We study a variety of combinatorial distance and dot product related problems in vector spaces over finite fields. First, we focus on the generation of the Special Linear Group whose elements belong to a finite field with \( q \) elements. Given \( A \subset \mathbb{F}_q \), we use Fourier analytic methods to determine how large \( A \) needs to be to ensure that a certain product set contains a positive proportion of all the elements of \( SL_2(\mathbb{F}_q) \).

We also study a variety of distance and dot product sets related to the Erdős-Falconer distance problem. In general, the Erdős-Falconer distance problem asks for the number of distances determined by a set of points. The classical Erdős distance problem asks for the minimal number of distinct distances determined by a finite point set in \( \mathbb{R}^d \), where \( d \geq 2 \). The Falconer distance problem, which is the continuous analog of the Erdős distance problem, asks to find \( s_0 > 0 \) such that if the Hausdorff dimension of \( E \) is greater than \( s_0 \), then the Lebesgue measure of \( \Delta(E) \) is positive.

A generalization of the Erdős-Falconer distance problem in vector spaces over finite fields is to determine the minimal \( \alpha > 0 \) such that \( E \) contains a congruent copy of every \( k \) dimensional simplex whenever \(|E| \gtrsim q^\alpha\). We improve on known results (for \( k > 3 \)) using Fourier analytic methods, showing that \( \alpha \) may be taken to be \( \frac{d+k}{2} \). If \( E \) is a subset of a sphere, then we get a stronger result which shows that \( \alpha \) may be taken to be \( \frac{d+k-1}{2} \).