A STUDY OF TEACHER KNOWLEDGE

AS SECONDARY MATHEMATICS TEACHERS USE A NEW TECHNOLOGY

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by
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The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

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ABSTRACT

Professional organizations (e.g., NCTM and NCSM) and educational leaders advocate for increased use of technology in high school mathematics. Educational researchers find that teachers' beliefs and knowledge influence use of technology and student learning (e.g., Hall & Hord, 1987, 1991; Mitchell, Bailey, & Monroe, 2007; Niess, 2005; Zbiek, Heid, Blume, & Dick, 2007; Zbiek & Hollebrands, 2008). Yet, we lack research examining what knowledge teachers need to effectively use specific technologies or how teachers enact this knowledge. Additionally, the conceptualization of teacher knowledge related to using technology in mathematics is at the early stages. Thus, the purpose of this qualitative case study was to investigate and analyze what knowledge secondary teachers draw upon as they enact a new technology (i.e., the TI-*n*spire™ calculator) in mathematics classrooms. Analysis of the data revealed: (1) Teaching with and reflecting on the use of the TI-*n*spire™ helps teachers to develop PCK with the TI-*n*spire™. (2) Teachers may develop specific components of their pedagogical content knowledge with technology before others, and (3) teachers consider the TI-*n*spire™ a "discovery-based" mathematics learning tool and believe students investigate and learn mathematics on the handhelds when they structure learning environments to support the nature of this type of instruction. The research findings can inform the design and implementation of teacher preparation and professional development programs and ultimately improve the teaching and learning of mathematics.

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CHAPTER 1: INTRODUCTION

For many years, the National Council of Teachers of Mathematics (NCTM), other national organizations, and professional committees have recommended the use of technology in the classroom (NCTM 1989, 2000, 2008; National Research Council [NRC] 1990; National Council of Supervisors of Mathematics [NCSM] 1998, 2007). Most recently, NCTM (2008) released a new position statement, *The Role of Technology in the Teaching and Learning of Mathematics*. They note, "Curricula and courses of study should incorporate instructional technology in learning outcomes, lesson plans, and assessments of students' progress" (p. 1). This statement also explains why the use of technological tools is important:

> With guidance from effective mathematics teachers, students at different levels can use these tools to support and extend mathematical reasoning and sense making, gain access to mathematical content and problem-solving contexts, and enhance computational fluency. In a well-articulated mathematics program, students can use these tools for computation, construction, and representation as they explore problems. The use of technology also contributes to mathematical reflection, problem identification, and decision-making. (p. 1)

In short, current technology should be used in mathematics classrooms as tools to support and extend students' mathematical understandings and thinking processes.

Historically, technology has not always been accessible to large numbers of classroom teachers although many have recommended the use of technology in the classroom, For example, when handheld calculators were first introduced, their cost prohibited widespread use. However, as the cost decreased, more students were given access to handheld calculators. It is estimated that it took 15 years (1975-1990) for most students to acquire scientific calculators and 10 years (1990-2000) for most students to access graphing calculators (Trouche, 2005). Based on a national survey, it has been reported that over 80 percent of U.S. high school teachers now use handheld graphing technology in their classrooms (Burrill, Allison, Breaux, Kastberg, Leatham, & Sanchez, 2002). The start of using the technology requires teacher knowledge. Zbiek and Hollebrands (2008) state, "The few studies that closely link practice with teaching acts suggest that teachers' conceptions and knowledge of mathematics and technology remain very influential for student learning" (p. 322). Thus, understanding this knowledge is important for preparing teachers to use technology in their classrooms and ultimately impact student learning.

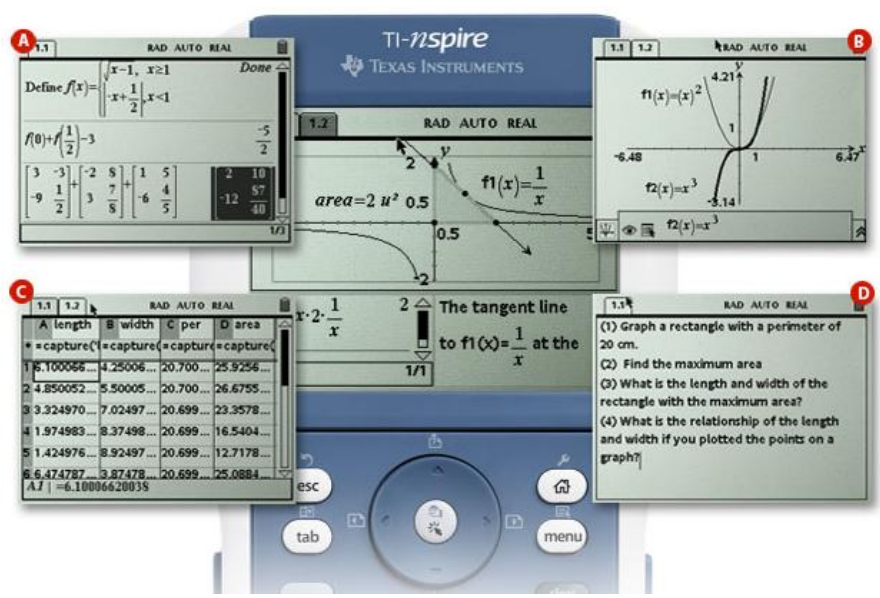
While technology has become more accessible to classroom teachers, it has presented challenges to teachers (e.g., time must be devoted to learning the technology as well as learning how to teach with the technology and learning how to help students learn mathematics with the technology) as it has evolved. For example, hand-held calculator technology has changed significantly from the first desk calculators that appeared in the 1970s when hand-held calculators such as the Ragen microelectronic calculators had the basic four operations (+, –, ×, ÷). Teachers must decide how to balance paper-and-pencil methods, mental computation, and computation with the calculator. In 1977, a Texas Instrument TI-30 hand-held calculator had four basic operations, other scientific functions, and basic memory. By 1980, scientific and programmable calculators appeared with fewer integrated circuits and with specialized modules (e.g., programs for insurance or marine navigation) installed on them. These devices handled trigonometric functions, combinations, permutations, factorials, percents, and absolute value. The advances in technology required teachers to decide whether or not, how, and when to use trigonometric tables, calculators, or a combination of these two in mathematics classrooms. Technology has become more sophisticated over time requiring a greater investment of time in learning how to use and implement the technology.

In 1985, engineers built graphing calculators with advanced display and visualization as well as with capabilities for plotting graphs, solving simultaneous equations and symbolic equations, and manipulating unnamed variables. The advent of this new technology forced teachers to decide when and how to use technology to graph and analyze functions and equations. In 1995 symbolic calculators with computer algebra systems and geometry software appeared (Trouche, 2005). These hand-held devices supported, and currently support, extensive symbolic manipulation (e.g., partial and total differentiation, the simplification of symbolic expressions with assumptions and constraints, and series expansion and summation). Just 25 years ago, these capabilities could only be found in massive research computers (Roschelle & Singleton, 2008), but now secondary mathematics teachers must make decisions about how and when to appropriately introduce mathematical ideas to students with these tools. Secondary teachers have had to learn to use and implement new calculator features and applications in their mathematics classrooms as calculator technology has evolved.

Not only has technology become more sophisticated over time but also we have seen an explosion of technological tools for mathematics classrooms in recent years. Currently, teachers and students use a number of technological tools such as SMART™ Boards, laptop computers, the Internet, personal digital assistants (PDAs) as well as calculators. In secondary mathematics classrooms, teachers integrate dynamic geometry software (e.g., Cabri geometry and Geometer's Sketchpad), calculators, and software permitting numerical or formal computation and algebraic manipulation (e.g., Doerr & Zangor, 2000; Huntley, Rasmussen, Villaburi, Santong, & Fey, 2000).

New technology (e.g., the TI-*n*spire™) highlights and displays mathematics in new and dynamic ways, which can challenge teachers' mathematical knowledge for teaching. For example, an algebra teacher must know more explicitly why and how an algebraic equation connects with a numerical and graphical representation or how the three representations inform one another to help build conceptual understanding while teaching and learning algebraic functions with algebra students since the students now have personal and easy access to all three representations. Currently, handheld graphing calculators assume the functionality of microcomputers (Kaput, 1992). With one of the newest technology tools, TI-*n*spire™ and TI-*n*spire CAS technology, educators and students can see multiple representations of a problem combined on a single screen or expressed on a single screen with dynamic links between representations (see Figure 1).

Figure 1. Four possible representations on the TI-nspire™.



These modern graphing calculators and hand-held computer algebra systems do calculations, create graphs, and display geometric figures thereby creating opportunities for further innovations in classroom use (Roschelle & Singleton, 2008). More specifically, the TI-*n*spire™ allows the user to access documents, a numeric and symbolic calculator, graphs, geometric figures, lists, spreadsheets, data plots, and statistics.

Representations (e.g., graphs, tables, and equations) can be dynamically linked, which means changes made to one representation of a problem are automatically and instantly reflected in other representations of the same problem on the same screen. With instant feedback and linked representations, teachers and students can make sense of mathematics such as how the change in a variable connects to the change in a graph in conceptual ways. With further visualization aids such as an overhead projector, the classroom discussions around the connections between these mathematical representations can foster shared conceptual understanding. Additionally, the dynamic linking of multiple representations provides real-time, interactive feedback for teachers and students to attempt and communicate about different problem-solving techniques. Ideally, the teacher should have knowledge of how to illustrate, exemplify, or model mathematical concepts with these representations.

As calculators have become more complex, they have placed more demands on teachers. For teachers to use technology effectively, they need more than access to the technology (Laborde, 2001; Mitchell, Bailey, & Monroe, 2007; Niess, 2008; Norton, 2006; Ruthven & Hennessy, 2002; Wachira, Keengwe, & Onchwari, 2008). Teachers need professional development opportunities, mathematics curriculum and educational policies that support the use of technology as well as knowledge related to how to integrate the technology in classroom practice. Wilson (2008) explains, "It is teachers who will make the difference between success and failure and it is teacher education that must serve as a major conduit that connects teachers with new technologies, research, curricula, and policies" (p. 415). Although there has been significant growth in the use of technology in mathematics classrooms, little research exists for how teachers use technological tools in secondary mathematics classrooms. However, researchers have begun to investigate how teachers engage students in mathematical reasoning, sense making, construction, representation, and problem solving with technological tools (e.g., Doerr & Zangor, 2000; Laborde, 2000; Marrades & Gutierrez, 2000; Mariotti, 2001; Niess, 2005). In order to utilize the potential of current technology in mathematically meaningful ways, further research is needed to understand what knowledge secondary classroom teachers draw upon. In the future, "reports need to provide description of teachers and teaching that occurs with technology in more detail than merely naming the type of technology involved" (Zbiek & Hollebrands, 2008, p. 336).

Teachers' beliefs and knowledge are critical to incorporating a technological innovation into one's teaching practices (e.g., Hall & Hord, 1987, 2001; Mitchell, Bailey, & Monroe, 2007; Niess, 2005; Zbiek, Heid, Blume, & Dick, 2007; Zbiek & Hollebrands, 2008). For instance, a teacher who is opposed to the use of a particular technological tool or who does not know how to use the tool will not likely attempt to use the technology (Niess, 2005). Whereas, a technological tool is more likely to be used when a teacher knows how to use it and values its use. Without knowledge about how, when, and in what ways to teach mathematics with technology, a teacher cannot possibly realize the vision (NCTM, 2008) of teaching and learning mathematics with technology. Unfortunately, successfully implementing a vision of a more technologically integrated approach is “yet to be clearly understood by the individual teacher” (Mitchell, Bailey, & Monroe, 2007, p. 76). Additionally, “substantial professional development and support is necessary for teachers to make informed decisions about how to best use handheld technology in their classroom” (Burrill et al., 2002, p. i).

Teachers’ success with technology is not immediate even with a lot of teaching experience (Heid & Blume, 2008). Success depends not only on teaching experience, but also on the integrated knowledge of: (a) the technology, (b) curriculum, (c) instructional strategies, (d) students’ mathematical understandings, and (e) assessment. This integrated knowledge is essential because teachers draw upon it in order to make decisions about how and when to use technologies, which directly impacts students’ mathematics learning experiences (Kendal & Stacey, 2001; NCTM, 2005). “Recent national studies in mathematics education and in related fields such as science (e.g., Martin, Mullis, Gonzalez, & Christowki, 2004) have left little doubt that pedagogy and content must be interwoven by teachers to achieve dynamic and effective environments with this evolving context for mathematics” (Grandgenett, 2008).

To improve the teaching and learning of mathematics, it is critical to facilitate and support the development and enactment of teacher knowledge related to the use of technologies. As we study teacher knowledge related to the use of technology, we must investigate the enactment of that knowledge in the classroom as well as the challenges that teachers face as they learn to enact new technologies. Knowing how to use a technological tool or knowing instructional strategies related to the use of the tool does not mean that that knowledge will be enacted in the classroom (Cuban, 2001; Trouche, 2005). Therefore, we need a better understanding of teacher knowledge necessary for effectively implementing new technologies, how to best facilitate the development of that teacher knowledge, how teachers use knowledge in implementing technology, and challenges teachers face as they enact this knowledge using new technologies in mathematics classrooms.

Research Purpose and Questions

The present study is designed to investigate and analyze what knowledge teachers draw upon as they enact a new technology in mathematics classrooms. More specifically, the study will focus on secondary mathematics teachers who are beginning to use TI-*n*spire™ calculators. The research questions underlying the study are: (a) What pedagogical content knowledge do secondary teachers draw upon when they begin to implement a new technology in their mathematics instruction? and (b) What orientations do secondary teachers hold about teaching mathematics with a new technology?

View of Teacher Knowledge

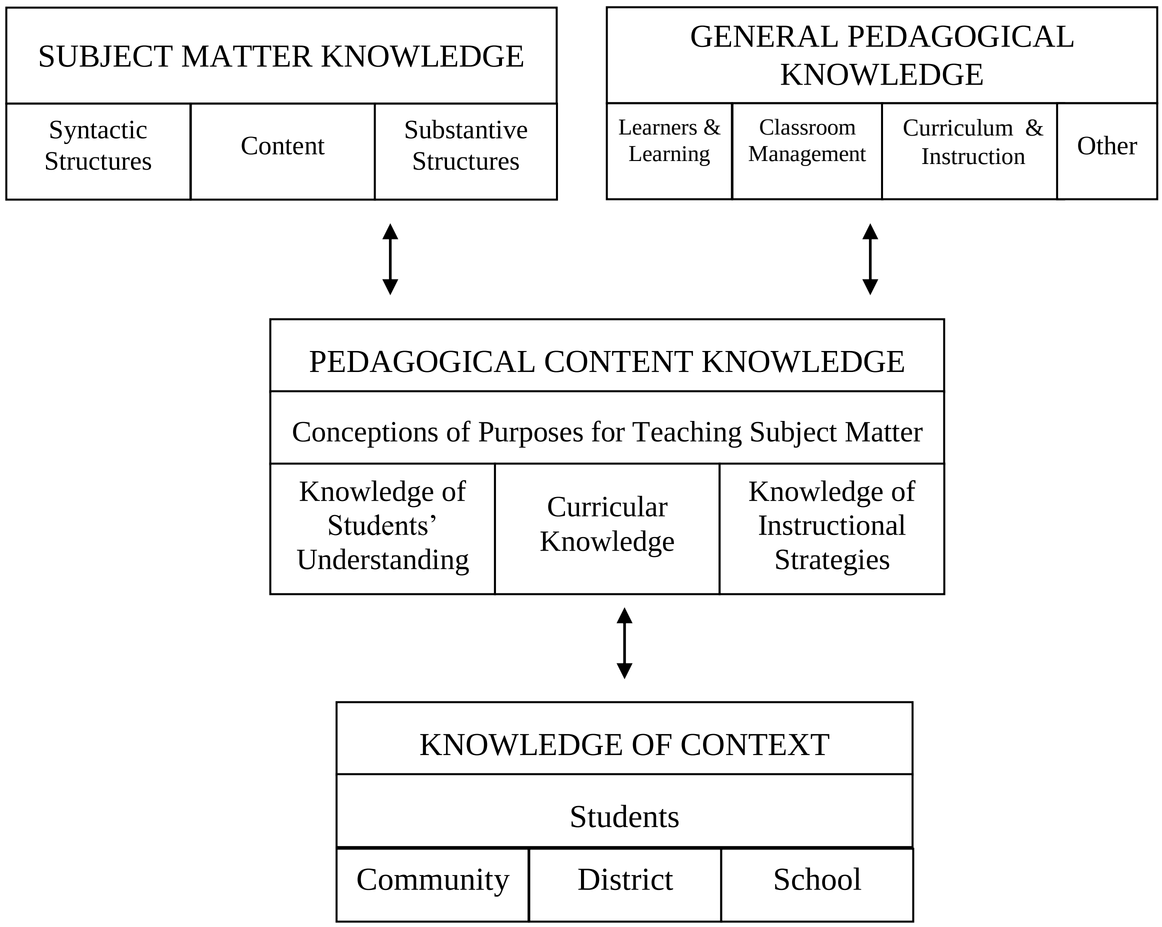
Teachers have different types of knowledge that they draw on when they teach. Shulman (1986; 1987) described different knowledge bases for teaching including content knowledge; general pedagogical knowledge; curriculum knowledge; knowledge of learners and their characteristics; knowledge of educational contexts; knowledge of educational ends, purposes, and values; and pedagogical content knowledge. He argued that too often "teaching is trivialized, its complexities ignored, and its demands diminished" (p. 225). As he considered the complexity of teaching, he theorized about a special kind of knowledge that teachers have—pedagogical content knowledge or PCK as it is widely known in the field. Shulman (1986) notes:

> In reading the literature of research on teaching, it is clear that central questions are unasked. The emphasis is on how teachers manage their classrooms, organize activities, allocate time and turns, structure assignments, ascribe praise and blame, formulate the levels of their questions, plan lessons, and judge general student understanding. What we miss are questions about the *content* of the lessons taught, the questions asked, and the explanations offered. From the perspectives of teacher development and teacher education, a host of questions arise. Where do teacher explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it and how to deal with problems of misunderstanding? (p. 8).

Shulman defined PCK as the "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 227) that includes the representation and formulation of concepts, pedagogical techniques, knowledge of what makes concepts difficult or easy to learn, knowledge of students' prior knowledge, and theories of epistemology. He argued PCK is "the dimension of subject matter *for teaching*" (p. 9, author's emphasis) and that it is necessary to be an effective teacher.

Grossman (1990) built on Shulman's initial conceptions of teacher knowledge and proposed a model of teacher knowledge containing four domains: a) subject matter knowledge; b) general pedagogical knowledge; c) PCK; and d) knowledge of context (see Figure 2). Subject matter knowledge refers to the facts, concepts, rules, and relationships among concepts within the field of the discipline. The syntactic and substantive structures of subject matter knowledge affect how the subject is validated and evaluated as well as organized and questioned, respectively. Furthermore, how one understands the field and the syntactic and substantive structures of the subject may influence how they represent the subject to their students (Grossman, 1990).

Figure 2. Grossman's model of teacher knowledge.



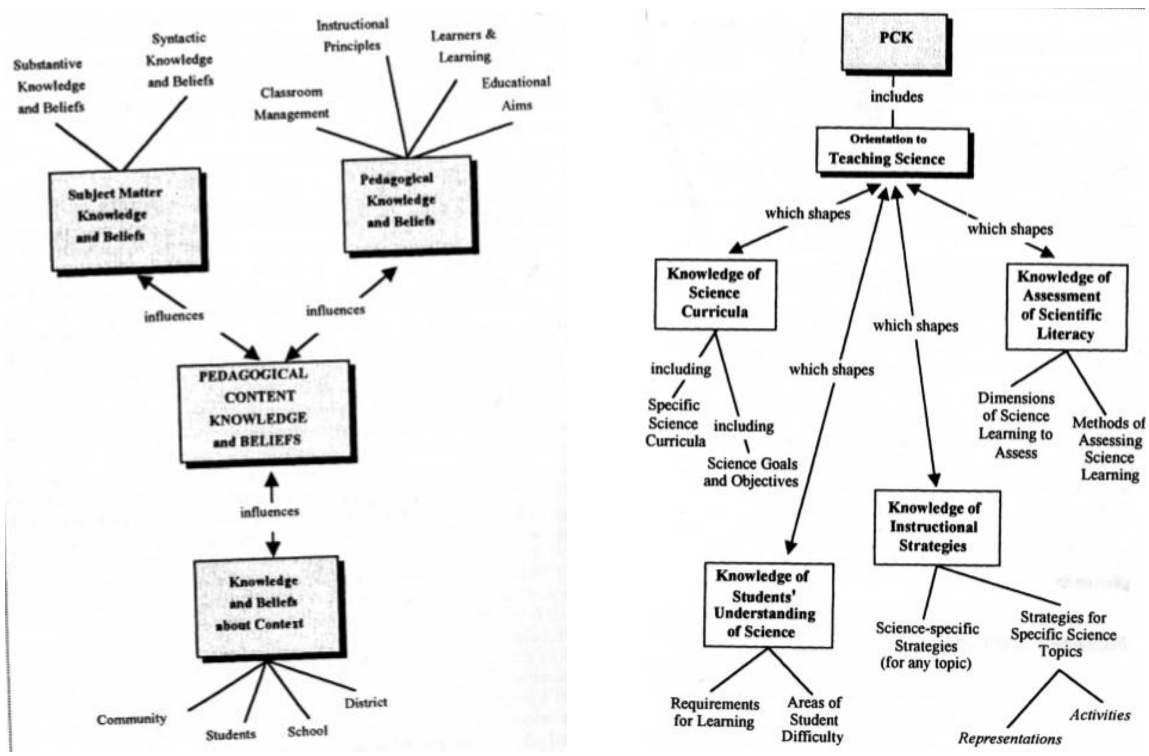
Pedagogical knowledge includes general knowledge, beliefs, and skills for teaching related to general instructional practices, wait time, classroom management, and cooperative learning. Historically, researchers (e.g., Good & Grouws, 1979) have identified and linked certain general beliefs and skills for teaching to student achievement in order to inform teacher education. More recently, however, Grossman

(1990) also recognized that the teacher's knowledge of context – an awareness and knowledge of the local district, community, school, and students – influences how more general knowledge is adapted in order to teach within specific school settings and to individual students.

According to Grossman (1990), PCK includes powerful representations, analogies, examples, illustrations, and demonstrations for a discipline-specific topic that make the subject matter comprehensible to others. As shown in the model, Grossman illustrated PCK using four components: conceptions of teaching, knowledge of students' understanding, curriculum, and instructional strategies. Grossman discussed how all four of these are critical components of PCK. For example, overarching conceptions of purposes for teaching are evident in a teacher's instructional and course goals. Ideas about what students will and will not understand influence the choice of representations and explanations or examples used by teachers demonstrating PCK. Curricular knowledge includes knowledge of curriculum materials for teaching particular content and knowledge of the horizontal and vertical curricula for a subject. A teacher with curricular knowledge can draw upon what they know a student has studied in 9th grade and will study in 11th and 12th grade in order to effectively teach a 10th grade student the content in an understandable way. The final component, knowledge of instructional strategies includes a teachers' repertoire of approaches for teaching a topic. This specifically includes explanations, representations, and experiments that can be used to effectively teach a particular topic.

Magnusson, Krajcik and Borko (1999) modified Grossman's model of teacher knowledge as shown in Figure 3.

Figure 3. Magnusson, Krajcik, and Borko's (1999) models of teacher knowledge.

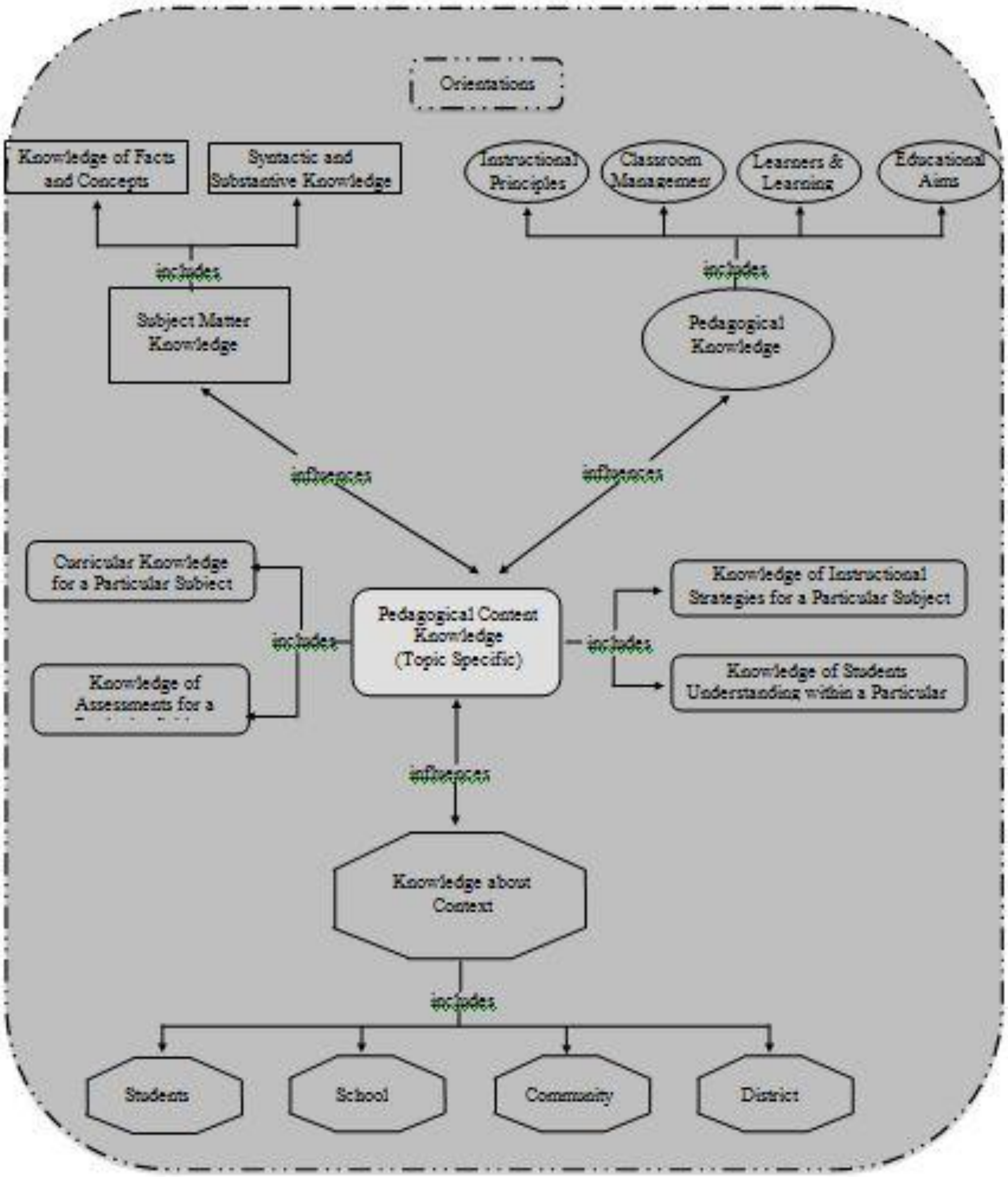


In this model, Magnusson, Krajcik, and Borko (1999) describe the conceptions of purposes for teaching as "orientations" and add knowledge of assessment. They define orientations as, "a general way of viewing discipline-specific teaching" (p. 97) that has been described by: (a) the goals of a teacher for teaching a particular subject and (b) the typical characteristics of the instruction practiced by a teacher. They argue that different emphases during instruction characterize different orientations of teaching, which is important because teachers' orientations influence teacher knowledge bases (Shulman, 1986) and how they are enacted in the classroom (Handal, 2003; Remillard & Bryan, 2004; Thompson, 1992; Thompson, Philipp, Thompson, & Boyd, 1994). Knowledge of assessment includes the knowledge of the dimensions of the subject important to assess and the methods for assessing the learning of these dimensions. They argue, "It is important for teachers to be knowledgeable about some conceptualization of scientific literacy to inform their decision-making relative to classroom assessment of science learning for specific topics" (p. 108). Additionally, teachers should know of specific procedures, approaches, and activities to assess important dimensions of subject matter learning during particular units of study. Moreover, knowing the advantages and disadvantages of particular instruments and techniques indicates knowledge of assessment.

The representation of the Magnusson, Krajcik, and Borko (1999) model of pedagogical content knowledge represent two important ideas. First, they argue teachers have topic-specific knowledge that is differentiated by the components of this knowledge; for instance, they may have a more elaborate knowledge of assessment for some topics than others. Through experience teachers develop knowledge of all components of pedagogical content knowledge for all topics they teach. Second, Magnusson et al. argue the components of pedagogical content knowledge function as parts of a whole. They write, "Lack of coherence between components can be problematic in developing and using pedagogical content knowledge, and increased knowledge of a single component may not be sufficient to effect change in practice" (p. 115). Consequently, interdependence exists amongst the components of PCK and "while it is useful to understand the particular components of pedagogical content knowledge, it is also important to understand how they interact and how their interaction influences teaching" (p. 115).

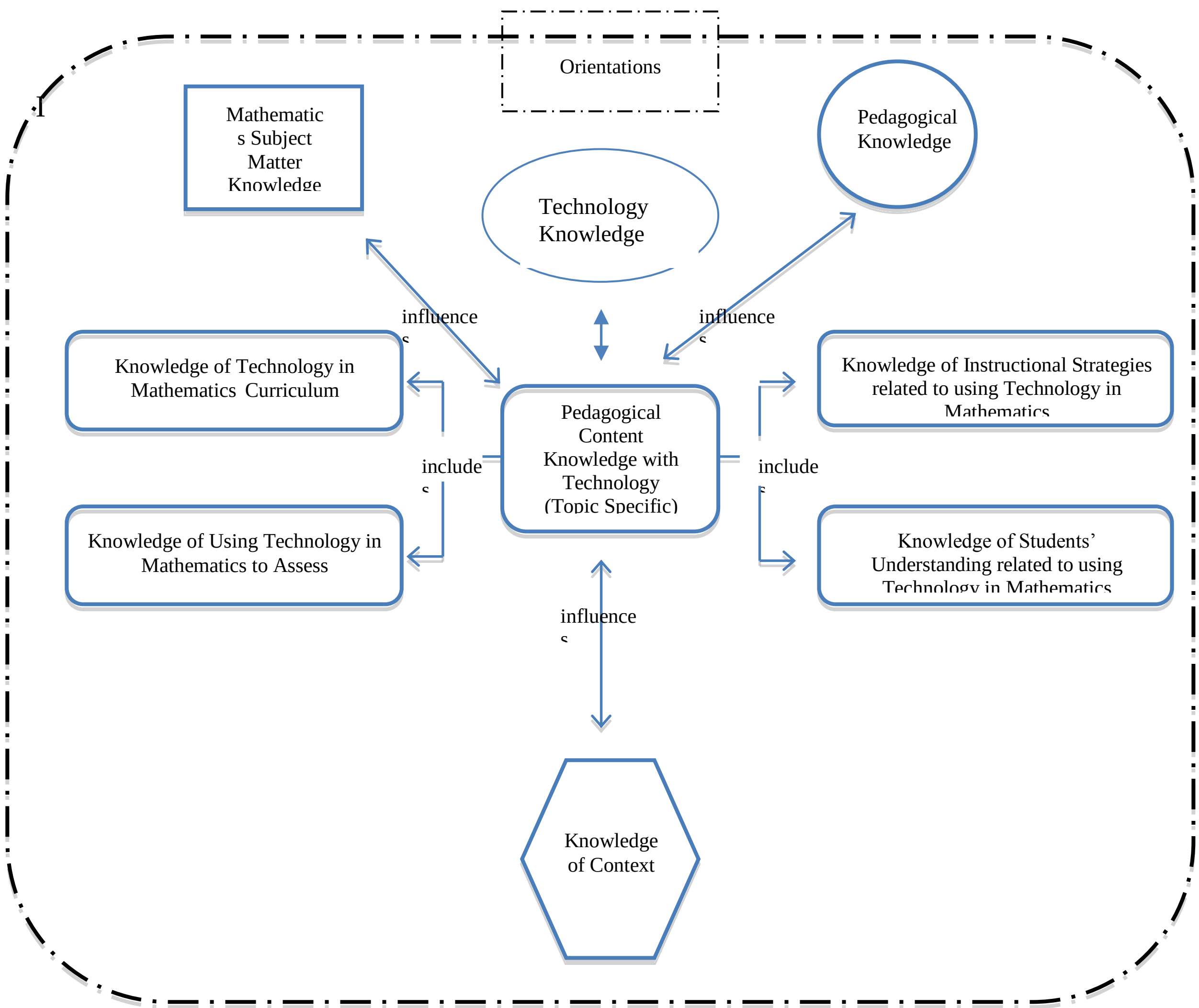
The Magnusson, Krajcik, and Borko (1999) model of pedagogical content knowledge was adapted by the Researching Science and Mathematics Teacher Learning in Alternative Certification Models (ReSMAR^{2}T) project team as shown in Figure 4.

Figure 4. ReSMAR2T model of teacher knowledge.



During the past three years, I have worked as a graduate assistant for the ReSMAR^{2}T project. This experience has provided me with opportunities to extensively use this framework during the research design phase of the project as well as during data collection and analysis activities. For this study, I draw on the ReSMAR^{2}T theoretical framework but make explicit references to mathematics and technology and add a technology knowledge component (see Figure 5).

Figure 5. Model of teacher knowledge explicitly using mathematics and technology.



I make technology an explicit focus because according to Mishra and Koehler (2006), "part of the problem [of studying how, when, and why teachers use technology] has been a tendency to only look at the technology and not how it is used" (p. 1018). Instead, research should focus on the complex roles of, and interplay among content, pedagogy, and technology (Mishra & Koehler, 2006; Koehler & Mishra, 2008).

Significance of the Study

Realizing the vision articulated by NCTM (2008) requires an understanding of teacher knowledge required for successfully integrating the use of technological tools in mathematics classrooms. New technologies will continue to be introduced and made accessible to secondary mathematics teachers. With these new advances teachers will be faced with additional demands as they learn how to use the technologies as well as how to integrate them into classroom instruction and assessment. Teacher preparation and professional development can influence the development of PCK related to technology, which in turn can influence classroom implementation, which in turn can influence student learning.

As a result, we need a better understanding of PCK necessary for effectively implementing new technologies, how to best facilitate the development of PCK, how teachers use PCK in implementing technology, and challenges teachers face as they enact this knowledge using new technologies in mathematics classrooms. Research that targets these questions will inform the design of teacher education and professional development initiatives and ultimately improve the teaching and learning of mathematics. Moreover, this study will influence future research related to teachers' use of technology, PCK in mathematics education, and teacher knowledge in action. In the following chapter, I review prior research in these three areas.

CHAPTER 2: REVIEW OF THE LITERATURE

The goal of this research study is to further understand secondary mathematics teacher knowledge as it relates to the integration of new technology (i.e., the TI-*n*spire™ calculator). Consequently, I focus my review of the research literature primarily on technology studies related to secondary teachers, rather than studies that focus on secondary students. In order to frame and inform the design of the proposed study, I organize this review into three major categories: (a) teachers' use of technology, (b) pedagogical content knowledge (PCK) in mathematics education, and (c) teacher knowledge in action as it relates to the use of technology. Prior to elaborating on each of these three categories, I begin with a brief discussion of the definition of technology and how research related to calculators has evolved.

Technology has multiple definitions. For example, technology is "(1) human innovation in action that involves the generation of knowledge and processes to develop systems that solve problems and extend human capabilities; (2) the innovation, change, or modification of the natural environment to satisfy perceived human needs and wants" (http://home.comcast.net/~pm1963/grade8/vocab.htm). According to NCTM (2005), technology refers to "all forms of electronic devices, including computers, calculators, and other handheld devices, telecommunications equipment, and the multitude of multimedia hardware, including software applications associated with their use" (p. ix). For example, technological teaching tools used in mathematics classrooms have included pencils, pens, paper, protractors, compasses, slide rules, chalkboards, and calculators. More recently, however, teachers refer to technology as personal digital assistants (PDAs), SMART™ Boards, laptop computers, the Internet, blogs, wikis, WebCT, computer software, applets, artifacts (movies, web sites, online courses), and more sophisticated handheld calculators like the TI-*n*spire™. Consequently, for the purposes of this study, I define *technology* as an electronic device used to change the environment of a mathematics classroom.

Research specific to the use of calculators in classrooms has been conducted over the last 40 years related to a wide range of topics. For example, early work focused on the relation between the use of calculators and student achievement (e.g., Ellington, 2003; Hembree & Dessart, 1992; Hollar & Norwood, 1999; O'Callaghan, 1998; Schwarz & Hershkowitz, 1999; Sheets, 1993). Recent studies focus on how calculators are used by teachers and students in classrooms (Chazan, 1999; Doerr & Zangor, 2000; Dwyer, Ringstaff, & Sandholtz, 1999; Goos, Galbraith, Renshaw, & Geiger, 2000; Hughes, 2005; Hollebrands, 2007; Ruthven & Hennessy, 2002; Mitchell, Bailey, & Monroe, 2007). Both students and teachers are important foci of study. However, this study's research questions focus on understanding pedagogical content knowledge as secondary teachers begin to implement a new technology (i.e., the TI-*n*spire™). As a result, in the following sections, I discuss research related to classroom use of technology, pedagogical content knowledge in mathematics education, and teacher knowledge in action as it relates to the use of technology.

Teachers' Use of Technology

Recently, researchers have examined the use of technology during mathematics instruction. These studies take place in different countries (e.g, Australia, England, France, United States), using different technologies (e.g., computers, dynamic geometry software, handheld graphing calculators), in different secondary mathematics courses (e.g., algebra, geometry, pre-calculus, calculus) and with different research purposes (e.g., how it is used, design and implementation of tasks, and change in teacher practice). Significant findings from specific studies are listed in Table1. I elaborate on each of these findings below.

Table 1

Research Findings and Citations Related to the Use of Technology during Mathematics Instruction

Findings	*Studies*
It takes time to integrate technology into mathematics classroom instruction.	Alejandre, 2005; Byrom & Bigham, 2001; Chazan, 1999; Dwyer et al., 1999
Technology changes the nature of the mathematics.	Laborde, 2000, 2001; Lampert, 1998; Slavit, 1996
Teachers within and amongst schools differ in their use of technology.	Burrill et al., 2002; Cuban, 2001; Kendal, Stacey, & Pierce, 2005
A teacher must learn new teaching techniques in order to effectively incorporate technology into teaching.	Byrom & Bigham, 2001; Doerr & Zangor, 2000; Laborde, 2001; Ruthven & Hennessy, 2002
Technology integration requires new teacher knowledge.	Chazan, 1999; Hughes, 2005; Mishra & Koehler, 2006; Niess, 2005

Each of these findings suggests that the teacher plays a central role in when, how, and why the technology is used in the classroom.

Technology Use in a Classroom Takes Time

In the 2005 NCTM Yearbook, Suzanne Alejandre advised teachers: “as you incorporate technology into your teaching build slowly…learn both the strengths and weaknesses of what you have available.” (p. 138). Within the research literature a number of examples exist that support this advice. For instance, Daniel Chazan (1999), an experienced teacher when he began integrating technology, took three years to feel like he successfully integrated technology into a high school, lower level Algebra I class. From 1995 to 2000, the SouthEast Initiatives Regional Technology in Education Consortium (SEIR-TEC) provided technical assistance and professional development to 12 schools referred to as intensive sites and leaders and researchers learned that the process of technology integration is slow (Byrom & Bigham, 2001). In a third study, it took several years for a teacher teaching with a computer algebra system (CAS) to move from an early emphasis on teaching about CAS as a tool and using it for difficult problems (e.g., to teach rules and procedures) to incorporating its use for primarily pedagogical aims (e.g., visualization and demonstration within lectures) (Kendal, Stacey, & Pierce, 2005). Nevertheless, over the course of a few units, Slavit (1996) was able to see how the instructional practices of an experienced teacher changed as he incorporated the graphing calculator into his high school Algebra II class. While these examples illustrate that incorporating technology into classroom instruction is a process that takes time, this process (i.e., integration of technology) can be investigated over relatively short periods of time.

Technology Changes the Nature of Mathematics

By innovatively supporting and encouraging students to use the calculators in mathematics classrooms, teachers see new approaches and an expanded repertoire of student strategies as confidence grows (Ruthven, 1992). For instance, when a teacher expected a symbolic formulation and algebraic evaluation of a word problem (i.e., A sum of $1000 is invested in an account in which interest of 1% is added at the end of each month. After how many months will the sum in the account exceed $1400), students responded with a building-up and guess-and-check strategy (Ruthven, 1992). Likewise, calculators make real data easier to handle and open opportunities for discovery learning and student projects (Rubenstein, 1992).

New technologies such as graphing calculators, the TI-*n*spire™ and dynamic geometry software dynamically link graphical, symbolic, and numerical representations of mathematical objects in ways that allow teachers and students to make connections among these representations. Furthermore, representations now reflect the variation of mathematical objects (C. Laborde, 2008). Such representations make it "possible to shift the locus of authority in the classroom – away from the teacher as a judge and the textbook as a standard for judgment, and toward the teacher and students as inquirers who have the power to use mathematical tools to decide whether an answer or a procedure is reasonable" (Lampert, 1989, p. 223–224). Kaput (1992) adds, "The locus of social authority becomes more diffuse; provision must be made for students to generate, refine, and prove conjectures; the teacher must routinely negotiate between student-generated mathematics and the teacher's curricular agenda" (p. 548). In other words, research and researchers' experiences tell us teachers must be prepared to consider mathematics as something students do and invent rather than observe as technology is incorporated into classroom teaching and learning experiences.

Technology Integration Varies Across Teachers

Given that technology shifts the nature of mathematics in classrooms, it is not surprising researchers find differences in how teachers use new technologies over time. For example, Cuban (2001) interviewed 21 high school teachers at two Silicon Valley High Schools, the location of thousands of computer and Internet companies where he thought he would find all teachers using the computers because they were available and considered powerful tools for teaching. At these schools, he found teachers and students had classroom access to one computer per 17-22 students during the 1998-1999 year, and only 13 teachers (approximately 60%) reported their teaching changed due to access to computers. Additionally, only four out of the thirteen teachers said they used computers to help them prepare for class and to help them make the classroom more student-centered.

Burrill, et al. (2002) also reported that how teachers used graphing technology in their teaching varied extensively after reviewing eight studies that asked what teachers do with handheld graphing technology. For example, Goos, Galbraith, Renshaw, and Geiger (2000) observed five secondary mathematics classrooms in Australia over the course of three years and found the teachers using the technology in four key roles: (a) to employ special features (as a *master*), (b) in creative ways to change the nature of activities (as a *servant*), (c) to increase student power over their own learning (as a *partner*) and/or (d) as a natural part of their pedagogical and mathematical skills (as a *extension of self*). For instance, in a classroom where the teacher admitted limited expertise in the use of the graphing calculator and little confidence, they found the teacher had an expert student share mathematical displays on the overhead projection but he (the teacher) gave commentary and explanations to the silent student displays, giving the calculator the role of *master*. Again, Goos et al. elaborated that the technology is the master of the mathematical knowledge when giving the calculator or technology the role of *master*. In contrast, a different teacher set small groups of students to the task of investigating transformations of several functions $y = x^2$, $y = 1/x$,

$y = |x|$ and then asked a person from each group to present their findings to the whole class via the overhead projector. In this case, the calculator plays the role of *partner* because the teacher gave students control of the mathematical exploration and learning by giving a task that required the use of the calculator and active student thinking. The findings from this study suggest that teachers give different authority and roles to calculators. This leads to different uses of calculators in secondary mathematics classrooms.

In addition to the fact that teacher use technology in different ways within mathematics classrooms, the work of Hall and Hord (1987; 2001) suggests that teachers go through different stages as they adopt innovations (see Table 2).

Table 2

Stages of Concern Encountered by Teachers While Incorporating Innovation into Teaching Practices

Stages of Concern about the Innovation		
Unrelated	0. Awareness	Little concern about or involvement with the innovation is indicated
Self	1. Informational	A general awareness of the innovation and interest in learning more detail about it is indicated. The person seems unworried about himself/herself in relation to the innovation. He/she is interested in substantive aspects of the innovation in a selfless manner such as general characteristics, effects, and requirements for use.
	2. Personal	Individual is uncertain about the demands of the innovation, his/her inadequacy to meet those demands, and his/her role with the innovation.
Task	3. Management	Attention is focused on the processes and tasks of using the innovation and the best use of information and resources. Issues related to efficiency, organizing, managing, scheduling, and time demands are most important.
Impact	4. Consequence	Attention focuses on impact of the innovation on the student in his/her immediate sphere of influence (i.e., relevance for students, evaluation of student outcomes, and changes needed to increase student outcomes).
	5. Collaboration	The focus is on coordination and cooperation with others regarding the use of the innovation.
	6. Refocusing	The focus is on exploration of more universal benefits from the innovation, including the possibility of major changes or replacement with a more powerful alternative.

* Adapted from Hall and Hord (1987), p. 61.

They refer to these stages as “stages of concern about the innovation.” With a concerns-based approach, importance is placed on understanding the teacher and their concerns throughout the process of change. Within the theory, the intensity of concern changes in two ways: in intensity and across stages as teachers and teacher leaders move through the change process. Hall and Hord’s framework suggests that teachers who begin to consider and then use the TI-*n*spire™ may experience these different levels of concern, especially if an adoption is made on a larger scale such as the district level.

Another framework that considers different stages of use, but is specifically related to the use of technology is the PURIA model (Beaudin & Bowers, 1997). Zbiek and Hollebrands (2008) argue this model is more powerful for researching the use of a new technology in high school mathematics classrooms. The PURIA model (see Table 3) is a useful tool to understand how teachers begin to use technology.

Table 3

The Elaborated and Extended PURIA Model (Zbiek & Hollebrands, 2008, p. 295)

PURIA Mode	*Nature of Activity During the Mode*
Plays with the technology	Uses technology for no clear mathematical purpose.
Uses technology as a personal tool	Uses technology in doing mathematics of one's own design. May be using it as a learner of mathematics but not using it with students.
Recommends technology to others	Recommends use to a student, a peer, or a small group of students or peers. This likely is not in a formal classroom setting and it is not an integrated part of instruction.
Incorporates technology into classroom	Integrates this technology into classroom instruction. This occurs to varying degrees.
Assesses students' use of technology	Examines how students use the technology and what they learn from using it.

Similar to the Concerns model, the PURIA model focuses on the teacher. Additionally, the actions of the teacher become the observable phenomenon for the researcher interested in understanding the small-scale use of new technologies in the classroom rather than researching the concerns as a phenomenon of the teacher.

I am interested in a high school teacher's actions and knowledge while using a new technology in their mathematics instruction. As a result, the research will investigate teachers who would be classified at the "Incorporates" mode within the PURIA model. This is an especially important mode to consider because as the PURIA model suggests and Mishra and Koehler (2006) point out, knowing how to use the technology is not the same as knowing how to teach with it. Thus, I select research subjects based on this criterion, which I will describe in more detail in Chapter 3.

Technology Integration Requires New Teaching Techniques

The effective implementation of technology involves more than just adding the use of technology to existing practices and mathematical tasks. It requires different mathematical tasks, different teaching practices and knowledge, and opportunities for professional development. Byrom and Bingham (2001) found that teachers in 12 southeastern U.S. schools, who were given new teaching approaches and tasks involving the use of technology and participated in professional development focused on those approaches/tasks, were eager to try them. Byrom and Bingham concluded effective use of technology requires changes in teaching and the adoption of a new teaching strategy can be the catalyst for technology integration.

When Collette Laborde (2001) worked with teachers in France over the course of three years to integrate dynamic geometry software, she found that teachers needed time to create tasks, use them in their classrooms, and then modify them based on reflections of their implementation. She found that a teacher's ability to modify tasks is part of the integration process. Moreover, in comparing teachers, she found that teachers differed in how they created and modified tasks, especially in relation to their experience with teaching in general and their experience using the technology. For example, differences existed in novice and experienced teachers. Teachers new to the technology and/or teaching tended to use technology in independent sessions rather than to coherently teach the content; they used the TI-92 or computers to visualize and conjecture about the mathematics more often than to experiment with the mathematical ideas. Although the teacher new to teaching did not do much revision of the task from year to year, the experienced teacher who was new to technology did; she removed the repetition of completing the same task with the technology and with paper-and-pencil and gave greater autonomy to the students in exploring the problem. On the other hand, teachers experienced with the technology (although this was the first time to integrate it into teaching) initially designed tasks centered on student exploration. Then, in their revisions, the tasks highlighted more efficient strategies than paper-and-pencil techniques and could only be completed in the computer environment by noting the geometrical and algebraic dynamically changing links of a construction. Laborde concluded these findings illustrate the gradation in the extent of integration of technology into teaching; in more advanced and effective uses of technology, the teacher introduced new content through technology and gave more student-centered tasks in which students had to make judicious use of computer technology. These new teaching techniques required changes in teaching and new knowledge of the technology and mathematics, which other researchers, too, have found and will be discussed next.

Technology Integration Requires New Teacher Knowledge

Teachers construct new knowledge as they integrate technology into instruction. For example, Daniel Chazan taught high school mathematics in northeastern United States and analyzed his own teaching before and after integrating technology into an algebra course. After analysis, Chazan (1999) concluded technology has a role to play in supporting teachers' understandings of mathematics. His knowledge of mathematics and mathematics teaching expanded as he used the graphing calculator as a tool to link input-output tables and graphs in an effort to teach functions as a unifying concept within an algebra course. With time, he found himself able to pose problems to students that allowed them to understand the desired goals. Chazan expanded his knowledge of mathematics and of teaching mathematics with the integration of technology.

More recently, Hughes (2005) investigated teacher learning during technology professional development. She studied two high school and two middle school teachers with 3-26 years of experience. Hughes collected data via three biographical interviews and three direct observations with field notes. She found that teachers with less professional knowledge needed more content-specific technology learning opportunities while teachers with more professional knowledge were able to develop innovative technology-supported pedagogy by bringing their own learning goals to professional development activities. While this research recognizes that overall teacher knowledge is central to technology integration, it is unclear what knowledge teachers draw on as they transition into using a new technology in mathematics instruction. To date, almost all studies that have investigated teacher knowledge in relation to using technology have been conducted through professional development settings, self-study, or teacher interviews/surveys. We lack research on teacher knowledge during instruction. In the next section, I review the research related to PCK in mathematics education.

Pedagogical Content Knowledge in Mathematics Education

As discussed in Chapter 1, several researchers have theorized about the highly complex nature of teacher knowledge and delineated the components of different knowledge bases, and more specifically pedagogical content knowledge (PCK) (e.g., Grossman, 1990; Magnusson, Krajcik & Borko, 1999; Shulman, 1986, 1987). In this chapter, I include a brief review of research studies focused on PCK in mathematics education. Then I discuss a review of the literature related to a new construct, technological pedagogical content knowledge (TPCK).

Pedagogical Content Knowledge

A number of researchers (e.g., Hill, Ball, & Schilling, 2008; Kinach, 2002; Marks, 1990) in mathematics education have examined teachers' pedagogical content knowledge with qualitative and quantitative methods. However, researchers studied different teacher populations without a shared conceptualization and identification of the same constructs of pedagogical content knowledge. While Hill, Ball, and Schilling (2008) assessed more than 5000 during three years and interviewed 26 Kindergarten through 6^{th} grade teachers, Kinach (2002) conducted a teaching experiment with 21 preservice teachers, and Marks (1990) interviewed 8 fifth-grade teachers. Nevertheless, researchers do agree that a teacher's knowledge of students' common errors for a specific mathematics topic is an indicator of a teacher's pedagogical content knowledge. However, as Graeber and Tirosh (2008) asserted, "differences and the lack, at least to-date, of a widely agreed upon characterization of PCK, suggest that while progress has been made, much remains to be done" (p. 124).

Hill, Ball, and Schilling (2008) conceptualized knowledge of content and students (KCS) as a subset of pedagogical content knowledge as a domain of mathematical knowledge for teaching. They defined KCS as content knowledge intertwined with knowledge of how students think about, know, or learn particular content such as number concepts and operations and examined measuring KCS via a multiple-choice item test, which was written by a group of teachers and teacher educators. With these multiple-choice items, they aimed to assess teachers' understandings of common student errors, students' understanding of content, identification of easier versus more difficult problems for students, and common student computational strategies related to number concepts and operations. Then, they piloted these multiple-choice tests with over 5000 teachers during California's Mathematics Professional Development Institutes (MPDIs) during three years with pre- and post-tests as part of evaluations. Hill, Ball, and Schilling analyzed the tests using statistical analyses (i.e., factor analysis and item response theory measure construction) as well as validity checks. In addition, they conducted cognitive interviews to determine whether they measured what they conceptualized as KCS. They found "exploratory factor analyses *did* indicate that the KCS items formed their own separate, interpretable factor but with a wrinkle: Some items meant to tap teachers' KCS scaled with items meant to tap content knowledge (CK), although no obvious differences emerged between these items and those that scaled on the KCS factor" (p. 385). In the end, multiple forms and interviews with teachers suggested familiarity with aspects of students' mathematics thinking (e.g., common errors and common strategies) and thus suggested that KCS is one element of knowledge for teaching.

While Hill, Ball, and Schilling (2008) studied the pedagogical content knowledge of in-service teachers, Kinach (2002) examined the pedagogical content knowledge of preservice teachers (PSTs). She investigated PSTs' understandings of integer subtraction and addition with an interest in their PCK development via a teaching experiment with 21 secondary preservice mathematics teachers (18 undergraduate, 3 graduate students; 16 female and 5 male students). The PSTs engaged in three main tasks during the teaching experiment, which aligned to what she called a 5 element cognitive strategy to develop PCK. The first task was to explain the addition and subtraction of the set of all integers (i.e., whole real numbers), (Z, +, -), in a context. The second task was to explain (Z, +, -) on the number line, and the third task was to explain (Z, +, -) in the algebra-tile context.

Kinach (2002) analyzed written journals, written homework on instructional explanations, and transcribed video-recordings of classroom discussions about what constitutes a "good" explanation. She noted the difference between an instrumental and relational understanding of PCK. That is, the teacher may know what and how to explain a mathematics concept to a student (i.e., instrumental PCK) but they may not have thought about why they explain it in the way they do in order to help the student understand why a mathematical rule exists as it does (i.e., relational PCK). After task 1, she found PST's explained their teaching as telling the rules, showing students how to use them, and subsequently letting students practice. After discussion of task 2, students were generally unsatisfied with the number line explanation as justification for integer addition. However, eventually students wrote about their preference for relational explanations such as one in an algebra-tile context over instrumental explanations in their written journals. She concluded the results of this teaching experiment help us understand the transformation process of subject matter knowledge to effective pedagogical content knowledge. As a result, she presented a cognitive strategy guide in her work to guide teacher educators in their work with preservice teachers as they aim to develop PSTs' PCK. She asserted teacher educators must identify, assess, challenge, transform, and then sustain PCK via the 5 element cognitive strategy.

Kinach (2002) collected and analyzed data to inform her teaching practices and other mathematics teacher educators whereas Marks (1990) researched fifth-grade teachers in order to articulate a description of fifth-grade mathematics teachers' PCK. Marks interviewed 8 (6 experienced and 2 novice) 5^{th} grade mathematics teachers about teaching equivalent fractions with task-based interviews for 45-90 minutes. Each interview was transcribed and then coded by topic (e.g., classroom management, textbooks, purposes, assessment). Then, he synthesized the codes into three main categories: subject matter knowledge, pedagogical knowledge, and PCK. He reported a discussion of PCK and elaborated on the areas of students' understanding (e.g., how students learn, typical understandings and errors, what is easy or hard for students) that teachers discussed when asked questions aiming to investigate their PCK. For instance, teachers cited students had the most trouble with knowing which procedures to apply to which situations such as when to multiply two fractions with cross multiplication or straight across or knowing when to find a common denominator. He concluded that PCK, the mathematics knowledge needed for teachability, consists of four closely connected components: students' understanding, media for instruction, subject matter, and instruction processes.

Although characterizations of PCK may not be agreed upon, researchers such as Hill, Ball, Schillings, Kinach, Marks, Magnusson, Krajick, Borko, and Shulman do agree that PCK is a multidimensional construct of subject matter knowledge special to teaching. What has not yet been explicitly addressed is how the use of technology when teaching impacts these models of teacher knowledge. However, some researchers are beginning to discuss technological pedagogical content knowledge. Therefore, I share a review of this literature in the next section.

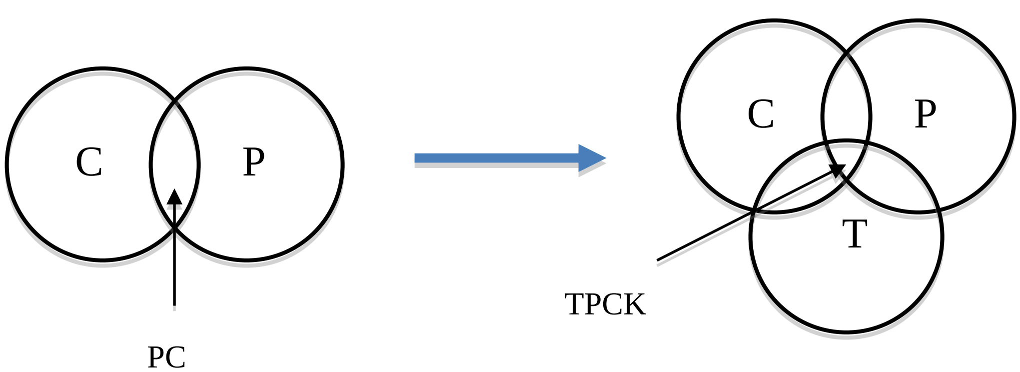
Technological Pedagogical Content Knowledge

As shown in the previous section, studies focused on PCK do not include "technology" in their discussions of teacher knowledge bases. Some would argue that technology is included implicitly in domains such as curriculum knowledge or knowledge of instructional strategies while others argue technology adds a new construct to the model of teacher knowledge. As a result a few researchers have begun to discuss models referred to as Technological Pedagogical Content Knowledge or TPCK. For example, Mishra and Koehler (2006) recognize,

> Teachers will have to do more than simply learn to use currently available tools; they also will have to learn new techniques and skills as current technologies become obsolete. This is a very different context from earlier conceptualizations of teacher knowledge, in which technologies were standardized and relatively stable. (p. 1023)

Therefore, like Shulman, they base their TPCK framework on the understanding that teaching is a highly complex activity that draws on many kinds of knowledge. Teaching occurs in an ill-structured, dynamic environment where it is a complex cognitive skill. Figure 6 illustrates their comparison of PCK and TPCK.

Figure 6. Mishra and Koehler's model of TPCK and how it compares to PCK.



Mishra and Koehler (2006) reason, “Technology is changing so fast that any method that attempts to keep teachers up to date on the latest software, hardware, and terminology is doomed to create knowledge that is out of date every couple of years” (p. 1032); thus, we need to teach conceptual ideas that are framed theoretically in terms of content, technology, and pedagogy and the interplay of these three components. They argue that TPCK requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students’ prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies (Mishra & Koehler, 2006; Koehler & Mishra, 2008). This model differs from the models shown in Chapter 1 as it considers TPCK at the intersection of technological, content and pedagogical knowledge. In other words, TPCK is a subset of content knowledge.

While thinking about TPCK in the same ways as Mishra and Koehler (2006, 2008), Niess (2005) examined the TPCK of 22 PSTs (2 physics, 5 mathematics, 4 chemistry, 5 biology, and 6 integrated science) during a one-year graduate level science and mathematics teacher preparation program. In this program, faculty and supervisors aimed to incorporate pedagogical concerns into the introduction of using technology for teaching and learning mathematics and science rather than just teaching PSTs *about* technology. They thought of TPCK as “the integration of the development of knowledge of subject matter with the development of technology and of knowledge of teaching and learning" (p. 510), and therefore, exposed PSTs to real-time data collection devices (calculator-based ranger (CBR) or calculator/computer-based laboratory (CBL) probes) before and during student teaching experiences. The PSTs then planned, taught, and reflected on teaching hands-on lessons with technology. Niess analyzed 5 PSTs' lesson plans, written reflections, and a written analysis of the use of technology in teaching science and mathematics. She found only some students recognized the interplay of technology and content throughout the program and were able to successfully integrate technology into teaching and learning experiences during their student teaching. For example,

> Terry extended his lessons to have his students investigate the effects of the external environment on the temperature reading, redesigning the probeware setup to improve the data collection. Yet, when the integration was a natural inclusion in the unit, Karen resisted using class time to explore the science embedded in the design of the technology. Denise simply rejected the consideration of the science of the technology thinking of the technology as a tool to do science rather than a tool embodying science. (p. 510)

She concluded teacher preparation programs must seriously consider specific directions to guide and support PSTs in expanding their understandings of the interactions of the knowledge of technology and the knowledge of subject matter while teaching and learning.

Investigating the teacher knowledge of preservice and student teachers who incorporate technology is important and informative work. However, also investigating in-service and experienced teachers as they incorporate technology is important and informative work. In the following section, I describe studies that focus on teacher knowledge during enactment within secondary mathematics in-service teacher populations.

Teacher Knowledge in Action

Only a few researchers have examined teacher knowledge as it is enacted in the secondary classroom with technology (e.g., Mitchell, Bailey, & Monroe, 2007; Ruthven & Hennessy, 2002). However, the studies that have been conducted do tell us that teachers must acquire new knowledge and orientations for teaching in order to be effectively incorporate technology into teaching.

Ruthven and Hennessy (2002) conducted group interviews with secondary mathematics teachers in England to investigate the enactment of technology. More specifically, they analyzed the pedagogical ideas underpinning teachers' accounts of the successful use of computer-based tools and resources to support teaching and learning of mathematics. Developing the use of information and communications technology (ICT) to support subject teaching and learning was identified as a priority across the participating secondary schools. Their primary interview prompt requested examples of ICT use that participants felt had been successful in supporting teaching and learning. Teachers' explanations of examples gave researchers a window into their knowledge of teaching with technology. From teachers' explanations, they developed a model with ten operational themes describing what secondary mathematics teachers conceive as the successful use of computer tools and resources to support mathematics teaching and learning. They learned success with technology in the classroom, according to current teachers, related to three main priorities: securing and enhancing student participation in class work, the pace and productivity of such work, and the progression in learning arising from use with technology in the classroom. Ruthven and Hennessy concluded technology appears to be a “fulcrum for some degree of reorientation of practice” (p. 85). In other words, when teachers work to integrate technology into their teaching, technology will play a central role in shifting their teaching approaches away from the transmission model of tell and practice and towards a more student-centered inquiry-based classroom.

Mitchell, Bailey, and Monroe (2007) worked with an experienced secondary teacher, who taught from a traditional pedagogy based on text and lecture in a geometry classroom, via a partnership between a high school and their university setting. The high school principal allowed the participating teacher one release period per day for planning with the expectation that the teacher would be a resource to other teachers who wished to integrate technology in their teaching in the future. The university team members regularly met with the teacher to problem-solve, encourage, and offer suggestions on integration of content and technology. They focused first on knowledge of the technology (presentations and Web page software) with the teacher and then supported implementation of pedagogical changes. They found the teacher struggled with logistics and the technology learning curve as well as the integration process. They noted, “It became obvious that the teacher was having difficulty in switching between the traditional teaching method and the technology-integrated approach” (p. 86). They concluded technology integration requires a paradigm shift for teachers as their knowledge and practices change.

While Ruthven and Hennessy and Mitchell, Bailey, and Monroe have began to study teacher knowledge by interviewing and working with secondary teachers outside of the classroom, there has been little research related to studying teacher knowledge related to enacting curricula. The studies that have been conducted have not collected data through the use of classroom observations.

Summary

I focus my review of the research literature primarily on teacher knowledge as well as technology studies related to secondary teachers. As discussed earlier, models of teacher knowledge bases, in recent years build on the seminal work of Lee Shulman (1986; 1987). I draw on this earlier work to inform and frame the research design for this study in which I investigate secondary mathematics pedagogical content knowledge as it relates to the integration of a new technology (i.e., the TI-*n*spire™).

Studies related to teachers' use of technology suggest that transitioning to the effective use of new technologies is challenging and takes time. Teachers must shift their orientations of teaching and knowledge bases of teaching away from approaches that support a transmission model and towards a student-centered inquiry based model in order to make effective use of the power of new technologies during classroom integration (Kaput, 1992; Laborde, 2001; Mitchell, Bailey, & Monroe, 2007; Ruthven & Hennessy, 2002). Yet, research related specifically to teacher knowledge in the context of integrating technology has been minimal. Moreover, there have not been any research studies related to teacher knowledge and technology integration in action (i.e., as secondary teachers teach mathematics lessons in classrooms), especially as they transition into the use of a new technology. Thus, with this study, I add to the research base on teacher knowledge and technology integration. In the next chapter, I describe the research methodology.

CHAPTER 3: METHODOLOGY

The design of this study employs qualitative methods in order to investigate the pedagogical content knowledge (PCK) of secondary mathematics teachers as they initially integrate a new technology into their teaching. I assume individual constructions of reality are worth investigation and exist in local experiences. In other words, I bring a constructivist perspective to this research design as I inquire about how US high school teachers understand teaching when they begin to use a new technology during mathematics instruction. After having been a US high school mathematics teacher myself, I recognize the complexities of knowledge for teaching with technology. As a research doctoral student in education, I have come to understand these complexities theoretically as pedagogical content knowledge. Consequently, within a constructivist paradigm, I use a case study methodology to view mathematics teacher knowledge enacted and displayed when integrating a new technology (i.e., the TI-*n*spire™) during instruction through a PCK theoretical framework as I aim to answer the following research questions:

- What pedagogical content knowledge do secondary teachers draw upon when they begin to implement a new technology in their mathematics instruction?
- What orientations do secondary teachers hold about teaching mathematics with a new technology?

In this chapter, I describe the case study methodology, theoretical framework, data collection, and data analysis for the study.

Case Study Methodology

The focus of case study research is to describe the unique cases and interpret emergent themes that differentiate or unite settings and/or participants (Yin, 2003). In case studies, the researcher uses multiple data sources to construct a holistic and meaningful representation of personal experiences (Denzin & Lincoln, 2005). Case studies are preferred when (a) exploratory, "how," or "why" questions are being posed, (b) the investigator has little control over events, and (c) the focus is on a contemporary phenomenon within some real-life context (Yin, 2003). Additionally, Yin asserts, "A *case study is an empirical inquiry that* investigates a contemporary phenomenon with its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (p. 13). Since boundaries do not distinctly separate teacher knowledge in action and the classroom context and since I am interested in investigating teacher knowledge enacted when using a new technology during high school mathematics instruction, a case study methodology is appropriate for the study.

According to Hatch (2002), "Defining the boundaries or specifying the unit of analysis is the key decision point in case study design" (p. 30). The unit of analysis decides what it is I want to be able to say something about at the end of the study (Patton, 2002). Therefore, cases include 3 experienced secondary mathematics teachers who said they were teaching mathematics topics for the first time with the TI-*n*spire™ calculator and were willing to be observed and interviewed while teaching at least one chapter/unit. I conducted an intensive observation and interview process to ascertain what knowledge secondary mathematics teachers draw upon while instructing with a new technology. Classroom observations were important because as Marks (1990), who researched teachers' pedagogical content knowledge, asserted, "Perhaps the most important limitation was that this [his] study gathered only verbal data and no observational data, thus slighting the contextual and interactive processes of teaching. Classroom observations might have provided areas or aspects of knowledge not identified in the interviews" (p. 11). Data sources included: (1) initial interviews, (2) video recorded classroom observations, (3) field notes, (4) classroom artifacts, (5) stimulated-recall interviews, and (6) closing interviews. As I collected and analyzed data for these cases, I constructed findings through a PCK with technology theoretical framework lens, which I describe in the next section.

Theoretical Framework

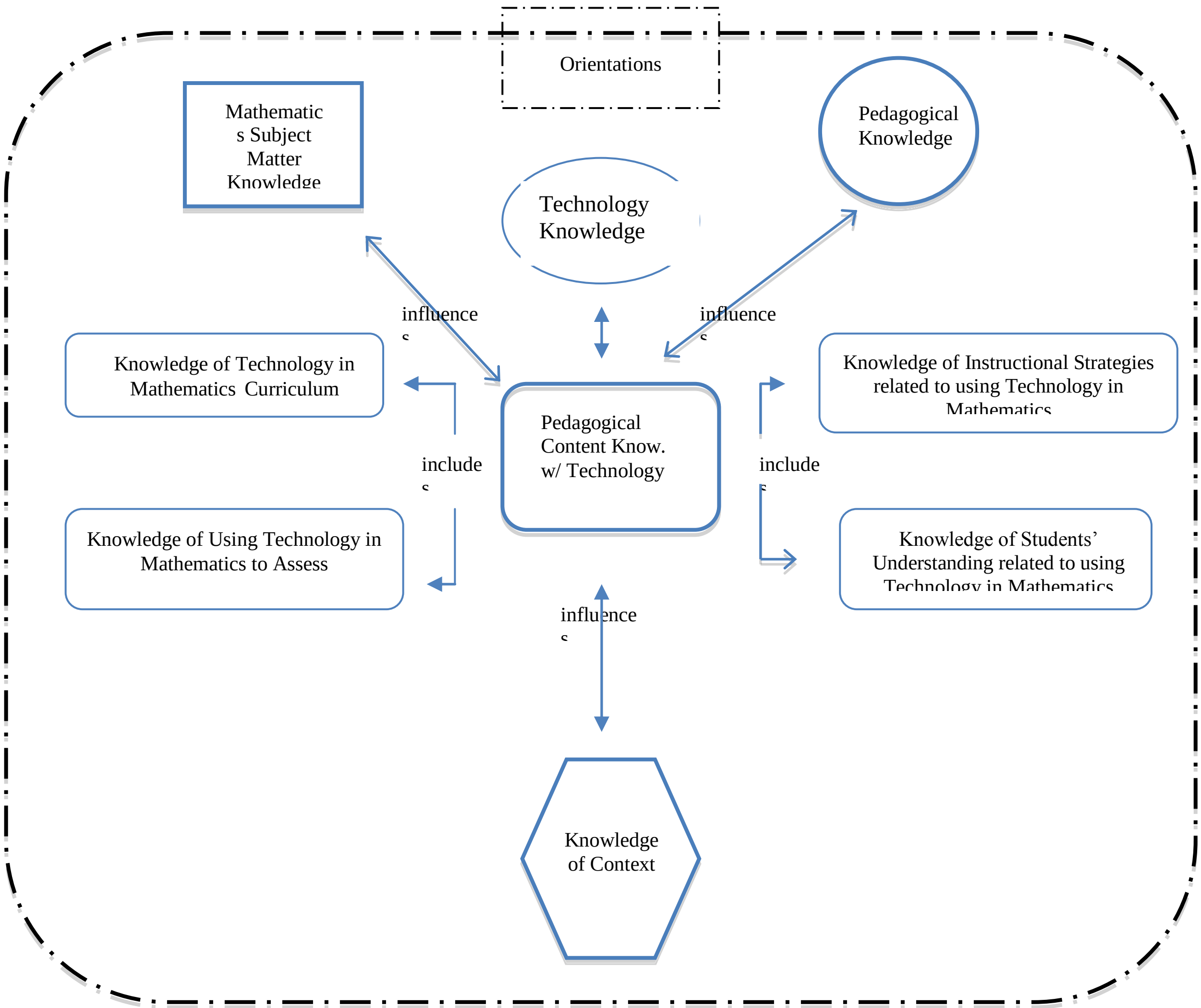
During the past several years, my interest related to researching teachers as they incorporate a new technology into their secondary mathematics instruction has grown. At the same time, I have worked as a graduate research assistant on an NSF-funded research project, Researching Science and Mathematics Teacher Learning in Alternative Certification Models (ReSMAR^{2}T), which has been focused on investigating teacher knowledge and orientations. In addition to the research literature base, this study draws on my interest in teachers' use of new technologies and my work with the ReSMAR^{2}T project. As a result, this study draws on the ReSMAR^{2}T model of teacher knowledge as well as data collection instruments and methods used by the project team. With modifications, I explicitly highlight technology and the subject of mathematics in a model of teacher knowledge as I examine what pedagogical content knowledge secondary mathematics teachers draw upon when they begin to implement a new technology (i.e., TI-*n*spire™) in their instruction.

With models of teacher knowledge, users assume teachers have different types of knowledge that they draw on when they teach. For example, they have knowledge about mathematics, general teaching strategies such as managing small groups or posing questions, and assessment. Moreover, they have a specialized knowledge that Lee Shulman (1986) described as pedagogical content knowledge. Lannin, Chval, and Arbaugh (under review) explain,

> Shulman (1986) conceptualized the specific knowledge that applies to content area specialists. In our work we focus on the knowledge that mathematics teachers need and use that distinguishes them from other middle and secondary teachers. This knowledge, pedagogical content knowledge (PCK), represents the transformation of several types of knowledge that is unique to mathematics teachers. PCK includes what teachers know about learners, curriculum, instruction, and assessment that help them transform content knowledge into effective teaching and learning (Grossman, 1990; Magnusson, Krajcik, & Borko, 1999). (p. 4)

Based on the work of Shulman (1986), Grossman (1990), and Magnusson, Krajcik, & Borko (1999), the model of teacher knowledge used by the ReSMAR^{2}T team (see Figure 4) relates subject matter knowledge, pedagogical knowledge, PCK, and knowledge of context. As discussed in Chapter 1, I modify the ReSMAR^{2}T theoretical framework (see Figure 7) to include a focus on technology for this research study.

Figure 7. Model of teacher knowledge explicitly using mathematics and technology.



The model of teacher knowledge framework captures interplay between content, pedagogy, and technology as it is based upon the Magnusson, Krajcik, and Borko model in which "by designating these components as part of a single construct – pedagogical content knowledge – we indicate that the components function as a whole" (p. 115).

As mentioned in chapter 2, Shulman (1986), Magnusson, Krajcik, and Borko (1999), and Mishra and Koehler (2006, 2008) conceptualized teacher knowledge; however, these frameworks are not specific to mathematics teaching. Furthermore, I use and make modifications to the PCK model rather than the TPCK model because PCK is more established as theoretical construct in the field of educational research. There is disagreement within the mathematics education community about what TPCK is and where it applies (personal communications at the Association of Mathematics Teacher Educator annual conference, 2008). As a result, the theoretical framework for the study utilizes a modified version of PCK within a teacher knowledge model.

In addition to looking at teacher knowledge, I investigated teachers' orientations. Theoretically, teachers filter their knowledge acquired while learning as well as their knowledge used when making decisions during teaching practices through their orientations. Teachers' orientations for teaching influence teacher knowledge bases (Shulman, 1986) and how they are enacted in the classroom (Handal, 2003: Remillard & Bryans, 2004; Thompson, 1992).

Orientations are part of the PCK construct that the ReSMAR^{2}T team defined more specifically by building upon the work of Magnusson, Krajcik, and Borko (1999) and mathematics education research literature. Magnusson et al. (1999) state orientations towards science teaching include "teachers' knowledge and beliefs about the purposes and goals for teaching science at a particular grade level… an orientation represents a general way of viewing or conceptualizing science teaching" (p. 97). In mathematics education research literature, a *conception*, the term most directly connected to an orientation towards mathematics teaching, is "a general notion or mental structure encompassing beliefs, values, meaning, concepts, propositions, rules, mental images, and preferences" (Philipp, 2007, p. 259). Philipp also notes,

> When using the term *conceptions*, Thompson, recognizing the important relationship between knowledge and beliefs, seemed less interested in drawing distinctions between these terms, and she stated, "To look at research on mathematics teachers' beliefs and conceptions in isolation from research on mathematics teachers' knowledge will necessarily result in an incomplete picture" (p. 131)…A.G. Thompson (1992), in her chapter on teacher beliefs, addressed the importance of considering beliefs together with knowledge and referred to this construct as *teachers' conceptions.* (pgs. 259, 262).

In the mathematics education literature, orientations for teaching mathematics, teachers' conceptions of mathematics and mathematics teaching as related to instructional practice (Thompson, 1984; Thompson, Philipp, Thompson, & Boyd, 1994), and the impact of beliefs on teaching mathematics (Ernest, 1989) appear linked. Thompson (1984) agreed with Magnusson et al. (1999) as she wrote, "any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by the teachers and how these are related to their instructional practices" (p. 106). Therefore, after reading, synthesizing, and analyzing mathematics research literature, the ReSMAR^{2}T team, in which I participate, settled upon the following definition: *orientations* are the goals and purposes related to the teacher's role, students' roles, and the nature of the discipline of mathematics. In this study, I utilized the ReSMAR^{2}T definition of orientation within the model of teacher knowledge shown in Figure 8 to guide data collection and analysis, which I describe in the following section of this chapter.

Data Collection

Three experienced teachers from 2 secondary schools participated in this study as I qualitatively investigated orientations to teaching mathematics with technology and pedagogical content knowledge with technology to further understand secondary mathematics teacher knowledge as it relates to the integration of new technology (i.e., the TI-*n*spire™ calculator). I used persistent observations and interviews to identify the characteristics and elements in the situations most relevant to understanding what pedagogical content knowledge secondary mathematics teachers draw upon as they begin using a new technology during classroom instruction. In the following subsections, I describe participant selection, the participants, and data collection methods.

Participant Selection

I sought out teachers who said they were transitioning into using the TI-nspire™ calculators within their secondary mathematics classroom instruction. I attended a week-long professional development session for grades 6-12 mathematics teacher leaders in June 2008 and a TI-nspire™ training workshop in November 2008 where I met teachers who fit into this category. Then, I requested permission to research in the school districts where these teachers worked. As a result, I obtained permission to research within two Midwestern school districts.

School administrators identified teachers to participate in my study. Then, I emailed these teachers identified by school administrators and invited them to participate in the study (see Appendix A). After they emailed agreement to participate, I met with each person and secured written consent in which they agreed to: (a) participate in an initial interview and closing interview, (b) allow me to observe and videotape at least six mathematics lessons, and (c) participate in a post interview following each videotaped lesson and respond to questions about the lesson after watching parts of the video (see Appendix B).

During the summer of 2008, I attended a week-long professional development session for grades 6-12 mathematics teacher leaders. As I interacted with the participants, I met Kate. Kate and two other teacher leaders told me they planned to incorporate the TI-nspire™ calculator into their high school mathematics instruction within the next year. When I sought out research participants in Spring 2009, Kate was the only teacher who had access to the new technology and used it in her mathematics classes. Kate agreed to participate in April 2009.

I also attended a TI-nspire™ calculator training session in Fall 2008 which facilitated my interest in studying the use of the TI-nspire™ calculator. Upon preparing my for data collection in Spring 2009, I contacted Wendy, one of the teachers who facilitated the TI-*n*spire™ calculator training session. Wendy invited me to visit her classroom and school. She also introduced me to her mathematics department colleagues and allowed me to survey them. Through this survey, I learned that Joe, Mary, and Wendy were the three teachers in this suburban school who were using the TI-*n*spire™ calculators in their teaching practices. As a result, I requested permission to research in their school district and got reconnected with Joe and Mary. Joe and Mary agreed to participate in the study in May 2009.

Participant Descriptions

Joe, Mary, and Kate, three experienced secondary mathematics teachers, agreed to participate in this study. Although I hoped to recruit three teachers teaching the same course because PCK is theorized as topic- and/or course-specific, I had to relax this criterion in order to find participants to conduct the study. Although, Joe, Mary, and Kate taught different mathematics curriculum, they met the other selection criteria. They had a minimum of three years of teaching experience, which was critical for the study because many novice teachers lack PCK. They were also in their first year of teaching with the TI-nspire™. I provide more descriptive information about each participant in Table 4.

Table 4.

Case Study Participants

Participant	Course Observed	Yrs. Teaching Experience	School Context	Technologies in Classroom
Joe	Algebra 1B	Math: 7 Course: 2	Suburban School Size: 1374 Grades: 9-12 90 minute periods 7 blocks and 1 Academic Lab, 4 block-day/alternating Administrators: 6 Faculty: 96 Counselors: 5 Support: 28	TI-nspire™, TI-Nspire software, TI-84, TI-84 software, TI-84 Plus, TI Navigator System, SMART board, internet, electronic copy of textbook, projector
Mary	Honors Geometry	Math: 8 Course: 8	Suburban School Size: 1374 Grades: 9-12 90 minute periods 7 blocks and 1 Academic Lab, 4 block-day/alternating Administrators: 6 Faculty: 96 Counselors: 5 Support: 28	TI-nspire™, TI-Nspire software, TI-84, TI-84 Plus, TI Navigator System, SMART board, document camera, internet, projector
Kate	Honors Integrated 4	Math: 3 Course: 1	Mid-sized City School Size: 1822 Grades: 10-12 95 minute periods 8 block schedule, 4 block-day/alternating Administrators: 7 Faculty: 115 Counselors: 7 Support: 27	TI-nspire™ CAS, TI-84, TI-84, SMART board, projector

Data Collection Methods

The primary data sources included interviews (initial, stimulated-recall, and closing interviews) and videotaped observations. Secondary data sources included handouts and field notes. I videotaped Kate's classroom in April/May 2009 as well as Joe and Mary's classrooms in August/September/October 2009. Overall, I collected the data listed in Table 5. I describe the interviews and videotaped observations in the subsections below.

Table 5.

Summary of Collected Data

Participant	Initial Interview	Videotaped Observations	Stimulated-Recall Interviews	Closing Interview
Joe	1	16	6	1
Mary	1	16	4	1
Kate	1	8	4	1

Initial interview. I designed a semi-structured interview (see Appendix C) to uncover secondary teachers' knowledge while using the TI-*n*spire™ to teach mathematics and their orientations towards teaching with the TI-nspire™ calculator. Through the initial interview, I gathered information regarding participant teachers' teaching background, professional development experiences, their pedagogical content knowledge, and orientation to teaching mathematics with technology. Participants engaged in an interview for approximately one hour.

Videotaped observations. I observed, videotaped, and took field notes during at least 2 secondary mathematics lessons per week for at least 4 consecutive weeks in each classroom to document and reveal how teachers enact knowledge when using the TI-nspire™ during mathematics instruction. I focused the camera on the teacher during instruction and purposefully captured all the images written or projected on the SMART board, overhead, or chalkboard. The teaching practices and images on the SMART™ board served as points of discussion in order to learn about the teacher's knowledge of assessment, curriculum instructional strategies, and students' understanding with technology within and for mathematics during stimulated-recall interviews. Each class period lasted 90 or 95 minutes. During each observation, I also collected classroom artifacts such as handouts when appropriate.

During the first week of filming, I videotaped classrooms without having discussions with the teachers about their knowledge or practice. This approach enabled me to capture a more naturalistic setting before sharing my theoretical lens with the teacher thereby influencing extra attention to instructional strategies, assessments, curriculum or students' learning. Alba Thompson used this approach in her 1982 dissertation work so that she could both capture a naturalistic setting and formulate questions specific to the teachers' contexts.

Stimulated recall interviews. Teachers draw upon their knowledge as they make decisions during instruction. As a result, I displayed and discussed specific segments from the video-recorded lessons with the teacher during a stimulated recall interview (Pirie, 1996; Schempp, 1995) each week of filming. I probed participant knowledge via playback of parts of the lesson. I selected clips where students made a profound comment and the teacher did or did not recognize it or misinterprets what the student said or did, where the teacher made an instructional decision that altered the flow of the classroom by asking a question or directing students to perform a particular task, where the teacher implemented assessment to ascertain student prior knowledge, or where the teacher demonstrated particularly strong PCK with the technology. By discussing specific aspects of their teaching with them for approximately one hour via 14 stimulated-recall interviews, I gained an understanding of their PCK with technology.

I asked questions such as (see Appendix D for a more complete list):

- Tell me about that (example/analogy/activity) with the TI-*n*spire™.
- How did this teaching strategy with the TI-*n*spire™ help you achieve your overall goals?
- How could you teach this topic with the TI-*n*spire™ in a different way?
- Tell me about how you found out about student learning with the TI-*n*spire™.
- How did the activities with the TI-*n*spire™ achieve the purpose you intended?
- How did your curriculum materials support or hinder you in implementing your plan with the TI-*n*spire™?

Closing interview. I conducted a semi-structured closing interview to reveal secondary teachers' experiences with using the TI-nspire™ to teach mathematics, what modifications they would make for the next year, the challenges they faced with learning about and using the TI-nspire™ during classroom instruction, and their orientations towards teaching with the TI-nspire™ calculator. Participants engaged in the interview for approximately 45-60 minutes. Questions during the closing interview included: (a) what did you learn about using the TI-*n*spire™ from teaching this unit, (b) what modifications would you make for next year, (c) what challenges did you face while integrating the TI-*n*spire™ into mathematics instruction, (d) describe your best day of teaching with a TI-*n*spire™, (e) what is the teacher's role with the TI-*n*spire™ in a typical lesson, (f) what is the student's role with the TI-*n*spire™ in a typical lesson (see Appendix E).

Data Analysis

I transcribed all the interviews, and after a prolonged period of engagement and persistent observations, I analyzed the data via triangulation of multiple data sources and data collection methods in order to claim credible findings. I describe this data analysis process in six phases.

Phase 1

While traveling after observations and interviews, I recorded initial impressions, reflections, and additional questions as I spoke into an audiorecorder. In these conversations and memos to myself, I tried to articulate how I would answer my research questions based on what I had seen or heard the teacher say that day. I generated follow-up questions as well as initial analyses while collecting the data.

Phase 2

After I had collected audio recorded interviews, I transcribed each interview. Then, using QSR NVivo 8 software, I imported the transcriptions into a project file and coded the transcripts line-by-line with nodes related to orientations and the four main dimensions of PCK with technology: (a) knowledge of technology in mathematics curriculum, (b) knowledge using technology in mathematics to assess, (c) knowledge of instructional strategies related to using technology in mathematics, and (d) knowledge of students' understanding related to using technology in mathematics. I used the coding dictionary shown in Table 6 to define each node and illustrate an example of each node with a sample from the data. While identifying components of PCK with technology three different individuals coded a complete interview transcript and resolved all discrepancies through adjustment of the examples.

Table 6. *Coding Dictionary for PCK with Technology*

Component of PCK with Technology	Description	Example
K. of Technology in Mathematics Curriculum	knowledge of curriculum materials for teaching particular content and knowledge of the horizontal and vertical curricula for a subject with technology	"I've done circles; I've done all the triangles. I've done vectors and all the transformations on the calculator. That is pretty powerful. I've done proofs using the calculator, proofs of theorems. We have done bell curves, normal distribution and stuff. And of course all of the regular, all of the topics like writing the equation of the lines and parabolas and stuff."[Mary, initial].
K. of Using Technology in Mathematics to Assess	knowledge of the dimensions of the subject important to assess and the methods for assessing the learning of these dimensions with technology; knowing the advantages and disadvantages of particular instruments and techniques	"I don't know that I was able to assess them. I guess just because I don't see what they see on their screen. All I can do is show them what their screen should look like" [Joe, 9/16].
K. of Students' Understanding related to Using Technology in Mathematics	ideas about what students will and will not understand with the use of particular representations and analogies with technology	"few things we did they really understood what the calculator was doing what they were asking it to do and what they were looking at because a lot of several of the things we did are similar to their 84 like comparing a table with a graph it's just now maybe you can see it at the same time or whatever. They still compared things in similar ways" [Kate, 5/8].
K. of Instructional Strategies related to Using Technology in Mathematics	a teacher's repertoire of approaches for teaching a topic with technology (e.g., explanations, representations, and experiments that can be used to effectively teach a particular topic)	"We did a unit on horizontal shifts and vertical shifts and horizontal and vertical compression and all of those things. We did look at them to look at graphs because you can actually grab the graph and drag it" [Kate, 5/18].
Orientation	"a general way of viewing discipline-specific teaching" (Magnusson et. al, 1999, p. 97) that has been described by: (a) the goals of a teacher for teaching a particular subject and (b) the typical characteristics of the instruction practiced by a teacher (Friedrichsen)	"my students are working with their groups and there is not a lot of whole class there is not even a whole lot of whole class discussion and there is virtually no whole class instruction" [Kate, 5/8].

Phase 3

After coding all of the interview transcripts, I watched the classroom observation videos and looked at field notes to identify instances when Joe, Mary, and Kate used the TI-nspire™ to teach or used the TI-84 and reflected on how they could have used the TI-nspire™ to teach. I found 5 episodes within which Joe used the TI-nspire™ handhelds and emulator to teach his Algebra 1B class. I noticed Mary used the TI-nspire™ calculator twice with students. Kate powerfully used the TI-84s five times and then reflected on how she could have and/or should have used the TI-nspire™ calculator. As a result, I analyzed what teacher knowledge they drew upon while teaching with a new technology within each of these episodes, and then wrote a description of their orientations for teaching and summarized each of the cases before making cross-case assertions.

Phase 4

During phase 4, I created profiles describing how Joe, Mary, and Kate drew upon different components of their PCK with technology knowledge. Within each profile, I described the participant and their background, discussed their orientation to teaching mathematics with technology, and then described the knowledge they drew upon while using the new technology within episodes before summarizing their PCK with technology. To do this, I queried knowledge component codes using QSR NVivo 8 software on relevant transcripts. I queried all the interviews for data related to orientations and selected the stimulated-recall interview to query by the date that the episode occurred. In other words, I reviewed interview data excerpts coded and separated different components of teacher knowledge within a QSR NVivo 8 project to describe orientations as well as enacted and displayed knowledge within each episode. I read field notes and watched video data before describing how the task was enacted. I used handouts and field notes to describe the task. Two fellow doctoral students coded a full interview transcript independently with the coding dictionary (Table 5) and then compared and discussed coding. All discrepancies, although there were few, were discussed until agreed upon. Additionally, a faculty member checked coded portions of all other data that supported the findings to ensure reliability.

Phase 5

Using post-it-notes, I wrote down instances of different components of PCK with technology while rereading each profile. I stuck these post-it-notes to a piece of chart paper for each participant. Then, I looked at the chart paper with post-it-notes to summarize each participant's PCK with technology. I used data from phase 4 to answer both of my research questions in terms of each case within this phase of analysis. I collected data from multiple sources and triangulated to increase the trustworthiness.

Phase 6

Joe, Kate, and Mary represented three cases of secondary mathematics teachers who were beginning to use the TI-nspire™ during instruction. They had classroom sets of TI-nspire™ calculators and told me that they were actively trying to incorporate the new technology during instruction although they did not use it every day that I videotaped and interviewed them. I deductively analyzed each case using my teacher model framework and then inductively analyzed across the cases to make assertions and discuss my findings of teacher knowledge as secondary teachers use a new technology in the final stages of analysis. To do this, I looked at the summaries of PCK with technology for each case and considered how PCK with technology appeared in similar ways or different ways across the cases. I wrote down assertions that emerged from patterns in the data and then looked for alternative explanations and contrasting data before finalizing assertions. I asserted explanations about PCK with technology across cases to develop and articulate ideas for further study.

Summary

I observed, videotaped, and interviewed three experienced secondary mathematics teachers during the first year they used a new technology, the TI-nspire™ calculator. I used videos, field notes, and interview transcripts to document the enacted and displayed pedagogical content knowledge with technology and orientations for teaching mathematics with technology for these three cases. I coded each transcript according to a coding dictionary and then made inferences regarding orientations for teaching with technology and teacher knowledge at the beginning of a technology integration process. I present my findings in the next chapter.

CHAPTER 4: FINDINGS AND DISCUSSION

In this chapter, I document the findings that resulted from this qualitative study on what teacher knowledge, situated within their orientation for teaching mathematics with technology, three experienced secondary mathematics teachers draw upon while incorporating a new technology during instruction. I view Joe, Mary, and Kate, the three teachers beginning to use the TI-nspire™ calculator, as the experts in this naturalistic inquiry using a case study methodology. They decided to use the new TI-nspire™ calculators during their instruction based on their own initiative, not as a result of my coercion or suggestion. I am simply a researcher, with secondary teaching experiences, knowledge of the technology, and knowledge about pedagogical content knowledge, who observes and listens to them as they teach. I reflect on their knowledge and teaching (with them during stimulated-recall interviews and without them after data collection) in order to be able to analyze, document, and start to articulate what teacher knowledge they draw upon while using a new technology during mathematics instruction. Examples described in this work do not necessarily represent best teaching practices with the TI-nspire™. Rather, these examples illustrate what secondary teachers do, try, and think about as they begin to integrate the TI-nspire™. I highlight secondary teacher knowledge as they integrate this new technology into their mathematics instruction in the first year. In this chapter, I first organize and present findings case-by-case. Then, in the final major sections, I summarize and discuss the findings.

Joe

> *"Each person [teacher in the building] has a white board. I would just rather have the technology because I think it's more powerful, and I think it relates to the students more because they are very technology savvy. You should see them with their phones and text messaging" (September 17, 2009).*

Joe teaches algebra, geometry, and computer programming classes at a suburban high school in the Midwest. He describes what prompted him to start using the new TI-nspire™ calculator in the following way:

> In my district, they initiated a program called tech demo teachers where in all of the four core areas they decided that they were going to spend as much money as needed to put all kinds of technology into the classroom and see if the technology made a difference in the students' learning. Their goal was to compile all this data and bring it to the board and say you know we need this money; we need this technology in the room. Look what it has done for these students in the classroom. Initially they just started giving me technology to go into my classroom, and I started out with the TI-84s with the TI Navigator. And since then, the new TI-nspire™ calculator came out and they decided that the tech demo teachers should have the newest stuff so they sent a classroom set of thirty of those. (initial interview)

Joe is a "tech demo" teacher at his school, meaning he is 1 of 4 people in the building (out of 15 mathematics teachers and out 63 teachers in the core curriculum areas) and 1 of 16 in the district who works to stay abreast of the latest technology and incorporate the newest technologies into his teaching practices. He cares about being a tech demo teacher because he believes the students learn better in a technology-enhanced classroom environment. He explains,

> I think technology changes constantly and I think that if we don't if we don't use the technology in the classroom, the students are definitely, the students are definitely using it. It might not be the graphing calculator but if you can relate to them in any way it just makes teaching so much easier and they I believe they learn better. (initial interview)

To learn how to use and teach with the TI-nspire™calculator and emulator, Joe attended the 2009 International Teachers Teaching with Technology (T^3) Conference in Seattle. Additionally, Joe attended and presented at regional T^3 conferences and district-wide tech demo teacher meetings and volunteered to pilot supplementary materials created by TI. He shares,

> I've gone to many Texas Instruments conferences, the regional ones. This past year we went to Seattle. And I've also presented at Texas Instruments conferences some of the regional. We did one at the end METC. We do some stuff in school, in district, too. There are other tech demo teachers. There are four high schools. There are four other teachers that we meet every summer kind of go over the new technologies and what we've done with it and share ideas with each other. (initial interview)

During these experiences, he has learned about activities that utilize some of the unique powers of the TI-nspire™ calculators (e.g., dynamic images and linked mathematical representations). As a result of these professional development opportunities, Joe has set a goal to incorporate this new technology into his geometry and algebra classes during the 2009-2010 academic year.

At start of the 2009-2010 school year, Joe was optimistic that the transition to using the TI-nspire™ would be worthwhile for his students. He stated, "I think it will help them think about the math more visually… I think I will be able to do a lot of the stuff that I did with the TI-84, and I will be able to do more with the TI-nspire™." (initial interview). Furthermore, he shared,

> Prior to having the new technology in the class [with the TI-84, Navigator, and SMART board but not yet the TI-nspire™] it was all up at the board, let's do this example, let me show you this, here let's do this example, are there any questions, okay good let's go on. I didn't have the visual learner in mind when I was planning my lessons. With the TI-nspire™because you can do so many activities right there at your hands, a person that is more hands-on and more visual gets to see use, and experience the math. (initial interview)

I attended Joe's Algebra IB class, a remedial class, for 8 weeks and observed 17 lessons during the beginning of the 2009-2010 school year. These observations provided an opportunity to study Joe's transition into using a new technology. During this time, Joe used the TI-nspire™calculators (without CAS), five times as follows:

1. He introduced a scavenger hunt activity so that students would become familiar with the new handheld.
2. He demonstrated using the TI-nspire™ while discussing a fractions problem.
3. He explored and rationalized the inverse property of multiplication using the TI-nspire™.
4. He used a student activity related to expressions and equations to more precisely define each of these terms and recognize similarities and differences in using these mathematical terms.
5. He engaged students in a distributive property activity to demonstrate how and why the symbolic representation of this property works and makes sense.

In the following sections, I will describe his orientation to teaching mathematics with technology and the pedagogical content knowledge (PCK) Joe enacts and displays during these five episodes.

Prior to this discussion, one should note the two contexts that influence Joe's use of the new technology. First, Joe's subject-specific textbook does not suggest ways in which he could or should use the TI-nspire™ calculator with the algebra curriculum. Therefore, Joe seeks out resources such as the TI Activity Exchange (http://education.ti.com/educationportal/activityexchange), which provides ideas and print materials to teach mathematics with technology. He describes,

> Some books are really good about including technology stuff in there. They'll have an activity for the 84. Even some of the newer books are starting to have [activities] for the TI-nspire™. A lot of the newer books are starting to have activities you can do with the Navigator, built right into the book. This book is the oldest book that we have, we were supposed to be getting a new book last year, but it never happened. They stuck with the book that they've been using for three or four years. So a lot of the technology stuff that we've been doing and we're going to do is stuff that we've made up, with the exception of the *Algebra Nspire* stuff that Wendy received over the summer. The book doesn't lend itself to technology at all. It's just not technology based. It doesn't have activities in it like some of the newer books do. (9/2 interview)

The fact that Joe did not have a textbook to support the use of TI-nspires™ influenced whether and how he used these calculators for specific lessons. A second important context was related to Joe's perception of the students in his Algebra IB class. Joe did not consistently use the TI-nspire™ with this class at the beginning of the year. He reported that he was hesitant about using this new technology with this particular group of students. Joe explained,

> I don't know if I'd dive into an Algebra 1B class, with the type of kids that we have, without something already premade. The reason behind that is that with that level of kid, you're so focused on getting them to be quiet, getting them in their seats, trying to teach them how to be good student, that if you do the calculator integration and you're still trying to learn at the same time, I think it can get overwhelming. (9/15 stimulated-recall interview)

Joe believes he can create TI-nspire™ activities and provide opportunities for students to independently explore with the new technology in classes where he is more comfortable (e.g., Geometry) and the students are at higher levels. Nevertheless, when asked about how he plans to build his knowledge and comfort level with using the TI-nspire™ during his Algebra 1B class, he commented:

> Leading up to an activity, we did the scavenger hunt with the TI-nspire™, I think that helped them get used to it. Some kids used the TI-nspire™ calculator as just an extra calculator on their desk… I think just having it on the desk for them to use as a calculator has been good because we get them out every day, and they at least have it on their desk every day. I think that it's going to help bring up the comfort level…I think just having it on their desk every day is going to help them become more comfortable with it even if we don't use it, even if they're just using it as a calculator. That way they're not overwhelmed when they get it out and they'll be seeing it for the first time, all the buttons can be overwhelming. (9/15 stimulated-recall interview)

Although Joe only used the TI-nspire™5 times during 17 lessons, the students have the TI-nspire™ handhelds as well as TI-84s with the Navigator system on their desks every class day and use handheld graphing calculators daily.

Orientation to Teaching Mathematics with Technology

Joe holds a didactic (Magnusson, Krajcik, & Borko, 1999) and activity-driven (Anderson & Smith, 1987) orientation to teaching mathematics with technology. He states, "My goal is to give each of my students some idea of how to problem-solve whether it's a math problem or a problem in everyday life. I want them to think logically and systematically through a problem" (initial interview). When asked about his ideal image of teaching mathematics with technology, he believes "a balance of teacher instruction along with student discovery using the technology" (closing interview) is what he strives to achieve in the classroom. During a typical 90-minute class period, Joe describes, "What we will do is go over the homework at the beginning of class, maybe take a little quiz. Then, I'll either present something to them and then do an activity or start right off with an activity" (initial interview). He wants the students to participate in "hands-on" activities with the technology and have a vested interest in learning and knowing the rules of mathematics.

Joe envisions the teacher as the guide and expert other in the classroom. Furthermore, he reflects,

> I like to see myself as a facilitator of the activity. I try not to get too involved. I try not to do the activity for them and answer all of the questions for them. I try to point them in the right directions…but then again I am also there to know if they're not getting it and hold their hand and walk them through it or further the discussion along. (closing interview)

He is in the classroom to guide the learning process and to share mathematical facts with the students.

He believes the students should be participants who do not disrupt class. He describes,

> The role of the students is like an athlete where they are to try their best all of the time and their goal is to try to learn something new every day and just be the best that they can be…They need to be an active participant. They need to play along. (closing interview)

Joe thinks students should problem-solve, think, use technology, and make sense of what they are doing while in their mathematics classroom.

Scavenger Hunt

The task. In order to familiarize students with using the new TI-nspire™ handhelds, Joe selects a scavenger hunt task from the TI Activity exchange and uses it with his Algebra 1B students during the third week of school. He describes the activity by saying,

> It did different things; it showed them how to copy a problem. It showed them how to get into the catalog where they can make a fraction type thing. A lot of it, they hadn't seen before. But it was good because it gave them a step-by-step example of, "Do this, now do this," and it explored the home key. "What do you see on the home key? How many icons do you see? Click on this icon, what does it tell you to do? Let's add a new page," and it shows them how to add a new page. Stuff like that. We try and navigate our way through it. (9/2 interview)

For instance, students do the following problems on the TI-nspire™ calculator with the following tips and guiding questions next to each problem (see Table 7).

Table 7

Sample problems from a TI-nspire™ Scavenger Hunt

Problem	Tip	Guiding Question
$3^8 - 4^5$	Make sure your screen matches the problem.	What did you do to get out of the exponent box?
$\sqrt{27}$	Try them both!	Did you use ctrl x| |{ or ctrl $\sqrt{\ }$ x^2 ?
$\lvert -28.87 \rvert - 2\lvert 19.3 \rvert$	None	Where was the absolute value sign?
$\left(\frac{4}{3}\right)\left(\frac{3}{5}\right)$	To get the decimal answer, press ctrl =(enter). What did you get?	Is your answer given as a decimal or a fraction?

Enacting the task. Joe wants to introduce the students to the TI-nspire™ without the stress of having to learn a new mathematical concept. As a result, he chooses this task. He asks each student to individually complete the scavenger hunt using the TI-nspire™ calculator. He does not demonstrate anything with the TI-NSpire before giving them the calculator. Rather, he expects the students read through the guiding worksheet and learn about using the TI-nspire™ on their own.

The students work for 60 minutes. They sit in three long horizontal (to the front of the room) rows with table-top desks next to each other. Although they are expected to individually complete the worksheet and learn how to use the TI-nspire™, they help one another as they work. Joe circulates the room and monitors their interactions.

Knowledge enacted. Joe enacts knowledge of curriculum materials for mathematics with technology and knowledge of students' understanding by finding and using the scavenger hunt. He finds the activity by searching the TI-Activity exchange. Then, after seeing that the activity guides students in learning how to use the TI-nspire™ to compute and graph, he decides to introduce the new technology to students the day they review evaluating expressions and multiplying real numbers during classroom instruction.

The Algebra 1B students are struggling to accurately evaluate expressions, simplify expressions, and solve equations. However, Joe knows the students need to stay motivated to continue practicing these basic skills, build confidence in their math abilities, and progress in their math learning. Consequently, he introduces the TI-nspire™ calculator as a tool they can use to verify their computational work.

Knowledge displayed during reflection. When asked what he thinks students learn from the Scavenger Hunt activity, Joe explains, "I've seen some kids use the fraction tool." Joe determines how the students use their handhelds by watching them work. He acknowledges that students do not regularly use the TI-nspire™ handheld, yet. However, Joe realizes that, "if they used the special feature more, they would start to realize how they can use it more often" (9/15 interview). He recognizes that using the technology more with new features such as copying problems or getting the reduced and exact form of irrational and fraction numbers will help the students learn the features and realize the value of the TI-nspire™ as a tool.

I asked Joe what he thought about modeling the use of the TI-nspire™ on the SMART board. Joe stated, "Whenever I do an TI-nspire™ activity, I do it with them at the board as well. That way, if they get lost, I tell them just to stop what they're doing and watch what's going on at the board" (9/15 interview). Joe knows that modeling how to use the TI-nspire™ calculator can be an effective instructional strategy. He also knows that projecting the TI-nspire™ on the SMART board provides an opportunity to present multiple representations to explain a concept, help students make sense of a problem, or discuss problem solving strategies. Although he demonstrated knowledge of these instructional strategies during the interview, he admitted that he had not used these approaches with the Algebra IB class: "I haven't pulled the calculator up on the board and shown them different ways to do a problem and different ways to type in the problem" (9/2 interview).

Upon further reflection and with a little more time, Joe shares,

> I hadn't thought about it. I have in the past, when talking about… I don't know if we did this or not when we did the parentheses, I guess we didn't. I had them, in the past, brought the calculator up, and showed them if we had the parentheses, if we didn't have the parentheses. (9/15 interview)

So, here although he displays knowledge of instructional strategies with the new technology and knowledge of students' understanding with the technology as he describes how it is important and helpful to talk about the use of parentheses and order of operations with algebra students, he has not used this knowledge while teaching Algebra I at this point in the semester.

Interestingly, Joe comes up with ideas about how he wants to and can use the TI-nspire™ calculator in his Algebra 1B class without a TI activity by reflecting on and discussing his knowledge of instructional strategies related to using technology in mathematics during one of the stimulated recall interviews. Consequently, Joe chooses to use the TI-nspire™ with problems involving fractions and operations with fractions during a later lesson, which I describe in the following section.

Fractions Problem

The task. During the tenth lesson I observed (during the fifth week of the semester), students multiplied real numbers and evaluated expressions. Some of the problems included:

MULTIPLYING REAL NUMBERS. Find the product.

30. -3.3(-1)(-1.5)

31. $15\left(-\frac{2}{15}\right)\left(\frac{3}{4}\right)$

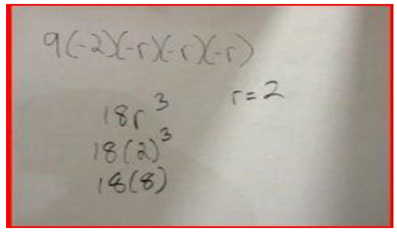
EVALUATION EXPRESSIONS. Evaluate the expression for the given value of the variable.

43. -3(-*a*)(-*a*) when *a* = -7

44. $9(-2)(-r)^3$ when $r=2$

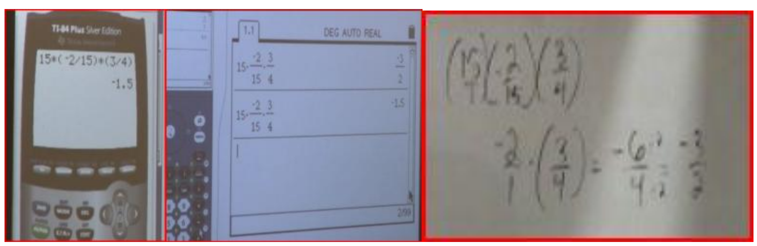
Enacting the task. Joe begins the lesson with a discussion about homework. He displays the answers on the SMART board and then fields students' questions. One student begins with, "What about #44?", evaluate $9(-2)(-r)^3, r = 2$. Joe responds by asking, "How many negatives are in the problem?" Joe poses this question because he assumes that some students will struggle with correctly evaluating $(-r)^3$. As students answer, he writes the representation as 9(-2)(-*r*)(-*r*)(-*r*) on the white board. With this representation, it becomes clear that there are four negatives in the problem so the expression, before evaluating with *r*=2, will be positive $18r^3$. Next, he replaces the *r* with 2, notes that $2^3= 8$ and then multiplies 18 and 8 to get the final answer, 144 (see Figure 8 below).

Figure 8. Simplification of an expression on the whiteboard.



Next, a student asks about #31: $15\left(\frac{-2}{15}\right)\left(\frac{3}{4}\right)$. Joe exclaims, “You don’t like the fractions? Then, pull out your calculator. You have the Nspire calculator on your desk. Right?” (video 9/15). Next, Joe opens the TI-84 SmartView software as well as the TI-nspire™ emulator software on the SMART board to answer this question while using the old and new technology. He represents the problem on the TI-84, then the TI-nspire™, and finally on the whiteboard (see the images in Figure 9).

Figure 9. Multiple representations of fractions with different technologies.



Knowledge enacted. During this lesson, Joe draws upon PCK with and without technology (i.e., digital devices). For example, Joe writes the representation 9(-2)(-r)(-r)(-r) on the white board after a student asks about problem #44 so that the students can see an equivalent algebraic representation of (9) (-2)$(-r)^3$ that can help students build meaning for $(-r)^3$. The second representation helps the students see that the solution will be positive because multiplying four negative numbers will result in a positive answer. Here he draws on his knowledge of students’ understanding with evaluating exponential expressions. He recognizes that students misinterpret or do not understand what $(-r)^3$

represents. Knowing this and using this knowledge, he expands the notation to 9(-2)(-r)(-r)(-r)(-r) in order to decide if the resulting expression will be positive or negative before evaluating the expression at r=2. This is an instructional strategy without the TI-nspire™ that he uses while helping students accurately learn to evaluate exponential expressions.

Although Joe utilizes PCK without the technology to answer the first homework question, he draws upon his PCK with technology when the next homework question is asked. A student asks about problem #31: $15\left(\frac{-2}{15}\right)\left(\frac{3}{4}\right)$. In this instance, Joe uses his knowledge of instructional strategies of mathematics with the technology to teach and respond. He shows and discusses solutions with the TI-84, TI-nspire™, and on the whiteboard. The TI-84 says the answer is -1.5. The TI-nspire™ displays -3/2 and -1.5 as the answer. Here Joe uses his PCK of instructional strategies with technology to evaluate fraction expressions with students using two types of calculators. A discussion of the different outputs from the two calculators leads into a teaching moment without the digital devices.

While it is helpful to use the technology as a tool to check computations, Joe also demonstrates that is important to know how and why the calculator(s) gave the output(s) given by modeling paper-and-pencil work with a think aloud at the whiteboard to simplify $15\left(\frac{-2}{15}\right)\left(\frac{3}{4}\right)$. Joe encourages by-hand computations and reasoning when he states, “What? You don’t trust your calculators?”(video, 9/15) and then takes the time to model accurate reasoning. He first explains the 15s cancel out because anything over itself is one. Then, to multiply fractions you can multiply the numerators and denominators. The result is -6/4, which is the same as -3/2 and -1.5. He draws upon his knowledge of students' understandings of evaluating fractions without the technology when deciding to use the TI-84, the TI-nspire™, and then the whiteboard to discuss how to solve and represent the answer to $15\left(\frac{-2}{15}\right)\left(\frac{3}{4}\right)$.

Knowledge displayed during reflection. Joe draws upon his knowledge of students' understanding with the technology while reflecting upon this episode. During the stimulated-recall interview, Joe recalls that a student said, "I got -3 over 2, but I don't think that's right" and remembers thinking "I think that's the kid who I saw typing it in the TI-nspire™" (interview 9/16). The student either does not realize that -3/2 = -1.5 (what the book says is the answer) or the student is indirectly asking for more help on how to solve this problem without the technology in a way that makes sense. Joe explains,

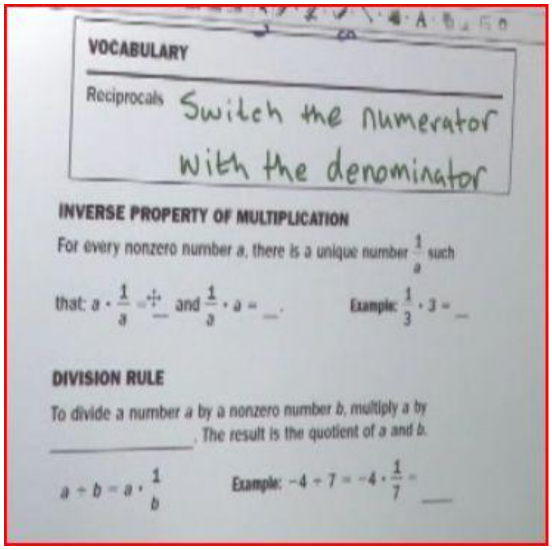
> I think we take it for granted that every student should know that a fraction is really a decimal. They probably really don't think about it. They probably don't really understand that. I think that was really helpful for them to see that if I copy the same problem, and then get the decimal equivalent, they are one and the same. It's the exact same problem so they should be the same. (9/16 interview)

Consequently, Joe intentionally displays multiple fraction answers as well as the decimal equivalent and shows how to compute on the technologies in addition to giving further explanation on the whiteboard. By choosing to respond to this student's comment in multiple ways, Joe draws upon his integrated knowledge of students' understanding, instructional strategies, and assessment (i.e., multiple dimensions of PCK with technology).

Inverse Property of Multiplication

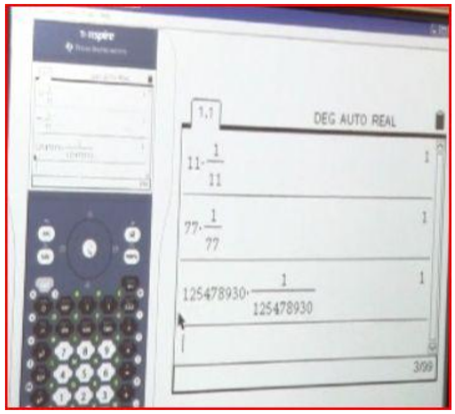
The task. After reviewing how to multiply real numbers and evaluate expressions, Joe and the student move forward and discuss algebraic properties and rules involving real numbers. They define reciprocal and the inverse property of multiplication as well as the division rule. An electronic copy of the textbook is on the SMART board as shown in Figure 10.

Figure 10. Textbook presentation of the inverse property of multiplication.



Enacting the task. Joe starts by defining the word reciprocal with the students. They write on the board: switch the numerator with the denominator. Next, it's onto the Inverse Property of Multiplication, $a \cdot \frac{1}{a} =$ ____ and $\frac{1}{a} \cdot a =$ ___. Joe begins by asking the class, "What is a variable?" (video, 9/17). The students agree that a variable is any number. Joe then asks, "If you take any number and multiply one over the number, what do you get?" (video 9/17). With this question, Joe launches into a TI-nspire™ calculator-based discussion from the SMART board (see image of TI-nspire™ screen in Figure 11 below) to help the students make meaning of this property.

Figure 11. TI-nspire™ screen while investigating the inverse property of multiplication.



With the TI-nspire™ on the SMART board, Joe tells students to pick their favorite number. The conversation goes as follows:

> Do you have a calculator page open? If you don't have a calculator page, you can do home, and then new document, that's number 6. It's going to ask you if you want to save, and then click "no". Then it will ask you what you want to add, you can add a calculator page. The home key is right here. It looks like a house. Here we go. The question that's being asked is here: "If you take a number, and you multiply it by 1 over that number, what's going to happen?" Everyone, pick your favorite number. Mine is 11. And multiply that number by the fraction 1 over your favorite number. Control, divide will give you a fraction. So you took your favorite number and you multiplied it by one over your favorite number. What do you get? (video)

He picks 11 and enters $11 \cdot \frac{1}{11}$ into the calculator while using the emulator and SMART view which allows the students to see the buttons he pushes as well as the input and output displayed on the calculator screen. Students have handhelds at their desks and Joe expects them to follow along and participate in this discussion and investigation with the calculator. They all hit enter together and get one! Joe tells them to select another favorite number and enter them on their TI-nspires™. Joe chooses 77. He enters $77 \cdot \frac{1}{77}$ and again gets one. Next, he tells students to enter the biggest number they can think of and multiply it by one over itself.

He then asks, “Did anybody get 1?” (video 9/17). The students verbally and nonverbally agree. At this point, Joe decides to go back to the property and ask the students what *a* times 1 over *a* equals. The students respond, “One.” Joe seems satisfied to move on but pauses for students’ questions. A student takes advantage of this opportunity and asks, “How is 67 times 1 over 67 one?” To answer, Joe moves away from the TI-nspire™ calculator on the SMART board and writes with a dry erase marker on the whiteboard.

On the whiteboard, Joe draws a circle and divides it into 8 equal pieces. He explains to the student that he is going to use 8 pieces rather than 67 here because he cannot easily draw 67, which he would need to do to directly answer the student’s question. Next, Joe asserts that 1/8 is one piece of the circle. The student can agree on that. Then, Joe says so if I give you 8 pieces, what do you have if you put them all together. The student responds, “The whole.” Right. Therefore, 1/8 times 8 equals one. Joe uses this circle analogy and representation as another powerful way, without the technology, to explain and teach the inverse property of multiplication.

Knowledge enacted and displayed. During the discussion (i.e., teaching portion of this lesson), Joe must use his PCK with technology as well as his PCK for the inverse multiplication property of multiplication. First, he uses his knowledge of instructional strategies with technology, knowledge of students’ understanding, and knowledge of the TI-nspire™ as he makes the decision to immediately use the calculator to investigate the math question of the day: "If you take any number and multiply one over the number, what do you get?" He explains,

> I always find that if you start with the letters, they get really confused. Some of them just don't understand what the letters are. If I were to start explaining to them that a times 1 over a is 1, I really don't know that they would have understood. I always like to start with random numbers. I always find that if you do just basic examples like that, before you start talking about the abstract, then the abstract will make a little bit more sense. (9/21 interview)

He knows the TI-nspire™ displays fractions as fractions just as students see in the textbook and realizes this might be a worthwhile way to teach this property. Likewise, Joe realizes that pattern finding with concrete numbers helps the students understand and explain the generalization of the formal property.

After the lesson, Joe further draws upon his knowledge of students' understanding of the inverse property of multiplication (PCK) as he explains,

> I'm hoping they were thinking, "Am I going to get the same thing? I probably should, if it happened twice before. The third time, I probably should get 1 again." I'm hoping they were thinking along those lines. With the biggest number thing, I think some of them were trying to argue about which number they thought was the biggest number. I think I might change that, and next time say make up a 4-digit number instead of the biggest number. But I'm hoping that they were thinking, "Maybe the pattern is going to be true. The pattern is 1, so hopefully the next one is going to be 1." (9/21 interview)

As a result, Joe starts with a number less than twenty. Next, he uses a number greater than 20 and then an even larger number, and asks students to predict what the calculator's output will be before hitting "enter." With the TI-nspire™, Joe provides the students an opportunity to think about multiple examples, predict, and test, which leads one to believe and begin to see a generalization (in this case the inverse property of multiplication).

Joe also draws upon students' understanding and instructional strategies with technology (PCK with technology) when he asks each student to enter their favorite numbers. Joe explains,

> I always like to start with random numbers. Anytime that I can let them pick a number, and they all pick a different number and they all still come up with the same answer, I think it's good. That way, they feel like they have a choice in it, they have a say in it, they're not being told what to do. (9/21)

Joe asks students to choose their favorite numbers. This choice actively involves them in the activity and allows them to see multiple examples instantaneously because most likely the person sitting next to them and the teacher will pick different numbers. In addition, this instructional strategy with the handheld facilitates an exploration that would lead to a generalization.

Joe also knows how he could have improved this activity in order to help teach generalization more effectively when he says,

> To show that any number, and fractions are numbers too, if you multiply any number by its reciprocal, you get one. And even throwing negatives in there. Negative 10 times 1 over -10 is still going to be 1, you're still going to get that positive 1 value. (9/21 interview)

By expanding the set of numbers used within the examples, the generalization is strengthened. In other words, algebraic properties typically hold true for all real numbers. Students often forget that negative numbers and fractions (within the set of rational numbers) are real numbers, too, and these are cases worth empirically testing before making a generalization about all real numbers. Here Joe does not consider or acknowledge irrational numbers, a larger set and subset of the real numbers.

Expressions to Equations Activity

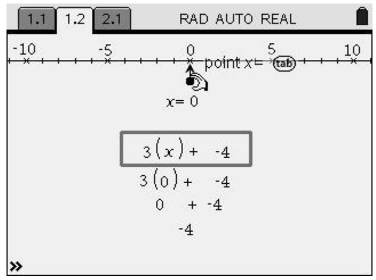
The task. As Joe reaches chapter 3 in the algebra textbook during the first month of the semester, he introduces a TI-nspire™ activity titled, *From Expressions to Equations*. This student activity (see Appendix F) is from the *Algebra Nspired* supplemental materials published by Texas Instruments. With this activity, Joe comments,

> It helps them [the students] see that an expression isn't just, it's just a number, it just represents a number, and then when you have an equal sign, you are talking about what does the expression equal and finding a true value for x. Then, with the slider on the number, the value of x which would make the expression or the equation true. (closing interview)

Enacting the task. Joe instructs with the *From Expressions to Equations* activity via teacher-led and worksheet-based discussion. He begins by reintroducing some functions of the calculator. For example, Joe models and tells students to hit the control button and then the right side of the "nav pad" to move to the next "page" (tab in the document screen). The students follow along by listening and watching Joe work on an enlarged image of the TI-nspire™ handheld on the SMART board. Each student has a TI-nspire™ handheld in his/her hands and a worksheet on his/her desk.

On the first page of the TI-nspire™, students see a number line (see Figure 12), a mathematical statement saying what x is equal to, and the expression $3x + -4$ below the number line as well as three lines showing simplification of the algebra when the expression is evaluated at the given value of x.

Figure 12. Expression and number line representation.



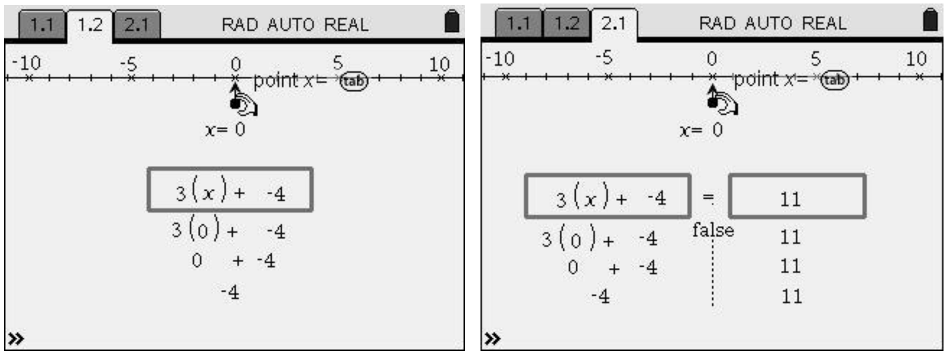
Joe demonstrates how to move the hand on the screen in order to grab and drag the point on the number line. When he drags the point on the number line the value of *x* changes as does the algebraic simplification of the expression below.

Joe asks the students, "What changes as you move the point to the right or the left on the number line? Describe that change." He expects them to answer this question individually in writing on the worksheet. Therefore, he gives students time to answer on their own and then he asks for students to volunteer their answers aloud to the whole class. He instructs the students to write down what others say if it is different than what they have written.

Students notice the value that *x* is equal to changes as the point moves along the number line. They also recognize that "the equation below changes." At this point, Joe recognizes the misuse of the word equation and controls the class discussion by asking others to say more and use different words here. He guides the students to more precise language by saying, "you are changing the value of *x* in the expression" (video, 9/29). Finally, as a class, they conclude the value of the expression changes as the value of *x* changes.

Next, Joe and the students determine the value of *x* when the value of the expression is 20 although they don't share much of their reasoning aloud. Then, they move to the second page of the document and describe what looks the same and what looks different between the pages (see Figure 13).

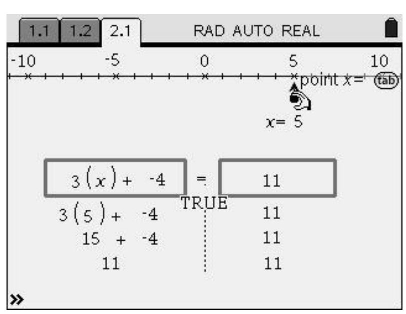
Figure 13. Document pages from the From Expressions to Equations activity.



The students notice that the number line representation at the top is the same and the expression 3*x* + -4 is the same. They mention the pages differ because on the second page a “FALSE” statement is below the equation, a bunch of 11s are on one side of a vertical line, and values for the expression 3*x* + -4 are on the other side of the vertical line.

Joe then asks the students to decide when the statement becomes true rather than false. Students move the point on the number line to 5 and the equation is marked as “TRUE” (see Figure 14).

Figure 14. TI-nspire™ screen shot when equation is true.



At this point, Joe asks the students to describe the process they used to decide 5 would make the equation true. One student asserts, “I took 11 plus 4 and divided by 3 to get 5” (video 9/29). Joe says okay but somewhat dismisses this statement and pushes students to write down that they guessed and checked by moving the point along the number line although he does not explain why he focuses on the guess and check method with students.

To assess towards the end of this lesson, Joe asks students to log onto the TI-84 calculators attached to the Navigator. He polls the class by asking them to respond yes or no: is five the only value of x that makes $3x + -4$ true? Twenty-one out of the twenty-seven students respond yes within a minute according to the display of the quick poll results on a side whiteboard. He then explains, "It always increases or decreases from five so there are no other values of x that makes this true" (video 9/29). Now feeling like the majority of the class is with him, he summarizes the lessons learned from this activity by saying, "Okay, we have learned: (1) there is no equal sign in an expression and (2) you have to know what an expression is equal to in order to be able to solve for x."

Knowledge enacted and displayed. While planning, Joe looks at the activity exchange or the TI *Algebra Nspired* book for relevant topic-specific activities since his textbook does not integrate TI-nspire™ activities and his goal is to do so. In order to connect the procedures and make important distinctions about the different vocabulary (expressions and equations), Joe inserts the *From Expressions to Equations* activity into the Algebra 1B curriculum materials. He explains,

> The expression activity helped them see that an expression isn't just, it's just a number; it represents a number. Then when you have an equal sign, you are talking about what does the expression equal and finding a true value for x. (closing interview)

Here Joe draws upon his knowledge of curriculum and is building his knowledge of mathematics curriculum materials with technology.

Joe clarifies and directs students' use of language during this TI-nspire™ activity and teacher-directed discussion. He uses pedagogical knowledge to draw out students' ideas and manage the 40-minute conversation in order to teach more precise and accurate mathematical language. Additionally, he uses his knowledge of students' understanding with technology to realize that individual students will notice different yet important changes while moving the point on the number line. For instance, the students observe and share that "*x* changes" and "the equation below changes" and Joe redirects and tells the students "the value of *x* in the expression is changing" (video 9/29).

By taking the time to answer the open-ended questions on the worksheet together, Joe models expectations while doing mathematical work as well as his mathematical thinking with the technology, which is significant because students, in this class, most often answer fill-in-the-blank or multiple choice questions during class lectures and on homework assignments. For example, when discussing the value of *x* that makes the equation true, one student explained an algebraic procedure while most students simply dragged the point along the number line until the statement changed to true. In this instance, Joe did not validate the vocal student's formal response and then ask for other student answers. Instead, Joe described an informal approach that he saw many of the students doing, guessing and checking, which is unlike most norms for communicating mathematical reasoning. As a result, Joe simultaneously draws upon his knowledge of students' understanding with technology and instructional strategies with the technology to share appropriate and new language for what the students did with the technology.

Joe does not have knowledge of assessment with the new technology as he explains,

> I don't know that I have really changed the way that I assess simply because we are still using the Navigator so we are constantly doing that assessment piece in every class. We are assessing them on almost every problem, every question that we ask them in class they are answering through their 84 calculator. I don't think that I have changed my view on assessment by using the Nspire in the classroom at all. (closing interview)

He is not yet thinking about how the summative assessments of the chapter might be influenced by his use of these types of discovery, technology-based activities. Moreover, to formatively assess, he is comfortable using the TI-Navigator system, and when I try to push him to be more forward-looking while hoping to learn more about what he knows and thinks about assessment, the following conversation occurs:

> I: If you start using the TI-nspire™ more often throughout and distribute more activities, how do you or do you see assessment changing because if it doesn't have the Navigator system and you spend more time doing the activities how does that change assessment in your mind?
>
> Joe: I mean it's just a thing where we are using the TI-nspire™. We are maybe not using the Navigator because we can't be asking questions throughout the whole activity of the TI-nspire™ what I would probably do is make sure it had a worksheet to go along with the activity to kind of make sure that they are staying on track with what they are supposed to be doing. Make sure that they are being held accountable for the questions that are being asked, not necessarily being able to ask them to think about. Asking them questions that they need to write the answers down to and being accountable for them in class. (closing interview)

All Joe can think about is how he will use worksheets to hold students accountable during class with the TI-nspire™ calculators. Again, his knowledge of assessment with the TI-nspire™ is limited at this point.

Joe concludes,

> The expression activity, I really liked that activity because it helped them see that an expression isn't just, it's just a number, it just represents a number, and then when you have an equal sign, you are talking about what does the expression equal and finding a true value for *x*. Then, with the slider on the number line, [the students could see] the value of *x* which would change the expression and make the equation true. I thought that was really good thing… I have been thinking more about the visual student and how the Nspire helps them understand the concepts because the Nspire is hands-on and visual. I have been thinking about that, especially after seeing the activity with the sliders of the expressions and the equations. (closing interview)

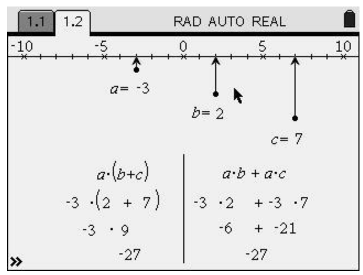
By doing the activity with the students, Joe is building PCK with the TI-nspire™ technology. He realizes that the number line representation in combination with the algebraic and numeric representations dynamically linked on one document screen helps students make sense of the mathematics and mathematical language (expressions and equations). He finds powerful and engaging activities by taking a risk and using new curriculum materials with his students. Although he may not exactly know the best time to insert these new and extra activities into the existing curriculum, he is learning and building knowledge of student understanding, instructional strategies, and curriculum with technology as he uses these new activities. As Joe begins to use these activities in coordination with his textbook, he realizes that he does not always introduce these activities at the appropriate time, which I describe in the next and final episode.

Distributive Property

The task. Joe uses the Distributive Property Student Activity from the *Algebra Nspired* supplementary materials to help the students make more sense of the distributive property. He has previously introduced the distributive property to these

Algebra 1B students, but they are struggling to apply the distributive property correctly and to know when to use it. On the TI-nspire's™ document page, a number line with two expressions below separated by a vertical line appeared on students' handheld screens (see Figure 15).

Figure 15. TI-nspire™ screen shot of the distributive property activity.



Joe also gives the students a worksheet with guiding exploratory questions (see Appendix G).

Enacting the task. Students return from lunch (they leave in the middle of math class for lunch and then return for 30 more minutes of math each day) to find Joe with a TI-nspire™ calculator page open on the SMART board. Joe uses 30 minutes of a 90-minute class period to engage students in this task. He begins by telling them to go to My Documents (the 7th option on the home screen) and then find the chapter 3 folder. The students open up the Distributive Property file that has been preloaded to their TI-nspire™ calculators. Students take six minutes to get in to class, settled, and at their desks with the activity open on their TI-nspires™.

Joe tells and shows them that they are going to move the points *a*, *b*, and *c* along the number line in a similar way that they moved the point on the number line for the *From Expressions to Equations* activity. Then, he directs the students to look at the worksheet, read it, and answer the questions and/or follow the directions as they follow along on the worksheet. He reminds students how to grab and drag points on the TI-nspire™ screen.

As they engage in the task, Joe asks, "As you are moving those points, what do you observe? As you move *b*, what do you observe? Write it down." He then invites some of the students to share what they wrote down. One student states, "The numbers get bigger." Joe clarifies, "If you move a point in one direction, the numbers increase, but if you move the point in the opposite direction, the numbers decrease." He suggests that a good thing for students to write down is that the numbers for *a*, *b*, and *c* are changing as you move the points on the number line.

Joe pauses to hear another student's answer, and a student then suggests, "The numbers at the bottom are changing." Joe agrees with the student and points out how each letter represents a number. He highlights how the numbers on the right are multiplied first and then added but the numbers on the left are added and then multiplied. Next, he moves to question two on the worksheet and instructs student to move the points so that $b+c$ is positive. To keep moving through the activity, he states, "To ensure we get a positive number make both *b* and *c* positive."

Next, they explore the value of *a* that makes both expressions zero. Joe moves the point *a* back-and-forth along the number line while giving the students time to answer on their own. Then, he asks one student for his answer and the student replies, "Zero." Joe responds, "Why?" They agree zero is the solution because *a* is multiplied by everything. Thus, they conclude when they make $b + c$ negative by making both b and c negative, the expressions will stay zero when a is zero. It does not matter whether $b+c$ is positive or negative when $a = 0$ to make the expressions equal to zero.

They turn the worksheet over and describe the process used to evaluate the expressions. Joe moves *a* off of zero and then talks through the steps involved in evaluating the expressions on each side of vertical line while noting how the order of operations differ yet produce the same result. He asks how the two expressions are the same or different, and ends by telling the students this is the distributive property. He states, “Anything that is on the left gets multiplied by what on the right.”

Joe intentionally sets $a = -7$ because a student struggled with problems when *a* is negative and notes how two negatives make a positive. The students struggle to use the property to write an equivalent expression for given expressions. Joe talks through all of the problems with the students.

Knowledge enacted and displayed. The *Distributive Property* activity seems to flop, and in an effort to make sense of why a “good” activity was ineffective with his students, Joe reflects upon his knowledge of students’ understanding with technology. He comments, “I think there was some level of understanding of the distributive property before we did it, which could have hindered the activity as well. The kids weren’t as into it. They were getting distracted” (closing interview).

He does not draw upon his knowledge of mathematics curriculum with technology because he is not critical of the task itself nor does he suggest that there might be a better activity or modifications he could make to improve this task. Joe shares,

> I think with the distributive property, I think they were having trouble making the connection between the two sides because one side was adding the two numbers then multiplying and the right side was multiplying and distributing the two numbers, then adding them together. I don’t know but I think they were looking at it as two separate things when I was trying to get them to look at it as one whole. The two sides were the same. (closing interview)

An equal sign did not appear across the vertical line or anywhere on the screen. The guiding questions on the worksheet associated with the activity discussed the algebraic representations as expressions and did not mention to the students that the expressions were equivalent until question 7, which asked students to use the property to write an equivalent expression for four given expressions. The students did not notice that $a(b+c)=ab+bc$ because for all input values a, b, and c the value of the two expressions are equal.

Additionally, Joe does not explicitly note with students that the values of the expressions were the same when comparing how the two expressions are similar and different (problem #5) during the teacher-led discussion of the questions. He does not realize students' understanding with technology for this topic or with this task until he implements it in the classroom. As a result, he lacks a flexible and effective repertoire of instructional strategies to teach the distributive property with the TI-nspire™ calculator. After the lesson, Joe realizes the activity should have been used earlier when the distributive property was first introduced to the students. He shares,

> I would probably just do the activity earlier as opposed to later to review the distributive property. We use this because they were having such a difficult time with the distributive property. I thought, "Oh let's try this activity even though we have talked about the distributive property. Let's try this activity." But I would like to do this earlier rather than later. (closing interview)

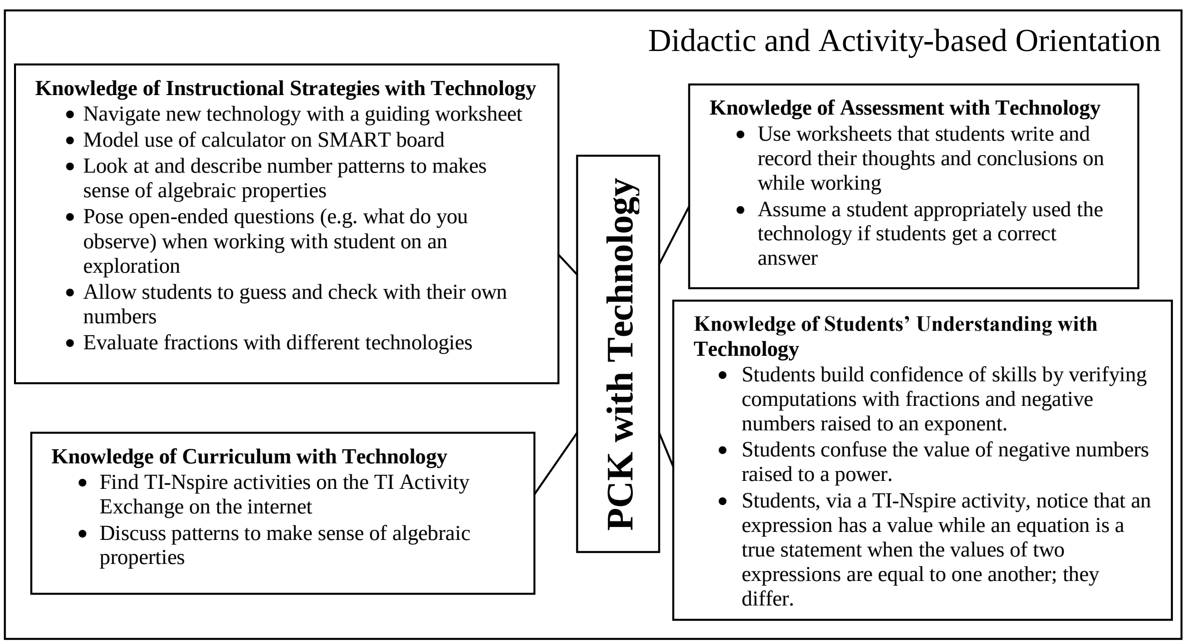
Joe's PCK with Technology

While Joe holds limited pedagogical content knowledge with technology to teach properties of algebra as well as concepts and skills related to expressions and equations, he begins to build his PCK with technology as he incorporates TI student activities into his teaching practices. Joe appears to build some knowledge of students' understanding with technology, knowledge of instructional strategies with technology, and knowledge of curriculum materials with technology. However, he does not seem to be actively building knowledge of assessment with the new technology. He shares,

> I have not really changed the way that I assess simply because we are still using the Navigator so we are constantly doing that assessment piece in every class. We are assessing them on almost every problem, every question that we ask them in class they are answering through their 84 calculator. I don't think that I have changed my view on assessment by using the TI-nspire™ in the classroom at all. (closing interview)

As a result, the knowledge of assessment dimension of Joe's PCK with technology does not advance through his use of the TI-nspire™ during this first semester. Figure 16 represents this summary of my findings.

Figure 16. Joe's pedagogical content knowledge with TI-nspire™ technology.



Mary

> *"The kids really do say it [the TI-nspire™] does make them [help them learn math], I watch them discover all the proofs first semester using the system and they were, they did amazing. They picked up on the theorems and postulates so*

quickly. Then, when I have talked to them about it as well when we've polled them, I did a survey with them to see what they thought, they all said, as well, that they thought it was, that it's a really cool way to learn math. It's more engaging."

Mary, like Joe, volunteered to work with me when I mentioned that I was interested in learning about the knowledge teachers draw upon while using a new technology to teach high school mathematics. I observed and interviewed Mary in fall 2009. I watched her teach honors geometry topics at the beginning of the year for the first time with the TI-nspire™ calculator.

Mary described the different calculator models she had used during her teaching career, "I have been using the TI-83 Plus and worked through the TI-84, the TI-84-plus, the silver, now onto the TI-nspire™" (initial interview). Wanting to communicate and connect with more of her students in the classroom, she was motivated to move to the TI-nspire™ by the positive things a co-worker mentioned about using the TI-nspire™ with students. She explained,

> One of my coworkers is a TI fast-track rep and she was telling me how neat it was in the classroom and how the kids are really connecting and thinking a lot more. I thought well I could try this being that I have freshmen honors. They are already engaged because they are honors students, but I wanted something so I could hear more from everybody. (initial interview)

Beginning in the summer of 2007, Mary had the opportunity to engage in professional development related to teaching with the TI-nspire™ calculator at international T^3 conferences. She shared, "I've been to two international T^3 conventions. Last year I went to Dallas and this year I went to Seattle" (initial interview). During these experiences, she learned about how the technology can be used to present activities to students in order for the students to be able to discover, learn and *do* more of the mathematics during a lesson.

However, Mary admitted that using technology to teach mathematics was not exactly easy or natural for her. When Mary used TI produced TI-nspire™ activities or asked students to use the TI-nspire™ to explore or make sense of course content (e.g., a geometric theorem), she had to give students some control of the class time and their own learning. Mary admitted that she was not quite comfortable with that yet.

Mary does not use a computer algebra system (CAS) on her TI-nspire™ handheld, and she continues to teach with the TI-84 handheld as well as TI-nspire™. She feels pressure to teach the basics of the TI-84 handheld because other teachers in her building use it and do not use the TI-nspire™. Nevertheless, she believes the use of the TI-nspire™ calculator enhances students' geometry classroom learning experiences and she enjoys using the new technology because as she describes,

> When I use the TI-nspire™ face plate, I do actually do more discovery with that because I think it just lends itself more for them to draw the pictures to see how it interacts. [We use the TI-nspire™] to measure things and stuff, which you can just do at your desk with paper and pencil without other instruments and [I don't have to mess with other] stuff or taking them to the computer lab which is a hassle in and of itself. I like to use the TI-nspire™ for that [discovery-based learning and geometric drawings]. (9/2 interview)

I attended Mary's honors geometry class for 8 weeks and observed 17 lessons during fall 2009. During this time, Mary used the TI-nspire™ calculators (without CAS) with her class two times. First, she loaded the classroom set of handhelds with an activity on conditional statements that students independently worked through while learning to use the new technology. Then, she used a TI-nspire™ Angle Relationship activity to discuss vertical angles, linear pairs, transversal, and parallel lines with her geometry students. I will use these two episodes to explain the pedagogical content knowledge (PCK) Mary draws upon while teaching mathematics with a new technology as well as the knowledge she demonstrates during the stimulated recall interviews after describing her orientation to teaching mathematics with technology.

Orientation to Teaching Mathematics with Technology

Mary holds a didactic (Magnusson, Krajcik, & Borko, 1999) and activity-driven (Anderson & Smith, 1987) orientation to teaching mathematics with technology. Namely, her goal of teaching mathematics is to transmit the facts of mathematics. She shares, “I like to stand up there and teach…I am one of those teachers that doesn’t like to give up power. I like to make sure everybody is on task and getting what I want them to accomplish out of the activity” (closing interview). Furthermore, Mary asserts, “It’s important that they have those basic skills before they can even add on the new dimension of discovering” (initial interview). However, because she thinks of the TI-nspire™ as a learning tool with discovery activities and she wants to incorporate the new technology into her teaching practices, she also aims to engage students in technology “hands-on” experiences via TI-nspire™ activities. She reflects, “I am sort of more of an old-fashioned teacher where I like to stand up there and teach. I am trying to test my limits on writing the lesson on the TI-nspire™ and then putting them on all of the calculators so the kids can go through and learn as they are going through each page” (closing interview).

Mary views the teacher as the content and technology expert in the room who directs and manages time as well as the learning process. She typically begins class with an introduction and overview of the lesson, followed by a presentation of vocabulary words, occasionally uses a TI-nspire™ activity with students, and then lectures before letting students practice problems and start their homework. She states that she, as the teacher, is in the classroom to do the following:

> To help them [students]; to help facilitate, you know, make sure that they are moving along; to help them to discover and get a little more confident that they can actually do math. And then also to just do the normal stuff where you are policing to make sure everyone is on task and doing what they need to do. (initial interview)

She must decide when to take and release control of the conversation and learning with the technology. Additionally, she, as the teacher, believes, "You have to know where you are headed and what the purpose of the lesson is" (closing interview), especially when using the TI-nspire™ calculator activities with students.

Mary describes the role of the students in the following ways:

> They need to be engaged. They need to be paying attention with how to manipulate the TI-nspire™ and then they need to be open-minded and just go with the flow because there is always quarks that happen during a lesson when you are using technology something can always go wrong or someone has an issue and you just have to roll with the punches…when math is being taught with the new technology… to be patient…think a little bit more. (initial interview)

According to Mary, students should: (a) participate, (b) follow the teacher's lead, be patient, and (c) think while doing mathematics and using technology to explore, verify, and discover mathematical relationships with tools such as the TI-nspire™ calculator. In the next sections, I describe how Mary uses two TI-nspire™ activities while instructing honors geometry and the teacher knowledge she draws upon during these two episodes.

Conditional Statements

The task. Honors geometry students in Mary's class first use the TI-nspire™ during the third week of school. Mary introduces the students to the new technology with a TI-produced TI-nspire™ activity titled *Conditional Statements.* The drawing exercises embedded in this activity help the students learn to use the TI-nspire™ calculator to construct and explore geometric objects and relationships.

With the TI-nspire™ calculators, the students complete a construction and then determine what is true about their construction. For example, students construct a line and a point not on the line. Then, they make a second line through the point that is parallel to the first line. Then, the students find the slope of both lines. After using the calculator to construct and compute the slopes of parallel lines, the students complete an if-then statement. For this problem, the conditional statement reads, "If two lines are parallel, then the slopes of the lines are _______" (worksheet). Their construction should verify and/or tell them that the slopes of two parallel lines are the same or equal. In other words, the students construct and observe parallel lines in order to make a true conditional statement and then to decide upon the truth of the converse, inverse, and contrapositive statements.

Enacting the task. At this point in the year, the geometry students are beginning to think about "truth" and proof within the geometry course. The students have been introduced to points, lines, rays, planes, angles, segments, angle bisectors, linear pairs, parallel and perpendicular lines, and associated theorems (e.g., Angle Addition Postulate). They have solved problems in which they applied geometric rules in order to set-up an algebraic equation and solve for an unknown. To begin Chapter 2, Mary "preteaches" for approximately 40 minutes and then engages students in using the new technology for the remaining 45-50 minutes of class. Mary describes,

> What I did was I pretaught all the definitions for conditionals – inverse, converse, contrapositive, biconditional…They came in and I had my

> SMART board up with slides like I was teaching a lesson. We went through about ten slides learning vocab words and practicing application. Then, we got the calculators out and they opened the program. (9/11 interview)

Mary tells students how to turn on the TI-nspire™ calculator and navigate to the activity she has preloaded onto their handhelds. She explains,

> We got out the TI-nspires™ and it was their first time on it so I was trying to give them a little bit of demonstration like this is the home key and this is how we get to the menu screen. (9/11 interview)

Then, she expects students to figure out how to construct points and lines on the pages of the calculator screen.

Mary gives students time on their own or with their peers to complete the activity at their desks. As they work through the activity, they fill-in-the-blank for the three conditional statements and write in the converse, inverse, and contrapositive of each statement. Then, they determine whether all of the statements are true or false. If the statement is false, they must produce a counterexample. On the last two problems, the students use a given diagram on their calculator screens to answer questions such as, "What is the relationship between AB + BC and AC?" and write a conditional statement to express their conclusions. Consequently, they write a version of the Segment Addition Postulate on their own.

After approximately 40 minutes of work time, Mary stops the students. They talk through the correct answers to the TI-nspire™ activity, and then summarize their work on the TI-nspire™. Next, Mary presents a few notes and assigns homework problems from the textbook. She assigns homework problems in which she expects the students to practice writing the converse, inverse, and contrapositive of conditional statements.

Knowledge Enacted and Displayed. Mary draws upon her knowledge of student understanding with technology, knowledge of curriculum, and knowledge of instructional strategies with technology when selecting and choosing to implement this activity. Mary knows that the students already have ideas about parallel and perpendicular lines as well as ideas about lines with the same y-intercept and collinear and non-collinear segments. Mary describes,

> I think it made it a lot more engaging because we are not just doing here's a statement, write these; they are actually making things and drawing things and then writing. Yeah, the drawing stuff wasn't exactly necessary overall but it did make some of the concepts that we have already talked about reinforced again in the classroom. (9/11 interview)

Therefore, she intentionally uses this activity to help students make sense of conditional statements while practicing and learning to write different versions of conditional statements. She also realizes this work will be helpful for formal proofs later in the course because this work uses and reinforces geometric relationships. Furthermore, the drawing exercises embedded in this activity help the students learn to use the new technology for geometry constructions and exploration.

After the lesson, Mary recognizes the problem with using the TI-nspire™ with content that honors students already know (e.g., parallel lines have equal slopes). Mary explains,

> They didn't have to explore to find out what the answer was…Once they figured out they were just filling in the statement with the drawing they were just like let's just fill in the statements, write in what we need to, then play on the calculator. (9/11 interview)

As a result, while some students used the technology in powerful ways to explore and make sense of the mathematics, most of Mary's honors students focused on exploring the tool itself. Nevertheless, Mary draws upon her knowledge of instructional strategies with the technology and builds new knowledge that will inform future practices.

During this episode, Mary also builds knowledge of student understanding with the technology and knowledge of instructional strategies with the technology while reflecting on instruction with the activity. After the lesson, she decides to give the students more time with the new technology when bringing it out for the first time. She says, “I think what I will do next year is possibly add some more pages to the document with the definitions of all of the words” (9/11 interview). She may not need to talk students through how converse, inverse, and contrapositive are defined. Instead, students can read these definitions for themselves on a document page of the TI-nspire™ calculator and then make sense of and apply the definitions. They can practice writing appropriate versions of if-then statements by working through the problems of the activity and communicating with others (i.e., peers and the teacher) at their own pace on the TI-nspire™. Upon reflection, she decides she should give them more time to control their own learning, especially for their first experience with the new technology.

To assess with the new technology, Mary walks around and reads what the students write on their worksheets. By doing so, she observes and learns that students typically mix up the converse and inverse. Mary knows that she could also assess with the TI-nspire™ by collecting students’ documents on the TI-nspire™. While students can save their work on the handhelds and then transfer their documents back to her computer after class, she did not do this for a variety of reasons. She states,

> I didn’t collect them all and grade them all. I didn’t do any of that today because it was the first time with the TI-nspire™. I wanted them to get a

> comfortable feel for them. Towards the end or when they get more comfortable I will start collecting it from them and getting all of the data and collect it. It would be a hundred times easier if I had the new Navigator. I don't do a lot of collection from TI-nspires™. I do a lot of collection from the regular 84 face plate with Navigator because it's just so much easier. (9/11 interview)

It is easier for Mary to think about assessing with the older technology. However, I imagine Mary will strengthen this knowledge as she further considers assessment with the TI-nspire™.

Angle Relationships

The task. Mary uses a TI-produced student activity, *Angle Relationships* (see Appendix H), to help students explore and learn about special angle relationships along a transversal when two lines along the transversal are parallel. The worksheet consists of three problems. In the first problem, students explore what is always true about vertical angles and linear pairs when two lines intersect. In the second problem, students examine what happens when a line intersects two other lines and they force a pair of angles to be congruent. Then, students generalize these relationships. Finally, in problem three, students apply their generalizations and problem solve.

Enacting the task. As usual, Mary starts class by allowing students time to check their homework with answers that she posts on the SMART board. Students can ask any questions they have while they are self-assessing at the start of the period. Mary answers as many questions as the students have and then moves into a lecture-style presentation of notes and vocabulary words for the day. She defines exterior angles, interior angles, corresponding angles, and consecutive interior angles at the SMART board with pictures and different colored markings. After providing definitions for the students (32 minutes after the start of class), Mary instructs the students to turn on their

TI-nspire™ calculators. She hands each student a worksheet to guide his/her thinking. She expects the students to record their observations and conclusions while working through an TI-nspire™ activity on angle relationships.

Mary verbally gives instructions for turning on the calculator and navigating to the activity while passing out the worksheet (see Appendix H). Then, she opens the TI-nspire™ SmartView on the SMART board and models how to measure angles and drag lines to see how angle measures change. For example, on page 2 of the activity, two intersecting lines are already constructed on the screen. Mary asks students to describe what is *always true* about vertical angles. They should already know this fact because of how they defined vertical angles but this gives them a chance to begin exploring with the new technology using existing mathematical knowledge. Mary shows them how to go to the measure tool and click the three points that make up the angle in order to see the angle measure displayed on the screen. Once both angles have been measured, she drags and moves a line to see how the measures change across multiple versions of intersecting lines. The students notice the measure of the vertical angles stay the same despite the angle measurement.

After approximately five minutes of demonstration at the SMART board, the students work individually at their desks. Mary circulates the room and answers students' questions. The students struggle through the activity displaying a lack of confidence. They ask Mary what they should write on their worksheets and how they should use the technology.

After 25 minutes, Mary brings the class back together to summarize the mathematics. She starts by saying, "You should have found the lines are parallel when you make the angles congruent" (video 10/8). She proceeds to use the document camera (another technology in the classroom, which projects images onto the SMART board like an overhead projector) and displays a copy of the worksheet with her answers listed. She encourages student involvement during this summary as she gets to problem three which requires students to find missing angle measures on a given diagram using the information from the previous problem. However, she does not involve the students in summarizing the mathematical generalizations with the new technology.

Knowledge enacted and displayed. With this activity, Mary hopes her students will build understanding and see how special angle relationships exist (e.g., corresponding angles are congruent) when two lines cut by a transversal are parallel. She explains,

> I want them to understand and that was the point of the TI-nspire™ that the lines must be parallel for the rules to hold true about the angles. We can identify an angle based on where they are located, but we can't prove anything congruent or supplementary and find angle measurements unless we know the lines are parallel.(10/8 interview)

She usually tells students the theorems and shows them how to apply them for mathematical problem-solving purposes. However, now that she knows about curriculum materials and accessible tools with which her students can visualize geometric relationships that are formalized as theorems, she approaches teaching differently by using more activities. However, Mary does not take the time to explain her expectations about using the TI-nspire™ with this activity. She tells the students that they have 20 minutes to get through the activity on their own, but she does not explicitly explain or show what it means to explore and conjecture with the TI-nspire™. This demonstrates that she lacks knowledge of instructional strategies with the TI-nspire™. For example, the students read, "Grab and drag line AE to see what remains true about the angle measures" and are asked to reflect upon, "What is always true about vertical angles?" The students should carefully and critically observe as they grab and drag in order to recognize generalizations and then communicate these generalizations in writing on their worksheet. However, Mary focuses on reminding them how to work the calculator (e.g. move to the next page by hitting control and then right on the nav pad and measure, grab, and drag the angles). As a result, she reflects,

> They [the students] were having problems moving the lines in order to make angles congruent, making the lines actually parallel. A couple of them did not understand which way to move the line so I was like, let's test our theory. Let's move it one direction and see if it gets us closer or farther away. We moved it back the other direction and you know and they were able to determine which way to go. (10/8 interview)

While walking around, Mary finds herself having to teach the students how to explore a problem using the new technology. Not all the students realize that when the problem asks, "What appears to be true about lines AF and BG when the measure of angle CDF equals the measure of angle DEG?" they should grab and drag line AF or line BG until the given angle measures appear equal. Then, they will be able to look at the picture and state what appears to be true about lines AF and BG.

She struggles to build knowledge of effective instructional strategies with the new technology. The students do not communicate with peers that are sitting next to them, but instead they follow Mary's instructions to work individually and fill out the worksheet. Mary's patience with the students decreases as they ask her task-related questions and struggle to use the TI-nspire™. She shares,

> They didn't know which page to be on at the problem. Then what else was there? What they were need to do on the worksheet like what do you expect me to write here. They always want to, I guess, it's just a middle school mentality or an honors mentality that they always want to

> have it perfect rather than trying it first and seeing how it goes, seeing what the issue is. (10/8 interview)

As she loses patience, she starts telling the students what to do rather than encouraging students to think on their own or with each other. For instance, seven minutes after she releases the students to individually work, Mary tells the students that there should be eight sets of angles that are congruent, but not vertical (under the second bullet of problem 2) and two sets of angles listed that are supplementary, but do not form a linear pair. Unfortunately, this tension between her giving the students control of the time to explore and make sense of the mathematics versus her telling them what they should do or see with the technology exists the entire 35 minutes they spend on this activity.

Mary does not realize that students need knowledge of generalizing with the technology in order to be able to move from the third problem on page one to the second half of the questions on page two. Mary lacks knowledge of student understanding with the new technology. For example, at approximately 50 minutes into the lesson, a student asks what to write in the blank for "the measures of the corresponding angles are ________." The student wants to write an angle measure to fill in the blank. Mary wants her to write "equal," that is a generalization about the measure of all pairs of corresponding angles based upon her observations and responses to the beginning of problem two in this activity. The student does not quite understand and Mary reflects,

> These guys were like, 'What?', kind of wanting an explanation. We didn't really even get too much of an explanation because we had to get to the end. Hopefully they go home tonight and they work on their homework and they make sense on why we did it all. We'll see. (10/8 interview)

At this point, Mary does not realize that the students do not know how to and need to know how to construct, manipulate, critically observe, and then communicate generalizations about the geometric relationships with the technology. If she had anticipated that students would struggle to explore the angle relationships and then generate appropriate generalizations for this activity, then she may have used different instructional strategies.

Mary needs to strengthen her knowledge of instructional strategies to reach her goal in teaching via this activity. She tries to figure out how to modify her instructional practices after the lesson:

> It might have been too long for them. I think I should have cancelled out problem one because vertical angles, linear pairs they don't need to worry about. They should know them already. I don't think they need to get reinforced again, but yet they still have to recognize it. And I did like the second problem where they had to move the line in order to make it parallel and then come up with a list of angles. I think the list was a little bit lengthy, having to list them all. They get tripped up on did I name that pair yet. They need to just list a few and move on. Then, I think the other concepts were good. (10/8 interview)

However, she becomes flustered when the students are not successful with the activity. With limited knowledge of instructional strategies, she resorts to telling the students rather than letting them engage with the activity on their own. Mary tries for a second time when she says,

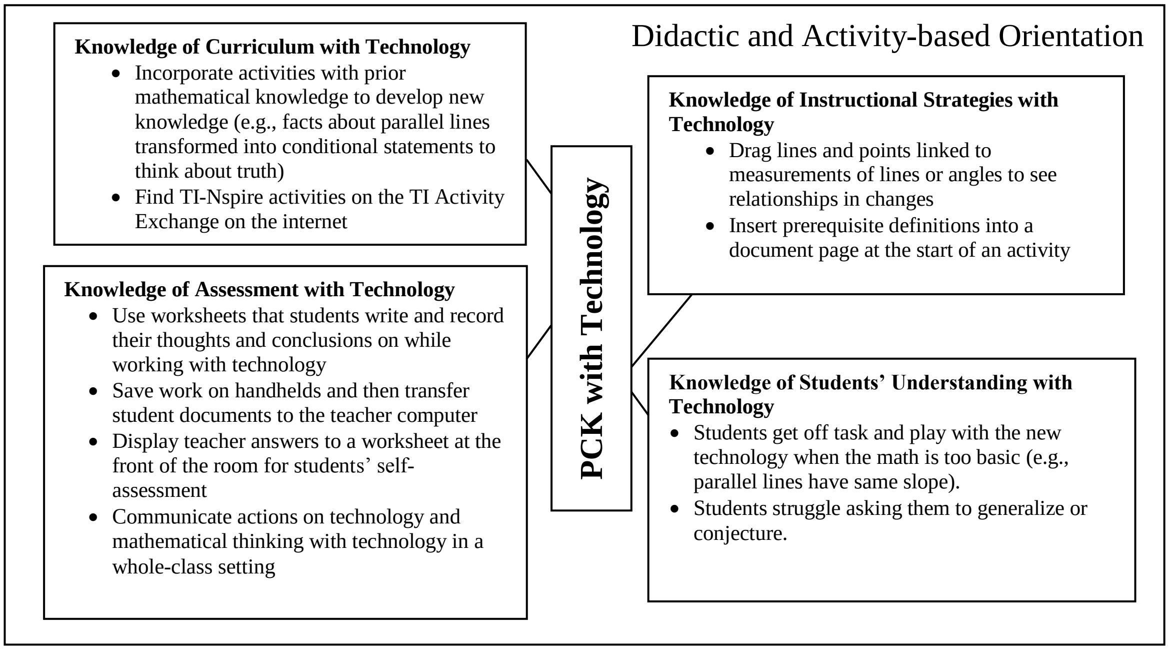
> Part of me wonders if it would be better to go through the activity. Like do definitions of what the actual corresponding angles are, what I did in the beginning. Give them the activity and go through it and then do it more teacher-led and then actually going and doing the hiding and showing. The problem is it just takes so much time to do all of that and would we be able to get to the proof, the theorem part, writing down what did we discover from this that two lines have to be parallel. We'll that's the theorem. (10/8 interview)

She does not know teaching strategies that will help the students make mathematical conjectures on their own or support them in pursuing their own thoughts.

Mary's PCK with technology

Mary, an experienced teacher who has been teaching the same course for six consecutive years, adjusts her thinking and begins to construct pedagogical content knowledge with technology in terms of her knowledge of instructional strategies with technology and knowledge of assessment with technology when she begins to use the TI-nspire™ calculator to teach mathematics. While using the new technology in her classroom, she begins to learn what students do and do not understand in terms of the mathematics with the new technology, but she develops this knowledge slowly. Likewise, Mary enacts and displays a limited knowledge of curriculum with technology, which builds as she makes time to consider inserting technology-active tasks into the curriculum. Although Mary had used calculators for a number of years and attended professional conferences related to using the TI-nspire™, she did not use it extensively after she made the decision to transition to using the TI-nspire™, Out of the 17 lessons I observed her teach, she only used the TI-nspire™ during two lessons illustrating the challenge teachers face as they transition to using a new technology. Figure 17 represents this summary of the findings directly related to Mary.

Figure 17. Mary's pedagogical content knowledge with TI-nspire™ technology.



Kate

> *"The teacher that taught in my room before me had attended this training and got a whole classroom set and she was a teacher of Integrated 4 and so she had got it basically for the Integrated 4 class. I thought this was really cool, and then I heard since my kids are Integrated 4 they will be taking calculus next year and they will be able to use them on AP calculus test…I don't really know why I chose to use it. I guess you know part of it is just those kids a lot them will go on to do things where they will be doing a lot of math. I feel like it's kind of my responsibility to just show them all of the different things that are available and out there and so just as I am working with the curriculum if there is something that I can think of that hey there is another way they can do this then I like to show them that. But I think that it's just like I wanted them to have an introduction to it" (initial interview, 5/8/09).*

Kate teaches mathematics in a mid-sized, small-city high school in the Midwest. There are 17 mathematics teachers at Kate's school, but Kate is the only teacher who teaches Honor Integrated 4 using the Core-Plus v2 curriculum materials and one of three teachers who uses the TI-nspire™.

I observed Kate during her third year of teaching, but her first year of using the TI-nspire™. When I interviewed Kate the first time, she said, “We use three main technologies everyday: the SMART board and the graphing calculator [TI-84+] and the TI-nspire™” (5/8 interview). Two of these technologies were new to her, the SMART board and the TI-nspire™ calculator. She attended a professional development session on the use of the SMART board with her school and district colleagues. However, she had not attended professional development related to the TI-nspire™ calculators prior to the study. She recalled,

> When we were at a district-wide leadership training with all of the teachers, they [PD leaders and local teachers] talked about it [the TI-nspire™ calculator] for a little bit. The two main local high schools had at least one classroom set. TI had given us like a few CDs about activities and stuff to use with it so I was like oh I guess I need to start looking at this and they brought the TI-nspire™ there so that everyone could have an opportunity to see it. Then I came to school and actually checked one out to my name because all of our things are through the media center. My classroom sets have a barcode through the media center so I had to go and check it out. All of mine are checked out to me. So, I went and checked one out and had it at home over the summer. I tried to turn it on a few times and do something and I was like I have no idea how to do this. (initial interview)

Nevertheless, Kate tried to read the calculator’s instruction manual to learn how to use it for specific mathematics lessons so that she could use the new tool to enhance her instruction. She explained,

> I haven’t read it [the TI-nspire™ calculator instruction manual] front to back but I just look at the book when I am trying to do something. Like I was trying to teach them how to divide polynomials or something and I was like you can multiply the polynomials in the Nspire to check your answer. Do these things multiply back to be the original, the numerator? So, I would just have to look that up every time. (initial interview)

Kate establishes her classroom environment differently than the two other participants in the study. When asking Kate to describe how she and her students use technology during math class, she emphasizes her use of group work and the different paces that resulted. She expressed,

> My students are working with their groups and there is not a lot of whole class there is not even a whole lot of whole class discussion and there is virtually no whole class instruction. So, when you come in it's going to be group by group what is going on not necessarily the whole class working on the same thing. That is probably different than most math classes and so there is a lot of times that I am showing each group how to do something at different times. They will be using both calculators like I said they usually have both out if I am saying some days I specifically say everybody pick up a TI-nspire™ today. On some days some kids try to use it and on some days they don't. It just depends on if it's a day when we're like I've taught them to use a graph. If they are graphing, I think they'll pick up a graph, they'll pick up an TI-nspire™ I mean because they are going to be graphing and they know how to do it, but then if it's something that I haven't really taught them then they probably won't try it in the TI-nspire™ because they get really frustrated…I designed this class to be more the students were deciding their own pace they are working through the material and everything. (5/8 interview)

Next, I asked her how her textbook helps or hinders the use of the TI-nspire™ calculator in her classroom. She stated,

> I think it helps a lot. The questions are already set-up asking them to explore and compare different graphs and what is happening with the table or how did you have to change the equation. It is already asking those questions and I don't have to create that so then they are able to use the TI-nspire™ to compare those things or when they are asked to factor or expand it will say those things. It was just something the book was asking them to do. Now, if you multiply this back out, will it take you back to the function where you started? The book set that up for me and I just showed them how the calculator can also do that. I didn't create anything new. (5/8 interview)

Kate used curriculum materials that included specific tasks involving the TI-nspire™. As a result of this supportive resource and Kate's description of her classroom environment, I expected to observe her students use the TI-nspires™ because their textbooks prompted them to verify or explore problems with different features of the

TI-nspire™ calculators. I also expected to observe Kate use the TI-nspire™ to solve problems or demonstrate strategies during her mathematics lessons.

Although I observed Kate teach eight lessons to her Honors Integrated 4 class for four weeks at the end of the 2008-2009 school year, Kate did not use the TI-nspire™ calculators (with CAS) even once with her class. However, she regularly used TI-84 graphing calculators during instruction and reflected upon how she could have used the TI-nspire™ calculators during stimulated recall interviews. Therefore, I choose to include the data collected from the interviews with and videotapes of Kate's teaching. I highlight five episodes in which she used the TI-84 graphing calculators and then discussed how she could use the TI-nspire™ for the following year:

1. Students used graphing calculators to decide whether given trigonometric identities were true or false.
2. Students investigated what the number *e* is.
3. Students worked to explain the connection between $\left(1+\frac{1}{n}\right)^n$ and *e* to frequent compounding of interest.
4. Students discussed exact versus approximate answers when solving exponential problems on their calculators.
5. Students explored how negative and fraction exponents influenced the value of an expression.

I will use these five episodes to explain the pedagogical content knowledge (PCK) with the TI-84 Kate draws upon while teaching mathematics as well as the teacher knowledge she demonstrates about the TI-nspire™ (the new technology) during the stimulated recall interviews. However, before describing Kate's PCK with technology via the five episodes, I characterize her orientation to teaching mathematics with technology through which she filters her knowledge.

Orientation to Teaching Mathematics with Technology

Kate holds a guided inquiry (Magnusson & Palisnear, 1995) and process (Magnusson, Krajcik & Borko, 1999) orientation to teaching mathematics with technology. She asserts, “They [students] are learning math in my room” (initial interview); however, she, as the teacher, is not the main voice of authority in her classroom. Instead, the students are responsible for making sense of the mathematics and thinking in the classroom. Furthermore, she comments, “I’m fine with them solving problems in whatever way makes the most sense to them” (5/18 interview). Kate typically introduces an investigation and then presents a timeline for students to complete certain problems within the textbook materials. Then, she gives the students space and time to work and circulates the room answering their questions, listening to student thinking, and sharing her expertise and guidance when needed. She shares, “I think they make a whole lot of sense out of it if I don’t say anything. I just let them work through the investigation” (5/18 interview). Consequently, she arranges for and allows students to work in groups of three or four as they engage in determining patterns, testing explanations, evaluating the utility and validity of their data, and evaluating the adequacy of their conclusions while developing thinking processes and integrated skills.

She sees herself as a resource for her students and an expert that structures student learning opportunities and introduces new technologies. She explains that she aims “to get them to use and understand the technology … be a resource for them if they are stuck with the technology or an expert to show them some things… be able to guide them if they need help with it" (initial interview). She introduces and poses questions to help students acquire new mathematical knowledge and learn how to problem solve with technology tools. She considers using the TI-nspire™ calculator most often when they look at multiple representations (5/15 interview).

Kate expects students to be problem-solvers, mathematical thinkers, and communicators with technology in her classroom. She explains,

> Students need to be people who are going to investigate and be problem solvers. They just have to think about that, think about what the technology can do for them when they're solving a problem… be really patient and not give up on it…help each other, they're going to have to work together using the new technology…be supportive of each other, and show each other and help each other. (closing interview)

She further shares, "I think their role when they are learning math is if they need or want the support from the technology they are going to have to try it and ask me how to do it if they want help" (closing interview). In short, the students should use the technology in ways the textbook suggests and consult peers and the teacher only after struggling to learn on their own. In the following sections, I describe Kate's use of technology and knowledge of teaching mathematics with the TI-84 and TI-nspire™ technologies as she teaches and reflects on the five episodes.

Trig Identities

The task. While working on Investigation 2, Kate asks students to, "Decide whether each of the following equations is or is not an identity. If an equation is an identity, use symbolic reasoning to prove it. If not, provide a counterexample" (p. 11). Then, the following equations are provided:

a. $sec\theta - tan\theta sin\theta = cos\theta$
b. $tan^2\theta - sec\theta sin\theta = tan\theta(tan\theta - 2)$
c. $cot\theta = cos\theta csc\theta$
d. $tan^2\theta = \frac{1-cos^2\theta}{cos^2\theta}$
e. $csc^2\theta - sec^2\theta = 1$
f. $sec^2\theta = \frac{sin^2\theta+cos^2\theta}{cos^2\theta}$

In this task, students learn how to write algebraic proofs with trigonometric functions and practice proving identities.

Enacting the task. Students work in groups of 3 or 4 as they work to prove trigonometric identities. Kate monitors students' progress and talks with groups of students as she circulates the room during a 90-minute class period. Students have textbooks, pencils, pieces of loose leaf paper, and graphing calculators on their desks. Some students work alone and then discuss problems with group members when they get stuck or want to verify their accuracy while other students talk aloud with one or two group members to make sense of every problem. One or two students within each group initiates using the TI-84 graphing calculator in some way to explore whether or not a trigonometric identity is true or false.

As Kate listens to students' conversations and questions, she notices, "A lot of students kept really being concerned with how do we know if this is even an identity to begin with because I don't want to spend a lot of time trying to prove it if it is not even provable" (5/15 interview). Students took time to discuss how they would decide whether an identity was true before they worked to symbolically reason and prove an identity's truth. At this point, students turned to their graphing calculators. Although each student had access to a TI-nspire™, he/she reached for a TI-84 handheld.

Kate asked one group of students, “What would you put into your calculator to see if *a* is an identity?” (field notes). Some students tried guessing and checking by computing with different values of theta to see if the statement held true amongst examples. Other students used their graphing calculators to graph the expressions on each side of the given equation in order to see if the graphs overlapped for all values of the domain. Only a few of the students entered the expressions as functions and then looked at a table representation of inputs and outputs to investigate common domains and ranges while looking for inconsistencies. Kate did not discuss using either calculator to decide whether a given statement is a true trig identity with the whole class at any point during the class period. Rather, she talked with most of the groups as they reached problem #5 of investigation two since all groups worked at different paces.

After students decided an identity was true and struggled to reason symbolically and to write an algebraic proof, Kate used the SMART board technology to discuss analogies for simplifying and rewriting fractions. For example, when students do not understand symbolic reasoning from $\frac{1}{sin\theta cos\theta} - \frac{cos\theta}{sin\theta} = tan^2\theta$ to $\frac{1}{sin\theta cos\theta} - \frac{cos^2\theta}{sin\theta cos\theta}$ on the left side of the equation, Kate asked students what they would do to find the least common denominator of $\frac{1}{2} - \frac{5}{6}$ while writing on the SMART board at the front of the room. The students recognized that six would be the common denominator of one-half and five-sixths. Thus, they would multiply one-half by 3 over 3 in order to express both terms with the least common denominator in the same way that the model student multiplied $\frac{cos\theta}{sin\theta} \cdot \frac{cos\theta}{cos\theta}$ to rewrite the term and find the least common denominator of the trig expression.

Knowledge enacted and displayed during reflection. Kate admits, "I hadn't really thought about them trying to use the technology that much for it [proving identities]. It's usually I try to think of it more when we are looking at multiple representations of something" (5/15 interview). Prior to the lesson, she does not consider how a table or graphical representation would be a way to explore the truth of a trig identity before reasoning symbolically. As a result, she does not use the TI-nspire™ when discussing the use of the calculator with students as they decide whether a trig identity was true or false although she talks with students about the math with the TI-84 as students use it at their desks.

Kate does not consider the potential role of the TI-nspire™ in teaching trig identities or proof. She explained, "Looking at all of the big ideas that I have seen the Calculus students working on, I thought, 'Okay we need to look at what the other three trig identities or trig functions are'" (5/15 interview). This quote conveys that although she has knowledge of the mathematics curriculum, she has not yet considered the curriculum with technology. As a result, she lacks knowledge of curriculum with technology.

However, Kate displays knowledge of instructional strategies with technology to teach reasoning about the truth of a trigonometric mathematical statement during the stimulated recall interview. I asked her how she explained or talked to students about using their calculators to decide on the equivalence of two expressions and how she could have explicitly instructed with the TI-nspire™. The conversation occurred as follows:

> Kate: I usually ask them first how they could check. Several of them said I could plug an angle in for theta and see if both sides are equal

> to each other. Then, I tried to get them to think what other ways they could see that on their calculator and how could they check for multiple points, not just the one angle. A lot of them then went to I can look at the graph. Very few of them, which was very shocking because they use the table a lot, said hey I could look at the table to see if the values are the same. They have used that a lot but none of them said that to me. I was kind of surprised but I tried to if they didn't realize it to try to point out that they needed to show and look at a lot of points, not just one.
>
> I: If you would have demonstrated for them or had the technology up on the board what would you have shown them or displayed at that point do you think?
>
> Kate: I think that if I would have demonstrated that these two sides are equal then I probably would have tried it for a few points and noticed that they are the same. So, can we look more efficiently at a lot of points and then probably both looked at the graph and the table together to notice that they were the same thing. (5/15 interview)

Kate realizes that she can engage students in using the TI-nspire™ at their natural entry points by looking at a few values of theta for which the expressions may or may not be true although she does not enact this knowledge. Then, she can push the students beyond a few examples in order to explore truth for a larger domain because when they reason symbolically, they will prove truth for an infinite domain. She also knows that looking at a table of input and output values for the expressions with the TI-84 calculator is the most efficient way to look at a reasonably large domain while looking for counterexamples before symbolically reasoning and she does push groups to consider this use of the TI-84 during this lesson.

Kate also enacts knowledge of instructional strategies with the TI-84 and knowledge of student understanding when teaching students to symbolically reason through proofs of trigonometric identities although this PCK with the TI-84 does not lead to the use of the TI-nspire™ calculator. Instead, she utilizes the SMART board and draws upon her PCK without technology in order to share fraction analogies and help the students rewrite and simplify trigonometric expressions. Nevertheless, when considering how the TI-nspire™ could be used in the situation, Kate explains and the conversation continues as follows:

> I: Is there a way that you could see yourself using the TI-nspire™ in this situation?
> Kate: For trig identities?
> I: I am just throwing this out there. Even for this representation, to get at this idea of finding the common denominator more specifically? There might not be. I am just curious.
> Kate: I don't know. I don't know how I would use it.
> I: I wonder what would happen, I am thinking out loud now, I am wondering what would happen if you typed that into the computer algebra system. What would the output be? I don't…just thinking out loud.
> Kate: If you added it would be eight sixths. I would think that the answer the calculator would give would be four-thirds. You know? So then that would be super confusing.
> I: It might add a whole new
> Kate: It might add a whole a different thing. You found this common denominator that is then not the denominator of your sum once you do that. I think that is probably what the CAS does because it usually, it will give you that fraction answer. I can't remember if it automatically reduces all of the time.

Therefore, she concludes that it was more appropriate to use the SMART board and mental math rather than the TI-nspire™ while considering simplifying and rewriting fractions. As a result, she gains knowledge of PCK with the TI-nspire™ as she enacts tasks and reflects upon students understanding with technology and instructional strategies with the TI-84 after the lesson despite the fact that this knowledge does not automatically transfer prior to or during classroom instruction.

What the Number e Is

The task. Unit 5 of the Core-Plus textbook focuses on Exponential Functions, Logarithms, and Equations. At the start, students find, use, and apply the compounding interest formula. The goal of lesson 1 is "to review and extend the properties of exponential and logarithmic functions to include the cases based on the transcendental number *e*" (p. 1). By finding the limit of the expression $\left(1+\frac{1}{n}\right)^n$, they will be introduced to *e*. Students discover the limiting value as they complete the following problem:

> You can think of the expression $\left(1+\frac{1}{n}\right)^n$ as representing the value of $1 credit card balance being charged 100% annual percentage interest rate that is compounded at *n* intervals in each year. Use your calculator to evaluate $\left(1+\frac{1}{n}\right)^n$ for values of *n* in the following table:

N	1	2	3	4	5	10	20	100	500	1000
$\left(1+\frac{1}{n}\right)'$										

Enacting the task. Kate opens class with a calendar on the SMART board and discussion of her expectations for student work this week. She expects they work through Investigations 1 (#1-8) and Investigation 2 (#1-7) in their groups or on their own this week. She reminds students that they must do whatever they need to do at home in order to be at the end of Investigation 2 by the end of the week. Then, Kate lets the student get to work. Sitting at their desks with group members and a textbook, a pencil, and calculator in front of them, they begin working through problems 1-8 within Investigation 1. Kate circulates the room and answers students' questions or listens to students' problem-solving strategies as she approaches each of the groups. Each groups of students works at a slightly different pace.

Knowledge enacted and displayed during reflection. As students investigate compounding interest for larger and larger values of *n*, they use the TI-84 to explore and determine that a limiting value exists. However, Kate does not anticipate students will use the TI-84 technology and does not prepare to lead a discussion of key problems like how *e* is introduced because of her course design (i.e., she wants students to work in small groups at their own pace and therefore doesn't know if and when all students will reach the introduction to *e*). She reflects, "I never really say for everyone I want you to type it into your calculator, or I want you to do it this way, so I actually thought I bet they don't use it very much today" (5/18 interview). As a result, she draws upon a limited range of pedagogical content knowledge with the TI-84 while preparing to facilitate and teaching this lesson. Nevertheless, as students work Kate notices how they use technology and builds teacher knowledge of students' understanding with the TI-84, instructional strategies with TI-84, and knowledge of curriculum with the TI-84 via reflection.

I ask Kate to describe specific events that stood out to her from the lesson. She explains,

> I noticed that when they [the students] were talking about the first few problems…they just went through and actually calculated each one of those with the specific numbers. They didn't write a general rule and put it into their table and just say, ok now if it was this number, this number or this number, they didn't get there yet. So really all they were using their calculator for was just when you raise it to this power, raise it to this power. And they were actually just typing that exponent into their equation. (5/18 interview)

Initially, she notices students use only the computational functions of their TI-84 graphing calculators to solve and think about the problems within the investigation. However, as students reach number seven, she observes some students using the technology differently. She explains,

> Number seven: it has, if you have started out with one dollar and the interest rate is one percent over the course of a year, and for the different *n*'s, it gives the question, like what's the value if you compound once a year, twice a year, ten times a year. I did notice several groups did put it in their calculator and look at the table, they just wrote the equation in the y equals. And then one girl said, yea, I'm looking at my table, but I heard a girl in the next group over say, man, why didn't I just do that. (5/18 interview)

As a result, Kate becomes more aware of how using the TI-84 graphing calculators within the lesson enables and enhances students' introduction to the value of *e* as a limiting value of compounding interest.

Kate displays knowledge of students' understanding with the TI-84 calculator as she further considers the role technology played within this lesson. She reflects upon how different students reason, communicate, and problem solve differently with the TI-84 graphing calculators by saying,

> For some people, they really understand like what a table, how a table works in the calculator. And some people, that's just not where their brain goes. It doesn't think, oh if I just type in this equation, I can just check all the x values cause it's what I'm plugging in, you know what I mean, and look at all my outputs. Some people just don't really think that way. (5/18 interview)

She concludes some students immediately think to enter the expression as a function in their graphing calculator and then look at a table or graph although others do not. However, the students that did use a tabular representation to think about the problem could more efficiently and more consistently visualize and determine the limiting value of compounding interest. She reflects,

> People who did look at the table kind of scrolled down and they definitely saw the table, they could really see how it was big jumps at first if you compound once a year, or twelve times a year. And they got all the way to a thousand you can see that it's barely changing. So it really was approaching some asymptote as it went to infinity… for those of them who put it in their calculator. That was something for us

> to talk about real quick, if you just scan through your table real quick what's happening. And they definitely got that really fast, so that helps them get there without actually calculating…a fast way for them to see hey its getting closer and closer to like, someone said "It looks like it's like 2.72 maybe" …That group that was right in front of you, they didn't look at the table in the calculator, I don't think, but they did notice how it was increasing at a decreasing rate. I don't know if you heard them actually say that. They said, well its increasing by less and less, so it's increasing at a decreasing rate. And so I think they kind of saw that it was leveling off without actually looking in their calculator…but I think it [the calculator] really could help them think a little bit more about *e*. (5/18 interview)

As a result, she realizes that guiding students to look at a tabular representation either within a whole class discussion or small-group collaboration is a worthwhile instructional strategy with the TI-84 that takes into account students' understanding with technology to facilitate the introduction of *e* as the limiting value of compounding interest.

Additionally, Kate concludes that presenting and discussing a graphical representation will be helpful to extend and solidify students' understandings of *e* as a limiting value of compounding interest. She shares, "Next time after they have thought about this a little more, I do want us to look at the graph of that and see how it does have a horizontal asymptote at *e*" (5/18 interview). Thus, she gains knowledge of instructional strategies with the TI-84 during analysis and reflection.

Kate's knowledge of curriculum with the TI-84 also expands during analysis and reflection. During the stimulated-recall interview, she says,

> It [the textbook] definitely helped because number seven was to make a table, and it gave you the function they wanted you to make a table from. So it said, "Hey, using this function when it is this and it is this make a table that shows these values," and so a lot of them are just really use to when this curriculum does that, they can just look at the table in their calculator and not actually have to calculate each

> one…this whole curriculum, when it asks them that, it kind of leads them to go ahead and use their graphing calculator. (5/18 interview)

She notes how the curriculum consistently integrates the use of a calculator with graphing capabilities (e.g., a TI-84 or TI-nspire™). By making it a norm within the course, she recognizes how the curriculum supports the developments of students' mathematical knowledge, reasoning, and problem-solving skills with the use of the technology. Explicitly realizing these things about the curriculum, she gains further and more comprehensive knowledge of the curriculum with technology for teaching.

Continuously Compounding Interest: Where the Formula Comes From

The task. One application of base *e* includes continuously compounding interest. In order to better understand what the number *e* is, students explore applications and contexts involving *e*. For problem 8 of investigation 2, students algebraically reason and justify how and why *e* is used within the continuously compounding interest formula, $A = A_0e^{rt}$. Problem 8 states:

> For moderately large values of *n*, the expression $\left(1+\frac{1}{n}\right)^n \approx e$.
> Use the relationship between $\left(1+\frac{1}{n}\right)^n$ and *e* and familiar properties of algebraic expressions to explain steps in the following connection to frequent compounding of interest.
>
> Step 1: $\left(1+\frac{1}{\frac{n}{r}}\right)^{\frac{n}{r}} \approx e$
>
> Step 2: $\left(1+\frac{r}{n}\right)^{\frac{n}{r}} \approx e$
>
> Step 3: $\left(1+\frac{r}{n}\right)^{n} \approx e^r$
>
> Step 4: $\left(1+\frac{r}{n}\right)^{nt} \approx e^{rt}$

Enacting the task. Students work in groups of 3 or 4. They encounter problem 8 at different times. Nevertheless, within a 30-minute range, Kate talks to each group of students about this problem. Many of the students struggle to write an algebraic proof and reason about the relationship between $\left(1+\frac{1}{n}\right)^n$ and *e*. They use graphing calculators to make sense of the first statement, why $\left(1+\frac{1}{n}\right)^n \approx e$. Then, Kate uses the SMART board to communicate and explain the algebraic proof.

Knowledge enacted and displayed during reflection. Kate realizes her students are struggling and recognizes that they do not all know how to use the TI-84 in ways that help them make sense amongst the problems (i.e., 7 and 8 where *e* is introduced and then connected to continuously compounding interest). Kate reflects,

> It was on number eight where they had actually looked and it was really confusing to most groups. They were wondering why we were setting this left side equal to *e*…a lot of students didn't really think about how things that are compounded more and more often it got closer to this number 2.71. They were just kind of like ok. (5/18 interview)

Then, she considers this knowledge of students' understanding and considers knowledge of students' understanding with the TI-84 as well as instructional strategies with the TI-84 or TI-nspire™ that she could use to help more students either next class or next time she guides students through this problem and investigation. She suggests,

> If they did type it in the calculator, looking at the graph they just might see a little more how that happened. And what the graph looked like might help them make a little more understanding of what it's doing [leveling off and converging at the value of *e*]. (5/18 interview)

In other words, she could use a graphing calculator to assist students in more explicitly understanding why for moderately large values of *n*, $\left(1+\frac{1}{n}\right)^n \approx e$. She lacks this knowledge of instructional strategies with a graphing calculator although she builds it as she considers students' understandings, students' understandings with TI-84, and modifications to her teaching with the TI-84 or TI-nspire™ (since both have similar graphing capabilities).

While she realizes the TI-84 or the TI-nspire™ may be helpful to discuss and understand the assumption of the problem, she recognizes that it would not be worthwhile to use for other parts of this problem. Rather, the new SMART board technology is a more effective use of technology when discussing and sharing algebraic reasoning. For instance, Kate notices how going from the assumption to step one is confusing for students. She describes,

> Step number one on number eight is the most confusing. They were having the most trouble with why do we divide this by *r*. I had a girl she worked through all of it and she said here is what I think. It was really good, I thought. She said so I know we are trying to by the end of this show that the rule we wrote back on number six that really all of this stuff right here is how like how it involves *e*. And she said so we are up here we had this is equal to this but if we are going to incorporate this rule somehow we needed an interest rate somehow so this was just their method of incorporating *r* into their equation so they just rewrote this variable right here as *n* over *r*. I was like okay so why would they do that and she said because one over that is just *r* over *n* which is where we are trying to go. (5/18 interview)

Kate writes on the SMART board as the student explains and uses different colored markers to highlight and circle changes described by the student. Here, she decides to use the SMART board technology rather than the graphing calculator technology to mathematically communicate and reason with students in her classrooms. She is beginning to consider and find times when it is most appropriate not to use the graphing calculator technology (either the TI-84 or TI-nspire™) during reasoning and justification. Instead, as she leads discussion of justification through the four steps, Kate instructs at the SMART board and presents analogies to explain algebraic manipulations. For instance, Kate recollects,

> I had to remind them that raising this whole thing to the *r* power is the same as raising it to the *n* times *r* because it would be like so I wrote that on the board with an example like three to the *x* to the fourth was really three to the four *x* [$(3^x)^4 = 3^{4x}$] and they remembered that but it took them a minute to get to what did they do to get from this step to this step and they had to see that. (5/18 interview)

She draws upon pedagogical content knowledge without technology as she presents a parallel example involving fewer variables (e.g., algebraic notation).

Rounding Issues while Solving Exponential Equations

The task. After the introduction to the meaning of *e*, students use properties of exponents and logarithms to write algebraic expressions in equivalent forms and solve equations involving logs and exponents. The textbook authors assume and note that students should have access to calculators or computer software with CAS capability while working through this lesson. In Kate's class, students use their TI-84 calculators while discussing problems involving exponential functions. Two such problems include:

1. Suppose that early in the flu epidemic the number of cases doubles every 5 days, and that the number of cases at any time *t* days after the epidemic begins can be modeled by a function $C(t) = C_0e^{rt}$.
    a. How can the fact that $C(5) = 2C(0)$ be used to write an equation in which the unknowns are C_0 and r?
    b. How can the equation in Part 1 be solved to find the value of *r*?
    c. If counting of flu cases begins when 250 have been reported, what number of cases is predicted for a time 14 days later? How about 28 days later?
2. All living matter contains both stable and radioactive forms of the element carbon. In all living matter, the ration of radioactive (^{14}C) and stable (^{12}C) carbon is the same. However, when any living matter dies, the radioactive isotope begins to decay and the ratio of radioactive to stable carbon declines exponentially. This idea is used in dating of archeological discoveries because (^{14}C) has a known half-life of about 5,730 years.
    a. If the function $R(t) = R_0e^{rt}$ gives the ratio of radioactive to stable carbon in an organic object that has been dead for *t* years, what value of *r* is implied by the half-life of 5,730 years for (^{14}C)?

b. Suppose that a fossil is discovered and its ratio of radioactive to stable carbon is $0.2R_0$. About how long ago was that object last "alive"?

For the first problem, students should find $C_0e^{5r}=2C_0e^{0r}$, r=0.14, C(14) = 1,775 , and C(28) = 12, 600. Likewise, students should find that r = 0.00121 because that is the solution of $0.5R_0=R_0e^{5730r}$ and t = 13,300 because that is the solution to $0.2R_0=R_0e^{-0.00121t}$ for the second problem.

Enacting the task. Students spend 45 minutes solving seven problems involving exponential functions. As usual, students work in groups of three or four at their own pace. Kate circulates the room and answers students' questions as they arise. After approximately 15 minutes, two girls ask Kate about their findings; they describe the same process of solving, but they generate two different answers. They want to know why. As Kate talks with the students, she determines that one girl entered rounded numbers into her calculator while the other keyed in "second answer" when using a previous computation on the calculator to find a more exact final answer. In the end, the girls share answers that differ by more than 1000. As a result, Kate advises students to use exact answers until the final step, when the result should be rounded to the hundredths place.

Knowledge enacted and displayed during reflection. Kate displays knowledge of the TI-84 for solving exponential growth and decay problems. However, this knowledge leads her to an incorrect prediction of what students do with the graphing calculators when they independently problem-solve. Kate anticipates students will use the tabular and graphical applications on their TI-84 calculators while problem-solving during this lesson. She is surprised when the questions related to the mathematics with the technology do not relate to the table or graph related to a given problem. She describes,

> I was surprised actually, I didn't see anybody use a table or a graph today in their calculator, and I kind of thought they would, that would help them. Maybe somebody look at the graph, or the table, and say, 'For this many years it's this,' and someone else could plug in the equation and that would just be different ways they'd be using it, but I didn't see a single person using it in those ways. (5/22 interview)

During class, Kate fields questions related to computing on the TI-84 rather than graphing or looking at a table on the calculator.

Kate also displays knowledge of instructional strategies with the TI-84 or TI-nspire™ although she does not enact it. She reflects,

> I could have asked or prompted them to somehow look at different representations in their calculator. We never graphed any of those functions today, to just kind of get an overall idea, or general picture of what is actually happening to Carbon-14 or how long it takes to get really small. We never really looked at that or thought about the broad picture of any of the situations, so that's something that with more time would have been cool to look at using the technology. (5/22 interview)

In other words, she realizes that by using a graphical representation of a sample problem like the decay of carbon-14 and dating fossils with this information, she could have led the class in a discussion to summarize the mathematics within applications of exponential functions. Furthermore, she could have used a graphing calculator to accurately and efficiently present a graphical representation that all students could access and replicate with the use of handheld graphing calculators. As she talked during the interview, she did not draw upon knowledge about how instructional strategies with the TI-84s would differ from the TI-nspire™. At this point, she is not using the TI-nspire™ regularly and her students choose to use the TI-84 rather than the TI-nspire™. Therefore, she does not enact knowledge of teaching with the TI-nspire™.

Kate has knowledge of students' understanding with the TI-84 graphing calculators although this understanding was not evidenced with the TI-nspire™ handhelds. With the TI-84, some students struggle when discussing final answers because they enter one computation at a time (e.g., ln 2 enter and then divided by 5 rather than ln 2 divided by five as one expression in the calculator). She comments,

> The technology brought up a problem where one kid put the whole answer straight into her equation for the rate, so she put the natural log of 2 divided by 5, and hit second answer. And for the rate in the next equation, then I brought up a problem of, if I round for the answer on the first part, for my rate, then when I use that rate, I get a different answer than someone who uses the exact answer. And that was a big discrepancy…She wouldn't have done that if she was just using pencil and paper. But, with the calculator, they noticed that it's really different if they use the exact answer. (5/22 interview)

Whether students use an exact rate (e.g., $\frac{ln2}{5}$) or an approximate rate (e.g., 5.2%) makes a difference in the value of the final answer. Use of the TI-84 prompts students and the teacher to consider when it is appropriate to round, however, this is further complicated because students have better number sense with approximate answers (i.e., decimal representations). Kate understands,

> They [the students] are able to check their answer [with the calculators]…when they don't have their graphing calculator, they're just stopping right there with their answer. They don't get to check first of all, "is this reasonable?" because they don't even stop to think what that would even be close to. When they get the decimal approximation with their calculator, they can stop and think if it would be a reasonable answer. (5/22 interview)

As a result, Kate needs knowledge of instructional strategies with a graphing calculator in order to teach and explicitly address this issue of when to use approximate or exact answers. Although she lacks this knowledge, she acquires capacity to build this knowledge as she reflects upon her knowledge of students' understanding with the TI-84.

Finally, just as Kate does not enact knowledge of instructional strategies with TI-84, she does not enact knowledge of assessment with the TI-84. She describes,

> If they were getting the right answer, I just assumed that they were putting it in the calculator correctly. A lot of times, I didn't really see what they were doing with the calculator. There were a couple of times I definitely looked in the calculator, and I was assuming if they had gotten the wrong answer it was still that parentheses issue somewhere, with exponents. But I didn't really notice anything. I had two groups [out of six] bring up the rounding, that same issue. Otherwise, I didn't really watch what they were doing with the calculator. I didn't really look at many peoples' calculators today. (5/22 interview)

Without accurately predicting what students will do with the TI-84 or TI-nspire™, it is impossible to instruct and assess in meaningful ways with the technology. Initially, Kate knows she can listen to students discuss their use of technology and look over their shoulders at the screens of their handhelds in order to assess with the technology. However, until she better understands how students use and think about the mathematics with the technology, she is not inclined to further develop knowledge of assessment with technology.

Negative Exponents: How They Change the Value of an Expression

The task. In a final activity with logarithms and natural logarithms, Kate hopes students will develop some procedural fluency and a better understanding of how to apply the definition of logarithms when simplifying expressions. She creates a 2-page worksheet to supplement the textbook materials and engage students in simplifying expressions without technology. I describe this episode because although Kate initially instructs students not to use calculators, she finds that students have forgotten about negative and fraction exponents. As a result, she encourages students to use TI-84 calculators to explore and re-identify what happens to an expression with a negative or fraction exponent although she still directs students to evaluate expressions involving logarithms without calculators. For example, students find the exact value of the following logarithms:

a. $log_5 1$
b. $ln\sqrt{e}$
c. $log_{\sqrt{3}} 9$
d. $log_{\frac{1}{2}} 16$

Applying the definition of logarithm and the rule involving negative exponents, students see that for part d $\frac{1}{2}^x = 16 \rightarrow \frac{1}{2}^x = 2^4 \rightarrow 2^{-x} = 2^4 \rightarrow -x = 4 \rightarrow x = -4.$

Enacting the task. Kate introduces the worksheet as an opportunity to practice procedural skills and the application of the definition of logarithms. She tells students they are expected to recognize and know how to work with the notation on the worksheet for more advanced mathematics classes. Then, she instructs students to turn off and put away their calculators; they need to use mental math to evaluate the given expressions.

She designates 30 minutes of class time for students to complete and discuss the worksheet. Students ask Kate to verify their reasoning, especially as they begin. After approximately 15 minutes, students have to pause, think, brainstorm, and question their ideas as they reach problem *d*. One student remarks, “We’re trying to figure this out. We have ½ to the power of x equals 16. So 0.5 to the power of x equals 16. We were thinking logically it would be about 8. 0.5 times 0.5 is 0.25, it just keeps getting smaller” (video 5/22). Then, her partner asks, “Can we take it to the power of a fraction?" (video 5/22). Consequently, Kate responds with a question and then directs students to explore their thinking and the mathematics on the TI-84 calculator. She replies, "If you put it to the power of a decimal would it make it get bigger instead of smaller? Why don't you open your calculator and try it" (video 5/22). Students enter a couple of examples such as $\frac{1}{2}^{1/8}$ and $\frac{1}{2}^{-2}$ into their calculator. With the TI-84, they determine ½ to the negative fourth power equals 16. Therefore, $log_{\frac{1}{2}}16$ = -4.

Knowledge enacted and displayed during reflection. Kate did not anticipate that students would struggle to remember how to use negative exponents. However, as she monitors students working in their groups and listens to their discussions, she understands, "They all had problems with the negative exponent" (5/22 interview). Likewise, she sees students are not confident about what they should do when the base number was a fraction. She explains, "They were having trouble understanding what was happening as the exponent got bigger when the base was a fraction" (5/22 interview). As a result, Kate makes an in-the-moment decision to allow and even encourage the use of TI-84 calculators for exploration and rediscovery of the resulting value with different types of exponents.

Kate does not plan the lesson with knowledge of instructional strategies with a graphing calculator or knowledge of students' understanding with the TI-84 or TI-nspire™, but as she engages students in problem-solving and communicating their mathematical reasoning, she becomes aware of how students can use the TI-84 to explore and make sense of the mathematics without her, the teacher, telling them the rules. She can continue to allow the students to have authority over their own learning by using the handheld calculators. Additionally, she recognizes that students learn when they have time to constructively struggle and the calculator provided this opportunity during this lesson. Kate shares,

> I think just letting them experiment with that a little more, prompting them to. That was the part where I had said no calculator, and then when I had people getting stuck with the negative exponent, and they wanted to try other things. They said, "I don't want to do the problem on the calculator, I just want to try other things so I can figure this out." Then I had groups go open their calculator because I felt like they would understand it if they had struggled with it for a while and then they had to try and figure out what was happening on their calculator, and they did. (5/22 interview)

As students use the TI-84s on their own, Kate observes what and how they are thinking about the math. She describes, "They decided they were going to raise a fraction to the power of a fraction. So she just picked a number. And it was being really weird, it was not what they thought was going to happen. She tried 0.5 to the power of 1/8 and was like, "That is not right." …I didn't watch everything she typed, because I knew she would eventually get to it" (5/22 interview). Kate learns how the TI-84 helps students think about the math and accurately identify rules while following their own intuitions. She builds knowledge of students' understanding with the TI-84 and instructional strategies with calculators.

When she pushes herself to further consider instructional strategies with technology, she decides,

> I could have had her graph 0.5 to the x…she could see what happens as the exponent changed from positive to negative…that would have been a good idea for them to think about what happens when the exponent is 1 or greater, when it's between 0 and 1, and when it's less than 0. Kind of think about what's happening there and what you have to do to it to make it get bigger. I mean, they got there by just trying stuff. But I think they might have gotten a better idea if they had looked at the graph. (5/22 interview)

After building PCK with the TI-84 by flexibly listening to and working with students while thinking as a teacher during class, Kate extends her knowledge for teaching logarithms and exponential expressions with calculators.

However, extending her knowledge for teaching with the new TI-nspire™ calculators is more of a challenge. She considers, "With the TI-nspire™, they could definitely be seeing multiple representations at once and see what's actually happening with the graph and the function as our exponents change. But I don't know what else" (5/22 interview). Inquiring further, the conversation continues as follows:

> I: Does it show different things with negatives that you know of, with negative exponents, with fractions, if they were working those calculations on there, would they be different?
>
> Kate: If you do the negative exponent, it would give you your answer as the fraction rather than the decimal. So if I had 2 to the -4, then the 84 is going to give me whatever 1 divided by 16 is. And I think that the TI-nspire™ would leave it as 1/16. I think it does, so that would help them understand how the negative exponent is the reciprocal, basically it's flipped over. That would help them understand that, because I know the 84 doesn't give us a fraction. When they see that decimal, they probably aren't going to stop and think, "That's 1/16. Oh, I see, 2 to the negative 4^{th} became 1 over 2 to the 4^{th}." So they aren't going to think of that.
>
> I: Do you think that would help or do you think that would hinder students in their thinking?
>
> Kate: I think that would definitely be something that would help, especially when I was teaching it. I didn't really think of that as something that I was trying to teach today.

The TI-nspire™ represents fractions and defaults to simplifying fractions as fractions where as the TI-84 represents evaluated fractions as decimals when first hitting enter. Kate begins to consider how seeing the negative exponent rewritten as a fraction when the student hits enter on the TI-nspire™ could help students more clearly and efficiently identify that negative exponents move the numerator to the denominator or vice-versa in an equivalent expression with a positive exponent. Thus, as she more consistently integrates the new technology (i.e., the TI-nspire™ calculator) into her teaching practices, she will need to be open to the growth of new PCK with technology.

Kate's PCK with Technology

Kate's development of PCK with technology begins with the predominant formation of knowledge of students' understanding with the TI-84 and instructional strategies with graphing calculators. Learning the curriculum and the role of graphing calculator (e.g., the TI-84 or TI-nspire™) within curriculum simultaneously is a larger task than she expects. Additionally, Kate considers graphing calculators and assessment but her knowledge of assessment with technology is limited. I illustrate these conclusions with an overview of the knowledge of teaching mathematics with the TI-84 Kate enacts and displays (Figure 18) and the knowledge of teaching mathematics with the TI-nspire™ Kate displays (Figure 19).

Figure 18. Kate's pedagogical content knowledge with the TI-84.

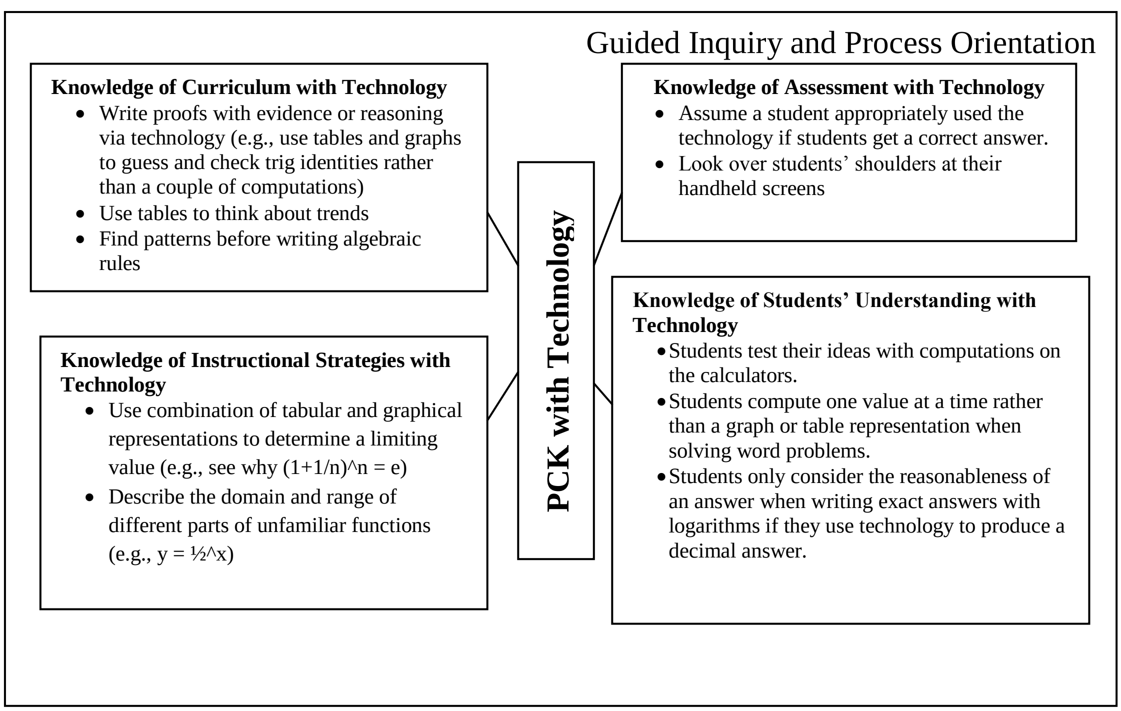
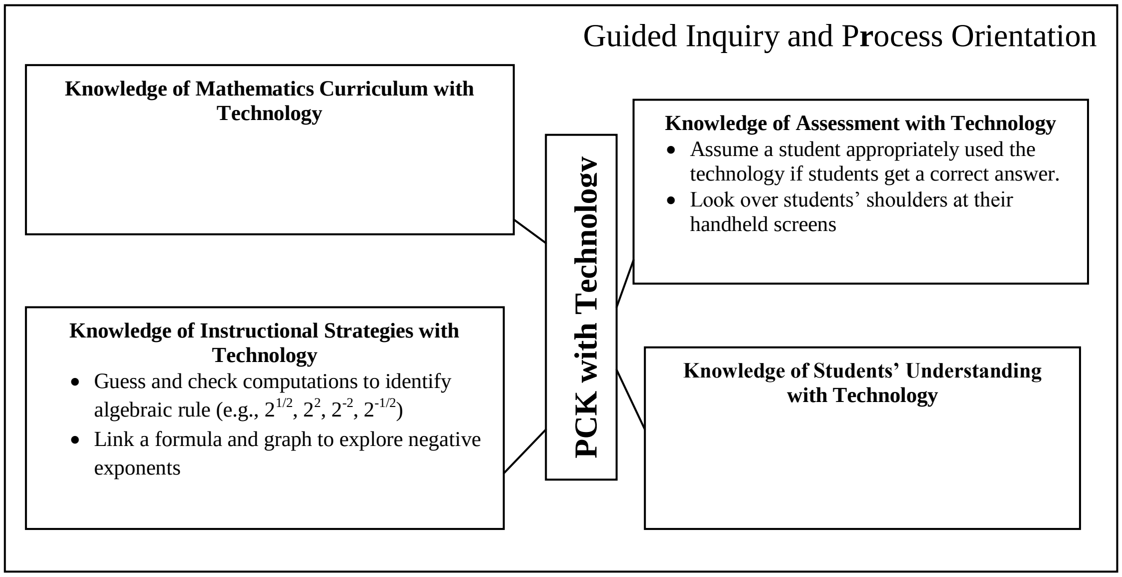


Figure 19. Kate's pedagogical content knowledge with the TI-nspire™.



Summary of Findings

Joe, Mary, and Kate displayed and enacted pedagogical content knowledge with technology in a variety of ways while instructing and reflecting upon their high school mathematics teaching practices. In the case study descriptions, I analyzed their orientations to teaching mathematics with technology and what knowledge they draw upon while instructing mathematics with a new technology (i.e., the TI-nspire™ calculator). I characterize the knowledge bases and orientations in order to be able to share findings that will support in-service professional development leaders, teacher educators, and educational researchers while expanding what we know about the knowledge teachers need for technology integration. The case study methodology (Yin, 1999) enabled me to develop descriptions of what PCK with technology looks like in practice, specifically examining components related to pedagogical content knowledge. Then, I considered and classified teachers' orientations for teaching as I considered the context within which teachers use and develop their knowledge for teaching with technology (i.e., pedagogical content knowledge with technology).

I summarize the orientations and knowledge enacted and displayed in Table 8 below.

Table 8.

Summary of Teachers' Orientation and PCK with TI-nspire™ Technology

	Orientation	K. of Instructional Strategies with Technology	K. of Students' Understanding w/ Technology	K. of Curriculum with Technology	K. of Assessment with Technology
Joe	Didactic & Activity -Driven	• Navigate new technology with a guiding worksheet • Model use of calculator on SMART board • Look at and describe number patterns to make sense of algebraic properties • Pose open-ended questions (e.g. what do you observe) when working with student on an exploration • Allow students to guess and check with their own numbers • Evaluate fractions with different technologies	• Students build confidence of skills by verifying computations with fractions and negative numbers raised to an exponent • Students confuse the value of negative numbers raised to a power • Students, via an Nspire activity, notice that an expression and an equation differ. An expression has a value while an equation is a true statement where the values of two expressions are equal to one another.	• Find TI-Nspire activities on the TI Activity Exchange on the internet	• Use worksheets that students write and record their thoughts and conclusions on while working • Assume a student appropriately used the technology if students get a correct answer
Mary	Didactic & Activity -Driven	• Drag lines and points linked to measurements of lines or angles to see relationships in changes • Insert prerequisite definitions into a document page at the start of an activity	• Students will play with the new technology when the math is too basic (e.g., parallel lines have same slope). • Students struggle to generalize or conjecture with the mathematical ideas generated with the TI-Nspire.	• Incorporate activities with prior math knowledge to develop new knowledge (e.g., facts about parallel lines transformed into conditional statements to think about truth) • Find TI-Nspire activities on the TI Activity Exchange on the internet	• Use worksheets that students write and record their thoughts and conclusions on while working • Save work on handhelds and then transfer student documents to the teacher computer • Display teacher answers to a worksheet at the front of the room for students' self-assessment • Communicate actions on technology and mathematical thinking with technology in a whole-class setting
Kate	Guided Inquiry & Process	• Guess and check computations to identify algebraic rule • Link a formula and graph to explore negative exponents			• Assume a student appropriately used the technology if students get a correct answer. • Look over students' shoulders at their handheld screens

As the table illustrates, Joe, Mary, and Kate have and draw upon all four components of PCK with technology although some components may be more developed than others. Their orientations to teaching differ by their school and textbook/curriculum context, which noticeably impacts their knowledge of instructional strategies with technology and knowledge of curriculum with technology. Furthermore, these findings lead me to conclude and make four assertions:

Assertion 1: Teachers begin to develop PCK with technology specific to the use of the TI-nspire™ calculator and software.

Assertion 2: Teachers with PCK for other technologies may not transfer that knowledge to PCK with the TI-nspire™.

Assertion 3: Teaching with and reflecting on the use of the TI-nspire™ helps teachers to develop PCK with the TI-nspire™.

Assertion 4: Teachers may develop specific components of their PCK with technology before others (e.g., students' understanding with technology before assessment with technology).

Assertion 5: Teachers consider the TI-nspire™ a "discovery-based" mathematics learning tool and believe students investigate and learn mathematics on the handhelds when they structure learning environments to support the nature of this type of instruction.

I elaborate on each of these assertions below.

Assertion 1: Teachers Begin to Develop PCK with Technology Specific to the Use of the TI-nspire™ Calculator and Software

Different technologies have different features that teachers can use to illustrate important examples or provide students with opportunities to investigate mathematical ideas in different ways. A teacher who develops PCK with a TI-84 graphing calculator may not necessarily have PCK with the TI-nspire™. Because the TI-nspire™, the new technology, affords new and dynamically linked representations, teachers begin to develop PCK with technology specific to the use of the TI-nspire™ calculator and software as they integrate or reflect upon the use of the TI-nspire™ during instruction. Two such examples from the three cases illustrate this point. First, Joe uses a document that links a number line representation with symbolic notation and dynamically changing numeric expressions that directly relate to points on the number line on the same screen. Calculator users could not create number line representations on handhelds before the release of the TI-nspire™ calculator nor could users dynamically link representations in order to explore multiple examples directly related to algebraic representations. In this case, Joe demonstrated and began to develop PCK that was specific to the TI-nspire™ as he used it during classroom instruction. Second, Kate developed PCK with the TI-nspire™ by reflecting on how the use of the TI-nspire™ would have impacted students' learning experiences had they used it during class. Kate could have asked her students to explore with the TI-nspire™ and discover that $2^{-4} = \frac{1}{16}$, which allows the students to immediately see the fraction and the connection to 2^4. Exploration on a TI-84 was not as powerful because it represents 2^(-4) as 0.0625 after hitting enter. However, she did not make this connection until she was confronted with it during the stimulated recall interview following the lesson. Consequently, PCK with TI-84 graphing calculators is different than PCK with TI-nspire™ calculators, and teachers begin to develop PCK with technology for specific technologies they use in the classroom by either teaching with it and then reflecting on how the use of it impacted students' learning experiences in the classroom or by considering how it could have changed students' representations and mathematical work in the classroom.

Assertion 2: Teachers with PCK for Other Technologies May Not Transfer that Knowledge to PCK with the TI-nspire™

In the first year of use with a new technology, experienced secondary mathematics teachers struggle to integrate a new technology into teaching practices even when they have a strong knowledge of how to teach, communicate with students, and do mathematics with an older and similar technology. All three cases demonstrate this struggle and how it resulted in limited use of the TI-nspire™ in the participating classrooms. Kate's case most clearly illustrates this point. Kate enacted pedagogical content knowledge with the TI-84. However, she did not enact pedagogical content knowledge with the TI-nspire™. For instance, Kate encouraged students to investigate negative exponents with the TI-84s because the students had these at their desks and used them regularly. However, had Kate picked up a TI-nspire™ to investigate the patterns of negative exponents with students, students would have been able to see more clearly that $2^{-4}=\frac{1}{2^4}$ (i.e., the negative exponent can be rewritten as a positive exponent with the term flipped to the denominator when the negative exponential term is a numerator). Kate's textbook consistently highlighted how and where a graphing calculator could and should be used to investigate and do the mathematics; she had experience and knowledge in teaching secondary mathematics with the old technology (i.e., the TI-84 calculator); she had a classroom set of the new technology (i.e., TI-nspire™ calculators), and Kate, on her own initiative, sincerely wanted to use the new technology throughout the year with the students. Nevertheless, when I observed her teach, I observed non-use of the new technology and only began to hear about how she was building PCK with the new technology via stimulated-recall interviews. Kate inconsistently used and at the end of the year did not use the new technology in her teaching practices despite the fact that she knew how to teach with a slightly older and similar version of the new technology.

Assertion 3: Teaching with and Reflecting on the Use of the TI-nspire™ Helps Teachers to Develop PCK with the TI-nspire™.

Teachers do not consider new instructional strategies let alone new student understandings with technology until they are able to work with it themselves and observe students discussing mathematics with the tools. For example, Mary's honors geometry students struggled to make conjectures (without the teacher telling) while exploring angle relationships on the TI-nspire™. They had never worked with dynamic geometry software nor been asked to communicate observations while moving and changing lines and angles. Mary instructed students to work individually because she wanted students to develop their own ideas. However, by using this instructional strategy, Mary made it impossible for students to compare findings across multiple examples and see that although different students had different angle measurements, they listed the same pairs of angles as congruent and supplementary when parallel lines are cut by another line and thus could make a generalization. After a disappointing lesson and reflection, Mary builds knowledge of instructional strategies with technology, students' understanding with technology, and curriculum with technology as she considers how to modify her instruction and the task in order to more effectively help the students understand and see that special angle relationships exist when parallel lines, not just any pair of lines, are cut by a transversal. In other words, after teaching with the technology and observing students work as directed, she generates different ideas about what instructional strategies she should use and what modifications she should make to the task in order to more effectively help students think about and learn the mathematics with the TI-nspire™.

Assertion 4: Teachers May Develop Specific Components of Their Pedagogical Content Knowledge with Technology Before Others (e.g., students' understanding with technology before assessment with technology)

All three case study participants acknowledged that they had either not yet considered or did not know how assessments might change with the use of the TI-nspire™ calculators. However, lack of knowledge of assessment with the TI-nspire™ did not hinder teachers from using the new technology with students, but rather challenged them to continue reflecting and constructing specialized teacher knowledge. Instead, teachers learned or first adapted either their knowledge of instructional strategies or knowledge of curriculum with technology as they began using the TI-nspire™. For instance, Mary arranged for activity-based learning rather than just lectures to begin occurring in her classroom because she knew about premade TI-nspire™ activities that she could insert into her curriculum. Joe did the same and more. He also talked through homework problems and mathematical reasoning while computing with the TI-nspire™ on the SMART board because he realized that notation and representations on the TI-nspire™ appear and operate differently than they do on the TI-84 graphing calculators. These examples show how didactic and activity-based secondary teachers developed knowledge of instructional strategies and curriculum with technology before developing knowledge of students' understandings with technology. Consequently, specific components of PCK with technology may develop before others.

In conclusion, Joe, Mary, and Kate displayed and/or enacted pedagogical content knowledge with technology specific to algebra, geometry, and integrated 4 courses, respectively. Through this exploratory study, I was able to begin to describe images of content-specific PCK with technology as teachers begin using a new technology. Likewise, I could classify teachers' orientations to teaching mathematics with technology in order to be able to begin consider in what ways these orientations relate to teacher knowledge. Clearly, teachers make teaching decisions based on their PCK with technology and orientations to teaching mathematics with technology. As a result, I expect content-specific PCK with technology grows and changes in different ways and at different rates in relation to the teacher's orientation to teaching, professional development, curricular resources, and school context.

Assertion 5: Teachers consider the TI-nspire™ a "discovery-based" mathematics learning tool and believe students investigate and learn mathematics on the handhelds when they structure learning environments to support the nature of this type of instruction.

"Discovery-based" learning and instruction is student-centered (Magnusson et. al, 1999); one in which students generate and investigate their own questions. Teachers in this study thought of the TI-nspire™ as a "discovery-based" learning tool because with dynamic geometry software and interactive activities students can conjecture about and explore the mathematics (e.g., the distributive property and angle relationships) for themselves on these handhelds. The following quote from Joe illustrates this idea,

> The nice thing with the TI-nspire™ is that students get the hands-on discovery of what we are doing in class, what we are trying to learn… they are doing more discovery in the learning as opposed to me trying to teach them what I know or what I want them to know. It's almost like they take more ownership in the learning with the technology… I know you can do discovery based learning without the TI-nspire™ technology however I feel like it does make it a lot easier. (initial interview)

In other words, now that Joe had access to a classroom set of TI-nspire™ handheld calculators and TI-nspire™ curriculum resources, he believed it was easier to structure student-centered classroom learning activities. However, as I observed Joe's instruction, this idea appeared to conflict with Joe's didactic orientation. Didactic and discovery orientations are dramatically different. These orientations lead to different student and teacher roles as well as classroom norms. Joe commented,

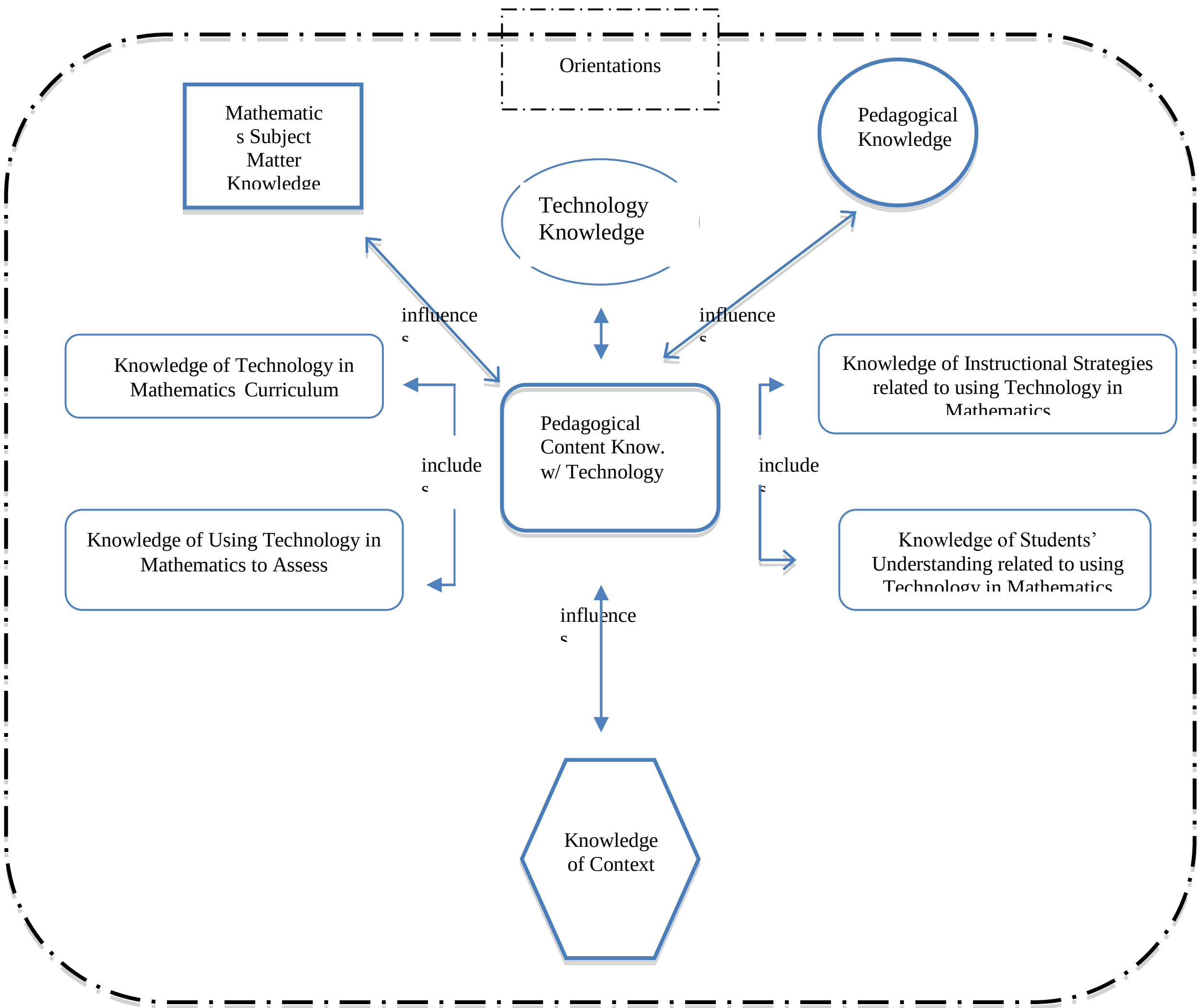
> I like to see myself as like a facilitator of the activity. I try not to get too involved. I try not to just do the activity for them and answer all of the questions for them. I try to just point them in the right direction. Hopefully, they can discover it on their own. But then again I am also there to know if they're not getting it and hold their hand and walk them through it or they are really understanding it, let me see if I can find something else that will further the discussion along. (closing interview)

Consequently, Joe struggled with how to enact a discovery-based activity with the TI-nspire™ (e.g., the distributive property activity). While he wanted to let them think about and learn the mathematics independently, he believed he needed to "hold their hand and walk them through it," a contradiction to a discovery-based orientation. Although Joe wanted them to use the TI-nspire™ to discover mathematical ideas, his didactic orientation served as a filter and conflicted with that idea. As Joe learns to use the TI-nspire™ calculator and uses it more often, will his didactic orientation shift to a more student-centered orientation? This raises the questions: (1) Could the TI-nspire™ (or any mathematical tool) act as a facilitator to begin to shift or challenge teachers' orientations for teaching? (2) Can teachers take on different orientations when teaching with and without technology? I suggest further research be conducted to examine these questions.

Discussion

A modified version of A Model of Teacher Knowledge (Abell et.al, www.resmar2t.missouri.edu) proved to be a useful framework to examine and help construct secondary mathematics teachers' knowledge for teaching as the interplay between content, pedagogy, and technology (see Figure 21). By reflecting upon dimensions of instructional strategies, curriculum, students' understandings, and assessment, teachers modified tasks and instructional practices, anticipated students' thinking more consistently, and more explicitly recognized the role of technology in teaching and learning secondary mathematics.

Figure 21. Model of teacher knowledge explicitly using mathematics and technology.



Additionally, analysis according to this framework, allowed me to identify and conceptualize the many kinds of knowledge that teachers draw upon when using and incorporating a new technology into a secondary mathematics classroom.

Teachers' beliefs and goals, related to teaching specific content with technology, influence how they use and assess with technology (Becker, 2000). All three case study teachers believed they should use the newest technologies in order to best prepare their students for the future and effectively engage their digital-age students in learning mathematics. However, Kate approached teaching with technology within a student-centered inquiry based modeled classroom while Joe and Mary occasionally attempted to enact student-centered inquiry based approaches with TI-nspire™ activities or TI-nspire™ based discussions within a course unit. Consequently, Kate had developed PCK about students' understandings of mathematics based on her experiences with listening to students' thinking while they used TI-84 graphing calculators. Based on her classroom learning environment, she could have developed PCK about students' understandings with the TI-nspire™. However, she did not have the necessary knowledge related to using this specific tool and she did not encourage her students to use this specific tool on a regular basis. On the other hand, Joe and Mary had opportunities to attend professional development specific to the use of the TI-nspire™ and worked with colleagues in their school buildings to discuss the use of the TI-nspire™. As a result, they began to use this specific tool (although still in limited ways) and began to learn instructional strategies related to this specific technology. These findings support the notion that a teacher's orientation to teaching mathematics with technology matters and impacts the components of pedagogical content knowledge with technology that they will first draw upon and develop as they incorporate a new technology into the classroom and provide students with technology-supported mathematical learning opportunities.

Technology integration requires new and complex teacher knowledge (Chazan, 1999; Hughes, 2005; Mishra & Koehler, 2006; Niess, 2005). I have documented that pedagogical content knowledge with technology differs from general pedagogical content knowledge. For instance, when students struggled to simplify trigonometric expressions in order to symbolically justify the truth of a trig identity, Kate drew upon her PCK *without* technology in order to ask students how they would solve a subtraction problem with fractions that have different denominators. The analogy and math question she posed to the students helped them to see how and why the students would multiply a term by cosine theta over cosine theta to get to a next step. Whereas, Kate knew not to use the TI-nspire™ CAS technology to help students justify symbolically because the CAS technology will generate a most-simplified output rather than multiple steps of the symbolic justification. Likewise, when Joe used the TI-nspire™ to lead an investigation and thought experiment related to the inverse property of multiplication, students could see, predict, and then generalize the pattern via 1/11 times 11 and the biggest number they could think of times one over itself, but when a student did not want to appeal to the calculator as an authority, Joe used his PCK *without* technology to explain that if you cut a piece of pie into eight pieces each piece would be one-eighth. Then, if put together 8 one-eighths you will have one whole. Thus, 1/8 x 8 = 1. Similar reasoning will work for all real numbers, and Joe justified this to *all* students in his class by using PCK with technology and PCK without technology.

The cases of Joe, Mary, and Kate show that teachers enact PCK with and without technology as they plan and teach with technology. In some cases, they transfer teacher knowledge with older technologies to PCK with new technologies and build new knowledge as they consider how new features, representations, and functions on the new technology impact teaching and learning in the classroom. In other cases, they do not transfer PCK from one technology to another. Although they drew upon PCK with the TI-nspire™ to plan their mathematics lessons, they also built PCK related to the TI-nspire™ as they taught with it. As they create tasks, use them in classrooms, and then modify them based on reflections of their implementation, they build PCK with technology, which parallels what Collette Laborde (2001) found when working to integrate dynamic geometry software with teachers in France for three years.

Also looking across the cases of Joe, Mary, and Kate, I captured very little use of the new technology within teaching practices during the first year of use. While I observed 40 lessons, I witnessed the use of the TI-nspire™ calculator only seven times with students. In the first year of using a new technology, the teachers did not regularly use it during instruction. This is not surprising as Alejandre (2005), Byrom & Bigham (2001), Chazan, (1999), Dwyer et al. (1999), and Mitchell, Bailey, & Monroe (2007) argue that teachers require multiple years to successfully integrate a new technology into their teaching practices. This suggests implications related to teacher preparation, professional development, and future research which will be discussed in chapter 5 along with the limitations and significance of the study.

CHAPTER 5: IMPLICATIONS, LIMITATIONS, and SIGNIFICANCE

"What a teacher might do in any situation is, of course, fundamentally shaped by the set of intellectual resources the teacher can bring to that situation – that is, the teacher's knowledge base." -Schoenfeld (1998)

"Teachers do what they do because they do (or do not) possess certain knowledge." – Sherin, Sherin, & Madanes (2000)

In this study, I qualitatively investigated secondary mathematics teacher knowledge as it relates to the integration of new technology (i.e., the TI-nspire™ calculator). I found that teachers who use the TI-nspire™ calculator believe that they should teach mathematics to their digital-age and tech-savvy students with the latest and greatest digital tools. This belief motivates teachers to learn about new technologies and begin to use them while teaching mathematics. However, their orientations for teaching mathematics with technology influence the knowledge they construct and enact during this transition. Nevertheless, the knowledge secondary mathematics teachers draw upon and build while using and reflecting upon the use of a new technology can be described as pedagogical content knowledge with technology (i.e., an integration of topic-specific knowledge of students' understandings, instructional strategies, curriculum, and assessment with technology). In this chapter, I discuss the implications, limitations, and significance of the study.

Implications

Implications for Future Research

I utilized a modified model of teacher knowledge that emphasized technology to investigate pedagogical content knowledge and orientations of secondary mathematics teachers as they began to use a new technology during mathematics instruction. The findings from this study suggest this is a useful model. However, further research is needed to test and refine the model with a larger sample of teachers from various backgrounds and contexts (e.g., different grade levels, mathematics courses, curriculum materials, and technologies). Moreover, the model should be tested with teachers across the professional continuum including novice teachers, established teachers, and teacher leaders, and teachers with varying levels of experience with specific technologies.

Students need opportunities to learn and think about mathematics using technology (Chazan, 1999; Doerr & Zangor, 2000; Heid & Blume, 2008; NCTM, 2000, 2005, 2008; NCSM, 2007; Roschelle & Singleton, 2008). Yet, these opportunities are only possible if teachers possess and use pedagogical content knowledge specific to the targeted technology. Thus, we need to provide opportunities for teachers to develop this knowledge and support their efforts as they begin to enact this specialized knowledge. This suggests further research that investigates how to facilitate the development of this knowledge as well as what contexts (e.g., curriculum or professional development) support and constrain the development and enactment of this teacher knowledge. Moreover, we must study the relationships between the use of technology and the development of teacher knowledge including an investigation of the transfer of knowledge from one technology to another.

Although future research is needed to refine the model, it should be noted that conducting studies based on the ideas above will be a challenge. When teachers begin to use a new technology, they become a novice again—a novice who has to learn PCK specific to the new technology. This reality hinders the enactment of mathematics lessons and as a result, teachers may decide to avoid using the technology. This was evident during my study, as the TI-nspire™ was only used on 7 occasions during 40 lessons. This suggests that researchers need to observe a large number of lessons in order to investigate how this knowledge develops over time as teachers learn to use it. As a result, researchers must carefully consider appropriate and worthwhile methodologies to effectively and efficiently collect data regarding the use of new technologies in mathematics classrooms. As we design future research related to teacher knowledge and technology, we must also consider the constant changes in technology. It takes time for teachers to develop specific knowledge, yet technologies are constantly evolving. For example, teachers may remember using the TI-82, then the TI-83, then the TI-84 graphing calculators before moving onto the TI-nspire™. These technological tools have a short life in mathematics classrooms suggesting that longitudinal investigations of teacher knowledge with specific technologies may not be possible.

Implications for Preservice Teacher Education

As technology evolves, mathematics teacher educators need to consider what preservice teachers need to know before they design appropriate learning experiences that will facilitate development of mathematical knowledge for teaching with technology (Laborde, 2001; Niess, 2008). As illustrated in Chapter 4, Joe, Mary, and Kate developed PCK specific to the technology as they taught mathematics with the TI-nspire™ calculator. This suggests that mathematics teacher educators will need to consider how to develop PCK specific to mathematics concepts and specific to technologies (e.g., the TI-nspire™).

As Joe, Mary, and Kate's cases illustrate, some components of PCK with technology, such as knowledge of curriculum with technology and knowledge of instructional strategies with technology, may develop before others. Focusing efforts to build knowledge with preservice teachers according to these dimensions may be a more appropriate place to begin. Then, as they reflect upon the implementation of these technology-supported strategies and curriculum materials, they will have the necessary prior knowledge to build capacity for and knowledge of students' understanding and assessment with technology. Preservice teachers should have opportunities to interview and work with students one-on-one with technology-active tasks in order to begin to recognize and know students' understandings and more explicitly consider assessment with technology. Preservice teachers should also have opportunities to observe and analyze classroom practice during field experiences or video segments so they have images of technology use in classrooms. Finally, they should have opportunities to analyze different curriculum tasks so they are aware of specific curricular resources as well as criteria of worthwhile tasks (Breyfogle & Williams, 2008).

Implications for Professional Development

Teachers need to develop PCK with technology specific to the use of the TI-nspire™ calculator and software and specific to the topics they teach. This requires a significant investment in professional development. Joe and Mary had opportunities to attend regional and national conferences to learn about and reflect upon teaching with technology. However, many teachers do not have these opportunities due to financial constraints. This suggests that local districts must provide resources for teachers to attend professional development opportunities either online or at the local level. These opportunities should be course, curriculum, and technology specific to help mathematics teachers construct knowledge for teaching with technology in terms of curriculum, instructional strategies, students' understanding, and assessment.

At the end of my time with Mary, Joe, and Kate, they thanked me for the invitation to participate in this research study. The teachers appreciated the questions specific to instructional strategies, curriculum, students' understanding, and assessment with technology. They thought the questions specific to each of these areas facilitated their reflection and their planning for future mathematics lessons. This suggests reflection on teacher practice with technology using questions specific to each component in the model may facilitate the development of PCK with technology.

Implications for Curriculum Design

Teachers need access to both professional development resources and curricular materials that are directly related to specific technologies. Mathematics curriculum must include lessons that use specific technologies. If districts adopt curriculum materials that do not use specific technologies, then teachers will either not use technology or they will search for lessons from other sources as Joe and Mary did. In this case, then teachers have the challenge of combining different curricular resources so that they are connected and coherent.

Limitations

This study was limited by a small sample of teachers. Moreover, this sample was not representative of secondary mathematics teachers. The participants in this study independently chose to use the TI-nspire™ calculator during their mathematics instruction. Therefore, these participants represent a group of teachers who were willing to take on the role of novice as they learned the technology and began to teach with it. They also represent teachers who are risk-takers who are interested in trying new innovations in their classrooms. Furthermore, they were not afraid to use technology as some teachers appear to be. Finally, all three participants were not beginning teachers. They all had a minimum of three years teaching experience. Therefore, the knowledge they draw upon during the transition may look different than the knowledge of beginning teachers. As discussed above, more research is needed with a larger and more diverse sample of mathematics teachers to examine what knowledge teachers need to learn so they can draw upon it to teach mathematics with technology.

I base my conclusions on data from three teachers who teach three different mathematics courses. Thus, I provide descriptions of what occurred with Joe, Mary, and Kate under the conditions that existed in their classrooms at the time of data collection. I purposefully selected teachers in their first year of using the TI-nspire™ calculators to teach mathematics. As a result, the findings are specific to the transition period. Teachers require multiple years to successfully integrate a new technology into their teaching practices (Alejandre, 2005; Byrom & Bigham, 2001; Chazan, 1999; Dwyer et al., 1999; Mitchell, Bailey, & Monroe, 2007). Therefore, longitudinal studies are necessary to investigate how PCK specific to the TI-nspire™ develops over time. In sharing the results, I provide descriptions of the components of PCK with technology in order to more concretely think about this specialized knowledge from the early stages of development. Although these results provide us with new insights, one should be careful about the extent to which they generalize these findings without further research.

Significance

Effective uses of technology can enhance students' mathematical learning (Zbiek & Hollebrands, 2008). However, effective use requires a teacher who is knowledgeable about how to use technology as well as how to integrate it during mathematics instruction (Kaput, 1992; Laborde, 2001). Unfortunately, knowing how to use a technological tool or knowing how to implement an instructional strategy related to use of technology does not mean instruction will reflect that knowledge (Trouche, 2005). While a number of researchers in mathematics education have examined teachers' pedagogical content knowledge (e.g., Hill, Ball, & Schilling 2008; Marks, 1990), few researchers have investigated teachers' knowledge in relation to teaching K-12 mathematics with technology (e.g., Koehler & Mishra, 2008; Niess, 2005). Consequently, I examined the pedagogical content knowledge of three secondary mathematics teachers as they began using and teaching with a new technology (i.e., the TI-*n*spire™ calculator). The results from this study demonstrate what three secondary mathematics teachers know and what knowledge they draw upon while teaching with a new technology.

Research on teaching mathematics with technology tends to look only at the technology and not *how* it is used (Mishra & Koehler, 2006). Furthermore, we know little about the knowledge teachers need to teach effectively with technology. Case study methods allowed for an exploratory investigation of holistic and meaningful characteristics of real-life events (Yin, 2003). In this instance, the collection and analyses of initial interviews, video recorded observations, stimulated-recall interviews, and closing interviews allowed for innovative work and the examination of secondary mathematics teachers' pedagogical content knowledge as they began to use the TI-nspire™ calculator during mathematics instruction. Documentation of what knowledge teachers draw upon and use while making teaching decisions can inform the design and implementation of teacher preparation and professional development programs and ultimately improve the teaching and learning of mathematics.

The expansion of technology resources in U.S. schools has been considerable in recent years. Yet, providing resources without sufficient attention to the development of teacher knowledge to use those resources is a poor investment. Teachers and their knowledge play a central role in providing our students opportunities to learn and think mathematically with technological tools. It is time to build theory and enrich the research knowledge base about pedagogical content knowledge with technology. Our children and teachers deserve nothing less.

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APPENDIX A

EMAIL RECRUITMENT SCRIPT

I am conducting a dissertation research study entitled, **"A Study of Teacher Knowledge as Secondary Mathematics Teachers Use A New Technology."** My dissertation committee members at the University of Missouri have approved the proposal for this study and **____________** administrators have given me permission to contact you as a potential research participant.

In order for me to investigate the knowledge that secondary mathematics teachers enact while they use a new technology (i.e., the TI-nspire™ calculator) in mathematics instruction, I will videotape mathematics lessons and interview participants.

I invite you to take part in this research study. I believe that the research findings will help other teachers who use the TI-nspire™ or other new technologies. I also believe that you will benefit from participation in the study as you reflect on the teaching and learning of mathematics with the TI-nspire™ calculator. Ultimately, this reflection will provide you with new insights into how to engage and motivate your students to learn in new ways.

Your participation is essential for my learning. I think that by learning more about your knowledge while using a new technology, I can share the findings with teacher educators and professional development designers in order to improve support for secondary teachers using technology to teach mathematics.

[Send a copy of the Consent Forms.]

The attached consent form explains the details of the research study and outlines the commitment of the work if you were to agree to participate.

Please respond to this email (sjphxf@mizzou.edu) or phone Sarah Hicks (573-882-1495) if you have any questions about the study or if you are willing to participate in the study.

APPENDIX B

UNIVERSITY OF MISSOURI
TEACHER INFORMED CONSENT

A Study of Teacher Knowledge
as Secondary Mathematics Teachers Use A New Technology

The purpose of this research study is to investigate what you know and are thinking about as you use the TI-nspire™ calculator in your high school mathematics classroom. The research study will begin in March 2009 and conclude in May 2009.

INFORMATION

You must be at least 18 years of age to be eligible to participate in the study. Your participation in this study is voluntary; you may choose not to participate and there will be no penalty. If you decide to participate, you may withdraw from the study at any time without penalty.

PARTICIPATION

1. Participate in an initial interview (March 2009) and closing interview (May 2009) in which you will be asked questions about teaching and learning mathematics with the TI-nspire™ calculator. I anticipate that each interview will last approximately 1 hour.
2. Allow the researcher to observe and videotape 8-10 lessons in one of your mathematics classes during April/May 2009.
3. Participate in a post interview following 6-8 lessons in which you will be asked to watch the video and respond to questions about the lesson. I estimate that each interview will last approximately 1 hour. These post lesson interviews will begin during the second week of observations.
4. Allow the researcher to display clips at professional research conferences and other professional meetings. (Your image may appear in these clips.)

BENEFITS

Your participation in this research study will provide insight into the knowledge needed for teaching secondary mathematics when using a new technology. The research findings will support other secondary mathematics teachers who use the TI-nspire™ calculators in mathematics instruction. The information gained in this study may be useful to designers of teacher education programs and professional development programs in mathematics education. The information gained in this study may be published and may also be useful to mathematics teacher educators at other universities and colleges.

CONFIDENTIALITY

Your identity will be kept strictly confidential. The data collected during the study will be stored in a secure area in Townsend Hall. In reporting the findings of this study, your name will be replaced with a pseudonym. You may view the videotapes on the University of Missouri campus and request that certain video segments not be used. You may choose to end your participation at any time during the study, and your data will be destroyed. Data will be stored for three (3) years beyond the completion of the study and at that time it will be destroyed.

RISKS

This project does not involve any risks greater than those encountered in everyday life.

This project has been reviewed and approved by the University of Missouri Human Subject Review Board. The Board believes the research procedures adequately safeguard your privacy, welfare, civil liberties, and rights. For additional information regarding human subject participation in this research, please contact the University of Missouri IRB officer at (573) 882-9585.

CONSENT

Please read the consent statement below and place an "x" next to the statement that describes your desire to participate in this study at this time. Sign and date the form.

I have read the information presented above and have had an opportunity to ask questions and receive answers pertaining to this project.

______ I hereby agree to participate in this research study. I am aware that my participation is voluntary and that I am free to withdraw participation at any time without any penalties to myself. I agree to allow my classroom instruction to be videotaped as part of my participation in this study.

______ I do **not** agree to participate in this research study.

Signed: ____ Date: ________________

Printed Name: ____

Thank you. If you have questions at any time, please call Sarah Hicks at the University of Missouri at (573) 882-1495.

APPENDIX C

INITIAL INTERVIEW

Background Questions

a. What hand-held graphing technologies have you used in the last 10 years of teaching?
b. Talk to me about what prompted you to start using this new technology.
c. Why are you using the new technology in class?
d. How did you first learn about the technology?
e. How did you first learn about using it?
f. Why did you become interested in using this new technology?
g. Talk to me about your professional learning experiences with this new technology.
h. What professional development experiences, related to using this new technology, have you participated in?
i. What challenges have you encountered while learning about and using this new technology?
j. Tell me about the reasons why you use this technology in your math classes. [Background and Orientation]
k. What does it allow you to do that you cannot do without it?
l. What did you have to learn in order to teach with this new technology?

Pedagogical Content Knowledge Questions

Knowledge of Instructional Strategies

a. How do you and your students use technology during math lessons?
b. How does the use of technology impact the strategies that you use when teaching?
c. How has the use of technology impacted how you teach particular topics in the curriculum?
d. What is the same or different about the ways you have taught with the old and new technology?
e. What new features do you know about for this technology that you do not yet use? (Why have you chosen to not introduce these features?

Knowledge of Students

a. How does using technology impact the learning of students?
b. What mathematical difficulties do students encounter while using the new technology?
c. What mathematical misconceptions arise while working with the new technology?
d. How does the new technology help or hinder student mathematical reasoning?

e. In what ways does the new technology help students think about the mathematics differently than the old technology?

Knowledge of Curriculum

a. Talk to me about the tasks you have used to teach with the new technology.
b. How are the tasks you use with the new technology different, if in any way, than the tasks you used with the old technology?
c. Talk to me about how you have or have not used standards documents or curriculum frameworks to design how you teach with the old technology.
d. How does the use of technology change what you teach? How does the use of technology impact the mathematical topics that you emphasize? How has the use of technology changed how you view the importance of topics in the curriculum?
e. Talk to me about how you do or do not use standards documents or curriculum frameworks to design how you teach with the new technology.
f. In what ways did you use your mathematics textbook to teach with the old technology?
g. In what ways do you use your mathematics textbook to teach with the new technology?
h. Describe how your knowledge of mathematics curriculum has changed while using a new technology.
i. What specific new features do you use during instruction?

Knowledge of Assessment

a. Talk to me about how you assessed using the old technology during mathematics instruction.
b. How does the use of technology impact what you assess? How does it impact the assessment strategies that you use? What do you see as the strengths and weaknesses of these strategies? How does it impact how you use the information that you collect via these assessment strategies?
c. How do you assess using the new technology during mathematics instruction?
d. Describe the similarities and/or differences in your assessment practices between using the old and new technology.
e. What have you learned about assessment now that you are using a new technology?
f. What do you still want to learn about assessment with the new technology?

Orientation Questions

a. What mathematics have you learned by using a new technology during instruction?
b. What are the roles of the teacher when instructing mathematics with the old technology?
c. Describe the roles of the teacher when instructing mathematics with the new technology.
d. What is the role of the students when mathematics is being taught with the _______________ (old technology)?
e. What is the role of the students when mathematics is being taught with this new technology?
f. Why was it important for students to study mathematics with the old technology?
g. Why is it important for students to now study mathematics with the new technology?
h. How do you describe what mathematics is?

Closing Question

a. Is there anything else you would like to share about your experiences regarding teaching a new technology?

APPENDIX D

STIMULATED RECALL INTERVIEW

Reflect on your use of the TI-nspire™ in today's lesson. What specific incidents or questions stand out to you?

- Why does that stand out to you? **[Orientation]**
- What do you think the students were thinking? **[K of Learners]**
- Why do you think the student was having difficulty at that point? **[K of Learners]**
- How did you respond? Why did you respond in that way? What are other ways you could have responded in that situation**? [K of Instructional Strategies]**
- What knowledge about students did you use to respond to this situation? **[K of Learners]**
- What do you think students got out of that portion of the lesson? **[K of Assessment]**
- How do you know what students got out of that portion of the lesson? **[K of Assessment]**

What were your purposes and goals for teaching with the TI-nspire™ today? **[Orientation]**

How did you decide on these purposes and goals? **[Orientation]**

Why are these purposes and goals important to you? **[Orientation]**

What challenges did you face while using the TI-nspire™ in today's lesson?

How did your curriculum materials support or hinder you in implementing your plan with the TI-nspire™? **[K of Curriculum]**

I have selected some parts of the instruction I found particularly interesting. I want to watch with you and ask you some questions about them.

- Tell me about that (example/analogy/activity) with the TI-nspire™.
- What do you think the students were thinking? **[K of Learners]**
- Why do you think the student was having difficulty at that point? **[K of Learners]**
- How did you respond? Why did you respond in that way? What are other ways you could have responded in that situation? **[K of Instructional Strategies]**
- What knowledge about students did you use to respond to this situation? **[K of Learners]**
- What do you think students got out of that portion of the lesson? **[K of Assessment]**

- How do you know what students got out of that portion of the lesson? **[K of Assessment]**
- How did this teaching strategy with the TI-nspire™ help you achieve your overall goals?
- How could you teach this topic with the TI-nspire™ in a different way? **[K of Instructional Strategies]**
- What modifications could you have made while teaching with the TI-nspire™? **[K of Instructional Strategies]**
- What knowledge about students did you use to make instructional decisions? **[K of Learners]**
- Tell me about how you found out about student learning with the TI-nspire™. **[K of Assessment]**

Where did you find the activities you used to teach with the TI-nspire™ for today's lesson? **[K of Instructional Strategies; K of Curriculum]**

How did the activities with the TI-Nspire achieve the purpose you intended? **[Orientation]**

What mathematics content did you learn while teaching this lesson with the TI-nspire™? **[SMK and Orientation]**

Describe the instructional strategies you used when teaching **______________** with the TI-Nspire. **[K of Instructional Strategies]**

How did students learn **_____________** with the TI-nspire™? **[K of Learners]**

How do you assess students' learning of _____________ with the TI-nspire™? **[K of Assessment]**

APPENDIX E

CLOSING INTERVIEW

Think about your mathematics instruction using the TI-nspire™ during the past month. Talk about your successes and your challenges.

What did you learn from using the TI-nspire™? (probe for learning related to mathematics, students, assessment, instructional strategies, and teaching)

What mathematics did you learn by using the TI-nspire™?

What did you have to learn in order to teach the chapter/unit on _______ with the TI-nspire™?

What modifications would you make for teaching _____________ with the TI-nspire™next year?

Think about a case where the TI-nspire™ was beneficial for students' learning.

Think about a case where the TI-nspire™ hindered students' learning.

Why will you continue to use the TI-nspire™?

How are you thinking differently about teaching ___________ (the topic taught during the observed lessons) with the TI-nspire™? *This question may be repeated when more than one topic was taught within a chapter/unit in order to probe about the teacher's thoughts specific to teaching a topic with the TI-nspire™.*

Describe the instructional strategies you used when teaching _____________ with the TI-nspire™. In what ways, if any, are your instructional strategies different than when teaching without the TI-nspire™? *This question may be repeated when more than one topic was taught within a chapter/unit in order to probe about the teacher's thoughts specific to teaching a topic with the TI-nspire™.*

How did students learn ____________ with the TI-nspire™? In what ways, if any, is this learning different than learning mathematics without the TI-nspire™? *This question may be repeated when more than one topic was taught within a chapter/unit in order to probe about the teacher's thoughts specific to teaching a topic with the TI-nspire™.*

How do you assess students' learning of ____________ with the TI-nspire™? In what ways, if any, are assessment practices different than assessing students' learning without the TI-nspire™? *This question may be repeated when more than one topic was taught within a chapter/unit in order to probe about the teacher's thoughts specific to teaching a topic with the TI-nspire™.*

What is role of the teacher when instructing mathematics with the TI-nspire™?

What is the role of the students when mathematics is being taught with the TI-nspire™?

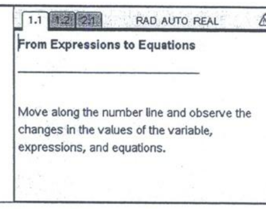
Why is it important for students to study high school mathematics with the TI-nspire™?

APPENDIX F

From Expressions to Equations

STUDENT ACTIVITY

Algebraic expressions and equations look quite similar. There are, however, important differences to keep in mind. This activity emphasizes the distinctions between the two.



- **Open the TI-Nspire document *From Expressions to Equations*.**
- **Press (ctrl) ▶ to move to page 1.2 and begin the lesson.**

1. Describe something that changes as you move the point to the right or left on the number line.

   ____

2. If the value of the expression is 20, what is the value of *x*? ____________________

- **Move to page 2.1 by pressing (ctrl) ▶**

3. What looks the same as the previous page? What looks different? ____________________

   ____

4. As you move the point, what changes? What stays the same? ____________________

   ____

5. Find a value of *x* to make the equation true. Describe the process you used. ____________________

   ____

6. Is that the only value of *x* that makes the equation true? Justify your answer. ____________________

____

7. The statement $3(x) + {}^{-}4 = 11$ on page 2.1 is called an equation. The left side of the equation $3(x) + {}^{-}4$ is called an expression.

   a. What is the difference between an expression and an equation? ____________________________

   ____

   b. Write an example of each. ____

   ____

8. What does it mean to solve an equation? ____

   ____

9. Why can't an expression be solved? ____

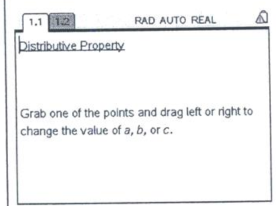
   ____

APPENDIX G

Distributive Property

STUDENT ACTIVITY

Distribution of multiplication over addition maintains equality of expressions. This activity allows you to explore that property of distribution.



- **Open the TI-Nspire document *Distributive Property*.**
- **Press ctrl ▶ to move to page 1.2 and begin the lesson.**

1. As you grab a point to move an arrow beneath the number line, what do you observe when you change the value of *a*? *b*? *c*?

   ____

   ____

2. Place *b* and *c* so that their sum is a positive number. For what value(s) of *a* are the expressions both equal to 0? Why?

   ____

   ____

3. Place *b* and *c* so that their sum is a negative number. For what value(s) of *a* are the expressions both equal to 0? Why?

   ____

   ____

4. Describe the first step used to evaluate each expression.

____

____

5. Compare the two expressions. How are they similar? How are they different?

____

____

6. Do you think these expressions will always have the same value? Why or why not?

____

____

7. The Distributive Property states $a(b + c)$ and $ab + ac$ are equivalent for all real numbers a, b, and c because they are equal for all possible values of the variables. Use this property to write an equivalent expression for each expression shown below.

a. $17(x + 2) =$ ____________

b. $15(c + d) =$ ____________

c. $ac + dc =$ ____________

d. $5a + 40 =$ ____________

TI-*nspire*™ **Getting Started with Geometry**

Angle Relationships

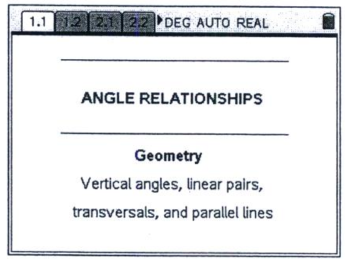
ID: 8670

Name ______________________

Class ______________________

In this activity, you will explore:

- *vertical angles and linear pairs*
- *relationships among angles formed by two parallel lines and a transversal*



Open the file *GeoAct14_AngleRelationships_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.

Problem 1 – When two lines intersect

On page 1.2, estimate and measure the angles formed by the intersecting lines. Grab and drag line *AE* to see what remains true about the angle measures.

- What is always true about vertical angles?

- What is always true about angles that form a linear pair?

Problem 2 – When a line intersects two other lines

Look at the diagram on page 2.1.

- What appears to be true about lines *AF* and *BG* when $m\angle CDF = m\angle DEG$?

- List pairs of angles that are congruent but not vertical.

- List pairs of angles that are supplementary but do not form a linear pair.

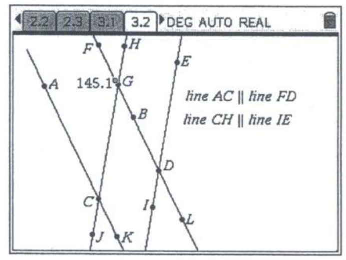
Advance to page 2.3. Hide or show certain angle measures as directed by your teacher. Then complete the following:

When two parallel lines are intersected by a transversal…

- …the measures of corresponding angles are ____________________.
- …the measures of alternate-interior angles are ____________________.
- …the measures of alternate-exterior angles are ____________________.
- …the measures of same-side interior angles are ____________________.

Problem 3 – Putting it all together

In the diagram to the right, $m\angle FGC = 145.1°$.

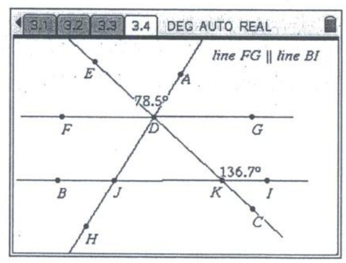


Use the diagram to find the following measures. Be prepared to justify your reasoning.

- $m\angle ACG$ = ___________
- $m\angle EDL$ = ___________
- $m\angle LDI$ = ___________
- $m\angle JCK$ = ___________

When directed by your teacher, check your answers by advancing to page 3.2 and using the **Measurement > Angle** tool.

In the diagram to the right, $m\angle EDA = 78.5°$ and $m\angle DKI = 136.7°$.



Use the diagram to find the following measures. Be prepared to justify your reasoning.

- $m\angle ADG$ = ___________
- $m\angle DKJ$ = ___________
- $m\angle EDF$ = ___________
- $m\angle DJB$ = ___________
- $m\angle HJK$ = ___________

When directed by your teacher, check your answers by advancing to page 3.4 and using the **Measurement > Angle** tool.

VITA

Sarah J. Hicks is currently an Assistant Professor of mathematics education in the School of Graduate and Professional Studies at Rockhurst University in Kansas City, MO. She earned a B.S. degree in Mathematics and Secondary Education from Rockhurst University in Kansas City, Missouri (2003), a M.Ed. degree in Curriculum and Instruction from the University of Missouri in Columbia, Missouri (2006), and a Ph.D. in Curriculum and Instruction with an emphasis in Mathematics Education from the University of Missouri in Columbia, MO (2010). Sarah's research interests include teacher knowledge, teacher preparation, and the effective use of technology during mathematics instruction.

Sarah taught high school mathematics for two years in Denver, Colorado prior to graduate studies. During doctoral studies, she instructed undergraduate mathematics and mathematics education courses, facilitated professional development with mathematics teachers, and conducted research in schools. In addition, she spent a year implementing a specially designed mathematics curriculum with a gifted elementary school student. As a graduate research assistant, she was actively involved with two longitudinal studies funded by the National Science Foundation.