USING NESTED STRUCTURES TO SELECT MODELS FOR DEVELOPMENTAL TRAJECTORIES OF COGNITIVE ABILITIES IN ADULTHOOD

A Dissertation

presented to

the Faculty of the Graduate School

at the University of Missouri-Columbia

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

by

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MAY 2010

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USING NESTED STRUCTURES TO SELECT MODELS FOR DEVELOPMENTAL TRAJECTORIES OF COGNITIVE ABILITIES IN ADULTHOOD

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and hereby certify that, in their opinion, it is worthy of acceptance.

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ACKNOWLEDGEMENTS

I would like to thank Dr. Phillip Wood, my doctoral advisor, for his guidance and advice with respect to my dissertation as well as my graduate career in general. Further, I would also like to thank the members of my dissertation committee, Dr.'s Denis McCarthy, Douglas Steinley, Jeni Hart, and Tim Trull, for their service, efforts, and feedback. Finally, I would like to thank Dr. Chandra Reynolds for allowing me to use her data in the current work.

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Abstract

Change over time can be modeled through a variety of statistical techniques, including linear slope-intercept, repeated-measures MANOVA, and single-factor growth models, to name a few. Researchers wishing to characterize developmental trajectories typically wish to model change over time. Choice of method for modeling longitudinal data is often based on convention or familiarity with a particular modeling approach. It is argued that this should not be common practice, but rather the data and model comparisons should be used to inform the choice of model. The current work shows how several growth models are actually special cases of the free curve slope-intercept (FCSI) model, and as such, the models are nested within the FCSI model. Given the nested structure, direct model comparisons can be made, using chi-square difference tests as well as comparisons of alternative fit indices (e.g., AIC, BIC, RMSEA), to determine the best model for a given dataset. This idea is illustrated through application to an example dataset that included cognitive ability information over adulthood. More specifically, the developmental trajectories of crystallized and fluid abilities over adulthood were modeled. Substantively, the results suggest that crystallized abilities change at a non-linear rate that is not adequately characterized by either a quadratic or linear slope-intercept model (which are commonly used). Fluid abilities, however, showed a more consistent rate of change than crystallized abilities, consistent with the literature. More specifically, fluid abilities were adequately modeled by a linear SI, quadratic SI, and single-factor growth model that illustrated a linear tendency.

Using Nested Structures to Select Models for Developmental Trajectories of Cognitive Abilities in Adulthood

Reynolds, Finkel, McArdle, Gatz, Berg, and Pedersen (2005) explored behavior genetics models for cognitive aging in a sample of older adult twins. That is, their work sought to decompose environmental and genetic variation in change in cognitive abilities over time. Regarding the form of phenotypic growth, they suggested that cognitive aging occurs in a quadratic fashion. That is, they argued that decline of multiple cognitive abilities, as existed in their data, was best characterized by a quadratic slope-intercept model, suggesting that change is a function of a linear and quadratic component. There has been much research on cognitive aging, beyond Reynolds et al.'s (2005) work. Indeed, researchers that focus specifically on the study of cognitive functioning in older adulthood are often interested in modeling the developmental trajectory of cognitive processes over time.

Previous research has repeatedly shown that the shape and rate of change varies amongst different cognitive abilities (e.g., Baltes, Staudinger, & Lindenberger, 1999; Schaie, 1994. 1996). That is, some cognitive processes, such as fluid abilities, decline relatively steadily with age, whereas others, such as crystallized abilities, appear to be relatively stable over much of the adult lifespan (e.g., Baltes, Staudinger, & Lindenberger, 1999). As such, there is not one particular model that has been accepted as the definitive form of growth for cognitive abilities. Rather, there are a variety of proposed trajectories. The differences in suggested rates and forms of change are due not only to differences in the actual trajectories, but also to experimental design, and the method selected for testing or modeling change over time. Given that other research has suggested that different abilities have different developmental trajectories, Reynolds et al.'s (2005) conclusion that cognitive decline occurs in a quadratic fashion is called into question. Indeed, Reynolds et al. (2005) acknowledged that there were several limitations to their chosen model. Specifically, they explicitly state that, "other latent growth models might better describe the data." (p. 15)

Moving beyond cognitive aging work, it is not uncommon for researchers in the psychological sciences in general to seek to test for, or characterize the shape of, change in psychological phenomena over time. Many models have been proposed for modeling change over time including, but not limited to: slope-intercept factor models, single-factor growth models, multilevel models, and repeated-measures MANOVA. Each of the mentioned models has it's own unique set of assumptions and restrictions. However, it can be show that there are similarities among the models such that they can be expressed as special cases of a free curve slope intercept model (Meredith & Tisak, 1990). As a result, the models are covariance-nested and direct comparisons of model fit can be conducted which provide information about which model is most appropriate for characterizing the form of growth in the data.

Substantively, the current work focuses on the developmental trajectory of cognitive processes in older adulthood; however, the purpose of the current work is not to draw a definitive conclusion about the nature of cognitive aging. Rather, the purpose of the current work is to illustrate how to use nested model structures and comparisons of relative model fit to inform the choice of model. In describing this process, we define each model in terms of the general factor model, compare and contrast the different approaches to modeling change over time, and use an example dataset to illustrate the technique. That is, we are attempting to show how this flexible modeling framework (i.e., defining the models in terms of the general factor model), and resulting nested model comparisons, could be applied to situations where researchers are seeking to accurately characterize developmental trajectories.

In application, model choices are determined by several factors. When choosing a modeling technique, researchers may select a model based upon assumptions or hypotheses about the nature of change over time or the nature of the data. Further, model choice may also be the result of familiarity with a particular modeling technique. Indeed, researchers may not be familiar with alternative modeling options, and thus might default to an approach that they have frequently used in the past. Further, while it is important to make sure that the choice of model reflects the research question and hypothesis, it is also important to allow for the possibility that a priori expectations about the nature of change and the data may not be accurate. That is, there may be more accurate or appropriate models for a data set than the one initially expected and selected for use by a research team. Further, if different researchers are using different modeling techniques, they are likely to get different results. Such actions can lead to contradictions in the literature. As such, if researchers were to adopt the proposed approach to model selection/testing, allowing the data to inform the model choice, then contradictions due to model selection might decrease, yielding more consistent results across studies.

In the following pages, the previously mentioned approaches to modeling longitudinal data are reviewed, the nested structure of the models is discussed, and model comparison procedures are described. To facilitate discussion of the nested structure, and model comparison procedures, the general factor model is defined and then each of the individual models is defined in terms of the general factor model. Following the discussion of growth models, the cognitive aging literature is briefly reviewed. Finally, the current study is introduced and the methods and results of the current work are reported.

The General Factor Model

The general factor model (Sörbom, 1974) is represented in matrix form as

$$X = \mu + \xi\Lambda + \varepsilon \tag{1}$$

where X is an *n* (number of cases) by *k* (number of manifest variables) matrix of observed scores, μ is an *n* by *k* matrix of intercepts (of the manifest variables), ξ is an *n* by *f* (number of factors) matrix of factor scores, Λ is an *f* by *k* matrix of factor loadings, and ε is an *n* by *k* matrix of errors. The factor score matrix, ξ, is further decomposed into means and deviations in the equation

$$\xi = \theta + \eta \tag{2}$$

where θ is an *n* by *f* matrix of factor means and η is an *n* by *f* matrix of the deviation scores (of the factor scores from the factor means). The implied covariance structure is then defined as

$$\Sigma = \Lambda'\Phi\Lambda + \Psi^2 \tag{3}$$

where Φ is an *f* by *f* matrix of factor variances/covariances and Ψ^2 is a *k* by *k* matrix of error variances.

Traditional confirmatory factor analytic approaches estimate the model so as to recreate the variance-covariance matrix. When factor models are fit based on the covariance matrix, the means are estimated at the level of the manifest variables, not at the factor level. Referring back to the equations above, this is indicated when the elements of the intercept matrix, μ, are estimated and the elements of the factor mean matrix, θ, are set to zero. However, it is also possible to specify a factor mean (FM) model that estimates the means at the level of the latent variable. With respect to the previously discussed equations, factor level mean estimation is indicated when the elements of the intercept matrix, μ, are set to 0 and the elements of the factor mean matrix, θ, are estimated. Such models are estimated based on the adjoined average sums of squares and cross products (SSCP) matrix rather than the covariance matrix (Wood and Jackson, 2008).[1] Analysis of factor level means is essentially based on the raw score matrix, because the adjoined average SSCP matrix is a matrix of sufficient summary statistics for the raw score matrix.[2]

$$\text{Adjoined Average SSCP Matrix} = \begin{bmatrix} \frac{\Sigma X^2}{N} & \frac{\Sigma XY}{N} & \frac{\Sigma XZ}{N} & \frac{\Sigma X}{N} \\ \frac{\Sigma XY}{N} & \frac{\Sigma Y^2}{N} & \frac{\Sigma YZ}{N} & \frac{\Sigma Y}{N} \\ \frac{\Sigma XZ}{N} & \frac{\Sigma YZ}{N} & \frac{\Sigma Z^2}{N} & \frac{\Sigma Z}{N} \\ \frac{\Sigma X}{N} & \frac{\Sigma Y}{N} & \frac{\Sigma Z}{N} & 1 \end{bmatrix}$$

Note that the last column and row of the adjoined average SSCP matrix includes the observed means for the manifest variables. It was previously stated that FM models estimate means at the level of the factor rather than the manifest variable. The observed means can still be calculated here because they are the observed sample statistics. Indeed, the observed sample means are accurate estimates of the population parameters; however, these values are not calculated with the express purpose of estimating the population parameters. Thus, the manifest means are not being estimated in the model.

Identification of the General Factor Model

Broadly speaking, there are three ways in which latent variable models must be identified in order to be both estimable and to achieve accurate parameter estimates. Models need be *logically*, *mathematically*, and *empirically* identified. In order for a model to logically identified, it must estimate fewer (free) parameters than the number of unique elements of the covariance matrix (or the number of elements in the average SSCP matrix). For a model to be mathematically identified, a unique solution to the model must exist. Mathematical identification (of each factor) in a factor model is typically achieved through the use of one of the following uniqueness constraints: (1) the variance of the factor is set to unity, (2) a factor loading is set to unity, or (3) the sum of the squared factor loadings is constrained to equal 1. Finally, a model is empirically identified if it is estimable with the data and/or sample at hand (Kenny, Kashy, & Bolger, 1998).[3]

With respect to mean estimation, we cannot simultaneously estimate all factor and all manifest level means and still have an identified model. One condition of identification is that the number of estimated means (in the μ and θ matrices) cannot exceed the number of manifest variables (or means that actually exist). Thus, estimating one or more factor level means, in addition to means for all of the manifest variables would result in the estimation of f (number of factors) means more than are allowed for an identified model. Furthermore, the factor model defined in equations 1 and 2 is not identified unless boundary conditions are imposed on the variance components and intercepts; at a minimum there must be f^2 constraints across the Λ and Φ matrices (Sörbom, 1974). For example, if there are two factors specified in the model, then there must be a minimum of 4 constraints across the factor loading and factor variance/covariance matrices. This can be achieved by setting factor loadings to constant values or to equality, or by setting factor variances or covariances to a constant value.

Modeling Growth or Change Over Time

Now that the general factor model has been described, we can move on to a more detailed discussion of the individual growth models, in terms of the general factor model. As noted by Bollen and Curran (2006), when analyzing observations over time, we are often trying to characterize an unobservable underlying pattern of growth or change (i.e., a latent trajectory). In characterizing the underlying trajectory, we typically seek to identify group-level patterns of change while also summarizing the individual-level variability in patterns of change. As previously mentioned, researchers may choose a statistical model based upon convention, ease of computation, the familiarity of the researcher with a particular type of growth model, the theoretical expectations of the researcher, and/or assumptions about the nature of the data. While multiple models may be *acceptable* for use with a particular dataset, it is generally desirable to identify the optimal (or most appropriate) model given the structure of the data. As such, it is arguable that model choice should not be dictated by convention; rather it can be argued that model choice should be based on the qualities of the particular dataset and theory.

The specific growth models that will be reviewed in the following paragraphs include free curve, linear, and quadratic slope-intercept models, repeated-measured MANOVA, multilevel (or hierarchical linear), intercept-only, and single factor growth models (both with and without an additive constant). Consistent with previous statements, several researchers (e.g., Meredith & Tisak, 1990; Voekle, 2007; Wood &

Jackson, 2008) have noted that the mentioned growth models are special cases of the slope-intercept model. Essentially, the discussed models of growth differ by assumptions (e.g., distributional or measurement-level assumptions with respect to either manifest or latent variables), estimated parameters, and the number of estimated factors. Thus, the purposes of this section are to: (1) review commonly used approaches to modeling growth or change over time, (2) define said models in terms of the general factor model, (3) compare and contrast the structures and assumptions of the given models, and (4) discuss how the nested structure of the model allows for model comparisons, and selection of the best model for a given dataset. The matrix forms for the relevant growth models, in terms of the general factor model, are shown in Table 1.

The Slope-Intercept (SI) Model

One common approach to characterizing growth or change over time is the slope-intercept (SI) model (e.g., Bollen & Curran, 2006; Meredith & Tisak, 1990). The SI model specifies a latent intercept and latent slope factor, each with freely estimated factor-level means and variances, and a freely estimated covariance between the two factors. The estimated intercept and slope means represent the average intercept for the latent trajectories (i.e., the average at the reference point across all cases) and the average rate of growth for the entire sample, respectively. The estimated factor variances represent the amount of variation in intercepts and slopes exhibited by individuals in the sample. Thus, individual growth trajectories are characterized by both fixed and random slope and intercept components. The estimated covariance between the intercept and slope factors represents the overall relationship between the starting points of the latent trajectories and the subsequent rates of growth.

For all SI models that will be discussed, the factor loadings on the intercept factor are set to unity, whereas the factor loadings on the slope factor are either set to predetermined values or estimated. The slope factor loadings set the metric of change. That is, the slope factor loadings represent the amount of change that is of interest, with respect to: (1) the actual amount of time that elapsed from measurement occasion to measurement occasion, and (2) the selected starting point of time. Thus, the choice of how to define the slope factor loadings reflects the assumptions about the nature of change over time in the data. The SI model is illustrated in path diagram form in Figure 1, whereas the matrix forms for the SI model (in terms of the general factor model) are defined in the first column of Table 1. Note that both the path diagram and matrix forms do not include necessary uniqueness constraints, but rather reflect the general structure of the SI model.

Identification requirements of the SI model. In accordance with the rule that the number of parameters estimated in a model cannot exceed the number of known parameters, the SI model requires a minimum of three measurement occasions to be identified (Bollen & Curran, 2006). Further, latent variable models require that f^2 constraints are imposed across the Λ and Φ matrices to be identified; in the case of the SI model, this means that a minimum of 4 constraints must be imposed across the Λ and Φ matrices. The mathematical identification requirements for the intercept factor are met by setting the factor loadings to unity. Similarly, any of the three approaches discussed above (e.g., setting a factor loading to unity) can be used to achieve mathematical identification of the slope factor.

The metric of change. As stated previously, the selection of factor loadings on the slope factor reflect the assumptions about the nature of change over time in the data. Researchers can choose to fix the factor loadings to constant values, based on theory, predictions, or assumptions, or they can choose to allow the data to inform the metric of change by freely estimating some, or all, of the slope factor loadings.

Freely estimated latent growth curves. When researchers set slope factor loadings to pre-determined constants, as they do in linear or quadratic SI models (which will be reviewed below), they essentially force a form on the shape or rate of change; this can result in poor model fit statistics and improper or inaccurate parameter estimates if the form of the model doesn't accurately reflect the developmental trajectory (Wood & Jackson, 2008). To counter these potential problems, researchers can choose to allow the data to inform the metric of change by specifying SI models wherein some, or all, of the slope factor loadings are estimated. More specifically, to this end, Meredith and Tisak (1990) introduced the Free Curve Slope Intercept (FCSI) model. The advantages of estimating slope factor loadings are that: (1) the model allows for either a linear or non-linear rate of growth, as indicated by the actual data, (2) researchers do not necessarily need to make a priori assumptions about the nature of change in the data, which may prevent model misspecification, and (3) allowing the data to set the metric of change would yield more accurate parameter estimates and better model fit when growth is occurring at a non-linear rate.

FCSI models can take on several forms, based upon the number of estimated parameters and factor loadings. First, as exemplified by Meredith and Tisak (1990), the factor loadings associated with the first and second measurement occasions (or two measurement occasions in general) can be set to zero and one, respectively, and then the remaining factor loadings are freely estimated. The estimated factor loadings from such a model are interpreted as the amount of change between the first and the given measurement occasions, relative to the amount of change that occurred between the first and second measurement occasions (Bollen & Curran, 2006). So, if λ_1 and λ_2 are set to zero and one, respectively, an estimate of $\lambda_3 = 1.5$ would indicate that the amount of change observed between the first and third measurement occasions is 1.5 times that observed between the first and second measurement occasions. In this example, by allowing the data to inform the metric of change, Meredith and Tisak (1990) were able to accurately recover a negative quadratic growth model from simulated data.

The FCSI model that was just described would be the oblique form of the model. The correlation between the slope and intercept factors is estimated, in the oblique model, and identification is achieved by setting a factor loading (the reference point) to 0 and applying one additional constraint to the slope factor (e.g., variance to unity, another factor loading set to a constant; Wood, Under Review). The oblique model is arguably appropriate when the researcher wishes to know if there is inter-individual variation at a given time point, and it performance at that time point is associated with changes across other time points.

One potential disadvantage to the oblique FCSI model is that researchers have to select a reference point for the model in the oblique form. When a slope factor loading for a measurement occasion is set to 0, that measurement occasion becomes the reference point for the metric (i.e., the starting point for time). That is, time is scaled, and parameter estimates are interpreted, relative to this reference point (Hancock & Choi, 2006; Mehta

& West, 2000). It has been argued that the reference point should correspond to the aperture point (Hancock & Choi, 2006; Wood & Jackson, 2008), as selecting a non-aperture reference point can result in inaccurate parameter estimates (e.g., significant covariance between the intercept and slope factors, even when the two are independent).

Given that researchers can estimate as many slope factor loadings as is desirable, provided that the minimum conditions for identification are met, an alternative FCSI model can also be specified wherein *all* factor loadings are estimated. Identification requirements can be met in such a model by either setting the slope variance to 1 or constraining the sum of the squared factor loadings to 1. The orthogonal FCSI model is the alternative to the oblique model described above; the covariance between the slope and intercept factors is set to zero in this case, and all slope factor loadings are estimated. By specifying the model in this manner, the model is scaled relative to the aperture, if it were to be observed. That is, the researcher does not have to identify the appropriate reference point, but rather the model should recover the reference point during estimation. The estimated factor loadings derived from the orthogonal FCSI model provide information about the aperture point as well as information regarding at which time points change occurs. For example, an estimated factor loading of zero indicates an aperture, consistently increasing positive factor loadings indicate that the aperture occurred prior to those time points, and factor loadings that are the same over several time points suggest an asymptote in growth (Wood, Under Review).

Given the zero correlation between the slope and intercept factors, the orthogonal form of the FCSI model is appropriate when one is interested in determining how much of observed variability is due to growth and how much is due to individual difference variables. It is important to note that the oblique and orthogonal forms of the FCSI model, are essentially equivalent and will yield the same model fit. However, there will be differences in the estimates of the slope factor loadings, intercept factor mean and variance, and the covariance between the two factors (Wood, Under Review). This is the result of the different scaling approaches. That is, the two models are scaled relative to different time points, and thus the parameter estimates reflect that difference. Indeed, Beisanz et al. (2004) and Hancock and Choi (2006) note that fitting two models that differ by choice of reference point yields a predictable change in parameter estimates in the case of linear SI models.

In sum, it is arguable that researchers who choose to utilize the FCSI model are better able to recover the true form of growth in a dataset than other alternatives (to be reviewed below) because assumptions about the nature of change over time are not required, nor does the researcher necessarily need to have a priori knowledge of the aperture. Freely estimating the slope factor loadings allows the researcher to achieve a metric of time that most accurately characterizes the rate and nature of change in the data. More specifically, the appropriate reference point for time (i.e., the aperture point) should be correctly identified and the surrounding factor loadings should reflect the rate of change as it actually exists in the data. Finally, all of the alternative growth models that were mentioned (and will be reviewed) can be shown to be a special case of the FCSI model, with the exception of the quadratic SI model. As such, the FCSI model has the additional advantage of allowing one to fit alternative models and compare their relative fits via nested model comparisons in order to ensure the best modeling choice.

Linear slope-intercept model. The conventional approach to the SI model (i.e., the linear latent trajectory model) assumes that change is linear with respect to time and as such, slope factor loadings are set to the constant values that reflect a linear rate of change. For example, it is common for researchers employing a linear SI model to set the slope factor loadings to (t – 1), where t is the measurement occasion. The slope factor scores that result from this model are interpreted as the amount of change that occurs over a single unit change in the observed metric of time. The coding of the factor loadings reflects the time period of interest with respect to the observed metric of time, and is not limited to the (t – 1) metric; any transformation on the (t – 1) coding scheme that preserves linearity in the factor loadings is allowable (Hancock & Choi, 2006). It should be noted that these linear factors loadings imply the assumption that there is equal amounts of time between measurement occasions (Bollen & Curran, 2006).

It is not uncommon that the first measurement occasion be set as the reference point. However, it is not required that the first measurement occasion be the reference point. If one chooses a reference point other than the first measurement occasion, the surrounding factor loadings would be transformed to reflect linear change relative to that reference point. Thus, instead of setting the factor loadings to (t – 1), the factor loadings are set to (t – r), where r is the selected reference measurement occasion and t is the measurement occasion associated with the particular factor loading. For example, in a situation where there are 4 measurement occasions and the third measurement occasion is selected as the reference point, the factor loadings would be set to (t – 3) and the vector of slope factor loadings would be $\lambda_S' = [-2 \quad -1 \quad 0 \quad 1]$.

Biesanz et al. (2004) argued that the choice of reference point should largely determined by substantive interest. For example, they suggest that if the researcher is interested in relationships at the beginning of the growth process or at the end of the growth process, then s/he should set the first or last loading to zero, respectively. As mentioned previously, the choice of reference point not only has implications for substantive interpretation, but also for model fit and parameter estimates. Biesanz and colleagues (2004) and Hancock and Choi (2006) note that transforming the time metric such that the reference point of the model is different though the interval of time remains in the same form (e.g., linear) does not affect overall model fit, but does result in predictable changes in mean and variance estimates based on the time transformation.[4] Again, the choice of a non-aperture reference point can also result in inaccurate parameter estimates (Wood & Jackson, 2008). Thus, researchers utilizing the linear SI model are faced with the burden of identifying the most appropriate reference point to ensure accurate parameter estimates.

It is also important to note that the choice to code time as linear can be problematic in terms of model fit and parameter estimation if the data do not actually conform to the assumption of linear change. More specifically, fitting a linear growth model to data that shows a non-linear rate of growth will be detrimental to model fit. With respect to parameter estimation, fitting a linear model to non-linear data is likely to yield improper parameter estimates, such as correlations greater than 1, negative error variances, and/or inflated factor variance and covariance estimates (Wood & Jackson, 2008).

Polynomial latent trajectory models. When fitting a linear latent trajectory model, a researcher assumes that change occurs at a uniform linear rate over time. However, based on either theory or the nature of the data, it is not always reasonable or appropriate to assume a constant rate of change over time. One alternative strategy is to employ polynomial SI models which are appropriate when the rate of change increases or decreases at a consistent rate. Polynomial SI models are an extension of the previously described linear SI model; additional slope factors are added for each polynomial term in the model, and the factor loadings are set to the polynomial function of the original slope factor loadings. The quadratic SI model is most commonly used, wherein an intercept, linear slope, and quadratic factor are specified. Factor means, variances, and covariances are freely estimated, whereas the factor loadings for the linear factor are specified as discussed above, and the factor loadings on the quadratic factor are the squares of their respective linear values.

As a more specific example, a quadratic SI model could include an intercept factor with loadings set to unity, a slope factor with factor loadings set to $(t - 1)$, and a second slope factor with factor loadings set to $(t - 1)^2$. The two slope factors represent the linear rate of change and the change in the rate of change, respectively. Similarly, a cubic model would include another additional slope factor with loadings set to $(t - 1)^3$. Though polynomial growth models have the advantage of allowing for non-linear rates of change, the interpretation of the models becomes increasingly complex with the inclusion of each additional slope factor (Bollen & Curran, 2006), and they still make the assumption that change (and the rate of change in change) occurs in a consistent fashion.

Additional forms of the SI model and metrics of change. The current work is in no way a comprehensive review of all possible methods for modeling growth. To start, there are several additional forms of the SI model, with alternatively defined metrics for change. These models include modeling change as an exponential function (i.e., the rate of change is non-linear, proportional to earlier rates of change, and the rate of change steadily increases or decreases towards an asymptote) or modeling growth as a cycle (such as with a sine or cosine function). It is also possible to capture non-linear growth within a linear model framework by applying a non-linear transformation to the factor loadings defined in a linear trajectory model; this allows for the amount of change to differ from measurement occasion to measurement occasion at a rate consistent with the transformation (Bollen & Curran, 2006). Further, it is also possible to specify piecewise linear latent trajectory models, wherein change is believed to occur at a constant rate, but it is also believed that the rate of linear change may change at a particular point (i.e., that the pattern of change can be explained by two sequential linear processes). However, in the interest of space, and given the forms of growth that are thought to occur in cognitive aging data (to be reviewed later), we have chosen to limit our discussions of SI models to the FCSI, linear, and polynomial SI model

Alternative (Non-SI) Models for Change Over Time

The models that we have described to this point have all been forms of the slope-intercept model. That is, the previously discussed models characterize latent trajectories in terms of fixed and random intercept and slope components. There are several alternative approaches to modeling change over time that are not conceptualized as slope-intercept models, in this sense. However, as stated before, we are interested in using the flexible framework of the general factor model to allow for nested model comparisons. As such, though the remaining models may not be conceptualized as slope-intercept models, they are defined in terms of the general factor model, with multiple factors, to allow for such model comparisons.

Intercept-Only Model

The previously reviewed SI models, by the very inclusion of the slope factors, assume there is change in the variable of interest over time. However, one possibility that has not yet been considered is that some characteristics may not change over time. Indeed, it may be that inter-individual variability in trajectories is due entirely to intercept variability. Such a case would be seen when the trajectories appear constant over time, but differ in level. That is, each individual shows the same amount of the characteristic over measurement occasions, but there are still differences between individuals in the amount of the characteristic. The intercept only model is not only a special case of the FCSI model, but is also a special case of the linear SI model. More specifically, the intercept-only model can be specified by zeroing out the slope factor mean and variance, and thus essentially removing the slope factor from the model.

Single-Factor Growth Model

Whereas the intercept-only model allows for the possibility that there is not actually change over time, nor variation in change over time, the Single Factor Growth Model (SFGM) allows for the possibility that there is not variation in intercepts. Wood and Jackson (2008) and McArdle and Epstein (1987) suggested that it in some cases it is not necessary to model change as a function of both a slope and intercept factor. Rather, it is possible to use a single slope factor, without the intercept factor, to model growth.

The estimated growth factor mean represents the average rate of growth across the sample. Consistent with the slope factor loadings from the SI model, the factor loadings represent the metric of time. McArdle and Epstein (1987) introduced this as the CURVE model and chose to identify the model by setting the first factor loading to the value of the observed manifest mean at the first measurement occasion. Individual factor scores were interpreted as an index of the individual's degree of similarity/dissimilarity to the general group level curve. Specifically, higher factor scores indicate greater similarity with the group trajectory and lower factor scores indicate lesser similarity with the group trajectory (McArdle & Epstein, 1987). The authors note that "dissimilarity" includes both variability due to differences in intercept (i.e., elevation) and trajectory shape (since this is a single factor growth model). In other words, intercept variability is confounded with slope variability. Thus, the estimated growth curve can only be viewed as a baseline curve to represent the group as a whole. Wood and Jackson (2008) suggested a similar single factor growth model (SFGM), but chose to identify the model by setting the slope factor variance to unity and estimated all of the factor loadings. Given the lack of an intercept factor, it is arguable that the SFGM would be appropriate when it is expected that individual trajectories all start at the same point (i.e., there is no intercept variability). The SFGM model can be achieved by zeroing out the intercept factor mean and variance within the FCSI model. The matrix forms for the SFGM proposed by Wood and Jackson (2008) is shown in the second column of Table 1.

When a researcher chooses to exclude the distinct intercept factor in this fashion, they are not only assuming that all trajectories share a common point of origin; the exclusion of a distinct intercept factor further implies that measurement has occurred at the ratio level (McArdle & Epstein, 1987). This is not always a reasonable assumption; indeed measurement scales are often arbitrary, resulting in an arbitrary zero point (i.e., interval level of measurement) (McArdle & Epstein, 1987; Wood & Jackson, 2008). Thus, if measurement occurs on the interval level, the SFGM is not appropriate. However, Wood and Jackson (2008) suggest that an additive constant can be used in the model to create a meaningful zero point and essentially convert the interval- to ratio-level variables. This additive constant can be estimated via a latent variable (termed a *shift operator*) with factor loadings set to unity, factor variance set to zero and a freely estimated factor mean. The estimated factor mean is the additive constant by which the measurements are “shifted” to create the meaningful zero point. This latent variable is assumed to be uncorrelated with all other factors in the model. Thus, the SFGM model with a shift operator (SFGM-Shift) essentially makes the assumption that measurement actually occurs on an interval level, which is often more reasonable, but then adjusts the observations so that they actually conform to the necessary level of measurement for accurate model estimation.

Recall that the general SI model estimates variances on both intercept and slope factors, as well as covariance between the factors. These estimates imply the assumption that there is significant interindividual variability in both the starting points (i.e., intercepts) and rates of change (i.e., slopes) of the latent trajectories. The SFGM-Shift model is essentially an SI model, that makes the assumption that there is not interindividual variability in intercepts, but rather than there is a constant non-zero starting point across all trajectories. The SFGM-Shift model can be achieved by zeroing out the intercept factor variance within the FCSI model. The matrix forms for the SFGM-Shift model is shown in the third column of Table 1.

Multilevel Models (a.k.a. Hierarchical Linear Models)

It has been noted by several researchers (e.g., Mehta & West, 2000; Voelkle, 2007; Wood & Jackson, 2008) that multilevel models can be viewed as an alternative approach to, or special case of, the linear trajectory (SI) model discussed above. Indeed, when we reframe the multilevel model as a factor model, we can see that the only differences between the Linear SI model and the multilevel model are that: (1) MLM assumes that the error variances are equal at each measurement occasion, whereas the chronological SI model discussed above does not, and (2) the factor loadings associated with the slope factor are set to t, the integer value associated with the measurement occasion in the MLM, whereas the values are set to $(t - 1)$ in the chronological SI model. The matrix forms for the multilevel model are shown in the fourth column of Table 1.

Repeated-Measures MANOVA

Similarly, Meredith and Tisak (1990) and Voelke (2007) noted that repeated-measures MANOVA can also be expressed as a special case of a factor model. The repeated-measures MANOVA model differs from the FCSI model in the Φ, Λ, and Ψ^2 matrices. Under the MANOVA model, only the variance of the intercept factor is estimated, and the slope variance and the covariance between the slope and intercept factors is set to zero. The Λ matrix differs in that in the MANOVA model, the first factor loading is set to zero, and the remaining slope factor loadings are estimated. As such, the estimated slope factor loadings are the observed means of the measurement occasions minus the mean of the first measurement occasion. So essentially, we can frame

MANOVA, which looks for differences in manifest means, as a factor model by using latent variables to estimate manifest level means. Finally, the Ψ^2 matrix differs in that the error variances are assumed to be equal across measurement occasions. Thus, MANOVA does not seek to characterize interindividual differences in patterns (trajectories) of change but rather examines mean differences across measurement occasions and lumps these interindividual differences into error variance (Voelke, 2007). The matrix forms for repeated-measures MANOVA, expressed as a factor model, are shown in the fifth column of Table 1.

Model Comparisons

Though it is arguable that the choice of modeling procedure should be at least partially determined by theory and a priori prediction, it is not uncommon for researchers to base their model decision on convention or familiarity with a particular model. As noted by Wood and Jackson (2008), researchers are provided little information about the relative merits of the different approaches to modeling growth. Thus, it may be that researchers do not necessarily have the knowledge necessary to make informed modeling decisions. On the other hand, it is further arguable that researchers should seek to employ a modeling technique that is best suited to the data, even if that model is not consistent with a priori hypotheses. Indeed, the question of which model is most appropriate to the data is empirically testable. That is, as we have shown, the various models for growth that were discussed above can be specified in terms of the general factor model, and some models are nested within others; this allows for direct model comparisons via chi-square difference testing. Furthermore, even when the models are not nested, alternative measures of fit, such as the AIC, BIC, and RMSEA can be used to compare relative model fit. Wood and Jackson (2008) suggest using this approach to compare the available growth models and select the model that is most appropriate (and parsimonious) for the given data, rather than making blind assumptions about the nature of growth.

Nested Structure of Reviewed Models

When the FCSI, SFGM, SFGM-Shift, Intercept-Only, Linear SI, Quadratic SI, and MANOVA models are expressed in terms of the general factor model, we can see that some of the models are covariance-nested. To review, a model is covariance-nested within another model when one model is essentially a constrained version of the greater (more estimated parameters) model. With respect to the reviewed models, the SFGM, SFGM-Shift, Intercept-Only, Linear SI, and Repeated-Measures MANOVA models are all nested within the FCSI model. The SFGM model is further nested within the SFGM-Shift model. Additionally, the Intercept-Only model is also nested within the Linear SI model, which is nested within the Quadratic SI model. The nested structure is illustrated in Figure 2.

Model Comparison Procedures

The relative fit of covariance-nested models can be statistically compared using a χ^2 difference test. That is, the difference test can be used to determine if the more complex model yields a significant improvement in model fit over the more parsimonious model. When a model is nested within another model, the test statistic can be calculated as the difference between the individual model χ^2 statistics. The degrees of freedom for the χ^2 difference test are the difference between the two models' degrees of freedom.

Given the nested structure of the growth models discussed above, χ^2 difference tests can be used to determine if the FCSI model yields significantly better model fit than the Linear SI model, the SFGM and SFGM-Shift models, MANOVA, and Intercept-Only approaches. Further, a χ^2 difference test can be used to determine if the SFGM-Shift model yields significantly better model fit than the SFGM model. Finally, difference tests can be used to compare the Linear SI and Intercept-Only models to the Quadratic model, and the intercept-only model to the linear SI model.

Based upon Wood and Jackson's (2008) discussion of the different models for change and nested model comparisons, it could be argued that the results of these comparisons should inform the final model choice. That is, by making these formal model comparisons and allowing the results to inform the final choice of model, researchers may be more likely to choose the model that is most appropriate for the structure of his/her data, rather than basing the model on convention, and avoiding model misspecification. It is worth noting that this is not a novel idea. Indeed, McArdle and Epstein (1987), when proposing the CURVE model, constructed several other (nested) growth models with a variety of alternative constraints on the factor loadings and error variances for comparison to the CURVE model. These comparisons included comparing the CURVE model with estimated factor loadings to a model with linear factor loadings.

We have described several different nested models in our discussion. However, not all models are nested within all other discussed models. For example, the SFGM and repeated-measures MANOVA are not nested within one another. In such cases, researchers may turn to alternative fit indices such as TLI, AIC, and RMSEA to inform model choice. Finally, researchers may also consider the issues related to model complexity in model decisions. More specifically, the Task Force on Statistical Inference suggested that in the face of new and more complex analytic options, researchers should still opt for the simplest model that still adequately answers the research question (Wilkinson, 1999).

Conclusion

In summary, the FCSI, SFGM, SFGM-Shift, intercept-only, linear SI, quadratic SI, and repeated-measures MANOVA models can all be expressed in terms of the general factor model. When doing so, it becomes evident that some of models are nested within other models, allowing for direct tests of the differences in model fit. Each model makes distinct assumptions about the nature of growth over time and about variability in change over time. Researchers wishing to model change over time are faced with the burden of selecting the best growth model for their data. Selecting the wrong model can lead to poor model fit, and improper or inaccurate (e.g., inflated) parameter estimates. However, to date researchers have not been provided much information to guide them in choosing amongst the model options. To this end, the nested structure of these models, as illustrated above, and the aforementioned comparisons of model fit can assist researchers in selecting the model that fits best to their data. When models are not nested, researchers can consider alternative fit indices and questions of complexity in model decisions.

Cognitive Processes in Older Adulthood

To this point we have discussed growth modeling procedures, in a broad sense. For the current work, we must consider how longitudinal data modeling techniques apply to cognitive aging, specifically. Hertzog (2008) argued that cognitive aging research has focused far too much on *describing* the course of cognition over time, and should be more focused on *explaining* or identifying the factors that contribute to, or determine, the developmental trajectory of cognitive processes. Sliwinski and Mogle (2008), however, point out that before researchers can adequately explain how outside variables relate to changes in cognitive processes over time, we must be able to adequately characterize the actual patterns of change that occur over time. Further, the researchers made similar arguments to those discussed previously in regards to model choice and time metrics, with respect to cognitive aging specifically. More precisely, they note that inappropriate model or metric choices can lead to erroneous conclusions, and question if model choice should be data driven as a result.

Similarly, Widaman (2008) pointed out that choosing to model developmental trajectories, in general, as a linear function may be inappropriate, and argues that current advanced statistical techniques should be taken advantage of to model non-linear developmental processes. Likewise, Ram and Grimm (2007) also point out that the flexibility of the growth modeling framework allows for relative ease of non-linear model fitting. Indeed, it has been repeatedly suggested that the developmental trajectories of cognitive abilities are non-linear (e.g., Baltes, Staudinger, & Lindenberger, 1999; McArdle, Ferrer-Caja, Homogami, & Woodcock, 2002; Schaie, 1996).

Before continuing with our discussion of the form of change over time of cognitive abilities, we should first briefly review some basic ideas in the study of cognitive abilities. Though at one time intelligence may have been conceptualized as a singular construct (e.g., Spearman, 1904), it has long been accepted, and repeatedly empirically illustrated, that there are multiple types of intelligence that would be subsumed under any general intelligence factor. In spite of this, some researchers persist in constructing variables that represent a general intelligence factor. However, multiple researchers have noted that the use of a general intelligence construct oversimplifies cognitive processes and obscures important differences in developmental trajectories among different types of abilities (e.g., Baltes, Staundinger, & Lindenberger, 1999; Cattell & Horn, 1978; McArdle, Ferrer-Caja, Hamagami, & Woodcock, 2002; Schaie, 1994).

Some commonly examined types of cognitive processing abilities or intelligence include, but are not limited to: fluid abilities, crystallized abilities, memory, processing speed, and spatial rotation abilities. These different types of intelligence may be differentiated by the type of abilities that are illustrated (as indexed by specific tasks), or by the developmental source of the abilities. Broadly, crystallized intelligence can be seen as acculturated knowledge, or knowledge that is acquired in a cultural context, and reflects important cultural constructs. Further, crystallized abilities can be acquired through repeated applications of fluid abilities that lead to a solidifying of the ability or process (Beauducel & Kersting, 2002; Cattell, 1963; Horn, 1968). Crystallized abilities are often acquired in formal learning environments or through structured learning opportunities. Fluid intelligence, on the other hand, is knowledge and abilities that are acquired outside of the culture, through personal experiences. Fluid abilities also include abstract and verbal reasoning abilities, and processing capacity (Beauducel & Kersting, 2002; Horn,Year). Thus, fluid abilities can be illustrated in tasks requiring adaptation to new situations (Cattell, 1963).

To bring our discussion back to the form of change over time, it has been well-documented that different forms of intelligence, or cognitive functioning, have different developmental trajectories. For example, McArdle, Ferrer-Caja, Hamagami, and Woodcock (2002) showed that fluid intelligence peaked earlier, had a slower initial growth rate, and a faster decline rate than crystallized intelligence. Indeed, multiple researchers have suggested that fluid intelligence peaks at an earlier age, and shows a much faster rate of decline, than crystallized intelligence, and that crystallized intelligence actually remains relatively stable through adulthood, showing some decline much later in life (e.g., Alwin & Hofer, 2008; Cattell, 1963; McArdle, Ferrer-Caja, Hamagami, and Woodcock, 2002; Schaie, 1996; Wang & Kaufman, 1993). The established variation in forms and rates of growth among different cognitive abilities supports the need for flexible modeling frameworks that allow the data to inform the metric of change. That is, as stated previously, specifying multiple models within the nested factor model framework allows researchers to test the possibility that models other than their selected model may be more appropriate to the data. This is especially appropriate when there are several variables of interest and it either cannot, or should not, be assumed that all variables have the same form of growth (as is the case with cognitive aging).

The Current Study

The idea that multiple models could be fit to in order to find the best fitting model is not new. Indeed, many researchers have utilized multiple models to such ends. McArdle, Ferrer-Caja, Hamagami, and Woodcock (2002) fit several different models before settling on a model that allowed for estimation of non-linear trajectories. However, they did not utilize the nested model structure for direct model comparisons as we have suggested, nor did they use the exact set of growth models that we have reviewed. To serve the purposes outlined in the previous sections, and to illustrate our points, we have acquired longitudinal cognitive process data utilized by previous researchers. More specifically, we acquired the Swedish Twin data used by Reynolds, Finkel, McArdle, Gatz, Berg, and Pedersen (2005) that was referred to at the beginning of this document. Recall, Reynolds et al. sought to decompose environmental and genetic components of change over time in cognitive abilities. What is relevant to the present work is that prior to fitting the behavior genetic model, Reynolds et al. (2005) had to determine the appropriate form for the phenotypic growth model. Ultimately, Reynolds et al. (2005) decomposed observed phenotypic variability into intercept, linear and quadratic growth components via a quadratic SI model for multiple cognitive abilities.

Given the criticisms of quadratic SI models outlined above, it is possible that a different model may be a more accurate reflection of the developmental trajectory suggested by the data. Reynolds et al. (2005) noted that other models may be better for the data and that the model may be unstable due to sparse data for later measurement occasions. Thus, while the primary purpose of the current work is to illustrate how to use the nested structure of several growth models to allow the data to inform the choice of model with developmental data, and ultimately recover the most accurate characterization of the data, it will also serve the ancillary purpose of reevaluating the appropriateness of Reynolds et al. (2005)'s selected phenotypic growth models and their conclusions based upon the use of this model selection procedure.

To be more precise regarding the work that is presented in the subsequent paragraphs, Reynolds et al (2005) used data from the Swedish Adoption/Twin Study of Aging (SATSA; Lichtenstein, deFaire, Floderus, Svedberg, Svedberg & Pedersen, 2002; Pedersen et al., 1991, 2002). The data set included information from a variety of measures of crystallized and fluid abilities, as well as memory, and perceptual speed abilities; there were multiple measures used to assess each of the four types of abilities, totaling 11 measures. The researchers decided upon quadratic SI models for all measures. It should be noted that, in alignment with our argument of allowing the data to inform the choice of model, Reynolds et al. (2005) did fit intercept only, linear SI, and quadratic SI models, and used nested model comparisons to settle upon the quadratic model.

Given that our primary purposes are not substantive, but methodological, we have chosen to limit the number of variables that we examined in order to keep the scope of the substantive work manageable. Thus, we chose to limit our work to examining the developmental trajectories of crystallized (3 measures) and fluid abilities (3 measures). To illustrate the application of these models, each of the growth models described above (i.e., FCSI, SFGM, SFGM-Shift, Intercept-Only, Linear SI, Quadratic SI, Repeated Measures MANOVA) were specified in terms of the general factor model, and fit to the data for each of the 6 measures. Nested model comparisons, as well as alternative fit indices and issues of complexity/parsimony, were then used to inform the choice of the most appropriate model for each measure.

Method

The dataset was acquired, with permission for use, from Dr. Chandra Reynolds at the University of California-Riverside. The dataset included information collected in the Swedish Adoption/Twin Study of Aging (SATSA) (Lichtenstein, deFaire, Floderus, Svedberg, Svedberg & Pedersen, 2002; Pedersen et al., 1991, 2002), and was used by Reynolds et al. (2005).

Sample

The sample was made up of nondemented twins from twin pairs reared together and twin pairs reared apart (defined as being separated before 11 years of age). Information from both complete and incomplete twin pairs was included. Participants were required to have provided information at least one time after having reached 50 years of age in order to be included in the sample. A total of 797 individuals were included in the final dataset. Age at first measurement occasion ranged from approximately 39 to 88 years of age (M = 61.11 , SD = 10.41). Three-hundred thirty (41.41%) participants were male, whereas 467 (58.59%) participants were female.

Measures

The dataset included information reflective of crystallized, fluid, and memory abilities, and perceptual speed. However, as stated previously, we chose to limit our work to an examination of crystallized and fluid abilities. Individual test scores were recorded as the percent of the possible points earned on each of the cognitive measures.

Crystallized abilities. Scores on the Swedish WAIS Information (CVB [Central Värnpliktsbyrån] scales; Jonsson & Molander, 1964), Synonyms (Dureman–Sälde Battery; Dureman, Kebbon, & Osterberg, 1971), and Analogies (Westrin Intelligence Test-III; Westrin, 1969) tests were used to quantify crystallized abilities.

Fluid abilities. Fluid abilities were indexed by scores on the Figure Logic (Dureman–Sälde Battery; Dureman et al., 1971), Koh's Block Design (Arthur, 1947), and Card Rotations (Educational Testing Service; Ekstrom, French, & Harman, 1976) tests.

Additional measures. Scores on the Digit Span (Jonsson & Molander, 1964), Thurstone's Picture Memory (Thurstone, 1938), and Names & Faces (Colorado Adoption

Project; DeFries, Plomin, Vandenberg & Kuse, 1981) tests were used to index memory abilities. Perceptual speed was quantified by scores on the Symbol Digit (Smith, 1982) and Figure Identification (Dureman–Sälde Battery; Dureman et al., 1971) tests.

Data Collection

Data was collected on four measurement occasions. The second and third measurement occasions occurred at 3 year intervals and the final measurement occasion occurred 7 years following the third measurement occasion. Thus, measurement occurred over a 13-year span. At each measurement occasion, participants engaged in a 4-hour in-person testing session during which they completed the previously identified measures of cognitive functioning. Participants were not required to complete all four testing sessions to be included in the sample. Approximately 38% (n = 299) of the sample completed four testing sessions, whereas 24% (n = 194), 16% (n = 124), and 23% (n = 180) of the sample completed 3, 2, and 1 of the testing sessions, respectively.

Analyses

Data Organization

Previous research has indicated that the rate of cognitive change is different at different ages. Thus, given the heterogeneity of age at first measurement in the sample, age must be included in the fitted models. Further, the measurement occasions during which data was collected occurred at unequal intervals. To address these two limitations of the data, we chose to restructure the data into equal intervals based upon age for each cognitive measure. We first established manifest indicators at each age point starting from age 38 years of age, and assigned the individual scores to the manifest variable that corresponded to the age when the score was obtained. Thus, there were manifest variables for ages 38 through 99, and each participant had values for up to 4 of those indicators for each cognitive measure. For example, if a participant was 50 years of age at measurement occasion 1, then his/her score for the particular cognitive measure would be assigned to the 50-years manifest variable. However, when attempting to fit the defined models to the data, the data were too sparse to allow for convergence using 1 year intervals. Thus, two year interval manifest variables were established instead, starting at age 38. To illustrate, if a participant completed cognitive measures at 39 years of age, his/her score would be assigned to the first 2-year time interval indicator (ages 38 and 39). The benefit of this structure is that the indicators were spaced at equal intervals and also account for the different ages of the participants in the sample.

Statistical Models

Statistical models were fit using MPlus version 5.2 (Muthen & Muthen, 1998-2006). Once the data were reorganized, FCSI, intercept only, SFGM, SFGM-Shift, linear SI, quadratic SI, and repeated-measures MANOVA models were fit to all three crystallized measures (i.e., information, synonyms, analogies) and all three fluid measures (i.e., figure logic, block design, card rotations). Though we reviewed the MLM in the previous sections, it was decided that it was unnecessary to fit the MLM model in addition to the linear SI model, as the MLM is a linear transformation of the linear SI, with the additional constraint of equal error variances.

Results

Though there were a total of 797 participants in the full dataset, each set of analyses (i.e., set of models for each variable) was based only on participants who had data for 3 or 4 measurement occasions. Further, though ages at first measurement occasion ranged from 39 to 88 years of age, and age at final measurement occasion ranged from 52 to 100 years of age, the age range included in the models was limited due to sparsity, and differed for each measure. That is, data were too sparse at certain time points (e.g., 38 through 43 years of age) to allow models to converge. Fit statistics for models of crystallized abilities are displayed in Table 2. Fit statistics for models of fluid abilities are displayed in Table 7.

Nested model comparisons were used to determine if: (1) the intercept only, SFGM, SFGM-Shift, linear SI, quadratic SI, and repeated-measures MANOVA models yielded significantly better fit than the FCSI model, (2) the quadratic SI fit significantly better than the linear and intercept only models, and (3) if the SFGM-Shift model fit better than the SFGM model. Results of the chi-square difference tests are displayed in Table 3 for crystallized abilities and Table 8 for fluid abilities. Unstandardized and standardized parameter estimates for the selected models for crystallized variables are shown in Tables 4 and 5, respectively. Unstandardized and standardized parameter estimates for the selected models for fluid variables are shown in Tables 9 and 10, respectively. Sample and model estimated means over time for crystallized abilities are shown in figure 3, and shown in figure 7 for fluid abilities. Observed and model estimated individual curves are shown in figures 4 and 5 for crystallized abilities, and figures 8 and 9 for fluid abilities. Plots of factor scores from the final models for crystallized abilities are shown in figure 6 and in figure 10 for fluid abilities.

Once the final model was selected, proportion of variance estimates were calculated to determine how much inter-individual variation at each time point was due to the change process as compared to more trait-like factors. More specifically, the proportion of variability at each time point that was accounted for, and not accounted for, by the selected model was estimated. Further, the proportion of variance accounted for by the model was also decomposed into the proportion of the change process variability due to each of the factors (e.g., intercept, slope), when more than one factor was estimated. The proportion of variance estimates are presented in tables 6 and 11 for crystallized and fluid abilities, respectively, and are all also displayed in figures 11 through 15.

Crystallized Abilities

Information. Once the dataset was limited to individuals with 3 or more measurement occasions, the sample size for the information variable was 487 participants. However, influence diagnostics indicated that 9 of the 487 participants were unduly influential.[5] Thus, the 9 outliers were excluded from the final analyses, yielding a final sample size of 478 participants. Performance on the Swedish WAIS Information scale was modeled from 45 to 87 years of age. The FCSI, Intercept Only, Linear SI, Repeated-Measures MANOVA, and SFGM models converged, whereas the Quadratic SI and SFGM models did not converge. Chi-square difference tests comparing the relative fit of the intercept only, linear SI, repeated measures MANOVA and SFGM model to the FCSI model indicated that all converged models showed significantly worse model fit than the FCSI model for the information measure. It should be noted that in order to achieve convergence for the FCSI model, the residuals had to be set to equality. As such, the remaining models were also fit with equal residuals to allow for nested model comparisons. The FCSI model was selected as the final model for the information variable. Alternative fit indices also supported this decision.

Results of the FCSI model indicate that the average intercept for the information trajectories was 76.33 with significant variance of 208.95. In other words, the average score at the aperature point (if it were to be observed) would be 76.33%. The average slope of the information trajectories was .64. Inspection of the factor loadings, sample means, model estimated means, and estimated individual curves suggest that the information trajectories are non-linear. More specifically, performance on the Swedish WAIS information task appears to increase somewhat during the earlier modeled years (e.g., 45 years of age), then become relatively stable for several years around the age of 55, and then starts to decline somewhere around 75 years of age. Note that this is a simplified description of the trajectories. Indeed, the trajectories would not have adequately been captured by a quadratic function. Further, as indicated by the significant variance on the intercept factor, there was notable variation among the starting points of the individual trajectories. Examination of the model estimated individual curves also suggest notable variation in the slopes among individual trajectories.

The proportion of interindividual variation in information scores at each time point accounted for by the FCSI model was high and relatively stable over time. The proportion of variance estimates were between 85% and 93%, indicating that the variation was largely change-related. Given that the proportion of variation not accounted for by the model ranged only from 7 to 15%, this indicates that variation was not so much trait-like as it was change-like.

Synonyms. In order to achieve convergence for the FCSI model, and thus allow for multiple nested comparisons, the sample had to be limited to individuals with 4 measurement occasions for the synonyms variable, and all residuals were set to equality.

The final sample size for the synonyms analyses, once individuals with less than 4 measurement occasions were excluded, was 268 participants. Performance on the synonyms task was modeled from 47 to 85 years of age. All models converged, with the exception of the SFGM-Shift model. Chi-square difference tests indicated that the intercept-only, SFGM, linear SI, and MANOVA models yielded significantly worse model fit than the FCSI model. Further, chi-square difference testing indicated that the difference in model fit between the quadratic and linear SI models was not significant. Results of the nested comparisons indicated that the FCSI model was the most appropriate model for the synonyms variable. Alternative fit indices also supported this decision.

Results of the FCSI model indicate that the average intercept for the synonyms trajectories was 66.52 with significant variance of 281.99. In other words, the average score at the aperture point (if it were to be observed) would be 66.52%. The average slope of the synonyms trajectories was .54. Inspection of the factor loadings, sample means, model estimated means, and estimated individual curves suggest that the information trajectories are non-linear. Performance on the synonyms task appears to be a somewhat more complicated curve. More specifically, performance appears to be relatively unstable (increasing and decreasing) prior to around the age of 59, at which time performance becomes relatively stable until around 69 years of age, at which time performance starts to decline. Further, as indicated by the significant variance on the intercept factor, there was notable variation among the starting points of the individual trajectories. Examination of the model estimated individual curves, however, did not clearly show notable variation in the slopes among individual trajectories, suggesting that the rate of change may be relatively consistent across individuals. These results are especially useful for illustrating the utility of the FCSI model, as such a trajectory could not be clearly characterized with a linear or quadratic function, but would need several different components in order to capture the large amount of fluctuation in the scores over time.

The proportion of interindividual variation in synonyms scores at each time point accounted for by the FCSI model was high and relatively stable over time. The proportion of variance estimates were between 87% and 91%, indicating that the variation was largely change associated. Given that the proportion of variation not accounted for by the model ranged only from 9 to 13%, this indicates that variation was not so much trait-like as it was change-like.

Analogies. The final sample size for the analogies analyses, after excluding individuals with less than 3 measurement occasions, was 435 participants. Performance on the analogies task was modeled from 47 to 85 years of age. All models converged, with the exception of the quadratic SI model. Chi-square difference tests indicated that the intercept only, repeated-measures MANOVA, linear SI, SFGM, and SFGM-Shift models yielded significantly worse model fit than the FCSI model. Thus, the appropriate model choice for the analogies variable was clearly the FCSI model. Alternative fit indices also supported this decision (see Table 2).

Results of the FCSI model indicate that the average intercept for the analogies trajectories was 51.94, with significant variance 105.03. That is, the average starting point of the trajectories across all individuals is 51.94% of the possible points on the analogies task. In other words, the average score at the aperture point (if it were to be observed) would be 51.94%. The average slope of the trajectories was .94 Inspection of the factor loadings, sample means, model estimated means, and estimated individual curves suggest that the analogies trajectories are non-linear. More specifically, it appears that performance on the analogies tasks is relatively stable over several years and then starts to decline somewhere around 69 or 70 years of age. Further, as indicated by the significant variance on the intercept factor, there was notable variation among the starting points of the individual trajectories. Examination of the model estimated individual curves also suggest notable variation in the slopes among individual trajectories.

The proportion of interindividual variation in Analogies scores at each time point accounted for by the FCSI model was rather unstable, almost cyclically increasing and decreasing over time. However, even with this instability, over half of the variability was accounted for by the growth model at all times. However, the proportion of variability not accounted for by the model ranged from .12 to .47; this indicates that at some time points there are likely other important trait-like characteristics that account for variation in observed Analogies scores.

Fluid Abilities

Figure Logic. The final sample size for the figure logic variable, once individuals with less than 3 measurement occasions were excluded, was 460 participants. Performance on the figure logic task was modeled from 45 to 85 years of age. All models converged. Chi-square difference tests indicated that the intercept only, repeated-measures MANOVA, SFGM, and SFGM-Shift models yielded significantly worse model fit than the FCSI model. However, the difference in model fit between the FCSI and linear SI models was not significant, based on the chi-square difference test. Thus, there was not enough evidence to conclude that the FCSI model fit significantly better than the linear SI model for the figure logic measure. Further, a chi-square difference test comparing the relative fit of the linear and quadratic SI models yielded a non-significant result, indicating that there is not enough evidence to conclude that the quadratic model fits significantly better to the data than the linear SI model. Given these results, it is arguable that the linear SI model is the most appropriate model choice for the figure logic data, as it showed comparable model fit to the FCSI model, yet is more parsimonious. Further, alternative fit indices supported the choice of the linear SI model for the figure logic data (see Table 7).

Results of the linear SI model indicate that the average intercept for the figure logic trajectories was 61.52, with significant variance 79.61. That is, the average starting point for the trajectories across all individuals was 61.52% of the possible points on the task. Given that the loadings were centered, the starting point for figure logic was around 65 years of age. The average slope of the trajectories was -.36, with non-significant variance .04. That is, the average rate of change across all individuals was a decrease of .36% in points over every 2 year period. As indicated by the significant variance on the intercept factor, there was notable variation in the starting points of the individual trajectories; however, the non-significant variance on the slope factor suggests that the rate of change is likely common across individuals. Further, the estimate of the covariance between the intercept and slope factor was non-significant.

The proportion of interindividual variation in figure logic scores at each time point accounted for by the linear SI model started out high, with a maximum of .86, and decreased relatively consistently over time, with a minimum of .29. Given that the proportion of variance not accounted for by the Linear SI model increased dramatically, this suggests that interindividual variation becomes more trait-like, and less change-associate, over time. Thus, there are likely several important individual difference variables that account for a significant proportion of variation in figure logic abilities later in life that were not examined in the current work.

Block Design. The total sample size for the block design variable after excluding participants with less than 3 measurement occasions was 462 participants. However, influential diagnostics indicated that there were 6 overly influential observations; the outliers were excluded from the final analyses, resulting in a final sample size of 456 participants.[5] Performance on Koh's Block Design task was modeled from 47 to 85 years of age. All models converged, with the exception of the quadratic SI model. Chi-square difference tests showed that the intercept only and repeated-measures MANOVA models yielded significantly worse model fit than the FCSI model. However, the differences in model fit between the FCSI model and the SFGM, SFGM-Shift, and Linear SI models were not significant. As such, there was not enough evidence to conclude that the FCSI model fit significantly better to the data than the SFGM, SFGM-Shift, or linear SI model. Further, a chi-square difference test indicated that the SFGM-Shift model did not yield a significant improvement in model fit relative to the SFGM. Thus, the candidate models for block design were the FCSI, Linear SI, and SFGM. Given that alternative fit indices were generally comparable across the 3 models, it is arguable that the SFGM is the most appropriate model choice for the block design data, as it is the most parsimonious of the three candidate models. Indeed, inspection of means and estimated curves suggest some non-linearity at the earlier ages, which supports the choice of SFGM over the linear SI model. However, alternative fit statistics support the linear SI model (see Table 7).

Recall that the SFGM model characterizes growth as a function of a slope factor only, and no intercept. The fact that the FCSI model did not yield significantly better model fit than the SFGM suggests that there was not significant variability in the intercepts of the trajectories, or that there was a common starting point for the trajectories across individuals. The results of the SFGM model indicate that the average slope of the block design trajectories was 3.45. Inspection of the factor loadings, sample means, model estimated means, and estimated individual curves suggest that the performance on the block design task declined over the entire age range that was modeled. However, there was some non-linearity to the rate of decline during the earlier modeled ages (approx. 47 to 53 years of age), after which the rate of decline appeared to become more linear. In spite of the common intercept among trajectories, there was notable variation in slopes across the trajectories that was evident in the model estimated individual curves.

The proportion of interindividual variation in block design scores at each time point accounted for by the SFGM model started out high (.94) and decreased relatively consistently over time, with a minimum of .54. Given that the proportion of variance not accounted for by the SFGM model increased dramatically, this suggests that interindividual variation becomes more trait-like, and less change-associate, over time. Thus, there are likely several important individual difference variables that account for a significant proportion of variation in block design scores later in life that were not examined in the current work.

Card Rotations. The final sample size for the card rotations data after exclusion of participants with less than 3 measurement occasions was 422 participants. Performance on the card rotation task was modeled from 47 to 85 years of age. All models converged. However, to achieve convergence for the FCSI model, the residuals for the last two indicators had to be set to equality. As such, the remaining models were also fit with these error terms set to equality to allow for nested model comparisons. Additionally, in order to achieve convergence for the SFGM-Shift and the Quadratic SI models, all residuals needed to be set to equality. Chi-square difference tests indicated that the intercept only, repeated-measures MANOVA, SFGM, and SFGM-Shift models yielded significantly worse model fit than the FCSI model. However, the difference in model fit between the FCSI and linear SI models was not significant, based on the chi-square difference test. Thus, there was not enough evidence to conclude that the FCSI model fit significantly better than the linear SI model for the card rotations measure. In order to conduct a chi-square difference test comparing the linear SI and quadratic SI models, the linear SI model was fit an additional time with all residuals set to equality. The results of said difference test were significant, indicating that when all residuals were set to equality, the inclusion of the quadratic term yielded significantly better model than stopping at the linear term only. However, we cannot determine if the quadratic model would yield significantly better fit without the equal residuals, as the model would not converge without the equality constraints.

Given these results, the *best* model choice is not necessarily clear. The candidate models include the FCSI, Linear SI, and Quadratic SI models. It is arguable that the linear SI model is the most appropriate model choice for the card rotations data, as it failed to show significantly different model fit from the FCSI model, yet is more parsimonious, and it did not require the extra equality constraints needed for the quadratic model. However, examination of the alternative fit statistics (e.g., CFI, AIC) show that all other fit statistics indicate the superiority of the Quadratic model relative to the FCSI or Linear SI models. Thus, the quadratic model was chosen as the final model for card rotations.

Results of the quadratic SI model indicate that the average intercept for the card rotations trajectories was 46.73, with significant variance 206.36. That is, the average starting point for the trajectories across all individuals was 46.73% of the possible points on the task. The starting point in this case, as it is a quadratic SI model, is the time point that was assigned a zero loading. Given that the loadings were centered, the starting point for card rotations was around 67 years of age. The average slope of the trajectories was -.87, with significant variance .24. That is, the average rate of change across all individuals was a decrease of .87% in points over every 2 year period. The average of the quadratic factor was -.02, with non-significant variance of .001. That is, the average rate of change in the slope across all individuals was a decrease of .02% over every 2 years. As indicated by the significant variance on both the intercept and slope factors, there was notable variation in the starting points and the rates of change of the individual trajectories. However, the non-significant variance on the quadratic factor suggests that the quadratic component may be constant across individuals. Finally, a significant correlation of -.48 was found between the intercept and quadratic terms, yet significant correlations were not found between the intercept and the slope, nor the slop and the quadratic factors. This indicates that the higher the individuals starting point was, the slower they declined. Substantively, this suggests that higher functioning individuals may be less likely to decline or have characteristics that buffer them from quicker rates of decline.

The proportion of inter-individual variation in card rotation scores at each time point accounted for by the Quadratic SI model was high and consistent over time, with a minimum of .73 and maximum of .8. Thus, the proportion of variance not accounted for by the quadratic model ranged from .2 to .27. This suggests that individual differences in observed scores are more change-process associated than trait-like. However, given that there was still 20 to 27% of variability not accounted for by the growth model, it also suggests that there is likely some other important individual difference that accounts for inter-individual variation.

Conclusion

In sum, by specifying the aforementioned models in terms of the general factor model, we were able to make model comparisons and attempt to determine the most appropriate model for each of the cognitive ability measures. Results indicated that the FCSI model was most appropriate to characterize cognitive aging with respect to crystallized abilities. Examination of observed and estimated means and curves clearly suggested non-linear trajectories. However, the FCSI model was not the model of choice for fluid abilities. Rather, a SFGM was selected for Koh's Block Design Task, a linear SI model was implicated for figure logic data, and a quadratic SI model was selected for card rotation abilities. Proportion of variance estimates indicate that the growth process accounted for a substantial proportion of variability at each time point. The proportion of variance accounted for by the growth model remained consistent across time for 2 of the

3 crystallized abilities, and 1 of the fluid, but decreased over time for 2 of the 3 fluid abilities.

Discussion

As noted earlier, researchers may, and often do, make erroneous assumptions about the shape or nature of change over time and, as a result, may inadvertently use inappropriate modeling techniques for characterizing developmental trajectories. This can lead to inaccurate conclusions about the nature of change over time or individual differences in such change, due to model misspecification and inaccurate parameter estimates. There are a plethora of possible modeling techniques that researchers can choose to use for longitudinal data. The models differ in complexity and assumptions, and researchers may not always know, a priori, which particular model best characterizes the data. As such, model comparisons can be used to inform the choice of modeling technique. We have defined several models for longitudinal data in terms of the general factor model, showing how some models are special cases of other models, and as such are nested. The current work illustrated how we can use chi-squared difference tests (when models are nested) and additional fit indices (e.g., AIC, BIC, RMSEA) to evaluate relative fit and select the best approach for the data.

Substantively, the purpose of the current work was to illustrate how this framework can be used to select the best model for developmental trajectories, specifically. To illustrate these points, we used an example dataset containing cognitive process information from older adult twins that had been previously analyzed by other researchers. Thus, there was an ancillary purpose of reevaluating the conclusions of those researchers with respect to the shape of the developmental trajectory of cognitive processes (i.e., crystallized and fluid abilities) over time in an older adult sample.

Though it was relatively easy to select the best model based upon the results of chi-square difference tests for some of the variables, it was not always the case that one particular model was clearly the *best* model choice based on chi-square difference tests. Further, in the introduction, it was argued that the FCSI model had some conceptual advantages over other models, because it requires the fewest assumptions about the shape or rate of change over time and structure of measurement errors over time. The FCSI model was clearly indicated as the best choice for all three of the crystallized abilities measures (i.e., Swedish WAIS Information, Analogies, and Synonyms); chi-square difference tests showed significant improvement in model fit over the other modeling options. The FCSI model was not the clear choice for fluid abilities, however. Rather, the linear SI model was chosen for one fluid ability (i.e., figure logic), the quadratic SI model was chosen for one fluid ability (i.e., card rotations), and the SFGM was chosen for the remaining fluid variable (i.e., Koh's Block Design). These conclusions are reasonable given the observed patterns of means for the fluid variables (shown in figure 7).

That clear evidence for the superiority of one model over all others did not always emerge also draws one to consider the role of theory in model selection. Indeed, it is not only important to allow the data to inform the choice of model, but also to have a theoretical basis for one's choice of model. That is, when the data do not clearly indicate that one particular model is best, but rather suggests that there may be multiple statistically appropriate models, theory can also aid the researcher in deciding among candidate models. Indeed, it is not our intention to minimize the importance of allowing theory to inform model choice. Good science should be based solidly in theory. That is, prior to collecting and analyzing data, researchers should develop a clear theoretical basis for their study, and have precise predictions that they wish to examine. As noted by Ram and Grimm (2007),

> "statistical models (including growth curve models) provide us with the opportunity to articulate and test our hypotheses against empirical data. At the same time, however, they offer only approximate renderings of our ideas about how and why individuals develop and change over time. Because of these constraints, special care must be taken to select and apply models that map, as directly as possible, onto the particular theory we are attempting to articulate and test." (p.311)

Thus, one should also consider the theoretical meanings of the models that s/he is considering, asking such questions as whether or not a candidate model makes sense given prevailing theory.

Substantively, the results suggest that some cognitive processes may change at a linear rate, whereas others show clear evidence of change at a non-linear rate that would not adequately be captured by a quadratic function. Broadly speaking, this supports prior arguments that different developmental processes and, more specifically, different cognitive processes/abilities change at different rates (e.g. Singer, Verhaeghen, Ghisletta, Lindenberger, & Baltes, 2003). Generally, crystallized abilities are thought to increase into adulthood, then level out for much of adulthood, then decrease very late in life. On the other hand, fluid abilities are generally described as steadily decreasing (i.e., at a linear rate) throughout adulthood (e.g., Baltes, Staundinger, Lindenberger, 1999). Our results are consistent with the general idea of how fluid and crystallized abilities change over the lifestyle. Indeed, our results showed that fluid abilities consistently decreased over the age ranges that we modeled; linear SI models were indicated for one of the three fluid measures (figure logic), a quadratic SI model was indicated for one measure (card rotations), and the SFGM model that was indicated for the third variable (block design) still showed a generally consistent decrease over adulthood. Similarly, the curves that we recovered with the FCSI model for crystallized abilities showed a more consistent scoring over adulthood than fluid abilities, with a drop off later in life. Returning to the issue of parsimony and theory informed decisions, one might question if a linear SI model was the appropriate choice for the block design variable, given that it fits with prevailing theory. Indeed, fit statistics were supportive of both the SFGM and linear SI models, and we chose the SFGM model due to parsimony.

Comparison with Reynolds et al.'s (2005) results.

With respect to the ancillary purpose of reevaluating Reynolds et al.'s (2005) conclusions regarding the form of growth, we failed to support the researchers' conclusions for most cases. Reynolds et al. (2005) concluded that the form of phenotypic growth for these data was quadratic. Their nested model comparisons indicated that the quadratic model fit significantly better than the linear SI model. Our results were not consistent with Reynolds et al.'s (2005) results. Indeed, our results failed to clearly indicate that the quadratic model was the most appropriate model for any but 1 of the 6 variables that we examined. Reynolds et al. (2005) pointed out that there were several alternative forms of growth that they did not attempt to model which may be more appropriate than the quadratic model. The FCSI and SFGM models are two such models. Thus, our conclusions that the FCSI model was most appropriate for the crystallized variables, and that the SFGM or linear SI model is most appropriate for the Koh's Block Design data are consistent with this sentiment. Further, our results indicated that for the figure logic variable the quadratic SI model did not yield significantly better model fit than did the linear SI model, contradicting Reynolds et al.'s (2005) results.

Limitations

Widaman (2008) noted that developmental trajectories are most likely non-linear, yet our results indicated that the linear SI model may be appropriate for some of the measures that we examined. There are multiple possible explanations for this. First, it may be that some abilities actually do decline linearly. Alternatively, the linear results may be an artifact of the properties of the dataset itself. That is, it is possible that the linear trajectories emerged as a result of sparsity. More specifically, as stated by Widaman (2008),

> "Many state-of-the-art modeling techniques require strong assumptions, including linearity of relations between variables and multivariate normality, assumptions that are unlikely to be met by many or any measures include in a particular study. In effect, modeling moves by leaps and bounds, rapidly extending beyond the quality of the measurements to which the models are fit." (p.55)

Further, it is possible that we failed to detect improvement in model fit when using the FCSI model, relative to other models, for the fluid abilities because the model contrasts were underpowered due to sparsity.

Note that we were not always able to achieve convergence in our initial modeling attempts, and in some cases we needed to set some or all residuals to equality in order to achieve convergence. Though this may be statistically acceptable, doing so has implications for model and parameter interpretations. That is, while setting residuals to equality may allow us to achieve convergence or improve model fit, it raises the issue of what such actions mean substantively. Related to previous arguments regarding the importance of allowing theory to inform model choice (e.g., Ram & Grimm, 2007), one must consider whether there is a theoretical basis for setting residuals to equality. One question that arises is: is it more important to improve model fit, or fit a more advanced model by setting residuals to equality, or is it more important for the model to be easily interpretable and consistent with theory?

Another consideration is the issue of experimentwise error. In the current work, and in work done by others such as Reynolds et al. (2005), multiple models were fit and then compared to determine which model was most appropriate to the data. Given that each model was fit separately, and experimentwise error was not controlled, it is difficult to know the probability of having made an incorrect decision among all of the models that were fit. However, critics of null hypothesis significance testing (NHST) might argue that this is not a limitation or truly a concern, as NHST has come increasing into disfavor among researchers and methodologists. Indeed, Rodgers (2010) argued that there has been a long developing epistemological change in views of statistical modeling that suggests that null hypothesis significance testing can be rejected in favor of model building and comparisons. The author points out that this approach allows or researchers to easily acknowledge when a model is inappropriate and identify a more appropriate model. Further, Rodgers (2010) essentially suggested that it is more appropriate to compare the fit of multiple candidate models, and consider gains in model fit relative to increases in complexity, than to analyze data using traditional NHST approaches. Finally, the author points out that NHST is subsumed under this model building/comparison approach in that it is used when estimating chi-square goodness of fit statistics. To bring this back to the current work, we did make multiple model comparisons using chi-square difference tests, yet still appealed to alternative indices of model fit when making our decisions. However, for those who still have concern for the experimentwise error rate, it is theoretically possible to remedy this issue (given the flexible structural equation modeling framework) by specifying one grand model that simultaneously estimates the nested models.

Further, applied researchers may criticize the FCSI model due to complexity of interpretation. Indeed, the linear SI model is extremely easy to interpret. However, if the model is inaccurate, ease of interpretability is unimportant. Another possible criticism of the FCSI model is that to identify the model, the slope and intercept components are specified as orthogonal (i.e., zero covariance). Thus, significant covariation between the slope and intercept factor is not captured through model estimation. However, the direction of the correlation between the slope and intercept factors can be inferred from the signs on the factor loadings. Additionally, as the orthogonal and oblique (i.e., estimate factor covariance and set a factor loading to zero) FCSI models yield equivalent model fit, simply rerunning the model in its oblique form will yield an estimate of the magnitude of the covariance and the significance level.

Additionally, some researchers may be critical of the manner in which we handled age in the present model. It is not uncommon for researchers to take age into account by either including age as an exogenous covariate, or assigning age at the measurement occasion as the factor loading. However, including age as a linear covariate implies a constant age effect, and linearly adjusting for age is not necessarily appropriate from a developmental perspective. That is, when attempting to model a developmental trajectory, it is possible that age does not have a constant affect on the rate of change, and one is likely interested in being able to identify periods of moratorium, rapid change, and so on (i.e., differential relationships between age and rate of change at different time periods). Including age in the model as we have allows us to see clearly (from factor loadings), time points where change occurs in an especially rapid fashion, or when change does not occur, and so on.

Finally, skeptics of the FCSI model might argue that in the cases where it was clearly indicated as having superior model fit, its superiority may be an artifact of the measurement instrument. More specifically, if the resulting data are non-normal, or there are floor or ceiling effects, the FCSI model would be shown to be the appropriate model. Item-level scaling or IRT analyses would need to be conducted to rule out such a possibility. As our data were secondary data, we did not have item-level information, but rather had aggregate scores for each measure, and such analyses were not conducted.

Future Directions

The current work focused on phenotypic growth models. The data that were used were from a genetically informative sample. Thus, future work should expand upon the present work to include the decomposition of the latent trajectories into genetic and environmental components, by combining the phenotypic growth modeling procedures discussed herein with behavior genetic modeling techniques, similar to the direction taken by Reynolds et al. (2005). Further, as the current data were limited due to sparsity, the present approach to modeling developmental trajectories should be applied to more full data. That is, while the current work had notably positive qualities such as the fact that it included a wide age range, there were very few measurements at each year of age. Having more full data will likely allow us to recover more non-linear developmental trajectories, as is suggested to be appropriate by Widaman (2008), and would likely resolve the convergence problems encountered.

Another possibility is that there actually exist multiple group slope curves rather than just one curve to for an entire sample. Modeling a single curve in the presence of multiple curves will result in model misfit due to model misspecification. As an extension, it may be that the sample includes multiple sub-groups of people (from multiple populations) rather than a sample from a single population (i.e., one group), and as such, there may be a different curve for each sub-group. One potential avenue of future research could explore this possibility by applying the nested structure discussed above to determine the best form of phenotypic growth in multigroup analyses. Additionally, as noted by Ram and Grimm (2009), multigroup growth models require a priori assumptions or knowledge about group membership. Given that group membership is always known, another direction of research could expand the current work to growth mixture models, which, as pointed out by Ram and Grimm (2009) allow for post-hoc extraction of groups (without prior knowledge of group membership). Further, our data were continuous, but this is not always the case. Indeed, the currently discussed framework and comparison procedures could be expanded to accommodate multiple groups and ordered categorical data, provided there are at least three ordered categories (Millsap & Yun-Tein, 2004).

Additionally, given the pattern of decline in the proportion of variance accounted for at each time period for the block design and figure logic variables, future research should seek to identify important individual difference variables associated with cognitive decline later in life for these abilities. For example, multiple researchers have suggested that cognitive decline later in life is associated with health concerns such as neurological and cardiovascular functioning (Piccinin & Hofer, 2008). Thus, the current approach to model selection should be repeated with other data and expanded to include exogenous covariates such as disease, lifestyle, and overall health.

Conclusion

It is not uncommon for researchers to choose a modeling technique based on familiarity or convention. We argue that such an approach may lead to model misspecification and erroneous conclusions. For example, linear and/or quadratic SI models are often utilized to model developmental trajectories. Multiple researchers have suggested that developmental trajectories are likely non-linear. Quadratic models, though they allow for a non-linear rate of change, still require strict assumptions about the rate of change over time. An alternative approach, the FCSI model, wherein slope factor loadings are estimated, can be used to model developmental trajectories and allows the data to inform the metric of change. That is, there are minimal assumptions about the rate and shape of change over time in the FCSI model, and the data essentially determine the rate and shape of change is over time.

The minimal assumptions of the FCSI model are advantageous in that it helps to reduce the chance of model misspecification and resultantly reduces the probability of inaccurate parameter estimates and poor model fit. However, the FCSI model is a complex model, and it may be the case that a simpler model would be sufficient to answer the research question. The FCSI model is further advantageous in that, when specified in terms of the general factor model, it can be shown that multiple, simpler, growth models (e.g., linear SI, repeated-measures MANOVA) are actually special cases of the FCSI model, and thus are nested within the model. This allows for direct model comparisons, via chi-square difference tests, to determine if the more complex FCSI model yields a significant improvement in model fit relative to the alternatives.

In the present work, we were able to successfully illustrate how to use this nested structure to compare the relative fits of different models for cognitive aging. Previous research by Reynolds et al. (2006) suggested that cognitive decline occurs in a quadratic fashion. However, utilizing the nested structure described previously, we have shown that the quadratic model was indicated as the most appropriate model for the developmental trajectory of only 1 of 6 cognitive abilities. Indeed, the FCSI model was shown to be the superior approach for modeling all three crystallized abilities. Fluid abilities, on the other hand, appeared to be more linear trajectories. Though these results contradict Reynolds et al.'s (2005) results, they are consistent with the overarching literature on crystallized and fluid intelligence. Rodgers (2010) argues that we should be, and are, moving towards using model comparisons for selecting models. We are in agreement with this argument and suggest that the current work is a step in helping cognitive aging research move in this direction.

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Footnotes

[1] Models that estimate factor level means, and are thus based on the adjoined SSCP matrix (which is the case for the SI, FM, FM-Shift, HLM and MANOVA models outlined) make the assumption that the observed data were all measured using the same (constant) metric. That is, whether we believe the data are ratio or interval, we think that all the variables are measured on the same scale. If this is not a reasonable assumption, and we think that the different variables should be treated as having different metric, then analyses need to be conducted based on the variance-covariance matrix.

[2] It should be noted that only when there is missing data, must we use the augmented raw score matrix as the input dataset, rather than the adjoined SSCP matrix because we cannot calculate the summary (sufficient) statistics in the SSCP matrix with missing data. That is, we need full augmented data matrix for FIML when there is missing data.

[3] It is important to note that it is possible for an unidentified model to be successfully fit to data with existing software, yielding what appear to be proper parameter estimates. When this occurs, the model yields unstable parameter estimates (Rindskopf, 1984; Kenny, Kashy, & Bolger, 1998), and therefore the solutions that result from such models are untrustworthy.

[4] As a result, Biesanz et al. (2004) point out that once a linear growth model is fit, it is not necessary to fit separate models with transformed time metrics, but rather estimates can be obtained directly from the original solution and the transformation information.

[5] Influential observations (i.e., outliers) were determined based on influence and mahalanobis distances. Influence statistics, mahalanobis distance, and the p-values associated with the mahalanobis distance were acquired from MPlus. Individuals with mahalanobis p-values less than .01 were considered outliers. Figures 16 through 22 show the probability plots the influence diagnostics (i.e., mahalanobis distance, influence, and log-likelihoods) from the FCSI model for each of the variables.

Table 1. Matrix formulations for reviewed growth models, assuming four measurement occasions

	SI	SFGM	SFGM-Shift	MLM	Repeated-Measures MANOVA
$\mu_{(\text{n x t})}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}$
$\xi_{(\text{n x f})}$	$\begin{bmatrix} \xi_{1i} & \xi_{1s} \\ \vdots & \vdots \\ \xi_{ni} & \xi_{ns} \end{bmatrix}$	$\begin{bmatrix} 0 & \xi_{1s} \\ \vdots & \vdots \\ 0 & \xi_{ns} \end{bmatrix}$	$\begin{bmatrix} \xi_{1i} & \xi_{1s} \\ \vdots & \vdots \\ \xi_{ni} & \xi_{ns} \end{bmatrix}$	$\begin{bmatrix} \xi_{1i} & \xi_{1s} \\ \vdots & \vdots \\ \xi_{ni} & \xi_{ns} \end{bmatrix}$	$\begin{bmatrix} \xi_{1i} & \xi_{1s} \\ \vdots & \vdots \\ \xi_{ni} & \xi_{ns} \end{bmatrix}$
$\Lambda_{(\text{f x t})}$	$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_{s1} & \lambda_{s2} & \lambda_{s3} & \lambda_{s4} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda_{s1} & \lambda_{s2} & \lambda_{s3} & \lambda_{s4} \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_{s1} & \lambda_{s2} & \lambda_{s3} & \lambda_{s4} \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & \lambda_{s2} & \lambda_{s3} & \lambda_{s4} \end{bmatrix}$
$\varepsilon_{(\text{n x t})}$	$\begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ \vdots & \vdots & \vdots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & e_{n4} \end{bmatrix}$	$\begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ \vdots & \vdots & \vdots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & e_{n4} \end{bmatrix}$	$\begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ \vdots & \vdots & \vdots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & e_{n4} \end{bmatrix}$	$\begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ \vdots & \vdots & \vdots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & e_{n4} \end{bmatrix}$	$\begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ \vdots & \vdots & \vdots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & e_{n4} \end{bmatrix}$
$\theta_{(\text{n x f})}$	$\begin{bmatrix} \theta_i & \theta_s \\ \vdots & \vdots \\ \theta_i & \theta_s \end{bmatrix}$	$\begin{bmatrix} 0 & \theta_s \\ \vdots & \vdots \\ 0 & \theta_s \end{bmatrix}$	$\begin{bmatrix} \theta_i & \theta_s \\ \vdots & \vdots \\ \theta_i & \theta_s \end{bmatrix}$	$\begin{bmatrix} \theta_i & \theta_s \\ \vdots & \vdots \\ \theta_i & \theta_s \end{bmatrix}$	$\begin{bmatrix} \theta_i & 1 \\ \vdots & \vdots \\ \theta_i & 1 \end{bmatrix}$
$\eta_{(\text{n x f})}$	$\begin{bmatrix} \eta_{1i} & \eta_{1s} \\ \vdots & \vdots \\ \eta_{ni} & \eta_{ns} \end{bmatrix}$	$\begin{bmatrix} 0 & \eta_{1s} \\ \vdots & \vdots \\ 0 & \eta_{ns} \end{bmatrix}$	$\begin{bmatrix} \eta_{1i} & \eta_{1s} \\ \vdots & \vdots \\ \eta_{ni} & \eta_{ns} \end{bmatrix}$	$\begin{bmatrix} \eta_{1i} & \eta_{1s} \\ \vdots & \vdots \\ \eta_{ni} & \eta_{ns} \end{bmatrix}$	$\begin{bmatrix} \eta_{1i} & \eta_{1s} \\ \vdots & \vdots \\ \eta_{ni} & \eta_{ns} \end{bmatrix}$
$\Phi_{(\text{f x f})}$	$\begin{bmatrix} \phi_i & \phi_{is} \\ \phi_{is} & \phi_s \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & \phi_s \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & \phi_s \end{bmatrix}$	$\begin{bmatrix} \phi_i & \phi_{is} \\ \phi_{is} & \phi_s \end{bmatrix}$	$\begin{bmatrix} \phi_i & 0 \\ 0 & 0 \end{bmatrix}$
$\Psi^2_{(\text{t x t})}$	$\begin{bmatrix} \varepsilon_1^2 & 0 & 0 & 0 \\ 0 & \varepsilon_2^2 & 0 & 0 \\ 0 & 0 & \varepsilon_3^2 & 0 \\ 0 & 0 & 0 & \varepsilon_4^2 \end{bmatrix}$	$\begin{bmatrix} \varepsilon_1^2 & 0 & 0 & 0 \\ 0 & \varepsilon_2^2 & 0 & 0 \\ 0 & 0 & \varepsilon_3^2 & 0 \\ 0 & 0 & 0 & \varepsilon_4^2 \end{bmatrix}$	$\begin{bmatrix} \varepsilon_1^2 & 0 & 0 & 0 \\ 0 & \varepsilon_2^2 & 0 & 0 \\ 0 & 0 & \varepsilon_3^2 & 0 \\ 0 & 0 & 0 & \varepsilon_4^2 \end{bmatrix}$	$\begin{bmatrix} \varepsilon^2 & 0 & 0 & 0 \\ 0 & \varepsilon^2 & 0 & 0 \\ 0 & 0 & \varepsilon^2 & 0 \\ 0 & 0 & 0 & \varepsilon^2 \end{bmatrix}$	$\begin{bmatrix} \varepsilon^2 & 0 & 0 & 0 \\ 0 & \varepsilon^2 & 0 & 0 \\ 0 & 0 & \varepsilon^2 & 0 \\ 0 & 0 & 0 & \varepsilon^2 \end{bmatrix}$

Table 2. Fit statistics for crystallized abilities

	FCSI	SFGM	SFGM-Shift	MANOVA	Intercept Only	Linear SI	Quadratic SI
				Analogies			
χ^2	**111.15**	117.40	114.82	160.55*	192.25**	140.00	NC
df χ^2	108	110	109	129	129	126	
CFI	**1.00**	0.99	0.99	0.96	0.91	0.98	
TLI	**1.00**	0.99	0.99	0.96	0.93	0.98	
AIC	11753.74	11755.98	11755.4	11761.14	11792.83	**11746.59**	
BIC	11928.98	11923.07	11926.57	11850.79	11882.49	**11848.47**	
RMSEA	**0.008**	0.012	0.011	0.024	0.034	0.016	
Low CL	0	0	0	0.008	0.023	0	
Up CL	0.026	0.028	0.027	0.035	0.043	0.029	
				Information			
χ^2	**193.36****	353.99**	NC	321.51**	475.96**	926.20**	NC
df χ^2	143	145		145	166	163	
CFI	**0.97**	0.88		0.90	0.82	0.55	
TLI	**0.97**	0.90		0.91	0.86	0.66	
AIC	**12580.80**	12737.43		12704.96	12817.40	13273.64	
BIC	**12689.21**	12837.50		12805.03	12829.91	13298.66	
RMSEA	**0.027**	0.055		0.05	0.063	0.099	
Low CL	0.016	0.048		0.043	0.056	0.093	
Up CL	0.036	0.062		0.058	0.069	0.105	
				Synonyms			
χ^2	**201.09****	222.71**	NC	217.56**	254.83**	238.14**	236.07**
df χ^2	121	125		125	142	139	137
CFI	**0.94**	0.92		0.93	0.91	0.92	0.92
TLI	**0.95**	0.94		0.94	0.93	0.94	0.94
AIC	**7875.97**	7889.59		7884.45	7887.71	7877.02	7878.95
BIC	7969.34	7968.59		7963.45	**7905.67**	7905.74	7914.86
RMSEA	**0.05**	0.054		0.053	0.054	0.052	0.052
Low CL	0.037	0.042		0.041	0.044	0.04	0.041
Up CL	0.06	0.07		0.06	0.07	0.06	0.06

Table 3. Chi-square difference statistics for crystallized abilities

		SFGM	SFGM-Shift	MANOVA	Intercept Only	Linear SI
				Analogies		
FCSI	$\chi^2_{difference}$	6.25*	3.67*	49.40**	81.10**	28.85*
	df_{χ^2}	2	1	21	21	18
SFGM-Shift	$\chi^2_{difference}$	2.58	-	-	-	-
	df_{χ^2}	1	-	-	-	-
Quadratic SI	$\chi^2_{difference}$	-	-	-	-	-
	df_{χ^2}	-	-	-	-	-
Linear SI	$\chi^2_{difference}$	-	-	-	52.25**	-
	df_{χ^2}	-	-	-	3	-
				Information		
FCSI	$\chi^2_{difference}$	160.63**	-	128.16**	282.61**	732.85**
	df_{χ^2}	2	-	2	23	20
SFGM-Shift	$\chi^2_{difference}$	-	-	-	-	-
	df_{χ^2}	-	-	-	-	-
Quadratic SI	$\chi^2_{difference}$	-	-	-	-	-
	df_{χ^2}	-	-	-	-	-
Linear SI	$\chi^2_{difference}$	-	-	-	Weird	-
	df_{χ^2}	-	-	-		-
				Synonyms		
FCSI	$\chi^2_{difference}$	21.62**	-	16.47**	53.74**	37.05**
	df_{χ^2}	4	-	4	21	18
SFGM-Shift	$\chi^2_{difference}$	-	-	-	-	-
	df_{χ^2}	-	-	-	-	-
Quadratic SI	$\chi^2_{difference}$	-	-	-	18.76**	2.07
	df_{χ^2}	-	-	-	5	2
Linear SI	$\chi^2_{difference}$	-	-	-	16.70**	-
	df_{χ^2}	-	-	-	3	-

Table 4. Unstandardized parameter estimates for crystallized abilities.

	Analogies		Swedish Information		Synonyms	
	FCSI		FCSI		FCSI	
Factor Scores						
λ	I	S	I	S	I	S
45-46	-	-	1.00	-15.08**	-	-
47-48	1.00	8.49*	1.00	-6.45	1.00	6.90
49-50	1.00	8.52**	1.00	-12.10**	1.00	2.85
51-52	1.00	7.38**	1.00	-5.93**	1.00	3.80
53-54	1.00	7.05**	1.00	-5.99**	1.00	0.55
55-56	1.00	8.86**	1.00	-2.67	1.00	3.77
57-58	1.00	4.81**	1.00	-3.35	1.00	-0.04
59-60	1.00	7.54**	1.00	-2.99	1.00	3.00
61-62	1.00	6.73**	1.00	-4.27*	1.00	0.81
63-64	1.00	6.44**	1.00	-2.85	1.00	0.68
65-66	1.00	5.27**	1.00	-1.76	1.00	0.60
67-68	1.00	6.95**	1.00	-1.04	1.00	2.66
69-70	1.00	3.75	1.00	-1.09	1.00	1.10
71-72	1.00	2.90	1.00	-1.19	1.00	-4.88*
73-74	1.00	3.24	1.00	-2.42	1.00	-1.12
75-76	1.00	2.82	1.00	-3.09*	1.00	-1.88
77-78	1.00	0.50	1.00	-5.46**	1.00	-1.00
79-80	1.00	-0.34	1.00	-8.50**	1.00	-0.02
81-82	1.00	-2.89	1.00	-10.14**	1.00	-7.26*
83-84	1.00	-2.91	1.00	-13.50**	1.00	-8.27**
85-86	1.00	-1.69	1.00	-17.39**	1.00	-8.42*
87-88	-	-	1.00	-23.09**	-	-
Means, Variances, and Covariances						
	I	S	I	S	I	S
θ	51.94**	.94**	76.33**	.64**	66.52**	.54**
Φ_I	105.03*	0	208.95**	0	281.99**	0
Φ_S	0	1	0	1	0	1

Note. **p < .01, *p < .05.

Table 5. Standardized parameter estimates for crystallized abilities.

	Analogies		Swedish Information		Synonyms	
	FCSI		FCSI		FCSI	
Factor Scores						
λ	I	S	I	S	I	S
45-46	-	-	0.67	-0.69	-	-
47-48	0.56	0.47	0.86	-0.38	0.88	0.36
49-50	0.71	0.59	0.73	-0.61	0.93	0.16
51-52	0.66	0.47	0.87	-0.36	0.92	0.21
53-54	0.69	0.47	0.86	-0.36	0.94	0.03
55-56	0.71	0.61	0.91	-0.17	0.92	0.21
57-58	0.76	0.36	0.9	-0.21	0.94	0.00
59-60	0.69	0.51	0.91	-0.19	0.93	0.17
61-62	0.72	0.47	0.89	-0.26	0.95	0.05
63-64	0.66	0.42	0.91	-0.18	0.93	0.04
65-66	0.75	0.39	0.92	-0.11	0.94	0.03
67-68	0.71	0.48	0.92	-0.07	0.93	0.15
69-70	0.75	0.27	0.92	-0.07	0.94	0.06
71-72	0.74	0.21	0.92	-0.08	0.91	-0.26
73-74	0.71	0.23	0.91	-0.15	0.94	-0.06
75-76	0.7	0.19	0.91	-0.19	0.93	-0.1
77-78	0.84	0.04	0.87	-0.33	0.94	-0.06
79-80	0.76	-0.03	0.81	-0.48	0.94	0.00
81-82	0.81	-0.23	0.78	-0.54	0.87	-0.38
83-84	0.75	-0.21	0.7	-0.65	0.85	-0.42
85-86	0.9	-0.15	0.62	-0.74	0.85	-0.43
87-88	-	-	0.52	-0.83	-	-
Means, Variances, and Covariances						
	I	S	I	S	I	S
θ	5.07**	.94**	5.28**	.64**	3.96**	.54**
Φ_I	1	0	1	1	1	0
Φ_S	0	1	0	1	0	1

Note. ***p* < .01, **p* < .05.

Table 6. Proportion of variance estimates for crystallized abilities.

	Analogies FCSI				Swedish WAIS Information FCSI				Synonyms FCSI			
λ	R^2_{Model}	R^2_{Inter}	R^2_{Slope}	R^2_{Error}	R^2_{Model}	R^2_{Inter}	R^2_{Slope}	R^2_{Error}	R^2_{Model}	R^2_{Inter}	R^2_{Slope}	R^2_{Error}
45-46	-	-	-	-	0.93	0.45	0.48	0.08	-	-	-	-
47-48	0.53	0.31	0.22	0.47	0.88	0.74	0.14	0.12	0.90	0.77	0.13	0.10
49-50	0.85	0.50	0.35	0.15	0.91	0.53	0.37	0.10	0.89	0.86	0.03	0.11
51-52	0.66	0.44	0.22	0.34	0.89	0.76	0.13	0.11	0.89	0.85	0.04	0.11
53-54	0.70	0.48	0.22	0.30	0.87	0.74	0.13	0.13	0.88	0.88	0.00	0.12
55-56	0.88	0.50	0.37	0.12	0.86	0.83	0.03	0.14	0.89	0.85	0.04	0.11
57-58	0.71	0.58	0.13	0.29	0.85	0.81	0.04	0.15	0.88	0.88	0.00	0.12
59-60	0.74	0.48	0.26	0.26	0.86	0.83	0.04	0.14	0.89	0.86	0.03	0.11
61-62	0.74	0.52	0.22	0.26	0.86	0.79	0.07	0.14	0.91	0.90	0.00	0.10
63-64	0.61	0.44	0.18	0.39	0.86	0.83	0.03	0.14	0.87	0.86	0.00	0.13
65-66	0.71	0.56	0.15	0.29	0.86	0.85	0.01	0.14	0.88	0.88	0.00	0.12
67-68	0.73	0.50	0.23	0.27	0.85	0.85	0.00	0.15	0.89	0.86	0.02	0.11
69-70	0.64	0.56	0.07	0.36	0.85	0.85	0.00	0.15	0.89	0.88	0.00	0.11
71-72	0.59	0.55	0.04	0.41	0.85	0.85	0.01	0.15	0.90	0.83	0.07	0.10
73-74	0.56	0.50	0.05	0.44	0.85	0.83	0.02	0.15	0.89	0.88	0.00	0.11
75-76	0.53	0.49	0.04	0.47	0.86	0.83	0.04	0.14	0.87	0.86	0.01	0.13
77-78	0.71	0.71	0.00	0.29	0.87	0.76	0.11	0.13	0.89	0.88	0.00	0.11
79-80	0.58	0.58	0.00	0.42	0.89	0.66	0.23	0.11	0.88	0.88	0.00	0.12
81-82	0.71	0.66	0.05	0.29	0.90	0.61	0.29	0.10	0.90	0.76	0.14	0.10
83-84	0.61	0.56	0.04	0.39	0.91	0.49	0.42	0.09	0.90	0.72	0.18	0.10
85-86	0.83	0.81	0.02	0.17	0.93	0.38	0.55	0.07	0.91	0.72	0.18	0.09
87-88	-	-	-	-	0.96	0.27	0.69	0.04	-	-	-	-

Table 7. Fit statistics for fluid abilities

	FCSI	SFGM	SFGM-Shift	MANOVA	Intercept Only	Linear SI	Quadratic SI
				Block Design			
χ^2	**167.88**	**170.22**	**169.23**	255.91	730.93	**196.59**	NC
df χ^2	108	110	109	129	129	128	
CFI	**0.95**	**0.95**	**0.95**	0.89	0.49	0.94	
TLI	**0.95**	**0.95**	**0.95**	0.91	0.56	**0.95**	
AIC	12074.02	12072.36	12073.37	12120.06	12595.08	**12062.74**	
BIC	12251.29	12241.38	12246.52	12210.75	12685.77	**12157.56**	
RMSEA	0.035	0.035	0.035	0.046	0.101	**0.034**	
Low CL	0.024	0.024	0.024	0.038	0.094	0.024	
Up CL	0.045	0.045	0.045	0.055	0.108	0.044	
				Card Rotations			
χ^2	**114.85**	155.29	153.34	153.51	443.47	**141.54**	**145.01**
df χ^2	109	111	128	129	130	127	141
CFI	0.99	0.95	0.97	0.97	0.65	0.98	**1**
TLI	0.99	0.95	0.98	0.98	0.70	0.99	**1**
AIC	11651.02	11687.46	11651.51	11649.68	11937.64	11641.72	**11617.18**
BIC	11820.91	11849.26	11744.55	11738.67	12022.58	11738.80	**11657.63**
RMSEA	0.011	0.031	0.022	0.021	0.076	0.016	**0.008**
Low CL	0	0.018	0	0	0.068	0	0
Up CL	0	0.042	0.033	0.033	0.083	0.03	0.035
				Figure Logic			
χ^2	**193.22**	**198.30**	**197.03**	277.40	298.78	**216.66**	212.47
df χ^2	114	116	115	136	136	133	129
CFI	**0.86**	0.85	0.85	0.75	0.71	0.85	0.85
TLI	0.86	0.85	0.85	0.78	0.75	**0.87**	**0.87**
AIC	12543.16	12544.24	12544.96	12583.34	12604.72	**12528.60**	12532.41
BIC	12729.07	12721.88	12726.74	12678.36	12699.73	**12636.01**	12656.35
RMSEA	0.039	0.039	0.039	0.048	0.051	**0.037**	0.038
Low CL	0.029	0.03	0.03	0.04	0.043	0.028	0.028
Up CL	0.048	0.048	0.049	0.056	0.059	0.046	0.046

Table 8. χ^2 difference statistics for fluid abilities

Base Model		SFGM	SFGM-Shift	MANOVA	Intercept Only	Linear SI
		Koh's Block Design				
FCSI	$\chi^2_{difference}$	2.34	1.35	88.03**	563.05**	28.71
	df_{χ^2}	2	1	21	21	20
SFGM-Shift	$\chi^2_{difference}$	0.99	-	-	-	-
	df_{χ^2}	1	-	-	-	-
Quadratic SI	$\chi^2_{difference}$	-	-	-	-	-
	df_{χ^2}	-	-	-	-	-
Linear SI	$\chi^2_{difference}$	-	-	-	-	-
	df_{χ^2}	-	-	-	-	-
		Card Rotations				
FCSI	$\chi^2_{difference}$	40.45**	38.50**	38.67**	328.62**	26.70
	df_{χ^2}	2	19	20	21	18
SFGM-Shift	$\chi^2_{difference}$	27.81**	-	-	-	-
	df_{χ^2}	1	-	-	-	-
Quadratic SI	$\chi^2_{difference}$	-	-	-	344.65**	24.31**
	df_{χ^2}	-	-	-	7	4
Linear SI	$\chi^2_{difference}$	-	-	-	320.34**	-
	df_{χ^2}	-	-	-	3	-
		Figure Logic				
FCSI	$\chi^2_{difference}$	5.08	3.81	84.18**	105.56**	23.44
	df_{χ^2}	2	1	22	22	19
SFGM-Shift	$\chi^2_{difference}$	1.27	-	-	-	-
	df_{χ^2}	1	-	-	-	-
Quadratic SI	$\chi^2_{difference}$	-	-	-	86.31**	4.19
	df_{χ^2}	-	-	-	7	4
Linear SI	$\chi^2_{difference}$	-	-	-	82.12**	-
	df_{χ^2}	-	-	-	3	-

Note. $^{**}p < .01$, $^{*}p < .05$.

Table 9. Unstandardized parameter estimates for fluid abilities.

	Block Design SFGM		Figure Logic Linear SI		Card Rotations Quadratic SI		
Factor Loadings							
	I	S	I	S	I	S	Q
45-46	-	-	1.00	-20	-	-	-
47-48	0	18.53	1.00	-18	1.00	-19	361
49-50	0	16.61	1.00	-16	1.00	-17	289
51-52	0	16.52	1.00	-14	1.00	-15	225
53-54	0	16.80	1.00	-12	1.00	-13	169
55-56	0	16.19	1.00	-10	1.00	-11	121
57-58	0	15.85	1.00	-8	1.00	-9	81
59-60	0	14.91	1.00	-6	1.00	-7	49
61-62	0	14.69	1.00	-4	1.00	-5	25
63-64	0	14.01	1.00	-2	1.00	-3	9
65-66	0	14.01	1.00	0	1.00	-1	1
67-68	0	13.07	1.00	2	1.00	1	1
69-70	0	12.88	1.00	4	1.00	3	9
71-72	0	11.99	1.00	6	1.00	5	25
73-74	0	11.49	1.00	8	1.00	7	49
75-76	0	10.84	1.00	10	1.00	9	81
77-78	0	10.57	1.00	12	1.00	11	121
79-80	0	9.71	1.00	14	1.00	13	169
81-82	0	8.65	1.00	16	1.00	15	225
83-84	0	7.95	1.00	18	1.00	17	289
85-86	0	7.76	1.00	-20	1.00	19	361
Means, Variances, and Covariances							
	I	S	I	S	I	S	Q
θ	-	3.45**	61.52**	-.36**	46.73**	-.87**	-.02**
Φ_I	-	-	79.61**	-.29	206.36**	-.07	-.18*
Φ_S	-	1	-.29	-.04	-.07	.24*	-.001
Φ_Q	-	-	-	-	-.18*	-.001	.01

Note. $^{**}p < .01$, $^{*}p < .05$.

Table 10. Standardized parameter estimates for fluid abilities.

	Block Design SFGM		Figure Logic Linear SI		Card Rotations Quadratic SI		
	Factor Loadings						
	I	S	I	S	I	S	Q
45-46	-	-	0.79	-0.34	-	-	-
47-48	0	0.97	0.82	-0.32	0.77	-0.50	0.51
49-50	0	0.93	0.81	-0.28	0.81	-0.47	0.43
51-52	0	0.96	0.79	-0.24	0.83	-0.43	0.35
53-54	0	0.93	0.80	-0.21	0.85	-0.38	0.27
55-56	0	0.95	0.68	-0.15	0.86	-0.32	0.19
57-58	0	0.91	0.73	-0.13	0.87	-0.27	0.13
59-60	0	0.88	0.75	-0.1	0.87	-0.21	0.08
61-62	0	0.87	0.72	-0.06	0.87	-0.15	0.04
63-64	0	0.86	0.74	-0.03	0.86	-0.09	0.01
65-66	0	0.89	0.76	0.00	0.86	-0.03	0.00
67-68	0	0.86	0.73	0.03	0.86	0.03	0.00
69-70	0	0.89	0.65	0.06	0.87	0.09	0.01
71-72	0	0.88	0.65	0.09	0.87	0.15	0.04
73-74	0	0.86	0.67	0.12	0.87	0.21	0.08
75-76	0	0.82	0.69	0.15	0.88	0.27	0.13
77-78	0	0.82	0.64	0.17	0.88	0.33	0.20
79-80	0	0.80	0.65	0.2	0.88	0.39	0.28
81-82	0	0.79	0.64	0.22	0.87	0.45	0.36
83-84	0	0.71	0.53	0.21	0.86	0.50	0.46
85-86	0	0.80	0.58	0.25	0.83	0.54	0.56
	Means, Variances, and Covariances						
	I	S	I	S	I	S	Q
θ	3.45**	-2.36**	6.90**	-1.88**	3.25*	-1.78**	-.60*
Φ_I	1	-.82**	1	-.17	1	-.01	-.48**
Φ_S	-.82**	1	-.17	1	-.01	1	-.13
Φ_Q	-	-	-	-	-.48*	-.13	1

Table 11. Proportion of variance estimates for fluid abilities.

	Block Design SFGM		Figure Logic Linear SI				Card Rotations Quadratic SI		
λ	R^2_{Model}	R^2_{Error}	R^2_{Model}	R^2_{Inter}	R^2_{Slope}	R^2_{Error}	R^2_{Model}	R^2_{Growth}	R^2_{Error}
45-46	-	-	0.83	0.62	0.12	0.17	-	-	-
47-48	0.94	0.06	0.86	0.67	0.10	0.14	0.80	0.20	0.58
49-50	0.86	0.14	0.81	0.66	0.08	0.19	0.78	0.22	0.45
51-52	0.92	0.08	0.75	0.62	0.06	0.25	0.76	0.24	0.34
53-54	0.86	0.14	0.74	0.64	0.04	0.26	0.75	0.25	0.24
55-56	0.90	0.10	0.52	0.46	0.02	0.48	0.74	0.26	0.16
57-58	0.83	0.17	0.58	0.53	0.02	0.42	0.74	0.26	0.10
59-60	0.77	0.23	0.60	0.56	0.01	0.40	0.74	0.26	0.05
61-62	0.76	0.24	0.54	0.52	0.00	0.46	0.74	0.26	0.02
63-64	0.74	0.26	0.56	0.55	0.00	0.44	0.74	0.26	0.01
65-66	0.79	0.21	0.58	0.58	0.00	0.42	0.74	0.26	0.00
67-68	0.74	0.26	0.53	0.53	0.00	0.47	0.74	0.26	0.00
69-70	0.79	0.21	0.41	0.42	0.00	0.59	0.74	0.26	0.01
71-72	0.77	0.23	0.41	0.42	0.01	0.59	0.74	0.26	0.02
73-74	0.74	0.26	0.44	0.45	0.01	0.56	0.74	0.26	0.05
75-76	0.67	0.33	0.46	0.48	0.02	0.54	0.74	0.26	0.08
77-78	0.67	0.33	0.40	0.41	0.03	0.60	0.73	0.27	0.13
79-80	0.64	0.36	0.42	0.42	0.04	0.58	0.74	0.26	0.20
81-82	0.62	0.38	0.41	0.41	0.05	0.59	0.74	0.26	0.29
83-84	0.50	0.50	0.29	0.28	0.04	0.71	0.75	0.25	0.40
85-86	0.64	0.36	0.35	0.34	0.06	0.65	0.76	0.24	0.52

Figure 1. The Slope-Intercept Model (without any imposed constraints). Note that the model is not identified as shown. Identification constraints would be imposed on the model appropriate to the assumptions of the particular form of the model. For example, in the linear trajectory model the slope factor loadings to (t – 1).

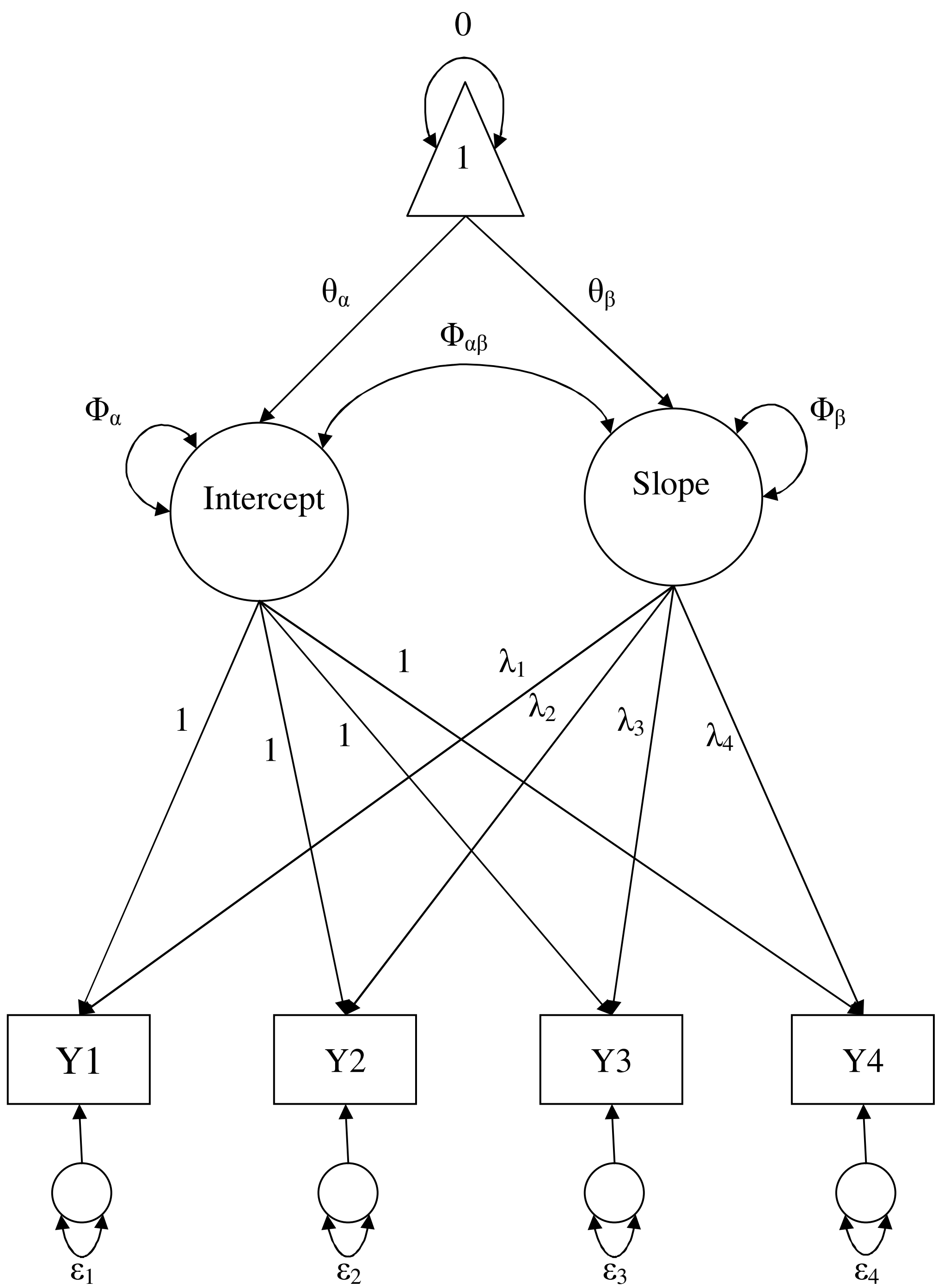


Figure 2. Nested model structure.

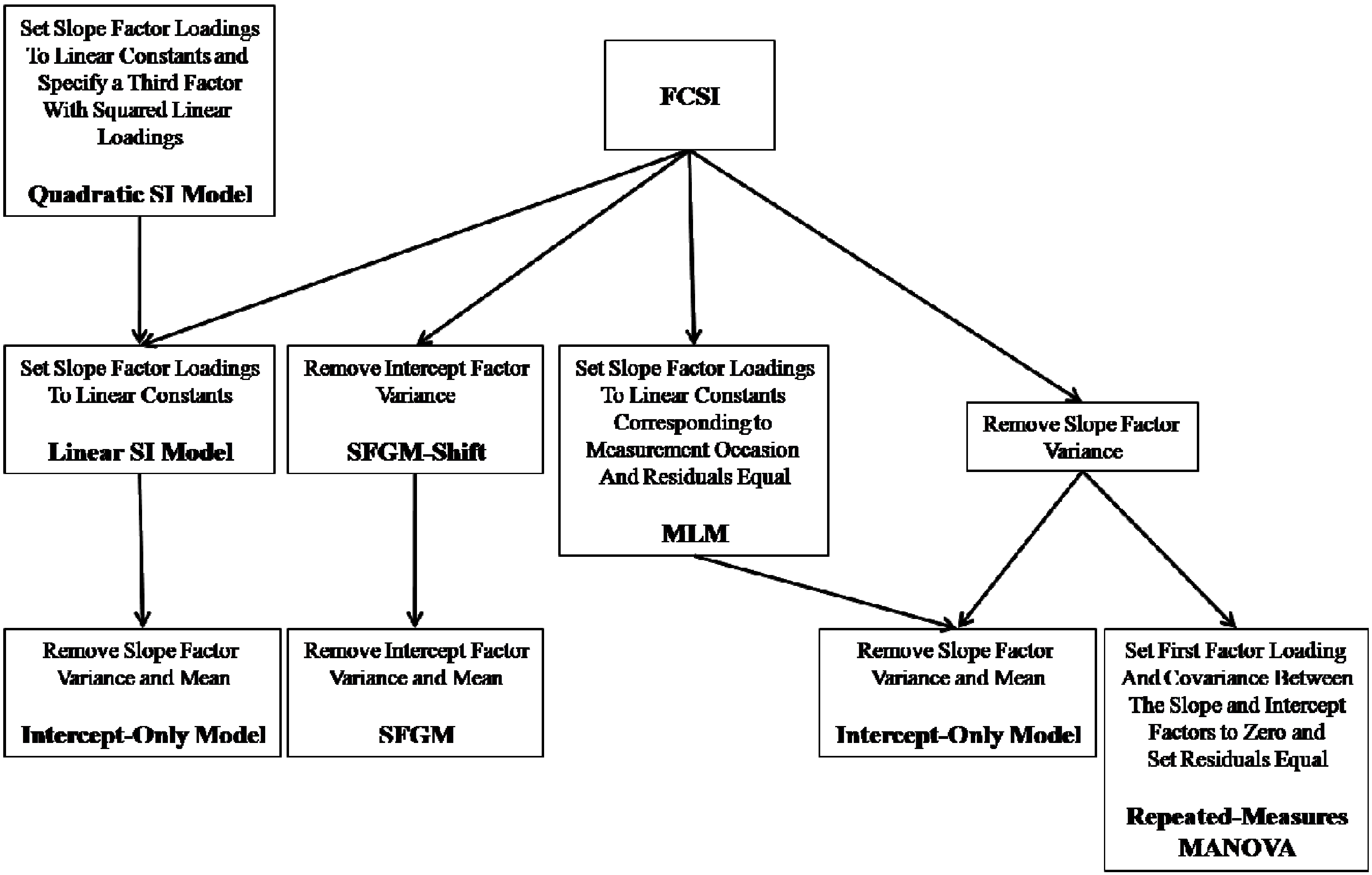


Figure 3. Observed and Model Estimated Means for Crystallized Abilities

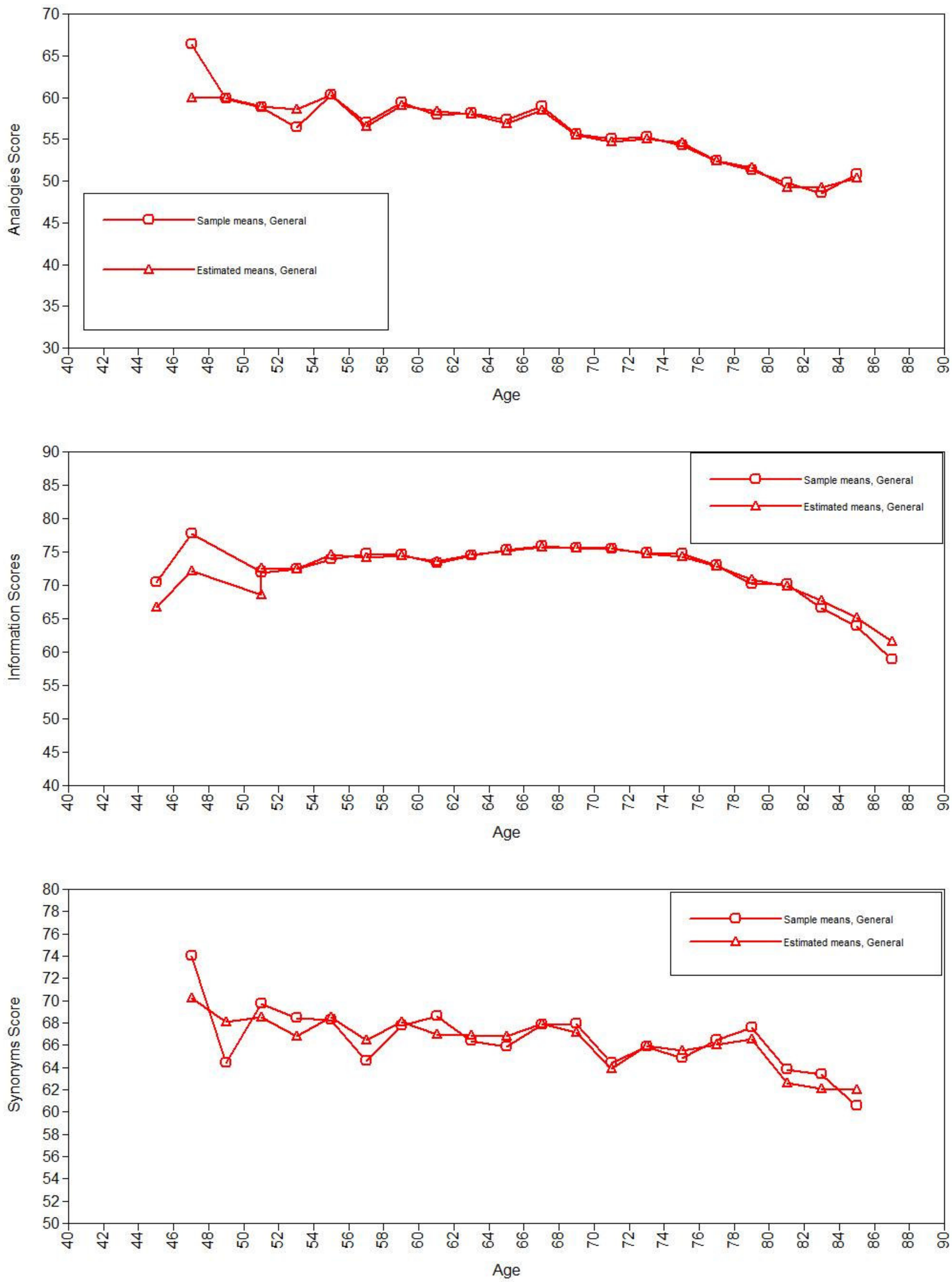
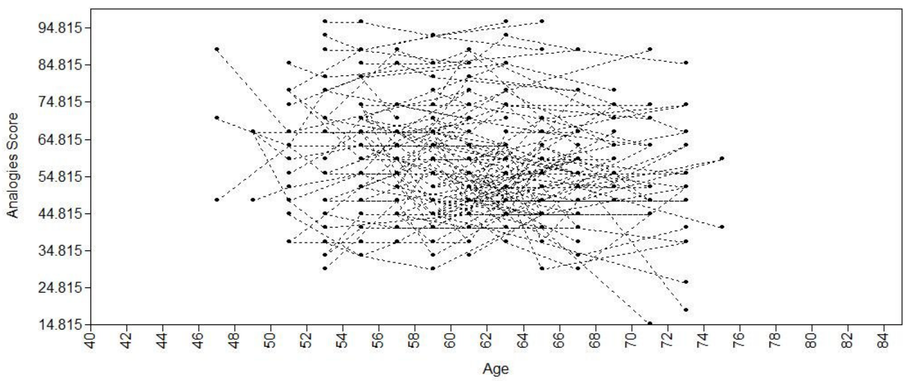
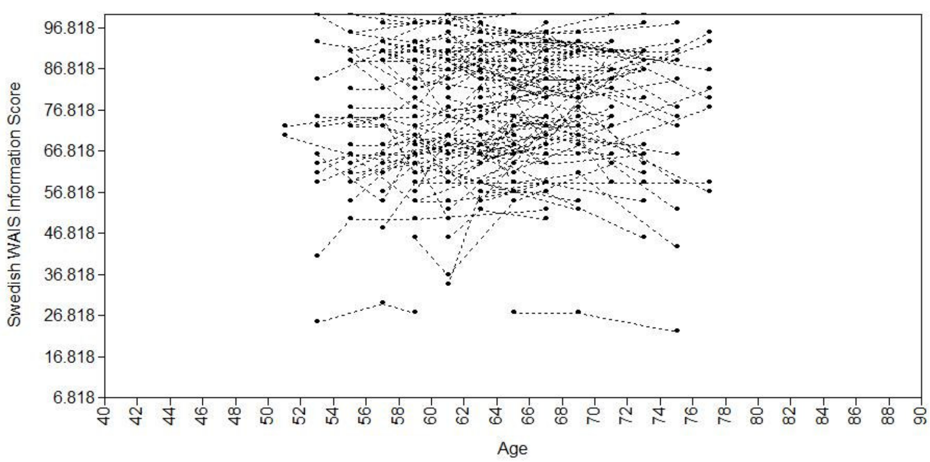


Figure 4. Observed individual curves for crystallized abilities.

(a) Analogies



(b) Swedish WAIS Information



(c) Synonyms

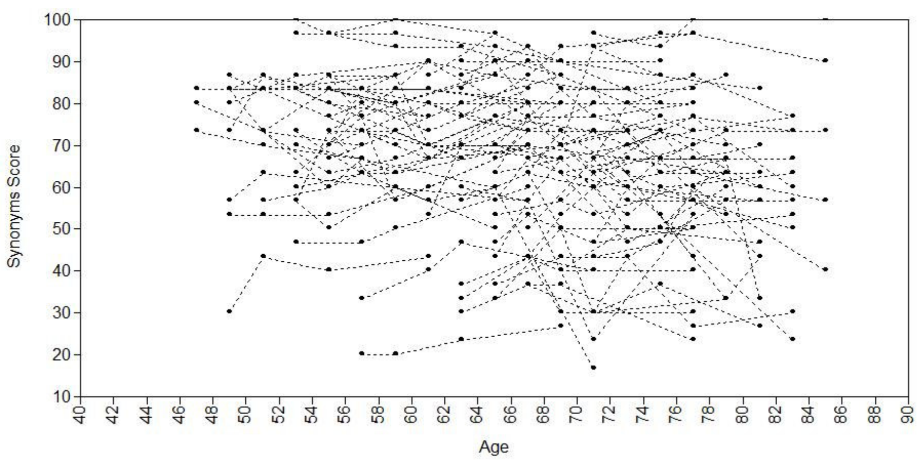


Figure 5. Estimated individual curves for crystallized abilities.

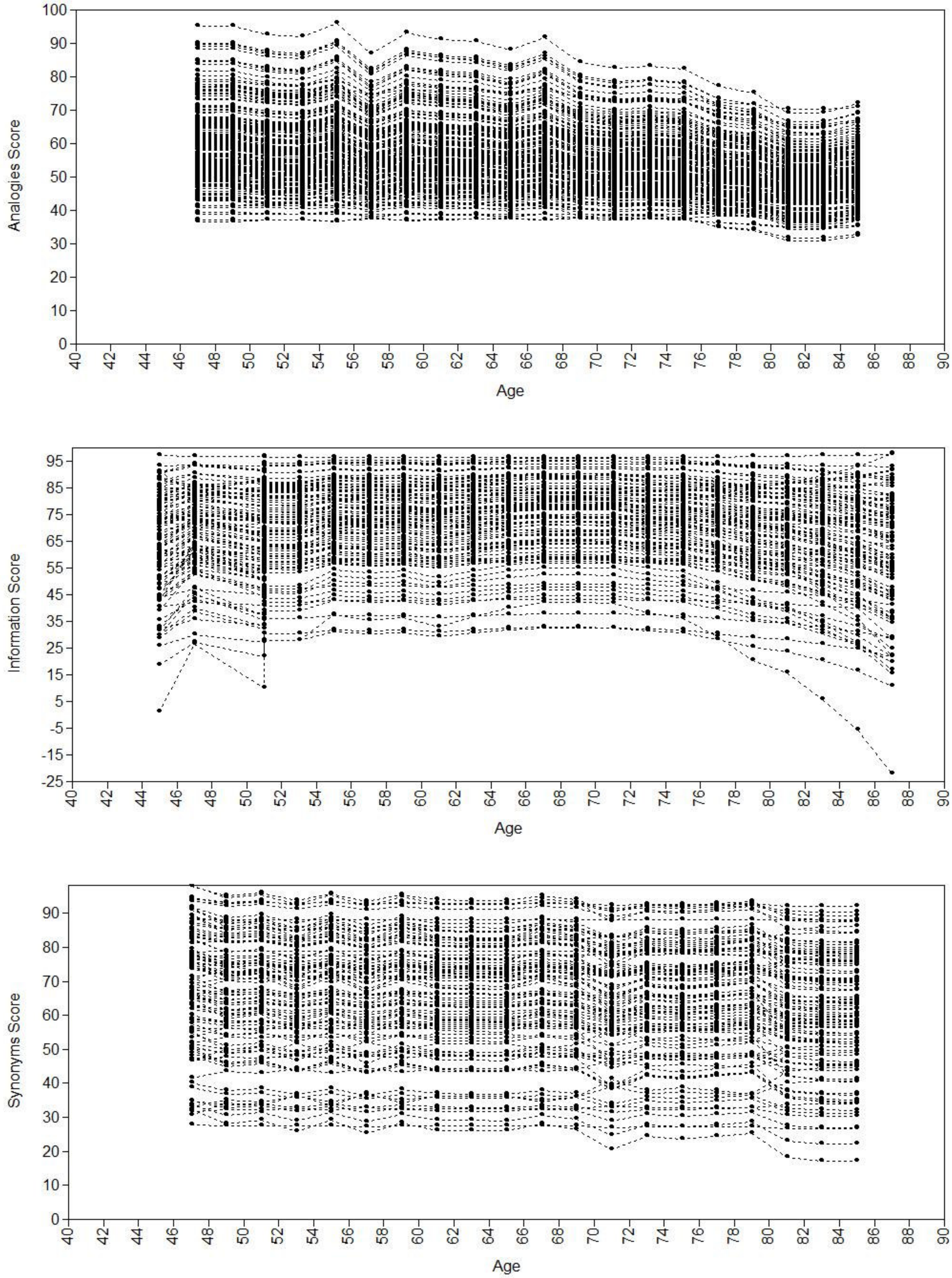
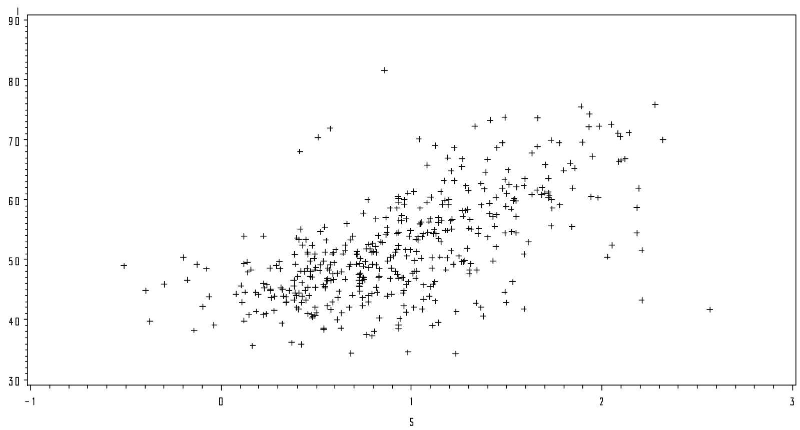
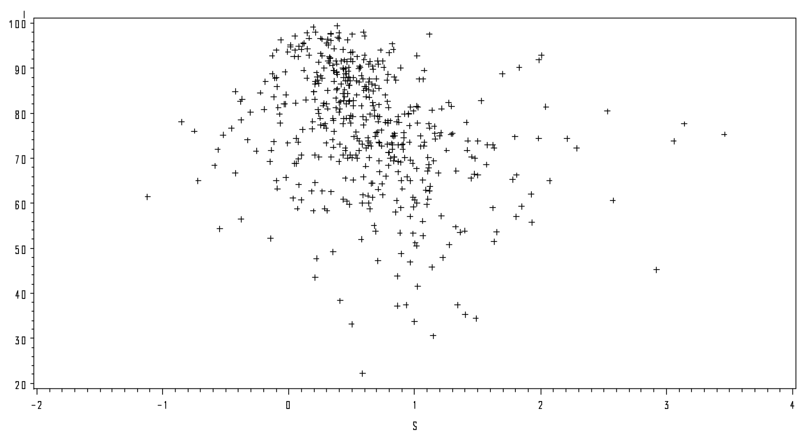


Figure 6. Plots of factor scores for crystallized abilities.

(a) Analogies



(b) Swedish WAIS Information



(c) Synonyms

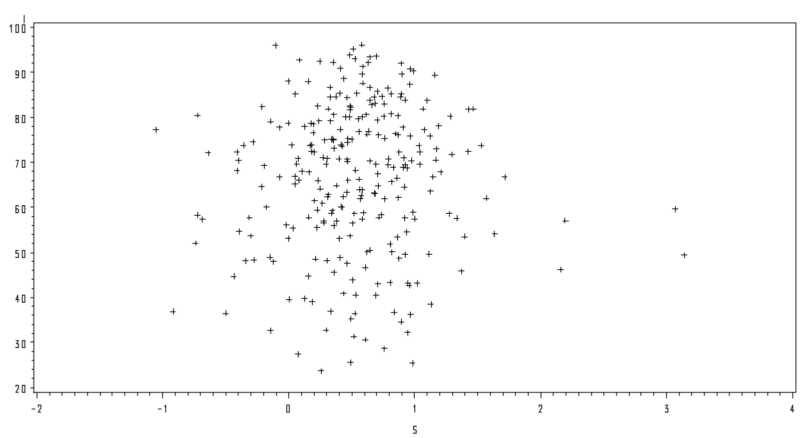


Figure 7. Sample and model estimated means for fluid abilities.

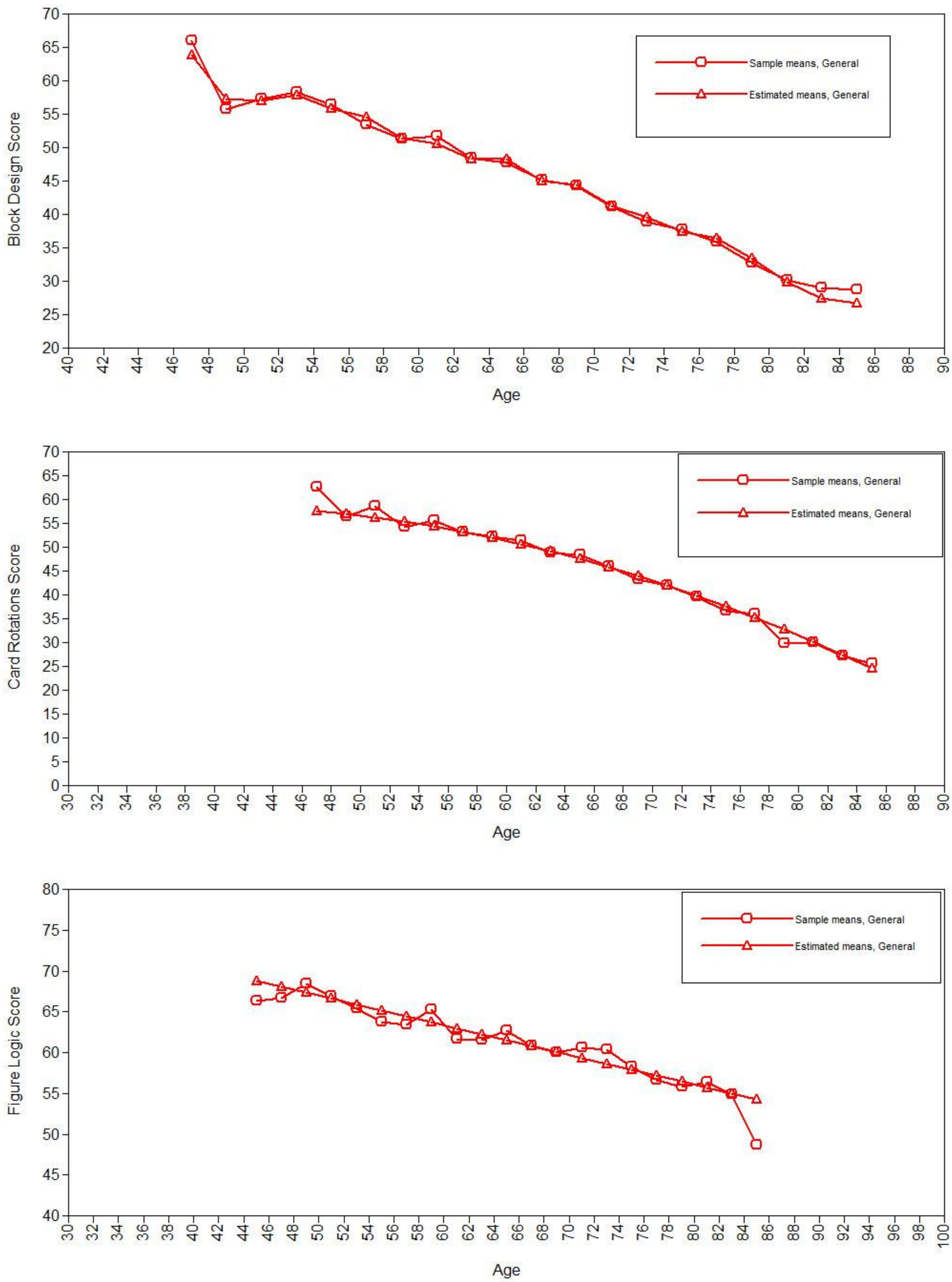
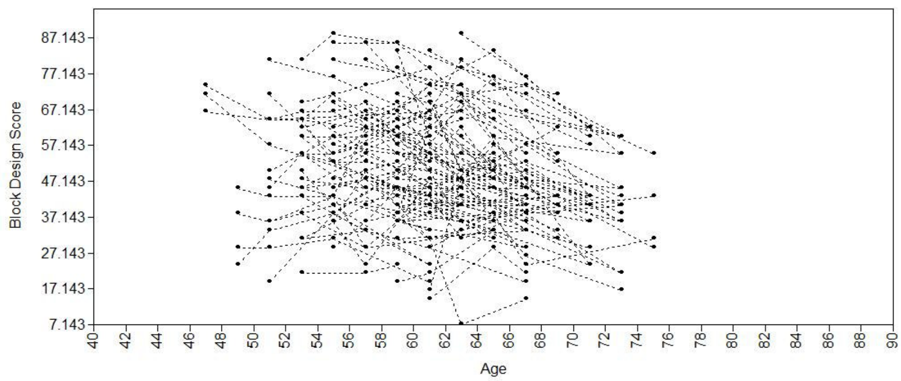
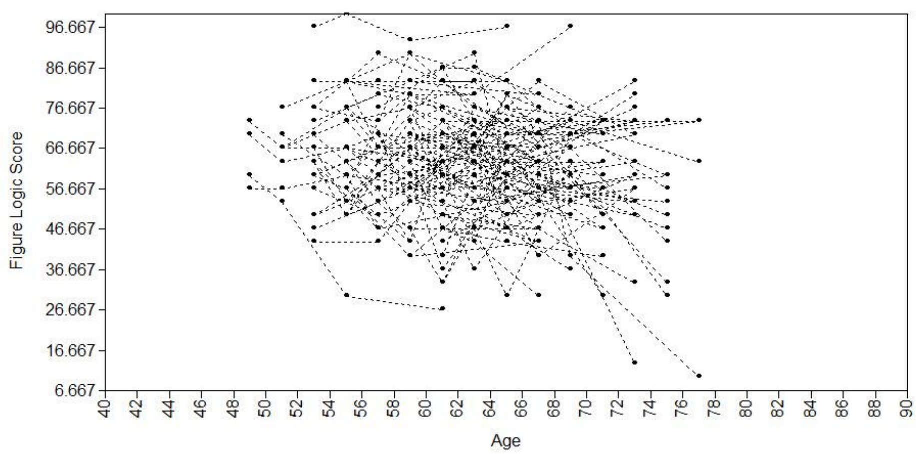


Figure 8. Observed individual curves for fluid abilities.

(a) Block Design



(b) Figure Logic



(c) Card Rotations

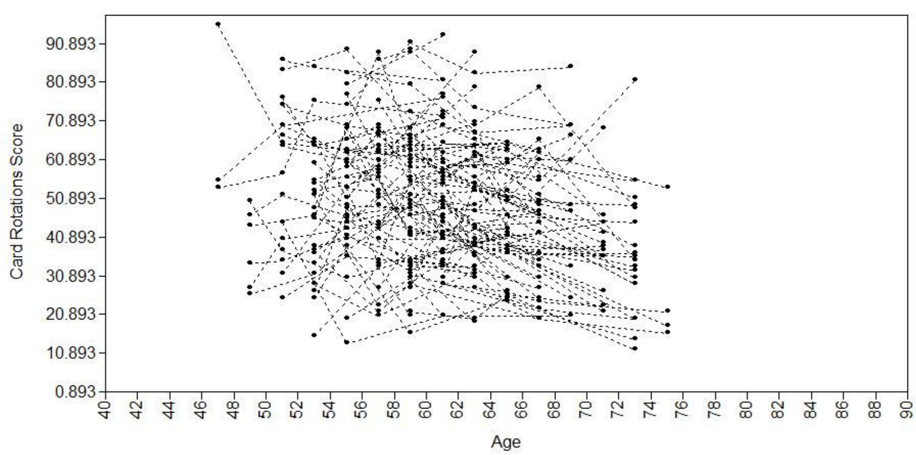


Figure 9. Estimated individual curves for fluid abilities.

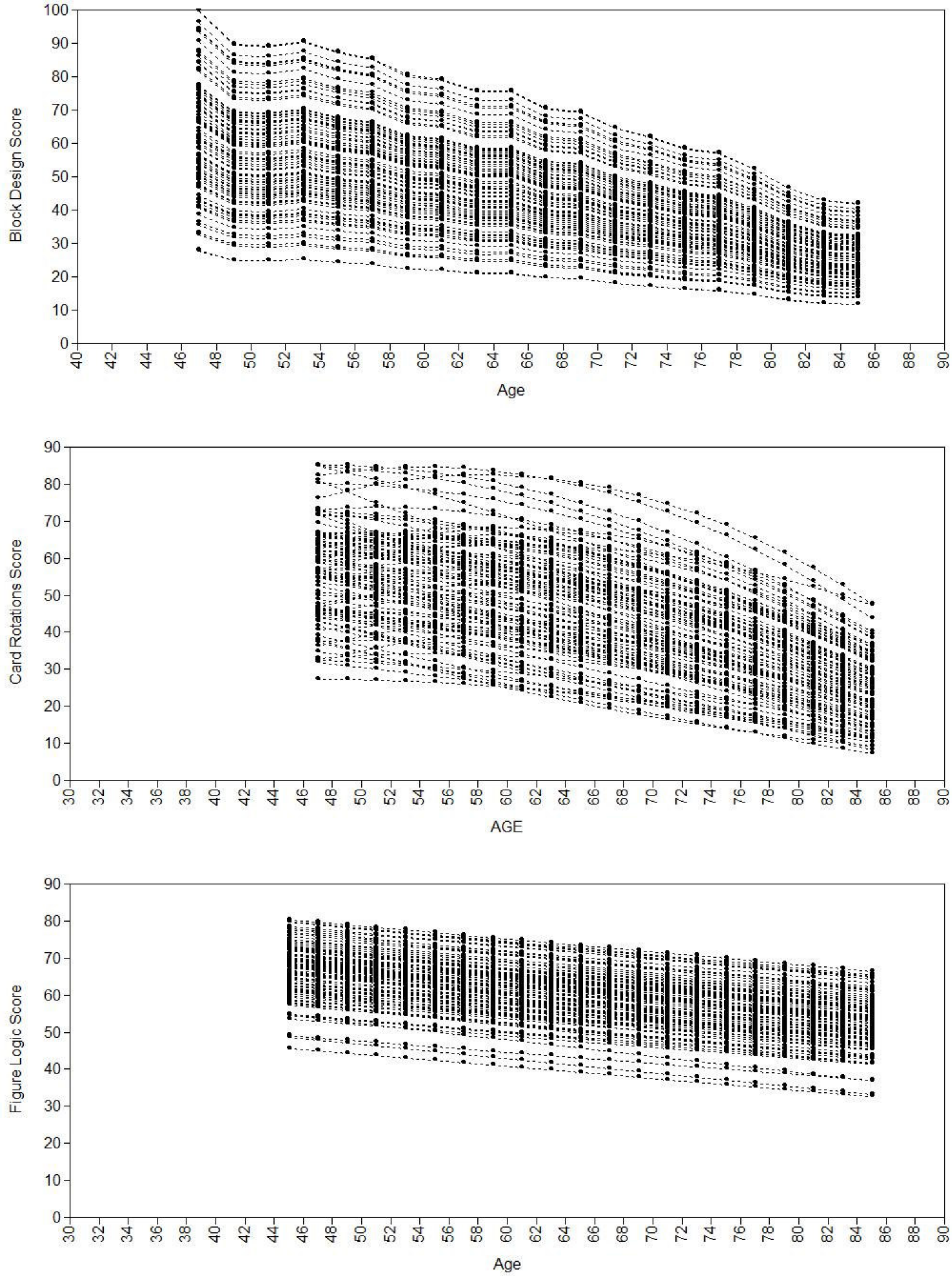
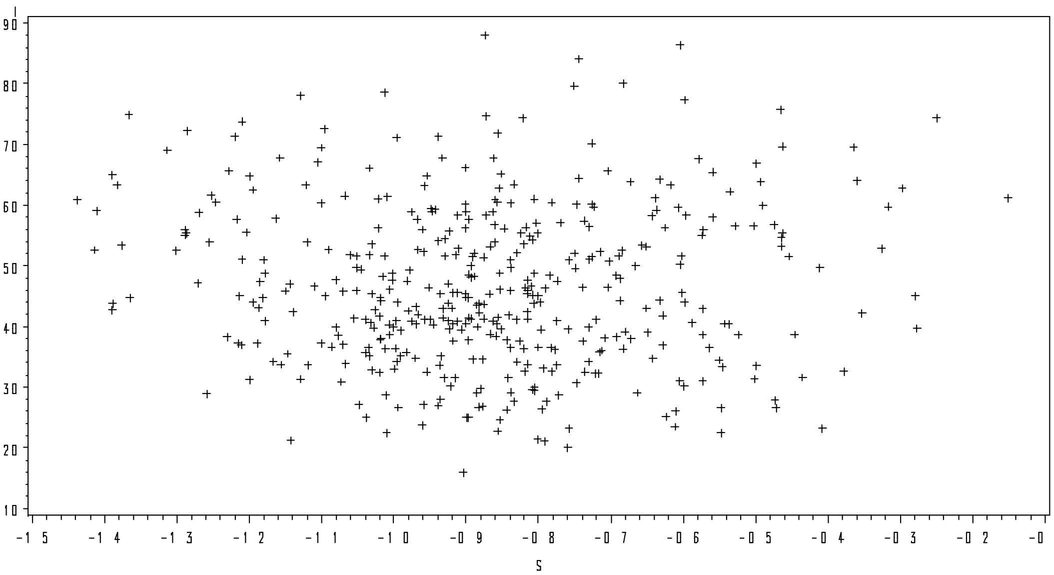


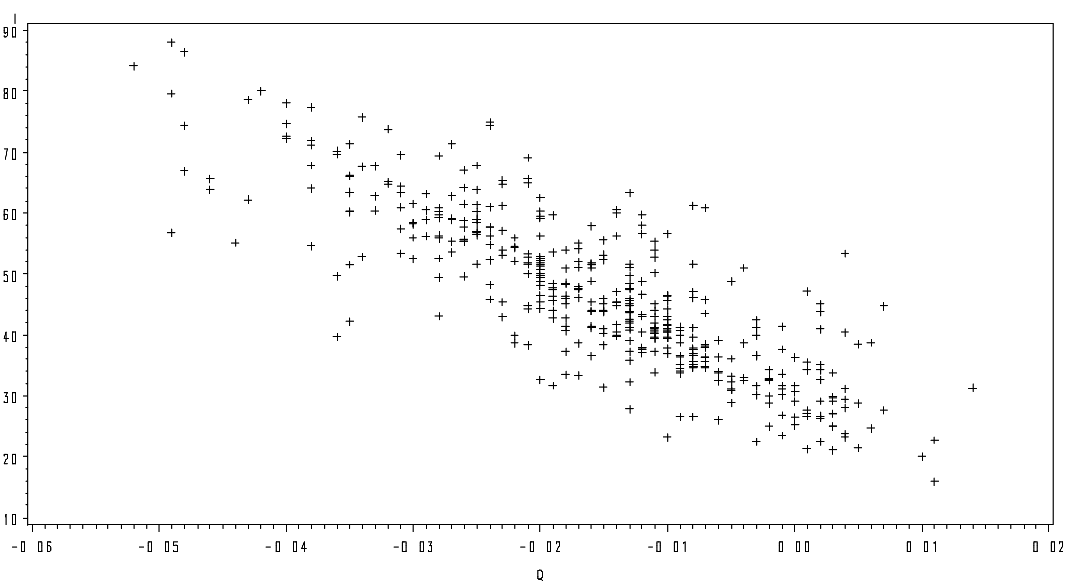
Figure 10. Plots of factor scores for fluid abilities.

(a) Card Rotations

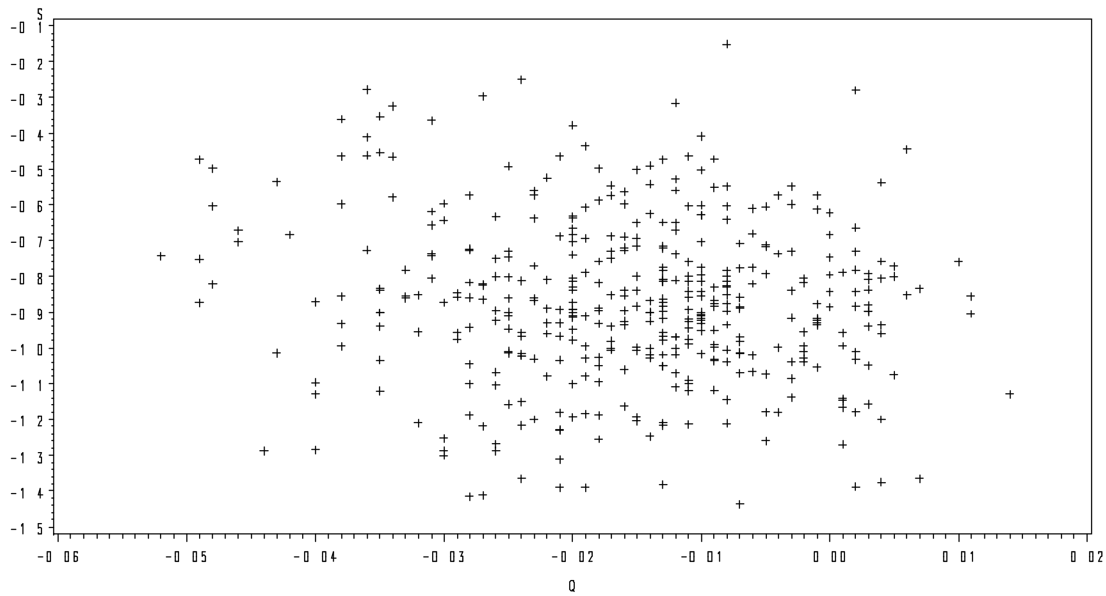
i. Plot of slope by intercept factor scores



ii. Plot of quadratic by intercept factor scores



iii. Plot of quadratic by slope factor scores



(b) Figure Logic: Plot of slope by intercept factor scores

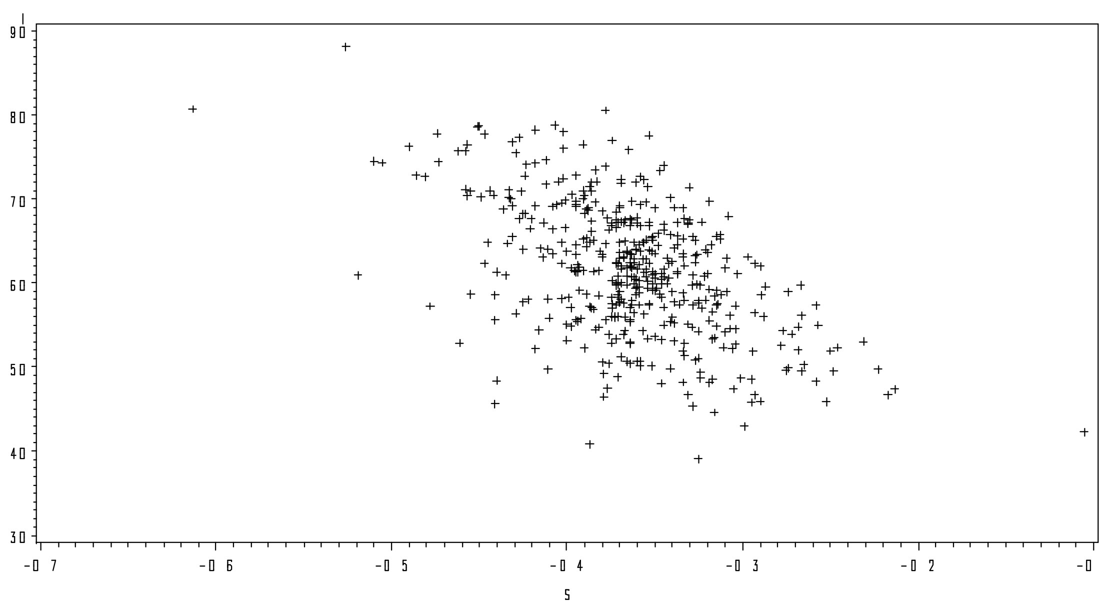


Figure 11. Proportion of variance estimates over time for crystallized abilities.

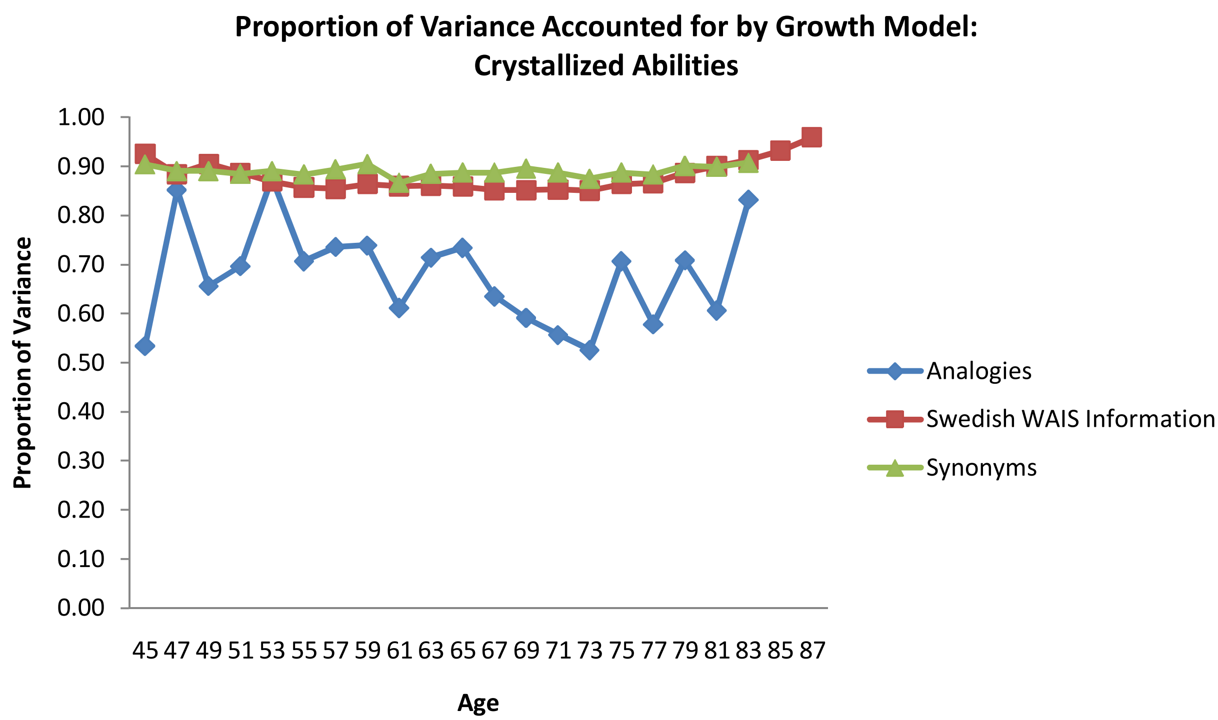


Figure 12. Proportion of variance estimates over time for fluid abilities.

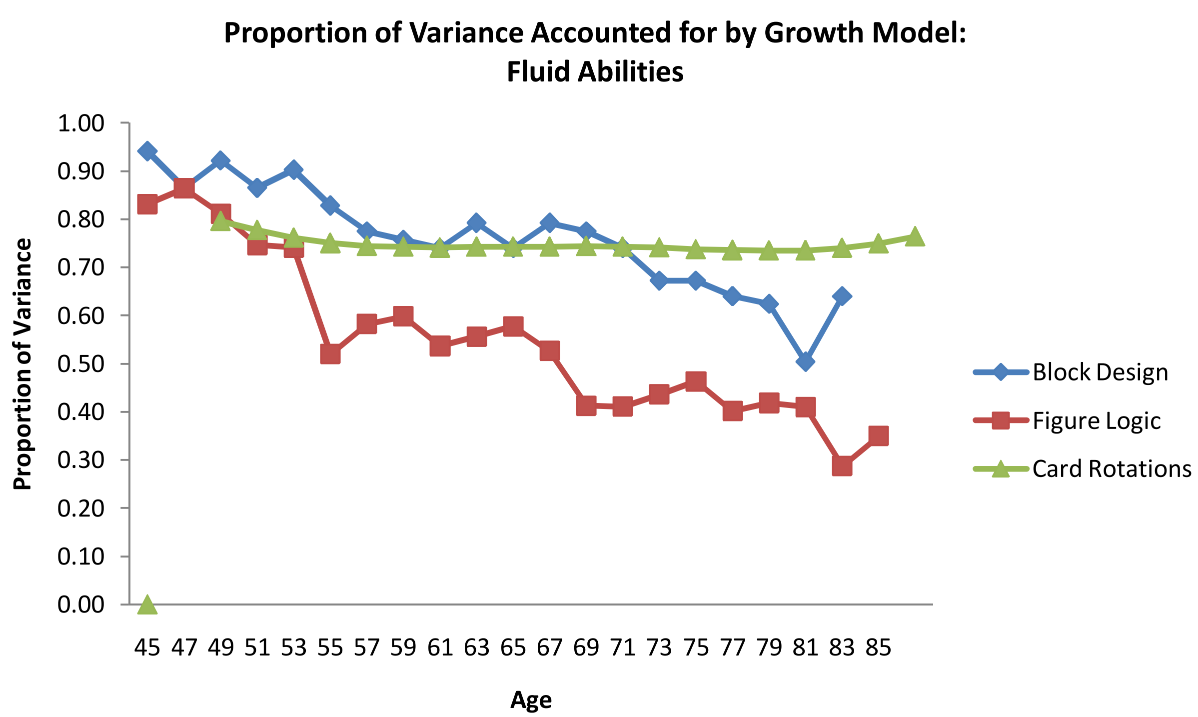


Figure 13. Proportion of variance accounted for by intercept factor.

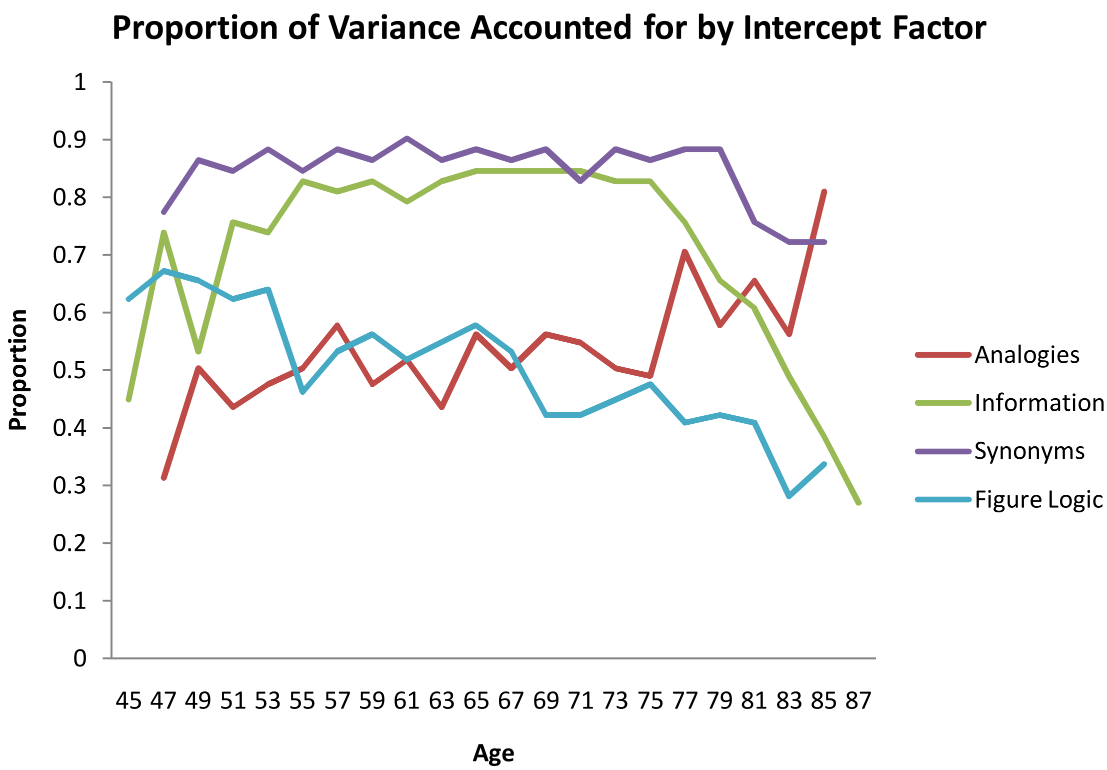


Figure 14. Proportion of variance accounted for by slope factor for crystallized abilities.

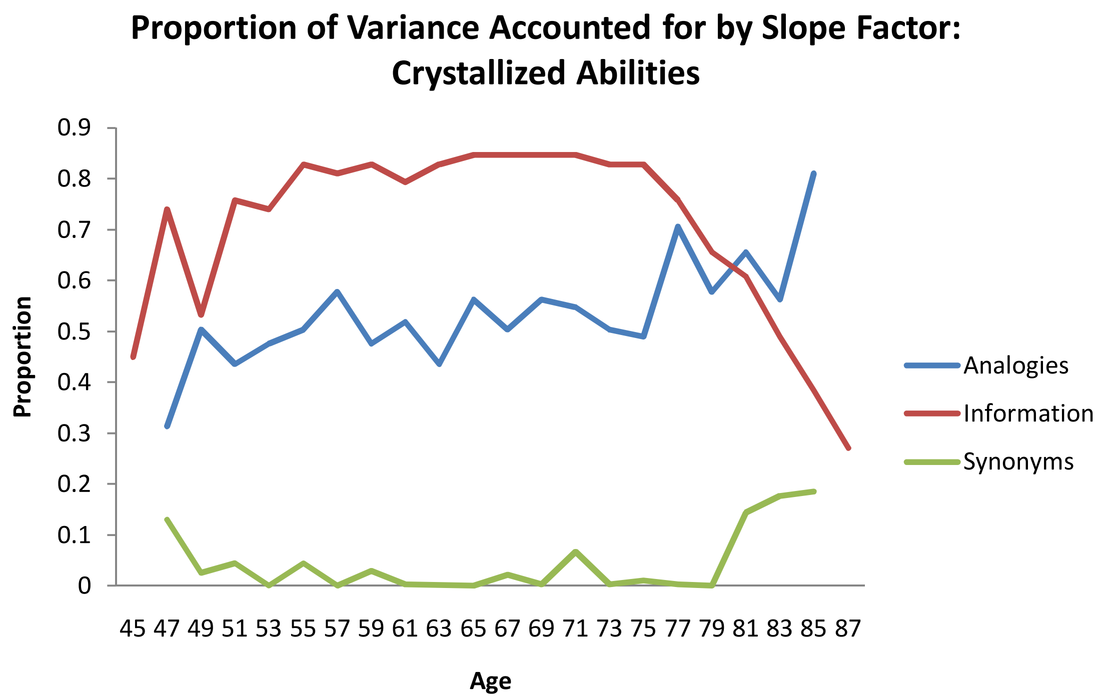


Figure 15. Proportion of variance accounted for by slope factor for fluid abilities.

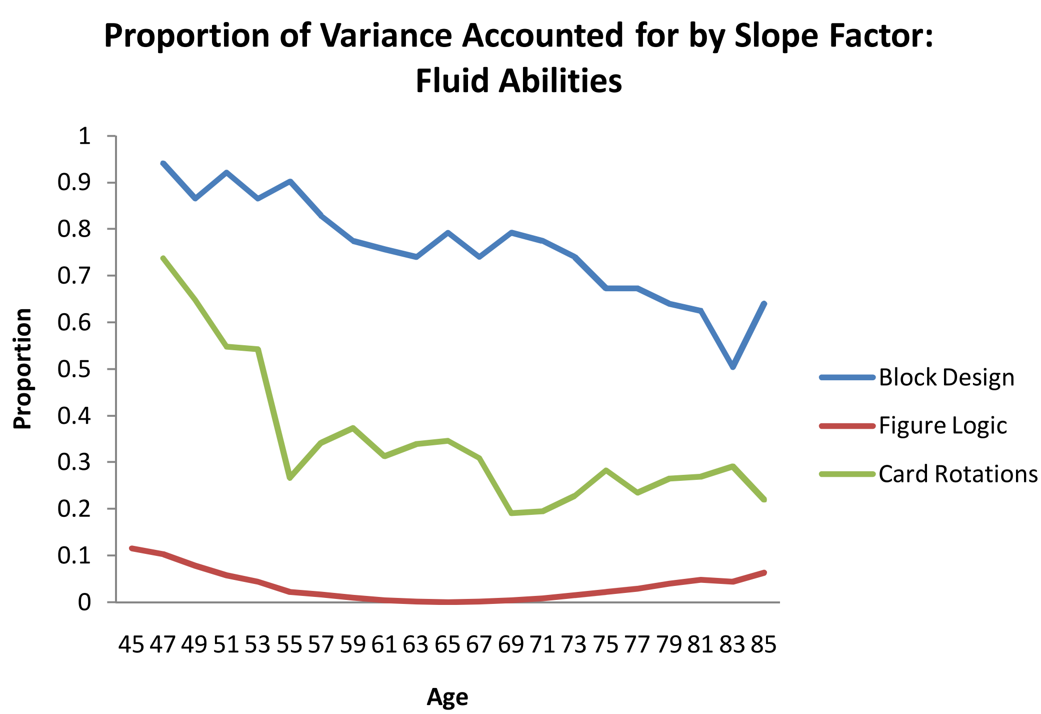
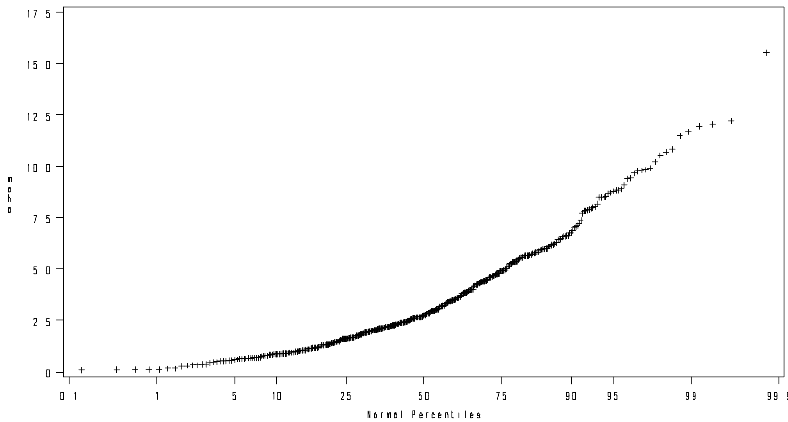
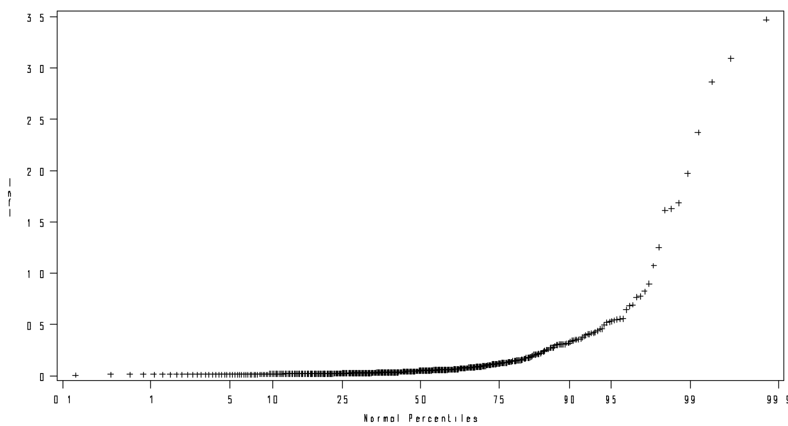


Figure 16. Probability plots of influence statistics for analogies.

(a) Mahalanobis distance



(b) Influence



(c) Loglikelihood

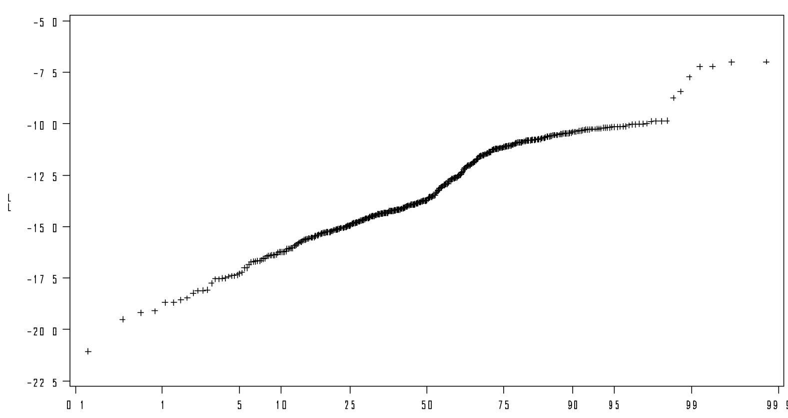
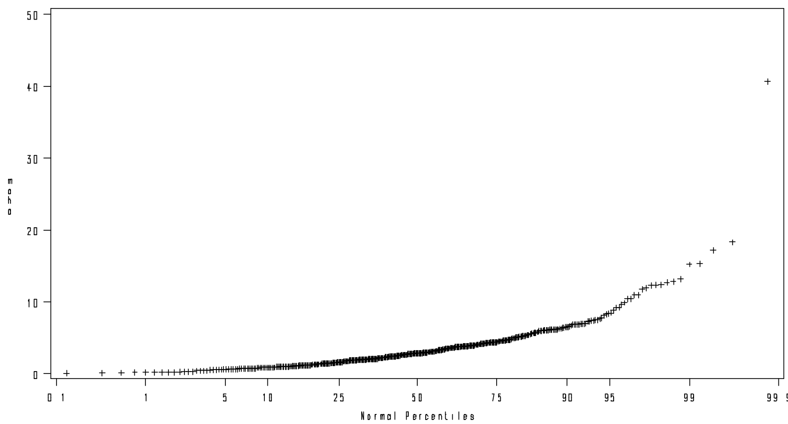
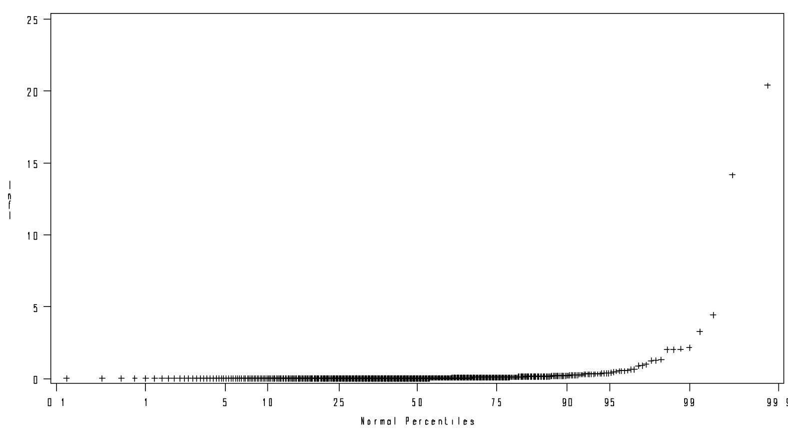


Figure 17. Probability plots of influence diagnostics for Koh's Block Design task (prior to removal of outliers).

(a) Mahalanobis distance



(b) Influence



(c) Loglikelihood

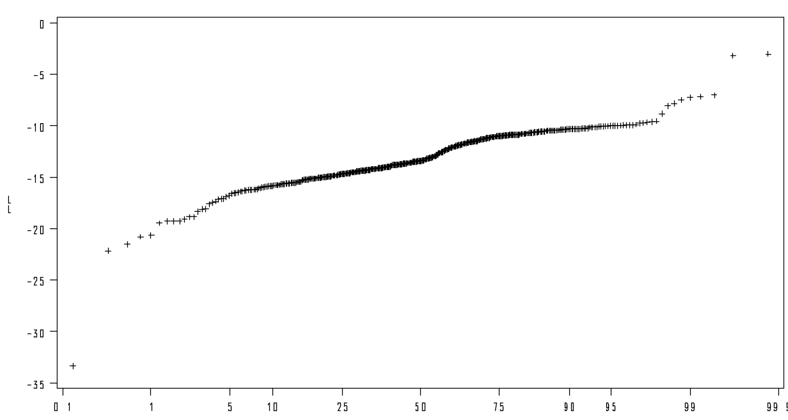
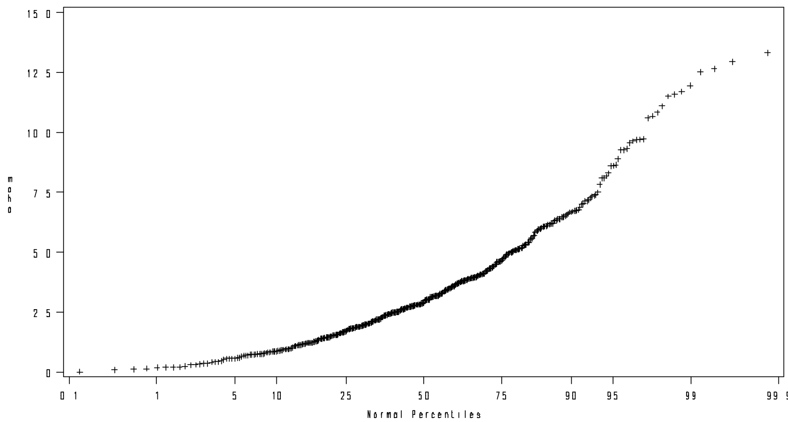
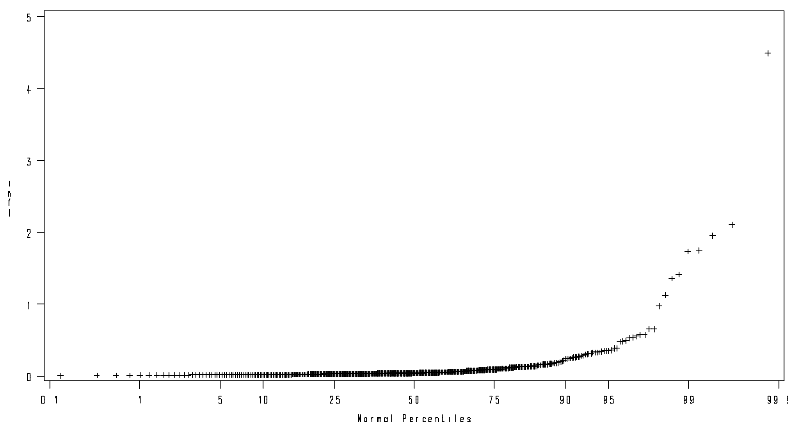


Figure 18. Probability plots of influence diagnostics for Koh's Block Design task (without outliers).

(a) Mahalanobis distance



(b) Influence



(c) Loglikelihood

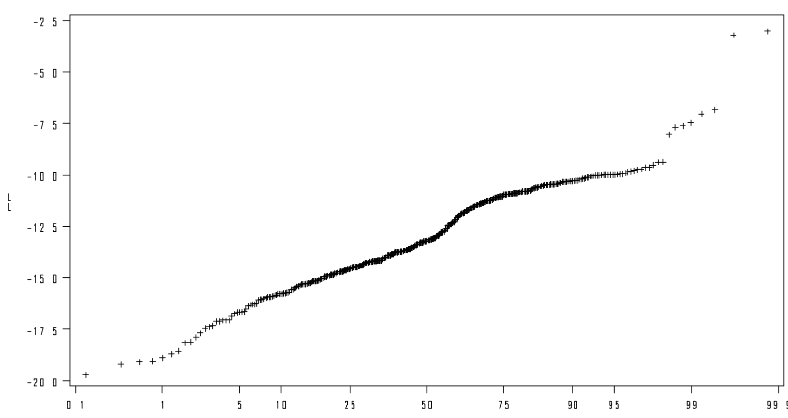
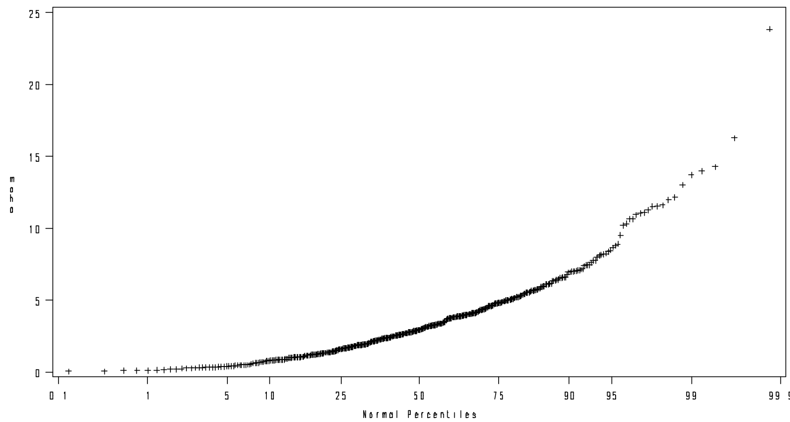
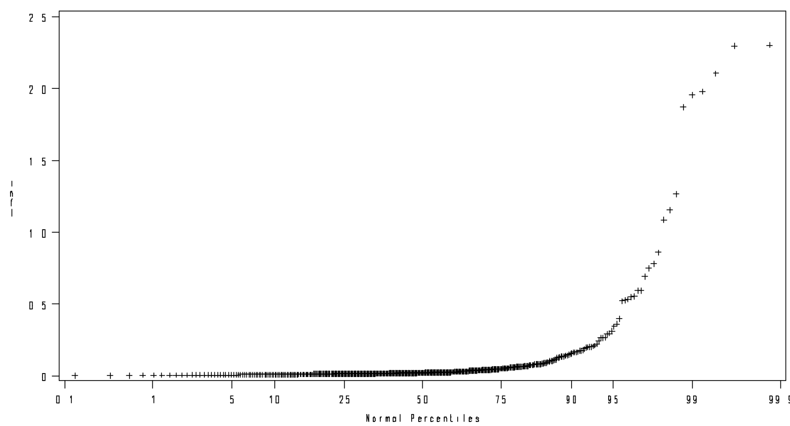


Figure 19. Probability plots of influence diagnostics for figure logic.

(a) Mahalanobis distance



(b) Influence



(c) Loglikelihood

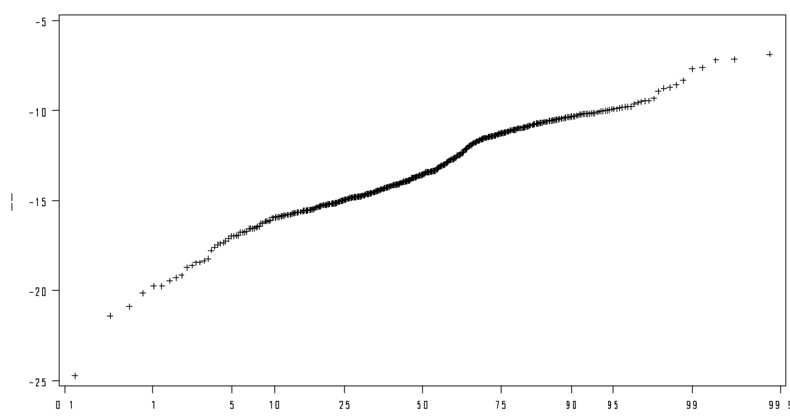
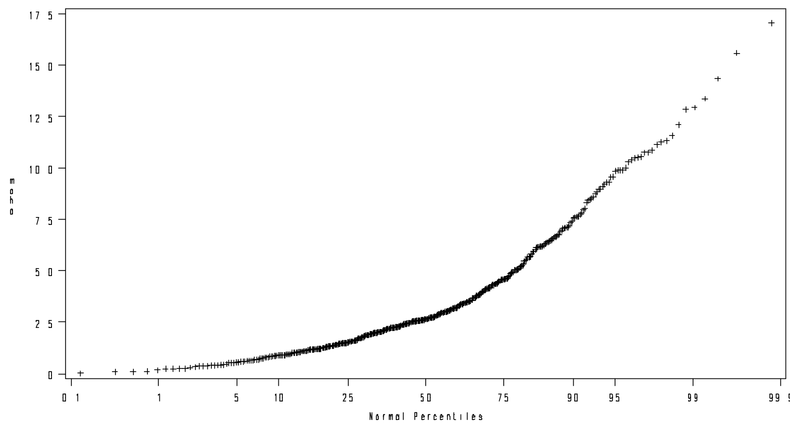
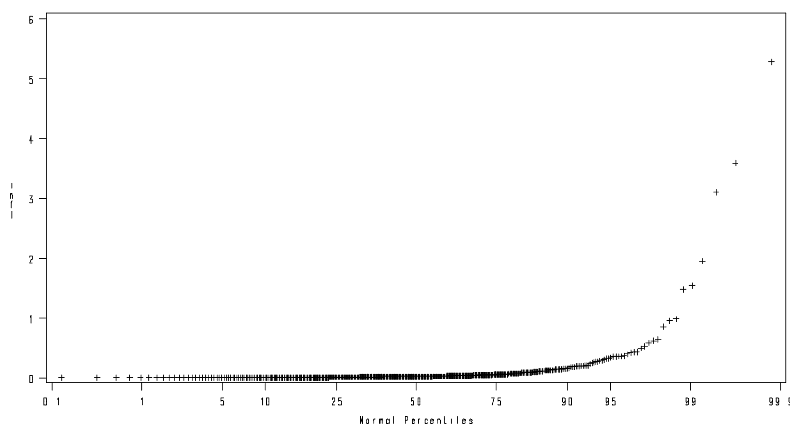


Figure 20. Influence diagnostics for Swedish WAIS Information (after removal of outliers).

(a) Mahalanobis distance



(b) Influence



(c) Loglikelihood

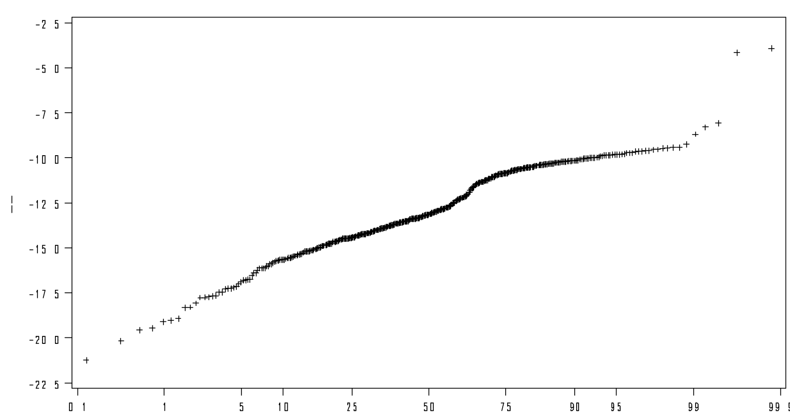
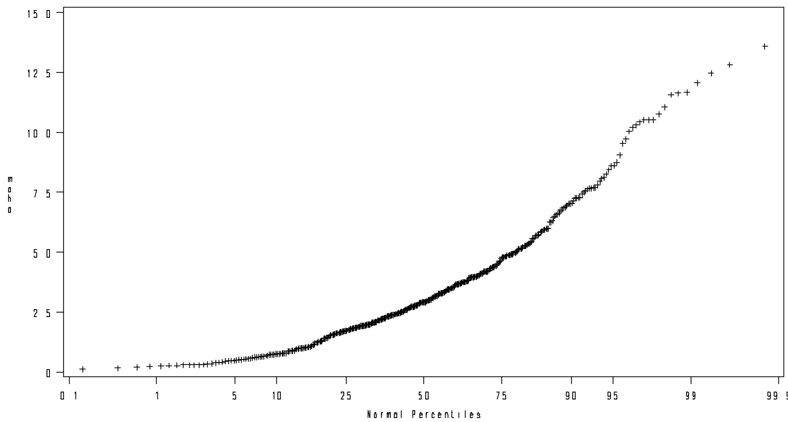
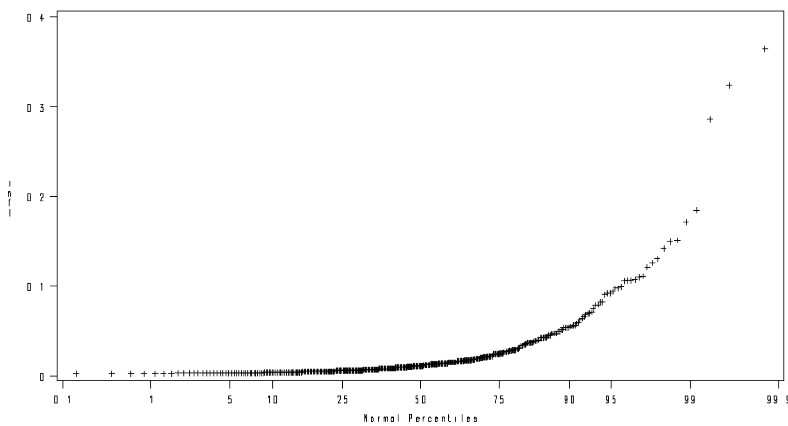


Figure 21. Probability plots of influence diagnostics for card rotations.

(a) Mahalanobis distance



(b) Influence



(c) Loglikelihood

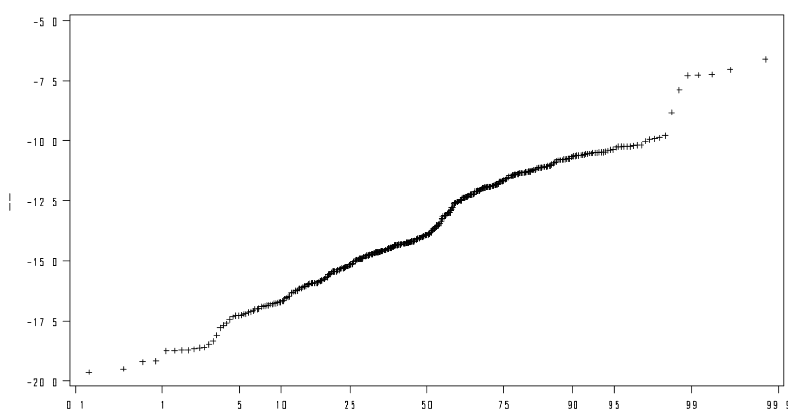
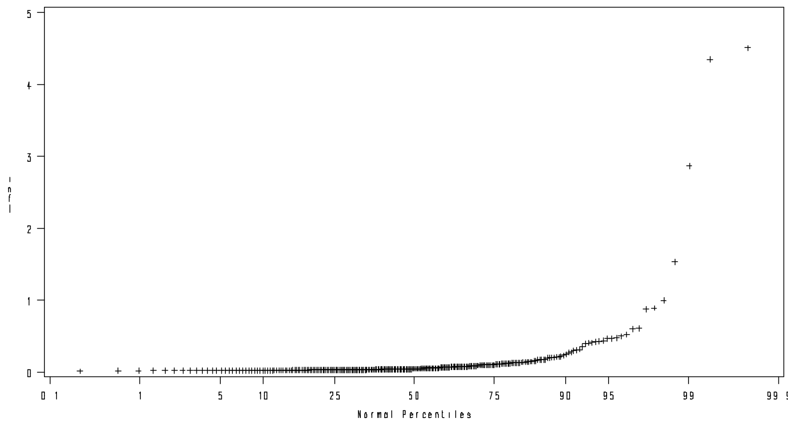
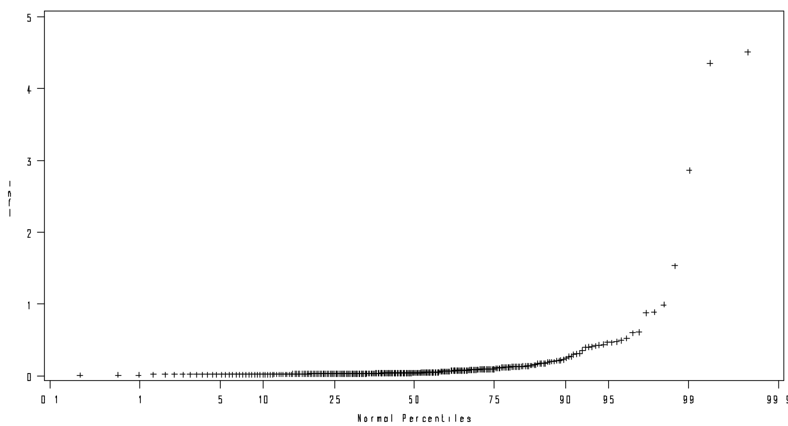


Figure 22. Probability plots of influence diagnostics for synonyms.

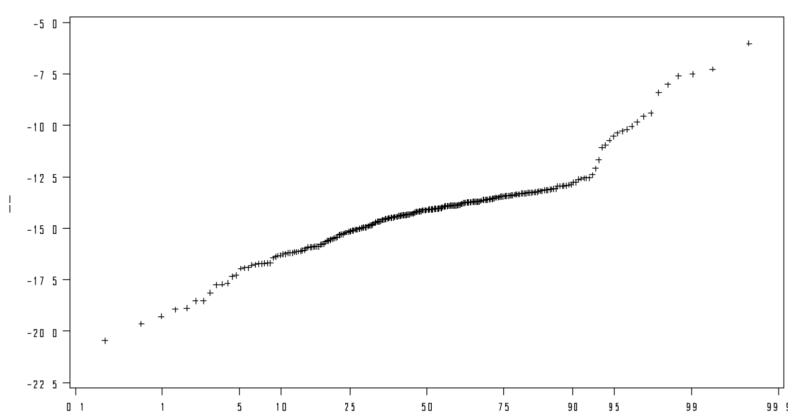
(a) Mahalanobis distance



(b) Influence



(c) Loglikelihood



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EDUCATION

University of Missouri-Columbia 2010
Ph.D. in Quantitative Psychology
Dissertation: *Using nested structures to select models for developmental trajectories of cognitive abilities in adulthood*
Minor: College Teaching

University of Missouri-Columbia
M.A. in Quantitative Psychology 2007
Thesis: *A factor structure with means confirmatory factor analytic approach to multitrait-multimethod models*

The College of Saint Rose
B.A. in Psychology 2002
Senior Seminar Project: *Now and later: The role of personality and cognition in considering the future*

The Sage Colleges
A.A. in Humanities 2000
Area of Concentration: Philosophy

HONORS AND AWARDS

Robert S. Daniel Teaching Fellowship	2007-2008
Research Grant, College of Saint Rose	2002
Outstanding Senior Award, College of Saint Rose	2002
Psi Chi Honor Society in Psychology	2001-2002

PROFESSIONAL MEMBERSHIPS

Society for Teaching of Psychology	2008-Present
Psychometric Society	2007-Present
Association for Psychological Science	2006-2007
American Psychological Association	2006
Eastern Psychological Association	2000-2003

SERVICE

Faculty Advisory Board, The Alliance, Westminster College	2009-Present
Faculty Advisor to Undeclared Majors, Westminster College	2009-Present
Graduate Advisor to Psi Chi, University of Missouri	2008-2009
Quantitative Search Committee, University of Missouri	2006-2007
Quantitative Search Committee, University of Missouri	2005-2006

TEACHING INTERESTS

Introductory (General) Psychology
Statistics for Psychology
Experimental Design in Psychology
Psychometrics (a.k.a., tests and measures)
Other, more advanced, research methods-related courses
Substantive courses as needed (e.g., social psychology, human sexuality, learning)

TEACHING EXPERIENCE

Westminster College, Fulton, MO 2010
Visiting Instructor – "PSY 411: Abnormal Psychology"
Developed syllabus and overall course structure, prepared and gave lectures, developed and graded assignments and tests.

Westminster College, Fulton, MO
Visiting Instructor – "PSY 113: Psychology as a Social Science" 2009-2010
Developed syllabus and overall course structure, prepared and gave lectures, developed and graded assignments and tests.

Westminster College, Fulton, MO
Visiting Instructor – "PSY 301: Human Sexual Behaviors" 2009
Developed syllabus and overall course structure, prepared and gave lectures, developed and graded assignments and tests.

University of Phoenix Online Campus 2009-Present
Instructor – "PSY 315: Statistical Reasoning in Psychology"
Facilitated independent learning of course material in an online setting through the use of discussion boards, assigned discussion questions, text-assignments and writing assignments, graded assignments and provided weekly feedback to students.

University of Missouri-Columbia, Columbia, MO 2007-2009
Instructor – "PSY 3020: Research Methods II"
Developed syllabus and overall course structure, prepared and gave lectures, developed and graded assignments and tests.

University of Missouri-Columbia, Columbia, MO 2006-2007
Teaching Assistant – to Melanie Sheldon "PSY 3020: Research Methods II"
Taught lab sections wherein students were instructed in the use of SPSS for Windows for statistical analyses, graded exams

University of Missouri-Columbia, Columbia, MO
Teaching Assistant – to Adam Hafdahl "PSY 3020: Research Methods II" 2005
Taught lab sections wherein students were instructed in the use of SPSS for Windows for statistical analyses, graded assignments

University of Missouri-Columbia, Columbia, MO
Teaching Assistant – to Colleen Sinclair "PSY 4975: Experiments in Social Psychology" 2005
Graded papers, met with and advised students on projects, grade database

University of Missouri-Columbia, Columbia, MO
Teaching Assistant – to Colleen Sinclair "PSY 3010: Research Methods I" 2005
Maintained grade database, took notes for disabled students

University of Missouri-Columbia, Columbia, MO
Teaching Assistant – to Gary Brase "PSY 3020: Research Methods II" 2004
Graded papers, helped develop tests, taught labs

University of Missouri-Columbia, Columbia, MO
Teaching Assistant – to Denis McCarthy "PSY 3860: Law and Psychology" 2004
Collaborated on curriculum and exam development, graded all written work.

Mildred Elley School of Business, Albany, NY
Instructor – "Introductory Psychology" 2003-2004
Developed syllabus, exams, lectures and assignments, evaluated students

RESEARCH INTERESTS

Substantive Topics
- Mediational/Moderational Relationships among Personality Variables
- Control (Desire or Motivation for, Locus of)
- Sexuality-related topics
- Education-related topics

Methodological Topics
- Structural Equation Modeling (i.e, latent variable or factor models)
- Latent growth models
- Psychometric models
- Two-part modeling

RESEARCH EXPERIENCE

University of Missouri-Columbia, Columbia, MO 2004-Present
Graduate Student
Learning and conducting research with mentor, manuscript preparation, study design, statistical analysis, development of models

University of Missouri-Columbia, Columbia, MO 2006-2007
Research Assistant to Dr. Kenneth J. Sher
Conducted statistical analyses, manuscript preparation, study design

Missouri Institute of Mental Health, Columbia, MO 2005-2007
Senior Research Laboratory Assistant to Dr. Lori McKinley
Conducted statistical analyses, gathered, entered and managed data, produced reports for dissemination to workers, evaluation of programs

Berkshire Farm Center and Services for Youth, Canaan, NY
Data Specialist 2003-2004
Entered, analyzed, and reported data for monthly and annual reports

The College of Saint Rose, Albany, NY
Research Assistant to Dr. Nancy Dorr 2000-2003
Collaborated on project designs, conducted literature searches, collected and analyzed data, prepared posters and manuscripts

PUBLICATIONS

Hafdahl, A. H., & Williams, M. W. (2009). Meta-analysis of correlations revisited: Attempted replication and extension of Field's (2001) simulation studies. *Psychological Methods, 14*(1), 24-42.

Williams, M. A., Sher, K. J., Wood, P. K., & Bartholow, B. D. (2007). A two-part model of alcohol sensitivity. *Alcoholism: Clinical and Experimental Research, 31*(6) supplement, 188A.

Williams, M. A. (2004). Now and later: The role of personality and cognition in considering the future. *Psi Chi Journal of Undergraduate Research, 9,* 82-88.

MANUSCRIPTS IN PREPARATION

Williams, M. W., Sher, K. J., Bartholow, B. D., & Wood, P. K. (In Preparation). A two-part model of alcohol sensitivity: An alternative psychometric approach to conditional response data.

Williams, M. W., & Wood, P. K. (In Preparation). A factor mean approach to multitrait-multimethod models.

POSTER PRESENTATIONS

Williams, M. A., Sher, K. J., Wood, P. K., & Bartholow, B. D. (2007, July). A two-part model of alcohol sensitivity. *Poster presented at the annual scientific meeting of the Research Society on Alcoholism.*

Williams, M. A., & Wood, P. K. (2007, May) A factor mean confirmatory factor analytic approach to multitrait-multimethod models. *Poster presented at the annual Association for Psychological Science Convention.*

Williams, M. A., Sher, K. J., Wood, P. K., & Bartholow, B. D. (2007, February). A two-part model of alcohol sensitivity. *Poster presented at the annual Guze symposium.*

Williams, M. A. (2004, April). Now and later: The role of personality and cognition in considering the future. *Poster presented at the annual meeting of the Eastern Psychological Association.*

Dorr, N., Williams, M. A., & Walpole, J. (2003, March). The relationship between locus of control and stress: Exploring the mediational role of cognitive appraisals. *Poster presented at the annual meeting of the Eastern Psychological Association.*

Williams, M. A., & Tedesco, C. (2003, March). Parenting, personality and well-being: The examination of a mediational hypothesis. *Poster presented at the Psi Chi portion of the annual meeting of the Eastern Psychological Association.*

Williams, M. A., Dorr, N., & Walpole, J. (2002, March). A diary study of perceived sexism. *Poster presented at the Psi Chi portion of the annual meeting of the Eastern Psychological Association.*

REFERENCES

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