ALMOST EVERYWHERE CONVERGENCE FOR MODIFIED

BOCHNER RIESZ MEANS AT THE CRITICAL INDEX FOR $p \geq 2$

Marco Annoni

Dr. Loukas Grafakos, Dissertation Supervisor

ABSTRACT

For $\lambda, \gamma > 0$, let the function $m_{\lambda,\gamma} : \mathbf{R^n} \to [0, \infty)$ be defined by

$$m_{\lambda,\gamma}(\xi) = \frac{(1 - |\xi|^2)_+^\lambda}{(1 - \log(1 - |\xi|^2))^\gamma}.$$

For $R > 0$, let the mean $B_R^{\lambda,\gamma}$ be defined by:

$$B_R^{\lambda,\gamma}(f)(x) = \int_{\mathbf{R^n}} m_{\lambda,\gamma}\left(\frac{\xi}{R}\right) \widehat{f}(\xi) e^{2\pi i \xi \cdot x} d\xi.$$

We have the following theorem:

Theorem 1.1 *For every $\lambda > 0$ such that $1 + 2\lambda < n$, let $p_\lambda = \frac{2n}{n-2\lambda-1}$. If $\gamma > \frac{1}{p'_\lambda} + \frac{1}{2}$ (where $\frac{1}{p_\lambda} + \frac{1}{p'_\lambda} = 1$) and $f \in L^{p_\lambda}(\mathbf{R^n})$, then we have:*

$$\lim_{R \to \infty} B_R^{\lambda,\gamma}(f)(x) = f(x),$$

for almost every $x \in \mathbf{R^n}$.