

# Inertia of Intrinsic Spin

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The state of a particle in space and time is characterized by its mass and spin, which therefore determine the inertial properties of the particle. The coupling of intrinsic spin with rotation is examined and the corresponding inertial effects of intrinsic spin are studied. An experiment to measure directly the spin-rotation coupling via neutron interferometry is analyzed in detail.

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## I. INTRODUCTION

In classical physics any inertial force acting on a particle is necessarily proportional to the particle's inertial mass. The principle of equivalence of inertial and gravitational masses would then suggest that inertial forces may be of gravitational origin—an idea (“Mach's principle”) that played a crucial role in the development of general relativity [1]. On the basis of the modern geometric interpretation of Einstein's theory of gravitation, however, the notion that inertial effects are of gravitational origin must be rejected [2, 3]. Moreover, in quantum theory the inertial properties of a particle are determined by its inertial mass as well as spin. Mass and spin characterize the irreducible unitary representations of the inhomogeneous Lorentz group [4]. The inertial properties of mass are well known [5]; therefore, this paper is devoted to the inertial properties of intrinsic spin.

Spin-rotation coupling and its consequences are described in sections II and III. An experiment to measure this effect for neutrons is discussed in section IV.

## II. SPIN-ROTATION COUPLING

It follows from the treatment of Dirac's equation in accelerated frames of reference that in the nonrelativistic regime the Hamiltonian of a Dirac particle contains  $-\mathbf{\Omega} \cdot \mathbf{J}$ , where  $\mathbf{\Omega}$  is the rotation frequency of the frame and  $\mathbf{J} = \mathbf{L} + \mathbf{S}$  is the total angular momentum of the particle [6]. The appearance of  $\mathbf{J}$  is natural, since in quantum theory the total angular momentum is the generator of rotations. The term  $-\mathbf{\Omega} \cdot \mathbf{L}$  involving the orbital angular momentum  $\mathbf{L}$  is responsible for the Sagnac effect, first demonstrated experimentally for slow neutrons by Werner et al. [7]. We are interested in the spin-rotation coupling term  $-\mathbf{\Omega} \cdot \mathbf{S}$ , which does *not* depend upon the inertial mass of the particle; indeed, it is an inertial effect of intrinsic spin [8]. It originates from the tendency of intrinsic spin to keep its aspect with respect to a global background inertial frame (“inertia of intrinsic spin”). From the standpoint of observers at rest in the rotating frame, the intrinsic spin therefore precesses in a sense that is opposite to the sense of rotation of the frame. This precessional motion can be described by the Heisenberg equation of motion with the Hamiltonian  $-\mathbf{\Omega} \cdot \mathbf{S}$ . Moreover, it can be shown that the coupling of spin with rotation is fully relativistic [9].

To illustrate the general nature of this coupling, consider an observer rotating uniformly with frequency  $\Omega$  about the direction of propagation of a plane electromagnetic wave of frequency  $\omega$ . It follows from the Fourier analysis of the electromagnetic field measured by the rotating observer that the frequency of the wave in the rotating frame is given by

$$\omega' = \gamma(\omega \mp \Omega), \quad (1)$$

where  $\gamma$  is the Lorentz factor of the observer. Here the upper (lower) sign refers to positive (negative) helicity radiation. Multiplication by  $\hbar$  results in  $E' = \gamma(E - \mathbf{\Omega} \cdot \mathbf{S})$ , where  $\mathbf{S}$  is the spin of the photon. This helicity contribution to the Doppler effect has been verified via the GPS [10]; in fact, the helicity-rotation coupling is responsible for the

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phenomenon of *phase wrap-up* [10]. An intuitive explanation of equation (1) involves the fact that in a positive (negative) helicity wave, the electromagnetic field rotates with frequency  $\omega$  ( $-\omega$ ) about the direction of propagation of the wave. Thus the rotating observer perceives positive (negative) helicity radiation with the electromagnetic field rotating with frequency  $\omega - \Omega$  ( $-\omega - \Omega$ ) about the direction of wave propagation. In addition, the Lorentz factor in equation (1) is due to time dilation. More generally, the energy of a particle measured by an observer rotating with frequency  $\Omega$  is given by

$$E' = \gamma(E - \hbar M \Omega), \quad (2)$$

where  $M$  is the total (orbital plus spin) “magnetic” quantum number along the axis of rotation ( $M = 0, \pm 1, \pm 2, \dots$ , for a scalar or a vector particle, while  $M \mp \frac{1}{2} = 0, \pm 1, \pm 2, \dots$ , for a Dirac particle). For fermions, there exists at present only indirect evidence for the existence of spin-rotation coupling [11, 12].

### III. ENERGY SHIFT

An interesting consequence of the spin-rotation coupling is the energy shift that would be induced when polarized radiation passes through a rotating spin flipper. To illustrate this effect, imagine positive helicity electromagnetic radiation of frequency  $\omega_{\text{in}}$  that is normally incident on a uniformly rotating half-wave plate. It follows from equation (1) that within the half-wave plate  $\omega' \approx \omega_{\text{in}} - \Omega$ , where  $\Omega$  is the rotation frequency and we have assumed that  $\gamma \approx 1$ . The spacetime in a uniformly rotating system is stationary; therefore,  $\omega'$  remains constant throughout the rotating half-wave plate. The radiation that emerges has negative helicity and frequency  $\omega_{\text{out}}$ , where  $\omega' \approx \omega_{\text{out}} + \Omega$  by equation (1). Thus  $\omega_{\text{out}} - \omega_{\text{in}} \approx -2\Omega$ , so that the photon energy is down-shifted by  $-2\hbar\Omega$ . Such an energy shift was first discovered experimentally in the microwave regime [13]. It follows from the general nature of spin-rotation coupling that an up/down energy shift

$$\delta H \approx -2\mathbf{\Omega} \cdot \mathbf{S} \quad (3)$$

occurs whenever spinning particles pass through a  $\pi$ -spin flipper. An experiment to measure this effect for slow neutrons was first suggested in [14]. This concept is elaborated in the rest of this paper. We expect that a longitudinally polarized neutron passing through a rotating  $\pi$ -spin flipper coil would experience an energy shift of  $\hbar\Omega$  that would result in a beat phenomenon in a neutron interferometer as in Figure 1.

A static  $\pi$ -spin flipper provides a region of constant uniform magnetic field  $\mathbf{B}$  over a definite width  $w$  such that the interaction  $H = -\boldsymbol{\mu}_n \cdot \mathbf{B}$  induces a reversal of the spin direction. Here  $\boldsymbol{\mu}_n = \gamma_n \mathbf{S}$  is the neutron magnetic dipole moment,  $\gamma_n$  is the gyromagnetic ratio and  $w \approx \pi v_n / (|\gamma_n| B)$ , where  $v_n$  is the neutron speed. For instance, in Figure 1,  $\mathbf{B} = B\hat{\mathbf{y}}$  and the outgoing neutron has a spin that is antiparallel to the direction of the incident longitudinally polarized neutron. Physically, we expect that a uniformly rotating spin flipper with  $\Omega \ll |\gamma_n| B$  would be essentially equivalent to a static coil with uniform but variable magnetic field  $\mathbf{B} = B(-\hat{\mathbf{x}} \sin \Omega t + \hat{\mathbf{y}} \cos \Omega t)$ .

### IV. DISCUSSION

We propose a neutron interferometry experiment using polarized neutrons to be carried out for two equivalent cases:

(1) Rotating 180°- spin flipper coil:

The experimental arrangement is shown in Figure 1. Neutrons, longitudinally polarized, pass through our interferometer with 180° - spin flipper coils (producing a uniform static magnetic field normal to the polarization axis of the neutrons), placed in each of the two coherent neutron beam paths. SF1 and SF2 are both optimized such that they flip the neutron spin by 180°. The phase shifter allows us to set the intensity at maximum when both coils are aligned parallel. Keeping the interferometer stationary in the inertial frame of the laboratory, we then rotate SF1 with angular velocity  $\mathbf{\Omega}$  parallel to the neutron wave vector and the resulting phase shift can be measured. After superposition of the two coherent beams, we expect to observe a sinusoidal intensity modulation (beating that arises from the spin-rotation coupling) with a beating period corresponding to  $\Omega$ , the rate of the rotation of SF1. In the actual experiment, we will rotate SF1 approximately 120° forth and back, since continuous full rotation would be impractical because of wires leading to the power supply and the cooling lines attached to the spin flipper coil.

(2) Quadrature coil:

The experimental arrangement is the same as in Figure 1, except that the rotating 180°-spin flipper coil SF1 is replaced by a stationary quadrature coil, which produces a rotating magnetic field  $\mathbf{B} = (-B \sin \Omega t, B \cos \Omega t, 0)$  normal to the polarization axis of the neutrons. The expected result should be the same, since the neutron cannot distinguish

if SF1 is physically rotated or the magnetic field is rotated. The proposed experiment, as described in case (1), is related to the neutron beat frequency measured by Badurek et al. [15] via the Larmor theorem; thus  $\boldsymbol{\Omega} \Leftrightarrow \gamma_n \mathbf{B}'$ , where  $\gamma_n$  is the neutron gyromagnetic ratio and  $\mathbf{B}'$  is the magnetic field of their experiment.

Contemplating an experimental arrangement similar to that employed by Badurek et al. [15], the rotation of the  $\pi$ -spin flipper with a period of about one minute would result in an energy difference of  $\sim 10^{-16}$  eV between the two beams of our interferometer. Using neutrons of wavelength  $\sim 2$  Å, we would expect an intensity modulation amplitude of  $\sim 30$  counts per second [15]. It is interesting to note that it may be possible to use magnetized foils [16] as  $\pi$ -flippers instead of some of the coils discussed in this section.

The proposed experiment will provide a direct interferometric test of spin-rotation coupling for fermions. The phenomenon of spin-rotation coupling is of basic interest since it reveals the rotational inertia of intrinsic spin.

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- [1] A. Einstein, The meaning of relativity (Princeton University Press, Princeton, 1955).
  - [2] B. Mashhoon, Phys. Lett. A 126 (1988) 393.
  - [3] W. Rindler, Relativity: special, general and cosmological (Oxford University Press, New York, 2001).
  - [4] E.P. Wigner, Ann. Math. 40 (1939) 149.
  - [5] G.F. Moorhead and G.I. Opat, Class. Quantum Grav. 13 (1996) 3129.
  - [6] F.W. Hehl and W.-T. Ni, Phys. Rev. D 42 (1990) 2045.
  - [7] S.A. Werner, J.-L. Staudenmann and R. Collela, Phys. Rev. Lett. 42 (1979) 1103.
  - [8] B. Mashhoon, Phys. Rev. Lett. 61 (1988) 2639.
  - [9] L. Ryder, J. Phys. A 31 (1998) 2465.
  - [10] N. Ashby, Living Rev. Relativity 6 (2003) 1.
  - [11] B. Mashhoon, Phys. Lett. A 198 (1995) 9.
  - [12] G. Papini, Phys. Rev. D 65 (2002) 077901.
  - [13] P.J. Allen, Am. J. Phys. 34 (1966) 1185.
  - [14] B. Mashhoon, R. Neutze, M. Hannam and G.E. Stedman, Phys. Lett. A 249 (1998) 161.
  - [15] G. Badurek, H. Rauch and D. Tuppinger, Phys. Rev. A 34 (1986) 2600.
  - [16] M.Th. Rekveldt, W.G. Bouwman, W.H. Kraan, T.V. Krouglov and J. Plomp, Physica B 335 (2003) 164.

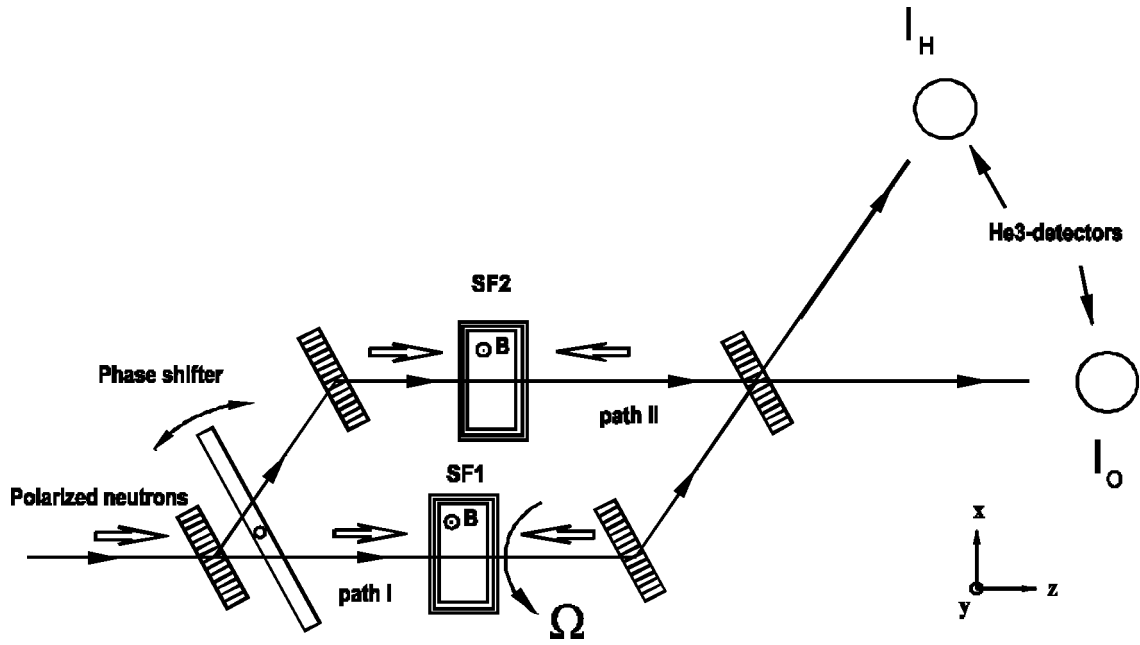


FIG. 1: Layout of the proposed experiment. Longitudinally polarized neutrons pass through a LLL-interferometer with a static  $\pi$ -spin flipper SF2 in path II, while there is a slowly rotating  $\pi$ -spin flipper SF1 in path I, having angular velocity  $\Omega$  (case (1)) [or replaced by a static spin flipper with a rotating magnetic field  $B$  (quadrature coil) in path I (case (2))].