience, modeling experience, and different conceptions of modeling. However, the data do allow for such analyses in my follow-up studies in the near future.

Analysis of Professional Obligations (RQ2)

Professional obligations could be found in participants' justifications as they (1) selected survey statements to explain their choices in the breaching experiment, (2) answered the survey's open-ended questions (i.e., participants' additional remarks about their choices), and (3) responded to the questions in the follow-up interview, in the case of the interviewees. The most robust data were found in 1, but 2 and 3 were used to corroborate and further interpret the findings from 1.

I recorded the selections of justification statements regarding each norm to determine the obligations perceived by the participants (see Table 9 for the example of Hypothesized Norm 1, factor specification). Then, I examined whether there was a connection between a participant's choice of the action and the justifications they used for that action; I created another table that provided information about the distribution of justification statements between participants who chose the canonical and noncanonical options. For instance, participants indicated how strongly they perceived the obligation of individual student thinking to justify their selection concerning Hypothesized Norm 1. See an example of Hypothesized Norm 1 in Table 10.

Table 9

Justification Statements for All Participants Regarding Hypothesized Norm 1 (n=88)

Hypothesized Norm 1 (factor specification)	**Obligation**							
	Individual		**Interpersonal**		**Disciplinary**		**Institutional**	
	JS 1.1	**JS 1.2**	**JS 2.1**	**JS 2.2**	**JS 3.1**	**JS 3.2**	**JS 4.1**	**JS 4.2**
Strongly considered	41	15	26	36	36	45	15	18
Considered	40	32	49	45	47	38	35	35
Did not consider	7	41	13	7	5	5	38	35

Table 10

Obligation of Individual Student Thinking Regarding Hypothesized Norm 1 – Factor Specification (n=88)

Obligation of individual student thinking regarding Hypothesized Norm 1	**Canonical option**		**Noncanonical option (NC)**	**Total**
	C1	**C0**		
Strongly considered	10	8	23	41
Considered	24	11	5	40
Did not consider	5	2	0	7
Total	39	21	28	88

If the participants entered additional remarks in response to the open-ended questions or a follow-up interview, I coded their responses by looking for the linguistic markers that indicated participants' rationale or justification, including words like "because," "since," "so," "in order to," and "so that" (see Table 11). I then categorized their justifications into different domains (i.e., individual, interpersonal, institutional, disciplinary, and other; Webel & Platt, 2015). For example, a participant might have mentioned that "I focus more on the mathematization part *because* it is what doing mathematics looks like." The use of "because" would flag this utterance for obligation coding. Based on what the utterance was related to, the justification statements were categorized into different domains: individual, interpersonal, institutional, disciplinary, and other. If the justification statement was related to a particular student or to student's individual needs, it was identified as an *individual obligation* (e.g., "I want this student to develop more advanced thinking, like being more critical, to recognize that his/her model has shortages and learn how to further revise it."). If the justification statement mentioned a group of students or the learning community, it was identified as an *interpersonal obligation* (e.g., "Each kid has his or her own way of solving this problem. They can learn about different ways of doing mathematics and practice being respectful."). If the justification statement related to an institutional requirement, it was identified as an *institutional obligation* (e.g., "Revisions are not required on the state tests."). If the justification statement was specific to the discipline, it was identified as a *disciplinary obligation* (e.g., "Mathematics is a rigorous and abstract subject, and it should be communicated with great precision."). Other justification statements not related to these four domains were identified as *other* (e.g., "I did that because I was trying out some new teaching methods.").

Table 11

Obligation Coding Scheme

Type of marker	Example conjunction	Code	Example statement
Causal – conditional conjunction	Because, since, so, in order to, so that, unless, without, if then	Individual	"I chose this action, so I could better support individual student thinking."
		Interpersonal	"To support the interactions among several students, I selected this option."
		Institutional	"This action best meets my department's pacing guide."
		Disciplinary	"I like this option because it best represents the concepts under consideration."
		Other	"This action best matches the teaching style I am comfortable with." or "I have been trying new teaching technique."

Note. The example statements are adapted from a previous study that used them as options for participants to select in a survey. From "Will teachers create opportunities for discussion when teaching proof in a geometry classroom?" by A. Erickson and P. Herbst, 2018, *International Journal of Science and Mathematics Education*, *16*(1), p. 173.

After open coding, I conducted a thematic analysis of the interview data at two levels, within each obligation and across different obligations (Creswell, 2013). For example, I further analyzed each interviewee's institutional obligations concerning modeling by analyzing the expectations of their department, school, and school district that impact their enactment of modeling. I then compared and contrasted how the four obligations were perceived differently by the interviewee. I later combined both the survey and interview findings to more fully answer the second research question regarding obligations and develop a more robust picture of teachers' practical rationality related to mathematical modeling. Notice that I placed the priority on this study's initial survey phase and used the subsequent interview phase to help explain the findings from the survey.

Trustworthiness of the Study

To increase trustworthiness, I conducted a pilot study before the study officially started. In addition, this study employed triangulation, reflexivity, advisor consultation, and memos with dense descriptions to ensure trustworthiness after its launch.

I conducted a pilot study with two experienced teachers to test the feasibility of the instruments (i.e., breaching experiment survey and interview protocol). This small-scale pilot study helped identify potential logistical problems that might occur and determine modifications needed in the research design. As the study progressed, I used the technique of data triangulation to compare the interview data with the survey responses to analyze response consistency and to check the accuracy of my interpretation of participants' obligations. I also kept reflections by bracketing my preconceptions and biases to ensure that I was aware of my impact on data interpretation (e.g., What I had considered to be a “real” implementation of modeling tasks might not have matched teachers’ perceptions. The disciplinary obligation might not have been the most prevalent type, as I had expected.). Additionally, I frequently consulted with my advisor to analyze the open-ended survey responses and the interview data to ensure that the analysis reflected participants’ expressed obligations. Lastly, I kept memos of dense descriptions regarding the process of data collection and analysis to show how repeatable or unique the study might be.

CHAPTER 4

Findings

In this chapter, I present the findings of two research questions:

1. What norms are perceived by secondary teachers in relation to mathematical modeling? (RQ1)
2. What professional obligations are perceived by secondary teachers in relation to mathematical modeling? (RQ2)

Hence, I lay out the findings in two major sections corresponding to these two research questions. In the first section, I primarily discuss the findings of the survey data from the study's fictional classroom scenarios, revealing the extent to which the data confirms or disconfirms the hypothesized norms described in the previous chapter. As a reminder, I hypothesized six norms to describe what I believe to be teachers' typical behaviors as they lead students through a modeling task. The secondary teachers who participated in this study's online survey selected from multiple-choice options of how they would behave in an illustrated fictional modeling lesson, as well as how they think other teachers would behave. During the lesson, the participants encountered four scenarios designed to elicit their perceptions about what is a normal classroom occurrence and what is a breach of the hypothesized norms. In this chapter, I first evaluate the participants' chosen actions in relation to the hypothesized norms. Then, in the chapter's second section, I share the findings of teachers' perceived obligations and how they are related to their selected actions in the classroom scenarios. To represent the findings, I primarily used the frequency counts for the selected options on the survey and representative excerpts from the follow-up interviews.

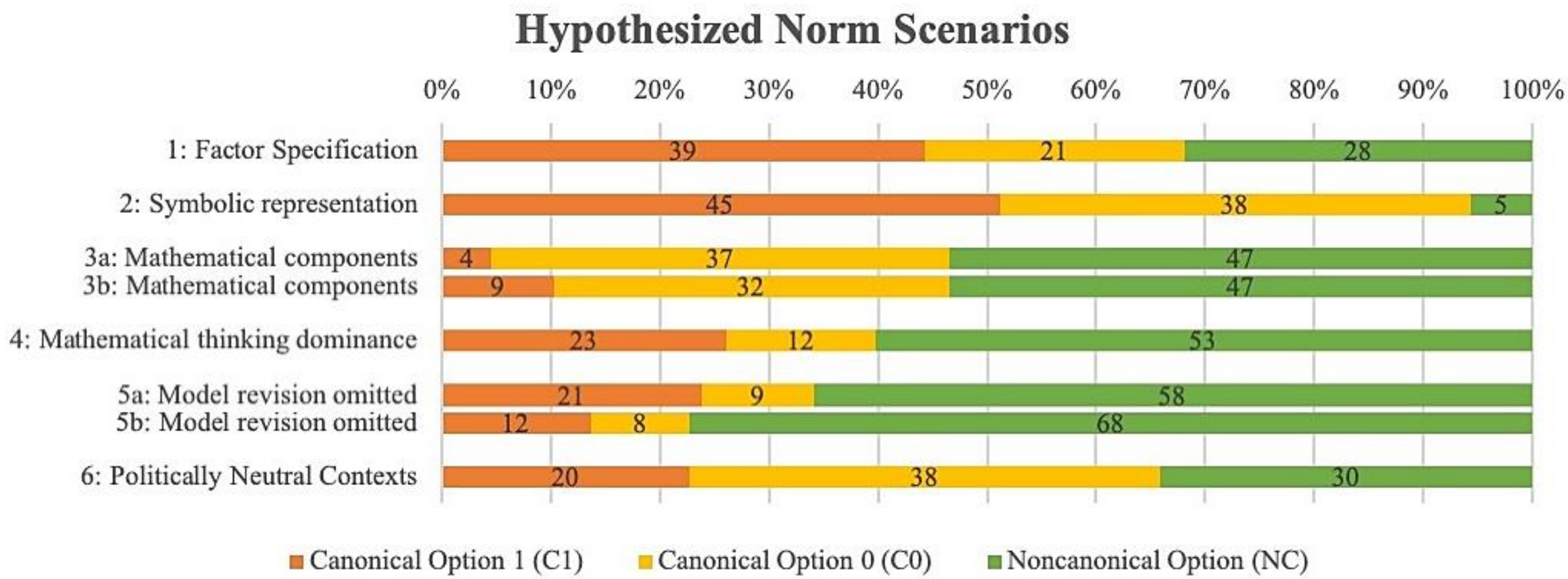
RQ1: What Are the Perceived Norms?

As described in the Method chapter, each scenario presented in the breaching experiment survey was designed to associate with one of this study's six hypothesized norms related to mathematical modeling. The scenarios were not necessarily part of a full modeling cycle, but each scenario involved potential opportunities that connected to at least one stage of the modeling process. Scenario 1 (factor specification) was related to the simplifying stage; Scenario 2 (symbolic representation) was related to the working mathematically stage; Scenario 3 (mathematical components) was related to the mathematizing stage; Scenario 4 (mathematical thinking dominance) was related to the constructing stage and the interpreting stage; Scenario 5 (model revision omitted) was related to the validating stage and the exposing stage; and Scenario 6 (politically neutral contexts) was related to the problem situation in which students would engage. Each scenario included three options (i.e., two canonical actions and one breaching action relative to the particular hypothesized norm), and teachers could pick one as their response to the classroom situation presented in the survey.

Figure 7 shows an overview of teachers' selections regarding the six hypothesized norms. Hypothesized Norms 1, 2, and 6 (factor specification, symbolic representation, and politically neutral context, respectively) had the highest rates of teachers selecting the canonical options (C1 or C0), whereas Hypothesized Norm 5 (model revision omitted) had the lowest rates of teachers selecting the canonical options. I present findings on each hypothesized norm in turn in this chapter; however, the teachers' selections of the scenario options constitute just the first indication of a norm. Further evidence, either confirming or disconfirming, comes from examining what they believe other teachers commonly do in those scenarios.

Figure 7

Teachers' Selected Options for the Six Hypothesized Norms (n=88)



Hypothesized Norm 1 (Factor Specification)

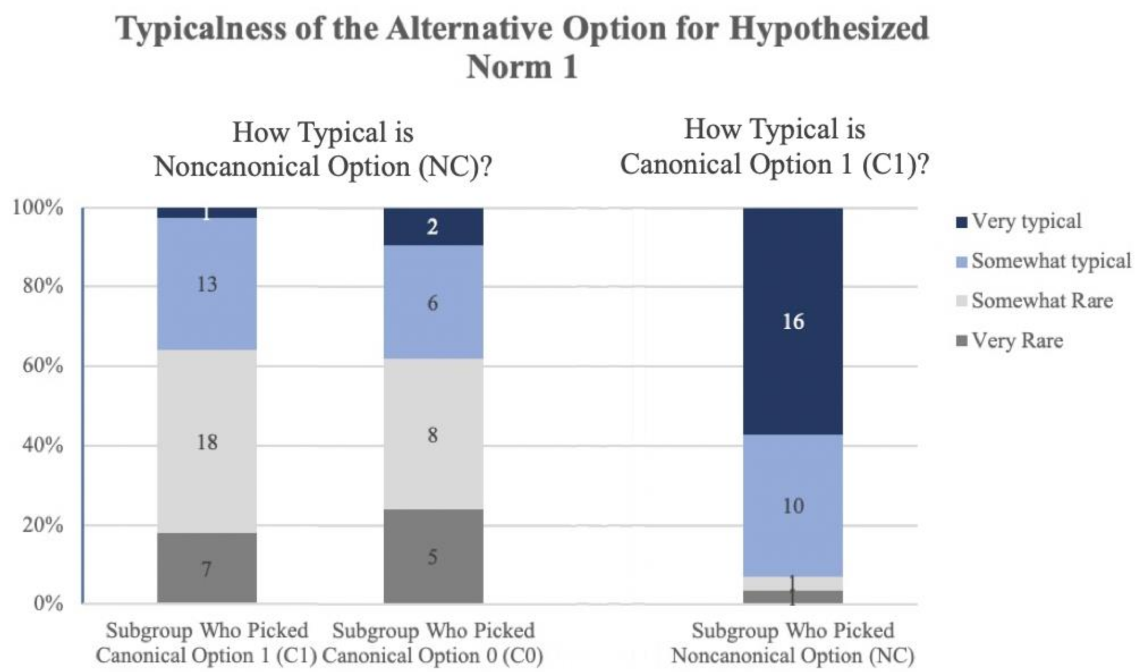
The first norm I hypothesized was that the teacher (rather than students) identifies precisely which information or factors should be included in the task.

Teachers participating in the survey were divided into two groups, and the participants in Group 1 (*n*=88) were shown a scenario related to which factors (e.g., initial amount, growth rate, temperature) should be included in their students' mathematical models of bacterial reproduction. For Hypothesized Norm 1, 39 (44.3%) of the teachers chose to identify precisely which factors students should include when developing their models, and 21 (23.9%) chose to give students directions about what should not be considered. On the other hand, 28 of 88 (31.8%) in Group 1 chose the NC (i.e., the option that departed from the hypothesized norm), encouraging students to decide for themselves what they want to include rather than directly leading them to include or exclude specific factors in their models. These results show that teachers reported being substantially more likely to give clear directions for factor selection in the study's scenario, rather than leave it up to students. These data are aligned with the hypothesized norm.

Recall that participants who picked one of the two canonical options were asked to indicate the typicalness of the NC, while participants who picked the NC were asked to indicate the typicalness of C1. Also, recall that the C1 choice for any given breaching scenario always followed the hypothesized norm more strictly than C0, fully adhering to the norm while C0 edged closer to the boundary between the norm and the breach. For Hypothesized Norm 1, of the teachers (60) who selected one of the canonical options (C1 or C0), 63.3% of them indicated that the NC is somewhat rare (26) or very rare (12). For the teachers (28) who selected the NC, 92.9% of them indicated that C1 is somewhat typical (10) or very typical (16). These findings are displayed in Figure 8. They provide more evidence to confirm Hypothesized Norm 1—that is, teachers (rather than students) identified precisely which factors should be included in the task during the modeling activities. Although a minority of teachers selected the NC, they reported that it was very typical or somewhat typical for teachers in general to pick C1.

Figure 8

Teachers Identified the Typicalness of the Alternative Option for Hypothesized Norm 1 – Factor Specification (n=88)



Hypothesized Norm 2 (Symbolic Representation)

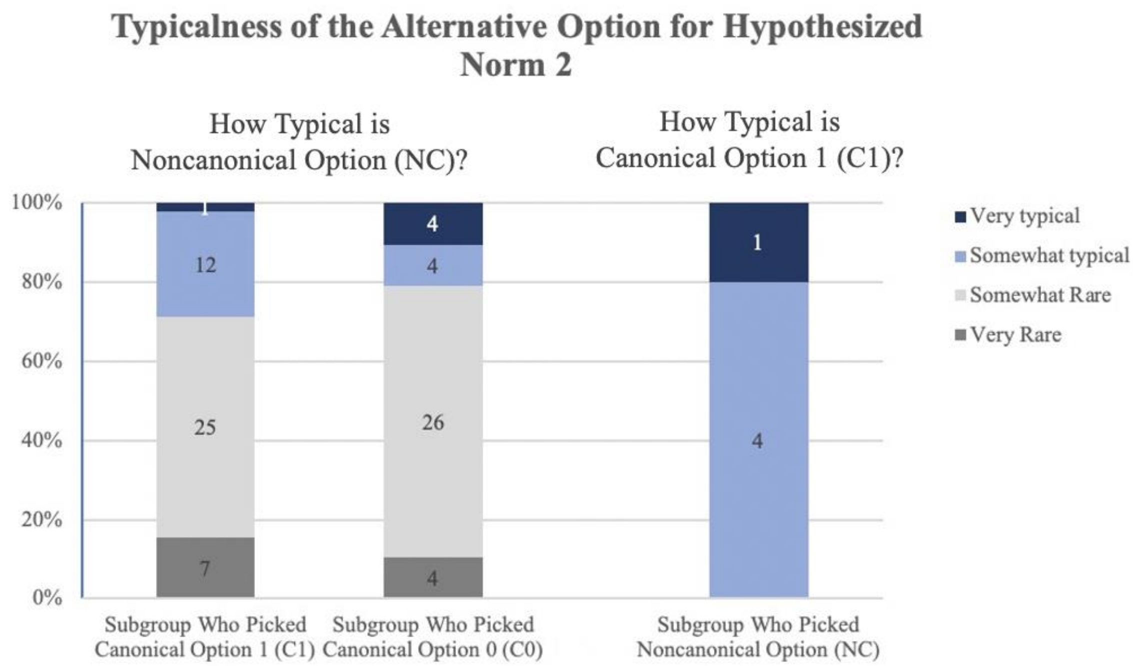
Per Hypothesized Norm 2, students are expected to find a symbolic representation (e.g., equations) as their model.

Teachers in Group 1 were shown a scenario in which the fictional teacher in the illustrated lesson discussed with students which type of representation could serve as a bacterial reproduction model. For Hypothesized Norm 2, only five (5.7%) of the 88 teachers chose the NC. The NC was the survey choice where the fictional teacher approved the simulated students' use of a tabular representation without requiring them to translate their table into a symbolic function. 45 (51.1%) suggested that finding a symbolic function or equation for the problem was necessary for modeling, and 38 (43.2%) expected students to find a symbolic representation. These results reveal that the vast majority of the study participants expected students to find a symbolic representation as their product in the modeling activity, which confirms the hypothesized norm.

As shown in Figure 9, of the teachers (83) who selected a canonical norm (C1 or C0), 74.7% of them indicated that the NC is somewhat rare (51) or very rare (11). In other words, they perceived the hypothesized breach to be, indeed, quite uncommon. For the teachers (5) who selected the NC, they indicated that C1 is somewhat typical (4) or very typical (1). These findings provide strong evidence confirming Hypothesized Norm 2—that is, students are expected to find a symbolic representation (e.g., equations) as their model, at least in the case of the bacterial reproduction task.

Figure 9

Teachers Identified the Typicalness of the Alternative Option for Hypothesized Norm 2 – Symbolic Representation (n=88)



Hypothesized Norm 3 (Mathematical Components)

For Hypothesized Norm 3, I postulated that the teacher gives students unambiguous directions on what they are expected to use in terms of the mathematical operations and components.

In the survey, I intentionally had two overlapping hypothesized norms for triangulation, Hypothesized Norms 3 and 5 (mathematical components and model revision omitted). Therefore, both Groups 1 and 2 saw some form of a scenario related to Hypothesized Norm 3. While Group 1 focused on a fictional modeling lesson concerning bacterial reproduction, Group 2 responded to survey questions concerning a fictional modeling lesson about water usage. Although the tasks were different, what was the same across the modeling scenarios was the simulated teacher directing students (or not) to use particular mathematical operations or components, such as the standard form of an exponential function (bacterial reproduction task) or a linear function (water usage task).

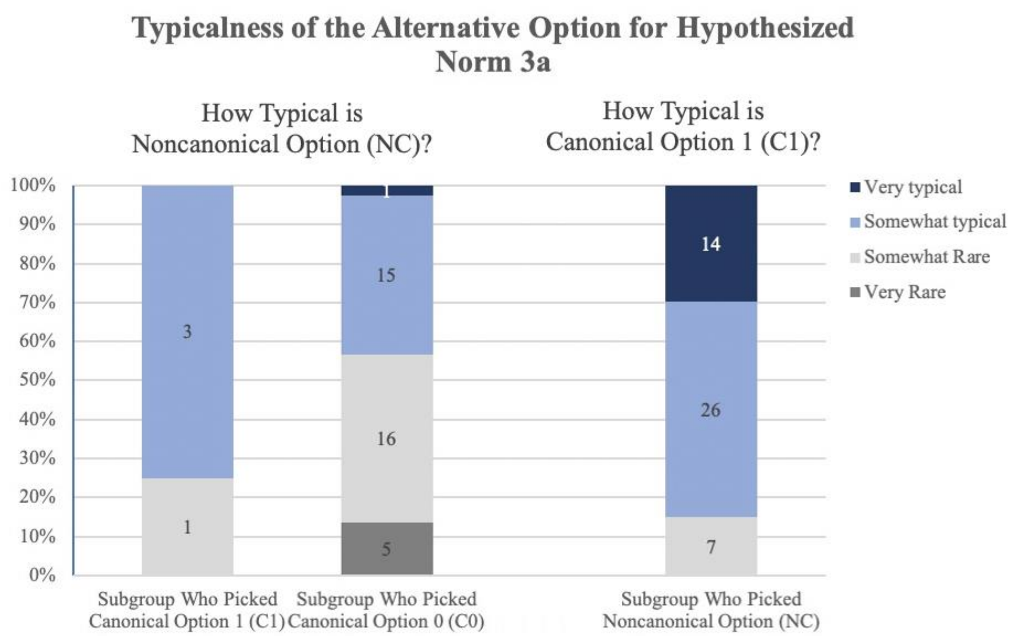
For Hypothesized Norm 3, in both Group 1 (*n*=88) and Group 2 (*n*=88), more than half (53.4%) of the teachers (47 for each group) chose the option that involved encouraging students to decide on the mathematical operations and components themselves (NC) rather than simply telling students the operations and components. For the rest of the teachers, most (37 teachers in Group 1 and 32 teachers in Group 2) chose to remind students what they had previously learned in class without giving the specific form. Only nine teachers (10.2%) in Group 1 and four teachers (4.5%) in Group 2 chose to give students a clear direction on what should be used in terms of explicit mathematical operations and components. These data do not confirm Hypothesized Norm 3; instead, the findings suggest that many of the study's teacher participants, at least when responding to a simulated situation, were open to the possibility of students considering different mathematical operations and components during the development of a mathematical model. It cannot be determined from these data whether the participants presumed that students would naturally select the fictional teachers' intended mathematical operation or whether the participants were open to a wide variety of resultant models.

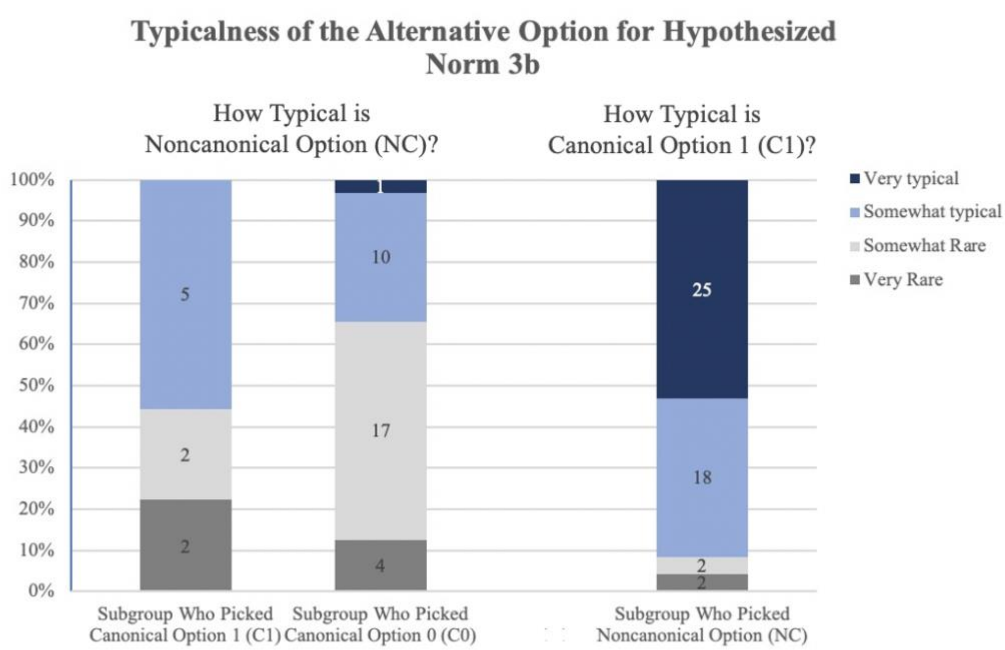
Figure 10 displays the teachers' responses about the typicalness of the action that they did not choose. Figure 10 is divided into two charts that show Norm 3a for Group 1 and Norm 3b for Group 2 to distinguish between the Group 1 participants who responded to the bacterial reproduction task and the Group 2 participants who responded to the water usage task. In Group 1, for the teachers (41) who selected one of the canonical actions (C1 or C0), 53.7% of them indicated that the NC is somewhat rare (17) or very rare (5). For the teachers (47) who selected the NC, 85.1% of them indicated that C1 is somewhat typical (26) or very typical (14). In Group 2, for the teachers (41) who selected one of the canonical actions (C1 or C0), 61.0% of them indicated that the NC is somewhat rare (19) or very rare (6). For the teachers (47) who selected the NC, 91.5% of them indicated that C1 is somewhat typical (18) or very typical (25). Taken together, these results provide evidence in favor of the hypothesized norm because a substantial number of teachers regarded C1 as typical and others regarded NC as rare.

The previous paragraph demonstrates that some evidence supports Hypothesized Norm 3. In other words, this study's evidence suggests that teachers do, indeed, direct students to incorporate specific mathematical operations or components into the students' mathematical models. The evidence in favor of Hypothesized Norm 3 was predominantly from the participating teachers' perceptions of *others'* typical behavior. However, because more participants chose *for themselves* the NC instead of the canonical actions, I could not fully confirm or disconfirm Hypothesized Norm 3. It is possible that the norm exists or that many teachers assume *other* teachers uphold the norm. Still, it is also possible, given these data, that teachers do provide flexibility to students in choosing the mathematical operations or components to use in their modeling.

Figure 10

Teachers Identified the Typicalness of the Alternative Option for Hypothesized Norms 3a and 3b – Mathematical Components (n=88)





Note. Norm 3a reports results from Group 1 participants concerning the bacterial reproduction task. Norm 3b reports results from Group 2 concerning the water usage task.

Hypothesized Norm 4 (Mathematical Thinking Dominance)

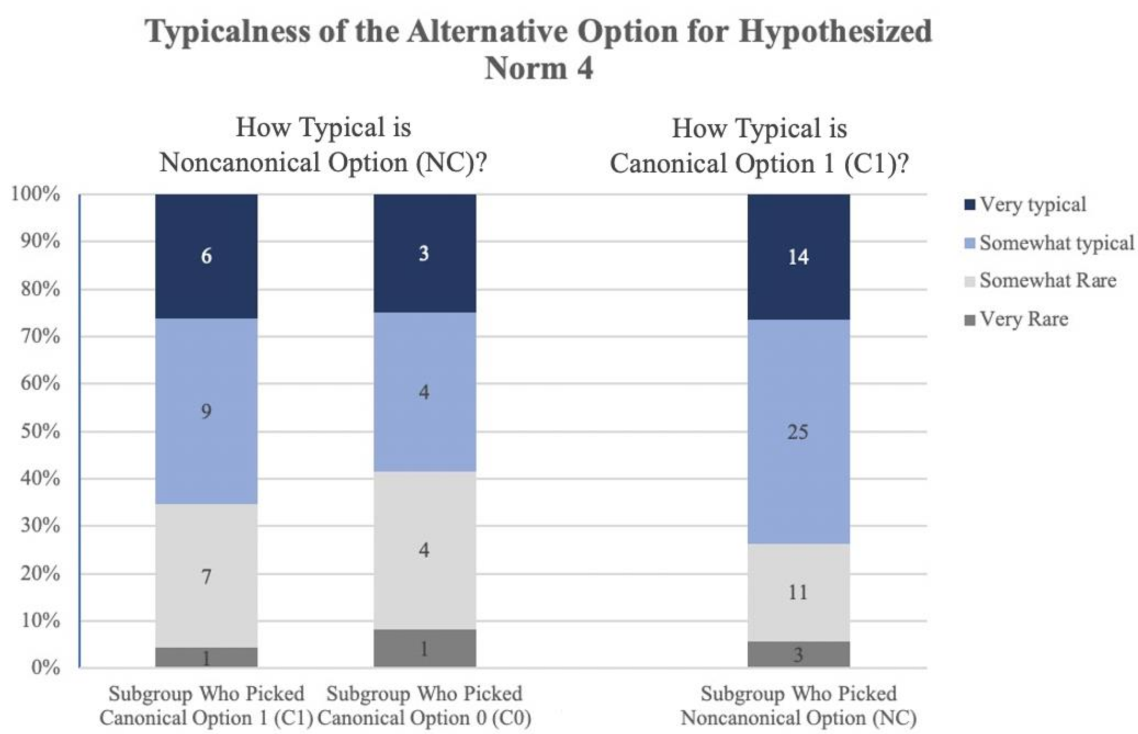
According to Hypothesized Norm 4, students are expected to primarily (or exclusively) engage in mathematical thinking rather than nonmathematical thinking.

Teachers in Group 2 (*n*=88) responded to a breaching scenario where students brought in nonmathematical thinking about water usage. For Hypothesized Norm 4, 53 (60.2%) of the teachers chose the teacher action that affirmed students' nonmathematical thinking by asking for additional real-world thoughts and experiences. That is, a majority of teachers selected the action that was hypothesized to be a norm breach. On the other hand, 12 (13.6%) of the teachers chose the action of steering students from nonmathematical thinking to mathematical thinking by asking them to consider two of the variables students had mentioned and put them into the equation. Also, 23 (26.1%) of the teachers even more directly asked students to merely consider an average person's water usage, rather than building from their own experiences, in order to develop a mathematical function. These results imply that many teachers, at least when confronted with this survey scenario, were open to the possibility of their students engaging in nonmathematical thinking during mathematical modeling, which runs counter to the hypothesized norm.

As shown in Figure 11, for the teachers (35) who selected one of the canonical actions (C1 or C0), only 37.1% of them indicated that the NC was somewhat rare (11) or very rare (2). In other words, even the relatively few teachers who followed the hypothesized norm often felt as though other teachers would breach the hypothesized norm. For the teachers (53) who selected the NC, 73.6% indicated that the C1 was somewhat typical (25) or very typical (14). These results do not confirm the hypothesized norm. They reveal the complexity of Hypothesized Norm 4, which I will discuss in the next chapter. For the moment, it suffices to state that, based on the survey data, I am unable to confirm or disconfirm this hypothesized norm.

Figure 11

Teachers Identified the Typicalness of the Alternative Options for Hypothesized Norm 4 – Mathematical Thinking Dominance (n=88)



Hypothesized Norm 5 (Model Revision Omitted)

Hypothesized Norm 5 states that all students are expected to arrive at the same model, and model revision (beyond resolving discrepancies between different models) is not expected.

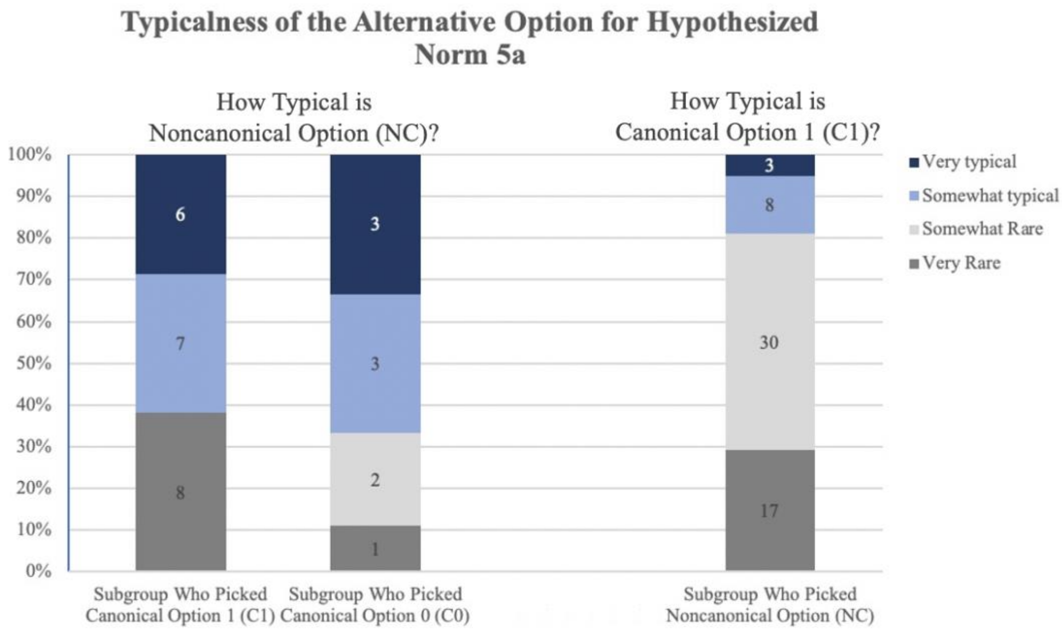
Both groups of teachers saw a scenario related to whether or not students would have an opportunity to revise their completed mathematical models. As a reminder, for triangulation purposes, both Groups 1 and 2 (*n*=88 for both groups) experienced different contexts of breaching scenarios related to Hypothesized Norm 5. Most of the teachers in the study chose the action of encouraging students to revise their models. More specifically, 58 teachers (65.9%) in Group 1 and 68 teachers (77.3%) in Group 2 chose the NC. Only 21 teachers (23.9%) in Group 1 and 12 (13.6%) in Group 2 chose to start a new lesson after all students shared their work. Even fewer—nine teachers (10.2%) in Group 1 and eight teachers (9.1%) in Group 2—chose to differentiate students by asking the rest of the class to revise and arrive at the same model that had been presented by one simulated group of students. These data run counter to Hypothesized Norm 5; they indicate that the teachers, at least in this simulated scenario, were open to the opportunity for model revision during mathematical modeling. Note that this does not necessarily imply that they *would* grant such opportunities for revision in their own teaching because, in reality, there may be different constraints than the conditions presented in the survey. These constraints and conditions are discussed further in the next chapter.

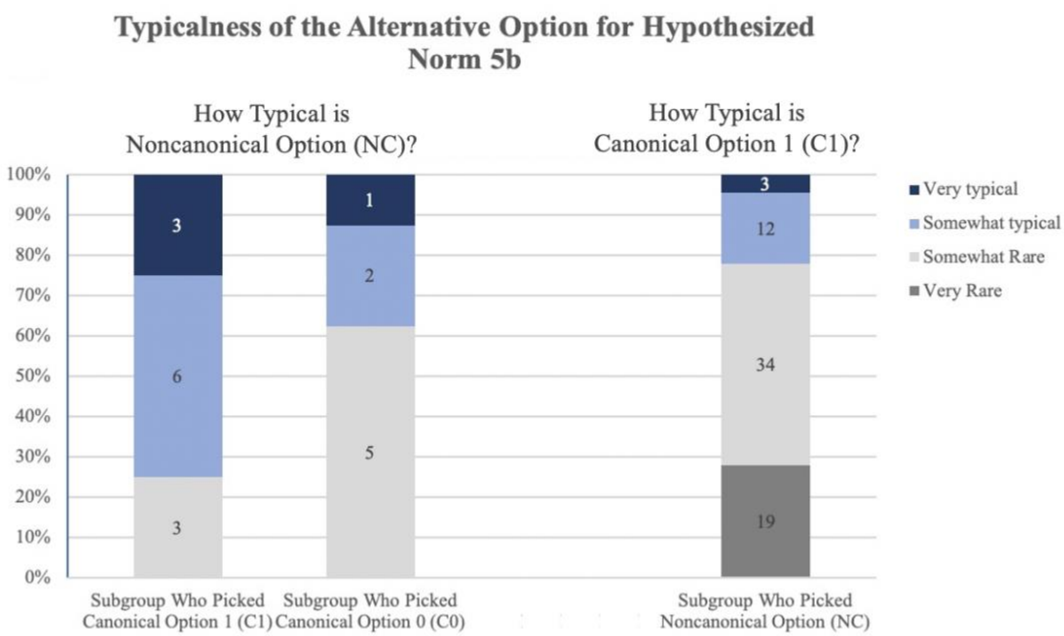
Figure 12 is divided into two charts that show Norm 5a for Group 1 and Norm 5b for Group 2 to distinguish between the Group 1 who responded to the bacterial reproduction task and the Group 2 who responded to the water usage task. When asking teachers to indicate the typicalness of the other option (i.e., the alternative they did not choose), in Group 1, only 36.7% of the teachers who selected one of the canonical actions (C1 or C0) indicated that the NC is somewhat rare (2) or very rare (9). For the teachers (58) who selected the NC, only 19.0% of them indicated that C1 is somewhat typical (8) or very typical (3). In Group 2, the results were similar. For the teachers (20) who selected one of the canonical actions (C1 or C0), 40.0% of them indicated that the NC is somewhat rare (8) or very rare (0). For the teachers (68) who selected the NC, only 22.1% of them indicated that C1 is somewhat typical (12) or very typical (3). See Figure 12 for details.

These data do not at all confirm Hypothesized Norm 5 and, in fact, suggest that the hypothesized breach is fairly typical. Many teachers chose to allow model revisions in the simulations, and they agreed that model revisions are somewhat typical or very typical in the classroom—or, at the very least, model revisions are perceived as desirable based on their responses to the survey. Hence, the data leads me to disconfirm the hypothesized norm.

Figure 12

Teachers Identified the Typicalness of the Alternative Options for Hypothesized Norms 5a and 5b – Model Revision Omitted (n=88)





Note. Norm 5a reports the results from Group 1 concerning the bacterial reproduction task. Norm 5b reports the result from Group 2 concerning the water usage task.

Hypothesized Norm 6 (Politically Neutral Contexts)

According to the sixth and final of this study's hypothesized norms, teachers expect students to primarily (or exclusively) work on politically neutral tasks instead of social-justice-oriented tasks.

Teachers in Group 2 (n=88) were shown two alternative versions of the original water task. One of the two included a racially infused context involving the water crisis that happened in a specific city, Flint, Michigan, which was intended to explore Hypothesized Norm 6. The other alternative task involved an environmental concern about the potential dangers of contaminated water in a generic city. This version of the task did not specifically mention race. For Hypothesized Norm 6, 30 teachers (34.1%) reported that they would implement the modeling task involving race and social justice rather than the task with a politically neutral context. For the rest of the teachers, 38 of them (43.2%) chose the alternative task with an emphasis on environmental concerns without racial issues, and 20 teachers (22.7%) chose the original politically neutral task.

This norm, more than the others, spurred written comments on the survey. I shared a few of the written responses here, as they illustrated the overall finding that most teachers avoided the racially infused version of the task. Several teachers showed strong resistance toward the racially infused version of the task. One of them mentioned, "I would not do any project that touched on race or politics. Not worth it." Another teacher commented, "Politics should not be included in this. There is another class for that!!" Even an interviewee who did choose the racially infused version described a modification that they would make:

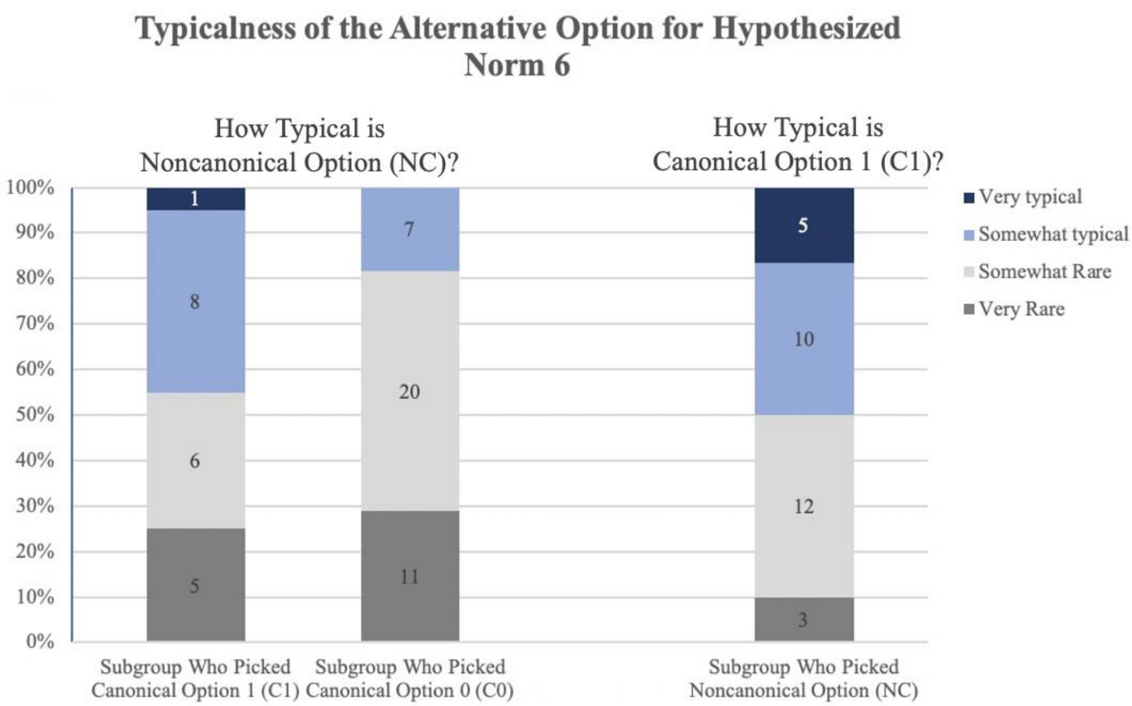
> You have to take out the portion of … [the] math problem [concerning African American issues]. We need to teach historical truth. However, that piece does not add anything to the problem. Instead, it divides the class and immediately takes the thought off math and shifts it to the racists that run Flint, Michigan.

These data reveal that most teacher participants chose tasks that did not involve racial concerns. Further insight into their justifications for doing so are presented in the next section.

When asking teachers to indicate the typicalness of the other option, for the teachers who selected one of the canonical actions (C1 or C0), 42 of 58 (72.4%) indicated that breaching the canonical action is somewhat rare (26) or very rare (16). See Figure 13. For the teachers who selected the NC, 15 of 30 (50.0%) indicated that it is somewhat typical (10) or very typical (5) for teachers to follow the hypothesized norm, that is, to implement the politically neutral tasks in class. These findings further confirm that teachers lean toward politically neutral tasks instead of social-justice-oriented tasks.

Figure 13

Teachers Identified the Typicalness of the Alternative Options for Hypothesized Norm 6 – Politically Neutral Contexts (n=88)



Summary of Findings for RQ1

As seen with the results above, strong evidence supports Hypothesized Norms 1, 2, and 6 (see Table 12). Within these three norms, the strongest evidence was seen for Hypothesized Norm 2 (symbolic representation); 94.3% of survey participants selected the canonical options, that is, the expectation of a symbolic representation in the students' mathematical modeling solutions. 68.2% and 65.9% of survey participants chose the canonical options in the Hypothesized Norm 1 scenario (teacher specifying which mathematical factors should be included) and Hypothesized Norm 6 scenario (the expectation of primarily or exclusively working on politically neutral tasks), respectively.

For Hypothesized Norms 3 and 4 (mathematical components and mathematical thinking dominance), the evidence was not strong enough to confirm or disconfirm the norms because less than half of the participating teachers picked the canonical options. Even more surprising was that Hypothesized Norm 5 (model revision omitted) revealed evidence for disconfirmation, suggesting that the opposite of the hypothesized norm might be true because 65.9% of Group 1 and 77.3% of Group 2 selected the NC, respectively. The teacher participants, at least in these survey-based scenarios, favored actions that allowed model revisions and indicated the NC as somewhat or very typical.

The previous paragraph summarizes the actions the participants would choose for themselves were they reacting to the simulated students in the survey. Only a portion of the survey questions focused on the participants' own behaviors. Other questions explored the participants' perceptions about what they thought was typical behavior for other teachers. Their responses are summarized in this and the next paragraph. From the results related to perceived typicalness, additional evidence confirmed Hypothesized Norms 1, 2, and 6 (factor specification, symbolic representation, and politically neutral context). For each of these three norms, a sizeable portion of participants who followed the hypothesized norms responded that the breaching actions are somewhat or very rare. In addition, most participants who departed from Hypothesized Norms 1, 2, and 6 regarded it typical among other teachers to do the canonical action.

For Hypothesized Norm 3 (mathematical components), both survey groups indicated that the breaching action is somewhat or very rare. Stated another way, the canonical action (C1) that I contrasted against the breaching action (NC) is somewhat or very typical, which supports the hypothesized norm. However, when considering the option the participants selected for themselves, I concluded that this study does not have robust evidence to confirm or disconfirm Hypothesized Norm 3. The data for Hypothesized Norm 4 (mathematical thinking dominance), however, showed mixed evidence in terms of typicalness. The majority of the teachers who departed from the hypothesized norm indicated that the canonical action C1 is somewhat or very typical, whereas most teachers who abided with the hypothesized norm indicated that the NC is somewhat or very typical. Finally, Hypothesized Norm 5 (model revision omitted) was disconfirmed; both groups indicated that it was the breaching action (NC) that is somewhat or very typical and that the contrasted canonical action (C1) is somewhat or very rare.

In sum, the three hypothesized norms most strongly confirmed by the survey data are the following:

- *Norm 1 (factor specification).* The teacher (rather than students) identifies precisely which information or factors should be included in the task.
- *Norm 2 (symbolic representation).* Students are expected to find a symbolic representation (e.g., equations) as their model.
- *Norm 6 (politically neutral contexts).* Students are expected to primarily (or exclusively) work on politically neutral tasks instead of social-justice-oriented tasks.

Conversely, the survey data presented strong evidence against Hypothesized Norm 5 (model revision omitted). With these findings in mind, I now turn to the obligations that the teachers perceived in relation to mathematical modeling.

Table 12

Summary of the Findings for RQ1

Norm	Evidence from the selected option	Evidence from typicalness	Conclusion for the hypothesized norm
1. Factor specification	Strong	Strong	Confirm
2. Symbolic representation	Strong	Strong	Confirm
3a & 3b. Mathematical components	Weak	Strong	Not able to confirm/disconfirm
4. Mathematical thinking dominance	Weak	Mixed	Not able to confirm/disconfirm
5a & 5b. Model revision omitted	Weak	Weak	Disconfirm
6. Politically neutral contexts	Strong	Strong	Confirm

Note. This study used 66.0% as the division between the categories of strong and weak.

RQ2: What are the obligations?

In this section, I address the second research question: What professional obligations are perceived by secondary teachers in relation to mathematical modeling? I hypothesized that teachers' preferred actions concerning mathematical modeling scenarios would draw upon their obligations in the following categories: obligations to individual students, interpersonal relationships and interactions, the discipline of mathematics, and their institutions. I have structured this section based on the findings from the previous section, meaning I first examine the obligations related to the confirmed norms (Norm 1 [factor specification] and Norm 2 [symbolic representation]). Then I present findings concerning the unconfirmed hypothesized norms (Hypothesized Norm 3 [mathematical components] and Hypothesized Norm 4 [mathematical thinking dominance]) and the disconfirmed hypothesized norm (Hypothesized Norm 5 [model revision omitted]). Last but not least, I present the findings associated with the last confirmed hypothesized norm (Norm 6 [politically neutral contexts]). I have separated this norm from the other confirmed norms because, out of all norms in the study, it is the only one associated with teachers' task selection decisions made prior to task enactment, whereas the others are part of the enacted modeling cycle from Blum and Leiß (2005).

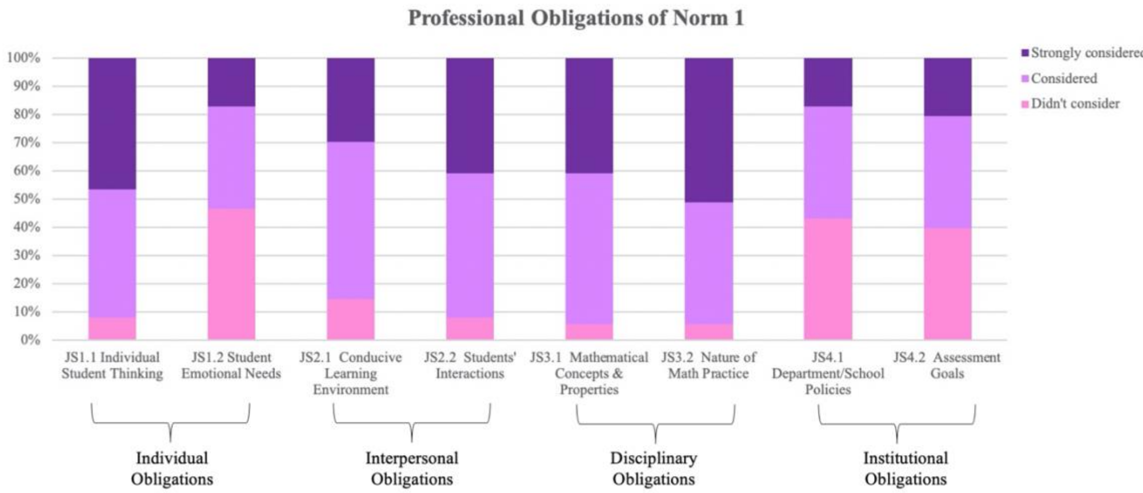
Regarding terminology in this section, for the hypothesized norms that have been confirmed by this study's results, I refer to them as *Norms* rather than *Hypothesized Norms,* whereas I keep the label *Hypothesized Norms* for those that have not been confirmed. I use *Norms* not to suggest that they are universal norms in classroom enactment but to support the reader in recollecting the findings for RQ1 as I now present findings for RQ2. Additionally, for ease of reading, I use *canonical group* to refer to those who chose either of the canonical options (C1 or C0) in the specific scenario being presented. For example, the canonical group of Norm 2 (symbolic representation) would be the participants who, when faced with Scenario 2, selected the option associated with requiring a symbolic function or the option associated with expecting a symbolic function. On the other hand, I use *noncanonical group* to refer to those who chose the option that breaches the hypothesized norm in the scenario. For example, the noncanonical group of Norm 2 would be the participants who selected the option that approves the simulated students' use of a tabular representation without requiring them to translate their table into a symbolic function. Note that the canonical group and the noncanonical group vary depending on the norm in question—they are not fixed subgroups.

Confirmed Norms 1 & 2: Factor Specification & Symbolic Representation

As explained in the Method chapter, participants were asked to select one or more from a list of eight justification statements after they selected their preferred action from the options given (C1, C0, and NC) for each of the survey's breaching scenarios. The goal was to understand how the participants justified their choices. As previously detailed in Table 5 of this dissertation, each obligation category—individual, interpersonal, disciplinary, and institutional—aligned with two of the eight justification statements. For the teacher participants who responded to survey questions about Norm 1 (teacher identifies precisely which factors should be included in their model), the obligations that they most strongly considered were those in the disciplinary categories. 96.6% of respondents considered or strongly considered both statements related to disciplinary obligations (see Figure 14). In the disciplinary category, the statements said that the action best (1) represented the underlying mathematical concepts and properties and (2) supported students' understanding of the nature of the mathematical practice. Furthermore, participants reported that they highly considered (93% considered or strongly considered) one statement in the individual category—i.e., the statement about promoting individual student thinking.

Figure 14

The Professional Obligations of Norm 1 – Factor Specification (n=88)



Because Norm 1 (factor specification) was confirmed, I looked specifically at the 60 teachers in the canonical group of the associated breaching scenario to better understand their justifications for their responses (see Figure 15). Of those teachers, 29 (48.3%) and 28 (46.7%) chose their action because they strongly considered how to support students' understanding of the underlying mathematical concepts and properties (JS 3.1) and the nature of mathematical practice (JS 3.2), respectively. One teacher, Aaron, explained why he picked the canonical option: "Since the problem is more mathematically based, I think adding items' temperature and humidity would make the problem more confusing for students especially if the problem is focusing on solving an exponential function." He emphasized the mathematical nature of the task and mentioned that "focusing on" the underlying mathematical concept (exponential functions) would be preferred to other things that may make "the problem more confusing." I interpreted

Aaron’s comment as noting a disciplinary obligation to the mathematics embedded in the modeling task, and he also indicated an individual obligation not to confuse students.

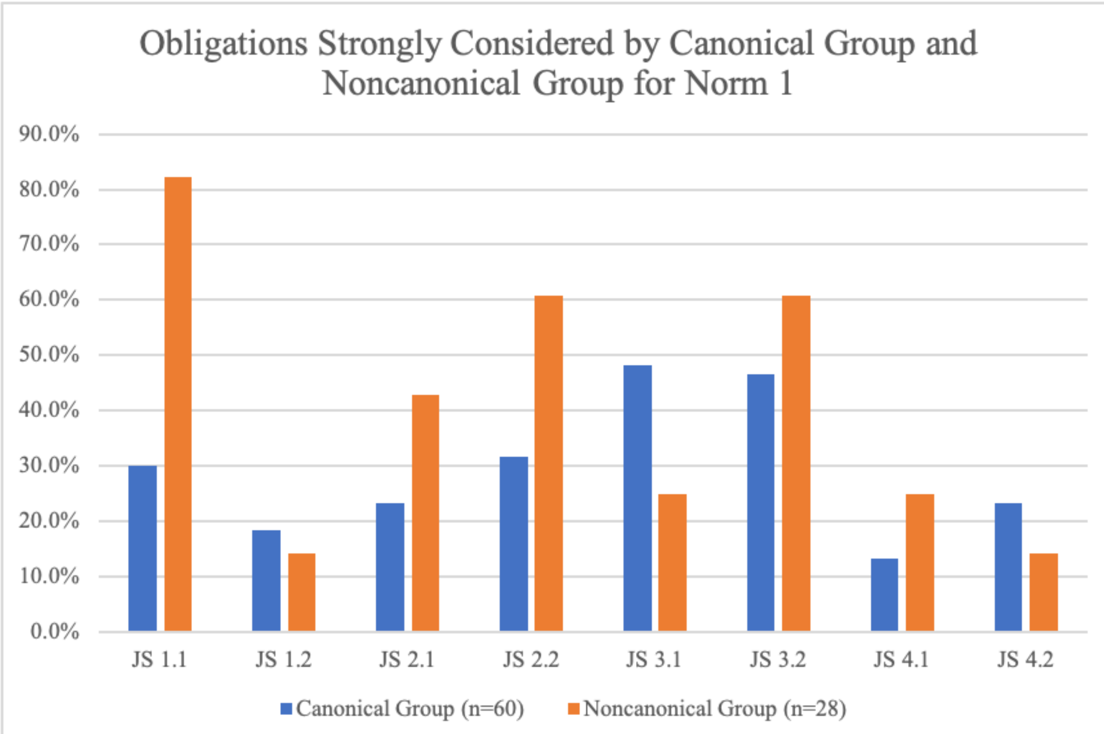
Of the 28 teachers in the noncanonical group, 23 (82.1%) indicated they strongly considered the obligation to develop individual student thinking (JS 1.1). For example, in the interview, Tyler mentioned the following:

> My personal goal is to help students become independent critical thinkers. If I just give them information, this whole process is lost, and I fail my goal. Giving them support without giving them answers is the best way to achieve this.

In this excerpt, Tyler talked about how telling students which factors to consider and which to eliminate ran contrary to his goal of developing individual students’ independent thinking. Thus, he viewed himself as having an obligation to attend to individual students and their thinking. What Tyler expressed was a different version of the individual obligation than what Aaron expressed when he said that he did not want to confuse students with too many factors. The differences correspond with the different actions these two participants chose for the Norm 1 scenario (Tyler chose to allow student determination, and Aaron chose to specify the factors for the students.).

Figure 15

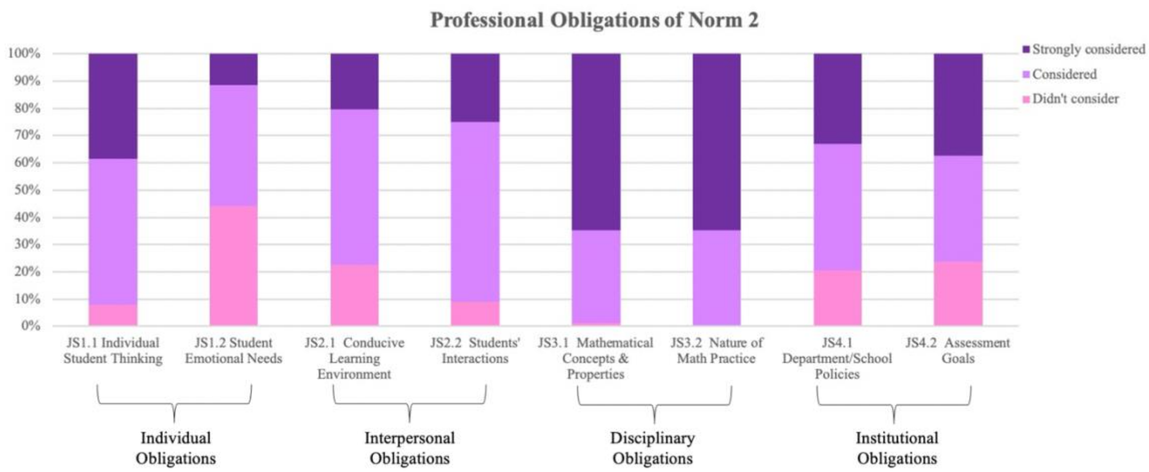
Professional Obligations Strongly Considered by Different Groups in Terms of Norm 1 – Factor Specification



The above findings suggest that disciplinary obligations were driving considerations for teacher participants who felt the need to identify precisely the factors students should include in the tasks. Similarly, for Norm 2 (symbolic representation), these same obligations were largely at play. When responding to Norm 2 on the survey, 64.8% of the participants felt strongly obligated to the mathematics discipline, that is, mathematical concepts and properties and the nature of the mathematical practice (see Figure 16). Furthermore, 53.4% and 38.6% considered and strongly considered the individual student thinking justification in the individual obligation category. Yet, 44.3% of the teachers did not consider the category's other justification statement—this action best supports individual students' emotional needs—when they picked their preferred action for the given breaching scenario.

Figure 16

The Professional Obligations of Norm 2 – Symbolic Representation (n=88)

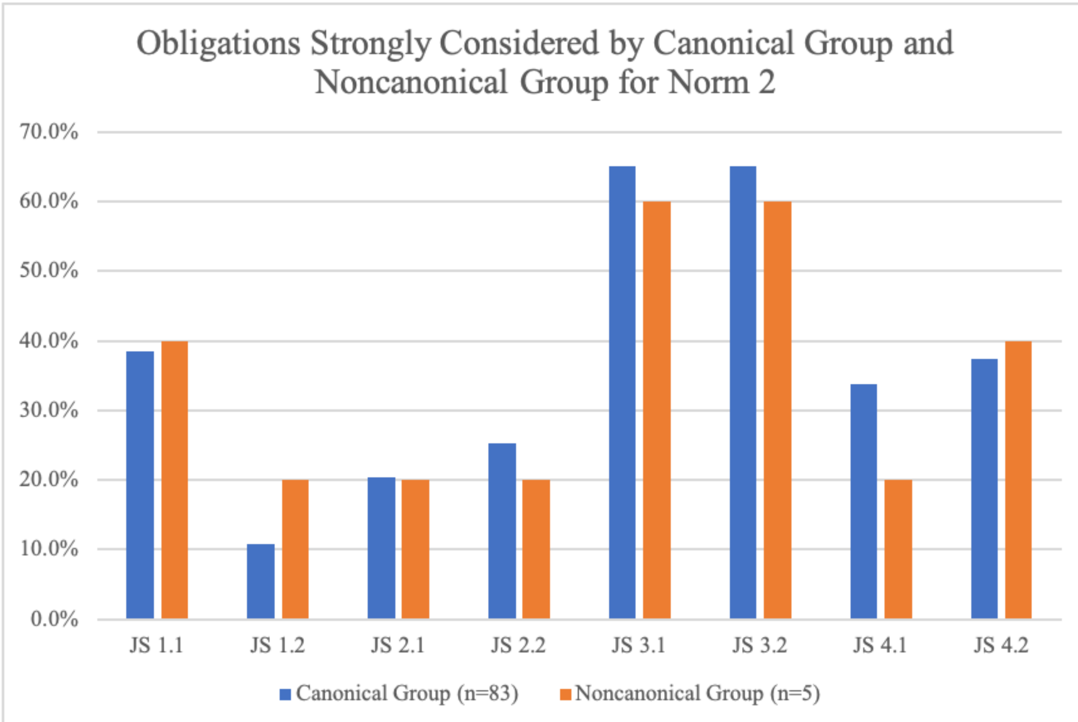


With regard to Norm 2, of the 83 teachers in the canonical group, 54 (65.1%) indicated that they strongly considered the disciplinary obligations (i.e., JS 3.1 mathematical concepts and properties and JS 3.2 the nature of the mathematical practice) (see Figure 17). Several commented that they would like students to understand that tables are not as efficient as equations. For example, after selecting the canonical option C1, a teacher Lena mentioned, "A table is a great start, but finding the function in most cases is the most efficient way to test a conjecture." She mentioned what is considered "the most efficient" in mathematics and even used the word "function" to mean a symbolic equation, indicating the perceived prevalence of symbolic representations in her view of the discipline (i.e., functions do not have to be symbolic, but Lena presumed this to be the case). Lena used efficiency as a criterion for selecting her preferred actions.

Disciplinary obligations were also at play for the teachers who chose a different action. Of the five teachers in the noncanonical group, three (60%) indicated that they strongly considered the underlying mathematical concepts and properties (JS 3.1) and the nature of mathematical practice (JS 3.2), although they may have had different perspectives on the discipline. Tyler mentioned that the medium to represent mathematical ideas does not have to be a particular type of representation. He also questioned the need of using symbolic representation if students have already come up with an appropriate way to express their thinking. He said, “If they came up with a great way to express their thoughts, why does it matter the medium they used to explain their thinking? It would seem redundant to also include an equation.” I considered the source of his obligation to be disciplinary. For Tyler, he had a more open view about what counted as an appropriate representation and expressed a desire for students to use any type of representation that made sense to them (not limited to symbolic representation) and served as a “medium” of thinking.

Figure 17

Professional Obligations Strongly Considered by Different Groups in Terms of Norm 2 – Symbolic Representation



These findings revealed that disciplinary obligations were a driving consideration for Norm 2 when teachers decided which type of representation was acceptable and desirable for the survey scenario. Teachers' responses illustrated various dispositions toward the disciplinary obligations, which led to different preferred actions. Some embraced more diverse representations, whereas the majority prioritized "efficiency" and favored one specific representation, in this case, symbolic representation.

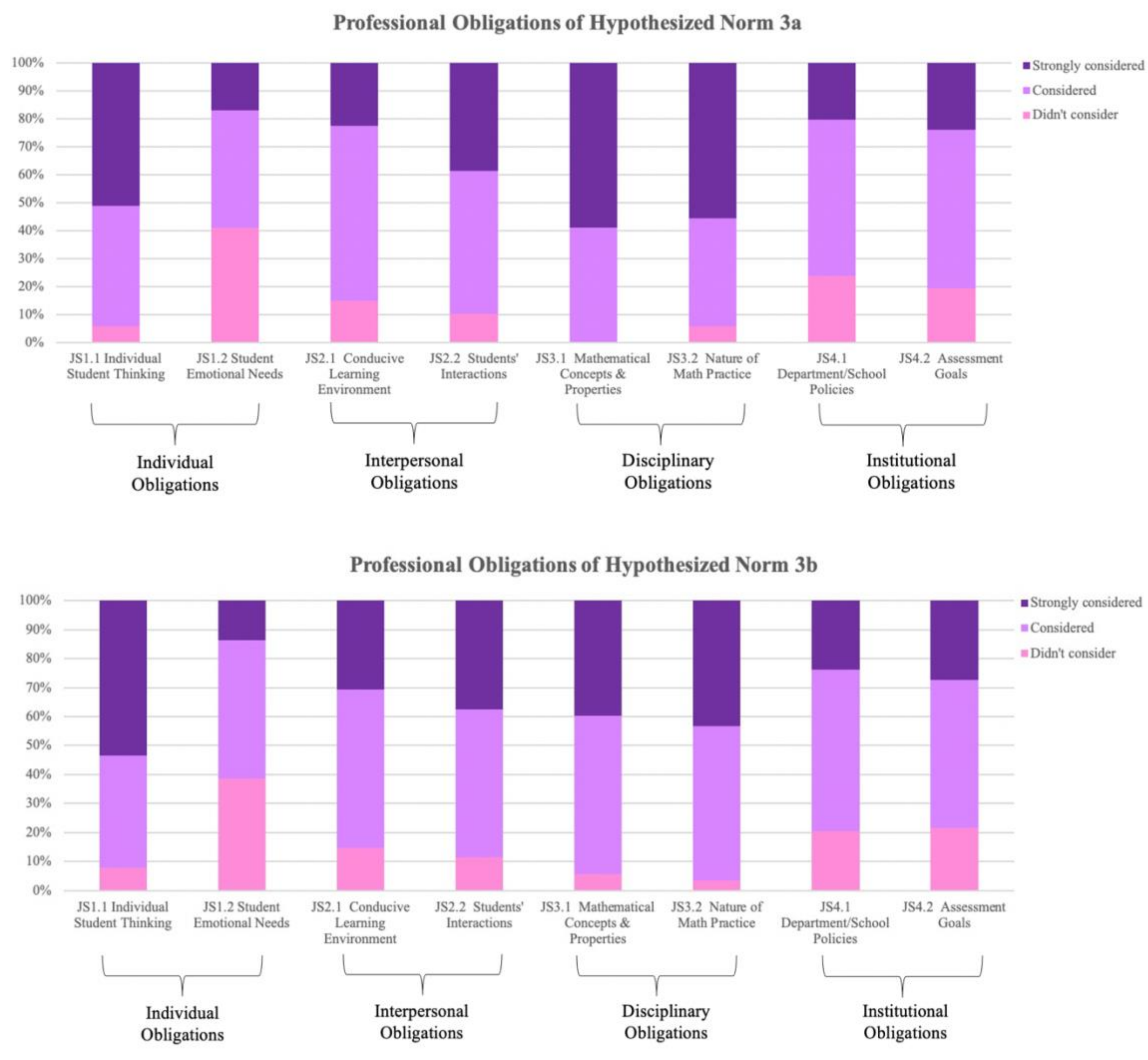
Unconfirmed Hypothesized Norms 3 & 4: Mathematical Components & Mathematical Thinking Dominance

All teacher participants were surveyed about Hypothesized Norm 3 (teacher gives students unambiguous directions on what they are expected to use in terms of the mathematical operations and components). Participants of Group 1, who read and responded to the bacterial reproduction task, encountered different mathematical content than those in Group 2, who read and responded to the water usage task. Even though the hypothesized norm was not confirmed, it is still worthwhile attempting to understand the participants' practical rationality when approaching the scenarios and what obligations they perceived as paramount in the midst of modeling-type situations. The two groups of respondents had broadly similar but slightly different results with regard to the obligations they considered as they selected their simulated actions. These two groups both reported that they strongly considered individual student thinking. Group 1 and Group 2 results are presented in Figure 18, denoted as Hypothesized Norm 3a and Hypothesized Norm 3b, respectively. The two groups considered the other individual obligation (i.e., students' emotional needs) the least often. Aside from students' emotional needs, the two justification statements in the institutional category (i.e., meeting department/school policies and matching the assessment goals) were considered less than all other justification statements. A slight difference was that teachers in Group 1 felt more obligated to the discipline of mathematics than Group 2.

For teacher participants in Group 1, 52 (59.1%), 49 (55.7%), and 45 (51.1%) of them reported strongly considering their obligations to underlying mathematical concepts and properties (JS 3.1), the mathematical practice (JS 3.2), and individual student thinking (JS 1.1), respectively. The least-considered obligation, with 36 (40.9%) of the teachers not considering it, was students' emotional needs (JS 1.2).

Figure 18

The Professional Obligations of Hypothesized Norms 3a and 3b – Mathematical Components (n=88)



Note. Norm 3a reports the results from Group 1 concerning the bacterial reproduction task. Norm 3b reports the results from Group 2 concerning the water usage task.

Focusing on specific subgroups in Group 1, the participants (41) in the canonical group preferred to specify a mathematical formula that students should use. In this group, the most highly considered obligation was to the mathematical concepts and properties (JS 3.1); 23 (56.1%) participants indicated that they strongly considered their obligations to the scenario's underlying mathematical concepts and properties (see Figure 19). For instance, a teacher mentioned, "If it is the exponential unit, I don't want students wasting time using a linear or quadratic function when I want them focusing on growth rates of exponentials." This comment shows evidence of teachers' disciplinary obligations toward the underlying mathematical concepts and properties, with an acknowledgment of the institutional expectations for the unit as well. On the other hand, of the 47 teachers in the noncanonical group, 38 (80.9%) indicated that they strongly considered individual student thinking (JS 1.1). One teacher explained: "I want students to become independent thinkers." Similar comments could also be found in other teachers' responses in the noncanonical group. They mentioned that they would not want to rob students of their opportunity to figure it out on their own and give students the opportunity to create their own functions. These comments indicated that some teachers held a commitment to their responsibility for teaching students how to become independent thinkers, which related to their decisions to allow the scenario's students to determine their own mathematical components.

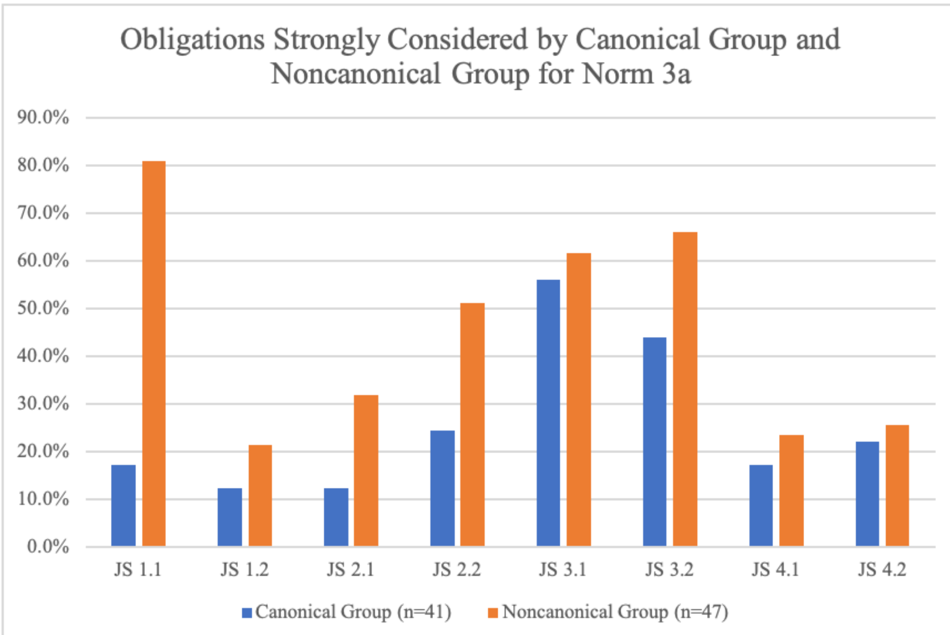
Shifting to the results from Group 2, 47 (53.4%) of teachers felt strongly obligated to individual student thinking (JS 1.1). Over 50% of them considered other categories of obligations including interpersonal, disciplinary, and institutional obligations, but they were not strongly considered obligations. About 34 (38.6%) of the teachers did not consider students' emotional needs (JS 1.2) when they picked their options in the scenario.

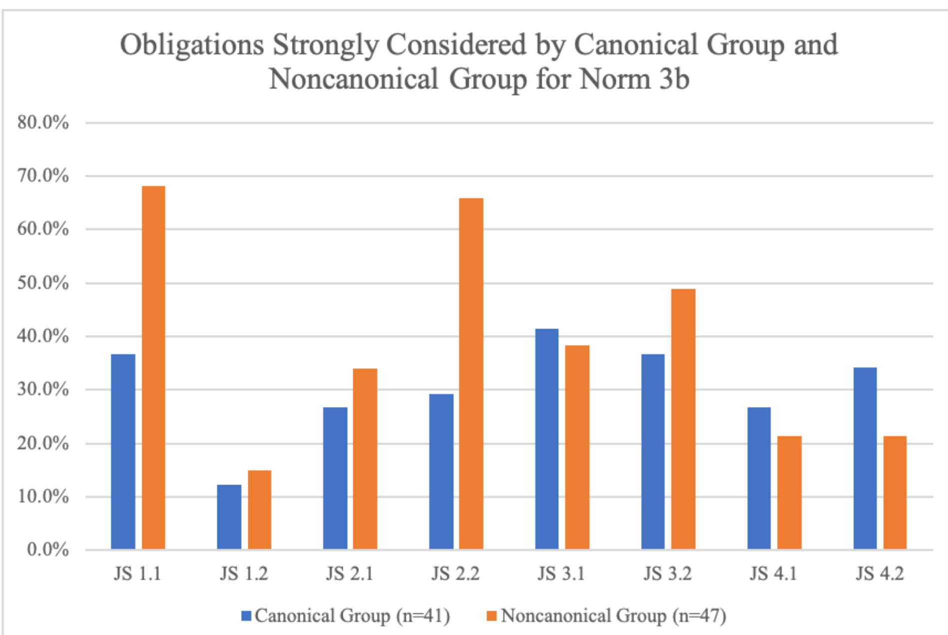
Among Group 2, of the 41 teachers in the canonical group, 17 (41.5%) indicated that the underlying mathematical concepts and properties (JS 3.1) were what they strongly considered, which is similar to Group 1. For the 47 teachers in the noncanonical group, 32 (68.1%) indicated that they strongly considered individual student thinking (JS 1.1). For instance, Pate said, "Students aren't going to have their math teacher beside them when solving problems in the real world. Allowing students to come up with their own functions provides them opportunities they will face in the real world." She felt it was important to support individual student thinking by not giving students too much direction on the mathematical components.

Although the hypothesized norm for the mathematical components was unconfirmed, there was somewhat of an association between teachers who chose to specify students' mathematical components or operations and the obligations those teachers felt to specify mathematical concepts and properties. They may have made a canonical choice in part due to their perceived obligations to the mathematics discipline during the modeling scenario. On the other hand, teachers who reported that they would encourage students to decide on their own mathematical operations and components tended to consider more strongly their obligations to individual student thinking.

Figure 19

Professional Obligations Strongly Considered by Different Groups in Terms of Hypothesized Norms 3a and 3b – Mathematical Components



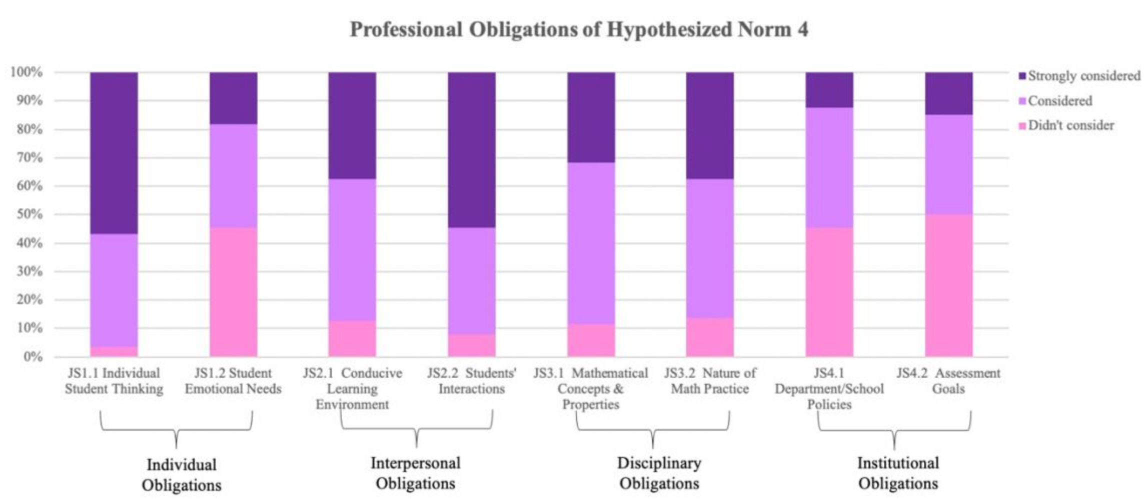


Note. Norm 3a reports the results from Group 1 concerning the bacterial reproduction task. Norm 3b reports the results from Group 2 concerning the water usage task.

The previous paragraphs focused on Hypothesized Norm 3. Now, I shift to the findings concerning another unconfirmed hypothesis, Hypothesized Norm 4 (mathematical thinking dominance). It involves the inclusion or exclusion of nonmathematical thinking and experiences in the mathematical modeling process. I hypothesized that the norm would be for teachers to expect students to exclude nonmathematical thinking and focus on mathematical thinking and results. This norm was not confirmed, but I still consider herein the obligations that were at play. For all teacher participants who were surveyed about Hypothesized Norm 4, over 50% felt strongly obligated to individual student thinking (one of the two justification statements in the individual category) and student interactions in class (one of the two justification statements in the interpersonal category). See Figure 20. The least-considered obligations were institutional. In particular, 44 (50.0%) of the teachers did not take the assessment goal into consideration when they chose their preferred action.

Figure 20

The Professional Obligations of Hypothesized Norm 4 – Mathematical Thinking Dominance (n=88)



Of the 35 teachers in the canonical group for Hypothesized Norm 4, 22 (62.9%) strongly considered their obligations to the underlying mathematical concepts and properties (JS 3.1, see Figure 21). For example, a teacher participant commented, “I usually try to make sure my conversations in these moments are guided by the learning objective and use the vocabulary discussed to help students model or practice socially. I feel like this selection might attempt to do that.” In the remark, he expressed a need to ensure that the classroom activities were guided by the objectives that he believed the lesson should achieve. This indicated that he was under a disciplinary obligation to focus on the underlying mathematical concepts rather than students’ nonmathematical thinking which was not in the scope of the lesson.

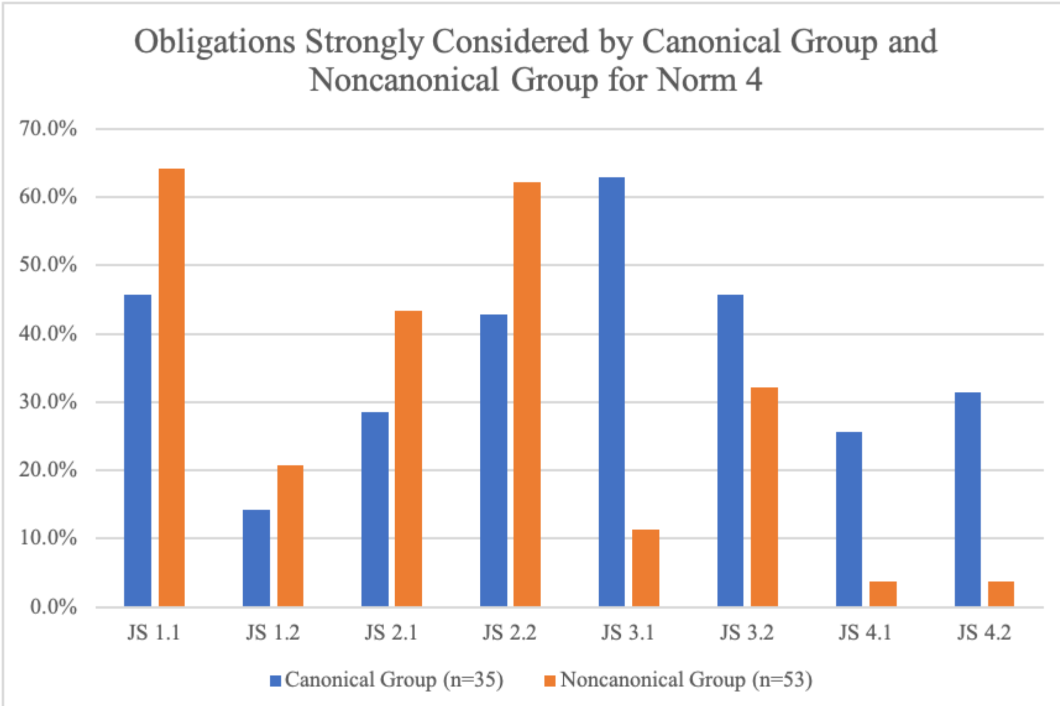
On the other hand, of the 53 teachers in the noncanonical group (who welcomed nonmathematical thinking), 34 (64.2%) and 33 (62.3%) justified their selections by strongly considering individual student thinking (JS 1.1) and student interactions (JS 2.2), respectively. A teacher mentioned:

> I wanted to encourage students to keep thinking about the problem. I did not feel they were stuck at a point where I would need to tell them to put those things into an equation. I believe the ability to analyze a situation and find relevant information to use in a problem is a highly important skill in today’s society.

In this excerpt, this teacher mentioned about encouraging students to continue thinking about the problem rather than directly leading them to apply specific mathematical knowledge or reasoning (i.e., “put those things into an equation”). He was concerned about developing students’ abilities to analyze problems and search for information, which he regarded as essential skills for their futures when they enter the society.

Figure 21

Professional Obligations Strongly Considered by Different Groups in Terms of Hypothesized Norm 4 – Mathematical Thinking Dominance



The findings suggest that teachers who favored the NC—the option to engage students in nonmathematical thinking during modeling—felt strongly obligated to student thinking and students' interactions with peers. Conversely, teachers who steered away from nonmathematical thinking and strictly adhered to mathematical thinking felt strongly obligated to teach the underlying mathematical concepts and properties. Looking across Hypothesized Norms 3 and 4 (mathematical components and mathematical thinking dominance), although this study could not confirm either one of them, the findings provided insights into the association between teacher participants' preferred action and their justification. Teachers who were mostly driven by the disciplinary obligation tended to abide by the Hypothesized Norms 3 or 4, whereas teachers who perceived the individual obligation as paramount tended to depart from Hypothesized Norms 3 or 4.

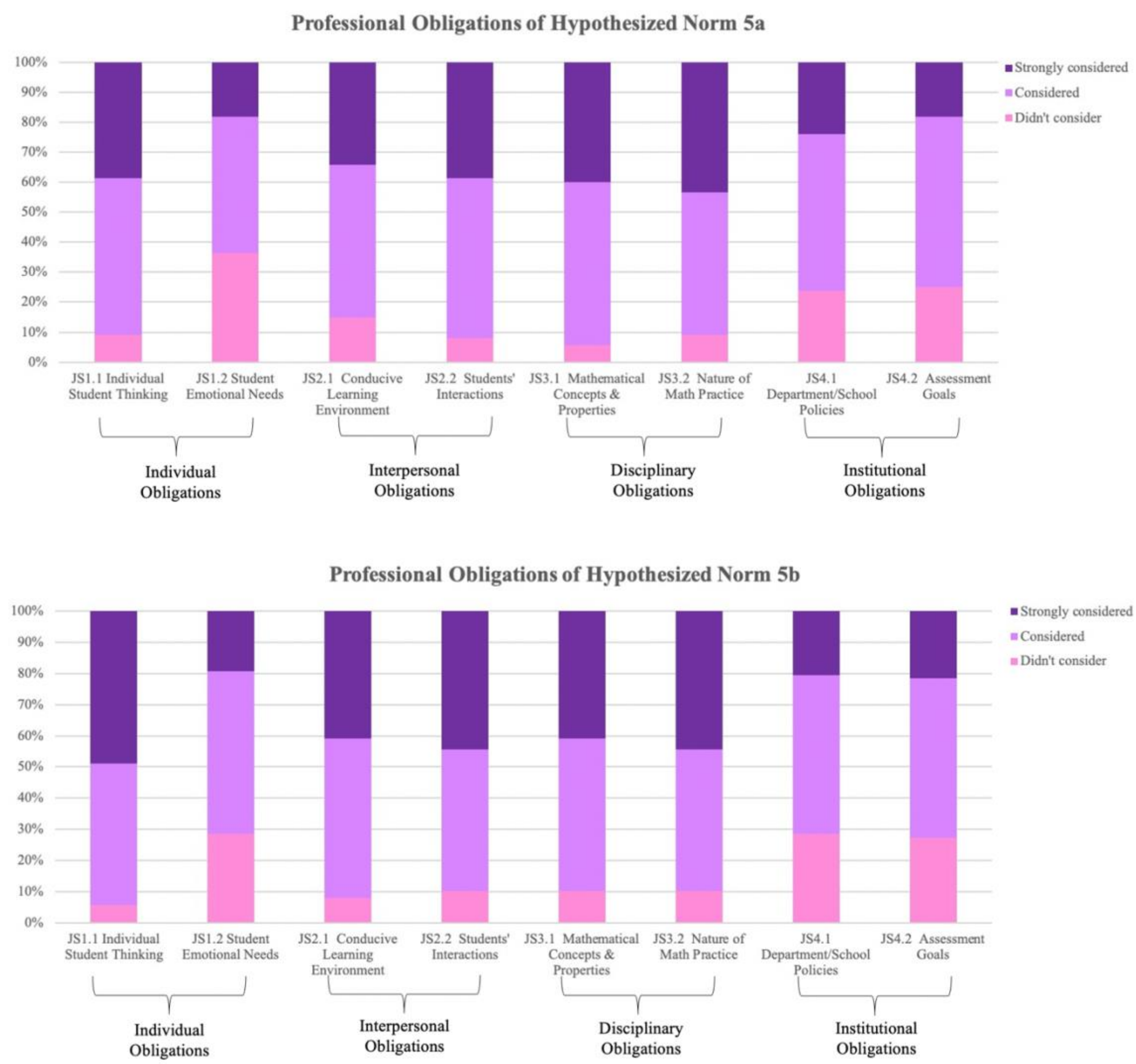
Disconfirmed Hypothesized Norm 5: Model Revision Omitted

Per Hypothesized Norm 5, students are expected to arrive at the same model, and model revision (beyond resolving discrepancies between different models) is not expected. All teacher participants responded to a scenario involving Hypothesized Norm 5, though the scenario for Group 1 (bacterial reproduction) was embedded within different tasks than the scenario for Group 2 (water usage). The two groups of participants had similar results with regard to the obligations they considered as they selected their preferred action from the simulated options.

As shown in Figure 22, specifically in the chart for Hypothesized Norm 5a, 83 (94.3%) and 80 (90.9%) of respondents in Group 1 strongly considered or considered the two disciplinary-related statements (mathematical concepts and properties and the nature of mathematics practice). Furthermore, 81 (92.0%) of respondents strongly considered students' interactions. The least-considered obligation concerned students' emotional needs, with 32 (36.4%) of the teachers reporting that they did not consider this justification statement when they picked their preferred action. As shown in the Figure 22 chart for Hypothesized Norm 5b, individual student thinking was the most considered obligation. 83 (94.3%) of the teachers strongly considered or considered individual student thinking, followed by interpersonal obligations and disciplinary obligations. Since this hypothesized norm was disconfirmed, I first present the findings of the noncanonical group, who indicated they would be open to model revisions. I then follow up by discussing the canonical group.

Figure 22

The Professional Obligations of Hypothesized Norms 5a and 5b – Model Revision Omitted (n=88)



Note. Norm 5a reports the results from Group 1 concerning the bacterial reproduction task. Norm 5b reports the results from Group 2 concerning the water usage task.

In Group 1's noncanonical group, 29 (50.0%) of 58 teachers indicated that they strongly considered individual student thinking (JS 1.1, see Figure 23). Similarly, among Group 2, for those in the noncanonical group (68), 37 (54.4%) indicated that they strongly considered individual student thinking (JS 1.1) and students' interactions (JS 2.2). These findings show that participants in both Groups 1 and 2 were more likely to use individual student thinking to justify their choice of an action that encouraged model revisions. Teacher Tyler in Group 1 mentioned the following:

> Let's try and play with that curiosity and creativity a little bit and see what we have …. That teacher doesn't really respond and say, you know, do you have to think about bacteria dying? Or you don't have to be—you know, there is no right or wrong about the bacteria dying piece, but is there anything else you can think about? And maybe it sparks a little creativity in the class, and then they keep working on it because clearly … with the discussion going … it would open up the kids … to actually do more work and [spark] more creativity.

Tyler specifically commented that mathematics teachers "open up" the task and provide more freedom for students to develop their curiosity and creativity. Some participants in the study, from a perspective of cultivating students' perseverance, mentioned that giving students time to thoroughly investigate a problem can help them develop "grit."

However, teachers also indicated that they selected the NC because they assumed that they were "not in a time crunch." These comments implied that time is an important institutional consideration during model revisions, and they suggested that teachers may make a different choice in their actual classroom than they did in the fictionalized survey scenario.

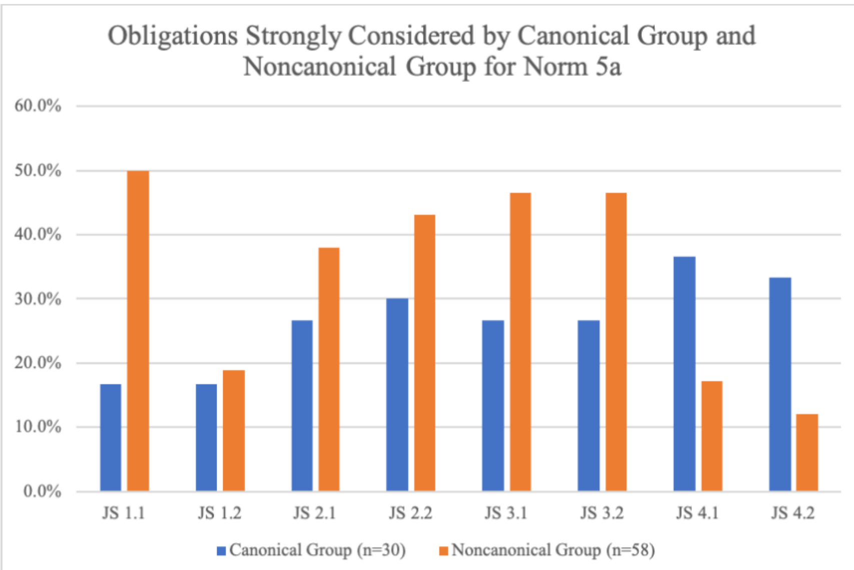
In terms of Group 1's canonical group (30), the participants' obligations to department or school policies (JS 4.1) constituted the justification statement most strongly considered, although it was not the most strongly considered justification for Group 1 overall (i.e., canonical and noncanonical groups combined). Of the 30 teachers in the canonical group, 11 (36.7%) indicated that they strongly considered department or school policies (JS 4.1). More specifically, a teacher mentioned, "Having spent 3 days on a single problem, it is time to move forward or the curriculum will be impossible to finish in the allotted time." She regarded coverage of the curriculum as important when selecting her preferred action. Group 2 yielded similar results. For the 20 teachers in Group 2's canonical group, eight of them (40.0%) indicated that they strongly considered their obligations to department or school policies (JS 4.1). Another teacher mentioned the following:

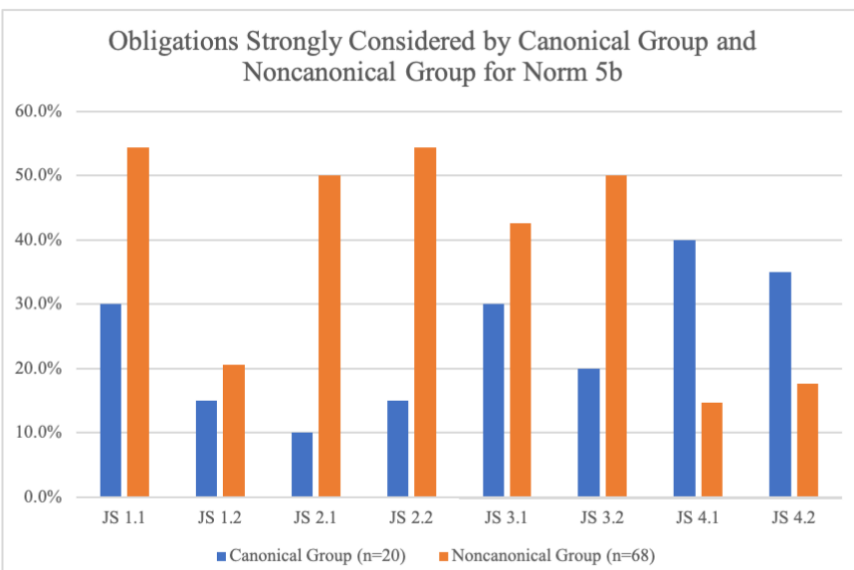
> I believe that curriculum is best when 80% have mastery over 80% of the material versus the normal 60% mastering 100% of the material. Still, it is imperative to push forward with content. The reality is that this is good ... but the state test is coming regardless of the water supply. This is important.

In this excerpt, this teacher also conveyed her concern about content coverage. Content coverage prevented teachers from spending too much time on one topic or model revisions. She even noted that she faced significant pressure to keep most of their students on pace so that students can meet the state testing requirements. Other teacher participants also mentioned their obligation to meet Missouri Learning Standards and the goals of standardized End-of-Courses assessment.

Figure 23

Professional Obligations Strongly Considered by Different Groups in Terms of Hypothesized Norms 5a and 5b – Model Revision Omitted





Note. Norm 5a reports the results from Group 1 concerning the bacterial reproduction task. Norm 5b reports the results from Group 2 concerning the water usage task.

Notably, a small portion of the justifications cited in the survey responses could not be captured by the obligation framework. For example, a teacher mentioned teacher knowledge:

> The time to delve into mortality rates of bacteria would be minimal, but you have to keep your class moving forward. The typical math teacher might not have sufficient knowledge to delve into epidemiology.

This teacher first used class time as a justification for limiting the model revisions which aligns with the category of institutional obligations. He further suggested that mathematics teachers, in general, were underprepared for the complexity of the task. More specifically, teachers were uncertain of how to move forward when students responded with unanticipated ideas, such as the mortality rates of bacteria, that might not be mathematical subject matter. The obligation framework could not capture teacher knowledge (or lack of knowledge) as a justification. These exceptions were rare in the data, as I found only two instances.

Before closing this section for Hypothesized Norm 5, there were a few more responses that I felt important to review. Even if teacher participants chose the NC, some still expressed in their comments that their selections were based on an ideal situation that is hard to achieve in reality. For example, one interviewee, Ben, said the following:

> If I’m 3 days into this [the modeling task], I’m already behind in my pacing guide. I won’t be able to even gloss over the 5 to 6 basic skills that I need to cover so they even have a slim chance of doing it correctly on the MAP [Missouri Assessment Program] test. I chose my selection this time based on what would be the most interesting to me and the students, but I would never choose to spend this

much time on one task unless it was an extension of math class or a gifted class who could have more time to explore mathematics in such a nontraditional way.

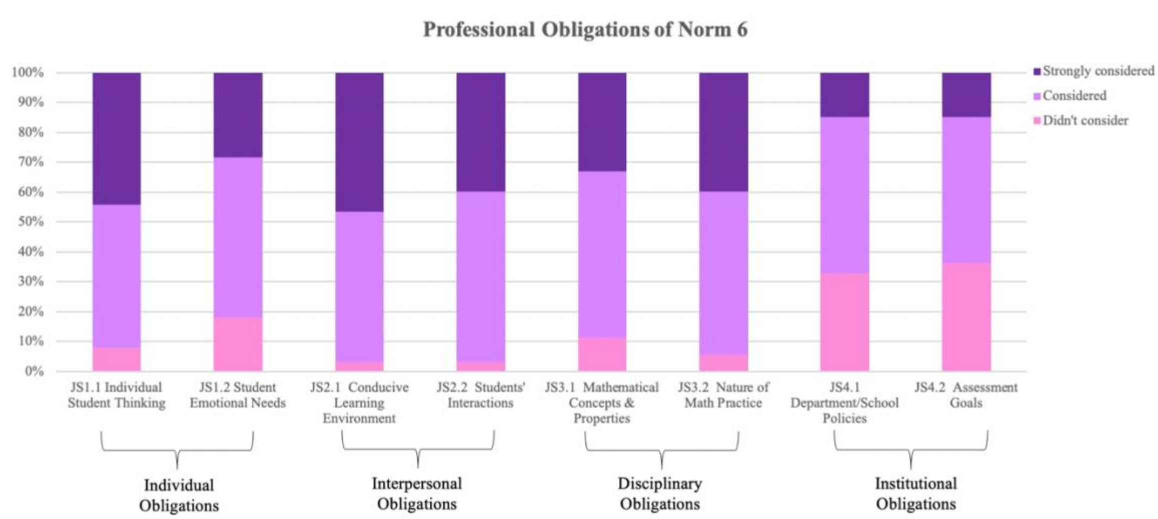
Ben's example showed the impact of the standardized test and time constraints on his decisions. He also conveyed his concern that the tasks were not appropriate for students of all levels. His response could be evidence that teachers consider more obligations in their actual teaching practice than when completing the survey.

Confirmed Norm 6: Politically Neutral Contexts

As shown in Figure 24, for all participants who responded to Norm 6, 85 (96.6%) of them strongly considered or considered their obligation to maintain a social environment that is conducive to learning (JS 2.1) as well as support productive interactions among students (JS 2.2). Moreover, 83 (94.3%) and 81 (92.0%) strongly considered or considered the nature of mathematical practice (JS 3.2) and individual student thinking (JS 1.1), respectively. However, over 33.0% of the participants did not consider institutional obligations, making it the least-considered category for this norm.

Figure 24

The Professional Obligations of Norm 6 – Politically Neutral Contexts (n=88)



In the canonical group, 58 teachers chose either the initial water usage task (Original Task) or the politically neutral alternative (New Task A). Of these, 25 (43.1%) indicated that they felt strongly obliged to maintain a conducive learning environment (JS 2.1, see Figure 25). Some mentioned that the racially infused task, New Task B, would lead to student distractions, which would not be helpful for mathematical learning. One teacher who picked New Task A said, “[New Task B] could cause the class to get into an emotional discussion that might make some students uncomfortable and shut down. … [New Task A] is more specific and interesting [than the original], but not as risky.” Another teacher said, “I believe New Task B would not be conducive to my current school population, which is not diverse, and many students would not see this as a problem that they have to deal with.” The focus in these comments was on the students’ experiences and how they would relate to or engage with the problem.

Some participants explained why they chose the Original Task by mentioning that mathematics classrooms should primarily focus on the main subject matter. For instance, a teacher provided the following comment:

> It is the school’s job to teach subject matter so that students are prepared for the future. Leave the parenting to the parents. Schools should stick to the job of creating critical thinkers that can look at facts and make their own judgments and not create thinly veiled scenarios to impose the … [teachers’] point of view.

This teacher distinguished between the roles of the school (or the mathematics discipline) and the students’ parents. She believed that mathematics teachers should cultivate “critical thinkers” without imposing their political views on students. She also felt obligated to teach the subject matter that can prepare students for their future, and that teaching political views should be parents’ responsibilities.

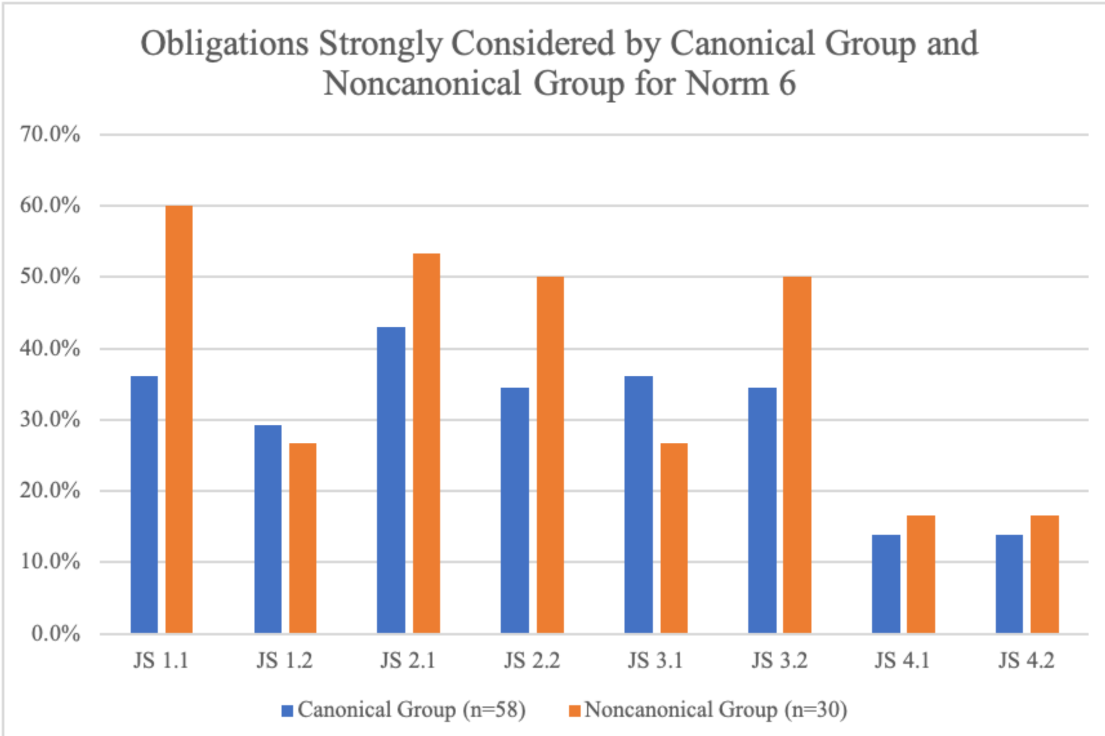
For the 30 teachers in the noncanonical group (those who chose New Task B), 18 (60.0%) of them felt strongly obligated to facilitate individual student thinking (JS 1.1). One teacher, Rebecca, in her follow-up interview, talked about the benefits of social-justice tasks:

> Those types of conversations should be normalized. There is no reason why we should shy away from having those conversations. People are scared of other races because people don’t talk to other races …. We don’t engage in other community cultures because where people grow up is so isolated to who they are and what they look like …. Students from communities with white privilege need to hear voices from other perspectives in order to grow their own thinking …. I think math, mathematics[,] is social justice.

In this excerpt, Rebecca conveyed a sense of responsibility to bring social justice topics into mathematics classrooms. She stated that students should be part of the conversation, and “voices from other perspectives” would help “grow individual thinking.” In her view, learning mathematics includes developing the ability to listen and understand the voices of those who have been systemically silenced.

Figure 25

Professional Obligations Strongly Considered by Different Groups in Terms of Norm 6 – Politically Neutral Contexts



The findings suggest that teachers who reported choosing the social-justice-oriented task felt strongly obligated to enhance individual student thinking. Conversely, teachers who expected to primarily (or exclusively) work on politically neutral tasks felt strongly obligated to create a conducive learning environment for their students.

It is noteworthy that a relatively large proportion of the Norm 6 respondents (81.8%) strongly considered or considered students' emotional needs when they were deciding whether to use a task with a racially infused context. In Hypothesized Norms 1–5, the obligation to support individual students' emotional needs was the least-considered obligation or among the least-considered. This issue is discussed in the next chapter.

Summary of Findings for RQ2

Overall, a pattern of professional obligations emerged among Hypothesized Norms 1 (factor specification), 3 (mathematical components), and 4 (mathematical thinking dominance). Disciplinary obligations were the most frequently considered justifications for teacher participants who followed the hypothesized norms. For teachers who departed from the hypothesized norms, they most strongly considered individual student thinking.

In terms of Norm 2 (symbolic representation), disciplinary obligations had an impact on teachers' preferred actions in both the canonical and noncanonical groups. However, disciplinary obligations operated differently in these two subgroups. Participants had different perspectives on the mathematics discipline. For example, some in the canonical group considered it a disciplinary fact that symbols were the most efficient mathematical representation, whereas others in the noncanonical group believed that mathematical representations do not have to be limited to one specific type if they can serve as a medium of thinking. These different interpretations of the disciplinary obligations are related to teacher participants' different responses to the scenario.

As for the only disconfirmed norm in this study (Hypothesized Norm 5, model revision omitted), one of the institutional obligations—department or school policies—was strongly considered for teachers who chose the survey option to exclude model revisions, whereas individual student thinking was strongly considered for those who opened up the opportunities for students to continue revising and refining their models.

With regard to Norm 6 (politically neutral contexts), participants who chose not to implement the racially infused task most strongly considered creating a conducive learning environment. Those who selected the racially infused task strongly considered the development of individual student critical thinking.

The RQ2 findings are summarized in Table 13. Note that the canonical and noncanonical groups are defined for each norm specifically and thus do not include the same participants across all six norms.

Table 13

The Strongest Obligation for Each Hypothesized Norm

Norm / Hypothesized Norm	Canonical group	Noncanonical group	All participants
	Obligation category		
Norm 1. Factor selection	Disciplinary obligations [a]	Individual student thinking	Disciplinary obligations
Norm 2. Symbolic representation	Disciplinary obligations	Disciplinary obligations	Disciplinary obligations
Hypothesized Norm 3. Mathematical components	Disciplinary obligations	Individual student thinking	Disciplinary obligations
Hypothesized Norm 4. Mathematical thinking dominance	Disciplinary obligations	Individual student thinking	Individual student thinking
Hypothesized Norm 5. Model revision omitted	Department/ school policies	Individual student thinking	Individual student thinking [b]
Norm 6. Politically neutral contexts	Conducive learning environment	Individual student thinking	Conducive learning environment

[a] The two justification statements of disciplinary obligations had a similar selection rate, so I listed them together as "Disciplinary obligations". [b] In terms of Hypothesized Norm 5, besides the obligation of individual student thinking, participants also strongly considered additional obligation categories in Groups 1 and 2, but those obligations were not present at the same level across the two groups.

CHAPTER 5

Discussion and Conclusion

Through my dissertation study, I attempted to understand the challenges and obstacles of enacting modeling tasks in the United States by analyzing teachers' practical rationality related to mathematical modeling, as revealed through a breaching experiment survey and interviews. In the past, researchers investigated instructional practice by studying teachers' knowledge and beliefs (e.g., de Oliveira & Barbosa, 2010; Goe, 2007; Kaiser & Maaß, 2007). However, as many studies have pointed out, the practice of teaching is not as simple as expressing one's knowledge, beliefs, or goals (e.g., Herbst et al., 2011; Webel & Platt, 2015). In fact, teachers must adapt to a sophisticated system of various agents, such as their students, schools, and society, and have to handle the complexity of interactions among their roles, goals, and expectations (Herbst & Chazan, 2012; Lampert, 1985).

My dissertation used the perspective of *practical rationality* (Herbst et al., 2011; Herbst & Chazan, 2012) to understand the mathematical modeling teaching practice by emphasizing the environmental impacts imposed on and perceived by teachers. The particular questions guiding this study were the following:

1. What norms are perceived by secondary teachers in relation to mathematical modeling? (RQ1)
2. What professional obligations are perceived by secondary teachers in relation to mathematical modeling? (RQ2)

In this chapter, I first summarize the major findings related to the research questions and connect these findings to past studies that formed or contributed to the study's conceptualization. Next, I discuss some limitations of the study. Finally, I describe implications for future research, teaching practices, professional development, and equity pedagogy concerning mathematical modeling.

Summary

To examine teachers' perceived norms related to mathematical modeling, I drew upon past research (e.g., Blum & Niss, 1991; Chazan et al., 2012; Gerofsky, 1996; Gould, 2013; Gutstein, 2003; Leiß, 2007) to formulate six hypothesized norms corresponding to various stages during mathematical modeling. I investigated whether, from my participants' points of view, the hypothesized norms were actual norms.

As described in Chapter 4, this study confirmed three norms, at least in the context of this study's tasks and scenarios. The three confirmed norms are Norm 1 (factor specification), Norm 2 (symbolic representation), and Norm 6 (politically neutral contexts).

With respect to Norm 1, this study confirmed that it is the norm for teachers, rather than students, to identify precisely which information or factors should be included in their solutions. Per Norm 2, this study revealed that teachers expect students to find a symbolic representation (e.g., equations) as their model. Should students offer a nonsymbolic solution—such as a table to demonstrate bacterial reproduction—the norm is that a teacher will ask them to use a symbolic representation. Finally, this study showed that teachers expect students to primarily (or exclusively) work on politically neutral tasks instead of social-justice-oriented tasks, per Norm 6.

This study also provided evidence that Hypothesized Norm 5 (model revision omitted) might not be a norm at all, at least according to teachers' responses to the survey's simulated classroom scenarios. I posited that teachers expect all students to arrive at the same model and avoid working on revisions or alternatives to their own solutions. Yet, this study's findings suggested the opposite. The participants did, indeed, indicate that they would be open for students to continue exploring various solutions.

While Norms 1, 2, and 6 were confirmed, and while Hypothesized Norm 5 was disconfirmed, the study could do neither for Hypothesized Norms 3 (mathematical components) or 4 (mathematical thinking dominance) based on the survey responses. With Hypothesized Norm 3, I posited that a teacher typically gives students unambiguous directions on what they are expected to use in terms of the mathematical operations and components needed for the task. Hypothesized Norm 4 suggested that most teachers expect students to primarily (or exclusively) engage in mathematical thinking rather than nonmathematical thinking. The findings did not support or disprove either of these hypotheses.

Discussion

Interpretation of the Confirmed and Disconfirmed Hypothesized Norms

The confirmation of Norms 1 and 2 (factor specification and symbolic representation) implies that students' model-construction process is unduly accelerated. When looking closely at the mathematical modeling framework (Blum & Leiß, 2005), one finds that these two confirmed norms happen at the start of the mathematical modeling cycle, when students are trying to understand and simplify the problem and select an appropriate representation to show their work. Norm 1 (factor specification) reveals that a sizable majority of teachers in the study supported specifying factors for their students and indicated that mathematics teachers, in general, were more inclined to converge students' thinking to specific factors. These findings align with the findings of an earlier study by Leiß (2007), which found that students were given too much support—they only needed to find one step by themselves to overcome the challenges. A study by Kaiser and Maaß (2007) showed that teacher beliefs about mathematics influenced how teachers thought modeling should be implemented. Given that, this study found that teachers felt strongly obligated to uphold what they perceived as the discipline of mathematics. Therefore, a possible explanation is that teachers supply students with necessary information for a given task to shorten their working time in the real world so that teachers can teach what should be taught in mathematics classrooms—the underlying mathematical concepts, properties, and practices.

The existence of Norm 2 (symbolic representation) shows that most teacher participants urged students to use one specific type of representation (i.e., symbolic representation). They also agreed that teachers, in general, would give precedence to symbolic representations. Likewise, previous studies have demonstrated that mathematics instruction in the United States predominantly focuses on symbolic manipulations (e.g., Gerofsky, 1996; Kieran, 2007). In this study, the teachers felt strongly obligated to their interpretation of the mathematics discipline. Some of them believed symbolic representation was the most efficient and was what it means to be mathematically competent. This idea is prevalent in mathematics teaching and aligns with *Mathematical Proficiency for All Students*, a book in which symbolic notions are regarded as "efficient" for "compress[ing] complex ideas into a form that makes them easier to comprehend and manipulate" (RAND Mathematics Study Panel, 2003, p. 37). Hence, teachers felt that they must move students toward what they perceived to be this more efficient type of representation. Similar to Norm 1, Norm 2 contributes to expediting the modeling process by taking away students' opportunities to interact with multiple types of representations that make sense to them.

Contrary to Norms 1 and 2, the disconfirmed Hypothesized Norm 5 (model revision omitted) is associated with the final stage of the modeling process, according to Blum and Leiß (2005). Whereas participant teachers tended to support a more restrictive set of norms in the beginning phases of the simulated scenario, they revealed at least a receptivity to opening up their teaching practices at the end-stage, giving students opportunities to revise and refine their models, not just limiting the students to one specific answer. However, these findings do not echo those of a previous study from Gould (2013), where he found that over half of the teachers perceived that the modeling process should result in an exact answer or answers. A couple of reasons may explain why the current study contradicts Gould's previous work. One reason is that the teacher participants rarely engaged in modeling. Survey responses indicated that modeling infrequently happens in their classrooms (e.g., as a special monthly event), so the teachers might be more willing to explore model revision with their students on these special occasions. Suppose they enacted modeling as a part of their regular classroom routine. In that case, they might find that practical constraints (e.g., limited planning and implementation time) made it more challenging to carry out model revisions. A second reason why the teachers in my study reported that they were willing to support students' revisions could result from social desirability, which I discuss further in this chapter's upcoming section about the limitations.

In terms of Norm 6 (politically neutral contexts), the majority of teachers did not select the task related to race and racism. They also suggested that mathematics teachers typically did not engage students in social-justice-oriented tasks. As described before, Gutstein (2003) discussed that teachers were concerned about introducing inequality issues in class because they could implicate power deconstruction and threaten authority, which causes teachers to shy away from an equity pedagogy. In this study, some participants also conveyed similar concerns. Moreover, the findings also suggested that the teacher participants chose not to involve students in social justice discussions to maintain a classroom environment conducive to learning mathematics. Notably, I only designed one task with a specific racially infused context (i.e., the Flint, Michigan, water crisis). The findings might be specific to the task. This study did not address many other aspects related to race, equity, and social justice. The teachers might have selected different options and justified them differently if presented with other scenarios.

Tensions Underlying the Hypothesized Norms That Could Not Be (Dis)confirmed

Although this study was unable to confirm or disconfirm Hypothesized Norms 3 (mathematical components) and 4 (mathematical thinking dominance), it is still worthwhile to attempt to understand them. Some teachers said that they would choose one of the given actions when they enact the modeling task but said that all teachers, in general, would choose the opposite action (i.e., the canonical vs. the noncanonical action). These kinds of contradictory responses highlight the potential tensions underlying the hypothesized norms in this study.

Hypothesized Norm 3 (mathematical components) happens in the middle stage of mathematization, where students work on generalizing the relationships among different factors and quantities. Based on the survey data, the existence of Hypothesized Norm 3 is still controversial. On the one hand, teachers felt inclined to remain vague about the mathematical components that students should use. On the other hand, they indicated that it would be typical for other teachers to provide students specific operations to use during modeling, which shows that they might feel pressed to specify to students what are the intended components. These findings suggest a tension between *specifying unambiguous operations within the solving process* and *keeping a productive ambiguity on operations that students need to undertake*. This tension might result from a potential competition between disciplinary obligations (ensuring students' proficiency in the mathematical practice) and individual obligations (deepening individual students' thinking).

As with Hypothesized Norm 3, this study was unable to confirm or disconfirm Hypothesized Norm 4 (mathematical thinking dominance). Most teacher participants indicated that they would allow students to introduce nonmathematical thinking, at least in the context of the survey scenarios. However, there is a divergence in terms of the typicalness of the canonical option. Some believed that teachers, in general, would allow nonmathematical thinking during such modeling scenarios, whereas the rest of them believed it is atypical to allow nonmathematical thinking. This conflict might involve a tension between *gearing the lesson toward the subject-matter goals with mathematical thinking exclusively* and *using students' nonmathematical thinking as resources for learning.* This tension can be explained by the competition between disciplinary obligations and individual obligations. Disciplinary obligations were what drove the focus of modeling activities toward mathematical thinking. In contrast, individual obligations led to the situation where students' nonmathematical thinking was valued, and their everyday knowledge and experiences were incorporated into the process of model building.

Same Obligation but Different Actions

My study found that even if two different teachers were under the same category of obligation, they could react differently to the same scenarios. This finding is aligned with Erickson and Herbst's study (2018) where some of their participants cited the disciplinary obligation as their justification for allowing more discussions whereas others used the disciplinary obligation to justify their decision of closing down discussions.

Take disciplinary obligation as an example: For Hypothesized Norm 4 (mathematical thinking dominance), among those teachers who felt obligated to the nature of mathematical practice, some chose to discourage students' nonmathematical thinking while others validated students' nonmathematical contributions beyond the hypothesized norm. Instead of explaining this phenomenon as a potential result of teaching experiences (e.g., Erickson and Herbst, 2018), the divergent approaches could stem from differing interpretations of mathematical modeling or from the teachers' own ideologies of what they think should be taught in modeling activities. Recall from Chapter 2 that there are two general modeling approaches: *modeling as content* and *modeling as vehicle* (Julie, 2002). The former uses modeling to understand natural and social phenomena, while the latter uses modeling to teach mathematical concepts and procedures. For the participants who regarded *modeling as content*, they viewed nonmathematical thinking as essential and incorporated students' nonmathematical thinking into the model-building process. On the other hand, those who considered *modeling as vehicle* used modeling as a way to develop students' understanding of mathematics in the problem context. Because some teachers did not regard modeling itself as a learning objective but rather a means to introduce new content or apply learned knowledge, they prioritized mathematical thinking over nonmathematical thinking.

This same kind of dichotomy appeared among the survey participants who cited individual obligations as the reason they selected the action they did. For Norm 1 (factor specification), the teachers who felt obligated to individual student thinking were on both sides of the canonical divide—some followed the norm, and some breached it. For example, teacher Aaron said that his obligation to individual student thinking compelled him to support the canonical approach because it was his job to avoid confusing students. Another teacher, Tyler, said that his obligation to individual student thinking compelled him to reject the canonical approach and support the noncanonical one because it was his job to encourage students to consider all possible factors and select the best ones for themselves.

Limitations of the Study

Note that the data of this study was collected through self-reported survey responses rather than actual classroom observations. The fictional scenarios designed for the survey might not actually arise in the participants' own teaching. Even if a similar scenario occurs in the participants' classrooms, it does not necessarily imply that teachers would offer students the simulated opportunities in their actual practice. For instance, teachers who supported the idea of allowing students to revise their models in the survey scenario might not do so for their own students due to other considerations (e.g., time constraints). Hence, one limitation of the study is that it can only shed light on perceived norms rather than enacted norms. The data collected might result from the overly ideal nature of the survey. Therefore, future studies might consider teachers' actual instructional practices to better understand the norms and related professional obligations.

Another constraint imposed by this lack of classroom observations concerns the institutional obligation category. The scenarios might have downplayed institutional obligations due to their fictionalized nature. It could have been challenging for teacher participants to feel their institutional obligations resonating in the scenarios, and their responses may not accurately reflect how they feel at work. Indeed, Sobolewski-McMahon (2017) found out that the institutional obligations are the most influential from classroom observations of teachers' enactment of other mathematical practices. Hence, it is reasonable to assume that in teachers' actual implementation of modeling, the institutional obligations might be stronger since they experience pressure from their evaluations, assessment goals, and other institutional stressors.

To understand the effects of institutional obligations that may have been underrepresented in the survey responses, I asked teachers in my follow-up interviews about how their colleagues, department, school, school district, and standardized tests influence their enactment of modeling. Several interviewees specifically mentioned that they were required to prepare students for exams (i.e., institutional obligation) as they worked to uphold the promise to their students' needs (i.e., individual obligation). These teachers negotiated various obligations and shifted their emphasis between fulfilling institutional mandates and committing to individual students' success.

A second limitation of the study is that the options within the scenarios may have inadvertently influenced the survey respondents. Admittedly, the teachers' choices may have been driven not only by the hypothesized norm purposefully embedded in the scenario but also by other aspects of the scenarios. For example, the way in which the fictionalized teacher communicated to the students might have influenced participants' choices because the participant may have wished to follow a similar course of action but would not have phrased their instructions in precisely the same manner. To eliminate the potential effects of other aspects of the scenarios and ensure that the study examined the intended norms, I attempted to vary the norm actions while keeping constant the other parts of the scenario design (e.g., wording and figures) across the three options (C1, C0, and NC). I also conducted a pilot study with two experienced teachers to see whether their selected options resulted from other aspects of the scenarios and further revised the scenarios based on their feedback. In addition, I included two of the same hypothesized norms (i.e., Hypothesized Norm 3, mathematical components, and Hypothesized Norm 5, model revision omitted) when designing the two different scenarios, which allowed me to examine the influences of other variates.

The third limitation of the survey responses could be that they reflect social desirability bias. Social desirability is the tendency to conform to what a respondent perceives to be more socially acceptable (e.g., popular opinions, politically correct responses) and to give the conforming answers rather than answering truthfully (Krumpal, 2013). The resulting untruthful answers can lead to skewed results. For example, a participant might try to project a favorable image of themselves by selecting a scenario's noncanonical option if they viewed the noncanonical option (e.g., giving students opportunities to choose the mathematical components) as a preferred action by the mathematics education community. The study's unconfirmed norms (Hypothesized Norms 3, mathematical components, and Hypothesized Norm 4, mathematical thinking dominance) or disconfirmed norm (Hypothesized Norm 5, model revisions omitted) could result from social desirability.

Since I sought to obtain information based on teacher participants' honest answers about what they would do when faced with the modeling activities, it is important to be aware of social desirability. I used three strategies to minimize social desirability bias as much as possible. First, I assured participants at the beginning of the survey that their responses would be anonymous. Their answers would not be shared with anyone or receive evaluations from the researcher so that they would know any responses were acceptable. Second, I kept the purpose of the survey vague to the participants (e.g., not asking their definition of mathematical modeling until the end of the survey) so that they were less likely to guess what might be the preferred responses to the survey questions. Third, I tried to eliminate the effect of social desirability by using a breaching experiment embedded in a survey where I confronted individual participants with representations of teaching that illustrated conceivable instructional scenarios. That way, the survey deliberately breached a familiar hypothesized norm of that action to "bring to the surface ... practitioners' sense of the norms of instruction" (Herbst & Chazan, 2011, p. 414).

A fourth limitation of this study was that cognitive bias and logical fallacy might have influenced teacher participants' judgment or decision making. One cognitive bias could have been a Dunning-Kruger effect in which people believed that they were more capable than they really are (Dunning, 2011). For example, if participants chose the noncanonical option and overestimated their teaching prowess in doing so, they might have underestimated the possibility of other teachers also choosing the noncanonical option. The Dunning-Kruger effect might explain why some teachers in the noncanonical groups (such as the two subgroups who answered the typicalness question for Hypothesized Norm 3, mathematical components) believed it was not typical for teachers to depart from the hypothesized norm.

A logical fallacy, on the other hand, could explain why some participants believed it was typical for teachers, in general, to follow the hypothesized norms. The mind projection fallacy, in particular, shows that people think that the way they see the world reflects the way the world really is (Taliaferro, 2018). For example, teacher participants might have judged the typicalness of the breaching action by projecting their actions onto other teachers' actions rather than reporting actual observations. In my study, I surveyed a large sample population to get a relatively large data set to lessen the impact of cognitive bias and logical fallacy. Nevertheless, I kept both in mind when interpreting the results, as should any reader.

Implications

Implications for Research

Although I tried to better understand modeling norms by designing two different tasks and randomly assigning participants into two groups, the survey design still invites questions about whether the features of the underlying modeling tasks affected which teaching practices the respondents regarded as typical. For example, the major mathematical content underlying Task 1 (i.e., bacterial reproduction) was exponential functions, and bacterial reproduction is a commonly used problem context in word problems about exponential growth. Therefore, the survey respondents may have favored symbolic representations more than other types of representations in ways that do not necessarily extend to other tasks. Teachers might have been more open to students' nonsymbolic representations if the task were set in other contexts or embedded with other subject matter knowledge (e.g., geometry content). Future research will be needed to find out if these norms apply in similar or distinct ways to different types of modeling situations, or if they are unique to the tasks and scenarios that I designed for the study.

Moving from the mathematical content to the participant context, although basic participant information (years of teaching, experience with modeling, subjects) was collected, differences between groups of teachers related to modeling norms have not yet been studied. As Bieda and colleagues (2015) suggested, perceived obligations are often in a state of flux and constantly evolve as teachers advance in their careers. Hence, one can assume that teachers with more modeling experience might feel different obligations toward modeling activities than those who have little to no experience. Teachers in different stages of their careers might also react to obligations differently. Moreover, teachers who have taught different subjects might feel obligated in different ways. For example, geometry teachers might not feel as obligated as algebra teachers to have students come up with symbolic representations. Furthermore, in terms of Norm 6 (politically neutral contexts), a teacher's individual factors, such as their ethnic background and their school's diversity, might have an impact on their preference for the task with a social justice emphasis but the present study did not conduct this analysis. Hence, future research might compare the perceived norms that various teacher populations selected and the obligations they identified as justifications for their choices. These comparisons might reveal more nuances about the practical rationality of modeling and, as a result, provide more concrete support to specific groups of teachers.

This study also found that a small group of teachers chose all noncanonical options and that another group of teachers chose all canonical actions across all scenarios. Future research might examine these specific cases and identify emerging themes. For example, it would be interesting to investigate if the members in one of these small groups shared similar instructional visions and explore how their visions connected to the norms and perceived obligations. This type of analysis would enable mathematical education researchers to specify other factors (e.g., teachers' visions) at play, explain how they contribute to the norms and professional obligations, and advance our understanding of the current instructional practice of mathematical modeling.

The professional obligations framework in this study helped understand how a teacher's environment impacts the manner in which mathematical modeling can be implemented in the classroom, rather than merely considering what knowledge the teacher brings with them or lacks. However, Herbst and Chazan's (2017) obligation framework did not cover all the dimensions that teachers perceived in relation to mathematical modeling. In the study, some teachers mentioned that it is a teacher's responsibility to develop students' lenses so that they can better perceive what is happening outside of school and better respond to that reality. A few teachers described their responsibility in cultivating the next generation to pursue solutions to challenges and improve society. This perceived obligation can be seen as a new dimension of the obligation framework: the teachers' obligation to society to prepare responsible citizens. As discussed in the previous chapter, the majority of teacher participants were hesitant to embrace tasks that touch on racism and social justice topics. Therefore, it is important to note that this societal obligation might have boundaries in terms of how politically charged teachers want their lessons to be or how comfortable they are in shaping social justice characteristics as they help to develop responsible citizens.

Implications for Instructional Practice

The findings of this study have implications not only for researchers but also for practitioners in the classroom. The first implication is that teachers' perceived obligations can be leveraged as an opportunity for teachers to adopt potential changes of the enactment of modeling tasks (or modeling-related tasks), leading to the expansion of the norms. Webel and Platt (2015) suggested that obligations should not only be seen as constraints but affordances as well, because encouraging teachers to reflect on the obligations behind their actions may cause teachers to pause and possibly consider changes in their practice, in this case the use of modeling tasks to achieve their educational goals. By recognizing teachers' obligations, rather than brushing them aside, they can be invited into meaningful changes to practice that they can buy into.

Take Hypothesized Norm 3 (mathematical components) as an example. Some teacher participants felt obliged to be prescriptive and gave students clear directions about the mathematical components they needed during modeling. These teachers seemed uncomfortable with sending students all alone down the potentially bumpy and muddy path of model building. They would rather carefully guide students down the preferred path until there is little chance that an alternative would disrupt the flow of the class. There is a rationality behind these teachers' decisions. They responded to individual obligations—they felt obliged to avoid confusing students. However, another group of teacher participants chose to put decision making in the students' hands. They were driven by individual obligations to deepen student thinking.

Suppose teachers' obligations could prompt them to examine students' confusion and recognize that struggles are a necessary part of learning mathematics (Hiebert & Grouws, 2007). Perhaps the obligation to not confuse students can be tweaked into guiding them through confusion. A different understanding of students' confusion might inspire teachers to breach normative teaching practices. In the case of Hypothesized Norm 3 (mathematical components), individual obligations can motivate teachers to share authority with students and give them opportunities to choose mathematical components. In addition to seeing individual obligations as affordances that can help hypothesized norms evolve, individual obligations might also help challenge or even expand confirmed norms.

Moving on to the second implication for instructional practice, let us discuss how teachers new to modeling can be successful. Considering the obligations that teachers have (such as disciplinary or institutional obligations), an incremental approach might better suit those teachers who are just starting to incorporate mathematical modeling into their current teaching. That is, rather than attempting to require teachers to implement an entire modeling activity, they should be encouraged to choose some parts of modeling, not necessarily the full modeling cycle (Blum & Leiß, 2005), that work for their lesson objectives. Since modeling tasks usually place a high cognitive demand on students and teaching with modeling tasks is regarded as a reformed way of teaching, it may be more achievable and less daunting for teachers to gradually add parts of modeling into their existing teaching practice. Doing so reduces the risk of radical change. Once teachers have experience successfully teaching part of the modeling process, they might add more elements, step by step, into their instruction and carry out modeling tasks more frequently. An incremental approach can benefit both novice modeling instructors and their students and each incremental step can be pursued with a recognition of the obligations that the teachers are feeling.

Implications for Professional Development and Teacher Education

The findings suggest that the enactment of modeling tasks can be improved by increasing teacher educators and education researchers' awareness of the variety of ways in which practicing teachers feel obligated. If an observer judges a teacher's action to be odd, the observer must seek to understand the action through the lens of the teacher's professional obligations. As teacher educators and education researchers, we might look empathetically into the practical rationality of teaching and the professionals who must contend with complex demands placed on them by their students, discipline, institutions, and society. More patience, time, and resources should be granted to teachers to allow modeling to occur more in the classroom.

There are some ways that teacher educators and education researchers can help teachers enact mathematical modeling or enact it more often. First, we can design and offer professional development about how teachers can use their limited time to integrate mathematical modeling into existing instructional practices, incorporating elements of modeling that do not severely conflict with the current norms.

Second, curriculum designers might take teachers' competing obligations into consideration and provide them with viable modeling tasks and tools. It might be helpful to systematically generate alternative modifications for proposed modeling tasks and to explain the variance among the tasks. For instance, consider the following word problem: "A population in a small town is 500, and it grows by 2% each year. Determine the population size after 3 years.” After the problem, textbooks could present different versions of the task to teachers: (1) Make the algorithm in the problem less clear for students so that they can determine which functions can help solve the problem. (2) Remove the information in the task (e.g., the initial population) and allow students to do some research. Then, textbooks could unpack the learning opportunities behind each modification and discuss how the modifications meet the learning standards or assessment goals.

Third, teacher educators might consider including modeling as part of their methods courses for teacher education because “substantive experiences engaging in mathematical modeling” can help teachers “understand mathematical modeling and its potential place in the curriculum” (Association of Mathematics Teacher Educators [AMTE], 2017, p. 124). As more prospective teachers are exposed to modeling in the early stages of their careers, they can better understand modeling, its different parts, and the benefits of implementing it in class. Teacher educators can show prospective teachers the obligations that practicing teachers have perceived and lead meaningful conversations about the nascent obligations the prospects already feel and how these obligations might be expressed in their future classrooms. Cultivating a mindful approach to teaching can help teachers realize how their approaches ultimately affect students’ learning.

Beyond teacher educators and education researchers, other stakeholders (e.g., the school, school district) should facilitate the incorporation of modeling. For example, administrators can include more flexibility in teachers’ schedules and allow more teacher-directed time for teachers to collaborate with their colleagues and share their modeling tasks and teaching experiences. Support from school administrators and collaborations with colleagues will help teachers manage institutional obligations.

Implications for Social Justice and Equity

In this study, the majority of the teacher participants were hesitant to use the racially infused task that referred to Flint, Michigan's water crisis. The teachers reported that they did not want to have "uncomfortable conversations" among their students; their desire to avoid such conversations was categorized as an interpersonal obligation, as part of their obligation to maintain a social environment conducive to learning. Moreover, the findings indicate that teachers' interpretations of mathematics as a discipline might prevent them from incorporating social justice goals into their practice. Some survey respondents did not consider race germane to mathematics, and some believed that parents are responsible for teaching children how to approach race and racism.

Still, achieving equity is a challenge facing mathematics educators (NCTM, 2014). As described in the Literature Review chapter, mathematical modeling has key attributes closely related to equity and social justice (Aguirre et al., 2013; Kaiser & Sriraman, 2006). We might use modeling as a tool to bring teachers into critical conversations about the diverse and divergent roles of mathematics in the social and public domains. These conversations might also help strategically integrate modeling tasks with a social justice emphasis into the teaching practice. It would be a small step toward a broader goal of achieving social justice and equity.

Conclusion

The body of research on mathematical modeling is rapidly growing in the field of mathematics education. A better understanding of how teachers perceive mathematical modeling and what professional obligations they feel can provide researchers and practitioners a foothold for the enactment of mathematical modeling. The findings of this study shed light on how the field can continue supporting secondary teachers and their practices, and ultimately achieve the desired goals for mathematical modeling in the classroom.

APPENDIX A

Teacher Recruitment Letter

The purpose of this survey is to gather information about your views on math tasks and how mathematics teachers interact with students as they work. Completing this survey does not obligate you to participate in any future activities in the study. If you are eligible for an additional study, the researchers will contact you for a follow-up interview with additional compensation.

Participant rights: Your name and email address will only be seen by the researchers. Your responses to the survey will NOT be shared beyond the researchers. Your participation is voluntary, and you have the right to withdraw from the survey at any time.

Compensation: Participants who complete the survey will be entered in a drawing to win one of fifty $20 Visa e-gift cards. Please complete the survey by Feb 21 and the result of the gift cards will be announced on Feb 23.

Contact information: If you have any questions or concerns about the research study, please feel free to contact the researcher: Wenmin Zhao, wenminzhao@mail.missouri.edu. If you have questions or concerns about your rights as a research participant, please contact the University of Missouri Campus Institutional Review Board at 573-882-9585 or umcresearchcirb@missouri.edu.

This survey involves 4 fictional classroom scenarios and should take you less than 20 minutes to complete. Thank you for your time!

Please select your desired choice for providing consent.

- Yes
- No

APPENDIX B

Mathematical Modeling Tasks in the Breaching Experiment

Task 1: Bacterial Reproduction Task

Droplets containing bacteria are released into the air when a person sneezes. You can get sick when you come into contact with these tiny droplets. If a person sneezes on his/her hand, how many individual bacteria are on his/her hand over time?

Task 2: Water Usage Task

Water is an important part of our daily lives. As vast as the water resources of the United States are, they are not endless. Water needs to be conserved and protected. People use water for a wide variety of purposes. How much water will be enough to meet the daily needs of a city?

APPENDIX C

The Options of Different Scenarios

Hypothesized Norm	Canonical option 1 (C1)	Canonical option (C0)	Noncanonical option (NC)
1. Factor specification	Good question. You can't consider everything, so for this problem, just focus on the initial amount of bacteria, the reproduction rate, and the length of time.	Good question. You can't consider everything, so you can narrow it down. You don't have to consider factors like temperature or humidity.	Good question. That is something your group needs to think about. Choose whatever makes sense to you.
2. Symbolic representation	You have a table, which is a good start, but you need to eventually find an equation for this problem.	You have a table, but it would be really nice if you could find an equation for the situation, too.	You have a table, and a table can certainly serve as the representation of this situation. You don't have to find an equation.
3a. Mathematical components	You can use an exponential function. Remember, it has the form $y = a \cdot b^x$.	Ms. Lee: What did we learn yesterday? Emma: Exponential functions? Ms. Lee: Then try some exponential functions.	You should decide on the function. Use whatever type of function or functions you think are appropriate.
3b. Mathematical components	You can use a linear function. Remember, it has the form $y = mx + b$.	Ms. Smith: What did we learn yesterday? Emma: Linear functions? Ms. Smith: Then try some linear functions.	You should decide on the function. Use whatever type of function or functions you think is appropriate.
4. Mathematical thinking dominance	I like how you are connecting the situation to your own experiences, but your family situation might not be typical. Try to	I like how you are connecting the situation to your own experiences. Let's put water for showering and	I like how you are connecting the situation to your own experiences. What else comes to mind about

Hypothesized Norm	Canonical option 1 (C1)	Canonical option (C0)	Noncanonical option (NC)
	quantify an average person’s water usage for showering and dishwashing when you develop your equation.	dishwashing into your equation.	water usage in your real life?
5a. Model revision omitted	Group 2 brought out an interesting idea about bacteria dying. That was part of their answer. There are multiple good answers to these kinds of problems, so we are going to wrap up. We are going to start the next lesson and finish this chapter by this Friday. (The class spent the rest of time on the new lesson.)	Group 2 brought out an interesting idea about bacteria dying. Other groups, go ahead and revise your current model to try to incorporate bacteria dying. Group 2, you can finalize it and then turn in your work. (Other groups spent the rest of time revising their models.)	Group 2 brought out an interesting idea about bacteria dying. Besides bacteria dying, what else can we add to the current model we have? Let’s think about a couple of other things and continue revising our model. (The whole class spent the rest of time revising the models.)
5b. Model revision omitted	Group 2 brought out an interesting idea that different seasons will affect the amount of water that we use daily. That was part of their answer. There are multiple good answers to these types of problems, so we are going to wrap up. We are going to start the next lesson and finish this chapter by this Friday. (The class spent the rest of time on the new lesson.)	Group 2 brought out an interesting idea that different seasons will affect the amount of water that we use daily. Other groups, go ahead and revise your current model to try to incorporate different seasons. Group 2, you can finalize it and then turn in your work. (Other groups spent the rest of time revising the models.)	Group 2 brought out an interesting idea that different seasons will affect the amount of water that we use daily. Besides the impact of seasons, what else can we add to the current model we have? Let’s think about a couple of other things and continue revising our model. (The whole class spent the rest of time revising the models.)

Hypothesized Norm	Canonical option 1 (C1)	Canonical option (C0)	Noncanonical option (NC)
6. Politically neutral contexts	Water is an important part of our daily lives. As vast as the water resources of the United States are, they are not endless. Water needs to be conserved and protected. People use water for a wide variety of purposes. How much water will be enough to meet the daily needs of a city?	Some cities in the United States changed their water source, causing high concentrations of lead in the local tap water. Exposure to lead is serious. It impacts public health and can be particularly harmful to young children. As a result, bottled water had to be brought in from outside the city. How much water will be enough to meet the daily needs of a city?	The government in Flint, Michigan, changed the water source for the city, causing high concentrations of lead to be found in its African-American neighborhoods. Exposure to lead is serious. It impacts the health of the communities, particularly among African-American children. As a result, bottled water had to be brought in from outside the city. How much water will be enough to meet the daily needs of Flint?

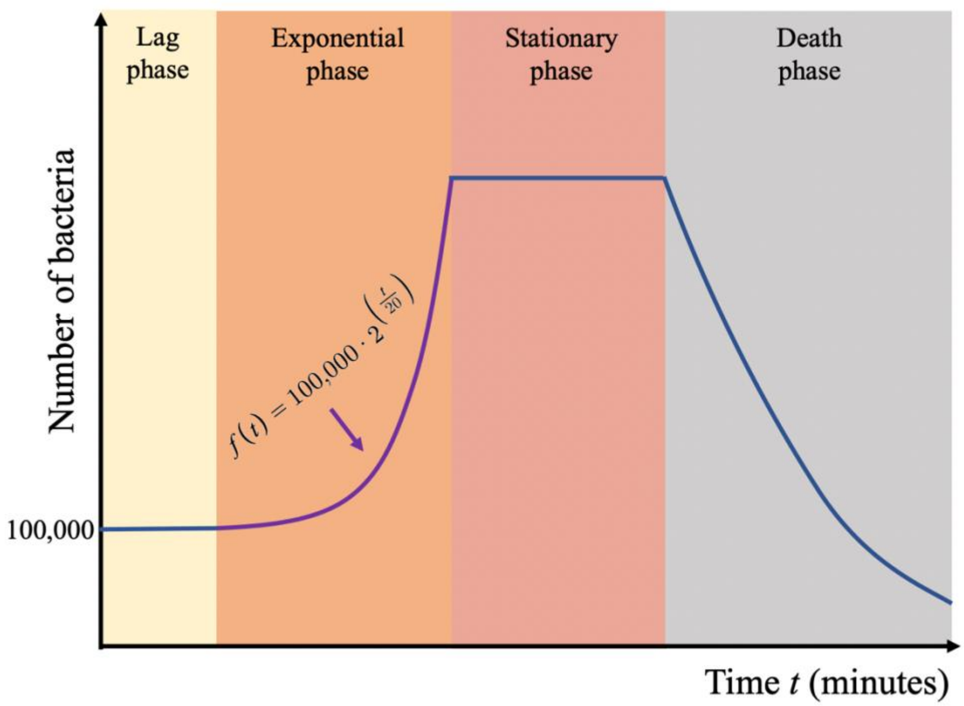
Appendix D

Possible Solutions for the Modeling Tasks

Solution for Task 1 – Bacterial Reproduction

Figure D1

The Four Phases of Bacterial Reproduction After One Sneeze



Note. Adapted from "Bacterial Growth Curve" by M. Komorniczak, 2020. (https://bio.libretexts.org/Bookshelves/Microbiology/Book%3A_Microbiology_(Bruslind)/09%3A_Microbial_Growth). Copyright 2020 by Michał Komorniczak.

The number of bacteria after one sneeze over t minutes is represented by the function $f(t)$. The initial number of germs after an average sneeze is 100,000 (University of Bristol, 2019). This bacterial reproduction model used 20 minutes as the bacterial doubling time (Allen & Waclaw, 2018). The four phases of bacterial reproduction include the lag phase, exponential phase, stationary phase, and death phase (Bruslind, 2020). The lag phase is when bacteria are adjusting to the new conditions after the sneeze. The exponential phase is when bacteria start to reproduce (doubling). The stationary phase is when the bacterial population runs out of nutrients or its growth is inhibited by its own waste products. The death phase is when the bacteria start to decrease.

Table D1

Connections Between Instructional Decisions and Hypothesized Norms in Task 1 – Bacterial Reproduction

Hypothesized norm	Noncanonical option (NC)	Aspects of the solution
1. Factor specification	The teacher encouraged students to decide what they wanted to include rather than directly leading them to include specific factors in their models.	This solution included not only factors defined by the teacher—such as the initial amount of bacteria, the reproduction rate, and the length of time—but also different phases and restrictions of bacterial reproduction.
2. Symbolic representation	The teacher approved their use of a tabular representation without requiring students to translate it into a function.	This solution was not limited to symbolic representation. It used both graphs and functions as its representations.
3. Mathematical components	The teacher encouraged students to choose the operations or components that made sense to them rather than directing them to specific equations.	This solution used an exponential function for its exponential phase, but it did not necessitate the use of exponential functions for other phases.
4. Mathematical thinking dominance	The teacher encouraged students to share a variety of personal experiences and incorporate them into the model-building process (not merely as motivation or an introduction to the task).	This solution brought in nonmathematical thinking. For example, bacterial reproduction was inhibited by its own waste products.
5. Model revision omitted	The teacher prompted the class to continue revising their models and allotted much in-class time for a more complex model.	This solution was revised by considering bacterial death.

Solution for Task 2 – Water Usage Task

$$z = 23{,}949(6x) + \ 108{,}500(2y),$$

where $x =$ number of dishwashing cycles, $y =$ average shower length (minutes)

Note. The amount of water that meets the daily needs of Columbia, Missouri, is represented by *z*. The population of Columbia, Missouri, is 108,500, and 23,949 is the number of families who resided in the city in the 2010 census. Water-saving showerheads produce about 2 gallons per minute. Energy-saving dishwashing machines use 6 gallons per wash cycle. The information on water usage is from the website of the United States Department of the Interior (DOI).

Table D2

Connections Between Instructional Decisions and Hypothesized Norms in Task 2 – Water Usage

Hypothesized norm	**Canonical option 1 (C1)**	**Aspects of the solution**
1. Factor specification	The teacher told students what they needed to include as factors (i.e., dishwashing and showers) in their model.	This solution was limited to two specific water usage activities, dishwashing and showers, which were offered by the teacher.
2. Symbolic representation	The teacher asked students to find an equation for this problem.	This solution used a symbolic representation as its representation.
3. Mathematical components	The teacher determined the specific type of function (i.e., a linear function) that students should use in their model.	This solution only used a linear function.
4. Mathematical thinking dominance	The teacher steered students from nonmathematical thinking to mathematical thinking by asking them to merely quantify two of the factors that students had mentioned and develop the equation.	This solution did not encourage students to bring in nonmathematical thinking, such as water usage in different seasons.
5. Model revision omitted	The teacher chose to start a new lesson after all students shared their work and did not allot time for the class to continue revising their models.	This solution did not explore a more refined model that considered other factors besides dishwashing and showers.
6. Politically neutral contexts	The teacher implemented a modeling task with a politically neutral context rather than a task with a social justice emphasis.	This solution did not incorporate social justice elements.

APPENDIX E

Survey Flow

Group 1

<table>
<tr><td colspan="2">Q1 Provide consent Yes/No</td></tr>
<tr><td colspan="2">Q2 Read the task below.
Bacterial Reproduction Task
Droplets containing bacteria are released into the air when a person sneezes. You can get sick when you come into contact with these tiny droplets. If a person sneezes on his/her hand, how many individual bacteria are on his/her hand over time?</td></tr>
<tr><td colspan="2">Q3 This task has a real-world context and does not specify what students have to do. Would you use this type of task as written with your students?
Yes/Maybe/No</td></tr>
<tr><td colspan="2">Q4 Explain your choice.</td></tr>
<tr><td>Display Q5 if Q3 = Yes:
Q5 How often would you use this type of task as written with your students?
Daily/Weekly/Monthly/Semesterly</td><td>Display Q6 if Q3 = Maybe Or Q3 = No:
Q6 Would you use the task if you could change it? Yes/No
Display Q7 if Q6 = Yes:
Q7 You indicated you would use this type of task if you could change it. What kind of changes would you make? And why?</td></tr>
<tr><td colspan="2">Q8 Here is the first scenario of a high-school lesson using the same task. View the scenario and then click the arrow → to the next page.</td></tr>
<tr><td colspan="2">Q9 Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.
Canonical Option 1 (C1)/ Canonical Option 0 (C0)/ Noncanonical Option (NC)</td></tr>
<tr><td colspan="2">Q10 Indicate the extent to which you considered these reasons as you selected your teacher action.
Didn’t consider/Considered/Strongly considered</td></tr>
<tr><td colspan="2">Q11 Are there other considerations when you selected the teacher action? If so, you may describe them here? Open responses</td></tr>
</table>

<table>
<tr><td colspan="2">Q12 Are there other ways you would respond to the students? And why? Open responses</td></tr>
<tr><td><u>Display Q13 if Q9 = Option NC:</u>
Q13 Read what another teacher, Ms. Johnson, did in that situation. (C1)
Q14 For mathematics teachers in general, how typical would it be to respond to the students as Ms. Johnson did?

Very typical/Somewhat typical/Somewhat rare/Very rare</td><td><u>Display Q15 if Q9 = C1 Or C0:</u>
Q15 Read what another teacher, Ms. Johnson, did in that situation. (NC)
Q16 For mathematics teachers in general, how typical would it be to respond to the students as Ms. Johnson did?

Very typical/Somewhat typical/Somewhat rare/Very rare</td></tr>
<tr><td colspan="2" align="center">The next three scenarios share the same flow from Q8-Q16</td></tr>
<tr><td colspan="2">Q17 What does the term “mathematical modeling” mean to you?</td></tr>
<tr><td colspan="2">Q18 Name (Known only to the researchers)</td></tr>
<tr><td colspan="2">Q19 Email address (Used only if you are selected to be invited for further participation in the study)</td></tr>
<tr><td colspan="2">Q20 School and location (Known only to the researchers)</td></tr>
<tr><td colspan="2">Q21 How many years of teaching experience do you have in total?</td></tr>
<tr><td colspan="2">Q22 What courses (and grade levels) have you taught?</td></tr>
<tr><td colspan="2">Q23 Have you ever enacted mathematical modeling tasks in your classroom?
Yes/No/Not sure</td></tr>
<tr><td><u>Display Q24—Q16 if Q23 = Yes:</u>
Q24 In what courses have you used modeling tasks?
Q25 For how many years have you enacted modeling tasks in your classroom?
Q26 What are the potential barriers for you to enact modeling tasks (or to enact modeling tasks more often)?</td><td><u>Display Q27 & Q28 if Q23 = No Or Not sure:</u>
Q27 Would you consider enacting modeling tasks in the future? Why or why not?
Q28 What would the potential barriers be for you to enact modeling tasks?</td></tr>
<tr><td colspan="2">Q29 What other ideas, if any, do you have about mathematical modeling that you would like to share?</td></tr>
</table>

Group 2

<table>
<tr><td colspan="2">Q1 Provide consent Yes/No</td></tr>
<tr><td colspan="2">Q2 Read the task below.
Water Usage Task
Water is an important part of our daily lives. As vast as the water resources of the United States are, they are not endless. Water needs to be conserved and protected. People use water for a wide variety of purposes. How much water will be enough to meet the daily needs of a city?</td></tr>
<tr><td colspan="2">Q3 This task has a real-world context and does not specify what students have to do. Would you use this type of task as written with your students?
Yes/Maybe/No</td></tr>
<tr><td colspan="2">Q4 Explain your choice.</td></tr>
<tr><td>Display Q5 if Q3 = Yes:
Q5 How often would you use this type of task as written with your students?
Daily/Weekly/Monthly/Semesterly</td><td>Display Q6 if Q3 = Maybe Or Q3 = No:
Q6 Would you use the task if you could change it? Yes/No
Display Q7 if Q6 = Yes:
Q7 You indicated you would use this type of task if you could change it. What kind of changes would you make? And why?</td></tr>
<tr><td colspan="2">Q8 Here is the first scenario of a high-school lesson using the same task. View the scenario and then click the arrow → to the next page.</td></tr>
<tr><td colspan="2">Q9 Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.
Canonical Option 1 (C1)/ Canonical Option 0 (C0)/ Noncanonical Option (NC)</td></tr>
<tr><td colspan="2">Q10 Indicate the extent to which you considered these reasons as you selected your teacher action.
Didn’t consider/Considered/Strongly considered</td></tr>
<tr><td colspan="2">Q11 Are there other considerations when you selected the teacher action? If so, you may describe them here? Open responses</td></tr>
<tr><td colspan="2">Q12 Are there other ways you would respond to the students? And why? Open responses</td></tr>
</table>

<table>
<tr>
<td><u>Display Q13 if Q9 = Option NC:</u>
Q13 Read what another teacher, Ms. Johnson, did in that situation. (C1)
Q14 For mathematics teachers in general, how typical would it be to respond to the students as Ms. Johnson did?

Very typical/Somewhat typical/Somewhat rare/Very rare</td>
<td><u>Display Q15 if Q9 = C1 Or C0:</u>
Q15 Read what another teacher, Ms. Johnson, did in that situation. (NC)
Q16 For mathematics teachers in general, how typical would it be to respond to the students as Ms. Johnson did?

Very typical/Somewhat typical/Somewhat rare/Very rare</td>
</tr>
<tr>
<td colspan="2" align="center">The next two scenarios share the same flow from Q8-Q16</td>
</tr>
<tr>
<td colspan="2">This is the Original Task from earlier in the survey:

 Original Task Water is an important part of our daily lives. As vast as the water resources of the United States are, they are not endless. Water needs to be conserved and protected. People use water for a wide variety of purposes. How much water will be enough to meet the daily needs of a city?

Now please read the two New Tasks below:

 New Task A Some cities in the United States changed their water source, causing high concentrations of lead in the local tap water. Exposure to lead is serious. It impacts public health and can be particularly harmful to young children. As a result, bottled water had to be brought in from outside the city. How much water will be enough to meet the daily needs of a city?

 New Task B The government in Flint, Michigan, changed the water source for the city, causing high concentrations of lead to be found in its African-American neighborhoods. Exposure to lead is serious. It impacts the health of the communities, particularly within the African-American children. As a result, bottled water had to be brought in from outside the city. How much water will be enough to meet the daily needs of Flint?

Q17 What stands out for you as the difference among the Original Task, New Task A, and New Task B? Please be specific.</td>
</tr>
<tr>
<td colspan="2">Q18 Which one would you be most likely to use in your class?
Original Task/New Task A/New Task B</td>
</tr>
</table>

<table>
<tr><td colspan="2">Q19 Indicate the extent to which you considered these reasons as you selected your teacher action.
Didn’t consider/Considered/Strongly considered</td></tr>
<tr><td colspan="2">Q20 Are there other considerations when you selected the teacher action? If so, you may describe them here? Open responses</td></tr>
<tr><td>Display Q21 if Q17 = Original Task Or New Task A:
New Task B
The government in Flint, Michigan, changed the water source for the city, causing high concentrations of lead to be found in its African-American neighborhoods. Exposure to lead is serious. It impacts the health of the communities, particularly within the African-American children. As a result, bottled water had to be brought in from outside the city. How much water will be enough to meet the daily needs of Flint?

Q21 For mathematics teachers in general, how typical is it for them to choose New Task B?

Very typical/Somewhat typical/Somewhat rare/Very rare</td><td>Display Q22 if Q17 = New Task B:

Original Task
Water is an important part of our daily lives. As vast as the water resources of the United States are, they are not endless. Water needs to be conserved and protected. People use water for a wide variety of purposes. How much water will be enough to meet the daily needs of a city?

Q22 For mathematics teachers in general, how typical is it for them to choose Original Task?

Very typical/Somewhat typical/Somewhat rare/Very rare</td></tr>
<tr><td colspan="2">Q23 Name (Known only to the researchers)</td></tr>
<tr><td colspan="2">Q24 Email address (Used only if you are selected to be invited for further participation in the study)</td></tr>
<tr><td colspan="2">Q25 School and location (Known only to the researchers)</td></tr>
<tr><td colspan="2">Q26 How many years of teaching experience do you have in total?</td></tr>
<tr><td colspan="2">Q27 What courses (and grade levels) have you taught?</td></tr>
<tr><td colspan="2">Q28 Have you ever enacted mathematical modeling tasks in your classroom?
Yes/No/Not sure</td></tr>
<tr><td>Display Q29—Q31 if Q28 = Yes:
Q29 In what courses have you used modeling tasks?</td><td>Display Q32 & Q33 if Q28 = No Or Not sure:</td></tr>
</table>

Q30 For how many years have you enacted modeling tasks in your classroom? Q31 What are the potential barriers for you to enact modeling tasks (or to enact modeling tasks more often)?	Q32 Would you consider enacting modeling tasks in the future? Why or why not? Q33 What would the potential barriers be for you to enact modeling tasks?
Q34 What other ideas, if any, do you have about mathematical modeling that you would like to share?	

APPENDIX F

Predesigned Scenarios for Breaching Experiment

Hypothesized Norm 1 (Factor Specification). The teacher (rather than students) identifies precisely which information or factors should be included in the task.

Scenario 1 – What factors should we use?

(After 2 minutes of launching the task, students are working in small groups and discussing what they want to include in their solution.)

Amanda: I think it depends on, like, how many bacteria you get as you sneeze.
Brandon: And how long a time should we go for? One minute, one hour, or like a whole day?
Cody: Also, different types of bacteria might reproduce differently.
Dan: I agree. How about temperature? You know, different temperatures can affect how bacteria reproduce.
Amanda: So does the humidity.
Brandon: OK, but this seems like too many things for us to consider. I feel kinda overwhelmed. Let's ask Ms. Lee.
(Ms. Lee walks around to check each group's progress)
Dan: Hey, Ms. Lee, we aren't sure what information we need to include.

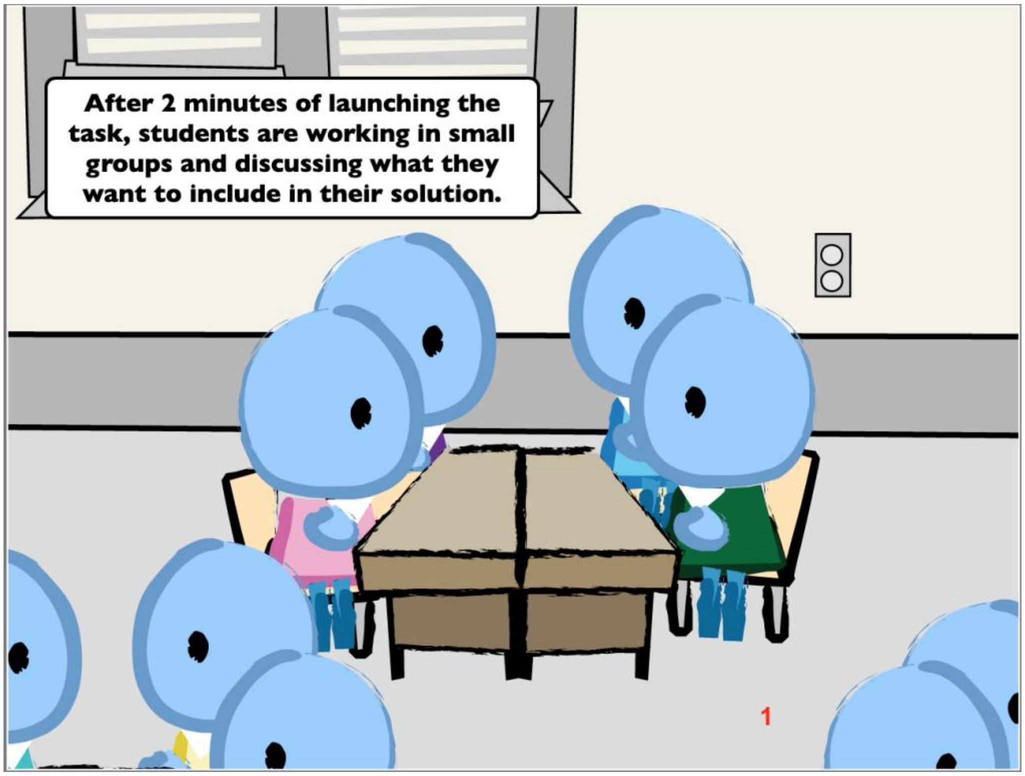
Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.

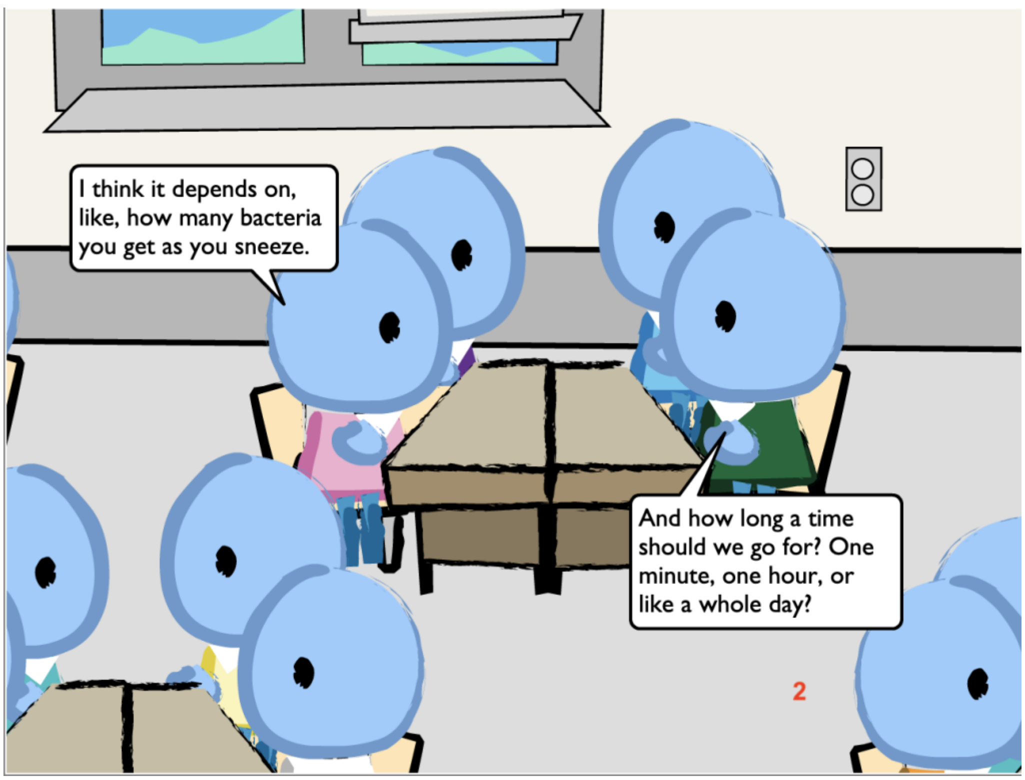
(NC) Ms. Lee: Good question. That is something your group needs to think about. Choose whatever makes sense to you.

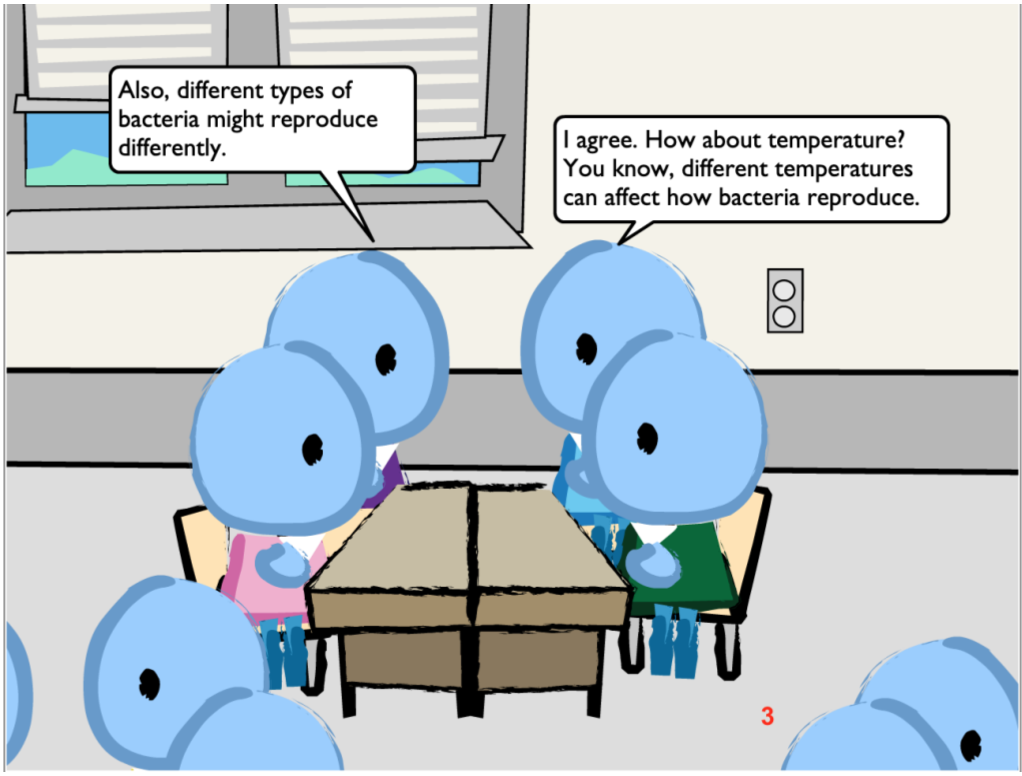
(C0) Ms. Lee: Good question. You can't consider everything, so you can narrow it down. You don't have to consider factors like temperature or humidity.

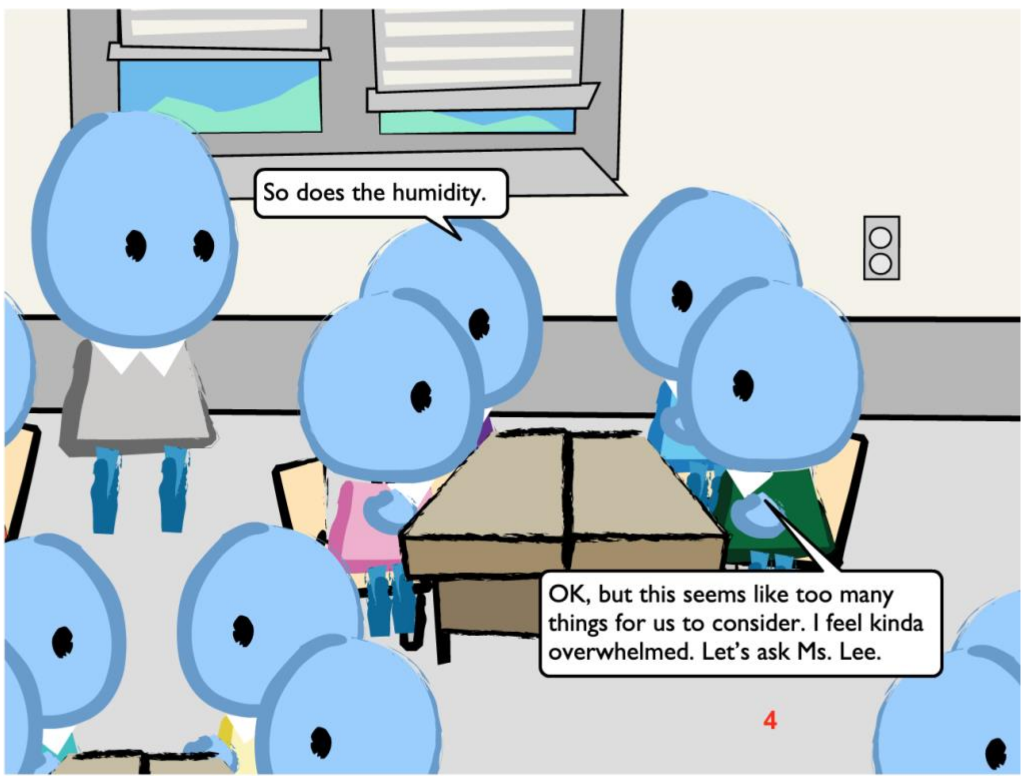
(C1) Ms. Lee: Good question. You can't consider everything, so for this problem, just focus on the initial amount of bacteria, the reproduction rate, and the length of time.

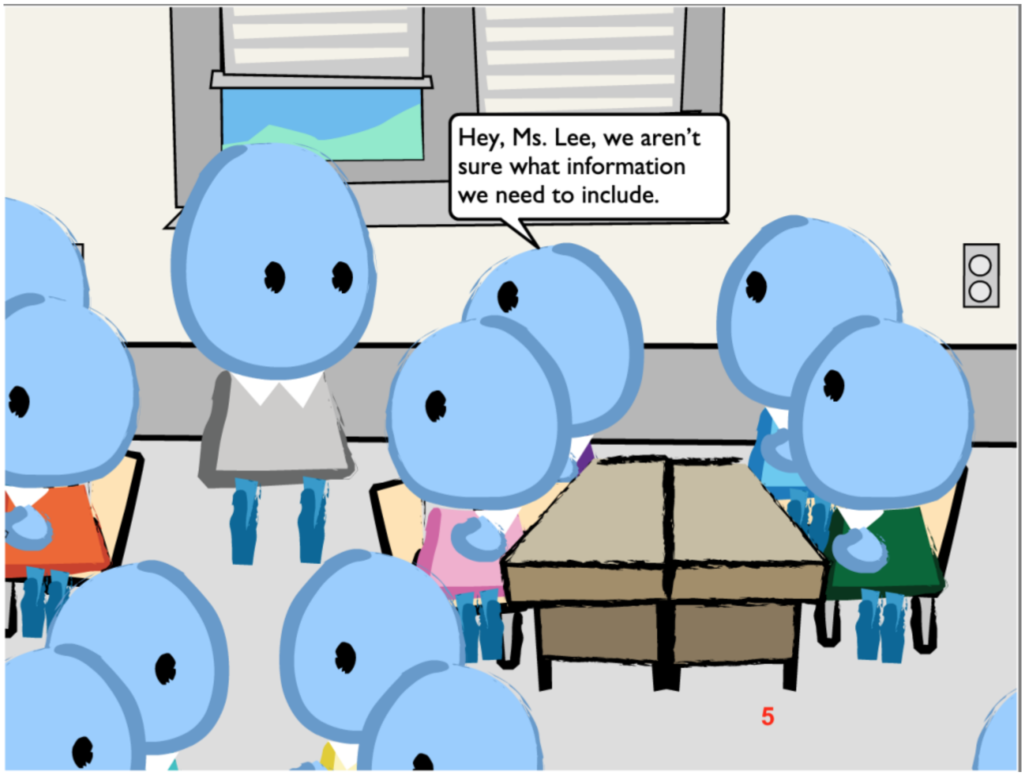
Scenario 1 – What factors should we use?
(5 pictures)





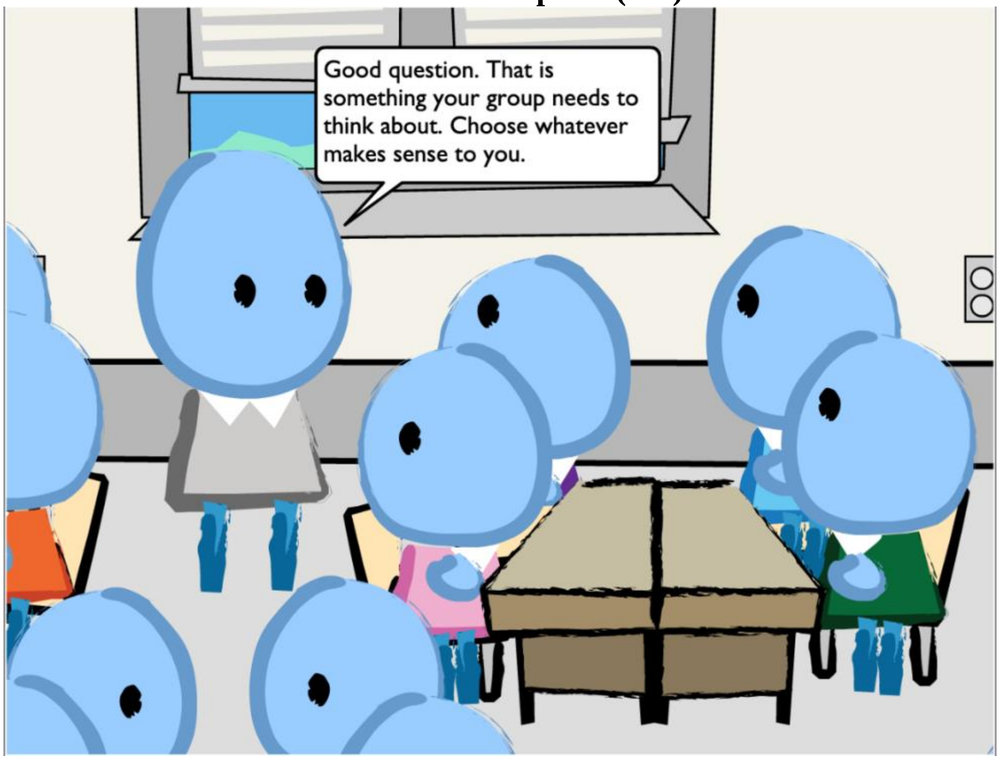

Also, different types of bacteria might reproduce differently.
I agree. How about temperature? You know, different temperatures can affect how bacteria reproduce.
3


So does the humidity.
OK, but this seems like too many things for us to consider. I feel kinda overwhelmed. Let's ask Ms. Lee.
4

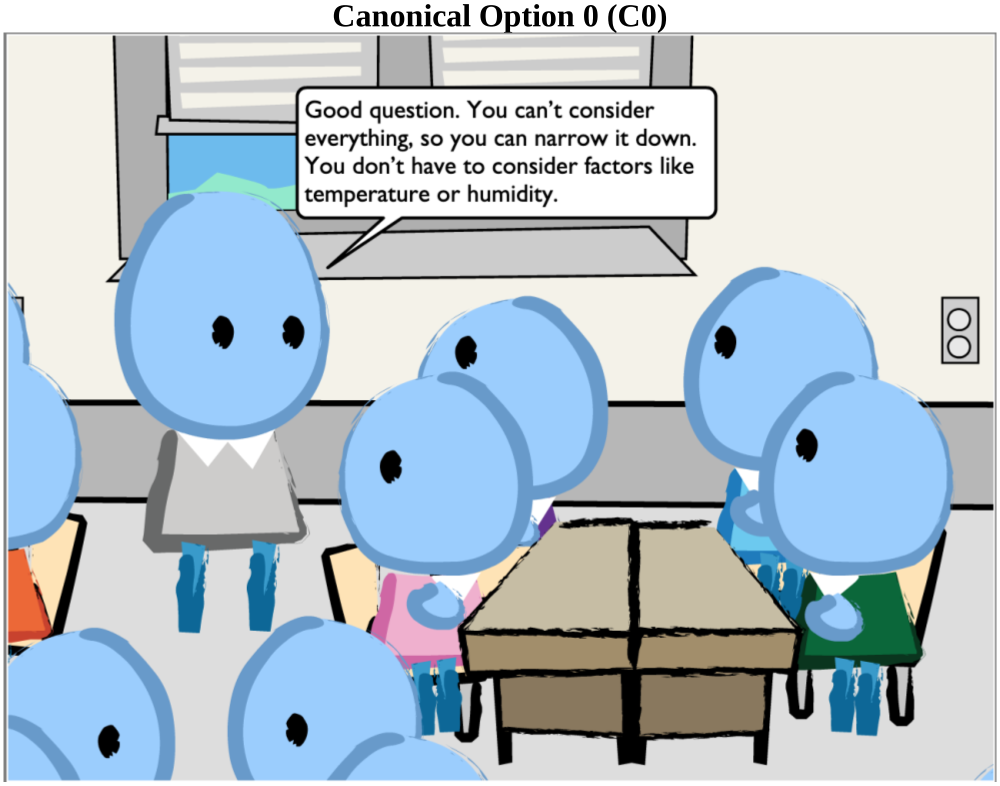

Hey, Ms. Lee, we aren't
sure what information
we need to include.
5

Three Options for Scenario 1

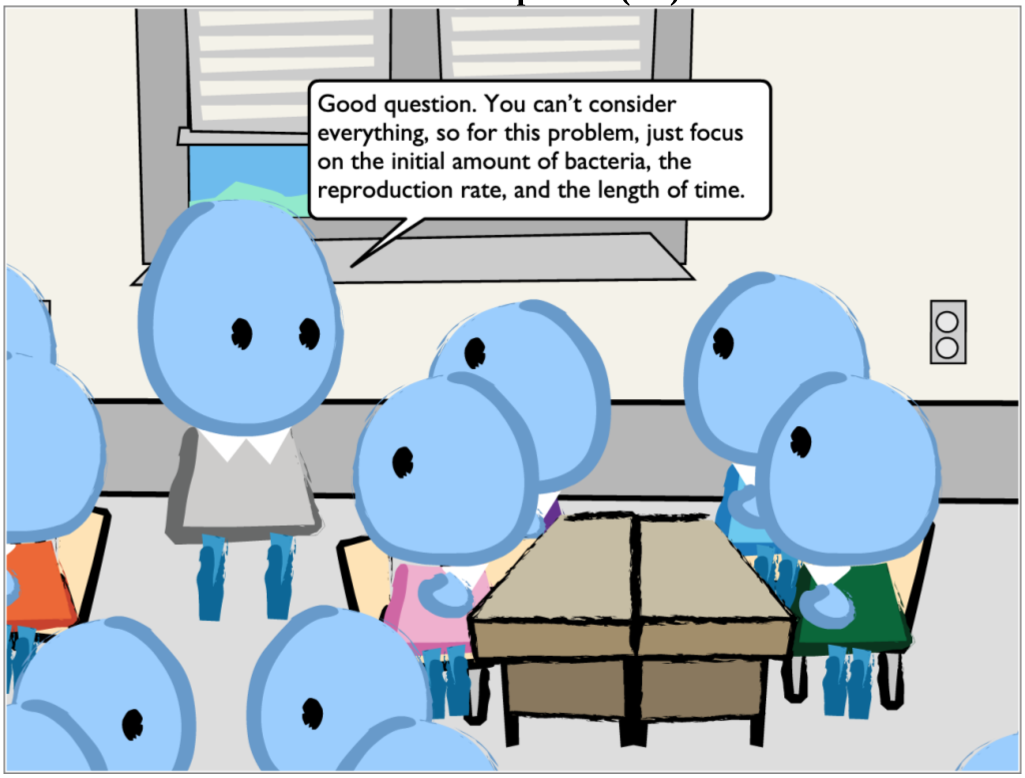
Noncanonical Option (NC)



Canonical Option 0 (C0)



Canonical Option 1 (C1)



Hypothesized Norm 2 (Symbolic Representation). Students are expected to find a symbolic representation (e.g., equations) as their model.

Scenario 2 – What functions should we use?

(Group 4 has decided that the initial amount of bacteria and reproduction rate per generation.)

n	
0	40,000
1	$40{,}000 \times 2$
2	$40{,}000 \times 2 \times 2$
3	$40{,}000 \times 2 \times 2 \times 2$
…	…

n = the n th generation

(Group 4 has created a table containing the amount of bacteria over several generations.)
Frank: Our table looks pretty cool. Oh, wait! Do we need to use equations?
(Ms. Lee walks around to check each group's progress)
Frank: Ms. Lee, do we need to use equations?

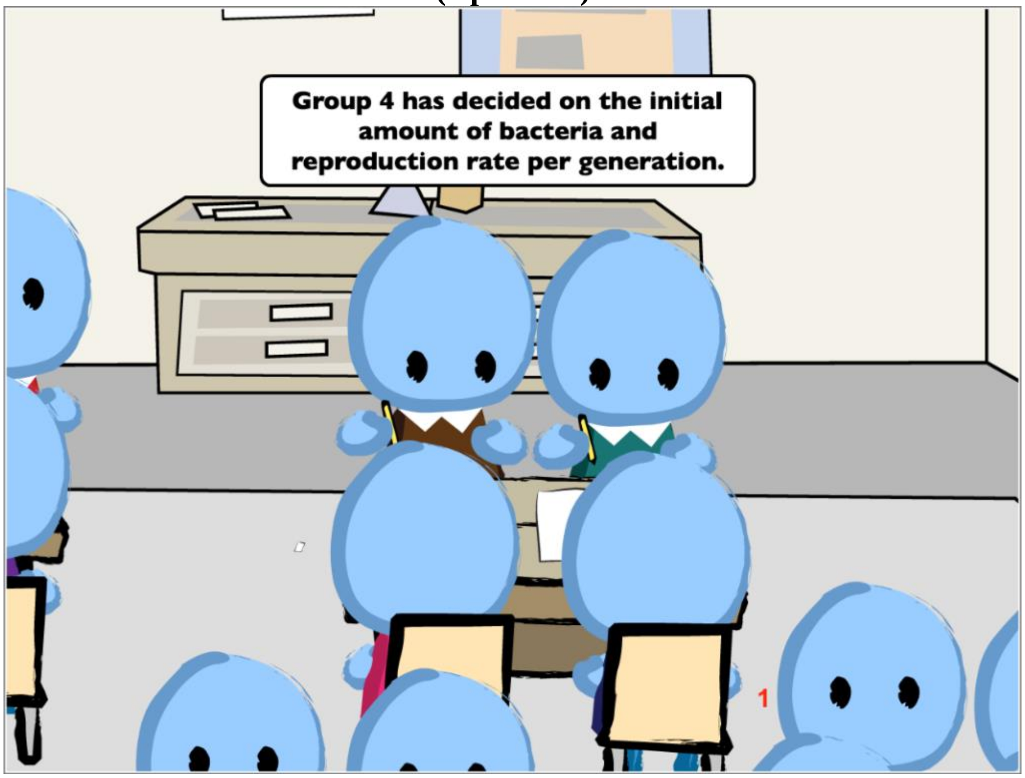
Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.

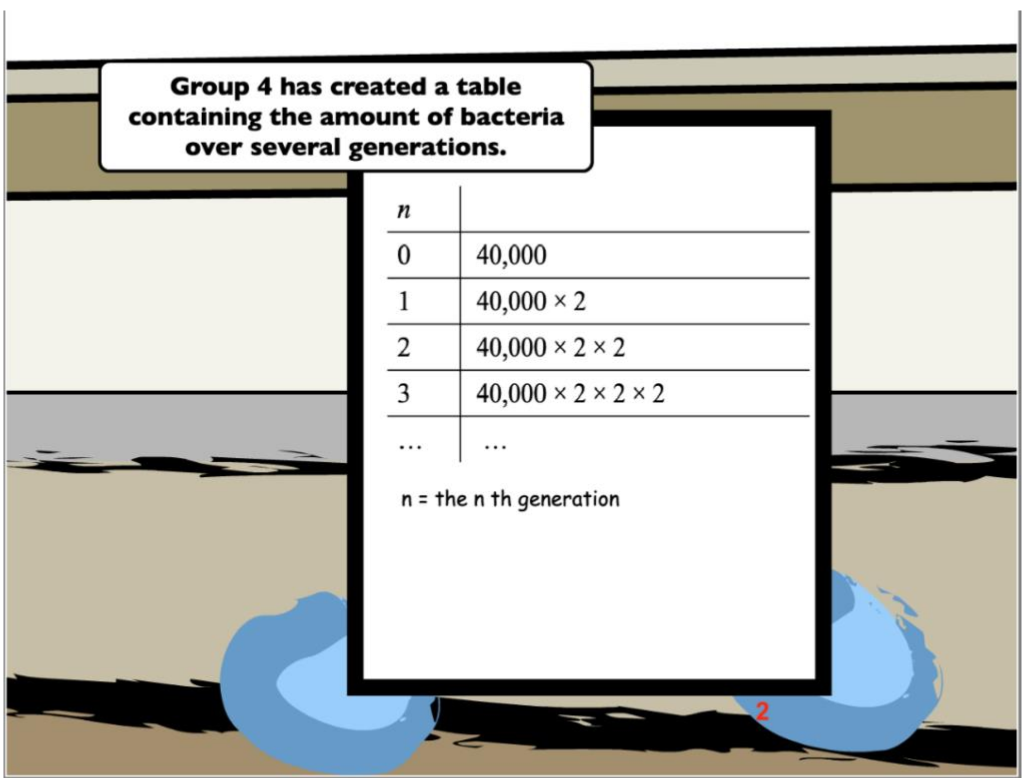
(NC) Ms. Lee: You have a table, and a table can certainly serve as the representation of this situation. You don't have to find an equation.

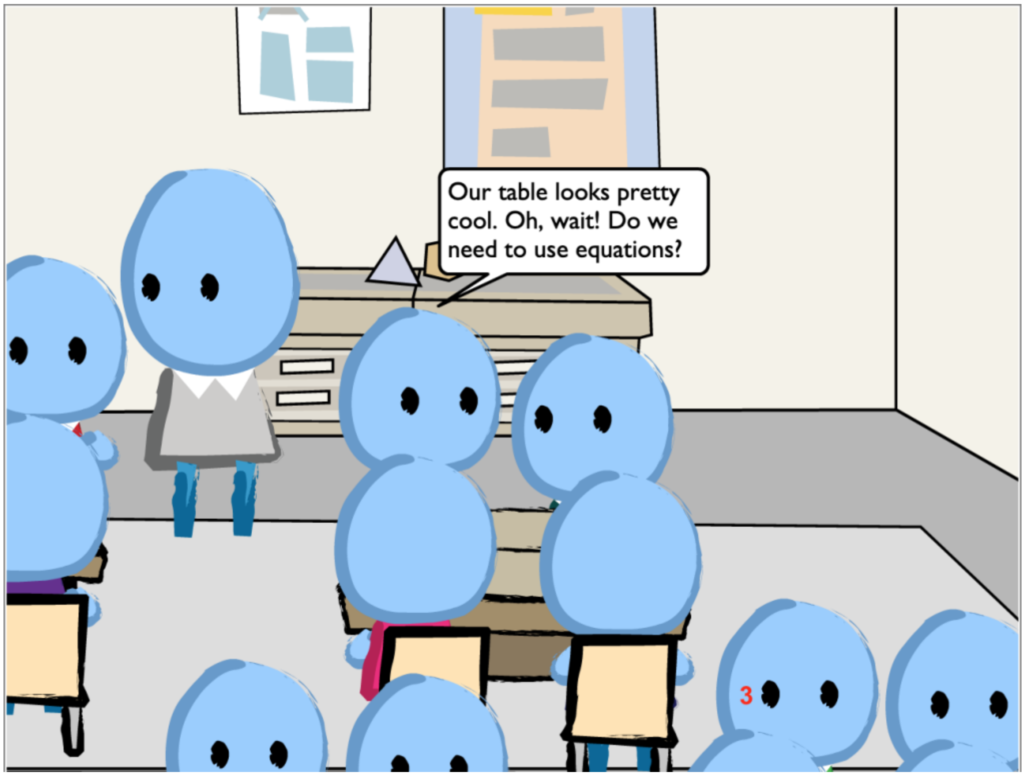
(C0) Ms. Lee: You have a table, but it would be really nice if you could find an equation for the situation, too.

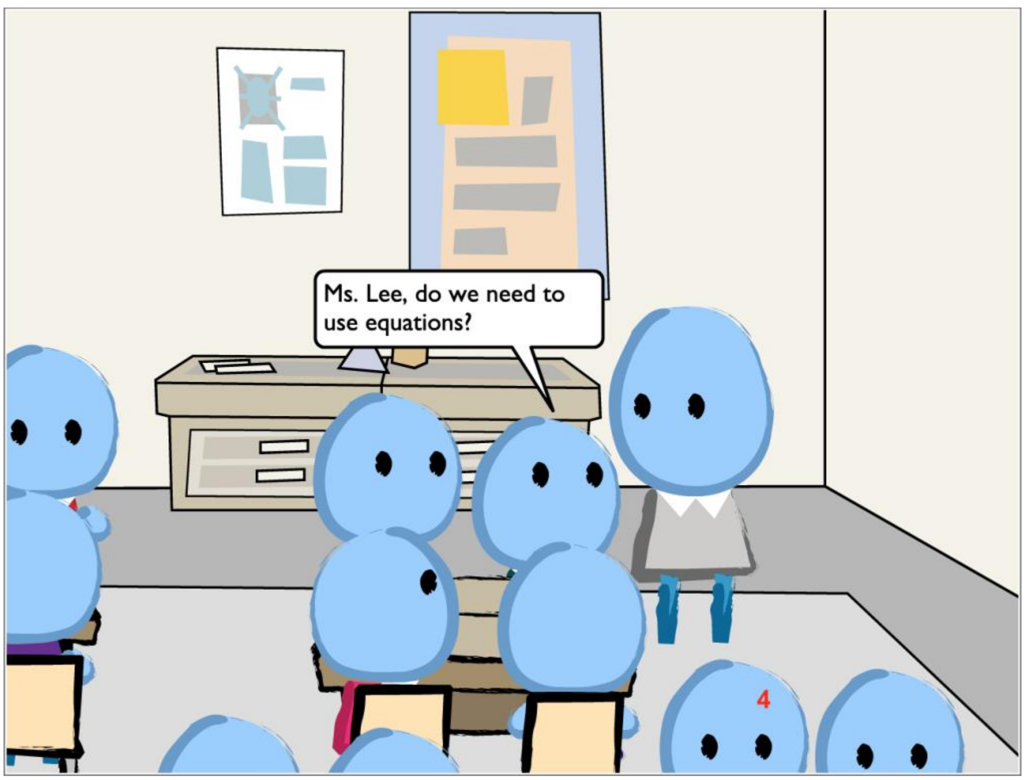
(C1) Ms. Lee: You have a table, which is a good start, but you need to eventually find an equation for this problem.

Scenario 2 – What functions should we use?
(4 pictures)



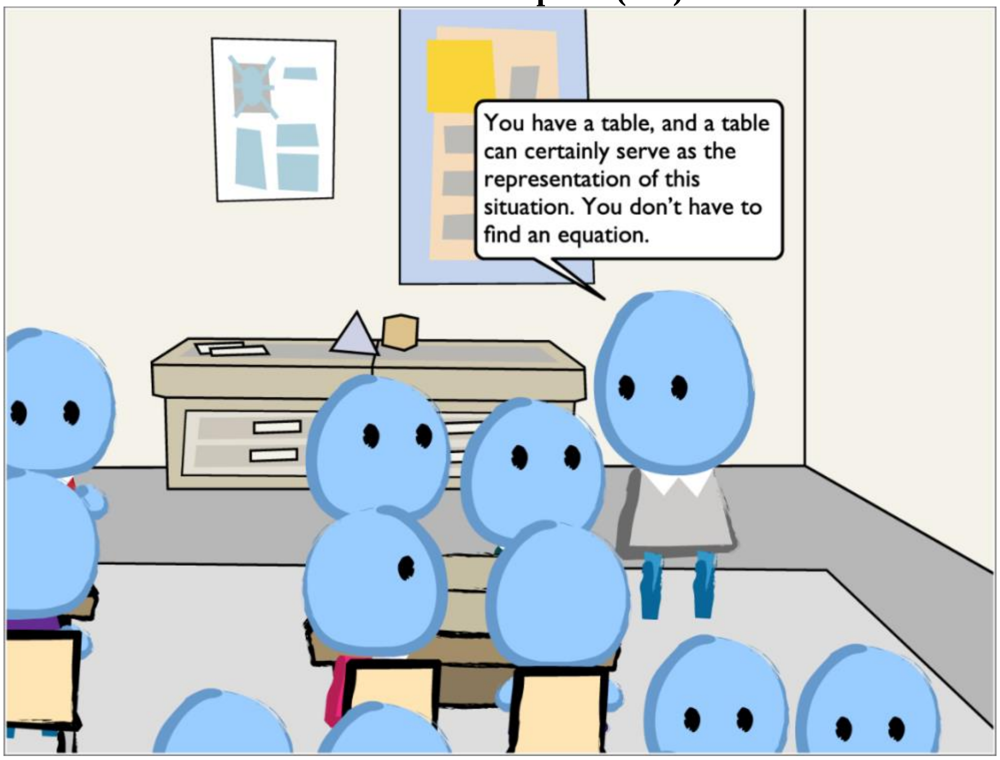



Our table looks pretty cool. Oh, wait! Do we need to use equations?
3

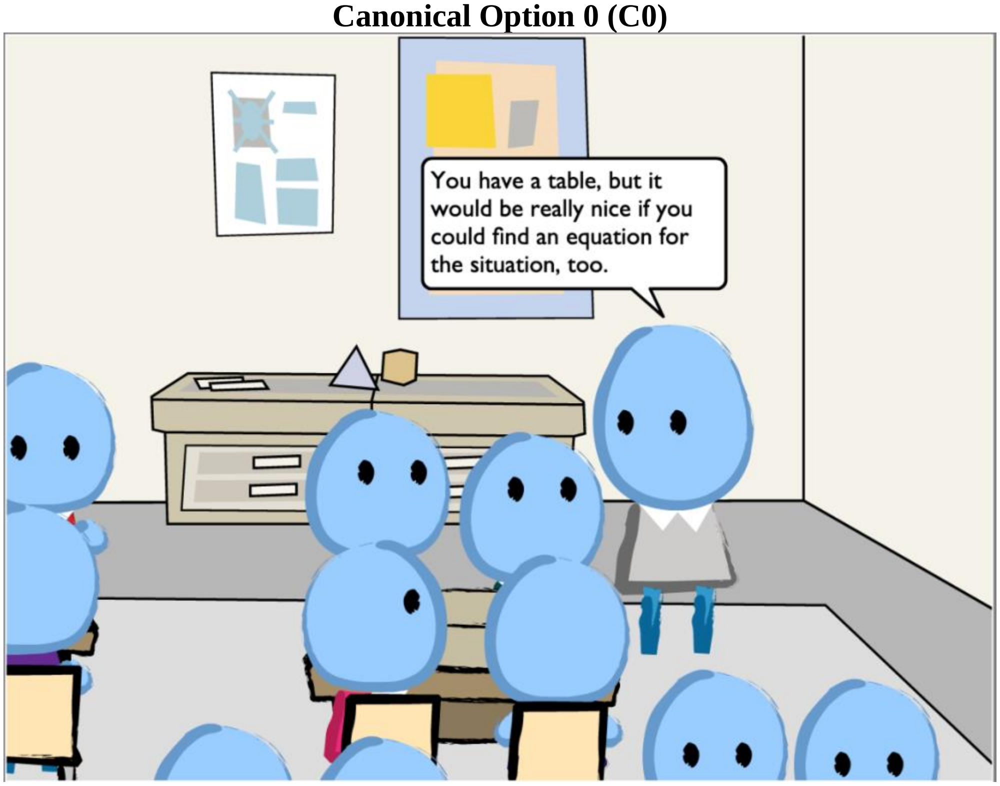

Ms. Lee, do we need to use equations?
4

Three Options for Scenario 2

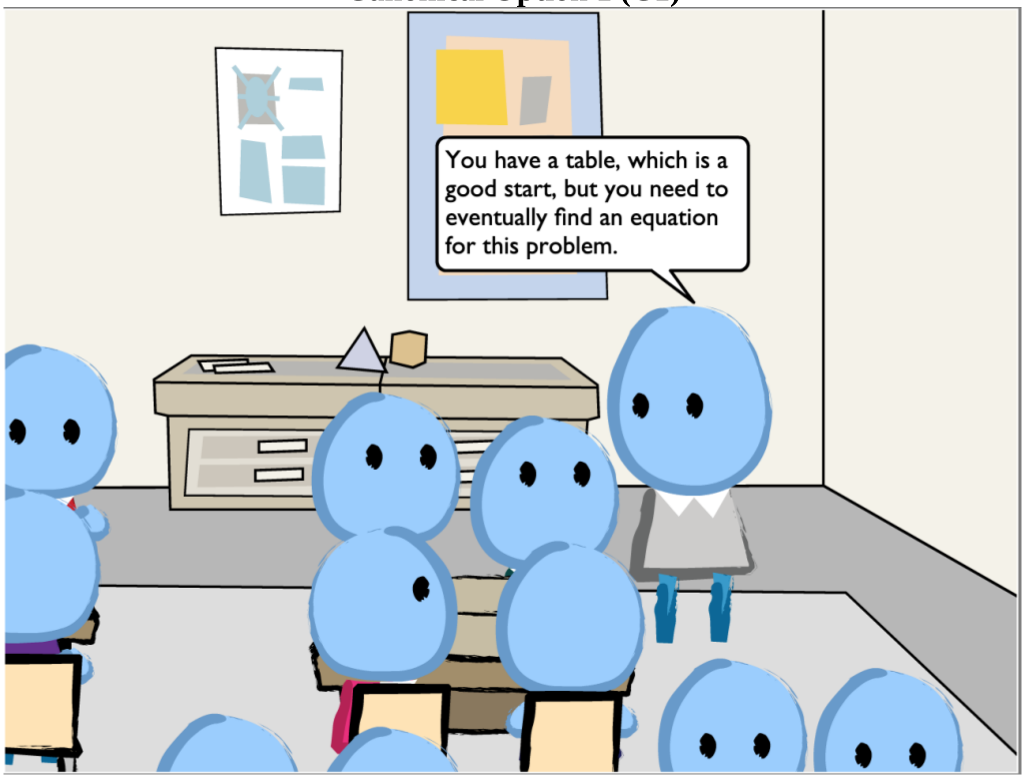
Noncanonical Option (NC)



Canonical Option 0 (C0)



Canonical Option 1 (C1)



Hypothesized Norm 3a (Mathematical Components). The teacher gives students unambiguous directions on what they are expected to use in terms of the mathematical operations and components.

Scenario 3a – Do we need to use equations?

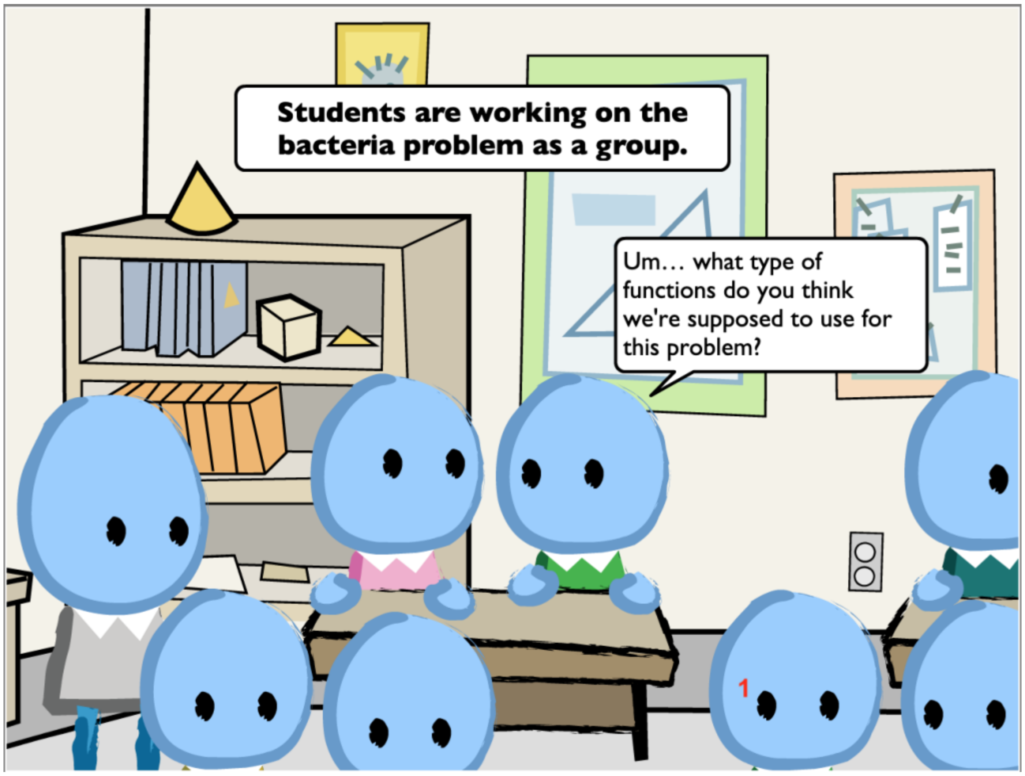
For Task 1:

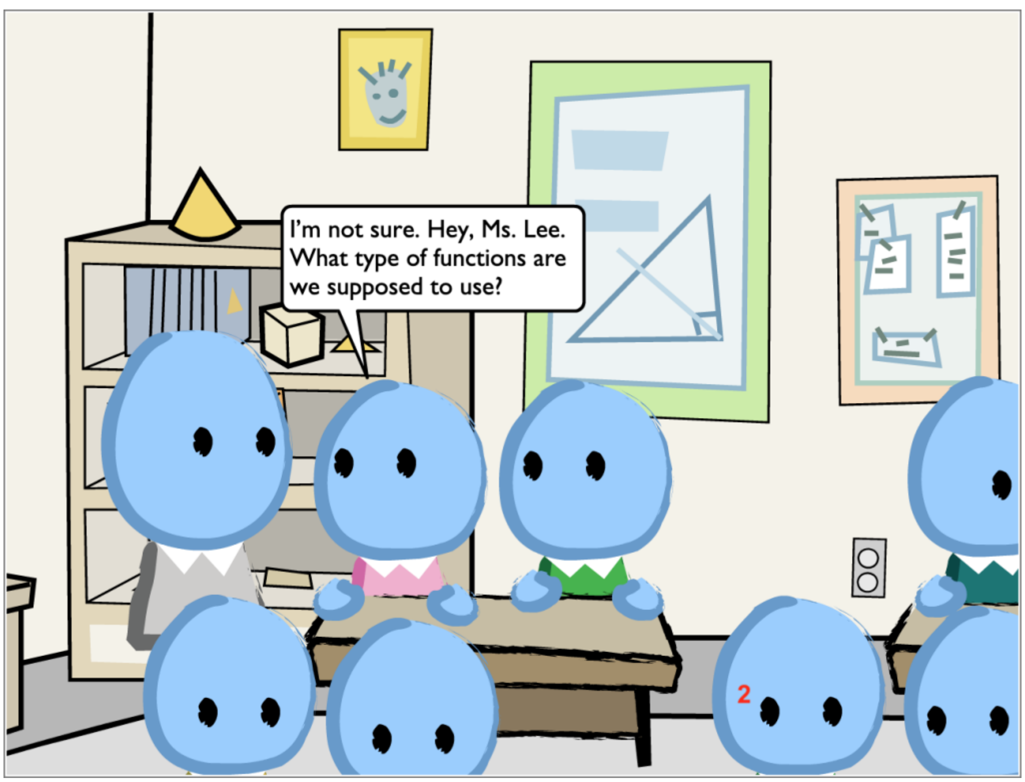
(Students are working on the bacterial problem as a group.)
Diana: Um… what type of function do you think we're supposed to use for this problem?
Emma: I'm not sure. Hey, Ms. Lee. What type of functions are we supposed to use?

Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.

> (NC) Ms. Lee: You should decide on the function. Use whatever type of function or functions you think are appropriate.
>
> (C0) Ms. Lee: What did we learn yesterday?
> Emma: Exponential functions?
> Ms. Lee: Then try some exponential functions.
>
> (C1) Ms. Lee: You can use an exponential function. Remember, it has the form $y = a \cdot b^x$.

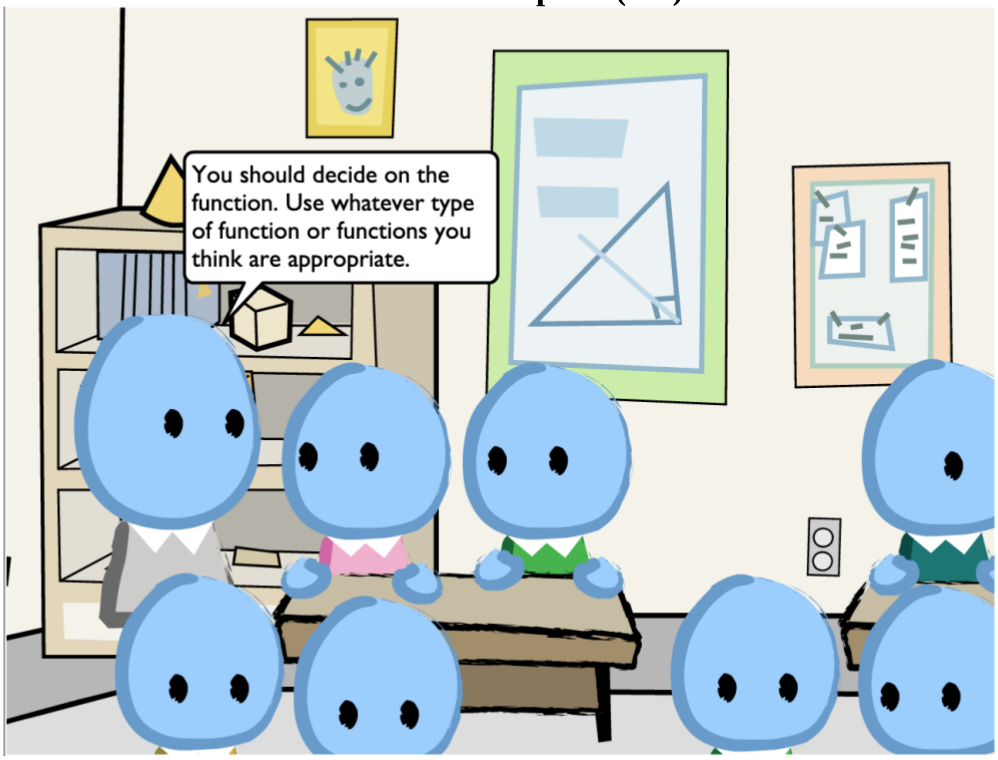
Scenario 3a – Do we need to use equations?
(2 pictures)



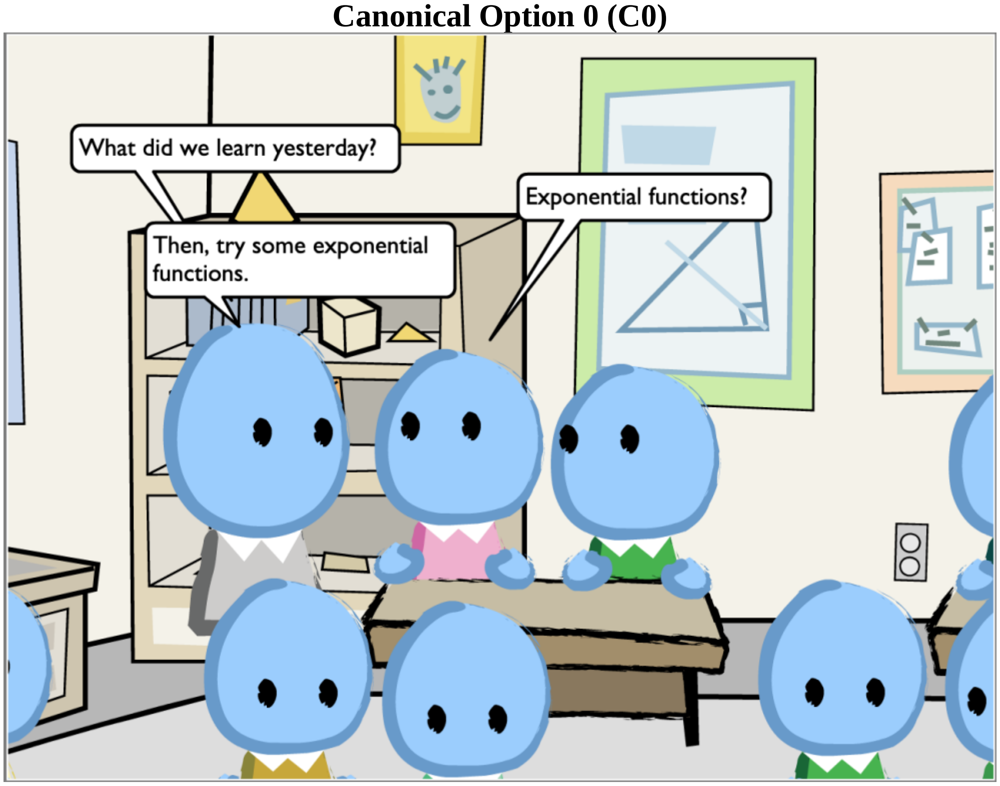


Three Options for Scenario 3a

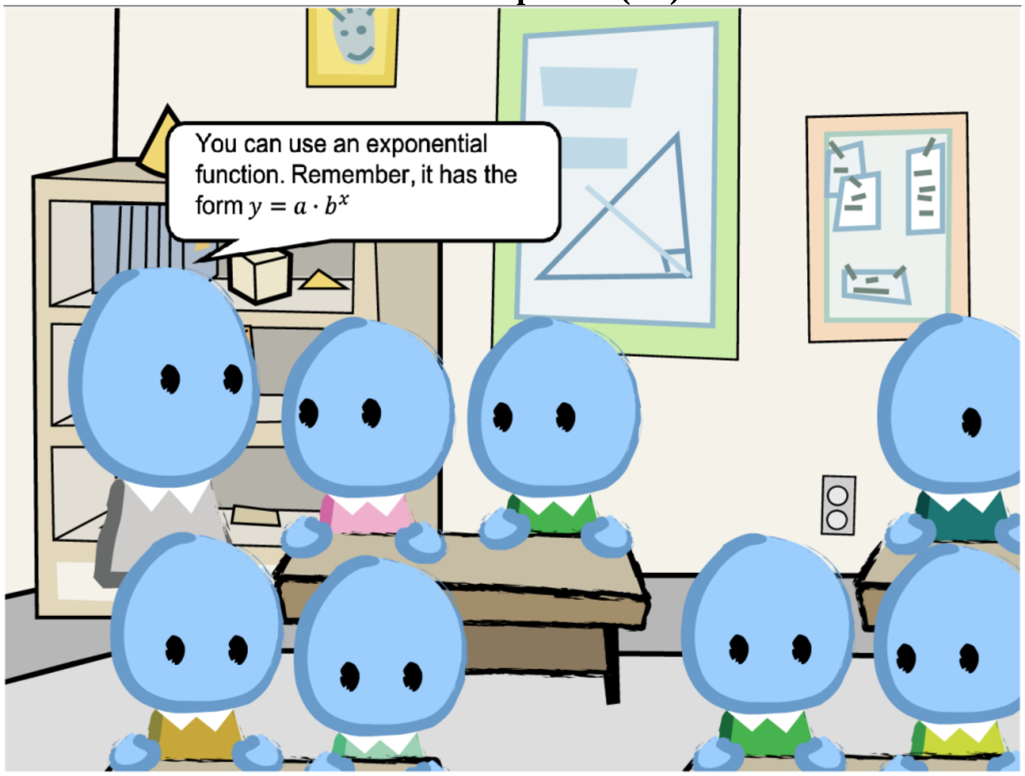
Noncanonical Option (NC)



Canonical Option 0 (C0)



Canonical Option 1 (C1)



Hypothesized Norm 3b (Mathematical Components). The teacher gives students unambiguous directions on what they are expected to use in terms of the mathematical operations and components.

Scenario 3b – Do we need to use equations?

For Task 2:

(Students are working on the water usage problem as a group.)
Diana: Um… what type of function do you think we're supposed to use for this problem?
Emma: I'm not sure. Hey, Ms. Smith. What types of functions are we supposed to use?

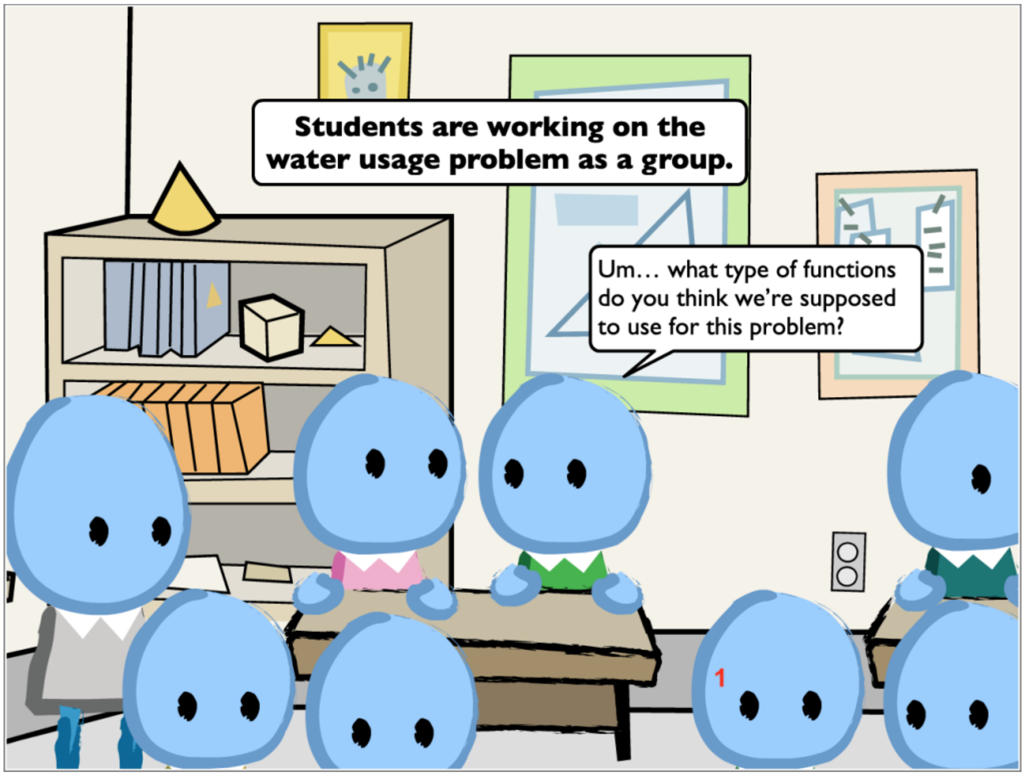
Imagine something like this happening in your classroom. For the below three options, indicate how likely YOU would do next.

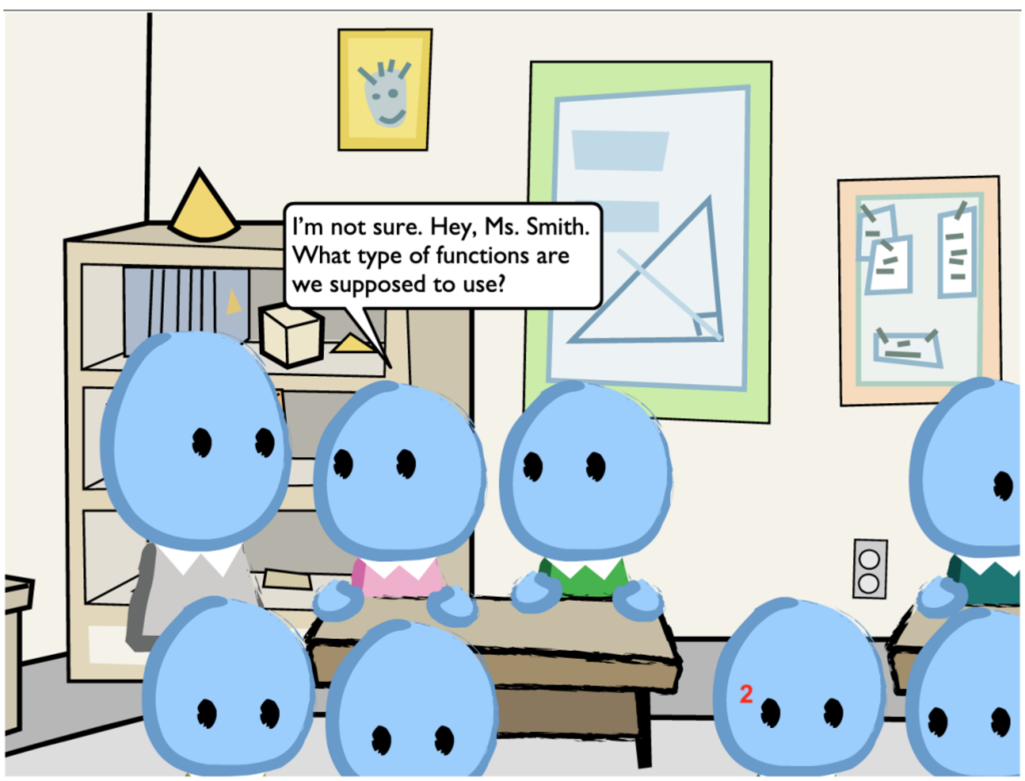
(NC) Ms. Smith: You should decide on the function. Use whatever type of function or functions you think is appropriate.

(C0) Ms. Smith: What did we learn yesterday?
Emma: Linear functions?
Ms. Smith: Then try some linear functions.

(C1) Ms. Smith: You can use a linear function. Remember, it has the form $y = mx + b$.

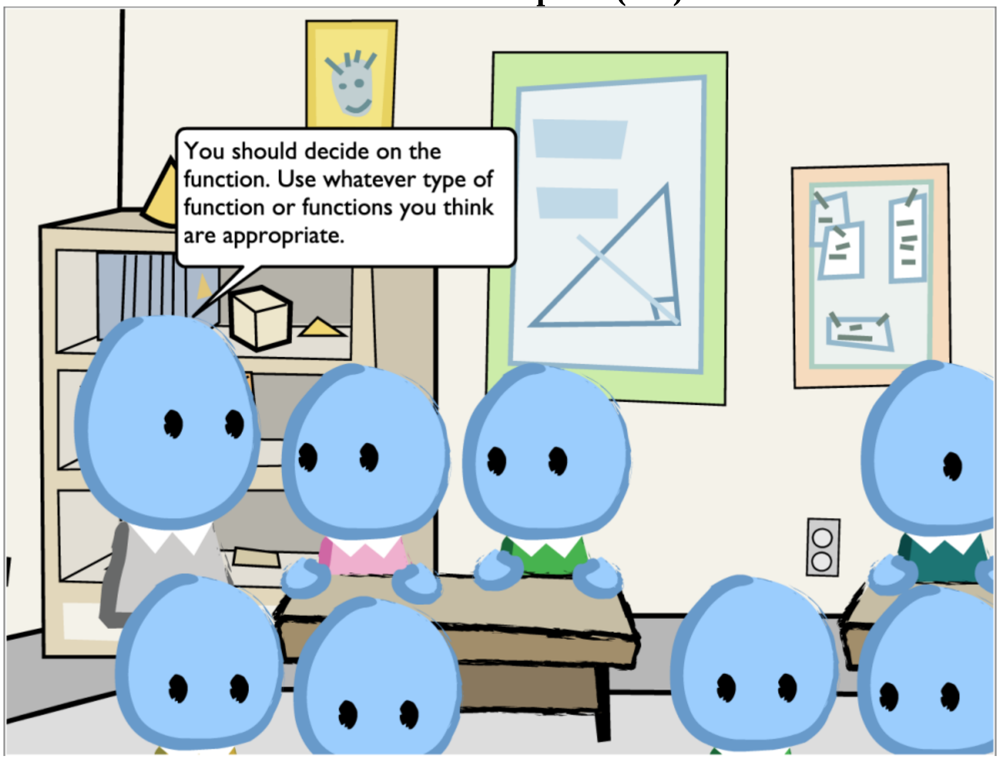
Scenario 3b – Do we need to use equations?
(2 pictures)



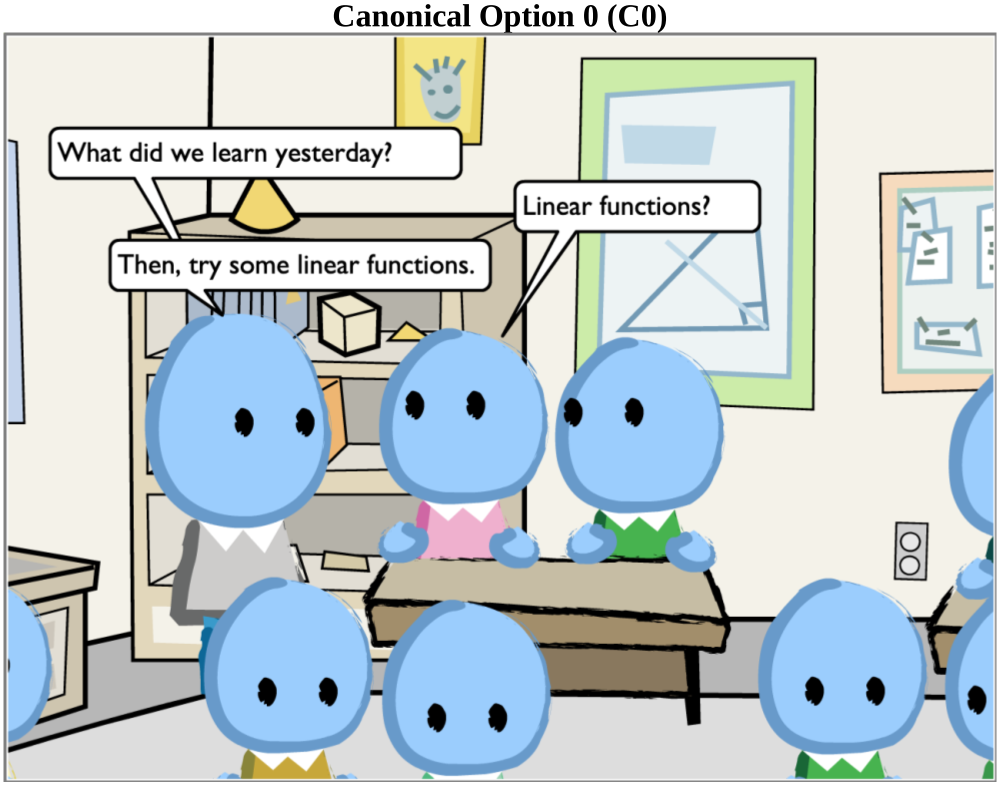


Three Options for Scenario 3b

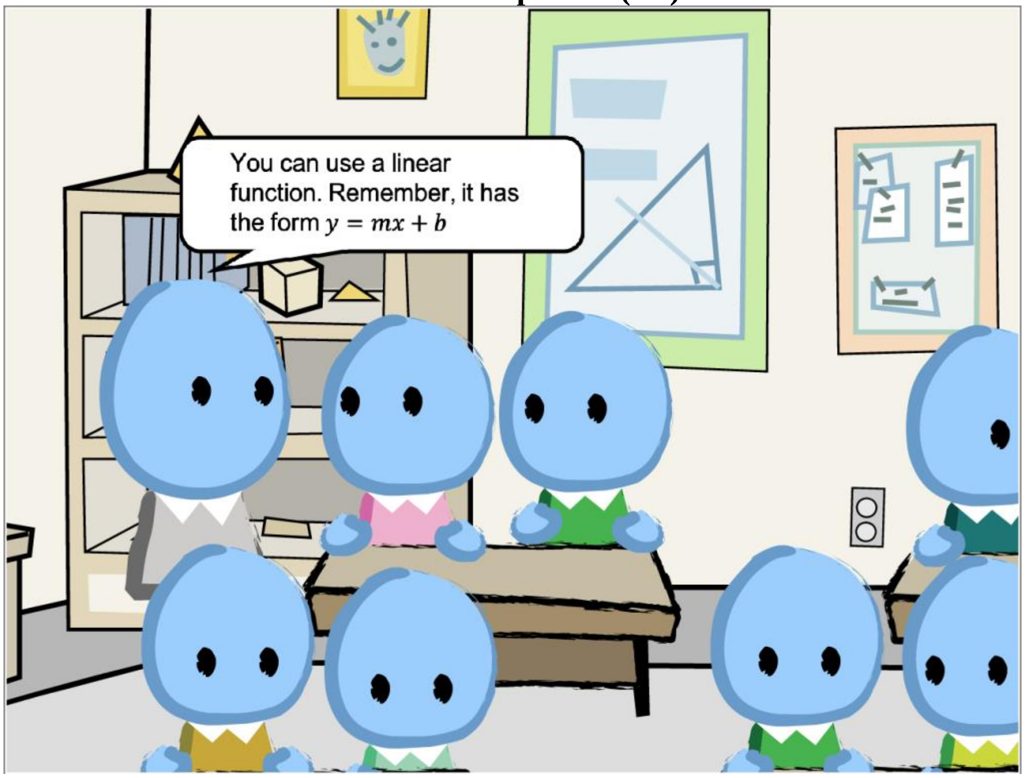
Noncanonical Option (NC)



Canonical Option 0 (C0)



Canonical Option 1 (C1)



Hypothesized Norm 4 (Mathematical Thinking Dominance). Students are expected to primarily (or exclusively) engage in mathematical thinking rather than nonmathematical thinking.

Scenario 4 – Your nonmathematical thinking matters.

(Students start working on the water usage task. They have a small-group discussion.)
Ivan: My younger sister always wastes a lot of water when she takes a shower. She doesn't care about saving water at all.
Jasmine: Maybe adults have better habits of using water. In my family, my mom doesn't use much water at all when she washes the dishes.
Ivan: Well, my dad uses a lot of water when he washes his car.
Jasmine: I think your dad is just a special case. I didn't see my dad use a lot of water.

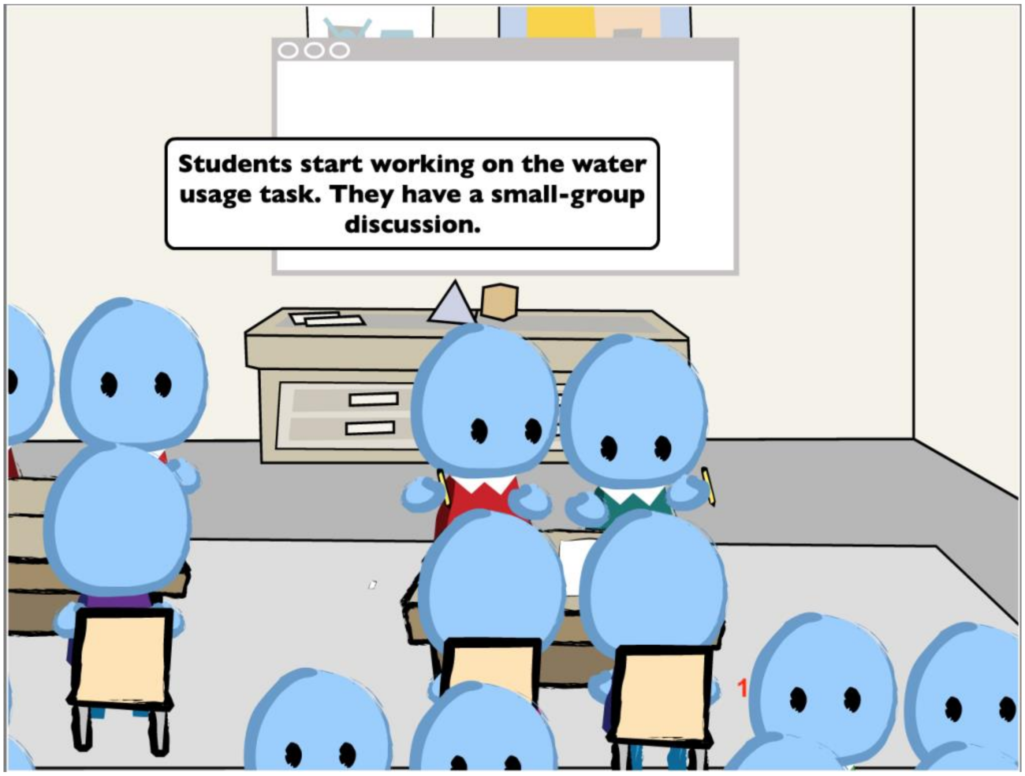
Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.

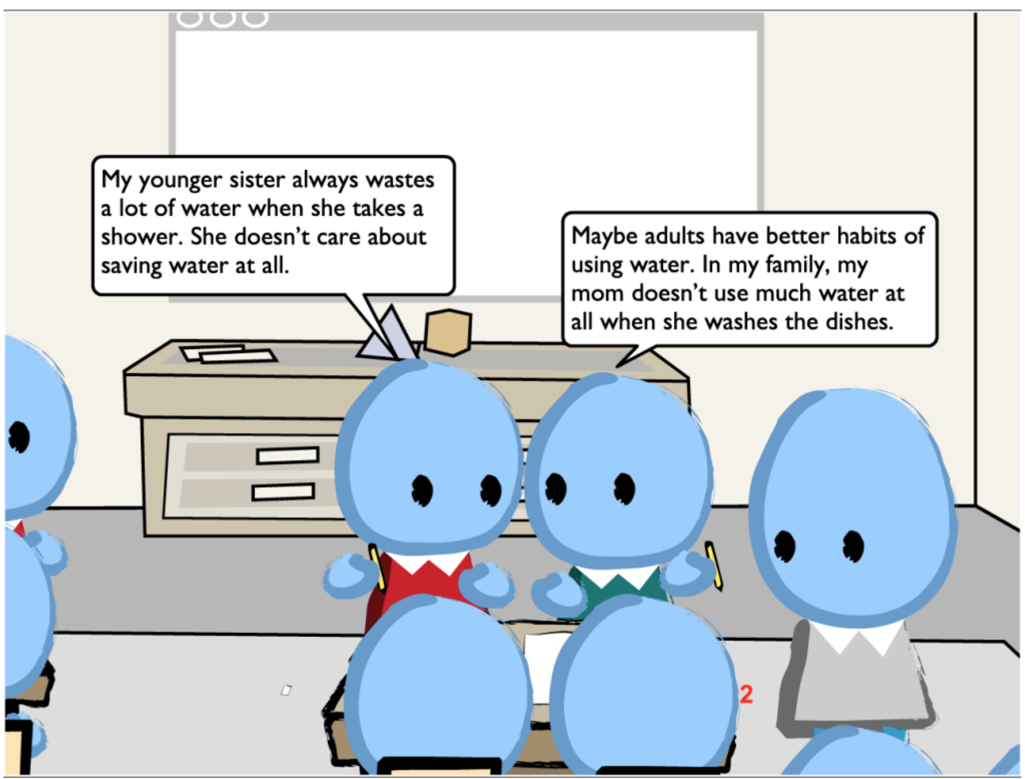
(NC) Ms. Smith: I like how you are connecting the situation to your own experiences. What else comes to mind about water usage in your real life?

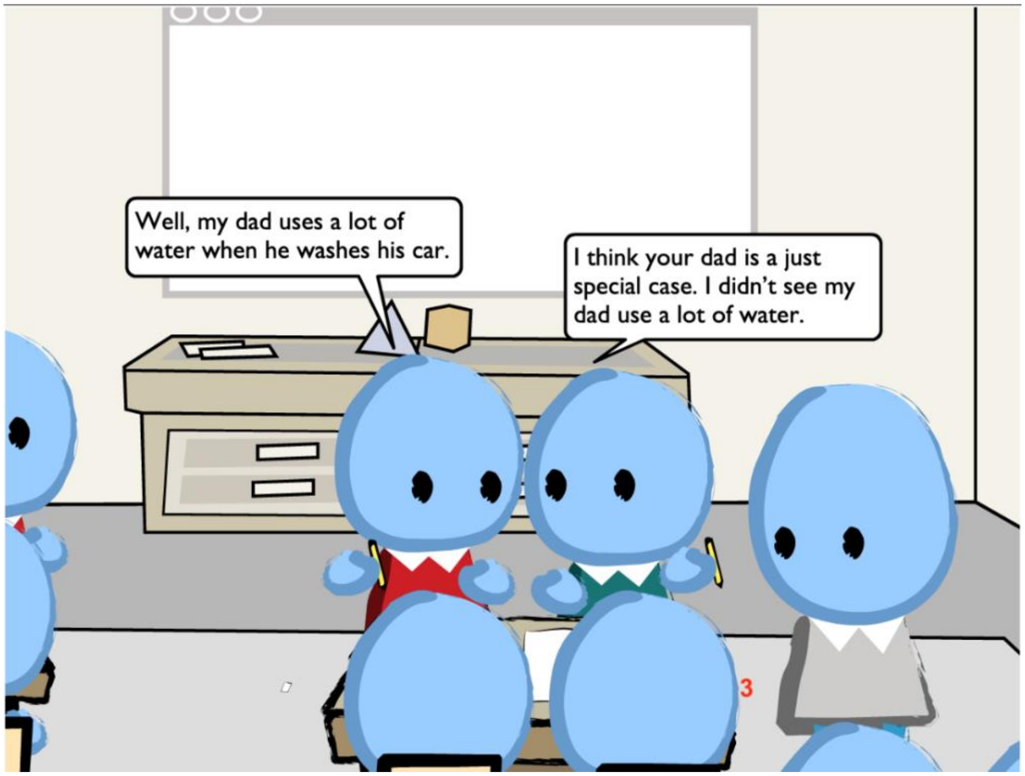
(C0) Ms. Smith: I like how you are connecting the situation to your own experiences. Let's put water for showering and dishwashing into your equation.

(C1) Ms. Smith: I like how you are connecting the situation to your own experiences, but your family situation might not be typical. Try to quantify an average person's water usage for showering and dishwashing when you develop your equation.

Scenario 4 – Your nonmathematical thinking matters.
(3 pictures)

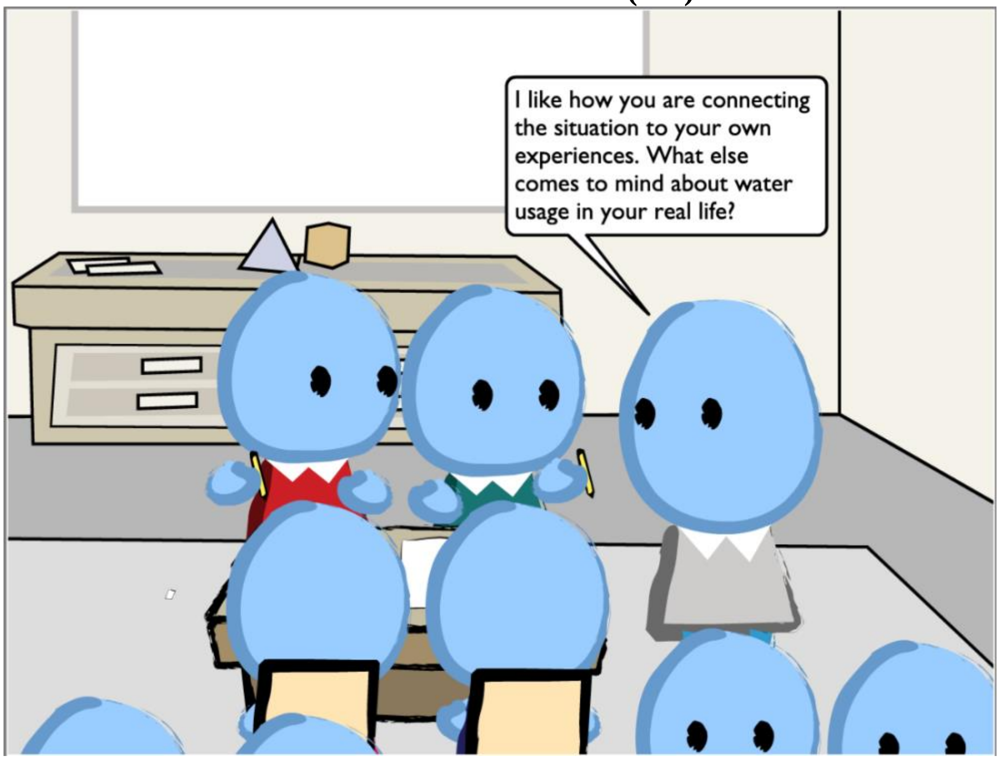




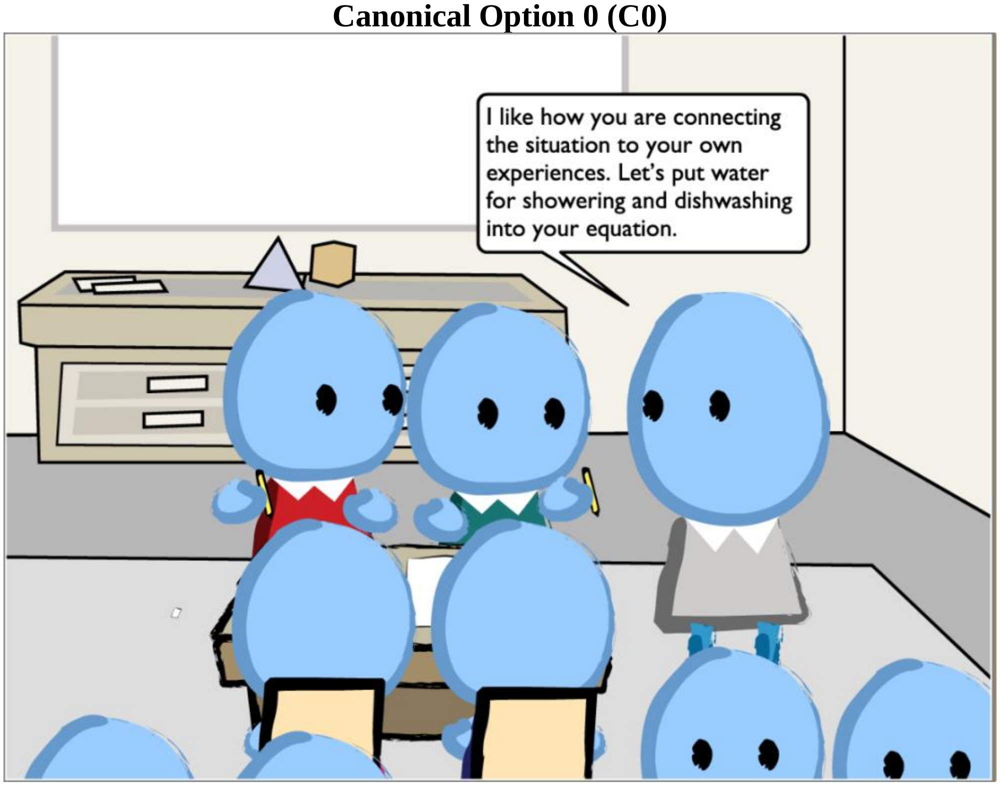

Well, my dad uses a lot of water when he washes his car.
I think your dad is a just special case. I didn't see my dad use a lot of water.

Three Options for Scenario 4

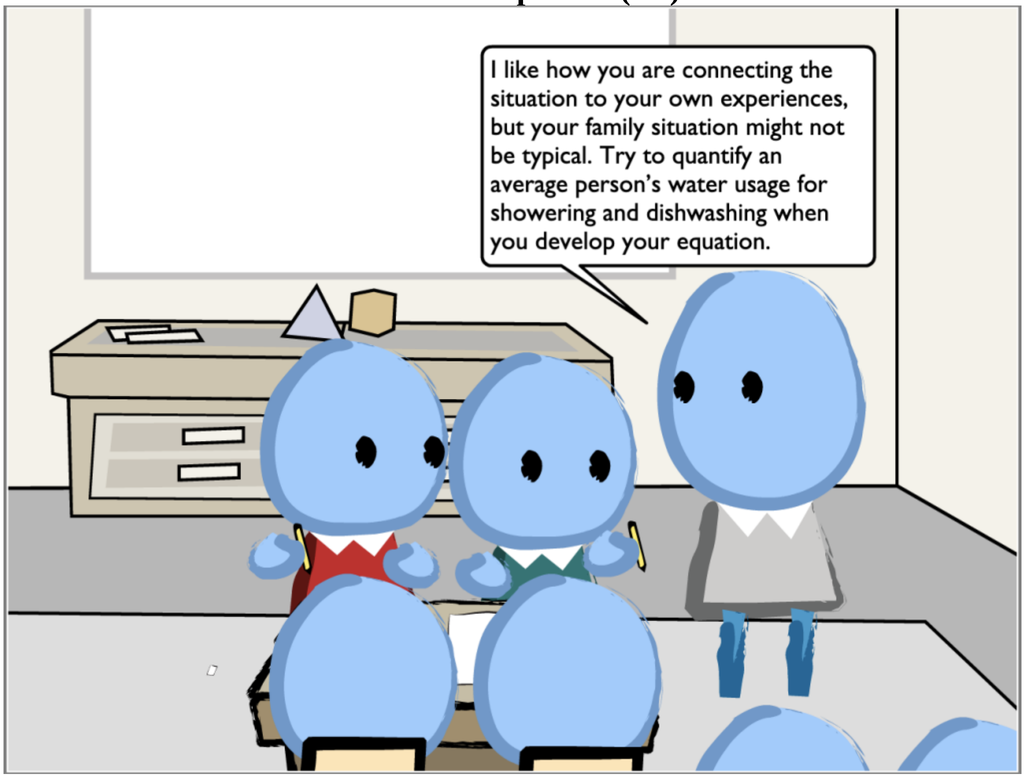
Noncanonical Scenario (NC)



Canonical Option 0 (C0)



Canonical Option 1 (C1)



Hypothesized Norm 5a (Model Revision Omitted). All students are expected to arrive at the same model, and model revision (beyond resolving discrepancies between different models) is not expected.

Scenario 5a – Let's revise our models!

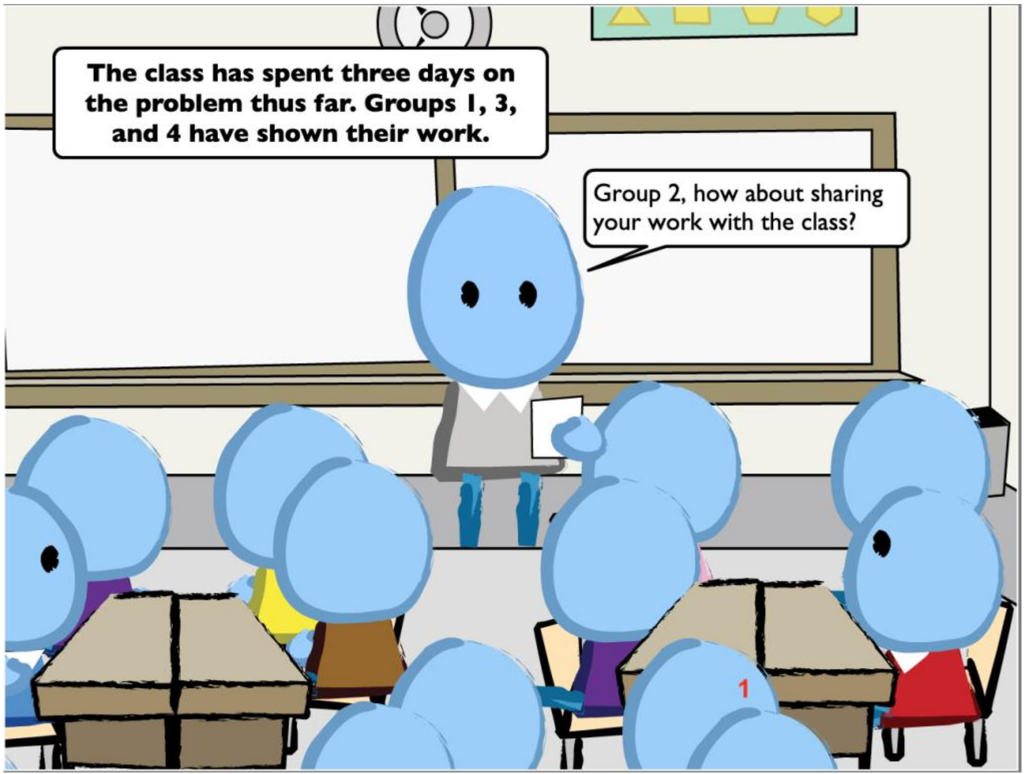
For Task 1:

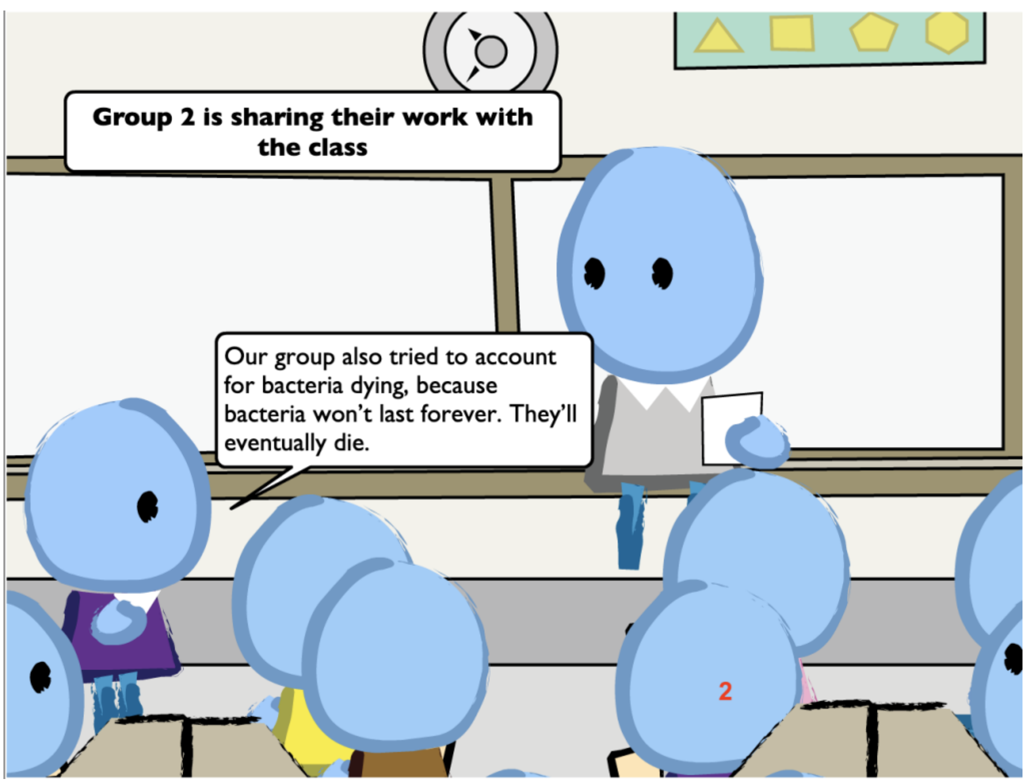
(The class has spent three days on the problem thus far. Groups 1, 3, and 4 have shown their work.)
Ms. Lee: Group 2, how about sharing your work with the class?
(Group 2 is sharing their work with the class)
Group 2: Our group also tried to account for bacteria dying, because bacteria won't last forever. They'll eventually die.

Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.

> (NC) Ms. Lee: Group 2 brought out an interesting idea about bacteria dying. Besides bacteria dying, what else can we add to the current model we have? Let's think about a couple of other things and continue revising our model.
> (The whole class spent the rest of time revising the models.)
>
> (C0) Ms. Lee: Group 2 brought out an interesting idea about bacteria dying. Other groups, go ahead and revise your current model to try to incorporate bacteria dying. Group 2, you can finalize it and then turn in your work.
> (Other groups spent the rest of time revising their models.)
>
> (C1) Ms. Lee: Group 2 brought out an interesting idea about bacteria dying. That was part of their answer. There are multiple good answers to these kinds of problems, so we are going to wrap up. We are going to start the next lesson and finish this chapter by this Friday.
> (The class spent the rest of time on the new lesson.)

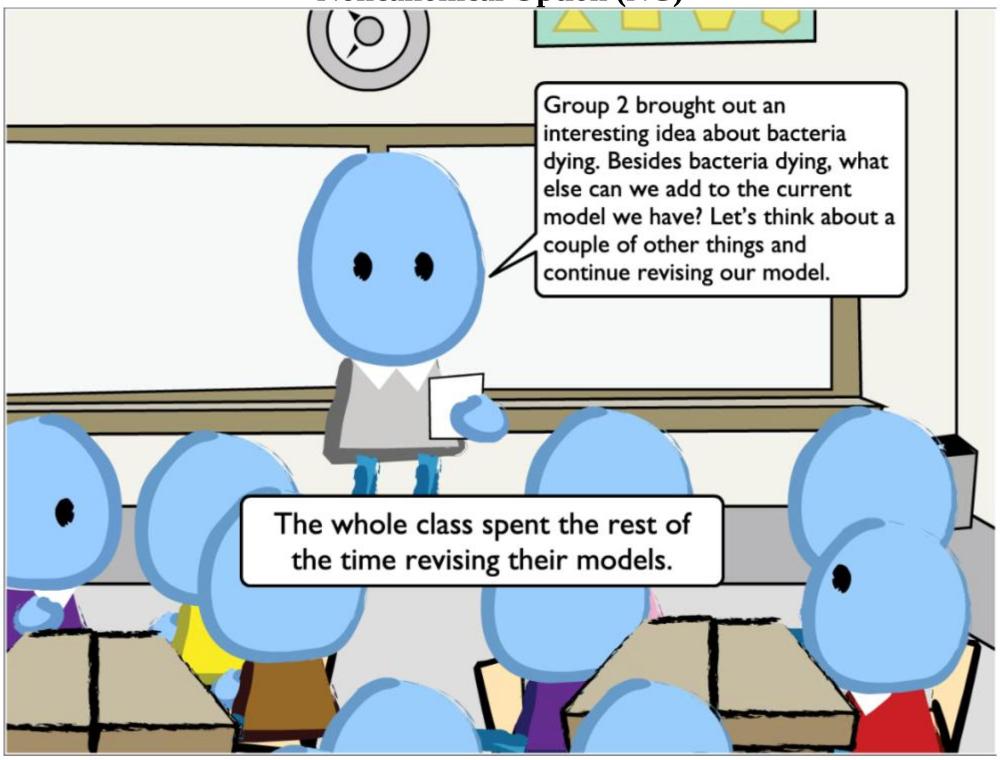
Scenario 5a – Let's revise our models!
(2 pictures)



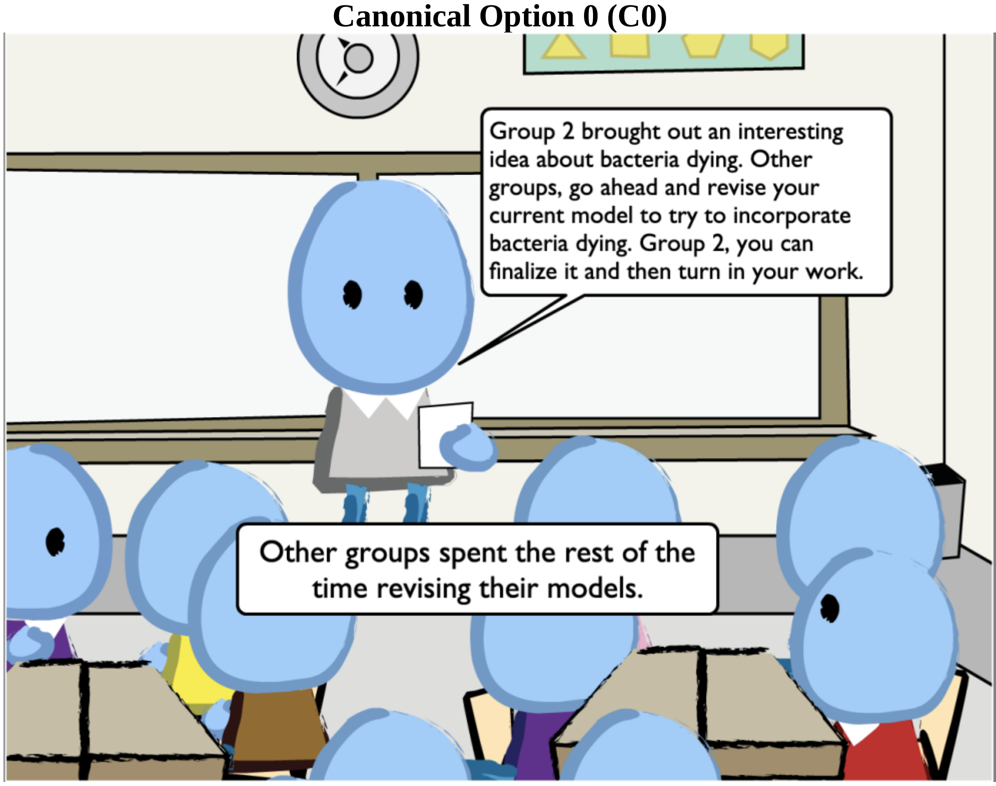


Three Options for Scenario 5a

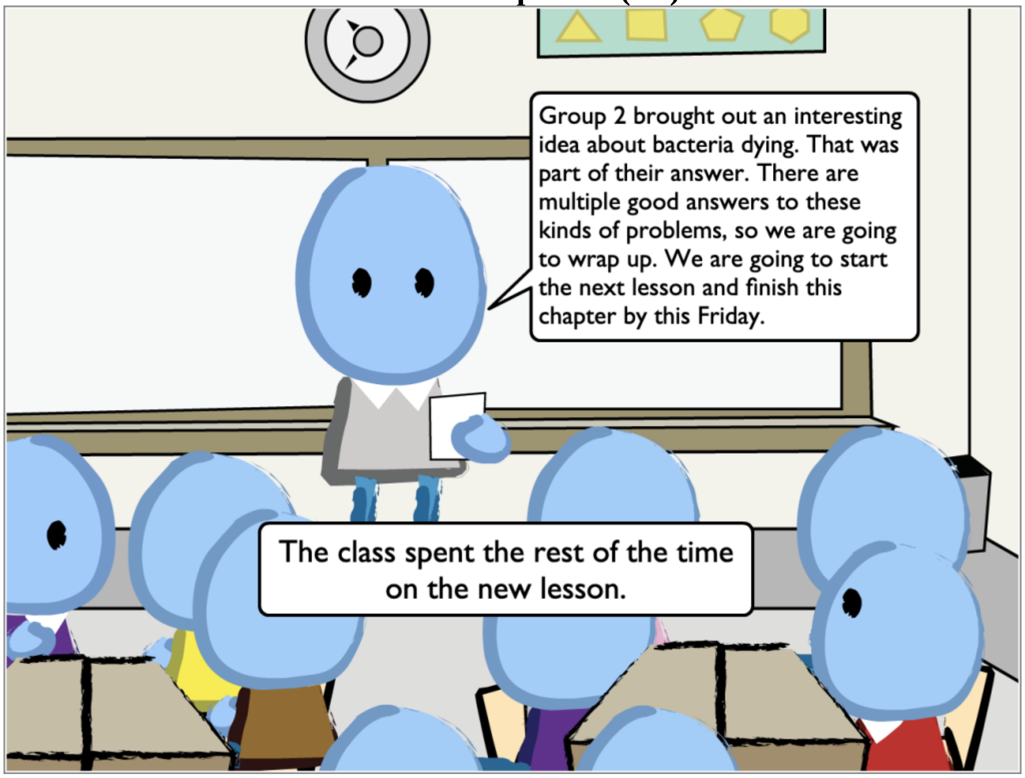
Noncanonical Option (NC)



Canonical Option 0 (C0)



Canonical Option 1 (C1)



Hypothesized Norm 5b (Model Revision Omitted). All students are expected to arrive at the same model, and model revision (beyond resolving discrepancies between different models) is not expected.

Scenario 5b – Let's revise our models!

For Task 2:

(The class has spent three days on the problem thus far. Groups 1, 3, and 4 have shown their work.)
Ms. Lee: Group 2, how about sharing your work with the class?
(Group 2 is sharing their work with the class)
Group 2: Our group also tried to account for water usage in different seasons. Like in summer, we usually use more water for drinking.

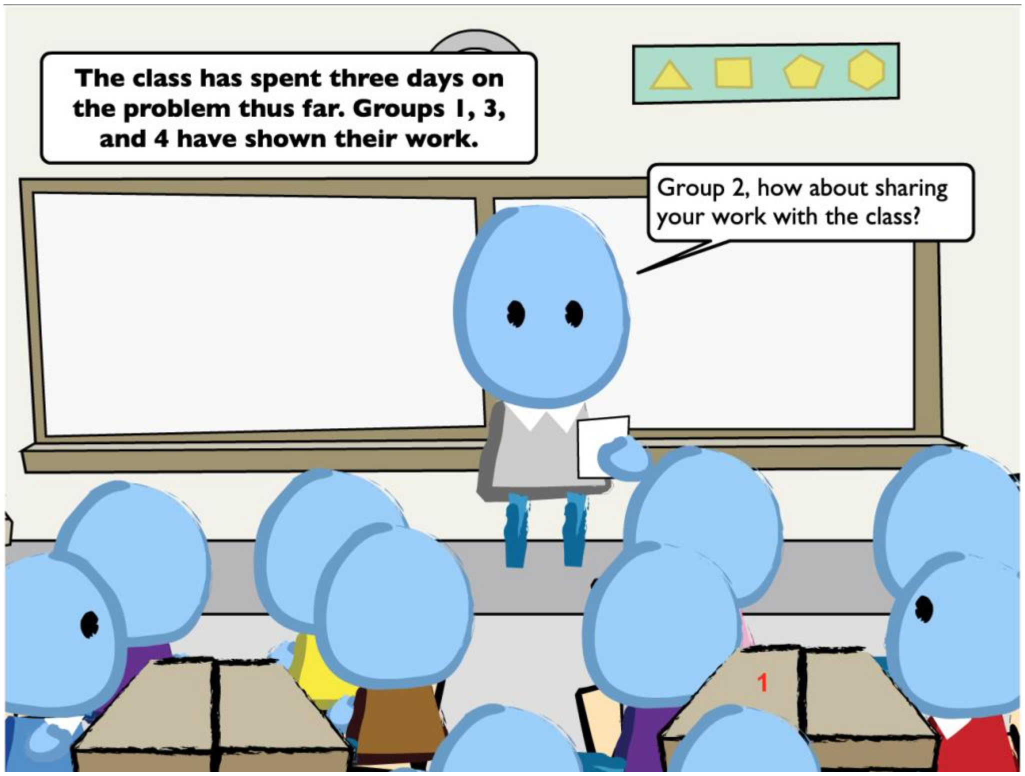
Imagine something like this happening in your classroom. From the three different options below, indicate which action YOU would most likely do next.

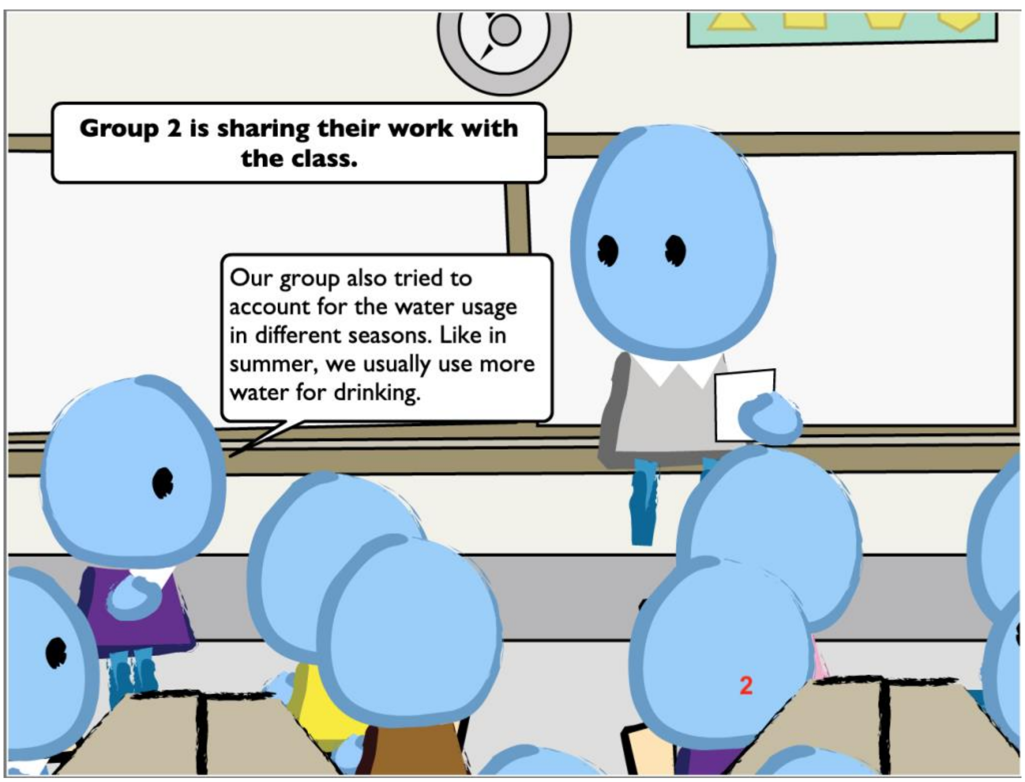
(NC) Ms. Smith: Group 2 brought out an interesting idea that different seasons will affect the amount of water that we use daily. Besides the impact of seasons, what else can we add to the current model we have? Let's think about a couple of other things and continue revising our model.
(The whole class spent the rest of time revising the models.)

(C0) Ms. Smith: Group 2 brought out an interesting idea that different seasons will affect the amount of water that we use daily. Other groups, go ahead and revise your current model to try to incorporate different seasons. Group 2, you can finalize it and then turn in your work.
(Other groups spent the rest of time revising the models.)

(C1) Ms. Smith: Group 2 brought out an interesting idea that different seasons will affect the amount of water that we use daily. That was part of their answer. There are multiple good answers to these types of problems, so we are going to wrap up. We are going to start the next lesson and finish this chapter by this Friday.
(The class spent the rest of time on the new lesson.)

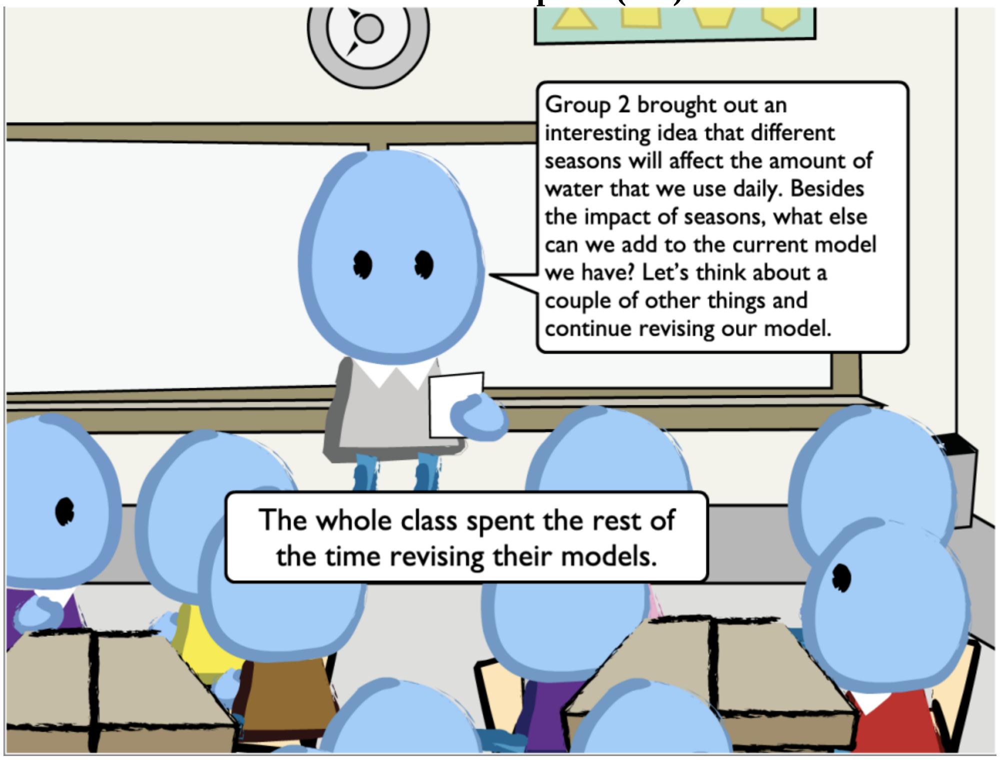
Scenario 5b – Let's revise our models!
(2 pictures)



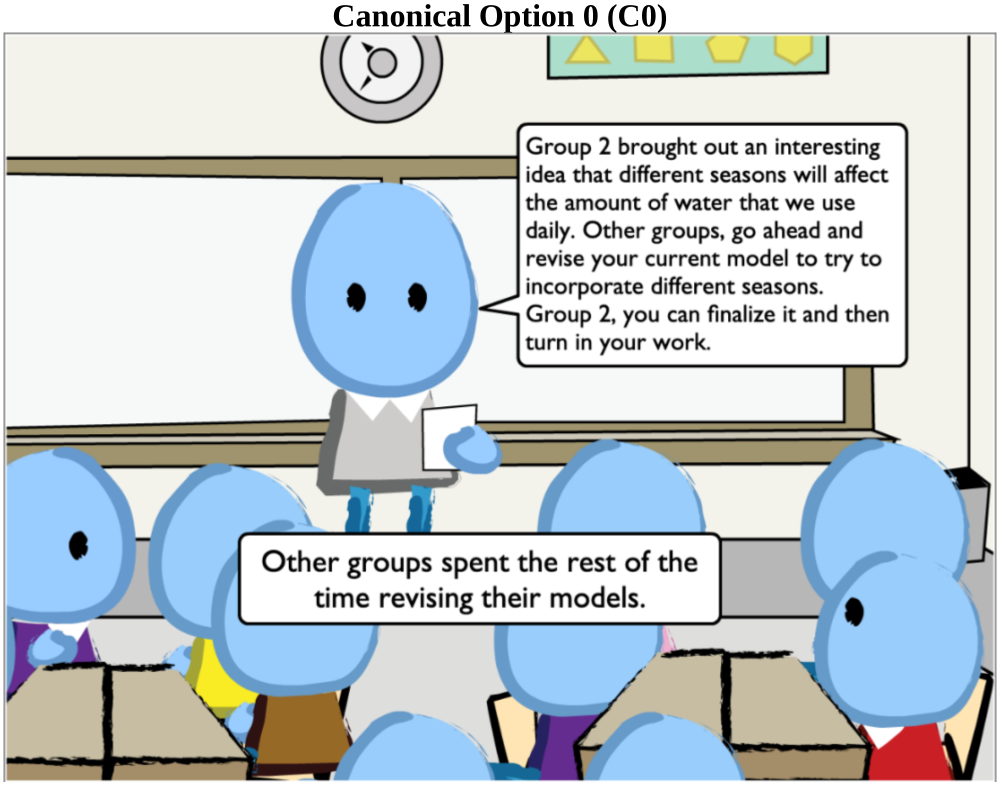


Three Options for Scenario 5b

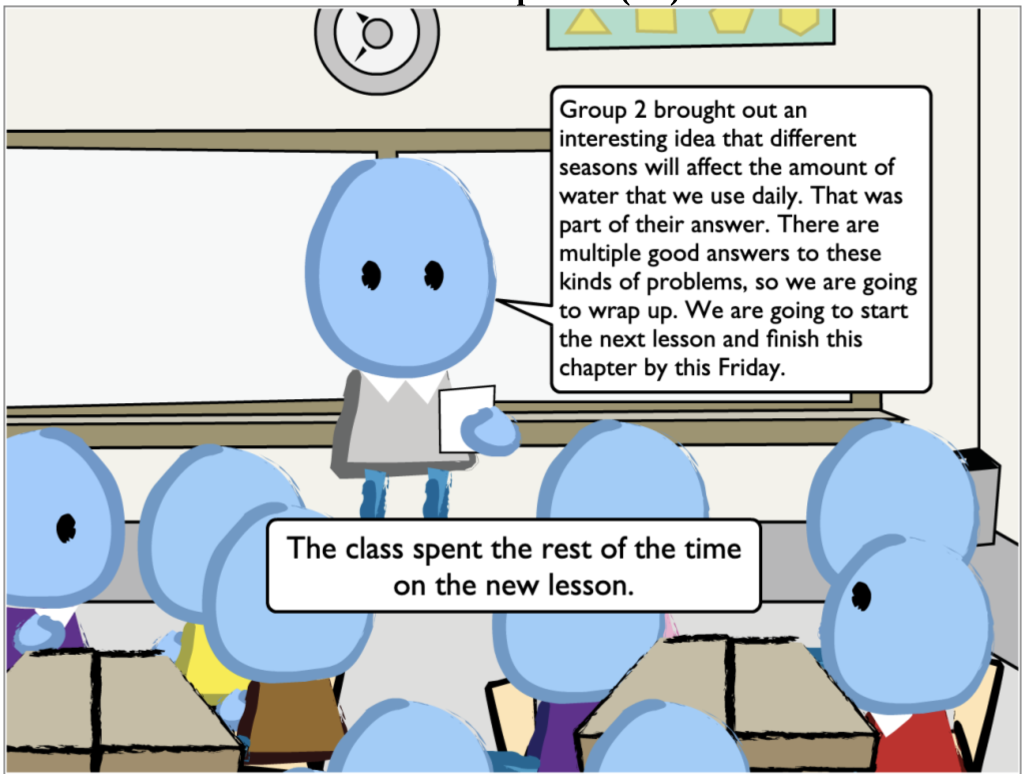
Noncanonical Option (NC)



Canonical Option 0 (C0)



Canonical Option 1 (C1)



Hypothesized Norm 6 (Politically Neutral Contexts). Students are expected to primarily (or exclusively) work on politically neutral tasks instead of social-justice-oriented tasks.

Scenario 6 – Social justice or not?

Which one would you be most likely to use in your class?

(NC) The government in Flint, Michigan, changed the water source for the city, causing high concentrations of lead to be found in its African-American neighborhoods. Exposure to lead is serious. It impacts the health of the communities, particularly within the African-American children. As a result, bottled water had to be brought in from outside the city. How much water will be enough to meet the daily needs of Flint?

(C0) Some cities in the United States changed their water source, causing high concentrations of lead in the local tap water. Exposure to lead is serious. It impacts public health and can be particularly harmful to young children. As a result, bottled water had to be brought in from outside the city. How much water will be enough to meet the daily needs of a city?

(C1) Water is an important part of our daily lives. As vast as the water resources of the United States are, they are not endless. Water needs to be conserved and protected. People use water for a wide variety of purposes. How much water will be enough to meet the daily needs of a city?

APPENDIX G

Follow-up Interview Protocol

Part 1:

1. You indicated that you strongly considered this justification statement (depends on what the teachers selected). Can you talk more about it?"
2. You indicated that you did *not* consider this justification statement (depends on what the teachers selected). Can you talk more about it?"
3. You indicated that it is very typical/somewhat typical for mathematics teachers in general to respond to the students in this way (depends on what the teachers selected). Can you talk more about it?"
4. You indicated that it is very rare/somewhat rare for mathematics teachers in general to respond to the students in this way (depends on what the teachers selected). Can you talk more about it?"

Part 2:

5. What kinds of experiences (or things) do you aim for your students to have (or learn) in this type of tasks? * [The purpose of this question is to understand the participant's individual and interpersonal obligations toward modeling.]
6. What do you think the task reflects what you consider as mathematics? [The purpose of this question is to understand the disciplinary obligations toward modeling.]
7. Has any of your colleagues ever enacted this type of tasks? In what ways, if any, do your colleagues influence your enactment of this type of tasks? [The purpose of this question is to understand the institutional obligations toward modeling.]
8. What, if any, are the expectations from your department/school/school district that impact your enactment of this type of tasks? * [The purpose of this question is to understand the institutional obligations toward modeling.]
9. What, if any, are the facilities (e.g., classroom space, tools/materials available) as well as support (e.g., administrative support, professional development) that impact your enactment of this type of tasks? How were you impacted? [The purpose of this question is to understand additional obligations toward modeling.]

* Herbst et al., 2018.

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