

Toward a Nonlocal Theory of Gravitation

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The nonlocal theory of accelerated systems is extended to linear gravitational waves as measured by accelerated observers in Minkowski spacetime. The implications of this approach are discussed. In particular, the nonlocal modifications of helicity-rotation coupling are pointed out and a nonlocal wave equation is presented for a special class of uniformly rotating observers. The results of this study, via Einstein's heuristic principle of equivalence, provide the incentive for a nonlocal classical theory of the gravitational field.

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I. INTRODUCTION

In a recent paper [1], the spin-rotation-gravity coupling has been worked out in detail for linear gravitational waves. In particular, it has been demonstrated that a plane monochromatic linear gravitational wave of frequency ω propagating in Minkowski spacetime has frequency ω' ,

$$\omega' = \gamma(\omega \mp 2\Omega), \quad (1)$$

as measured by an observer rotating uniformly with frequency Ω about the direction of propagation of the incident radiation. Here γ is the Lorentz factor of the observer and the upper (lower) sign corresponds to positive (negative) helicity radiation. More generally,

$$\omega' = \gamma(\omega - m\Omega), \quad (2)$$

where $m = 0, \pm 1, \pm 2, \dots$, is the total (orbital plus spin) angular momentum parameter in the case of oblique incidence [1].

Equations (1) and (2) are the spin-2 analogues of similar results for electromagnetic radiation that have been discussed in detail in [2]. In Eq. (2), ω' can be zero or negative. A negative ω' cannot be excluded due to the absolute character of the observer's rotation. However, in the case of $\omega' = 0$, there is no experimental evidence to suggest that a basic radiation field could ever stand completely still with respect to any observer. In the derivation of Eqs. (1) and (2), the standard theory of relativity based on the hypothesis of locality has been employed. A consequence of this assumption is that the gravitational wave could stand completely still for $\omega = m\Omega$ in Eq. (2). For instance, by a mere rotation of frequency $\omega/2$ in the positive sense about the direction of propagation of a normally incident positive helicity gravitational wave, the field becomes completely static in accordance with Eq. (1). Under Lorentz transformations, however, a linear gravitational radiation field can never stand still with respect to an *inertial* observer; indeed, this is the case for all basic radiation fields. Generalizing this circumstance to all observers, a nonlocal theory of accelerated observers has been developed that goes beyond the hypothesis of locality and is consistent with all observational data available at present [2, 3]. Specifically, the nonlocal electrodynamics of rotating systems has been successfully tested indirectly via the agreement of the nonlocal theory's predictions with standard quantum mechanical results for the electromagnetic interactions of rotating electrons in the correspondence limit [2]. Moreover, the postulates of the nonlocal theory forbid the existence of a fundamental scalar (or pseudoscalar) radiation field in nature in agreement with observation.

Let $\psi(x)$ be a basic radiation field in Minkowski spacetime and imagine an accelerated observer that measures this field as a function of proper time τ along its worldline. Let $\hat{\Psi}(\tau)$ be the result of such a measurement. According to the hypothesis of locality, the accelerated observer is pointwise equivalent to an otherwise identical hypothetical momentarily comoving inertial observer. Let $\hat{\psi}(\tau)$,

$$\hat{\psi}(\tau) = \Lambda(\tau)\psi(\tau), \quad (3)$$

where Λ is a matrix representation of the Lorentz group, be the measured field according to the infinite set of such momentarily comoving inertial observers; therefore, the hypothesis of locality would require that $\hat{\Psi}(\tau) = \hat{\psi}(\tau)$. On the other hand, the most general linear relationship between $\hat{\Psi}$ and $\hat{\psi}$ that is consistent with causality can be expressed as

$$\hat{\Psi}(\tau) = \hat{\psi}(\tau) + \int_{\tau_0}^{\tau} K(\tau, \tau')\hat{\psi}(\tau')d\tau' \quad (4)$$

for $\tau \geq \tau_0$, where τ_0 is the instant at which the acceleration is turned on. Equation (4), which expresses the nonlocality of field determination by an accelerated observer, involves a weighted average over the past worldline of the observer and is thus compatible with the ideas put forth by Bohr and Rosenfeld [4]. The kernel K must clearly vanish in the absence of acceleration. It follows from the results of Volterra [5] and Tricomi [6] that the relationship between $\psi(\tau)$ and $\hat{\Psi}(\tau)$ is unique in the space of functions of physical interest.

To ensure that a basic radiation field never stands completely still with respect to an accelerated observer, i.e. $\hat{\Psi}$ is variable when ψ is, we associate a constant $\hat{\Psi}$ with a constant ψ . The Volterra-Tricomi uniqueness theorem [5, 6] then excludes the possibility that a constant $\hat{\Psi}$ could ever result from a variable ψ . Our postulate thus implies the following integral equation for the kernel K

$$\Lambda(\tau_0) = \Lambda(\tau) + \int_{\tau_0}^{\tau} K(\tau, \tau') \Lambda(\tau') d\tau'. \quad (5)$$

The solutions of this equation have been investigated in detail [7, 8]. It turns out that the only acceptable solution is given by $K(\tau, \tau') = k(\tau')$,

$$k(\tau) = -\frac{d\Lambda(\tau)}{d\tau} \Lambda^{-1}(\tau). \quad (6)$$

The consequences of this theory for nonlocal electrodynamics have been worked out in detail [2, 3]. Moreover, $k = 0$ for a scalar (or pseudoscalar) field, which is therefore always local and hence subject to the difficulty involving scalar (or pseudoscalar) radiation standing completely still with respect to certain rotating observers. It would be interesting to extend the nonlocal ansatz to linear gravitational waves in Minkowski spacetime and explore some of the consequences of the resulting theory. This is done in the rest of this paper.

The spacetime metric associated with a linear gravitational wave is given by $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x^\alpha)$, where $(\eta_{\mu\nu}) = \text{diag}(-1, 1, 1, 1)$ is the Minkowski metric tensor, $x^\alpha = (ct, x, y, z)$ and $h_{\mu\nu}$ is a sufficiently small perturbation subject to the gauge transformation

$$h_{\mu\nu} \mapsto h_{\mu\nu} + \epsilon_{\mu,\nu} + \epsilon_{\nu,\mu} \quad (7)$$

due to an infinitesimal transformation of inertial coordinates $x^\mu \mapsto x^\mu - \epsilon^\mu$. The gauge-invariant field strength is given by the Riemann curvature tensor

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}(h_{\mu\sigma,\nu\rho} + h_{\nu\rho,\mu\sigma} - h_{\nu\sigma,\mu\rho} - h_{\mu\rho,\nu\sigma}). \quad (8)$$

In what follows, we will regard $h_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}$ as fields defined in a global inertial frame in Minkowski spacetime. It proves useful to introduce the trace-reversed wave amplitude $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$, where $h = \eta^{\mu\nu}h_{\mu\nu}$. Imposing the transverse gauge condition $\bar{h}^{\mu\nu}{}_{,\nu} = 0$, the source-free gravitational field equation reduces to the wave equation $\square\bar{h}_{\mu\nu} = 0$ in this case [9]. The remaining gauge freedom is usually restricted by introducing the transverse-traceless gauge in which the conditions $h = 0$ and $h_{0\mu} = 0$ are further imposed.

It is well known that the treatment of gravitational waves outlined above, namely, the linear approximation of general relativity for free gravitational fields on a Minkowski spacetime background admits of an alternative interpretation: It can be regarded as a Lorentz-invariant theory of a free linear massless spin-2 field in special relativity. This latter approach — to which the nonlocal theory of accelerated systems [2, 3] is directly applicable — is adopted in the rest of this paper.

It is important to recognize that the nonlocal ansatz can be applied either to the gravitational field ($R_{\mu\nu\rho\sigma}$) or the gravitational wave potential ($h_{\mu\nu}$ or $\bar{h}_{\mu\nu}$) resulting in two distinct but closely related approaches. The situation here is completely analogous to the electromagnetic case [10]. For the sake of simplicity, we choose the latter alternative in what follows. The potential as measured by an arbitrary accelerated observer in Minkowski spacetime is given by

$$h_{\hat{\alpha}\hat{\beta}} = h_{\mu\nu}\lambda^{\mu}_{\hat{\alpha}}\lambda^{\nu}_{\hat{\beta}}, \quad (9)$$

where $\lambda^{\mu}_{\hat{\alpha}}$ is the orthonormal tetrad associated with the observer. Our nonlocal ansatz (4) for the $h_{\mu\nu}$ then takes the Lorentz-invariant form

$$H_{\hat{\alpha}\hat{\beta}}(\tau) = h_{\hat{\alpha}\hat{\beta}}(\tau) + \int_{\tau_0}^{\tau} k_{\hat{\alpha}\hat{\beta}}{}^{\hat{\gamma}\hat{\delta}}(\tau')h_{\hat{\gamma}\hat{\delta}}(\tau')d\tau', \quad (10)$$

where $H_{\hat{\alpha}\hat{\beta}}$ is the gravitational wave amplitude as measured by the accelerated observer.

In general, the symmetric tensor $h_{\mu\nu}$ (or $\bar{h}_{\mu\nu}$) has ten independent components. We arrange these in a column vector ψ such that Eq. (9), or the analogous one for $\bar{h}_{\mu\nu}$, can take the form of Eq. (3) with a 10×10 matrix Λ . Specifically, $\hat{\psi}_A = \Lambda_A{}^B\psi_B$, where indices A and B belong to the set $\{00, 01, 02, 03, 11, 12, 13, 22, 23, 33\}$. For the sake of definiteness, we will henceforth assume that ψ represents $h_{\mu\nu}$.

It is worthwhile to work out explicitly the nonlocal theory of linear gravitational waves for certain accelerated observers. Section II is devoted to uniformly rotating observers; then, the nonlocal results are employed in Section III to re-examine the status of helicity-rotation

coupling for gravitational radiation. For a special class of uniformly rotating observers, the nonlocal gravitational wave equation is derived in Section IV. Section V explores the case of translationally accelerated observers. The implications of the nonlocal treatment of linear gravitational waves via Einstein's principle of equivalence are discussed in Section VI. Some of the computational details are relegated to the appendices.

II. UNIFORMLY ROTATING OBSERVER

Consider an observer that for $t < 0$ moves uniformly in the (x, y) plane of an inertial system of coordinates such that $x = r$ and $y = r\Omega t$, where r and Ω are positive constants. Suppose that at $t = 0$ the observer begins to move on a circle of radius r with $x = r \cos \phi$, $y = r \sin \phi$ and $z = 0$. Here $\phi = \Omega t = \gamma \Omega \tau$, where γ is the observer's Lorentz factor that corresponds to $\beta = r\Omega/c$. For $\tau > 0$, the observer's orthonormal tetrad frame [1, 2] is given by

$$\lambda^\mu_{\hat{0}} = \gamma(1, -\beta \sin \phi, \beta \cos \phi, 0), \quad (11)$$

$$\lambda^\mu_{\hat{1}} = (0, \cos \phi, \sin \phi, 0), \quad (12)$$

$$\lambda^\mu_{\hat{2}} = \gamma(\beta, -\sin \phi, \cos \phi, 0), \quad (13)$$

$$\lambda^\mu_{\hat{3}} = (0, 0, 0, 1). \quad (14)$$

In this case, Eq. (9) is written out in detail in Appendix A and thereby Λ can be immediately constructed. The kernel k can then be determined using Eq. (6). The general case will not be treated here; instead, we focus attention on the simple case of the observer that is at rest at the spatial origin of coordinates and refers its measurements to uniformly rotating axes, i.e. $r = 0$, so that $\beta = 0$ and $\gamma = 1$ in Eqs. (11) and (13). In this case Λ has a block diagonal form, $\Lambda = \text{diag}(1, R, 1, T, 1)$, where R is the 2×2 rotation matrix

$$R(\phi) = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \quad (15)$$

and T is the 5×5 matrix

$$T(\phi) = \begin{bmatrix} \cos^2 \phi & \sin 2\phi & 0 & \sin^2 \phi & 0 \\ -\frac{1}{2} \sin 2\phi & \cos 2\phi & 0 & \frac{1}{2} \sin 2\phi & 0 \\ 0 & 0 & \cos \phi & 0 & \sin \phi \\ \sin^2 \phi & -\sin 2\phi & 0 & \cos^2 \phi & 0 \\ 0 & 0 & -\sin \phi & 0 & \cos \phi \end{bmatrix}. \quad (16)$$

We note that $\det R = \det T = 1$ and

$$R^{-1}(\phi) = R(-\phi), \quad T^{-1}(\phi) = T(-\phi). \quad (17)$$

The kernel k can be easily determined in this case, since $\Lambda^{-1} = \text{diag}(1, R(-\phi), 1, T(-\phi), 1)$. It then turns out that k is a constant matrix $(k_{m,n})$, $m, n = 1, \dots, 10$, with nonzero elements given by

$$k_{2,3} = k_{6,8} = k_{7,9} = -\Omega, \quad (18)$$

$$k_{3,2} = k_{6,5} = k_{9,7} = \Omega, \quad (19)$$

$$-k_{5,6} = k_{8,6} = 2\Omega. \quad (20)$$

The kernel can be used to determine the nonlocal relationship, based on the ansatz (10), between the measured components of the gravitational potential, denoted by $H_{\hat{\alpha}\hat{\beta}}$, and the components $h_{\hat{\alpha}\hat{\beta}}$ that are obtained from the hypothesis of locality. The end result can be expressed as

$$H_{\hat{0}\hat{0}} = h_{\hat{0}\hat{0}}, \quad H_{\hat{0}\hat{3}} = h_{\hat{0}\hat{3}}, \quad H_{\hat{3}\hat{3}} = h_{\hat{3}\hat{3}}, \quad (21)$$

$$H_{\hat{0}\hat{1}} + H_{\hat{0}\hat{2}} = h_{\hat{0}\hat{1}} + h_{\hat{0}\hat{2}} + \Omega \int_0^\tau (h_{\hat{0}\hat{1}} - h_{\hat{0}\hat{2}}) d\tau', \quad (22a)$$

$$H_{\hat{0}\hat{1}} - H_{\hat{0}\hat{2}} = h_{\hat{0}\hat{1}} - h_{\hat{0}\hat{2}} - \Omega \int_0^\tau (h_{\hat{0}\hat{1}} + h_{\hat{0}\hat{2}}) d\tau', \quad (22b)$$

$$H_{\hat{1}\hat{1}} + H_{\hat{2}\hat{2}} = h_{\hat{1}\hat{1}} + h_{\hat{2}\hat{2}}, \quad (23a)$$

$$H_{\hat{1}\hat{2}} = h_{\hat{1}\hat{2}} + \Omega \int_0^\tau (h_{\hat{1}\hat{1}} - h_{\hat{2}\hat{2}}) d\tau', \quad (23b)$$

$$H_{\hat{1}\hat{1}} - H_{\hat{2}\hat{2}} = h_{\hat{1}\hat{1}} - h_{\hat{2}\hat{2}} - 4\Omega \int_0^\tau h_{\hat{1}\hat{2}} d\tau', \quad (23c)$$

$$H_{\hat{1}\hat{3}} + H_{\hat{2}\hat{3}} = h_{\hat{1}\hat{3}} + h_{\hat{2}\hat{3}} + \Omega \int_0^\tau (h_{\hat{1}\hat{3}} - h_{\hat{2}\hat{3}}) d\tau', \quad (24a)$$

$$H_{\hat{1}\hat{3}} - H_{\hat{2}\hat{3}} = h_{\hat{1}\hat{3}} - h_{\hat{2}\hat{3}} - \Omega \int_0^\tau (h_{\hat{1}\hat{3}} + h_{\hat{2}\hat{3}}) d\tau'. \quad (24b)$$

These equations, combined with the results of Appendix A, can be employed to determine the nonlocal modifications of the helicity-rotation coupling for gravitational waves incident on the special rotating observer that occupies the origin of spatial coordinates.

III. HELICITY-ROTATION COUPLING

Consider the reception of a plane monochromatic gravitational wave of definite helicity incident along the z axis by the rotating observer that is fixed at the origin of spatial coordinates. In the transverse-traceless gauge, the wave amplitude is given by the real part of

$$(h_{ij}) = A(e_\oplus \pm ie_\otimes) e^{i\omega(-t+z/c)}. \quad (25)$$

Here A is a constant amplitude, the upper (lower) sign corresponds to positive (negative) helicity radiation and the two independent linear polarization states [1] are given by

$$e_\oplus = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad e_\otimes = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (26)$$

For wave functions, the complex representation is employed throughout as all operations involving gravitational waves are linear. Thus only the real parts of the relevant quantities are of physical interest.

It follows from a detailed examination of the results of the previous section that in this case

$$h_{\hat{\alpha}\hat{\beta}} = e^{\pm 2i\phi} h_{\alpha\beta} \quad (27)$$

and

$$H_{\hat{\alpha}\hat{\beta}} = F_\pm(\tau) h_{\hat{\alpha}\hat{\beta}}, \quad (28)$$

where

$$F_{\pm}(\tau) = \frac{\omega \mp 2\Omega e^{i\omega'\tau}}{\omega \mp 2\Omega}, \quad (29)$$

$\omega' = \omega \mp 2\Omega$, $\tau = t$ and $z = 0$. Equations (27)-(29) are simply the spin-2 analogues of the corresponding results that have been discussed in detail in nonlocal electrodynamics—cf. Eqs. (12)-(17) of [2]. Specifically, for the case of resonance involving an incident positive helicity wave of frequency $\omega \mapsto 2\Omega$, we find that as $\omega' \mapsto 0$, $F_+ \mapsto f_+ = 1 - 2i\Omega\tau$; this linear divergence with time can be avoided with a finite incident wave packet. On the other hand, for an incident negative helicity wave of $\omega = 2\Omega$, $\omega' = 4\Omega$ and F_- becomes $f_- = \cos(2\Omega\tau) \exp(2i\Omega\tau)$.

Another direct consequence of nonlocality, evident in the factor F_{\pm} , is that the amplitude of a positive helicity gravitational wave of $\omega > 2\Omega$ as measured by the rotating observer is enhanced by a factor of $\omega/(\omega - 2\Omega)$, while that of a negative helicity wave is diminished by a factor of $\omega/(\omega + 2\Omega)$.

The nonlocal aspects of linear gravitation developed here may be extended to an inertial observer in the gravitomagnetic field of a rotating mass via the gravitational Larmor theorem [1]. Moreover, it should be remarked that for Earth-based gravitational-wave antennas the effective rotation frequency is about 10^{-5}Hz . The nonlocal effects discussed here would then be ordinarily very small for incident high-frequency gravitational waves with $\Omega/\omega \ll 1$; in fact, nonlocality could only become significant near resonance.

The results that have been obtained thus far for the rotating observer fixed at the origin of spatial coordinates may be simply extended to a whole class of such observers that are fixed in space and differ from each other only through their spatial positions. It turns out to be simpler to deal with this class of uniformly rotating observers than the class of observers whose tetrads are given by Eqs. (11)-(14). In the following section, we present the nonlocal gravitational field equation for the class of spatially fixed rotating observers.

IV. NONLOCAL WAVE EQUATION

Imagine observers that are always at rest in a global inertial frame and refer their measurements to the standard inertial axes for $-\infty < t < 0$; for $t \geq 0$, however, they employ axes that rotate uniformly about the z axis with frequency Ω . Thus for $t \geq 0$, each such

observer carries a tetrad frame given by Eqs. (11)-(14) with $\beta = 0$ and $\gamma = 1$. The purpose of this section is to develop the Lorentz-invariant nonlocal gravitational wave equation for this special class of noninertial observers.

It is a general consequence of Eq. (10) that for $\tau > \tau_0$,

$$h_{\alpha\beta}(\tau) = H_{\alpha\beta}(\tau) + \int_{\tau_0}^{\tau} \tilde{r}_{\alpha\beta}{}^{\gamma\delta}(\tau, \tau') H_{\gamma\delta}(\tau') d\tau', \quad (30)$$

where \tilde{r} is a variant of the resolvent kernel and has been discussed in detail in [10]. It has been shown in Appendix C of [10] that if k is a constant kernel, then \tilde{r} is constant as well and is given by

$$\tilde{r} = -\Lambda^{-1}(\tau_0)k\Lambda(\tau_0). \quad (31)$$

For the special class of rotating observers under consideration here, $\tau = t$, $\tau_0 = 0$, k is a constant matrix and its nonzero elements are given in Eqs. (18)-(20). Moreover, it is clear from Eqs. (15)-(16) that $\Lambda(0)$ is the identity matrix; hence, it follows from Eq. (31) that in this case

$$\tilde{r} = -k. \quad (32)$$

Thus the explicit form of the ten independent equations contained in Eq. (30) may be obtained from Eqs. (21)-(24b) by making the formal replacement $(H_{\hat{\alpha}\hat{\beta}}, h_{\hat{\alpha}\hat{\beta}}, \Omega) \mapsto (h_{\alpha\beta}, H_{\alpha\beta}, -\Omega)$. It is interesting to note that the form of Eqs. (21)-(24b) remains the same if all of the indices are raised; the same is true for the explicit form of Eq. (30) in the case under consideration here.

To express Eq. (30) for the *special class of rotating observers*, it proves convenient to write

$$h^{\alpha\beta}(t, \mathbf{x}) = H^{\alpha\beta}(t, \mathbf{x}) + \tilde{r}^{\alpha\beta}{}_{\gamma\delta} \int_0^t H^{\gamma\delta}(t', \mathbf{x}) dt' \quad (33)$$

for $t > 0$, since the observers are fixed in space. Here the components of \tilde{r} are all constants proportional to Ω . The substitution of $h^{\alpha\beta}(t, \mathbf{x})$ in the equations that it satisfies would then result, via Eq. (33), in the corresponding equations for the nonlocal wave amplitude $H^{\alpha\beta}(t, \mathbf{x})$.

The wave function $h^{\alpha\beta}(t, \mathbf{x})$ is subject to the gauge condition

$$\left(h^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} h \right)_{,\beta} = 0 \quad (34)$$

and satisfies the wave equation

$$\square h^{\alpha\beta} = 0, \quad (35)$$

which follows from $\square \bar{h}_{\alpha\beta} = 0$. Thus for $t > 0$, $H^{\alpha\beta}$ is subject to the gauge condition

$$\left(H^{\alpha\beta} + \tilde{r}^{\alpha\beta}_{\gamma\delta} \int_0^t H^{\gamma\delta}(t', \mathbf{x}) dt' \right)_{,\beta} = \frac{1}{2} \eta^{\alpha\beta} H_{,\beta}, \quad (36)$$

where $H = \eta_{\alpha\beta} H^{\alpha\beta}$ and it turns out that $H = h$ in this case. Moreover, $H^{\alpha\beta}$ satisfies the nonlocal wave equation

$$\square H^{\alpha\beta} = \tilde{r}^{\alpha\beta}_{\gamma\delta} \left(\frac{\partial}{\partial t} H^{\gamma\delta} - \Delta \int_0^t H^{\gamma\delta}(t', \mathbf{x}) dt' \right) \quad (37)$$

for $t > 0$.

The measurements of a class of linearly accelerated observers are considered in the next section. In particular, we will rule out the possibility of existence of a direct nonlocal coupling between the helicity of gravitational radiation and linear acceleration.

V. LINEARLY ACCELERATED OBSERVERS

Consider the class of observers at rest in the background global inertial system for $-\infty < t < 0$. At $t = 0$, the observers accelerate from rest with acceleration $g(\tau) > 0$ along the z axis. Here τ is the proper time and $\tau = 0$ at $t = 0$. For $\tau \geq 0$, the orthonormal tetrad frame of the observers is given by

$$\lambda^\mu_{\hat{0}} = (C, 0, 0, S), \quad \lambda^\mu_{\hat{1}} = (0, 1, 0, 0), \quad (38)$$

$$\lambda^\mu_{\hat{2}} = (0, 0, 1, 0), \quad \lambda^\mu_{\hat{3}} = (S, 0, 0, C), \quad (39)$$

where $C = \cosh \theta$, $S = \sinh \theta$ and

$$\theta = \frac{1}{c} \int_0^\tau g(\tau') d\tau'. \quad (40)$$

It follows from Eq. (9) that the wave amplitude, as measured by the momentarily comoving observers, is given by

$$h_{\hat{1}\hat{1}} = h_{11}, \quad h_{\hat{1}\hat{2}} = h_{12}, \quad h_{\hat{2}\hat{2}} = h_{22}, \quad (41)$$

$$h_{\hat{0}\hat{0}} - h_{\hat{3}\hat{3}} = h_{00} - h_{33}, \quad (42a)$$

$$\frac{1}{2}(h_{\hat{0}\hat{0}} + h_{\hat{3}\hat{3}}) = \frac{1}{2}(h_{00} + h_{33}) \cosh 2\theta + h_{03} \sinh 2\theta, \quad (42b)$$

$$h_{\hat{0}\hat{3}} = h_{03} \cosh 2\theta + \frac{1}{2}(h_{00} + h_{33}) \sinh 2\theta, \quad (42c)$$

$$h_{\hat{0}\hat{1}} - h_{\hat{1}\hat{3}} = (h_{01} - h_{13})e^{-\theta}, \quad (43a)$$

$$h_{\hat{0}\hat{1}} + h_{\hat{1}\hat{3}} = (h_{01} + h_{13})e^{\theta}, \quad (43b)$$

$$h_{\hat{0}\hat{2}} - h_{\hat{2}\hat{3}} = (h_{02} - h_{23})e^{-\theta}, \quad (44a)$$

$$h_{\hat{0}\hat{2}} + h_{\hat{2}\hat{3}} = (h_{02} + h_{23})e^{\theta}. \quad (44b)$$

The details of the calculation of the kernel are presented in Appendix B. We find, based on Eq. (10), that the components of the wave amplitude as measured by a linearly accelerated observer are given by

$$H_{\hat{1}\hat{1}} = h_{\hat{1}\hat{1}}, \quad H_{\hat{1}\hat{2}} = h_{\hat{1}\hat{2}}, \quad H_{\hat{2}\hat{2}} = h_{\hat{2}\hat{2}}, \quad (45)$$

$$H_{\hat{0}\hat{0}} - H_{\hat{3}\hat{3}} = h_{\hat{0}\hat{0}} - h_{\hat{3}\hat{3}}, \quad (46a)$$

$$H_{\hat{0}\hat{0}} + H_{\hat{3}\hat{3}} = h_{\hat{0}\hat{0}} + h_{\hat{3}\hat{3}} - \frac{4}{c} \int_0^\tau g h_{\hat{0}\hat{3}} d\tau', \quad (46b)$$

$$H_{\hat{0}\hat{3}} = h_{\hat{0}\hat{3}} - \frac{1}{c} \int_0^\tau g (h_{\hat{0}\hat{0}} + h_{\hat{3}\hat{3}}) d\tau' \quad (46c)$$

$$H_{\hat{0}\hat{1}} - H_{\hat{1}\hat{3}} = h_{\hat{0}\hat{1}} - h_{\hat{1}\hat{3}} + \frac{1}{c} \int_0^\tau g (h_{\hat{0}\hat{1}} - h_{\hat{1}\hat{3}}) d\tau', \quad (47a)$$

$$H_{\hat{0}\hat{1}} + H_{\hat{1}\hat{3}} = h_{\hat{0}\hat{1}} + h_{\hat{1}\hat{3}} - \frac{1}{c} \int_0^\tau g (h_{\hat{0}\hat{1}} + h_{\hat{1}\hat{3}}) d\tau', \quad (47b)$$

$$H_{\hat{0}\hat{2}} - H_{\hat{2}\hat{3}} = h_{\hat{0}\hat{2}} - h_{\hat{2}\hat{3}} + \frac{1}{c} \int_0^\tau g (h_{\hat{0}\hat{2}} - h_{\hat{2}\hat{3}}) d\tau', \quad (48a)$$

$$H_{\hat{0}\hat{2}} + H_{\hat{2}\hat{3}} = h_{\hat{0}\hat{2}} + h_{\hat{2}\hat{3}} - \frac{1}{c} \int_0^\tau g (h_{\hat{0}\hat{2}} + h_{\hat{2}\hat{3}}) d\tau'. \quad (48b)$$

An immediate consequence of these equations may be noted: For an incident gravitational wave of definite helicity (25) propagating along the direction of motion of the observer, $H_{\hat{i}\hat{j}} = h_{ij}$, as follows immediately from Eqs. (41) and (45). Thus there is no coupling of the observer's acceleration with the helicity of the incident gravitational radiation. This is a generalization of previous results [3] to nonlocal gravitation.

An important consequence of the results of this section should be noted: The general character of the nonlocal relations for observers that are linearly accelerated in the z direction makes it possible in this case to develop the nonlocal wave equation for linear gravitational waves following the same steps as in [10] for nonlocal Maxwell's equations. Einstein's heuristic principle of equivalence may then be employed to argue intuitively that nonlocality should extend to purely gravitational situations as well resulting in nonlocal as well as nonlinear gravitational field equations.

VI. DISCUSSION

There is only indirect evidence at present, based on the orbital decay of certain binary pulsars, for the existence of gravitational waves. Assuming that gravitation involves a basic radiation field, the nonlocal theory of accelerated observers has been extended in this paper to include linear gravitational waves. Following the approach presented in [10] for electrodynamics, it is in principle possible to develop nonlocal field equations for linear gravitational waves in Minkowski spacetime. This has been done in the present paper for a rather simple class of uniformly rotating observers. Invoking Einstein's principle of equivalence, the results of this paper may be considered to be a step in the direction of a nonlocal classical theory of gravitation.

APPENDIX A

For a general uniformly rotating observer considered in Section II, Eq. (9) may be written out in component form as follows:

$$h_{\hat{0}\hat{0}} = \gamma^2[h_{00} + \beta(-\sin\phi h_{01} + \cos\phi h_{02}) + \beta^2(\sin^2\phi h_{11} - \sin 2\phi h_{12} + \cos^2\phi h_{22})], \quad (\text{A1})$$

$$h_{\hat{0}\hat{1}} = \gamma \left[\cos\phi h_{01} + \sin\phi h_{02} + \frac{1}{2}\beta(-\sin 2\phi h_{11} + 2\cos 2\phi h_{12} + \sin 2\phi h_{22}) \right], \quad (\text{A2})$$

$$h_{\hat{0}\hat{2}} = \gamma^2\beta \left[h_{00} + \left(\frac{1}{\beta} + \beta \right) (-\sin\phi h_{01} + \cos\phi h_{02}) + \sin^2\phi h_{11} - \sin 2\phi h_{12} + \cos^2\phi h_{22} \right], \quad (\text{A3})$$

$$h_{\hat{0}\hat{3}} = \gamma[h_{03} + \beta(-\sin\phi h_{13} + \cos\phi h_{23})], \quad (\text{A4})$$

$$h_{\hat{1}\hat{1}} = \cos^2\phi h_{11} + \sin 2\phi h_{12} + \sin^2\phi h_{22}, \quad (\text{A5})$$

$$h_{\hat{1}\hat{2}} = \gamma \left[\beta(\cos\phi h_{01} + \sin\phi h_{02}) + \frac{1}{2}(-\sin 2\phi h_{11} + 2\cos 2\phi h_{12} + \sin 2\phi h_{22}) \right], \quad (\text{A6})$$

$$h_{\hat{1}\hat{3}} = \cos\phi h_{13} + \sin\phi h_{23}, \quad (\text{A7})$$

$$h_{\hat{2}\hat{2}} = \gamma^2[\beta^2 h_{00} + 2\beta(-\sin\phi h_{01} + \cos\phi h_{02}) + \sin^2\phi h_{11} - \sin 2\phi h_{12} + \cos^2\phi h_{22}], \quad (\text{A8})$$

$$h_{\hat{2}\hat{3}} = \gamma(\beta h_{03} - \sin\phi h_{13} + \cos\phi h_{23}), \quad (\text{A9})$$

$$h_{\hat{3}\hat{3}} = h_{33}. \quad (\text{A10})$$

These equations are equally valid for uniformly rotating observers at any fixed value of the vertical coordinate z .

For the incident radiation field (25) as measured by rotating observers at z , Eqs. (A1)-(A10) imply that

$$(h_{\hat{\alpha}\hat{\beta}}) = A \begin{bmatrix} -\beta^2\gamma^2 & \pm i\beta\gamma & -\beta\gamma^2 & 0 \\ \pm i\beta\gamma & 1 & \pm i\gamma & 0 \\ -\beta\gamma^2 & \pm i\gamma & -\gamma^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{\pm 2i\phi} e^{i\omega(-t+z/c)}, \quad (\text{A11})$$

where the temporal dependence is of the form $\exp(-i\omega'\tau)$ with ω' given by Eq. (1). Equation (A11) should be compared and contrasted with the measured components of the Riemann tensor in this case given by Eqs. (2.10) and (2.11) of [1]. Moreover, we note that for the special rotating observer with $\beta = 0$ and $\gamma = 1$, Eq. (A11) simply reduces to Eq. (27).

APPENDIX B

The purpose of this appendix is to compute Λ^{-1} and the kernel k for the linearly accelerated observers of Section V. Inspection of Λ reveals that the entries in its fifth and sixth rows and columns vanish except for the diagonal elements that both equal unity. Let the 8×8 matrix $\tilde{\Lambda}$ be the reduced form of Λ in which the fifth and sixth rows and columns have been ignored; then,

$$\tilde{\Lambda}(\theta) = \begin{bmatrix} M & N \\ P & Q \end{bmatrix}, \quad (\text{B1})$$

where

$$M = \begin{bmatrix} C^2 & 0 & 0 & 2CS \\ 0 & C & 0 & 0 \\ 0 & 0 & C & 0 \\ CS & 0 & 0 & C^2 + S^2 \end{bmatrix}, \quad N = S \begin{bmatrix} 0 & 0 & 0 & S \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & C \end{bmatrix}, \quad (\text{B2})$$

$$P = S \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ S & 0 & 0 & 2C \end{bmatrix}, \quad Q = \text{diag}(C, 1, C, C^2). \quad (\text{B3})$$

Here, as before, $C = \cosh \theta$ and $S = \sinh \theta$. We note that $\det M = \det Q = C^4$ and $\det N = \det P = 0$. It can be shown that

$$\tilde{\Lambda}^{-1}(\theta) = \begin{bmatrix} U & V \\ X & Y \end{bmatrix}, \quad (\text{B4})$$

where

$$U = (M - NQ^{-1}P)^{-1}, \quad V = -M^{-1}NY, \quad (\text{B5})$$

$$X = -Q^{-1}PU, \quad Y = (Q - PM^{-1}N)^{-1}. \quad (\text{B6})$$

From

$$M^{-1} = \frac{1}{C^2} \begin{bmatrix} C^2 + S^2 & 0 & 0 & -2CS \\ 0 & C & 0 & 0 \\ 0 & 0 & C & 0 \\ -CS & 0 & 0 & C^2 \end{bmatrix} \quad (\text{B7})$$

and the explicit evaluation of the matrices in Eqs. (B5) and (B6), one finds the simple relation

$$\tilde{\Lambda}^{-1}(\theta) = \tilde{\Lambda}(-\theta). \quad (\text{B8})$$

The same relation holds for Λ , i.e. Λ^{-1} has the same form as Λ but with $(C, S) \mapsto (C, -S)$. Moreover, the nonzero elements of k in this case turn out to be

$$k_{1,4} = k_{10,4} = -\frac{2}{c}g(\tau), \quad (\text{B9})$$

$$k_{2,7} = k_{3,9} = k_{4,1} = k_{4,10} = k_{7,2} = k_{9,3} = -\frac{1}{c}g(\tau). \quad (\text{B10})$$

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