FINITE POINT CONFIGURATIONS AND PROJECTION THEOREMS IN VECTOR SPACES OVER FINITE FIELDS

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ABSTRACT

We study a variety of combinatorial distance and dot product related problems in vector spaces over finite fields. First, we focus on the generation of the Special Linear Group whose elements belong to a finite field with q elements. Given $A \subset \mathbb{F}_q$, we use Fourier analytic methods to determine how large A needs to be to ensure that a certain product set contains a positive proportion of all the elements of $SL_2(\mathbb{F}_q)$.

We also study a variety of distance and dot product sets related to the Erdős-Falconer distance problem. In general, the Erdős-Falconer distance problem asks for the number of distances determined by a set of points. The classical Erdős distance problem asks for the minimal number of distinct distances determined by a finite point set in $\mathbb{R}^d$, where $d \geq 2$. The Falconer distance problem, which is the continuous analog of the Erdős distance problem, asks to find $s_0 > 0$ such that if the Hausdorff dimension of E is greater than s_0, then the Lebesgue measure of $\Delta(E)$ is positive.

A generalization of the Erdős-Falconer distance problem in vector spaces over finite fields is to determine the minimal $\alpha > 0$ such that E contains a congruent copy of every k dimensional simplex whenever $|E| \gtrsim q^\alpha$. We improve on known results (for $k > 3$) using Fourier analytic methods, showing that α may be taken to be $\frac{d+k}{2}$. If E is a subset of a sphere, then we get a stronger result which shows that α may be taken to be $\frac{d+k-1}{2}$.