MOTIVATING MATHEMATICIANS: A THEORETICAL FRAMEWORK OF PROFESSIONAL DEVELOPMENT TO SUPPORT EFFICACIOUS BEHAVIORS FOR TEACHERS OF ELEMENTARY MATHEMATICS

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MOTIVATING MATHEMATICIANS: A THEORETICAL FRAMEWORK OF PROFESSIONAL DEVELOPMENT TO SUPPORT EFFICACIOUS BEHAVIORS FOR TEACHERS OF ELEMENTARY MATHEMATICS

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ABSTRACT

The K-12 mathematics experiences of today’s elementary teachers often were one-dimensional, or traditional, in nature. Many elementary teachers recall negative, isolating mathematics experiences from their earliest memories of math, and these experiences have shaped their sense of efficacious behaviors for mathematics and teaching mathematics. This study utilized constructivist grounded theory to generate theory for how the sense of efficacy for elementary teachers of mathematics is changed by professional development. The inquiry extends current research on educational leadership for elementary mathematics education by examining multiple data, including sources of quantitative and qualitative data for analyses: a mixed survey that utilized Likert scale items and open-ended questions, interviews, observations, reflective writing documents, and focus group interviews for generating theory. Nine teacher participants shared their experiences: both from their K-12 education, as well as during the mathematics professional development, and how these have shaped their motivation, efficacious behaviors, and pedagogical practices.

The theory of math efficacy and motivation for teachers of elementary mathematics as well as the math professional development for efficacious behaviors theoretical framework
were developed because of this study. Implications of the findings for efficacious behaviors of elementary mathematics teachers include recommendations to utilize categories from the math professional development for efficacious behaviors theoretical framework for educational leaders when constructing future mathematics professional development for practicing elementary teachers.
The undersigned, appointed by the Dean of the School of Education, have examined a dissertation titled “Motivating Mathematicians: A Theoretical Framework of Professional Development for Teachers of Elementary Mathematics,” presented by Margaret Helwig, candidate for the Doctor of Education degree, and certify that in their opinion it is worthy of acceptance.

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CHAPTER 1

INTRODUCTION

I have not always been a math girl. As a kid, I was pretty good at school. My mom, well-intentioned as she was, would frequently tell me I was smart. I believed her, and throughout elementary school, performed as such. Sometime around middle school, things started to change when it came to math—the meaning began to break down for me in class. If school was always something that had come easy because I was “smart,” what did it mean when it no longer came easy? When I asked my parents for help with my math homework, my mom would always refer me to my dad, and she comforted me that it was okay; I was good at reading and writing—not all people are math people.

Rattan et al. (2011) found that students responding to comfort-oriented feedback not only perceived the instructors’ entity theory (fixed mindset) and low expectations, but also reported lowered motivation and lower expectations for their own performance. “There’s a widespread myth that some people are math people and some people are not. But it turns out there’s no such thing as a math brain,” Boaler (2016) explained. Recent brain research is now showing a strong connection between the attitudes and beliefs students hold about themselves and their academic performance (Boaler, 2016). Preckel et al. (2008) found that girls scored lower on measures of academic self-concept, interest, and motivation in mathematics. Boys earned significantly higher test scores, but there were not gender differences in grades. Additionally, Gunderson et al. (2013) found that the type of parent praise given as early as ages one through three years old predicts children’s motivational frameworks and mindsets, whether fixed or growth in nature, five years later.
The mathematical mindset that I had accepted for myself continued throughout middle and high school. The only math teacher I remember connecting with from sixth to twelfth grade was my Algebra II teacher, who encouraged me to come before and after school for extra help and taught me songs to remember abstract concepts like the quadratic formula. She, one of the few female math teachers I had during this time, was a ray of light in my dark math tunnel.

Still, at this point, several years of misconceptions and surface level understandings had caught up with me. While I was able to collaborate with my peers and pass these high school courses, college was another story. I took a remedial math class upon arriving at Mizzou my freshman year, and I again sought help at tutoring sessions and from friends. My geometry class was so abstract that I found myself re-taking it during summer school with a group of sorority girls who, like me, did not connect with our male teacher. Our summer school teacher was another ray of light. She brought snacks to small study sessions for us where we could ask questions until it made sense to us, and in the end, invited us to her apartment to cook us Lebanese food, sharing her culture with us. Then there was Statistics, in a lecture hall of 400 with a teacher scribbling on a dry-erase board I could not see. I played Snakes on my phone after getting lost and giving up and ended up re-taking that in summer school at a neighboring community college. My action research statistics class in my master’s degree program was another example of nonsensical scribbling on a dry erase board, and again, I received a lot of help from my classmates that got me through. “It’s okay…I’m not a math person.”

When I became a teacher, the way math was introduced to students was different than the math I had encountered in school. I appreciated the focus Math Investigations placed on
finding multiple strategies, and suddenly, the concepts I was teaching students made more sense to me than the algorithms I had been taught to memorize as an elementary student. It is likely, though, I still comforted some students who struggled by saying, “It’s okay. Not everyone is a math person. It’s hard for me, too, I get it. You’re really good at XYZ.”

In my years as an elementary teacher, much of the professional development (PD) I received and helped to plan and facilitate was focused on literacy. After spending several years teaching various elementary grades, I landed a job as an instructional coach. My first year as a coach was also my first year participating in professional development with a national mathematics consulting firm with whom our district had partnered and implementing Number Talks. I learned every bit as much from the visits I coordinated with our consultant as the teachers I was coaching. As I worked to help teachers and learned more about elementary math instruction, I started to ask questions: What is our aim when coaching teachers how to teach math? If content knowledge is essential to effective facilitation in a constructivist/inquiry-based classroom, where do the instructional tools we have available for reading and writing exist for elementary math teachers? I was sent with a group of coaches, teachers, and district administrators to the National Conference for Teachers of Mathematics (NCTM) that spring. I went on a mission: to find the same kind of tools we have for reading and writing, but for math. I had figured out many elementary teachers were just like me: math was not their favorite or best subject as a student growing up, and now they either did not have a great depth of conceptual understanding of math to help their students or had a limited level of self-efficacy about their ability to teach mathematics. I wanted to bring them tools that would help them. I did not find the tools I was looking for, but I did find Dr. Deborah Loewenberg Ball and Dr. Jo Boaler, who were both speaking at the conference.
Ball, who serves as the director of TeachingWorks and a research professor in the Institute for Social Research at the University of Michigan and has written over 150 publications in the area of elementary mathematics, was the Lucy Calkins of mathematics to me. (Lucy Calkins, the Founding Director at the Teachers College Reading and Writing Project at Columbia University, had been a huge influence on my pedagogy as a teacher of reading and writing workshops. All teachers I have coached know full well that “I love Lucy.”) I signed up for an online class with Dr. Jo Boaler that summer. Boaler is a Professor of Education at Stanford University, was formerly the Marie Curie Professor of Mathematics Education in England, and won the national award for educational research in the UK with her PhD. Her research on mathematical mindsets helped me learn that there aren’t, in fact, “math people.” Anyone can be a “math person.” That same summer, our district adopted a new program at the elementary level: Everyday Mathematics. I went to every training because again, I needed to help teachers teach a math program I had never taught myself. As we dug into this resource, I realized it, too, was not perfect and did not contain the magic instructional tools I had hoped it promised. We still needed to build capacity so we could work smarter than the resource.

I have not always been a math girl. This dissertation is the result of my own ongoing process—and conscious choice—to become one. As an instructional coach, and now as a building principal, I have been transparent about this journey (and sometimes, struggle) to address my own old, fixed math mindset in hopes that it invites others to join me in addressing theirs. It is my hope this research has addressed these stereotypes by developing a theoretical framework of recommendations for professional development for elementary
math teachers that will address the mindsets and efficacy teachers hold so our future students will have a different experience than I did.

**Problem Statement**

Successfully evaluated math programs include a focus on how teachers use instructional process strategies, such as: using time effectively, keeping children productively engaged, giving children opportunities and incentives to help each other learn, and motivating students to be interested in learning mathematics (Slavin & Lake, 2008). These process strategies, while found successful from a programming standpoint, can be applied to any content area and are not specific to teaching mathematics, nor the specific knowledge about mathematics teachers need to be successful. Teachers’ mathematical knowledge is significantly related to student achievement gains after controlling for key student and teacher-level covariates (Hill et al., 2005). Many elementary teachers have a lack of conceptual understanding about mathematics and therefore are often unable to design meaningful mathematics learning experiences. The field as a whole still “lacks a theoretically grounded, well-defined, and shared conception of mathematical knowledge required for teaching” (Hoover et al., 2016, p. 1). However, there seems to be some agreement that there is a need for teachers to have mathematical knowledge that is teaching-specific (Hill et al., 2008; Hoover et al., 2016; Shulman, 1986). Teachers’ self-confidence as mathematics teachers has also been found significantly associated with their students’ self-confidence as mathematical learners (Stipek et al., 2001). Research has shown that many college students who are anxious about mathematics choose to become K-6 teachers to avoid further mathematics requirements at colleges and universities (Boyd et al., 2014; Jackson, 2015). In a quantitative research study of 70 pre-service primary school teachers utilizing independent
samples t-test and ANCOVA to analyze results from a survey instrument, Peker (2009) found statistically significant differences between participants who were exposed to traditional instruction and those exposed to an approach focused on problem-solving strategies. The participants who were exposed to only traditional instruction had higher teaching anxiety levels in the area of mathematics than those exposed to instruction using problem-solving strategies. Mathematics anxiety was attributed to the strict and structured instructions and problem solving. Using different problem-solving strategies decreased the teaching anxiety level of pre-service teaching, and the pre-service teachers were better able to understand teaching and learning of mathematics (which included use of concrete manipulatives and small group collaboration).

When teachers have not gained the mathematical content knowledge that is necessary to teach effectively, they do not gain the self-confidence that is also necessary to teach it effectively, thus continuing the cycle of mathematics anxiety (Gresham, 2018). While a wealth of research has been conducted regarding training of pre-service elementary mathematics teachers, there is not as much empirical research available about professional development in the area of elementary mathematics that addresses mathematics anxiety and efficacies of in-service, or currently practicing, teachers.

Often, new teachers have the highest level of mathematics anxiety (Kim, 2014). In a mixed methods study, Gresham (2018) utilized a Likert-type survey to revisit in-service teachers who had previously participated in a research study during pre-service teaching. Gresham (2018) found that while experience can help lessen mathematics anxiety for teachers somewhat, for those teachers who start out with high math anxiety, this is often still present after five years in the field. In a quantitative research study using statistical analyses
(ANOVA and multiple regression model) of results from an adapted version of the McAnallen Anxiety in Mathematics Teaching survey of 400 randomly selected teachers, Kim (2014) found that South Korean elementary teachers’ anxiety for teaching decreased after their first five years (during which time they participated in 90 hours of professional development, with “at least 4 hours of lectures on mathematics education” [p. 73]), but increased again after 16 years in the profession. Kim (2014) recommended that further studies are warranted to explore the effectiveness of professional development programs in elementary mathematics education in reducing elementary teachers’ anxiety for teaching mathematics and ultimately students’ academic achievement in mathematics. Gresham (2018) concluded that “more investigation of the need for continued mathematics professional development opportunities that specifically address mathematics anxiety…is needed” (p. 105). Currently, there is a gap in existing literature around what professional development practices teachers perceive as valuable specific to the area of mathematics for practicing, in-service elementary teachers (Kim, 2014; Martin et al., 2018; Polly et al., 2014; Wei et al., 2009; Yoon et al., 2007).

Most teachers agree that the role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics (National Council of Teachers of Mathematics, 2014). However, this principle is often easier to believe than to put into action. There are several possible explanations for this phenomenon.

One possible explanation is that the mathematical understandings that prospective teachers bring are inadequate for teaching mathematics for understanding (Ball, 1990; Hoover et al. 2016; Ma, 1999). Many undergraduate teacher preparation programs do not
adequately provide teachers with the content knowledge to facilitate mathematics classrooms that are multidimensional (Boaler, 2016) and encourage discourse when problem solving (Boerst et al., 2011; Chapin et al., 2013; Stein et al., 2008). Once in the workforce, professional development opportunities for teachers are often few and far between. In the 1996 NAEP mathematics assessment, teachers were asked how many hours of professional development they had received in the previous 12 months. Nationally, only 28% of the fourth graders in the sample had teachers who had received 16 or more hours of professional development in mathematics (Kilpatrick et al., 2001).

Another possible explanation for the lack of alignment between teachers’ beliefs about the nature of mathematics and instructional behaviors in regard to mathematics instruction are teachers’ own past experiences with mathematics. In America, it is widely accepted that people either are “math people” (Rattan et al., 2011, p. 733) or are not. Jo Boaler (2016) explained in the introduction of her book, Mathematical Mindsets: “Students have such strong and often negative ideas about math that they can develop a growth mindset about everything else in their life but still believe that you can either achieve highly in math or you can’t” (p. ix). When teachers have had negative past experiences with mathematics, it is likely they may carry these experiences into the classroom. Sun (2015) found that if teachers have closed (Boaler, 1998) or one-dimensional (Boaler, 2006) views of math and instruct in one-dimensional ways, they are also more likely to communicate fixed messages about math ability to students. In contrast, teachers who hold more multidimensional views of math and instruct in ways that align with these views are more likely to communicate growth mindset messages. Educators cannot separate having a growth mindset from a
multidimensional (Boaler, 2006; Cohen et al., 1999) perspective about the nature of mathematics (Sun, 2015, p. 182).

A third potential explanation for the lack of alignment between teachers’ beliefs about the nature of mathematics and instructional behaviors in regard to mathematics instruction can be attributed to contextual elements (Sun, 2015), or a teacher’s compliance with building and/or district initiatives, which may not align to their own views or beliefs about the nature of mathematics.

The lack of alignment between teachers’ beliefs about the nature of mathematics and instructional behaviors in regard to mathematics instruction has led to surface level conceptual understanding of mathematics on the part of many teachers; and thus, shallow mathematics experiences for the students in many American elementary school classrooms. Classroom research studies have shown teachers often overemphasize computational practice rather than students’ exploration of mathematical concepts (Stein et al., 2007). In a review of the research on mathematics curriculum use over the last 25 years, Remillard (2005) found that teachers’ curricula enactment was linked to teacher characteristics, such as mathematics content knowledge, beliefs about mathematics teaching and learning, as well as contextual factors, such as support from their administrators and colleagues. As such, Remillard (2005) cautioned that adopters of curriculum or resource programs must carefully consider how they frame and support the teacher-curriculum relationship. High-quality, effective professional development for teachers in the area of elementary mathematics is needed, but what components do such professional development designs entail?

Little research has been done on the effect of determining teacher beliefs and background knowledge when planning for professional development opportunities.
Considering Vygotsky’s (1978) Zone of Proximal Development and its potential implications for adult/teacher learning, one might assume that teachers enter the teaching profession with varying degrees of experience with mathematics, particularly in regard to their previous history in how it was approached in their own learning as a child. Limited research has been conducted to inquire into how teachers’ varying backgrounds with mathematics have shaped their conceptual understanding, beliefs about the nature of mathematics, self-efficacy, and motivation regarding the professional development opportunity itself. Hattie (2008) identified that the most effective teachers use formative assessment to continuously inform themselves of where students fall within a given learning progression of a concept and give specific feedback accordingly. Similarly, should not teachers’ pedagogical knowledge, beliefs, and past experiences be considered when planning professional development? Specific to the area of mathematics, where fixed mindset beliefs (Boaler, 2016) are so prevalent, one might argue it is of the utmost importance to take teachers’ prior experiences into account if attempting to move their instructional behaviors past compliance into commitment; or even more, empowering and motivating teachers from principles to actions (NCTM, 2014) by facilitating opportunities that help them grow new, more productive beliefs. A need exists to determine which aspects of elementary mathematics professional development are most powerful for teachers (Boston & Smith, 2011; Elliott et al., 2009; Polly et al., 2014).

**Purpose Statement**

The purpose of this constructivist grounded theory study was to develop a theoretical framework of nine teacher participants’ sense of mathematics efficacy including K-12 schooling and throughout professional development. The primary objective of grounded
theory as a research methods framework is “to develop a theory of this process or action” (Creswell, 2013, p. 85). My pragmatic goal was to develop a framework of strategies that can be used in professional development to support the efficacious behaviors and beliefs about the nature of mathematics of elementary teachers. Constructivist grounded theory was an appropriate theoretical tradition for this research project, as it connects theorizing and research practice, and moreover, this method joins critical analysis with people’s lives (Charmaz, 2017). Multiple realities were presumed, as different teachers experience professional development opportunities differently (Grbich, 2013). The research focus explored ways teacher participants interact with the professional developer as well as “the way these interactions modify participants’ constructed understandings” (Grbich, 2013, p. 7) and their interactions with students. This study sought to develop a theory of nine teacher participants’ sense of efficacy for a mathematics professional development. Participants were defined as nine elementary teachers who teach kindergarten through fifth grade mathematics as well as three facilitators of elementary mathematics professional development. The units of analysis in this study were teacher participants’ theories related to their self-efficacy in mathematics. Professional development practices were defined through categories identified from participant mixed survey results, documents, interviews, observations, and focus groups. Self-efficacy was defined as individuals’ confidence in their ability to organize and execute a given course of action to solve a problem or accomplish a task; characterized as a multidimensional construct that varies in strength, generality, and level (or difficulty) (Bandura, 1997).

I set out to develop a theoretical framework of nine teacher participants’ sense of mathematics efficacy including K-12 schooling and throughout professional development.
These strategies or practices could then be replicated by educational leaders. If professional development facilitators utilize the practices found to be helpful to teachers in this research study, they may further impact the beliefs and efficacious behaviors to motivate teachers who have had negative experiences with mathematics in the past to modify their instructional practices for the benefit of students. Elementary teachers would then be better able to help students learn math concepts adopting a growth mindset, as opposed to passing on math anxiety and fixed mindset beliefs about the nature of mathematics to students, thereby breaking the negative cycle and improving the mathematics educational experiences for students.

Slavin and Lake (2008) conducted a best-evidence synthesis of 87 mathematics programs across the United States and found that “there is limited high-quality evidence supporting differential effects of different math curricula” (p. 445). One surprising observation was the lack of evidence that it matters much what textbook schools choose. Studies supported instructional programs that focused on helping teachers introduce mathematics concepts effectively. This review suggested that in terms of outcomes on standardized tests and state accountability assessments, “curriculum differences appear to be less important than instructional differences are” (p. 482). The research on instructional process strategies noted that the key to improving math achievement outcomes is changing the way teachers and students interact in the classroom.

While this best-evidence synthesis suggested that changing the way teachers and students interact in the classroom is of the utmost importance in the area of mathematics, little research has been conducted that has identified just what, specifically, it is that educational leaders might do to address the existing mindsets, self-efficacy, beliefs, and
corresponding behaviors of teachers of elementary mathematics. This research study will generate theory identifying what specific professional development strategies lead to change in teachers’ beliefs about the nature of mathematics, self-efficacy, and corresponding behaviors/pedagogy as perceived by the participants.

**Research Questions**

**RQ1:** What is the theory that explains the process for enhancing the efficacies of teachers in a large suburban school district?

- Which parts of the mathematics professional development process do participants attribute to their theories regarding efficacy in math?
- What theories do they generate about the nature of mathematics because of the professional development process?

**RQ2:** What professional development theories contribute to shifting mathematics pedagogical practices?

- What motivates teachers to engage in professional development?
- In what ways do their pedagogical practices change?

**RQ3:** What do findings suggest about leadership theories to support the facilitation of mathematics professional development?

- How do leaders facilitate safe environments for enhancing the efficacies of teachers?
- What support do participants need for trying out theory-based pedagogical practices in classrooms?
Theoretical Framework

Miles and Huberman (1994) defined a conceptual framework as a visual or written product, one that “explains, either graphically or in narrative form, the main things to be studied—the key factors, concepts, or variables—and the presumed relationships among them” (p. 18). Maxwell (2013) used the term in a broader sense to refer to the actual ideas and beliefs that (the researcher) holds about the phenomena studies, whether written down or not; this may also be called the “theoretical framework” or “idea context” of the study (p. 39). Maxwell (2013) described four recommended sources for constructing the theoretical framework: (1) the researchers’ own experiential knowledge, (2) existing theory and research, (3) the researchers’ pilot and exploratory research, and (4) thought experiments of the study. This framework provides the foundation on which the research study is built, and a researcher must carefully consider what it will entail; as Heinrich (1984) concluded, “Even carefully collected results can be misleading if the underlying context of assumptions is wrong” (p. 151).

Many teachers have a lack of conceptual understanding about mathematics and therefore are often unable to design meaningful mathematics learning experiences. Currently, there is a gap in existing literature around professional development practices specific to the area of mathematics for practicing elementary teachers. The purpose of this constructivist grounded theory study was to develop a theoretical framework of professional development for efficacious behaviors for elementary teachers.

I brought several assumptions to this study. First, that there is a general lack of depth of conceptual understanding of mathematics in America (Ma, 1999), knowledge for teaching (Hoover et al., 2016) and the pedagogical understandings necessary to effectively facilitate
quality mathematics instructional experiences for our youngest learners. I experienced this phenomenon as a classroom teacher and continued to experience it as an instructional coach and school administrator when working closely with elementary teachers. Secondly, I believe that more research needs to be conducted to determine what professional development practices can be utilized to address the common stereotypes and negative mindsets that accompany mathematics mindsets teachers carry with them into the classroom. Third, I believe that these professional development practices likely need to integrate constructivist/constructionist qualities in order to help teachers, both individually and collectively, make meaning of the new mathematics experiences being provided to them via effective professional development. Fourth, I propose that the combination of these factors would further motivate teachers undergoing such professional development to put newly learned conceptual understandings into practice to enhance the mathematics instructional tasks facilitated in their classrooms, which in turn might motivate our youngest mathematicians and change the negative mindset, anxiety, and stereotypes that often accompany math.

My experiences as an elementary teacher, instructional coach, and school administrator in multiple suburban elementary schools have given me a unique insight into the current state of mathematics education at the elementary level. These experiences have led me to the selection of the following topics to form a theoretical framework that underpin and inform the methodology in this research study: (1) Teachers’ conceptual understanding of mathematics, (2) Constructionism/constructivism, (3) Motivation, and (4) Leadership for Professional Development (PD). Each of these topics has been chosen in part due to my personal experiences, and in part from reviewing the current literature regarding elementary
mathematics instruction and professional development. I chose these topics because I hypothesized that teachers’ conceptual understanding of mathematics could be strengthened through professional development experiences that are constructivist in nature, and that this process could in turn build their self-efficacy and change their beliefs about the nature of mathematics.

**Teachers’ Conceptual Understanding of Mathematics**

In many cases across the United States, the mathematical understandings that prospective teachers bring are inadequate for teaching mathematics for understanding (Ball, 1988). Many undergraduate teacher preparation programs do not adequately provide teachers with the content knowledge to facilitate mathematics classrooms that are multidimensional (Boaler, 2016) and encourage discourse (Chapin et al., 2013) when problem solving. Many teacher preparation programs require much less coursework in the area of mathematics than the requirements in English courses for students pursuing an elementary education degree. For example, the University of Missouri-Columbia’s general education requirements for transfer students include only one course (three hours) in mathematics and three courses (nine hours) in written and oral communications. The general education requirements require only one course in math and quantitative reasoning, while requiring two courses in English/writing, with the option to satisfy the math requirement by demonstrating proficiency in college algebra or providing minimum ACT or SAT Math sub scores to be exempt from the math requirement altogether (University of Missouri-Columbia, 2020). Once students are accepted into programs specifically focusing on teaching elementary education, many universities still require disproportionate amounts of courses in mathematics as compared to English. Only four percent of elementary mathematics teachers
have college degrees in mathematics or mathematics education, and while 55% have completed coursework in college algebra/trigonometry/functions, less than half of K-5 teachers have completed coursework in statistics, probability, college geometry, or calculus (Malzahn, 2013). As a result, college students can successfully pursue a career as an elementary teacher even if they want to avoid math (Beilock et al., 2010). Hill et al. (2008) found in their mixed methods study that there is a powerful relationship between what a teacher knows, how they know it, and what teachers can do in the context of instruction. They found that there were significant, strong, and positive associations between levels of Mathematical Knowledge for Teaching (MKT) and Mathematical Quality of Instruction (MQI) and theorized that it is likely to be the case that classrooms with high MKT lead to increased MQI. The connection between the formal education of mathematics teachers and the content understanding important to them effectively teaching students is not straightforward (Hoover et al., 2016). Hoover et al. (2016) argued that “the specialized mathematics that teachers need to learn appears to be constituted in ways that span blocks of the student curriculum” (p. 19), and assert that a special kind of mathematical fluency is required for mathematics teaching, one that:

requires the teacher to ask questions in the moment, explain in response to a students’ puzzlement, listen to, interpret, and respond to a child’s explanation—each of these involves hearing and making sense of others’ mathematical ideas in the moment, speaking on one’s feet while seeking to connect with others. (p. 25)

While researchers continue to determine and further understand what specific mathematical knowledge for teaching is necessary and will predict or ensure increased mathematical quality of instruction, the literature paints a bleak picture of the inconsistent state of mathematical knowledge for teaching that exists across our country (Hoover et al., 2016;
Slavin & Lake, 2008). Particularly among elementary education majors, of whom the vast majority are female, research has found that not only is there often the highest levels of math anxiety, but also that there is a statistically significant relationship between the levels of a teacher’s math anxiety and girls’ math achievement (Beilock et al., 2010). There is a need to provide opportunities for in-service elementary teachers to develop knowledge about content and students’ mathematical thinking, as well as skills related to enacting standards-based pedagogies (Sykes & Darling-Hammond, 2009).

**Constructionism and Constructivism**

Patton (2015) highlighted Michael Crotty’s (1998) work that makes a distinction between these two concepts: “It would appear useful, then, to reserve the term constructivism for the epistemological considerations focusing exclusively on ‘the meaning-making activity of the individual mind’ and to use constructionism where the focus includes ‘the collective generation (and transmission) of meaning’” (p. 122). Both of these concepts apply to the topics of this research study. When a pre-service teacher begins taking mathematics courses with the intent to teach the content to children, they, themselves, come to this experience with previous experiences with mathematics, including the social and individual learning experiences of their own education up to that point. Once hired as working professionals and participating in a mathematics professional development experience, these teachers must continue to make meaning within their individual mind of the experience as they live it. Additionally, the teachers, as part of a grade level team or school staff, will make meaning collectively through the shared experience and transmission of their own individual meanings. This is how shared meaning is built, upon which culture is built for learning organizations. Lambert (2002) defined constructivist leadership as “the reciprocal processes
that enable participants in an educational community to construct meanings that lead toward a shared purpose of schooling” (p. 36). According to Piaget (1952, 1978), learners move through stages of cognitive development—from concrete to abstract—prompted by a discrepancy or “disequilibrium” between what they previously believed to be true and their new insights and experiences. Vygotsky (1978) asserted that when children learn, functions appear twice: first, on a social level, then later on an individual level. “That children’s learning begins long before they attend school is the starting point of this discussion. Any learning a child encounters in school always has a previous history” (p. 84). Out of this, he developed the concept of the Zone of Proximal Development, defined as: “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86). Bruner and Haste (1987) built on previous theory, determining that “Making sense’ is a social process; it is an activity that is always situated within a cultural and historical context” (p. i). Fast and Hankes (2010) conducted a quantitative, experimental pre/post design research study with 63 college mathematics education students and found that intentional integration of mathematics content instruction with constructivist pedagogy in instructional design as used in a college elementary mathematics education course showed significant improvement in mathematics content knowledge and confidence in that knowledge with a better understanding of constructivist pedagogy.

As previously discussed, in America the perspective is widely accepted that people either are “math people” (Rattan, Good, & Dweck, 2011) or are not. When teachers have had negative past experiences with mathematics, it is likely they may carry these experiences into
the classroom. Boaler (2016) referred to these experiences as “mathematics trauma” (p. x). Specific to the area of mathematics, where fixed mindset beliefs (Boaler, 2016) are so prevalent, one might argue it is of the utmost importance to take teachers’ prior experiences into account if attempting to move behaviors past compliance into commitment; or even more, empowering and motivating teachers from principles to actions (NCTM, 2014) by facilitating opportunities that help them grow new beliefs.

**Motivating Mathematicians**

Rattan et al. (2011) conducted a quantitative research study and found that implicit theories of math intelligence play a causal role in the early diagnosis of ability and pedagogical practices that follow. Undergraduate students completed online surveys to determine their implicit theories of math intelligence and beliefs about mathematics. An implicit theory of math intelligence scale was embedded in the survey, where participants were asked to agree or disagree with four statements that portrayed math ability as fixed. They also included four items to assess the participants’ sense of belonging to math and twelve items asking about their belief in the usefulness of math. Each measure used six-point scales ranging from strongly disagree (1) to strongly agree (6). Participants were then asked to imagine themselves as seventh grade math teachers and how they would respond to instructional situations. Through analyses where the variable of interest was regressed on mean-centered implicit theories of math intelligence scores, it was found that the more participants endorsed an entity theory (fixed mindset) about the nature of mathematics, the more they agreed that students’ poor performances were attributed to “lack of math intelligence” rather than “lack of hard work” (p. 732). In the second portion of the study, researchers had participants read an article that presented evidence indicating math
intelligence was either fixed or malleable, and after reading, were again asked how they might respond to particular instructional situations. Again, participants in the entity theory condition endorsed a significantly more fixed belief about math intelligence than those in the incremental theory condition. Participants in the entity theory condition were significantly more likely to endorse the overall index of comfort-oriented strategies and strategies that could reduce engagement and future achievement in math. The same findings were found when researchers extended the study to graduate students in math-related areas who were instructors or teaching assistants in undergraduate courses in their field of study.

Studies have researched motivation in regard to the area of mathematics by focusing on college students, teachers (Stipek et al., 2001) as well as children (Gilbert et al., 2014; Preckel et al., 2008; Skaalvik & Valas, 1999); however, there is a gap in the literature about what specific practices have motivated teachers to implement new mathematical knowledge for teaching gained from professional development programs (Polly et al., 2014). Pink (2009) introduced the motivation trifecta: autonomy, mastery, and purpose. Assuming that teachers are already part of a culture that creates a shared vision for the shared purpose of the organization, autonomy and mastery are two components of motivation that deserve consideration in planning professional development opportunities for teachers. Providing teachers choice (autonomy) in how they put mathematical knowledge for teaching into practice, as well as in regard to in which professional development they participate or how they participate, may contribute to effectiveness in shifting teachers’ beliefs and behaviors as a result of such professional development. Similarly, professional development that provides support for teachers working towards mastery of effective facilitation of mathematics learning experiences for students might also be found more motivating for teacher
participants. In creating professional development experiences that motivate teachers to change their beliefs and behaviors, one might assert that those teachers might additionally influence the motivation and mindsets of the student mathematicians they encounter each day.

**Leadership for Professional Development**

Because many elementary teachers enter the profession with limited mathematical knowledge for teaching, there is a need for educational leaders to provide support to practicing teachers through ongoing professional development (PD). However, at the elementary level, professional development intended to improve literacy often takes priority over mathematics. For example, among in-service teachers in South Korea who participated in a 90-hour professional development program within their first five years of teaching were only required to take four of their 90 hours in math (Kim, 2014).

In their phenomenographic research study of mathematics and science professional development projects, Rogers et al. (2007) found through coding of transcribed interviews of 72 teachers and 24 facilitators of mathematics professional development that teachers’ and professional development facilitators’ views of effective mathematics professional development included classroom application, teacher as learner, and teacher networking. Criteria of effective PD also included (1) challenging teachers’ content and pedagogical content knowledge with transformative learning experiences, (2) encouraging teacher leadership for sustained support, and (3) focusing on student learning by instructing teachers on how to use student data to inform their teaching practice (Rogers et al., 2007). Teacher participants ranged from elementary, middle, and high school levels, and most participated in science professional development (48, as opposed to 24 who participated in math
professional development). In their multi-methods approach, Polly et al. (2014) studied teacher participants in a task-focused, year-long elementary mathematics professional development program and found that a PD program focused on ongoing task-focused mathematics professional development led to statistically significant gains in teachers’ mathematical knowledge of teaching, self-reported enactment of standards-based pedagogies, and teachers’ beliefs about both mathematics as a content area and the overall teaching and learning of mathematics. Additionally, teacher observations across the year indicated growth in their abilities to enact high-level mathematical tasks and questions at the end of the study more than what was visible at the beginning. However, Polly et al. (2014) also concluded further research was needed to indicate which specific components of the task-focused professional development were most effective at influencing teachers.

**Design and Methods**

Qualitative inquiry is especially powerful as a source of grounded theory that is inductively generated from field work; that is, theory that emerges from researchers’ observations and interviews out in the real world rather than in the laboratory or the academy (Patton, 2015, p. 18). While the literature suggests that changing the way teachers and students interact in the classroom is of the utmost importance in the area of mathematics (Hill et al., 2008; Slavin & Lake, 2008), limited research has been conducted that has identified just what, specifically, it is that educational leaders might do to change the beliefs and corresponding behaviors of instructors of elementary mathematics (Hoover et al., 2016; Polly et al., 2014). This research study sought to identify what specific professional development practices are attributed to change in teachers’ efficacy, beliefs about the nature of mathematics, and corresponding instructional behaviors as perceived by the participants,
facilitators, and through classroom observations. This constructivist grounded theory study sought to explain and build an in-depth understanding of findings in regard to teachers’ perceptions of the professional development experiences they undergo.

The qualitative research approach lends well to studying PD strategies utilized by facilitators in elementary mathematics for several reasons. First, Patton (2015) stated that “qualitative research inquires into, documents, and interprets the meaning-making process” (p. 3). This research study intended to make meaning of strategies utilized by a facilitator of professional development experiences as perceived by teachers. Second, qualitative research is “studying how things work” (Patton, 2015, p. 6), which is what this study intended to do. Third, Creswell (2013) stated: “qualitative researchers have underscored the importance of not only understanding the beliefs and theories that inform our research but also writing about them in our reports and studies” (p. 15). As the researcher, I brought several assumptions and beliefs to this study, and qualitative research was fitting because it enabled me to develop the theoretical framework accordingly. Fourth, in grounded theory research studies, the researcher “focuses on a process or an action that has distinct steps or phases that occur over time… some action the researcher is attempting to explain” (Creswell, 2013, p. 85). Because the field lacks specificity regarding strategies are utilized, and, successful, grounded theory was an appropriate approach. Theory generated in this research study was “grounded” in the actual data collected, in contrast to theory that is developed conceptually and then simply tested against empirical data (Maxwell, 2013, p. 49).

The study took place in a large suburban school district located in the Midwest. This site was appropriate for the research study because this school district concurrently had several different elementary mathematics professional development programs in the last five
years, each led by various facilitators. Some included specific grade level foci and require teachers from several buildings to collaborate; whereas others generalize to K-5 teachers housed within one school building.

The participants were defined as teachers of kindergarten through fifth grade mathematics who are in a professional development program, as well as the professional development facilitators of the various elementary math PD programs offered in this district. Teacher participants were those who had participated in ongoing mathematics PD programs within the district in the last five years. “Ongoing” was defined as a series of PD sessions spanning across the course of one school year (not an isolated PD session). Some participants participated in a series of 6-8 one-hour PD sessions located at the building level with a local mathematics consultant, while others had participated in full-day PD held four times across the school year (28 hours) with teachers from across the district within their same grade level.

This grounded theory study utilized a specific type of purposeful sampling (Creswell, 2013) known as theory-based sampling (p. 158). Patton (2015) defined theory-focused sampling (also called “inductive theoretical sampling”) as “select cases for study that are exemplars of the concept or construct that is the focus of inquiry to illuminate the theoretical ideas of interest” (p. 269); or, “sampling on the basis of emerging concepts, with the aim being to explore the dimensional range or varied conditions along with the properties of concepts vary” (Strauss & Corbin, 1998, p. 73). Teachers were asked to respond to open-ended questions as part of a mixed survey to explain the journey from their earliest memories to now. Their responses were analyzed to identify teachers with a range of self-efficacy beliefs when it comes to the area of mathematics. Nine teachers were asked to participate in
the research study, representing three different theoretical groups: (1) generally high efficacy in regard to mathematics throughout their K-12 educational experience; (2) mixed efficacy in regard to mathematics throughout their K-12 educational experience; and (3) generally low efficacy in regard to mathematics as a child throughout their K-12 educational experience. This sampling strategy allowed the researcher to build theory around how the professional development program was experienced by each of these respective groups.

The teachers who were selected as participants in the research study were then asked a series of interview questions regarding both their reflective responses to the open-ended survey questions and their perceptions of the elementary professional development experiences in which they had participated.

Data collection methods employed in this research study included use of a mixed survey, in-depth interviews, observations, and reflective documents. After data collection, coding was utilized for data analysis. As part of the theory-sharpening process (Patton, 2015), follow-up focus groups with participants and facilitators regarding the findings from participant interviews were held. In these focus groups, I took the findings from data analysis back to the participants and facilitators for input to further sharpen the theory generated.

**Significance of the Study**

Mathematics education in America has made progress, but research shows we still have a way to go in ensuring equitable, high-quality mathematics instructional opportunities in schools nationwide (Hoover et al., 2016; Ma, 1999; National Council of Supervisors of Mathematics, 2008). Silva and White (2013) found that approximately 60% of college students in the United States are in two-year colleges, and 75% of them are required to take remedial math, repeating the courses they had in high school. Only one in ten of the students
pass the remedial courses; the rest have to leave without any college degree. Slavin and Lake (2008) stated:

The mathematics achievement of American children is improving but still has a long way to go. …The United States remains behind other developed nations in international comparisons of mathematics achievement. …The achievement gap between children of different ethnicities, and between U.S. children and those in other countries, gives us no justification for relaxing our focus on improving mathematics for all children. …it is essential to know what specific programs are most likely to improve mathematics achievement. (pp. 427–428)

Just as educational leaders must know what specific programs are most likely to improve mathematics achievement, similarly, it is necessary to develop an understanding of what it takes to support the mathematical and pedagogical knowledge of practicing teachers. Ball (1990) and Hill et al. (2008) have found that prospective teacher education programs often do not adequately prepare educators with the pedagogical knowledge needed to facilitate elementary mathematics experiences for students. If this is the case, the need for effective professional development for practicing elementary teachers to ensure they can facilitate these learning experiences for students is of utmost importance. Just as students need effective instruction to be successful in learning, teachers, too, need effective professional development experiences to ensure their success at facilitating learning experiences for students. But how do educational leaders ensure the experiences they are providing practicing teachers are, in fact, effective? How do educational leaders know whether the learning experiences in which they engage teachers are making a difference? Are the teachers changing practice on only a surface features level, or are their background experiences and knowledge addressed as they learn new pedagogy and instructional strategies so that their beliefs and self-efficacy are also altered?
Hoover et al. (2016) conducted a review of the empirical literature concerned with distinctive mathematical knowledge requirements for teaching. Criteria for papers included in this review included potential empirical grounding (as characterized by the American Educational Research Association, 2006), papers that appeared between the years 2006 and 2013, and papers including search terms “mathematics,” “content knowledge,” and “teaching” (with asterisks utilized as placeholders for derived terms in all three cases). Six databases were included: PsycInfo, Eric, Francis, ZentralBlatt, Web of Science, and Dissertation Abstracts. Abstracts of more than 3,000 articles were read, and based on reading of abstracts, 349 articles were identified as potentially empirical. These 349 articles were read in their entirety and coded for the following categories: (1) Genre of the study, (2) Research problem used to motivate the study, (3) Variables used, (4) Whether or not and how causality was addressed, and (5) Findings. Three major strands of research stood out in regard to mathematical knowledge for teaching (abbreviated SM in this review): (1) Nature and composition of SM, (2) Improvement of SM, and (3) Contribution of SM. Among these three major strands, the second, “Improvement of SM,” included a sub-category of studies that addressed the question: “What professional development improves teachers’ SM?” Twenty-eight such papers were found that met inclusion criteria. Conclusions of the review of empirical papers regarding professional development that improves teachers’ SM included:

Attention needs to be given to dynamics regarding teachers’ motivation, the timing of different activities, and specific mathematical opportunities arising from specific pedagogical activities. In reading these reports, one gets the sense that really smart enactment of the professional development was key to success and that replicating effects might be challenging….it would seem important to discern the essential design features and elaborate the necessary character of facilitation. (Hoover et al., 2016, p. 14)
Further discussion of the studies that focused on improving teachers’ SM through professional development highlighted that while moving between studying mathematical content and pedagogical practice is central to many of the professional development models, one issue facilitators struggled with was:

the fact that teachers positioned themselves and others as better or worse in mathematics. These dynamics got in the way of productive mathematical discussions and frustrated facilitators….This led researchers to see a need for facilitators of professional development to develop detailed purposes for doing mathematics in professional development in ways that teachers would see as relevant. (p. 14)

Analysis of this study (Elliott et al., 2009) also demonstrated the dynamics between mathematics and motivation and use of that mathematics was key to effective teacher learning of professionally relevant mathematics.

This research study sought to fill the existing gap in the empirical literature by providing further explanation of specific professional development strategies that might address the efficacy, motivation, and beliefs of practicing elementary school mathematics teachers. This theoretical framework of practices in elementary mathematics professional development can be used to make future programming decisions by educational leaders. The intent and significance of this study will lead to professional development opportunities for teachers that address teachers’ existing beliefs in order to facilitate more meaningful learning experiences for teachers, which could in turn lead to increased quality in educational experiences offered to students on a large scale in the area of elementary mathematics.

Chapter 1 provided an overview of the study through a discussion of theories, concepts, and empirical studies that support the prevalence of the problem. The problem statement laid the foundation for Chapter 2, which digs deeper into the conceptual framework
that underpins this study including teachers’ conceptual understanding of mathematics, motivation, constructivism, and leadership for professional development by providing a literature review. A review of the methodology was included here and is further expanded in Chapter 3.

Chapter 4 provides an in-depth understanding of how data analysis led to the generation of theory, grounded in the experiences of teacher participants they provided throughout the data collection process. Answers to the research questions, recommendations, and suggestions for future research are provided in Chapter 5, along with reflections on my own mathematics experiences and efficacy and how they have shaped my efficacious behavior as demonstrated in my leadership.
CHAPTER 2
LITERATURE REVIEW

Providing high-quality elementary mathematics learning experiences for students is critical as we prepare today’s students for tomorrow’s 21st century workforce. The workforce they will enter has evolved greatly since most of today’s educators learned elementary mathematics and will continue to evolve at rapid rates. To consider how educational leaders can better support in-service (currently practicing) teachers in meeting our students’ needs, I first sought to gain an understanding of the related empirical research that exists. A search of the literature was conducted utilizing online search engines of education and psychology journals such as JSTOR; EBSCO; International Journal of Science and Mathematics Education; Early Childhood Education Journal; SAGE, Psychology, Society, Education; and others. Search terms used included: “elementary mathematics professional development,” “constructivist professional development,” “conceptual understanding of mathematics and elementary teachers,” “mathematical knowledge for teaching,” “mindset,” “motivation and mathematics,” “effective professional development,” “self-efficacy,” “pre-service teachers,” “inservice teachers,” “task-focused,” and more. The search was narrowed by focusing primarily on research within the last decade (2006-2020); save for a few seminal works. At times, search terms were limited by specifying “NOT preservice teachers,” “NOT science” to gain a more focused result list. Another search was conducted using the terms “elementary teachers” and “self-efficacy” and “theory” and only 19 results were found, with most referring primarily to math anxiety and self-efficacy of students.

Dissertations conducted within the last decade on similar topics of interest were also reviewed to gain an understanding of the most recent research, as this dissertation sought to
build upon and contribute to the theory and literature in the field. Recent books written by leaders in the field of elementary mathematics were reviewed. While research in the field of elementary mathematics has been conducted, I found there were gaps regarding what specific mathematical knowledge for teaching is important for elementary teachers to develop conceptual understandings to facilitate effective learning experiences for students.

Additionally, while professional development in elementary mathematics has been the focus of some studies, there was a lack of specificity regarding what aspects of these professional development experiences led to change (regarding teachers’ efficacy, beliefs, instructional behaviors, motivation, and conceptual understanding of mathematical knowledge for teaching). Research in motivation and mathematics is more promising, often pointing to a correlation between mindset and learning experiences. Additionally, this research often pointed to stereotypes and misconceptions about the nature of mathematics, and the negative effect these messages have historically had on students and teachers. This review of literature led to the inclusion of the following four topics that conceptualize the theoretical framework that underpin this research study: (1) Teachers’ Conceptual Understanding of Mathematics, (2) Constructionism/Constructivism, (3) Motivation, and (4) Leadership for Professional Development. My review of the literature supported the idea that a gap existed in the empirical literature when connecting these four topics. This literature review helps connect these four topics, which can be used to explain meaningful learning experiences for practicing elementary teachers around elementary mathematics that could make the difference for the elementary teachers who are currently teaching the mathematicians of our future. However, further research was needed to build theory and add to the empirical literature in the field to make this happen.
The first topic chosen to review in the literature; Teachers’ Conceptual Understanding of Mathematics, was selected because many elementary teachers were not taught in a way that led them to gain a deep conceptual understanding of mathematics, and thus find difficulty designing tasks that allow students to make deep connections between mathematical ideas. It is this common lack of conceptual understanding of mathematics in America that necessitates more effective professional development practices in elementary mathematics.

Constructionism/Constructivism was selected because elementary math teachers today are being asked to teach in ways that help students construct mathematical ideas, when teachers often need more experience with these mathematical ideas themselves to further construct their own understanding before they can proficiently facilitate high level mathematical discussions of students.

Motivation was significant to this research because I hypothesized that teachers would be further motivated to teach math in ways that lend to students’ depth of conceptual understanding if they experienced the content this way themselves first. Many teachers still hold fixed mindsets about their own math abilities as well as those of students. Until given the opportunity to grow their own math brains—something that some of them may not believe is even possible at this point in their lives—teachers likely will not be as motivated to modify their own instructional practice, and it is likely the mindsets they model for students will reflect their own beliefs about the nature of mathematics and their own abilities.

As of yet, little consideration has been given as to how educational leaders could design professional development experiences for elementary teachers that would take their existing self-efficacy and beliefs into account while concurrently developing a deeper
conceptual understanding of the mathematics needed to break this cycle of negative mindset, stereotypes, and misconceptions about the nature of mathematics when designing and facilitating learning experiences for future generations of students. As a result, Leadership for Professional Development was selected as the final topic for review for this research study.

**Teachers’ Conceptual Understanding of Mathematics**

In this section, a discussion of the literature regarding Teachers’ Conceptual Understanding of Mathematics is included. Sub-sections include a discussion of teachers’ beliefs about the nature of mathematics to highlight how often teachers’ lack of conceptual understanding is a result of being taught in traditional/one-dimensional classrooms, how teachers often struggle to fully adopt multidimensional beliefs and pedagogical practices because this is often not how they were instructed in their K-12 math experience, as well as the role of multidimensional beliefs and pedagogy towards equity in mathematics.

The seminal work of Shulman (1986) referred to the absence of content focus so prevalent in teacher education as a “missing paradigm” and called for increased attention to be paid to subject matter research and practice. Research regarding the connection between the formal education of mathematics teachers and their content understanding has been varied. Analyses to correlate between teachers’ certifications in math or degrees, as well as courses taken, and the student achievement gains in these teachers’ classrooms revealed no advantage at grade levels K-8 and only a slight advantage at the secondary level (Thames & Ball, 2010). However, Kim (2014) found statistically significant differences between teachers’ degrees related to mathematics education. Teachers who graduated with a bachelor’s degree in mathematics education had lower anxiety for teaching mathematics than did teachers with a bachelor’s degree in general elementary education. Similarly, teachers
who had master’s degrees in mathematics education had lower anxiety for teaching mathematics than did both those who had bachelors’ degrees in mathematics education and those who did not, and a negative correlation between teachers’ degree levels and their anxiety for teaching mathematics was found.

Beilock et al. (2010) discovered that gender differences in math achievement did not exist at the beginning or end of the school year. However, using a mediation analysis that depicts a model of a causal chain of events, they determined female students’ math achievement at the end of the school year was negatively affected by the way in which their teachers’ math anxieties altered these girls’ gender ability beliefs, as girls who confirmed traditional gender ability roles performed worse than girls who did not and worse than boys more generally.

Shulman (1986) suggested teaching requires professional knowledge that is distinctive for the teaching profession, which he referred to as “pedagogical content knowledge” (PCK). Commonly referred to as Mathematical Knowledge for Teaching (MKT) (Hoover et al., 2016), it is commonly agreed upon that in addition to knowing math content or how to solve problems, there is another kind of teaching-specific mathematical knowledge that successful teachers must possess, along with pedagogical knowledge, to be successful in teaching mathematics so that students understand the concepts, not just rules of math (Hill et al., 2005; Hill et al., 2008; Hill & Ball, 2004; Thames & Ball, 2010).

Many research efforts have been made to define and measure the impact of such knowledge on student achievement to make meaningful progress. Hill et al. (2004) developed a survey of multiple-choice scale items to measure knowledge for teaching mathematics. This measure, often referred to as the Learning Mathematics for Teaching (LMT) instrument,
includes items in categories that were modified versions of a culmination of previous research, including those proposed by Shulman, Wilson, Ball and others (Hill et al. 2004), and includes questions on what they described as Common Knowledge of Content (CKC), Knowledge of Content (KC), Specific to Specialized Knowledge of Content (SKC), and Knowledge of Student Concepts (KSC). Copur-Gencturk et al. (2019) explored this dimensionality of MKT, and argue that MKT is unidimensional, that there are not separate dimensions of knowledge teachers need for teaching, but instead they are all interconnected and cannot be disentangled. Regardless, their results showed the two models did not differ statistically when a confirmatory factor analysis to compare two models was utilized. Hill et al. (2008) utilized the MKT survey measures and developed a system for coding responses in teacher observations, interviews, and curriculum materials to determine the teachers’ Mathematical Quality of Instruction (MQI) and found a strong, significant, and positive association between levels of MKT and the MQI. Hill et al. (2008) also identified other factors that could mediate this relationship, such as teacher beliefs about how mathematics should be learned and curriculum materials and how they should be used, as well as the availability of curriculum materials to teachers.

Hoover et al. (2016) reviewed empirical research regarding teacher content knowledge and found three themes of studies: (1) those that investigate the nature and composition of teacher content knowledge; (2) those that investigate approaches to increasing teacher knowledge, both with pre-service and practicing teachers; and (3) those which investigate the effects of teachers’ knowledge on both teaching and student learning. Several studies have created additional measures/instruments to measure Teacher Content Knowledge (TCK) to define it. In addition to the LMT, these include those designed for
broad consensus or widespread use: The Diagnostic Teacher Assessment in Mathematics and Science (DTAMS) instruments for practicing middle school teachers (Saderholm et al., 2010), The Teacher Education and Development Study in Mathematics (TEDS-M) instruments for pre-service primary and lower secondary teachers (Senk et al., 2012; Tato et al., 2008), and the Knowledge for Teaching Early Elementary Mathematics (K-TEEM) (Schoen et al., 2017).

Other instruments have been developed for more specific or localized use, including the COATIV instrument for practicing secondary teachers (Kunter et al., 2013), instruments designed for use with pre-service teachers at the university level (Fast & Hankes, 2010), and an instrument designed specifically for use with fourth through eighth grade teachers (Campbell et al., 2014). There are also several instruments developed to focus on mathematical knowledge related to a specific topic, such as algebra (McCory et al., 2012) continuous variation and covariation (Thompson, 2015), fractions (Izsak et al., 2012), and geometry (Herbst & Kosko, 2012). Additional instruments have been developed to focus on aspects of teaching and the mathematical knowledge required for these specific practices, like choosing examples (Chick, 2009; Zodik & Zaslavsky, 2008) and scaffolding whole-class discussions to address mathematical goals (Speer & Wagner, 2009). Other studies have developed instruments related to specific lines of research or in response to perceived issues with more established instruments (Kersting et al., 2012; McCray & Chen, 2012; Thompson, 2015). Researchers have also looked for consistency among instruments and found that different instruments measured aspects of MKT differently without clarifying how these facets of knowledge were different or categorized items to measure aspects of MKT inconsistently (Copur-Gencturk & Lubienski, 2013; Kaarstein, 2014).
While inconsistencies exist within the research when determining what specific mathematical conceptual knowledge is necessary to quality teaching, there is an overall agreement that teachers’ conceptual knowledge can influence their students’ learning, through both increased quality of instruction provided by teachers with higher MKT as well as evidence of increased student achievement. Hill et al. (2005) used a linear mixed-model methodology and found teachers’ mathematical knowledge was significantly related to student achievement gains in both first and third grades after controlling for key student- and teacher-level covariates.

Campbell et al. (2014) conducted a correlational study in which several data sources were analyzed, including: student achievement data for approximately 17,000 students nested within the classrooms of approximately 450 teachers across grades 4-9, paper/pencil teacher-knowledge assessments to measure teachers mathematical Content Knowledge (CK, 80 items) and Pedagogical Content Knowledge (PCK, 40 items), a beliefs and awareness survey composed of 40 Likert-format items addressing teachers’ mathematics teaching and learning beliefs and teachers’ claimed awareness of student dispositions, and surveys on the teachers’ professional background and instructional context surveys. For this study, only teachers who had six or fewer years of teaching experience were selected to control for teacher experience and its role in affecting teacher knowledge and student achievement. An exploratory factor analysis was completed using teachers’ responses on the 40-item beliefs and awareness survey. To determine whether there was a relationship between teachers’ mathematical content and pedagogical knowledge, teachers’ perceptions, and student achievement, Hierarchical Linear Modeling (HLM) with a two-level, random intercept model was utilized. Computed Pearson’s correlation coefficients indicated teachers’ CK and PCK scores were
correlated. A significant relationship between upper-elementary teachers’ mathematical content knowledge and their students’ mathematics achievement, after controlling for student- and teacher-level characteristics, was found. Students taught by teachers who held certification in special education had statistically significantly lower mathematics achievement scores on the state assessments than did other students, even after controlling for the identification of individual students receiving special education services. The impact of teachers’ CK on student achievement was influenced by teachers’ beliefs regarding modeling of solutions to mathematical tasks and organizing instruction to support incremental mastery of skills. Campbell et al. (2014) reflected that due to the quantitative design of this study, the following aspects were not addressed:

How teacher knowledge and perceptions were translated into instructional practice within the teachers’ classrooms; how school contexts influenced not only student assignment to teachers but also school culture; how local school administrators and teachers defined instructional priorities; or the duration, quality, local support for, and perceived relevance of, professional development and early career mentoring experienced by these teachers. (p. 450)

Further, Campbell et al. (2014) recommended: “Qualitative studies addressing these issues are needed if education research is to contribute to a more nuanced understanding of the role that knowledgeable teachers assume and the support they need” (p. 452).

As mentioned in the Campbell et al. (2014) study, teachers’ beliefs about both the nature of mathematics as well as their students’ abilities also play a role in how teachers’ conceptual understanding of mathematics are enacted in their pedagogical practices. Some in the field of mathematics education theorize that as the conceptualization of mathematical proficiency becomes more complex, teachers’ belief systems will also become more complex
(Campbell et al., 2014; Kilpatrick et al., 2001). For this reason, a discussion on teachers’ beliefs about the nature of mathematics is also included in the next section.

**Teachers’ Beliefs about the Nature of Mathematics**

There is a general agreement that when it comes to approaches to mathematics instruction, two approaches are widely referred to, including: (1) closed, or one-dimensional, and (2) open, or multidimensional. One-dimensional, or closed mathematics classrooms, have been the historic norm in America. One-dimensional mathematics can be categorized as supporting learning by rote, emphasis on arithmetic and numeracy, and the following of set methods and rules (Ball, 1990; Boaler, 1998). These classrooms are described as one-dimensional because in them a single practice is valued above all others—executing procedures correctly; often, with a focus on memorizing the teacher’s presentation of procedures (Boaler, 2006). The belief that teachers should model activities and approaches followed by student practice, emphasizing incremental mastery of procedural skills prior to solving application problems, termed by Battista (2001) as the universal script, is also characteristic of one-dimensional beliefs about the nature of mathematics. In contrast, a key aspect of classrooms where multidimensional mathematics pedagogy is present include: focuses on asking good questions, helping others, using different and multiple representations, rephrasing problems, explaining ideas, being logical, justifying methods, bringing different perspectives to a problem, collaboration, heterogeneous grouping, and group-worthy problems (Boaler, 2006). Classrooms that could be described as one-dimensional also are often referred to as traditional, textbook-based, quiet, and closed; whereas classrooms facilitating multidimensional mathematics would be rich with discussion (Boaler, 2006; 2016). The belief that teachers should allow students to struggle or grapple
with solving problems on their own before teacher intervention, aligned with Hiebert and Grouws’s (2007) description of a key feature in teaching mathematics focused on students’ conceptual understanding and meaning making, is also more characteristic of multidimensional beliefs of the nature of mathematics. Hiebert and Grouws (2007) use the word “struggle” to mean that students expend effort to make sense of the mathematics, to figure something out that is not immediately apparent (p. 387).

Research has shown that often students who learn math in a more one-dimensional or traditional classroom setting did not transfer these procedures to real-world mathematical situations (Boaler, 1998; Lave, 1988; Lave et al., 1984; Masingila, 1993; Schoenfeld, 1988), whereas students learning mathematics in a multidimensional classroom often have increased ability to transfer concepts learned to real-world contexts in authentic activities, giving them more access to mathematical careers, higher-level jobs, a more secure financial future, and also teach students to reason and critique one another’s reasoning—central to today’s high-tech workplaces (Boaler, 1998, 2006, 2016; Cognition and Technology Group at Vanderbilt, 1990). Multidimensional beliefs about the nature of mathematics and aligned pedagogical practices can support equity in mathematics education.

The Role of Multidimensional Beliefs and Pedagogy in Mathematics Equity

The literature suggests teachers who hold multidimensional beliefs, and whose pedagogical practices align with the characteristics of multidimensional classrooms, are likely also helping to close the gap and increase equity for all students in mathematics. Boaler (2006) found that transformation of a high school mathematics program from one-dimensional to multidimensional, with a focus placed on particular norms (such as discussing ideas, exploring different methods, and taking responsibility for others), helped students in
these mathematics classrooms appreciate the insights of their peers from different cultures and circumstances. At this school, practices focused on equity were included in the shift to pedagogy that reflects multidimensional beliefs about the nature of mathematics, including “helping students see mathematics as a part of their future by providing them with the quantitative reasoning capabilities need to function in an increasingly technological and global economy” (p. 365). Upon entering high school, students at this school were achieving at significantly lower levels than the students at the other two, more suburban, more affluent schools in the study. Within two years, students here were significantly outperforming the students at the other schools, with a greater percentage (41%) of students taking advanced mathematics courses as compared to 27% of students in the other schools (p. 365). The approach taken in this school to multidimensional mathematics was through utilizing complex instruction designed to counter differences of social and academic status. Students at this school felt success because there were many more ways than one to be successful, which helped students’ motivation, efficacy, mathematical mindsets, and ultimately, helped them to develop higher levels of understanding (p. 366).

Research also supports the idea that multidimensional beliefs about the nature of mathematics, if engaged in pedagogical practices that could be categorized as multidimensional in the classroom, can further support the mathematics learning of English Language Learner (ELL) students. Chval et al. (2015) conducted a case study of one third-grade teacher as she learned how to focus on language while designing and teaching mathematics lessons to facilitate participation of ELLs. Results of their study demonstrated the importance of professional development that emphasizes language in the teaching of mathematics. The intervention included instructional strategies that could be characterized as
multidimensional, including (1) developing relevant contexts for mathematical problems for her students in which they were successful to increase their engagement and interest and focus on that one context, (as opposed to moving between contexts daily as the math resource program did, often confusing ELLs if they were unsure of the vocabulary used),
(2) establishing expectations for students that they think through their own struggles metacognitively rather than approach the teacher first when struggling, (3) giving students opportunities to work collaboratively in small groups, and (4) facilitating discussion over relevant vocabulary in the context of the real-world task they would solve. As the teacher began to learn about the components of the intervention, she developed specialized knowledge.

Similarly, Khisty and Chval (2002) examined a teacher who had a record of assisting fifth grade Latinx students in making significant academic gains in mathematics and contrasted this teacher with another teacher whose instructional talk was not as mathematically rich. As previously discussed, classrooms rich with discussion and discourse are characteristic of multidimensional beliefs of the nature of mathematics (Boaler, 2006, 2016). Khisty and Chval’s (2002) observations suggested that teachers who facilitate mathematical discourse among students who intentionally use mathematical talk support students in learning the meaning of mathematics vocabulary and becoming literate in mathematical discourse, particularly in the learning of racially, ethnically, and linguistically diverse students. Additionally, Pinnow and Chval (2014) argued the use of the Standards for Mathematical Practice, particularly practice one: Make sense of problems and persevere in solving them (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), can be used to set high expectations for ELLs and their
teachers, as “these expectations emphasize the vital role of language and communication in solving mathematical problems” (Pinnow & Chval, 2014, p. 22).

Boaler (2016) also offered the following strategies to make math education more equitable and inclusive, all of which are consistent with practices seen in multidimensional classrooms:

1. Offer all students high-level content,
2. Work to change ideas about who can achieve in mathematics,
3. Encourage students to think deeply about mathematics,
4. Teach students to work together,
5. Give girls and students of color additional encouragement to learn math and science, and
6. Eliminate (or at least change the nature of) homework. (pp. 102–107)

While the literature thus far helps provide some indication of the role of teachers’ conceptual understanding of mathematics to impact student achievement, and supports the notion that multidimensional beliefs and pedagogy can increase mathematics equity for students, Campbell et al. (2014) noted that the two factors (the belief teachers should allow students to struggle before teacher intervention, and the belief that teachers should model activities first followed by student practice before application), “should not be categorized as pedagogical opposites because it is possible for teachers to hold both beliefs strongly and simultaneously” (p. 439). Even in classrooms where teachers believe they should teach in ways that are multidimensional, it is common to find some elements that would still suggest a more one-dimensional or traditional/closed. Often, it is difficult for teachers who were taught in traditional, one-dimensional ways, who may lack the deep conceptual understanding of the math content they are responsible to teach, to consistently maintain multidimensional pedagogical behaviors without sustained professional development and support.
Helpful to the understanding math is the role of constructionism/constructivism in the learning process. Whereas social constructivism constitutes knowledge the individual brings to the learning process through their social and psychological worlds (Duffy & Jonassen, 1992), social constructionism purports that knowledge is always shaped by history and culture (Burr, 2015; Young & Collin, 2004).

Constructionism/Constructivism

This section reviews the literature on the role of constructionism and constructivism in mathematics education, beginning with a brief history of constructivism and mathematics pedagogy, then looking at the pedagogical implications of constructivism for mathematics as well as cautions, clarifications, and a summary of constructivist pedagogy, then moves to take a closer look at constructivist leadership in education and its challenges.

History of Constructivism and Mathematics Pedagogy

Research on teaching mathematics suggests a constructivist approach is key to building the deep mathematics understanding promoted by the National Council of Mathematics (NCTM) Standards (Fast & Hankes, 2010; NCTM, 2000). Recent researchers suggest constructivism involves students engaging in an active search for meaning that is constructed rather than achieved through memorization (Amineh & Asl, 2015; Martin & Polly, 2019). The widespread interest in constructivism among mathematics education theorists, researchers, and practitioners has led to a plethora of different meanings for “constructivism” (Simon, 1995, p. 115). Simon (1995) named the most divisive issue in the epistemological debates as being whether knowledge development is seen fundamentally as a social or individual cognitive process. The difference in the two positions depends on the focus of the observer. A radical constructivist view focuses on the individual process of
constructing knowledge and refers to the specific cognitive and psychological process. A sociocultural constructivist view focuses on the social process of constructing knowledge. Social constructivist views posit that an individual constructs meaning and knowledge through their social environment and interactions (Beck & Kosnik, 2006; Vygotsky, 1978). A social constructivist view combines both psychological and sociological views, refers to an individuals’ knowledge of/about mathematics, their understanding of the mathematics of others, and sense of functioning as part of the mathematics class (Simon, 1995).

Piaget noted that schemas are “under construction,” meaning that the cognitive structures evolve as individuals interpret, understand, and come to know (Piaget, 1971). John Dewey (1916) first gave voice to this view of learning, noting that students must learn and make sense of new knowledge together, based on their individual and collective experiences. Piaget (1971) expanded the understanding of learning in ways that support and contribute to constructivism. According to Piaget, learners move through states of cognitive development— from concrete to abstract—prompted by a discrepancy or “disequilibrium” between what they previously believed to be true and their new insights and experiences. Bruner and Haste (1987) built on Piaget’s theory, determining that “Making sense’ is a social process; it is an activity that is always situated within a cultural and historical context (p. i). Fosnot (1992) added: “Some ideas require cognitive developmental shifts in perspective as they are constructed and thus have an aspect of structure within them…referring to conceptual structure, an idea having ‘wholeness’… in that it explains how the parts within the system interrelate” (p. 168). In the next section, I discuss the pedagogical implications this research has for mathematics instruction.
Pedagogical Implications for Mathematics

Fosnot’s (1992) view of constructivism asserted that new understandings cannot be understood simply by transmission, because the transmission is interpreted/assimilated and demanding of a mental reordering. One must give up an old perspective, at times reintegrating it, to construct new perspectives or understandings. Fosnot (1992) analyzed the interpretations of constructivism regarding pedagogy of her co-authors. One concept discussed is regarding different types of constructivism, introduced by Bruner (1973). Beyond the Information Given (BIG) Constructivism, as seen in Duffy and Jonassen (1992), refers to the kind of instruction that gives some information/direction to students, allows for them to extend upon given information. Engages learners in a number of thought-oriented activities that challenge them to apply & generalize their initial understandings, refining them along the way. (pp. 49–50)

Without the Information Given (WIG) instruction, on the other hand, holds back on direct instruction, provides heavy scaffolding if needed, but without directly providing answers. “Official” characterizations (or definitions) of concepts would never be offered, or only late in the instructional game. Learners encouraged to explain phenomena with intuitive notions. Anomalies emerge. To make sense of anomalies, learners are encouraged to devise better models of what was occurring. (pp. 49–50)

Fosnot (1992) generally took a Piagetian approach, leaning towards a WIG model of constructivist pedagogy. Duffy and Jonassen (1992) drew this effective conclusion between these two types of constructivism:

Education without any WIG episodes would rarely let students engage in and learn about processes of discovering and idea construction. However, education given over entirely to WIG constructivism would prove growly inefficient and ineffective, failing to pass on in straightforward ways the achievements of the past (p. 50).
When analyzing the various viewpoints on constructivism introduced by her colleagues, Fosnot (1992) summarized three principles of instruction: (1) Cognitive developmental research shows that learners progress from concrete explorations in meaningful contexts, to symbolic representations of these actions, to abstract models; (2) Learning is always a case of building with, and from, initial assimilatory structures; and (3) Teaching for conceptual understanding.

Fosnot’s (1992) Principle 1 can be seen throughout the Common Core Standards for Mathematical Practice (SMPs), and in mathematics classrooms as teachers facilitate opportunities for students to reason abstractly and quantitatively (SMP 2), model with mathematics (SMP 4), and use available tools strategically (SMP 5) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Another example, given by Fosnot (1992): “For conceptual structures—such as decimals and fractions—to be developed, instruction must at some point go beyond the concrete problem experience” (p. 171).

Principle 2 (Fosnot, 1992) is exemplified when mathematics teachers consider instruction from a viewpoint that considers the trajectory, or learning process, of conceptual understandings.

Learning is always a case of building with, and from, initial assimilatory structures. Only through challenges to these structures, will gaps, insufficiencies, or contradictions become apparent to learners—thus facilitating reflective abstraction and accommodation. Thus “entry level understanding” is of utmost importance (Fosnot, p. 169).

Pedagogy that considers Principle 2 would take a constructivist view because it encompasses students’ schema and background experiences regarding mathematics concepts, and projects learning experiences accordingly while providing the right amount of scaffolding and support
for students to construct new understandings. “Learners are always making meaning, no matter what level of understanding they are on” (Fosnot, 1992, p. 172).

Principle 3 (Fosnot, 1992) provides a solution for an often-seen problem in one-dimensional (Boaler, 2006) mathematics instruction. Often mathematics pedagogy still maintains a focus on specific, isolated skills and students’ mastery of these skills. Fosnot (1992) spoke of this disconnect between skills and conceptual understanding, arguing that while skills have a place in learning, too frequently, conceptual understanding is given less instructional time:

Understanding is not “mastered”; it can always be deepened as ideas and models are extended to new experiences. Skills and strategies are behaviors that are representative of the conceptual structures employed… I believe that we have focused on behaviors because (a) we have used a behaviorist psychology in instructional design and (b) they are easy to assess. I have seen children skillfully doing arithmetic computations but having little understanding of place value, and teachers having to re-explain it with the introduction of each new operation and algorithm for the next five years because learners never understood it in the first place. Practice and reinforcement are appropriate strategies if our goal is to train for mastery of skills.” If our goal is truly to educate, the teach for understanding, then much rethinking is needed. (p. 173).

Mathematics pedagogy that is multidimensional (Boaler, 2016) in nature will support learning experiences for students that build greater conceptual understandings. There is also a need for teachers who want to increase students’ deep, conceptual understandings in math to use authentic measures of ongoing formative assessment to document student thinking to guide instruction.

In a constructivist model it makes more sense (to me) to document learning, rather than to assess it…. Just as physicists, when they moved away from objectivism, began to study patterns of movements, shapes, interactions, we need to do the same. Portfolios of students’ writing, mathematics problem solving, or recordings of science investigations can be kept, as well as individual journals and clinical interviews. (Fosnot, 1992, p. 174)
Simon’s (1995) work, which proposed a Mathematics Teaching Cycle model, also supports these views, as it introduces an ongoing cycle where teachers construct and project instructional activities based on what they learn from student thinking.

Acar and Yilmaz (2015) conducted a collaborative action research study with third graders by coding transcriptions of the students’ work as well as interactions while solving problems in addition, subtraction, multiplication, and division in groups. Their research findings determined group talk and discussions had positive effects on constructing learning how to solve math problems. Interestingly, like findings regarding the role of adult elementary mathematics PD facilitators, they found that the type of leadership role displayed by student leaders in each group either created a productive constructivist learning environment or constrained a constructivist learning environment and affected the discussion and learning of students in different groups.

While constructivist pedagogy in mathematics can help provide a context for multidimensional beliefs and pedagogy, it is important to include a discussion of the cautions and clarifications given in the literature regarding the use of constructivist pedagogy in the classroom as well, to help one gain a full understanding.

**Cautions and Clarifications of Constructivist Pedagogy**

Hattie (2008) cautioned that constructivism is a form of knowing, not a form of teaching, and it instead should mostly be applied to the process teachers take when considering learning from the learner’s viewpoint first and constructing conceptions, beliefs, and models about how they teach and how students learn. He emphasized the role of the teacher in being actively and directly involved in the teaching and learning process as
students are actively creating and recreating knowledge in a social construct. Similarly, Simon (1995) stated that while constructivism offers one set notion of how to teach, it is more important not whether teaching is constructivist, but whether it is effective, and that we should instead ask “In what ways can constructivism contribute to the development of useful theoretical frameworks for mathematics pedagogy?” (p. 117). The following section gives a summary of constructivist pedagogy in mathematics.

**Constructivist Pedagogy: A Summary**

While constructivism provides a helpful framework for building mathematics pedagogy, it is not an instructional model in and of itself. The review of existing literature on mathematics and constructivist views highlights the importance of teacher pedagogical knowledge when facilitating learning experiences that help build deep conceptual knowledge for students. Fosnot (1992) highlighted the importance of clarifying and further defining what a constructivist approach to mathematics instruction would truly entail: “A constructivist approach to teaching will need to identify what instructional strategies will facilitate concept construction and when they are needed” (as cited in Duffy & Jonassen, 1992, p. 169). While the focus in constructivism is on the learner constructing their own knowledge, Fosnot (1992) also attributed student success of developing deep, conceptual understandings of mathematics to the teachers: “Empowering Teachers…the key is the teacher” (as cited in Duffy & Jonassen, 1992, p. 175). Simon (1995) asked: “How can mathematics teachers foster students’ construction of powerful mathematical ideas that took the community of mathematicians thousands of years to develop?” (p. 118). Richards (1991) asserted: “Students will not become active learners by accident, but by design, through the use of the plans that we structure to guide exploration and inquiry” (p. 38).
Similar to Hattie’s (2008) argument that we can take effective instructional strategies from one pedagogical model (i.e., direct instruction) and utilize what we know to be true about constructivist theory in regard to learners constructing knowledge, Fosnot (1992) closed with this: “Through discourse, argument, and debate we will negotiate meaning—differentiating constructivism from other models—and construct new models of pedagogy together” (p. 175).

Because the theoretical framework that underpins this research study includes leadership through professional development for mathematics elementary teachers, in the next section, I review the literature regarding constructivist leadership and its implications for mathematics professional development.

**Constructivist Leadership**

Lambert (2002) defined constructivist leadership as “the reciprocal processes that enable participants in an educational community to construct meanings that lead toward a shared purpose of schooling” (p. 36). Lambert outlined key ideas of constructivist leadership: (1) The lives of children and adults are inextricably intertwined; (2) Constructivism is the primary basis of learning for children, adults, and organizations; (3) Communities that encourage the growth of human potential are based on the principles of ecology; (4) Patterns of relationships form the primary bases for human growth and development; (5) Diversity provides complexity, depth, multiple perspectives, and equity to relationships, thereby extending human and societal possibilities, and (6) Leadership as critical social and intellectual transformation is achieved through reciprocal, purposeful learning in community. Regarding the first key idea, she stated, “It is important that we come to understand and interpret the learning needs of children and adults as patterns that recur in complex ways—
the learning patterns of all humans. If something is worthy for children, it is also worthy for adults. Authentic work must be experienced by adults as well as children, as must authentic relationships and possibilities” (p. xvi). Lambert (2002) emphasized the importance of recognizing that “Individuals and organizations bring past experiences and beliefs, as well as their cultural histories and world views, into the process of learning; all of these influence how we interact with and interpret our encounters with new ideas and events” (p. xvi).

This research project sought to understand how facilitators of professional development could construct learning experiences for teachers that will account for the background experiences and knowledge that they bring with them into learning organizations. Lambert (2002) provided a framework through which one can view constructivist leadership of a community of learners, as well as a community of leaders. According to Lambert (2002), in a community of constructivist learners, students work cooperatively and share knowledge. Classroom reward structures encourage this cooperative learning, and assessments determine both group and individual progress. Process is as important as content during learning. The students and teachers learner together, with group skills and interdependence emphasized (Lambert et al., 2002). The teacher’s role in a constructivist community of learners is that of a facilitator, as the classroom processes are more democratic than autocratic. The learning environment is viewed as interrelated with how the students learn and teachers teach. Students’ abilities and intelligence are not viewed as innate, but part of the educational context (Lambert et al., 2002). The professional community of the school is also an important part of a constructivist community of learners, as teachers and principals work together to positively impact student learning.
According to Lambert et al. (2002), in a community of constructivist leaders, leadership is viewed as a process that is shared among principals and teachers, with the principal as the “leader among leaders” who facilitates the growth of others (p. 13). The organizational structure of a constructivist community of leaders is flattened and integrated, with participants sharing common values and purposes. The nature of this community is interactive, promoting continuous improvement, with assessment guiding the work of the community. Democratic processes are emphasized (as in the constructivist community of learners), and comes from several theoretical constructs, including human relations and systems theory.

According to Lambert et al. (2002), constructivist learning is characterized by students constructing meaning from personal values, beliefs, and experiences. Throughout this process, it is key that students develop personal schema and the ability to reflect over one’s experiences. This viewpoint believes knowledge exists within the learner, and the activity at the center of the learning is shared inquiry. Multiple outcomes are expected and encouraged, with assessment being integral to the process. From a constructivist learning perspective, human growth is morally imperative. Implications of brain research and its implications for teaching and learning support constructivism. Additionally, when considering how one’s culture influences their personal experiences and schema, one could argue principles of constructivist learning can help resolve issues of equity in learning (Lambert et al., 1995).

While there are many empirical studies about constructivist approaches to learning, empirical studies focusing on constructivist leadership in education are lacking in the literature. Shapira-Lishchinsky (2015) conducted a qualitative study with the purpose of
reflecting leadership strategies that may arise using a constructivist approach based on organizational learning, including 50 teacher participants who serve in school leadership roles in addition to their teaching jobs. These teachers participated in 50 simulation sessions with other participants to construct new knowledge based on their current and past knowledge and experiences around the simulations. Research assistants were careful to phrase their comments and feedback to participants during discussions in a supportive and nonjudgmental manner to create a supportive, nonjudgmental environment. This supportive atmosphere was then reflected in the participants’ discussion as well. Research assistants used questions to probe into simulations that covered sensitive ethical issues to facilitate discussion among participants while maintaining a safe environment. This study found that combining constructivist approach and organizational learning theory with simulation provided these educational leaders effective learning experiences and found that as the participants discussed the implications of the scenarios through engaging in collegial constructivist dialogue, it gave them access to quality learning experiences (p. 985).

MacCallum and Morcom (2008) conducted an in-depth study of a classroom over a school year using ethnographic approaches (observation; videotaping; interviews/informal dialogue with children and parents; written surveys for both students and parents; reflective accounts of children, teacher, co-researcher, parents; documentation instructional practices; sociometric surveys, and school records of behavior) sought to learn more about how teachers create a classroom that supports students to participate fully in social and cultural practices of the classroom and ultimately, society. They found that the teacher used (1) class agreements, (2) social circles to develop social and emotional awareness, (3) weekly class meetings to build shared knowledge and understanding, and (4) scaffolded collaborative
leadership opportunities within small groups. The combination of these factors helped motivation to develop as students accepted leadership responsibilities with their peers, supported by the values that had been negotiated and underpinned the classroom culture (p. 193). Mutual respect and tolerance became embedded in the classroom culture.

While this research was conducted on a classroom, when one considers the implications for constructivist leadership, it could be asserted that it may be likely if these same structures were in place for adult learners, they could support the constructivist leadership of organizations such as schools or school districts. Rogoff (2003) stated, “humans develop through their changing participations in the sociocultural activities of their communities, which also change” (p. 11). In this way, as teachers participate together in learning activities during professional development, the learning community will also change. MacCallum and Morcum (2008) stated, “From a sociocultural perspective learning is conceptualized as primarily a social activity and motivation emerges from the social context that is manifested through both collaborative and individual action” (p. 193). In this way, their research suggests learning through social activities like collaboration can increase motivation and the actions of individuals.

In the next section, a discussion of the challenges of implementing constructivist leadership is warranted to help demonstrate the factors that compete for the attention of leaders in the context of educational leadership.

**Challenges of Constructivist Leadership**

For leaders who hold and value constructivist views and beliefs about learning, it can be a challenge to implement structures that align with those constructivist beliefs with leadership behaviors that take place within the learning organization each day. Regardless of
if those leaders are teachers, or building or district administrators, the challenge is still the
same. Lambert (2002) identified some of these challenges as rules, schedules, accountability
policies, hierarchal roles, timeworn practices, increasing emphasis placed on singular data
with accompanying superficial dialogue. There are several competing demands in
educational organizations today. In today’s era of accountability, school leaders are
challenged to simultaneously prepare students for statewide assessments while also placing
student learning in a broader context that has students learn at deep levels and perform well
on state assessments (Lambert, 2002). Bouck’s (2004) empirical study “Striving to be a
Constructivist Leader,” found:

> With underlying district expectations and teacher/government negotiations bearing heavily on all that she had to do, (the principal’s) constant “striving” and “struggle” toward constructivist leadership demonstrates the impact that bureaucracy can have on a leader’s efforts to be a constructivist leader. (p. 158)

Bouck’s (2004) qualitative study suggested that “those who strive to be constructivist leaders
will struggle with contradictions in policy statements that call for results in standardized
academic excellence as well as for sharing the responsibility of leadership” (p. 159). Lambert
(2002) further argued: “Schooling must be organized and led in such a way that these
learning processes provide direction and momentum to human and educational development”
(p. 35). After a close case study of a principal who consciously worked to change her
traditional leadership style into one of constructivist leadership, Bouck (2004) concluded:
“Constructivist leadership is a process of becoming; there is really no final arrival date. It is a
combination of continuous learning and leading that is unending” (p. 160).

While several theorists point to constructivism as a form of knowing and constructing
conceptual knowledge, further research and development are needed regarding putting
leadership behaviors into practice that reflect constructivist beliefs within a classroom, school, or district. Simon (1995) supported this view: “There has been a lack of connection between research on learning (which has focused on constructivism) and research on teaching (which has focused for the most part on traditional instruction)” (p. 118). These ideals, philosophies, and beliefs sound appealing in an idealistic sense, but what does it really look like in action? How do we further define the qualities leaders operating from this belief system should employ in daily leadership behaviors? How do leaders synthesize all the demands and expectations imposed on them while still facilitating learning experiences that allow learners to build on their background experiences to construct new deep, conceptual understandings? How do we move the principles, beliefs, and views of constructivist leadership into actions in a way that is still defined specifically towards effective leadership, rather than broadly and ideologically?

Motivation

Motivation has been widely studied, with researchers often elaborating on various types, effects, and costs. Sometimes hard to define, motivation is said to be the study of why people think and behave as they do (Graham, 2020; Graham & Weiner, 2011). The Latin root of the word “motivation” means “to move”; hence, in this basic sense the study of motivation is the study of action. There are several types and qualities of motivation for those studying it in psychology and education, such as needs, drives, goals, aspirations, interests, and affects. These processes have been studied from multiple perspectives in psychology (such as cognitive, developmental, educational, and social), with extensive lists of constructs and theoretical frameworks and research literature—much of which will be shared in this review. Susan Harter’s (1981) study on intrinsic versus extrinsic motivation asserted that often,
school systems gradually stifle children’s innate intrinsic interest in learning, specifically with regard to challenge, curiosity, and independent mastery. Many school cultures reinforce a more extrinsic orientation. More recently referred to as the Jenkins’ curve (Jenkins, 2015), which showed 95% of five-year-olds want to come to school and are motivated to learn what teachers want them to learn, dips to 37% by Grade 9, with a slight increase after. By the end of elementary school, two in three students do not see their classrooms as motivating and worth coming to learn (Hattie et al., 2020).

One possible hypothesis/explanation: As content becomes more abstract when students move up through the grade levels, teachers may tend to lean on more procedural, algorithmic, step-by-step instruction (one-dimensional) versus multidimensional instruction that engages students in relevant, engaging tasks to build conceptual understanding and problem-solving skills.

If we, as educators, wish to continue to engage students in a way that builds intrinsic motivation, we must provide learning opportunities for students that help them build meaning and conceptual understanding. “We want students to feel free as they work on math, free to try different ideas, not fearing that they might be wrong” (Boaler, 2016, p. 14). “When students develop interest in the ideas they are learning, their motivation and their achievements increase” (Boaler, 2016, p. 147).

Study after study shows that students who develop intrinsic motivation achieve at higher levels than those who develop extrinsic motivation (Pulfrey, Buchs, & Butera, 2011; Lemos & Verissimio, 2014) and that intrinsic motivation spurs students to pursue subjects to higher levels and to stay in subjects rather than drop out (Stipek, 1993). (Boaler, 2016, p. 147)

Our instructional model for students must be one that operates out of an incremental, or growth mindset (Rattan et al., 2011), and carefully facilitates learning experiences that are
designed to engage students with the appropriate levels of scaffolding and instructional support. Similarly, if leaders hope to further motivate and empower the adults who are responsible to do such for students, we must provide appropriate levels of mastery scaffolding and support, as well as specific feedback to adult learners, through the professional development learning experiences school leaders design, implement, and facilitate.

Motivational theory underpins the research interests for this study. A review of the literature on motivation supports the theory that if students and staff are provided the right amount of scaffolding and support, this success and strategic thinking builds agency/self-efficacy, and they are more likely to be highly engaged and empowered by consequent learning experiences. Currently, a lack of motivation appears to commonly exist in many elementary teachers and students, whether during professional development or the classroom (respectively), specifically regarding the subject of mathematics. While all classrooms and organizations work to increase employee motivation (be it that of the children or adults), I propose that this lack of motivation comes from many causes: (1) fixed mindset views about the nature of mathematics (Rattan et al., 2011); (2) inadequate mastery support (Pink, 2009) via specific, meaningful feedback (Hattie, 2008; Hattie et al., 2020); (3) lack of autonomy (Hattie, 2020; Pink, 2009); (4) low self-efficacy beliefs (Hattie et al., 2020); (5) little understanding of relevance/purpose (Hattie et al., 2020; Pink, 2009), and (6) lack of agency.

If people are motivated by working towards mastery (Pink, 2009) and making sense of their world (Boaler, 2016; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), how can we expect students and teachers alike to be highly motivated by and engaged in mathematics that is often one-dimensional (Boaler,
We must build teachers’ understanding of conceptual mathematics and pedagogy that lends to multidimensional (Boaler, 2016) mathematics instruction if they are to give effective feedback (Hattie, 2008) to students, thus providing more mastery support (Pink, 2009) to achieve learning goals (Locke & Latham, 2002). Therefore, a review of the literature on motivational theory is necessary in building an understanding of how professional development opportunities that will enhance multidimensional mathematics instruction are to be constructed in a way that will effectively provide this system of mastery support for both teachers, and in turn, students.

**Principles and Beliefs of Motivation**

“Modern theories of motivation focus more specifically on the relation of the beliefs, values, and goals with action” (Eccles & Wigfield, 2002, p. 110). Hattie et al. (2020) reviewed five recent articles by several seminal researchers/theorists in the field of education and psychology motivation (Eccles & Wigfield, 2020; Graham, 2020; Ryan & Deci, 2020; Schunk & DiBenedetto, 2020; Urdan & Kaplan, 2020) and found that four major dimensions cut across the most up to date research, including: (1) person factors comprised of self (success expectations, self-efficacy, and sense of competence); (2) social; and (3) cognitive (self-regulation, attributions, agency, and intrinsic/extrinsic regulation). This literature review contains the seminal works and empirical studies that serve as a foundation for the motivational theory that underpins the strategic design of professional development to motivate teachers from beliefs to a shift in behavior, past compliance into commitment. “But a belief about motivation or a specific guide to conduct is far from a theory” (Graham et al., 1996). Therefore, a comprehensive analysis of motivational theory and empirical studies is
necessary to inform strategic design of professional development that will shift teachers’
beliefs in ways that impact their own mindsets, self-efficacy, and pedagogy.

**Theories of Motivation: Expectancy Theories**

In this section, a review of literature regarding expectancy theories including self-
efficacy theory and control theories is warranted to provide a foundation for the research of
efficacious behaviors of teachers in this study.

**Self-Efficacy Theory**

Like mindset views of intelligence (Boaler, 2016; Dweck, 2006), Bandura (1997)
proposed a social cognitive model of motivation self-efficacy theory defined as individuals’
confidence in their ability to organize and execute a given course of action to solve a problem
or accomplish a task; characterized as a multidimensional construct that varies in strength,
generality, and level (or difficulty).

Thus, some people have a strong sense of self-efficacy and others do not;
some have efficacy beliefs that encompass many situations, whereas others
have narrow efficacy beliefs; and some believe they are efficacious even on
the most difficult tasks, whereas others believe they are efficacious only on
easier tasks. (Eccles & Wigfield, 2002, p. 110)

Bandura distinguished between two kinds of expectancy beliefs: (1) outcome expectations, or
beliefs that specific behaviors will lead to specific outcomes, and (2) efficacy expectations,
or the belief that one can efficiently perform those behaviors necessary to produce the
outcome. Bandura proposed that one’s efficacy expectations are the major determinant of
goal setting, activity choice, willingness to expend effort, and persistence (Eccles &
Wigfield, 2002). Marsh et al. (2002) found students as young as 4-5 years of age have at least
partially established a multidimensional structure of self-concept, and Valeski and Stipek
(2001) found primary grade children’s task-specific self-concept is related to their
achievement and teacher-rated engagement. Wilson and Trainin (2007) found even first
graders were able to differentiate between effort and ability, and that literacy attributions
have a significant and positive effect on self-efficacy and perceptions of competence in even
our youngest learners.

Bandura (1977, 1986) theorized that performance accomplishments provided the
strongest predictors of efficacy beliefs. According to his social cognitive theory, people gain
self-efficacy beliefs from four different sources: Mastery experiences, vicarious experiences,
social persuasions, and physiological states. Additional research suggested that perceived
performance accomplishments or perception of one’s level of mastery, are good predictors of
self-efficacy beliefs (Lopez et al., 1997). Through semi-structured interviews with 10
elementary school teachers in a rural school district who identified as having high
mathematics and mathematics teaching self-efficacy as measured by a Mathematics and
Mathematics Teaching Self-Efficacy Beliefs Survey, Ramirez (2015) found that vicarious
experiences, social persuasions, and physiological states were the most impactful factors in
the formation of mathematics efficacy beliefs. Vicarious experiences were the most
influential factors in the formation of mathematics teaching efficacy beliefs (p. xiii).

Control Theories

An understanding of control theories can help us to better understand the way
contextual factors impact the instructional decisions teachers make, and whether they are
motivated to change practice in their classrooms accordingly. Whether teachers feel it is
within their control to make pedagogical changes, as well as whether they expect their
instructional behaviors to help their students find mathematical success, can impact their
motivation for learning in professional development. Locus of control theories focus on one’s
expectancy to succeed to the extent that one feels in control of one’s successes and failures (Eccles & Wigfield, 2002, 2020). Connell (1985) demonstrated that not knowing the cause of one’s successes and failures undermines one’s motivation to work on the associated tasks. Connell and Wellborn (1991) also integrated control beliefs into a broader theoretical framework in which they proposed three basic psychological needs: competence, autonomy, and relatedness. They linked control beliefs to competence needs: “Children who believe they control their achievement outcomes should feel more competent” (Eccles & Wigfield, 2002, p.111). Skinner and colleagues (1998) proposed three critical beliefs: Means-ends beliefs, control beliefs, and agency beliefs. Agency beliefs are the expectations that one has access to the means needed to produce various outcomes. Children with strong belief in their own agency work harder, focus their attention better, are more interested in their studies, and are less likely to give up when they encounter difficulties than children with a weaker sense of agency (Skinner et al., 1998).

Engagement Theories

Because part of the problem this research sought to solve is the lack of motivation for mathematics that is often found in America, this section includes a discussion of the literature on engagement theories including intrinsic motivation theories. It is important to better understand what motivates people, including teacher participants, to engage in new learning if we are to better understand leadership for engagement in professional development.

Intrinsic Motivation Theories

Self-Determination Theory (Deci & Ryan, 1985) proposes humans are motivated by an optimal level of stimulation as well as a basic need for competence. Like the concept of mastery (Pink, 2009), Deci and Ryan (1985) argued that people seek out optimal stimulation
and challenging activities and find these activities intrinsically motivating because they have a basic need for competence, and people maintain intrinsic motivation when they feel competent and self-determined (White, 1959). Ryan and Deci (2020) later expanded by specifying six types of regulation on a continuum that reflects the degree to which the regulation of the behavior is autonomous, which includes (1) Intrinsic motivation, as well as five forms of extrinsic motivation: (2) external regulation (driven by external rewards and punishments), (3) introjected regulation (extrinsic motivation that has been partially internalized), (4) identified regulation (the person has consciously identified with the value of an activity), (5) integrated regulation (the person not only recognizes and identifies with the value or worthwhileness of the activity, but also sees it as congruent with other core interests/values), and (6) amotivation (lacking intentionality, either from a lack of competence, value, or interest).

Flow theory was defined by Csikszentmihalyi (1988) as intrinsically motivated behavior in terms of the immediate subjective experience that occurs when people are engaged in an activity. Recent research has shown that both the challenges and skills must be relatively high before a flow experience becomes possible (Massimini & Carli, 1988). According to Csikszentmihalyi, the repeated experience of flow is only possible when individuals seek out increasingly challenging tasks and expand their competencies to meet these challenges. Researchers have additionally defined enduring intrinsic motivational orientation in terms (a) preference for hard or challenging tasks, (b) learning that is driven by curiosity or interest, and (c) striving for competence and mastery, with the second component being most central to the idea of intrinsic motivation. “Empirical findings suggest that the
three components are highly correlated” (Eccles & Wigfield, 2002, p. 114). The next section reviews the literature on interest and the role it plays in motivation.

**Interest Theories**

Researchers who study the concept of interest differentiate between individual and situational interest regarding motivation (Alexander et al., 1994; Hidi & Harackiewicz, 2001; Murphy & Alexander, 2000; Schiefele, 1999). *Individual interest* refers to a relatively stable evaluative orientation towards certain domains; *situational interest* is an emotional state brought about by specific features of an activity or task. Two aspects of individual interest are distinguishable: feeling-related and value-related valences, both of which are directly related to the object/activity (Schiefele, 1999). Both feeling and value-related valences are directly related to the object or activity. Even though they are highly correlated, “it is useful to differentiate because some individuals interested are likely based primarily on feelings, whereas other interests are more likely to be based on personal significance” (Eccles & Wigfield, 2002, p. 115). Interest is more strongly related to indicators of deep-level learning than to surface-level learning (Schiefele, 1999). The next section provides a brief review of literature on goal theories and their role in motivation.

**Goal Theories**

Many researchers and psychologists have studied children’s achievement goals and their relation to achievement behavior (Ames, 1992; Anderman et al., 2002; Covington, 2000; Dweck, 1999; Lazowski & Hulleman, 2016; Linnenbrink, 2005; Pintrich, 2000; Urdan & Kaplan, 2020). Many highlight the differences in achievement regarding the nature of goals set and whether the goal is more intrinsic or extrinsic nature (Ames, 1992; Dweck, 1999; Elliot & Church, 1997; Midgley et al., 1998; Murphy & Alexander, 2000; Nichols et
al., 1990; Ryan & Deci, 2020; Skaalvik, 1997). Wentzel’s (1991) views on goals differs from the views of theorists like Dweck and Nichols in that she focuses on the content of children’s goals, rather than on mastery versus performance criteria of success (Wigfield & Eccles, 1992). The basic contention of achievement goal theory is that depending on their subjective purposes, achievement goals differentially influence school achievement via variations in the quality of cognitive self-regulation processes (Covington, 2000). Cognitive self-regulation refers to students being actively engaged in their own learning. In effect, then, one’s achievement goals are thought to influence the quality, timing, and appropriateness of cognitive strategies that, in turn, control the quality of one’s accomplishments (Covington, 2000).

Mägi et al. (2010) found that children’s self-reported performance-avoidance goals were related to their achievement outcomes. Math achievement seemed to influence children’s achievement goal orientation rather than vice versa in elementary school (the study referred to students in second through fourth grades). It was suggested that the earlier success experiences children are offered in their math classes in the early grades, the less they will be oriented toward performance-avoidance goal orientation in learning math and the more effort they will invest in their class work. They found that early failure reflected in low grades and an often-competitive classroom context may lead to children adopting a performance-avoidance orientation in other academic domains, as well.

In a meta-analysis of motivation interventions in education, Lazowski and Hulleman (2016) found that goal setting (specific, difficult task goals produce higher commitment and performance than vague goals that are easy to attain) had an average (mean) effect size of 0.67, while achievement goals (students’ goals for engaging in an activity which shape how
they approach, experience, and react to achievement situations) had a 0.38 average effect size. Other meta-analyses (Hattie, 2008; Hattie et al., 2020) have determined found that the highest effect sizes on enhancing student motivation toward achievement tasks come from those with mastery goals and strategic motivation.

Urdan and Kaplan (2020) went beyond learning and performance goals to also include avoidance and approach, including moderators such as beliefs, contextual factors, and definitions of success, including a students’ identity influenced partially by their cultural background and how it may affect their perceptions of the context, task, themselves in that situation, and their achievement goals. They proposed future research: (1) Examine integrated systems of perceptions, beliefs, goals, and affect; (2) focus on authentic contexts; and (3) understand how ethnicity, culture, gender, and intersectionality affect achievement goals.

**Expectancy/Value Theories**

This section reviews literature pertinent to expectancy, or value, theories, and the role one’s expectations and values play in motivation. It is likely that teachers’ past experiences with mathematics influence what they expect before participating in professional development, as well as whether they find the experience valuable. Much of the research in this study sought to determine what strategies used by facilitators of mathematics professional development were found valuable to teacher participants regarding supporting their efficacious behaviors and pedagogy, so it is important to develop an understanding of expectancy and value theories of motivation to understand how they can affect motivation for teachers as adult learners. These theories include attribution theory, modern expectancy-value theory, and self-worth theory.
Attribution Theory

Attribution models include beliefs about ability and expectancies for success, along with incentives for engaging in different activities, including valuing of achievement (Graham & Taylor, 2001). Attribution theories emphasize that individuals’ interpretations of their achievement outcomes, rather than motivational dispositions or actual outcomes, determine subsequent achievement strivings. Weiner (1992) argued that the individual’s explanations for achievement outcomes determine subsequent strivings and as such, are key motivational beliefs (Eccles & Wigfield, 2002). A unique contribution of attribution theory to the field of educational psychology regarding motivation is that it “addresses the antecedents and consequences of both intrapersonal attributions (how one perceives the self) and interpersonal attributions (how one perceives other people)” (Graham, 2020, p. 1). Attributions are cognitions that answer “why” questions students may have, such as “Why did I fail the exam?”

Graham (2020) reviewed the main principles of attribution theory, which is concerned with perceived causes of success and failure and discussed how teacher behaviors such as praise versus blame, help versus neglect, and pity versus anger are among the antecedents or determinants of attributions that can indirectly function as a low ability cue. Consequences are reviewed considering three properties or dimensions of causes, including locus, stability, and controllability. Each dimension is uniquely linked to particular psychological (pride, self-esteem, expectancy of success, anger, pity, guilt, shame) and behavioral (i.e., achievement, strivings of choice, intensity, persistence) outcomes, and Graham (2020) presented empirical evidence in support of each causal dimension-consequence linkage. Graham (2020) suggested future research think less about dimensional extremes (for
example, whether the cause of motivation attributed to behavior was external or internal) and more about how causes vary along any one dimension. She forwarded that most attribution research is based on participants’ recall of how they once felt or how they might feel, think, or do if particular conditions were present, but that future research methods should instead include non-experimental methods such as daily diary or event sampling procedures, longitudinal methods, and studying unconscious motivational processes.

**Modern Expectancy-Value Theory**

Eccles et al. (1983) developed and tested an expectancy-value model where choices are assumed to be influenced by both negative and positive task characteristic, and all choices are assumed to have costs associated with them precisely because one choice often eliminates other options. Consequently, the relative value and probability of success of various options are key determinants of choice. “Expectancies and values are assumed to directly influence performance, persistence, and task choice,” and assumed to be “influenced by task-specific beliefs such as perceptions of competence, perceptions of the difficulty of different tasks, and individuals’ goals and self-schema” (Eccles & Wigfield, 2002, p. 118). Eccles and colleagues defined expectancies for success as individuals’ beliefs about how well they will do on upcoming tasks, either in the immediate or longer-term future. Drawing on self-schema and identity theories (Markus & Wurf, 1987), they also linked attainment value to the relevance of engaging in a task for confirming or disconfirming salient aspects of one’s self-schema (Eccles et al., 1983). The four components of task-value involved in Eccles’ modern-expectancy-value theory include attainment value, intrinsic value, utility, and cost.

Wigfield et al. (1997) empirically demonstrated that over time, competence-related beliefs and values should become positively related to one another. Feather (1988, 1992)
found that values and ability perceptions are positively rather than inversely related. Heckhausen (1991) distinguished four different types of expectancies: situation-outcome, action-outcome, action-by-situation-outcome, and outcome-consequence. In his expectancy-value model, “motivation to act depends mainly on the value attached to the consequences of one’s behavior” (Eccles & Wigfield, 2002, p. 121).

Eccles and Wigfield (2020) have moved from expectancy-value theory to situated expectancy-value theory, or SEVT, a developmental, social cognitive, and sociocultural perspective on motivation that includes costs and benefits of doing this rather than that and includes a consideration of how individuals interpret their own performance and the messages received from different socializers regarding their activity participation and performance. For students, this could be from parents, teachers, or other students. The SEVT also considers situative and culturally focused aspects of the model, stressing the importance impact of situation and cultural background on children’s developing expectancy and value hierarchies. Costs included in the SEVT model are as follows: (1) effort costs, (2) opportunity costs, and (3) emotional costs. Benefits include (1) external rewards, (2) intrinsic satisfaction, (3) identification of core values, (4) compliance (usually leading to reduced or absent negative consequences), (5) autonomy or agency, and (6) a sense of competence.

Eccles and Wigfield (2002) discussed possible integrations of self-regulatory and expectancy-value models:

Different goals may be more or less useful to the individual or more or less interesting. We have predicted that the relative value attached to the goal should influence its placement in a goal hierarchy, as well as the likelihood that the individual will try to attain the goal. (p. 127)
Self-Worth Theory

Covington (1992, 1998) defined the motive for self-worth as the tendency to establish and maintain a positive self-image or sense of self-worth. Covington and Omelich (1979) demonstrated that both college students’ and younger students’ most-preferred attributions for success are ability and effort; the most-preferred attribution for failure was not trying. Attributing failure to lack of ability was a particularly problematic attribution that students preferred to avoid. Covington (1992) discussed strategies students develop to avoid appearing to lack ability, including procrastination, making excuses, avoiding challenging tasks, and not trying. If children try and fail, it is difficult for them to escape the conclusion that they lack ability, and if failure seems likely, some children will not try, precisely because trying and failing threatens their ability self-concepts (Eccles & Wigfield, 2002). Covington (1992, 1998) suggested that reducing the frequency and salience of competitive, social comparative, and evaluative practices, and focusing instead on effort, mastery, and improvement, would allow more children to maintain their self-worth without having to resort to these failure-avoiding strategies. Hattie et al. (2020) reinforced these concepts by naming the person factor of self (including expectations, self-efficacy, and sense of competence) as one of the major dimensions cutting across the most recent research on motivational psychology in the field. Self-worth theory is included as it may also play a pertinent role in teacher participants who have had particularly low self-efficacy regarding mathematics due to past experiences who would prefer to avoid math professional development altogether to avoid failure, particularly in front of colleagues or supervisors in the workplace.
The next section focuses on a review of literature of motivational theories that link motivation and cognition. These are included as they connect to the mathematical mindset (Boaler, 2016) one holds for themselves. If a teacher participant has a fixed, or entity mindset about their abilities to do mathematics, they likely would not be motivated to continue when faced with failure; whereas a teacher participant with a growth, or incremental mindset about their mathematics abilities may see failure as helping them in their growth process.

**Theories Linking Motivation and Cognition**

Motivational researchers and psychologists have studied how motivation and cognition interact and influence self-regulated learning. Zimmerman (2000) determined when self-observation, self-judgment, and self-reactions are favorable, particularly in response to failure, students are more likely to continue. The favorableness of one’s reaction to failure is determined by how individuals interpret their difficulties and failures (Eccles & Wigfield, 2002). Schunk (1990) emphasized the reciprocal roles of goal setting, self-evaluation, and self-efficacy. Schunk and Zimmerman (1994) suggested that self-efficacy should be higher under learning than under performance goals; some research supports this claim (Elliot & Dweck, 1988; Meece et al., 1988). Learners who feel efficacious about learning are more likely to engage in cognitive activities that improve their learning such as setting goals, using effective learning strategies, monitoring and evaluating their goal progress, and creating effective physical and social environments for learning (Schunk & DiBenedetto, 2020).

Hattie et al. (2020) included cognitive aspects as one of the four major dimensions that cut across the most up-to-date research, including self-regulation, attributions, agency, and intrinsic/extrinsic regulation in this dimension. Hattie et al. (2020) suggested a greater focus
on metacognitive monitoring in future research related to how the individual assesses the quantity and quality of their motivation during goal pursuit and attainment.

The next section reviews the literature focusing on theories of motivation and volition. These are included as they have implications to better understanding the ways in which teachers’ motivation to participate in professional development is affected by the contextual factors that surround them, such as district, state, and building administrator expectations. These forces can either support, if aligned, or distract from teachers’ new learning and implementation of new pedagogies.

Theories of Motivation and Volition

The term “volition” refers to both the strength of will needed to complete a task and the diligence of pursuit (Corno, 1993). Kuhl (1987) proposed several volitional strategies to explain persistence in the face of distractions and other opportunities: cognitive control strategies that help individuals stay focused on the relevant information, avoid distracting information, and optimize decision-making. These include emotional, motivational, and environmental control strategies. Ryan and Deci (2020) specified six forms of extrinsic motivation that systemically vary in their degree of autonomy or the extent to which the motivation is fully internalized and is volitional (external, introjected, identified, integrated, intrinsic, and amotivation).

In Appendix A (Theories of Math Motivation), a summary of the research relevant to motivation critical to the literature in this section is included. In the next section, a discussion of the literature on motivation and its role in mathematics learning—specifically, the oft-found lack of motivation and acceptance of a belief of fixed math ability widely held in America—is included.
Motivation and Mathematics Instruction

“The mean decline of academic intrinsic motivation over the ages for Math, Science, and Reading, and the absence of decline for Social Studies, indicates that academic intrinsic motivation is related to school curriculum” (Gottfried, 1985, p. 8). The Third International Math and Science Study in 1995-1996 (Ozgun-Koca & McCann, 1999) found declining math and science motivation, particularly in 8th and 12th grades. Additionally, Goffried et al. (2001) found:

As the greatest decline in the present study occurred in math, this further supports our previous findings that math is a unique subject area regarding academic intrinsic motivation. Perhaps the conditions that are detrimental to other subject areas, and to school in general, are more detrimental for math. (p. 10)

There are several possible causes for the decline in academic intrinsic motivation as students continue to grow older. Math has been perceived by high school students to be harder than other subjects (American Association of University Women, 1994; Eccles et al., 1984; Stodolsky, 1988). Fifth-grade students reported that they would be able to learn social studies, but not math, on their own, indicating that social studies is perceived as easier (Stodolsky et al., 1991). In a survey of high school teachers (Stodolsky & Grossman, 1995), those teaching math experienced less autonomy regarding course content than did social studies teachers. Possibly teachers communicated their lack of autonomy to students or the curriculum allows for less student autonomy (Gottfried et al., 2001). Stodolsky (1988) found that there were more diverse routes to learning social studies than math, which is typically directed by teachers. For these reasons, students may perceive greater autonomy in learning social studies (Gottfried et al., 2001). Grouws and Lembke (1996) stated that, for math in comparison with other subject areas, individuals have lower expectations of success, do not
believe that most people can do math in comparison with reading or writing, and do not see the need for math outside of school. Such views may be related to lower academic intrinsic motivation (Gottfried et al., 2001).

Other researchers determined that entity motivational frameworks predicted higher math anxiety, even in first and second grade students. High math achievement was a particularly strong predictor of lower math anxiety and less entity-oriented motivational frameworks. Reciprocal effects are already present in the first two years of formal schooling (Gunderson et al., 2018).

In a four-year longitudinal project with 865 first, second, and fourth grade children attending ten elementary schools, Eccles et al. (1993) found that even first grade students held more positive beliefs about their perceptions of confidence in math, reading, sports, and instrumental music. Overall, boys had more positive competence beliefs and values than girls for sports and mathematics, while girls held more positive competence beliefs and values for reading and music activities.

In their quantitative study that utilized path analysis with 350 undergraduates (229 women and 121 men), the majority of whom were enrolled in classes in the College of Education, Pajares and Miller (1994) found:

math self-efficacy was more predictive of problem solving than was math self-concept, perceived usefulness of mathematics, prior experience with mathematics, or gender. Self-efficacy also mediated the effect of gender and prior experience on self-concept, perceived usefulness, and problem solving. ...Men had higher performance, self-efficacy, and self-concept and lower anxiety, but these differences were due larger to the influence of self-efficacy, for gender had a direct effect only on self-efficacy and a prior experience variable. (p. 193)

The study methods included administering math self-efficacy scales, a perceived usefulness rating instrument, a math anxiety scale instrument, math self-concept self-description
questionnaires, reported prior experience in math coursework, and a math problems performance scale to participants, utilizing path analysis. Results of this study supported the hypothesized role of self-efficacy as in Bandura’s (1986) social cognitive theory.

Turner et al. (1998), through their research of self-reported involvement in mathematics of 42 students in upper-elementary classrooms, found that students in high-involvement classrooms reported a significantly more positive affect; reporting challenges and skills as above average, whereas students in low involvement classrooms reported skills as exceeding challenges. Their analyses of instruction in the high involvement classrooms found that teachers scaffolded instruction by negotiating understanding, transferring responsibility, and fostering intrinsic motivation in students. Conversely, instruction in the low involvement classrooms was characterized by Initiation-Response-Evaluation sequences, emphasis on procedures, and extrinsic motivation strategies. Their results suggested involvement can be socially constructed through whole class instruction.

Regarding teachers of mathematics, several studies have concluded that teachers find value in learning content that helps them with daily lessons, whether that is PD focused on utilizing a newly adopted resource (Hill et al., 2020) or content aligned to other contextual elements (instructional initiatives being required of them by their principal, district, or state). Elliot et al. (2009) determined that when leaders of mathematics PD can clearly articulate for teachers how the math conceptual knowledge is tied to classroom teaching in accessible ways, this will help them assure relevance in their PD. One way they suggested this can be done is by cultivating sociomathematical norms and employing practices for orchestrating discussions with clearly articulated ties to the mathematics concepts they will be responsible for teaching each day. Other studies have shown mathematics PD is successful when teachers
are involved in solving tasks like those they will then facilitate with their students (Koellner et al., 2011), when teachers feel the PD has deepened their content knowledge and given them engaging strategies that can be used in the classroom (Martin et al., 2018), and provides them opportunities to collaborate with their colleagues (Rogers et al., 2007). Other research has shown that even in undergraduate college students, use of “Mathematical Mindset,” (MM) or “rich-task” type math problems (Boaler, 2016, p. 57) can increase motivation in problem solving, while problems formulated in a standard way produced a decrease in reported motivation levels, even in participants who were not informed about or explicitly aware of MM theory (Daly et al., 2019).

However, studies have also shown that teachers find PD experiences that deepen their content knowledge and give them engaging strategies to be used daily in the classroom are limited, and the details around such PD experiences were limited as well. If teacher motivation towards math PD and mathematics instruction is based on the relevance the PD has to their daily math instruction and these opportunities are limited, it is critical that further research shed light on just how facilitators of math PD can provide effective PD experiences that motivate teachers to impact student learning.

It may be that only the most intrinsically motivated can forge through higher level math (Gottfried et al., 2001). We can no longer allow this to be the case. If we wish to truly close the achievement gap that currently exists in mathematics in America for students, we first must move teacher beliefs and views about the nature of mathematics so that we can ensure conceptual, multidimensional mathematics experiences for ALL students.

Instructional leadership, through strategically designed professional development opportunities that motivate and empower teachers, is the first step in shifting mathematics
mindsets and motivating mathematicians. Future research is needed to determine the effect of various constructs of strategically designed professional development activities on the motivation and mindset of teachers, as well as their students. For example, how might PD opportunities that take teachers’ current beliefs/views about the nature of mathematics into account affect teachers’ motivation and mindset about the nature of mathematics? What constructs would need to be part of the professional development to make it effective? How would one measure and consider effectiveness regarding teacher buy-in and commitment, as well as student growth? A review of the literature about the nature of mathematics education and motivational theory indicated this was a content area in great need of deeper conceptual understanding, not only regarding the nature of mathematics, but also in regard to the types of opportunities we provide students to ensure deeper conceptual understanding is achieved. Before we can provide students with these learning opportunities, we must start by experiencing them ourselves—the leaders of learning organizations, as well as the leaders of students: their teachers.

**Leadership for Professional Development**

A 2016 report found that over $18 billion is spent each year in the United States on teacher professional development programs, and that teachers spend 68 hours each year on professional development activities directed by school districts. When self-guided professional learning and courses are included, the annual total comes to 89 hours (Gates Foundation, 2016). Much research has been conducted to determine key features of effective PD. Desimone and Garet (2015) reviewed evidence from cross-sectional studies, longitudinal studies, literature reviews of qualitative and quasi-experimental studies, as well as some randomized control trials that documented the success of such PD across different ages and
subjects to present a conceptual framework for effective professional development that includes five key features that should be in place for the PD to be effective in regard to improving teaching practice and student learning: (1) content focus, (2) active learning, (3) coherence, (4) sustained duration (ongoing throughout the school year and including 20 hours or more of contact time), and (5) collective participation (teachers from the same grade, subject, or school participate together). From this five-feature conceptual framework, five ideas were proposed to help districts and schools put effective PD into practice, including (a) changing procedural classroom behavior is easier than improving content knowledge or inquiry-oriented instructional techniques; (b) teachers participating in the same PD vary in responses; (c) PD is more successful when explicitly linked to classroom lessons; (d) PD research and implementation must account for “urban contexts,” (defined, for example, by teacher and student mobility); and (e) leadership plays a key role in supporting and encouraging teacher implementation of what is learned at the PD. Desimone and Garet (2015) also noted current trends in PD, including (1) movement away from one-time workshops to more sustained, content-focused PD; (2) the linking of PD to teacher evaluations; (3) increased use of video as a tool for classroom observation and coaching, as well as cataloguing resources for teachers; and (4) development of use of high-quality classroom observation rubrics.

Challenges in PD included (1) tensions between having multiple providers and achieving coherence, as well as having multiple purposes; (2) multiplicity of PD, or the idea that it is conducted in so many ways that it is difficult to draw conclusions about what specifically contributes to the success or failure of PD efforts, which also leads to difficulties; (3) tracking PD for individual teachers to determine whether, and which, patterns of
participation led to results for teachers and students; and (4) explicitly stating what teachers
should learn from PD by linking ideas or behaviors taught to the materials teachers are using
in the classroom (Desimone & Garet, 2015).

Darling-Hammond et al. (2017) conducted a study examining research on professional learning that has proven effective in changing teachers’ practices and improving student outcomes to identify elements prevalent in successful PD models. Thirty-five studies were reviewed that met the following methodological criteria: the study (1) featured a careful experimental or comparison group design, or (2) analyzed student outcomes with statistical controls for context variables and student characteristic. Each study was coded to generate the elements of the effective PD models. Their review confirmed and expanded upon the previously named five components of high-quality professional learning for teachers articulated by Desimone (2009) and Desimone and Garet (2015). Seven characteristics of effective PD were identified: (1) content focused; (2) incorporates active learning utilizing adult learning theory; (3) supports collaboration, typically in job-embedded contexts; (4) uses models and modeling of effective practices; (5) provides coaching and expert support; (6) offers opportunities for feedback and reflection; and (7) is of sustained duration.

The next section specifically focuses on the literature regarding leadership for professional development recommendations that have been made specific to the area of mathematics.

**Leadership for Professional Development for Teachers of Mathematics**

Hill et al. (2020) conducted a meta-analysis of research on STEM teacher development programs to find out how much of an effect programs have on student outcomes, as well as which aspects of these programs are associated with learning gains.
They analyzed 89 research studies of programs that included professional PD for STEM teachers. Of these, 71 also included new curriculum materials for teachers to use in their classrooms. They also included six research studies that only focused on curriculum materials, but not professional development. They then read through these studies and created a data set with information on the size of the program’s impact on student achievement, the type of assessments used to measure student outcomes, and the type of programs studied. Information on the length, focus, activities, and program format of the PD was also included. They found that the PD programs that significantly associated with above-average student gains included (1) those that focused on new curriculum materials, (2) aimed at improving teachers’ knowledge of content, pedagogy, and/or how students learn, and (3) included meetings to troubleshoot and discuss classroom implementation of the program, same-school participation and collaboration, and/or summer workshops that allowed for concentrated learning time. They noted that limitations of this meta-analysis included being unable to determine whether each of these programs would be successful in another context— for example, in a high-poverty setting, or in another district or school context.

Martin et al. (2018) conducted a research study to examine teacher perspectives on focus areas for literacy and mathematics PD that teachers identified as beneficial, as well as how PD experiences in literacy and mathematics compared to one another, and how teachers believed PD influences students’ learning. Participants included 98 elementary and middle school teachers from two southeastern states. A survey was conducted, and results were coded according to the focus of the professional development experience that the teacher described as most beneficial for both content areas over the last three years. In both the areas of literacy and mathematics, they found teachers appeared the most positively impacted by
PD that deepened their content knowledge and provided engaging strategies for the classroom. The last largest coded category in the responses was the category of non-specific and no notably beneficial professional development. This was the largest category in mathematics. Teachers responded that while they were interested in having a greater and deeper understanding of these content areas and learning engaging strategies to directly be implemented in the classroom, they found these opportunities are lacking in both literacy and mathematics PD. Most teachers in the study viewed PD as positive for their students. One limitation of this study was that details as to what the teachers experienced during PD was limited. Researchers recommended more studies are needed to examine the influence of research-based PD experiences that connect what teachers’ value as well as what administrators and PD facilitators value. Many teachers in the study indicated their voices are not a part of the planning and they were therefore unable to identify beneficial PD experiences in the last three years.

Mathematics leaders have made empirically-based recommendations for teachers’ mathematics instruction, including posing cognitively demanding math tasks, assessing students’ understanding and using that information to implement tasks, and enacting a comprehensive curriculum of cohesive activities that support students’ mathematical understanding (Polly et al., 2015). However, other research has shown teachers struggle to implement these recommendations in their classrooms (Boston & Smith, 2009; Carpenter, Fennema, & Franke, 1996; Polly et al., 2014; Stein et al., 1996). Much research has been conducted with recommendations made regarding how to teach pre-service elementary teachers how to teach math at the university level (Boerst et al., 2011; Fast & Hankes, 2010; Kutaka et al., 2018).
Specific to the area of PD as it relates to in-service/practicing teachers of elementary mathematics, several studies have sought to identify characteristics of influential facilitators of, define effective mathematics PD, and conceptualize the work of leading mathematical tasks in PD (Elliott et al., 2009; Linder et al., 2013; Rogers et al., 2007); measure the effects of PD on teacher knowledge and students’ evidence of math understanding (Bell et al., 2010; Polly et al., 2014; Polly et al., 2015); as well as to describe how teachers interacted with and take from the PD experience (Farmer et al., 2003; Rogers et al., 2007). Empirically based research suggests that effective mathematics PD include the following elements:

(1) Inclusion of authentic and readily available student-centered mathematics learning activities; (2) rich opportunities for discussion and reflection; (3) an open, learner-centered implementation component; and (4) an inquiry stance taken by the facilitators (Farmer et al., 2003). Research providing teachers’ perspectives supports these characteristics, as teachers reported finding math PD effective when opportunities and resources were provided for them to practically apply what is learned to the classroom, experience activities and learn concepts in a manner like their students, and when the PD provided teachers support systems as well as collaboration with colleagues (Rogers et al., 2007). Facilitators of math PD report it to be effective when similarly, the PD provides teachers opportunities to be learners, providing resources and classroom applicability, as well as adding the need for PD to improve teachers’ knowledge of teaching mathematics (Rogers et al., 2007).

Koellner et al. (2007) utilized the knowledge domains identified in Ball et al. (2008) to implement a PD model designed to help teachers deepen their MKT through a cycle of solving a math problem, teaching that the problem, and analyzing first teacher questioning and then student thinking in video observations of their own teaching. The researchers then
analyzed artifacts from two years of a series of monthly, full-day workshops with ten middle school math teachers, including workshop videos and interviews with facilitators. Through analysis of teacher interactions, it was found that different learning opportunities were afforded by different activities. Specialized Content Knowledge (SCK) was developed by comparing, reasoning, and making connections between the various solution strategies; knowledge of content and teaching was developed by analyzing teacher questioning in the video clips from the teachers’ lessons, and knowledge of content and students was developed by analyzing students’ strategies. The researchers also found that the teachers’ own reflection and discussion of the nature of student thinking and teacher questioning of students led teachers to extend their MKT as they re-engaged with the math task and reconsidered how they might teach through the lens of their new learning for how students might approach it.

Research has demonstrated that while it can be effective for facilitators of math PD to utilize tasks that enable teachers to serve as student learners to facilitate mathematical discussions to deepen mathematical content knowledge, often leaders of math PD feel discomfort with facilitating this discourse with adult learners for fear it may uncover their own lack of understanding or create shame in teachers who may be unable to do so (Elliot et al., 2009). Similarly, while research has shown it is important facilitators are able to carefully articulate purposes for pedagogical moves or mathematical content strategies, facilitators also acknowledged they sometimes have difficulty doing this at the level of specificity they know will support teachers in learning to support students (Elliot et al., 2009). Researchers noted that teachers often made comments, and had a stance or belief system regarding their fixed ability regarding the nature of mathematics, such as whether they were good at math, and sometimes facilitators did, as well. This verbal positioning sometimes opened further
mathematical discussion, and other times, shut it down. The facilitator had to establish norms and be skilled in sequencing and structuring discussion and discourse during PD sessions, as well as understand the mathematical content knowledge in order to move the discussion along productively (Elliot et al., 2009).

Given what we do know regarding effective PD for elementary math teachers, a gap remains regarding what specifically facilitators can do to take into account the teachers’ existing beliefs and conceptual understanding of mathematics to support teachers in helping students learn mathematics concepts. Empirical research often recommends broad features of successful math PD experiences, which is at times, conflicted in support among various studies.

Copur-Gencturk et al. (2019) found a statistically significant relationship between a focus on curricular content knowledge and examining students’ work during PD and increases in teachers’ MKT. Their findings did not support other previously introduced empirically grounded categories for content focus (Desimone et al., 2002; Garet et al., 2001; Scher & O’Reilly, 2009), including: (1) subject matter knowledge, (2) knowledge of students’ thinking in a specific subject matter area, and (3) general pedagogical knowledge, or these categories of PD enactment strategies (Garet et al., 2001): (1) observing or being observed, (2) planning classroom implementation, (3) presenting, leading and writing, and (4) solving mathematics problems (Bell et al., 2010; Koellner et al., 2011). However, they acknowledged that the facilitators of these PD experiences did not often use observation, so this could be an explanation for why this was not found to have a large effect size on teacher increase in MKT. They also found that it was not so much the duration of the PD as the content focus and enactment strategies utilized that seemed to matter in relationship to
increases in teacher MKT because of participation. Silver et al. (2007) suggested that PD experiences in which participants have opportunities to build or strengthen connections among related mathematical ideas and consider them in relation to student thinking about the ideas and to a range of pedagogical actions and decisions that could affect the students’ opportunities to learn are beneficial. However, their work cannot disentangle several factors that likely influenced these outcomes (use of Professional Learning teams, or PLTs focused on mathematics, coordination of PD sessions around a core set of foundational ideas and frameworks, the roles of facilitators in keeping a focus on mathematical ideas and the mathematical basis for pedagogical decisions, high levels of participant engagement and interaction, the willingness of participants to work and reflect on their math knowledge and instructional practice publicly). Their study, like previously mentioned studies which called for further research to clarify PD constructs/strategies, suggested further research with a more complex research design would allow analysis of and contribution of each factor.

**Professional Development for Equity in Mathematics**

A review of the literature on equity in mathematics classrooms was conducted using search terms “math professional development” and “elementary education” and “equity” or “inclusion.” Much of the literature found referred to gender inequities or those of English Language Learners who struggle to understand the mathematics vocabulary used. Recommendations for PD that focused on elementary mathematics was limited. However, below is a summary of connected literature and themes I found that suggest that the following practices would be beneficial to include in elementary mathematics PD towards equity.
**Safe Environments for Classroom Discussions/Discourse**

Teaching elementary teachers how to set productive norms to support classroom discussions and discourse that allow for all student voices to be heard is one way to ensure equitable access to mathematics thinking for all students. Boaler (2006) also found using norms (exploring multiple methods, discussing ideas) helped students to appreciate insights from their peers of different cultures. Similarly, Knott (2007) found professional development that trained K-12 math teachers to address issues of status that occur in the classroom by teaching them about community agreements, group roles, and protocols to use in small groups helped teacher participants see positive results among their students after two years, including equal time for all students to have a voice in their mathematics classroom. Teachers can also support a safe environment in their classrooms by setting new expectations, including praising students for sharing wrong answers to help the group understand misconceptions, and facilitating class discussions to develop norms that acknowledge fears about sharing questions, answers, or ideas with the group, then committing to staying positive and supporting one another (Allen & Chval, 2009).

**Multidimensional/Constructivist Classroom Pedagogy**

Berryman et al. (2015) provide a culturally responsive framework for social justice which includes the following themes that align with multidimensional beliefs about the nature of mathematics and constructivist approaches to classroom math instruction:

1. When people can determine their own questions and sense-making processes, we may all be better places to learn;
2. Learning is dialogic, active, problem based, holistic, and spiraling when learners raise their own questions and evaluate answers and when knowledge is collaboratively co-created through relationships and interactions of reciprocity. (p. 158)
Similarly, Boaler’s (2016) strategies for making math education more equitable include offering all students equal access to high-level content through low floor/high ceiling tasks (have multiple access points), teaching students to work together collaboratively, changing ideas/mindsets about who can achieve in math, and giving girls and students of color additional encouragement to learn math and science. Boaler (2006) found that allowing students to read the math problems aloud together and discuss their ideas when problem-solving led to the group finding the task sufficiently challenging and “group-worthy,” which further engaged students to work together to support one another by asking questions and persisting (p. 366).

**Integration of Real-world Experiences that are Relevant to Students’ Culture**

Research supports the idea that experience-centered instruction intersects with best practices in teaching mathematics. Sheppard (2011) recommended teachers predicate their instruction on the experiences and interests of African American students, as well as scaffold based on students’ interests, which in turn demonstrates a commitment to their students as learners and demonstrates the effectiveness of using the students’ prior knowledge as a bridge to new learning, including an understanding of the deep structures of students’ interests and experiences (Ladson-Billings, 1997). Connecting mathematics with students’ life experiences and existing knowledge is also a supported practice towards mathematics equity with English Language Learners (ELLs) (Barwell, 2003; Chval et al. 2015; Secada & De La Cruz, 1996).

**Focus on Language with English Language Learners (ELLs)**

As seen in Chval et al. (2015), research recommends teachers support ELL’s math learning by (1) creating classroom environments rich in language and mathematics content
(Anstrom, 1997; Khisty & Chval, 2002); (2) by emphasizing meaning and the multiple meanings of words—at times, by using gestures, drawings, or their first language while developing command of the target language and mathematics (Moll, 1988, 1989; Moraeles et al., 2003; Moschkovich, 2002); (3) through the use of visual supports such as concrete objects, videos, illustrations, and gestures in classroom conversations (Moschkovich, 2002; Raborn, 1995); (4) by connecting language with mathematical representations (Khisty & Chval, 2002); (5) by writing essential ideas, concepts, representations, and words on the board without erasing so students can refer back to it throughout the lesson (Stigler et al., 1996); and (6) by discussing examples of students mathematical writing and providing students opportunities to revise their writing (Chval & Khisty, 2009). In general, teachers can do this by supporting the development of mathematics, supporting the development of language, enhancing mathematical tasks in curriculum materials, and by establishing, facilitating, and maintaining productive classroom interactions (Chval et al., 2015). This can also be accomplished through teachers’ use of rich mathematical talk when modeling and facilitating discussions in their mathematics classrooms (Khisty & Chval, 2002).

Each of these recommendations towards mathematics equity in elementary classrooms, if integrated into professional development for elementary math teachers, would support equitable mathematics pedagogy.

Chapter 3 gives an overview of the methodology used for this research study to construct grounded theory for the sense of efficacy for teachers of elementary mathematics.
CHAPTER 3

METHODOLOGY

Many elementary teachers have a lack of conceptual understanding about mathematics and therefore are often unable to design and/or facilitate meaningful mathematics tasks. There was a gap in existing literature providing insight into the theory behind professional development practices specific to the area of mathematics for practicing elementary teachers about how their past experiences and self-efficacy impact their participation and mindset in mathematics. Most mathematics PD studies had been conducted with secondary or pre-service teachers, and/or did not provide specifics as to which aspects of PD were powerful for teachers (Boston & Smith, 2009, 2011; Elliott et al., 2009; Polly et al., 2014).

Most teachers agree that the role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics (National Council of Teachers of Mathematics, 2014). However, agreement with this principle is often not enough to impact pedagogy. While high-quality, effective professional development for practicing elementary mathematics teachers is needed, much of the current literature recommendations do not account for how a teacher’s own past experiences and self-efficacy impacts their participation in elementary mathematics professional development experiences. Teachers’ pedagogical knowledge, beliefs, and past experiences must be taken into account by facilitators when planning PD. Specific to the area of mathematics, where fixed mindset beliefs (Boaler, 2016) are so prevalent, one might argue it is of the utmost importance to take teachers’ prior experiences, mindsets, and beliefs about the nature of mathematics into account if attempting to move behaviors past compliance into
commitment; or even more, empowering and motivating teachers from principles to actions (National Council of Teachers of Mathematics, 2014).

Improving the level of rigor and cognitive demand required of students in math classrooms in America requires that educational leaders provide teachers with professional development opportunities that are transformative in nature, leading to changes in teachers’ long-held, underlying beliefs about effective teaching and learning of mathematics (Thompson & Zeuli, 1999). A need exists to determine which aspects of professional development are most powerful for teachers (Polly et al., 2014), particularly when considering their background experiences, mindset, and self-efficacy around mathematics upon entering PD experiences.

The purpose of this constructivist grounded theory qualitative research study was to develop a theoretical framework of nine teacher participants’ sense of mathematics efficacy including K-12 schooling and throughout professional development. My pragmatic goal was to develop a framework for implementing efficacious professional development. The units of analyses for this study are teacher participants’ theories related to their self-efficacy in mathematics. Teacher participants are defined as elementary teachers who teach kindergarten through fifth grade mathematics. The research questions were:

RQ1: What is the theory that explains the process for enhancing the efficacies of teachers in a large suburban school district?

- Which parts of the mathematics professional development process do participants attribute to their theories regarding efficacy in math?

- What theories do they generate about the nature of mathematics because of the PD process?
RQ2: What professional development theories contribute to shifting mathematics pedagogical practices?

- What motivates teachers to engage in professional development?
- In what ways do their pedagogical practices change?

RQ3: What do findings suggest about leadership theories to support the facilitation of mathematics professional development?

- How do leaders facilitate safe environments for enhancing the efficacies of teachers?
- What support do participants need for trying out theory-based pedagogical practices in classrooms?

The intent and significance of this study may lead to PD opportunities for teachers that address the background experiences, self-efficacy, and beliefs teachers bring to facilitate more meaningful learning for teachers. A theoretical framework to explain teacher participants’ sense of efficacy for a mathematics professional development program could in turn contribute to increased quality in educational experiences offered to students by all teachers on a large scale in elementary mathematics. The framework is intended to aid educational leaders in designing PD experiences for all teachers that are meaningful and personalized based on their existing self-efficacy, mindsets, and beliefs about the nature of mathematics. Additionally, the study will help fill the existing gap in the empirical literature by providing further explanation of specific strategies that might address the motivation, mindsets, and pedagogical practices of practicing elementary school teachers.

This chapter gives a rationale for qualitative research, an overview of the design of this research study and data analysis procedures, and it describes limitations, validity,
reliability, and ethical considerations for the study. I begin with a rationale for the choice of qualitative methodology for this research study.

**Rationale for Qualitative Research**

A qualitative approach to research how elementary teachers learn new mathematics concepts probably seems like an intriguing choice, since many would associate mathematics with quantitative research—numbers. However, here, it is not the numbers themselves that were of interest. It is the teachers: elementary teachers, many of whom may not have taken great pride in their ability to do mathematics at high levels prior to stepping into the classroom. After all, is not one of the great math (teaching) myths that if you can do the math, you can teach it? Research has found this is not always the case. Often teachers themselves hold misconceptions and a limited conceptual understanding of mathematical topics, and they may not understand how working with different media and manipulatives can contribute to student thinking and learning in mathematics (Walker, 2007). Further, Hoover et al. (2016) found that “while the field lacks a theoretically grounded, well-defined, and shared conception of mathematical knowledge required for teaching, there appears to be broad agreement that a specialized body of knowledge is vital to improvement” (p. 1). Additionally, studies of analyses of correlations between teacher certification in math or a degree in math reveal no advantage at grades K-8 (Thames & Ball, 2010).

Patton (2015) stated, “Qualitative research inquires into...documents, and interprets the meaning-making process” (p. 3). Qualitative research studies how things work by getting inside the phenomenon of interest to gain detailed, descriptive data. A major way to do this is “to capture people’s stories about how things work” (Patton, 2015, p. 6).
Qualitative research was chosen for this study because it seeks to study how adult learners, specifically elementary mathematics teachers, acquire the conceptual understandings necessary to facilitate meaningful mathematics instruction because of participating in PD. Each teacher has their own mathematics background story they bring with them into professional development. Each teacher also has a story about how the professional development experience works for them. It is the intersection of these stories, and what makes the PD experiences meaningful for these teachers, that is at the heart of this research study. Teachers’ background experiences cannot be quantified. Their student achievement data can be, but qualitative data, as utilized in this study, takes us beyond the numbers to explain the theory of this process. Turner (2009) explained:

> theory helps us to build an edifice of concepts and explanations to understand social reality…an argument in which the social theorist strives to convince others about the nature of social reality by the use of evidence, narratives, hunches, concepts, and even material objects as “exhibits.” (p. 4)

In the same way, theory through concepts and explanations aids the understanding of participants’ social realities regarding the development of mathematics efficacy through an illumination of their overall experiences, including K-12 schooling and professional development (Turner, 2009). Maxwell (2013) maintained theory also operates to organize the study in the following manner:

- Shapes the study’s design.
- Supports the assessment and refinement of its goal(s).
- Helps in the formation of realistic and relevant research questions.
- Informs the methods.

Theory requires empirical data and without data it is hollow; yet, empirical data also needs theory; “without theory we are blind” (Turner, 2009, p. 4).
Qualitative researchers do not feel that the numbers themselves hold no value, but instead, the qualitative researcher asks what the numbers tell about the assumptions of the people who use and compile them. Qualitative researchers, then, are concerned with how enumeration is used by subjects in constructing reality (Bogdan & Biklen, 2007). In this way, qualitative research is the perfect fit for describing the way teacher participants construct reality during professional development.

Grbich (2013) provides a background summary of the historical progression of the way people have generally accepted the acquisition of knowledge, how each worldview/paradigm evolved over time, and what types of research fall under each (pp. 6–9). Grbich (2013) began the exploration of the history of qualitative research with the 18th century positivism and move to realism (more recently, postpositivism), which included types of research such as Classical ethnography and Straussian grounded theory. Grbich (2013) then depicted industrialization at the turn of the 20th century, at which point reality was being viewed as “power directed and multiply constructed” (p. 7), and types of research included “any qualitative approach that has taken a critical stance, including grounded theory, phenomenology, ethnography, hermeneutics, sociolinguistics, narratives, and feminist research” as part of the “critical emancipatory grouping” (p. 7). These approaches are rooted in constructionism and rely heavily on the idea that different people experience things differently; that there are multiple realities. “Powerful individuals…with an authoritative point of view became rejected in favor of anti-heroes and complex multidimensional individuals” as part of the postmodern and poststructuralism era, occurring during the last decades of the twentieth century” (Grbich, p. 8). Again, the idea of reality as being multiply constructed and transitional is adherent in poststructuralism research types, which “seek to
deconstruct discourses that have been established to control ways of thinking” (Grbich, p. 8) and include critical stances. Most recently, mixed/multiple method approaches often integrate qualitative and quantitative traditions to best answer the research question. This history of these theoretical traditions is important for qualitative researchers to understand when considering their methodology so they can select the research paradigm that best fits their study’s interest, which will help researchers determine and use appropriate techniques to ensure validity and reliability of their studies. For these reasons, grounded theory was the best fit for generating theory regarding the experiences of elementary mathematics teachers.

**Grounded Theory: A Historical Background**

Grounded theory was purposefully selected for this research study as it focuses on the “phenomena of human experiences within the world of social interaction. The assumptions come originally from symbolic interactionism and presume reality is a constructed and shifting entity and that social processes can be created and changed by interactions among people” (Grbich, 2013, p. 80). Similarly, the way teacher participants experience the phenomenon of professional development is also a constructed and shifting reality that involves social processes among colleagues.

Grounded theory, as a qualitative research methodology, was originally an approach that emerged in the 1960s from two academics: Barney Glaser (a psychologist) and Anselm Strauss (a sociologist) (Glaser & Strauss, 1967).

[Glaser & Strauss’ grounded theory] was viewed as a way to shift researchers from theory directed to theory generating research using observations of reality to construct both meaning and theories that were relevant to the mid-twentieth century, rather than those that had been previously developed during the late nineteenth-century. The primary focus [was] the investigation of the context of the setting within which the day-to-day lives of people were occurring—their interactions, their behaviors and
their constructions of reality, which were further reconstructed through researchers’ frames of reference. (Grbich, 2013, p. 80)

While Glaser and Strauss jointly devised this approach, they shortly after went separate ways, emphasizing different components in their approaches to data analysis (Glaser, 1978; Strauss, 1987). Regarding style, Glaser’s approach focused on discovery, while Strauss’ focused on verification. About the research question(s), Glaser’s approach focused on problem and variations, while Strauss’ focused on dimensionalising and critiquing.

Regarding the research process, Glaser focused on emergent directions, while Strauss focused on coding and hypothesis testing. Glaser’s literature review was ongoing from first category identification, while Strauss’ emerged when categories did—if desired by the researcher. Glaser’s coding approach is one of constant comparison of words, lines, and sections; Strauss’ has three levels of data fracturing involved and focuses on words, lines, and paragraphs. Glaser views axial coding as unnecessary, while Strauss utilizes a meticulous procedure for coding. Selective coding for Glaser’s approach refers only to core variables, while Strauss’s approach applies core categories to other categories. Glaser’s focus is on theory generation, while Strauss’ is on theory verification (adapted from Grbich, 2013, p. 81). Glaser also held stronger views about theoretical sensitivity than Strauss, with processes such as theoretical sampling, memoing, saturation, and substantive coding central to his approach (Glaser, 1978). Strauss, on the other hand, included processes such as dimensionalising and subdimensionalising, open coding, theoretical memos, axial coding, selective coding, integration, and formal theory.

Constructivist grounded theory, as introduced more recently by Kathy Charmaz (2006, 2014), differs from Glaser and Strauss regarding the relationship between researcher
and researched and data accountability. Charmaz challenged the previously “objective” relationship between researcher and participants and instead suggested they are partners in data generation. She refocused on the critical role of the researcher’s reflection, recognition, and management of their own biases (Grbich, 2013, p. 88). Charmaz also suggested that “immersion in and transposing raw data into memos is one way of keeping close and accountable to data to maintain participant voices,” and that “using non-scientific writing styles” that are more literary and postmodern will “forefront the voices of the researcher” (Grbich, 2013, p. 88). Criticisms of Charmaz’ approach by Glaser (2002) were that her approach was a misrepresentation of his notion of bias, and that constructivism legitimizes “forcing” the data into the researcher’s view. He also more recently (2009) accused Charmaz of “jargonising,” or creating meaningless labels to his original grounded theory approach for her own benefit.

Other combinations of grounded theory are now present, including “quasi” grounded theory, in which the original Straussian approach has been modified so that perhaps only open coding or some form of this is incorporated. Glaserian grounded theory and Husserlian phenomenology have also been combined (Baker et al., 1992), in which “bracketing is followed by open coding from the emerging codes and categories, which are then linked to a conceptual framework” (Grbich, 2013 p. 89). Regardless of the criticisms various grounded theorists have of one another’s’ methodology or definitions, grounded theory is still said to be “the most influential paradigm for qualitative research in the social sciences today” (Denzin, 1997, p. 18). Patton (2015) agreed and asserted that it is influential because it is an approach that “comfortably incorporates and applies quantitative concepts like validity,
reliability, causality, and generalizability to qualitative inquiry, and because the procedures are systematized and prescriptive as well as contextually adaptable” (p. 109).

The following section outlines the purpose behind the selection of a primarily constructivist grounded theory tradition of qualitative research methodology that utilizes a sociolinguistic approach to narrative analysis for data collection.

**Constructivist Grounded Theory**

This grounded theory research study was situated in a constructionist/interpretivist paradigm, as the reality (knowledge acquisition) was “fluid and changing, and jointly constructed in interaction by the researcher and the researched through consensus” (Grbich, 2013, p. 7). Constructivist grounded theory connects theorizing and research practice, and moreover, this method joins critical analysis with people’s lives (Charmaz, 2017). Multiple realities are presumed, with different people experiencing these professional development opportunities differently (Grbich, 2013, p. 7). This research focus theorized the ways teachers interact with their students, because of the way teachers interact with the professional development facilitators, and the way these interactions modify participants’ constructed understandings (Grbich). Patton (2015), for example, stated that “a constructionist evaluator would expect that different stakeholders involved in a welfare program would have different experiences and perceptions of the program, all of which deserve attention and all of which are experienced as real” (p. 123). Similarly, as the researcher, I expected teachers would have different background experiences and perceptions of the PD program. Grounded theory was the research approach that best fit my interest and research topic of study, as the construct of professional development itself presumes that “reality is a constructed and shifting entity and
social processes can be created and changed by interactions among people” (Grbich, 2013, p. 80).

Further, Grbich (2013) depicted the postmodern influence on societal changes regarding literature, specifically her explanation of Mary Swann (Shields, 1990): “Mary’s continuing ‘existence’ becomes constructed by and interwoven with the lives of the four characters, impacting significantly on each. When all four come face to face for the first time, reflection, refraction and distortion construct new possibilities for the lives of all five characters” (Grbich, p. 109). The nature of professional development requires that teacher participants construct new understandings that connect with their existing realities, truths, and beliefs as educators. Additionally, professional development is often a social process, where participants interact with one another and the facilitator. Similarly, Grbich (2013) comments:

social construction evolves from similar views of reality and truth. We construct, deconstruct, and decentre ourselves. The self is not a fixed entity, it can change from situation to situation, moment to moment. Individuals experience and recognize multiple and conflicting identities, both in themselves and in others. (p. 111)

My research has taken a subjective inquiry approach (Grbich, 2013), as my experiences also played a part in my analysis of the data. I occupied a dual role: that “of the researcher and researched” (p. 17). To help with reflexivity, I maintained detailed and critically reflected written records of my experiences and subjected myself to regular periods of debriefing with colleagues and supervisors (Grbich, 2013).

My research topic was both inductive and deductive. My hypothesis was both theory-derived and personal, based on my own experiences as an instructional coach working with elementary teachers in mathematics during a new math resource adoption, as well as an
elementary school principal. I believe that constructivist and constructionist theory can help explain the process of professional development. Crotty (1998) explained constructivism as the meaning-making of the individual and constructionism as the collective generation (and transmission) of meaning.

Much research and/or specific theory exists on applying these theories to mathematics learning experiences for practicing teachers. Boaler (2016) referred to the phenomenon in America where people accept that they simply are not math people, and the acceptance of math myths; for example, that some people are born with more ability to successfully “do” mathematics, or that males are often predisposed to be better at math than females. She implied that people’s past experiences have shaped their predisposition, beliefs, and attitudes toward the nature of mathematics. Rattan et al. (2011) supported the idea that constructivism, (or the experiences of people) shape their ideas, as they found teachers’ theories about math intelligence affected the type of feedback they gave students, and students responded accordingly to motivation and their self-efficacy beliefs about themselves regarding their abilities as mathematicians. While the literature lays a strong foundation for the theories or hypotheses I created regarding the ways teachers approach new learning experiences, specifically in the area of mathematics, I was unable to find specific theories that exist applying this mix of theory to adult learners (specifically, practicing elementary teachers). In that way, one might argue that the findings from this study generate new concepts, explanations, results, or theories (a more inductive approach). Grounded theorists Strauss and Corbin (1998) considered generating theoretical propositions or formal hypotheses after inductively identifying categories to be deductive analysis, stating: “Anytime that a researcher derives hypotheses from data, because it involves interpretation, we considered
that to be a deductive process” (p. 22). Patton (2015) proposed that grounded theorizing, then, involves both inductive and deductive processes (p. 542). Strauss and Corbin (1998) agreed: “At the heart of theorizing lies the interplay of making inductions (deriving concepts, their properties, and dimensions from data) and deductions (hypothesizing about the relationships between concepts)” (p. 22). Taylor and Bogdan (1984) described this process as analytic induction—an approach that begins with the researchers’ deduced propositions or theory-derived hypotheses and “is a procedure for verifying theories and propositions based on qualitative data” (p. 127). Similarly, Charmaz (2015) proposed that researchers “can engage in theory construction from the start of research, sharpen their comparative analysis throughout it, and improve any subsequent theoretical sampling” along the way (p. 1619).

**Study Design**

**Introduction**

This section describes the research study design, including: (1) setting and participants, (2) data sources, (3) data analysis procedures, and (4) limitations, including validity, reliability, and ethical considerations.

**Setting and Participants**

The sites/setting for data collection for the mixed survey that utilized Likert scale items and open-ended questions, interviews, observations, reflective writing documents, and focus groups took place in the same large suburban school district. I took on a role as a participant observer, as I have worked in this school district for 15 years serving in the roles of Teacher, Teaching and Learning Coach, Assistant Principal, and Principal. I have never served in an administrative supervisory role with any of the teacher participants of this research study. As a teacher, I experienced working with two of the participants several years
ago (prior to the PD that was the focus of this research study). I have worked in two of the six buildings in the past, one as a teacher and one as an Assistant Principal. I am familiar with all six buildings due to my experience in the district. All teachers consented to participation in the research study (see Appendix B, Consent to Participate in Research), and pseudonyms have been utilized to protect their identities in data reporting. Data were kept on a password protected computer for confidentiality.

The selection of the research sites were identified using purposeful sampling (Patton, 2015), as the cases for study were selected because they were “information rich and illuminative…they offer useful manifestations of the phenomenon of interest” (p. 46). Theory-focused sampling (Patton, 2015, p. 269) was utilized, as teacher participants were chosen who display varying levels of comfort and self-efficacy regarding mathematics. The district where they are situated has been committed to providing ongoing math PD as defined for the purposes of this study for several years, and several of the schools chosen had also previously identified a need for PD in mathematics.

The six elementary school sites identified for this research serve approximately 2,413 students in grades kindergarten through fifth and are situated in a large suburban school district. Full demographic details of students in participating schools can be found in Table 1.
Nine teacher participants were purposed through theoretic sampling for full participation in this research study. These teacher participants ranged in number of years of teaching experience, grade levels taught, age, as well as amount of professional development experience. Most participants were white females ages 30–50. This was not intentional, as the purposeful sampling focused on those who had participated in ongoing PD of the various efficacy categories but does but was reflective of my own demographics. Full teacher participant demographics are found in Table 2.
Table 2

Teacher Participant Demographics

<table>
<thead>
<tr>
<th>Participant Pseudonym</th>
<th>Age</th>
<th>Gender</th>
<th>Race</th>
<th>Ethnicity</th>
<th>Highest Education Level (Degree)</th>
<th>Job Title</th>
<th>School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tabitha Clayton</td>
<td>34</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Masters</td>
<td>4th Grade Teacher</td>
<td>4</td>
</tr>
<tr>
<td>Elle Taylor</td>
<td>50</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Masters</td>
<td>5th Grade Teacher</td>
<td>5</td>
</tr>
<tr>
<td>Rebecca Williams</td>
<td>41</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Specialist</td>
<td>2nd Grade Teacher</td>
<td>6</td>
</tr>
<tr>
<td>Leah Adams</td>
<td>49</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Specialist</td>
<td>3rd Grade Teacher</td>
<td>6</td>
</tr>
<tr>
<td>Marie Cahill</td>
<td>35</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Masters</td>
<td>1st Grade Teacher</td>
<td>2</td>
</tr>
<tr>
<td>Ella Simmons</td>
<td>30</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Masters</td>
<td>2nd Grade Teacher</td>
<td>1</td>
</tr>
<tr>
<td>Maggie Mills</td>
<td>34</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Masters</td>
<td>2nd Grade Teacher</td>
<td>3</td>
</tr>
<tr>
<td>Mario Mason</td>
<td>36</td>
<td>Male</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Masters</td>
<td>5th Grade Teacher</td>
<td>6</td>
</tr>
<tr>
<td>Lucy Ingram</td>
<td>47</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Masters</td>
<td>Kindergarten Teacher</td>
<td>3</td>
</tr>
</tbody>
</table>

The three facilitator participants were representative of most of the district PD experiences in which the teachers in this study had participated. Micheals was one facilitator of many from a national firm who, as one of his many roles as an independent math consultant, had been hired as part of a national consulting firm to support the district-level on-going PD experiences in which some of the teachers in this study had participated. Russell, who worked as a district administrator, had worked closely with both Taibi and Russell to design and facilitate PD experiences for the teachers in this district who participated in this study, and additionally works with other district administrators and
coaches who support teachers in implementing district curriculum, resources, and instructional models, as well as being responsible for coordinating the math curriculum and assessment for the district. Taibi, who also teaches pre-service elementary teachers how to teach mathematics at a local university, had been hired by individual schools as an outside consultant; typically, with the focus to teach teachers how to use math manipulatives to build conceptual understandings with students, but also, at times, by the district to support the PD of demonstration teachers. She had worked with some, but not all, of the schools where teacher participants from this study worked. I had previously worked with all three facilitators through my various roles as a coach and administrator in the district. Facilitator participant demographic details are found in Table 3.

Table 3

Facilitator Participant Demographics

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Age</th>
<th>Gender</th>
<th>Race Identification</th>
<th>Ethnicity</th>
<th>Highest Education Level</th>
<th>Job Title</th>
<th>Number of Years Teaching/Professional Experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chad Michaels</td>
<td>51</td>
<td>Male</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Specialist</td>
<td>Professional Development Consultant</td>
<td>25</td>
</tr>
<tr>
<td>Charles Russell</td>
<td>46</td>
<td>Male</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Doctorate</td>
<td>District Mathematics Instructional Coordinator</td>
<td>24</td>
</tr>
<tr>
<td>Tonja Taibi</td>
<td>51</td>
<td>Female</td>
<td>White</td>
<td>Not Hispanic or Latino</td>
<td>Doctorate</td>
<td>Professor of Education; Elementary Mathematics Professional Development Consultant</td>
<td>28</td>
</tr>
</tbody>
</table>
Data Sources

This research study utilized five sources of quantitative and qualitative data for analyses: a mixed survey that utilized Likert scale items and open-ended questions, interviews, observations, reflective writing documents, and focus group interviews for generating theory. Maxwell (2013) recommended using multiple data collection methods to ensure validity of findings. The following sections give background as to why these five data types were selected, as well as providing further description of how each source was collected for this research study.

Mixed Survey

Teacher participants for the research study were identified through purposeful theory-focused sampling through analysis of their written document responses on the participant selection survey. Potential participants were asked to complete a survey that was utilized for participant selection for the research study. This survey included questions from the NCTM Teaching and Learning Belief Survey (see Appendix C, Participant Survey Selection Tool) and questions created by the researcher that were inspired by Dweck (2006). Permissions were obtained from NCTM for use of their survey questions (see Appendix D, Copyright License). This survey was sent to certified teachers in six participating elementary schools. To minimize issues power dynamics can create, I purposefully did not send the participant selection survey to the teachers in the building where I serve as Principal (which is also the same building where I previously served as TLC and had greater involvement with their past math PD); and instead offered the research study to surrounding schools.

The criteria for participating in the survey was any K-5 teacher who had participated in ongoing mathematics professional development in the last five years. The last question on
the participant selection survey asked teachers to think back to their earliest experiences with math, from childhood until becoming a teacher, and write a reflective narrative describing their memories and experiences. In the narrative, participants were to include responses to the following questions:

(1) Why were each of the chosen memories meaningful to you?

(2) Upon becoming a teacher, how did you feel about your own ability to do mathematics?

(3) What about your ability to teach math effectively to your students?

(4) What did you believe was the best way to teach math when you first started teaching?

(5) What about now?

These responses, along with other data collected with three different Likert-survey style questions about their beliefs related to the nature of mathematics, own mathematics abilities, and Teaching and Learning Beliefs were utilized to classify nine teacher participants who were asked to participant into one of the following three categories: (1) generally high efficacy in regard to mathematics throughout their K-12 educational experience; (2) mixed efficacy in regard to mathematics throughout their K-12 educational experience; and (3) generally low efficacy in regard to mathematics as a child throughout their K-12 educational experience. Potential participants had five days to respond. The survey was sent either by their building principal or by me as the researcher. A reminder was sent by either the building principal or me one day before the survey window closed.

The Likert scale items were analyzed using descriptive statistics and by K-12 efficacy category to determine beliefs about the nature of mathematics (see Appendix E, Teaching
and Learning Beliefs Participant Selection Survey Results) and math mindsets of participants (Appendix F, Math Mindset Participant Selection Survey Results). The open-ended questions were analyzed and coded for nine participants who were selected for the study. Participant responses to the open-ended questions indicated that most teacher participants had some point in time during their K-12 or undergraduate college experience that included both negative and positive math experiences. However, efficacy categories were identified based on the majority of each participants’ K-12 math educational experience (see Table 4). All teacher participants had participated in ongoing PD in mathematics for elementary teachers within the past five years. Ongoing, for the purposes of this study, was defined as six hours or more of math PD within one school year.

All these teachers had participated in either math PD at the building level through a series of 6-8 one-hour sessions with a local consultant, and/or math PD at the district level for four full-day sessions (28 hours) with a district coordinator and math consulting firm. Teacher participants additionally had received coaching via grade level meetings and classroom observations by their instructional coach and building administrators. Some teacher participants also had additional math PD experiences (working with another nationally known math expert at their building, close study of grade level specific video/lesson content created for a virtually learning platform by district instructional coaches during the COVID-19 pandemic, participation on a district math curriculum development team, and/or serving the district through the role of a math demonstration teacher).

Interviews

Interviews have long been used as a tool for collecting qualitative data. deMarrais (2004) used the label qualitative interviews as “an umbrella term for those methods in which
researchers learn from participants through long, focused conversations. Qualitative interviews are used when researchers want to gain in-depth knowledge from participants about particular phenomena, experiences, or sets of experiences” (p. 52). Patton (2015) added that “qualitative inquiry for research and evaluation is but one of many uses of interviewing in the Information Age” (p. 423) and asserted that because qualitative inquiry can take place in any of the settings where other kinds of interviewing are occurring, it is important that the qualitative researcher utilize interviews as a means to understand and capture people’s experiences, beliefs, fears, triumphs—any and all aspects of their stories. Authors such as Kvale and Brinkmann (2009) and Rubin and Rubin (2012) have suggested specific steps necessary in conducting qualitative interviews. While Kvale and Brinkmann (2009) offered a specific sequence of stages, Rubin and Rubin (2012) offered a similar scope but allowed flexibility in sequencing that allows the researcher to change sites, questions, and situations to study (Creswell, 2013, p. 163). Creswell (2013) recommended the interviewer complete a process that takes the following steps:

1) Decide on research questions;
2) Identify interviewees;
3) Determine the type of interview;
4) Use adequate recording procedures;
5) Design and use an interview protocol or interview guide;
6) Refine interview question and procedures through pilot testing;
7) Determine the place for conducting the interview;
8) Obtain consent from the interviewee to participate in the study by completing a consent form; and
9) Use good interview procedures. (pp. 163–166)

The nine participants selected (three of each efficacy category) were then interviewed utilizing the Interview Protocol–Elementary Mathematics Professional Development Teacher Perceptions (see Appendix G) to expand on their responses to the open-ended questions
focused on their background experiences with and earliest memories of mathematics in the selection survey. These interviews could also be described as Patton’s (2015) explanation of “the active interview” (p. 462), or “social constructionist interviewing” (p. 433), as the goal of the interview was “to examine how knowing subjects…experience or have experienced particular aspects of life as they are coconstructed through dialogue” (Koro-Ljunberg, 2008, p. 430). Through serving in various instructional leadership roles in our district, I was also an active participant in the professional development that was being researched, which meant at times during the interview, the teacher participants and I were constructing experiences together. Mishler (1986) stated interviews are not simply exchanges of questions and answers by researchers and participants, but a form of discourse where the researcher and participant engage in co-constructing meaning within a particular type of social relationship. He argued that “even questions that are apparently simple in both structure and topic leave much room for alternative interpretations by both interviewer and respondent” (p. 45). As a result, the interviewer and participants engage in a process where both are working toward shared meanings simultaneously. I used this to guide me in writing the interview protocol. However, I also left opportunity for openness in the interview questions. In this way, the interview protocol itself was set up as Patton (2015) suggested:

> you can combine a guide approach with a standardized format by specifying certain key questions exactly as they must be asked while leaving other items as topics to be explored at the interviewer’s discretion. This combined strategy offers the interviewer flexibility. (p. 441)

Specific to constructivist grounded theory, Charmaz (2015) recommended researchers “treat interview guides as initial frameworks for opening the interview conversation rather than as recipes to follow” (p. 1613). In line with Creswell’s (2013) recommendation to start by
deciding on the research questions, after the interview protocol was designed for this project, the interviewees were then identified. Intensive qualitative interviewing was used, as questions were “open-ended yet directed, shaped yet emergent, and paced yet unrestricted” (Charmaz, 2014, p. 85). By this, I mean that at times during the interview, follow-up questions were also asked about specific portions of their reflective narrative document from the selection survey. Charmaz (2014) suggested intensive interviewing can be utilized as the primary form of data collection for grounded theory research, or as a “complement to observations, surveys, focus group interviews, and research participants’ written accounts” (p. 85), as is the case in this study. Charmaz (2014) explained that intensive interviewing is appropriate to elicit responses from participants that “may echo a shared discourse tied to one or more identities” and that these can often be contradictory in nature (p. 85). This was the case in this study, as data between participants surveys, responses in interviews, and reflective narratives sometimes conflict between one-dimensional and multidimensional approaches to mathematics pedagogy, as well as whether they maintain a growth or fixed mindset for themselves and their students. People are not just one way, and this study sought to explain the complexities that exist as participants’ background experiences determine their math efficacy, and as such, pedagogy. Grounded theorists “strive to gather participants’ accounts as completely as possible and to represent data and research participants fairly,” while using data “to construct inductive conceptual categories” (Charmaz, 2014, p. 87). The interviews were audio recorded and transcribed and were about 30 minutes in length. They were conducted either in the classroom of each elementary teacher or via online video conference. Before analysis, the transcription was sent to each participant for review to ensure accuracy.
**Observations**

Observations of classroom instruction, specifically, a math workshop, were then conducted in each of the nine teacher participants’ classrooms. Creswell (2013) stated, “Observation is one of the key tools for collecting data in qualitative research” (p. 166).

Patton (2015) outlined three specific kinds of qualitative data: interviews, observations and fieldwork, and documents. Observations are defined as:

fieldwork descriptions of activities, behaviors, actions, conversations, interpersonal interactions, organizational or community processes, or any other aspect of observable human experience are documented. Data consist of field notes: rich, detailed descriptions, including the context within which the observations were made. (p. 14)

Maxwell (2013) asserted. “observation provides a direct and powerful way of learning about people’s behavior and the context in which this occurs” (p. 103). Observation has been characterized as “the fundamental base of all research methods” in the social and behavioral sciences (Adler & Adler, 1994, p. 389). Angrosino (2005) pointed out that “even studies that rely mainly on interviewing as a data collection technique employ observational methods to note body language and other gestural cues that lend meaning to the words of the persons being interviewed” (p. 729). Grbich (2013) recommended that once researchers determine they can observe the cultural phenomena they have chosen for a qualitative research study, they must then identify the best type of data for the research question. Observational data can be collected by a qualitative researcher as an open-ended narrative, or through the utilization of published checklists or field guides (Angrosino, 2005). If observational data is included, Grbich (2013) recommended the researcher address whether the data collection will be overt or covert, considering which type of observational research will best answer the research questions at hand.
I conducted these overt observations as a participant observer. Creswell (2013) defined a complete participant as a researcher who is “fully engaged with the people he or she is observing” (p. 166). As I have served as both a teacher, the Teaching and Learning Coach (TLC), Assistant Principal, and Principal over the past 15 years in the same district in which the research was conducted, I have participated with teachers and students in the process of learning on a regular basis. In the role of TLC, I had facilitated and co-facilitated building-based PD. I partnered with the outside math consultant who facilitated the building-based PD, but then was not present for it as I took a role as an Assistant Principal in another school. I also participated in or observed some of the district-level PD and supported the work of teachers, the outside consultants, and district staff during both my time as TLC and administrator. In my time as Principal, I continue to serve as a facilitator, co-facilitator, and organizer of PD for our building.

At the end of the research interview, I prepared teacher participants by sharing the information in the Classroom Observation Protocol (see Appendix H). To conduct the classroom observations, I communicated to teacher participants I was not serving in the role of Principal and would not be an active participant in that day’s learning, but instead was there to observe the instructionally process holistically for the purpose of this research. I ensured teachers knew the purpose of this specific observation was for this research project and not connected to their Performance-Based Teacher Evaluations (PBTE). Creswell (2013) defined a participant as observer as a researcher who “is participating in the activity at the site. The participant role is more salient than the researcher role. This may help the researcher gain insider views and subjective data” (p. 167). Angrosino (2005) stated that “the interactive, membership-oriented researchers are by definition intrusive—not in the negative
sense of the word, to be sure, but they are still deeply involved in the lives and activities of the community members they study” (p. 736). Brayboy and Deyhle (2000) provided valuable insight into the paradoxical nature of the participant observer, stating that on one hand, “Insiders, it seems, miss out on some things that they take for granted or what is referred to as ‘over-rapport’ (Hammersley & Atkinson, 1996, p. 110),” (p. 165), stating that Hammersley and Atkinson ultimately argue the analysis is distorted by the participant observer’s connection the informants. Conversely, Brayboy and Deyhle (2000) stated, “this is a position we question. From our own experiences, it is this lack of distance that has enhanced our own research” (p. 165). They instead challenge that a participant observer may not be able to be objective and should not claim objectivity. Instead, they assert that insiders have issues with which they must deal, but these issues do not mean they cannot conduct good, rigorous research. Rather, they must address the issues in a manner that shows integrity and an awareness of some of the complicated issues facing them. (p. 166)

Further, they stated: “it takes (participant observers) themselves to understand the depth of meaning incorporated…to ask appropriate questions and find appropriate answers” (p. 166).

Each classroom observation was about 50-60 minutes in length. I was interested in how what the participants named as helpful, beneficial, or motivating parts of the PD process in their reflective narrative and interviews would be observed in their classroom pedagogy; and/or whether there would be times where their spoken beliefs conflicted with their instructional behavior. Classroom observations were also audio recorded and transcribed, then sent to participants for review to be returned to the researcher before data analysis began. Photographs were also taken of student work or other evidence around the classroom of tools that are used for learning (whether math manipulatives, anchor charts of past math
learning, or other helpful tools used by the teacher or students). I also took notes during the interviews and observations. These photographs and observation notes were mostly for my own use, as when analyzing transcriptions, it was helpful at times to look back at the hard copy of the interview protocol or the observation notes for anything I jotted about what was happening or I was noticing that was not captured in the transcription data. Photographs taken during classroom observations were then included with observation notes that were taken to describe what was seen in the classroom (in addition to the audio transcriptions).

**Documents**

Creswell (2013) recommended the use of documents in data collection for narrative, ethnographic, and case study approaches to qualitative inquiry. Though the primary approach in my research study is grounded theory, case study, by Creswell’s (2013) definition, traditionally studies sites or individuals; a bounded system, such as a process, an activity, an event, a program, or multiple individuals; all of which can be used to describe a PD program. After the classroom observation portion of data collection, participants were sent a link to complete a reflective document (see Appendix I, Observation Reflective Document Protocol) regarding that day’s lesson/instruction. The interviews, classroom observations, and reflective narratives were conducted and collected over the course of a two-week period.

**Focus Groups**

After initial analyses of data from the mixed survey, interviews, observations, and documents, the initial findings were shared with two focus groups: one consisting of the nine teacher participants, and one consisting of three PD facilitators who had served as facilitators of much of the PD processes in which the teacher participants had participated. Eight teacher participants were able to join the initial teacher participant focus group, and one teacher
participant met separately with me afterwards due to a conflict in scheduling. In the end, I was able to gain feedback on the initial findings from all teacher participants, as well as all three of the facilitator participants.

The following sections specify data analysis procedures for each of the five data sources: mixed survey, documents, interviews, observations, and focus groups.

Data Analysis Procedures

Data from each source were analyzed throughout the two-month period, both immediately following the data collection experience, soon after (within the same week), and again each time new data sources were added. Constructivist grounded theory approaches were utilized to analyze the data. Charmaz (2014) “chose the term ‘constructivist’ to acknowledge subjectivity and the researchers’ involvement in the construction and interpretation of data” and to “signal the differences” between her approach and “conventional social constructionism of the 1980s and 1990s” (p. 14). Her position aligns with the social constructivists whose influence includes Vygotsky (1962) and Lincoln and Guba (2013), who “stress social contexts, interaction, sharing viewpoints, and interpretive understandings” (Charmaz, 2014, p. 14). Below, I describe the coding process used for this constructivist grounded theory research.

Coding the Data

In analyzing all written documents (two reflective narratives for each of nine participants, nine interview transcriptions, nine transcriptions of participant classroom observations) line-by-line coding (Charmaz, 2014) was utilized for a total of 27 data sets. When analyzing data, Microsoft Excel was utilized for organizing the participant documents, interview, and observations; thus clustering initial and focused codes into sub-categories,
then sub-categories into categories or themes. The two main phases of Charmaz’s (2014) grounded theory coding were followed: “(1) an initial phase involving naming each word, line, or segment of data, followed by (2) a focused, selective phase that uses the most significant or frequent initial codes to sort, synthesize, integrate, and organize large amounts of data” (p. 113). Coding generates ideas to explore and expand (Charmaz, 2015). Line-by-line coding was used to open “data up to deeper and more analytical readings than occurs during a passive perusal of them” (Charmaz, 2015, p. 1615).

There were times where a section might be identified as fitting one initial code, as multiple sentences may be needed to give the context for the phenomena (for example, for the initial code of “sense making”: “Teacher: Right, takes grapes away. I want to show you this. This means we take it away. Okay? Take away. THIS is take it away” (Ingram). Other times, a few words could be identified as fitting an initial code (for example, for the initial code of “sense making”: “That make sense?”). As I conducted interviews, and soon after, observations, I coded the data across a two-week time period. Coding became more focused during the analysis phase. I analyzed for frequency and developed the sub-category codes by grouping similar initial focused codes together, then identifying categories or themes to describe what I was seeing in the data. Part of my data analysis process was during the writing, as well; as I would mine through the data again and add new data that may have been missed initially, or revise where each piece of coded data fit best. Additionally, after analyzing the data through the lens of the four literature review topics that conceptualized the theoretical framework of this study, I looked for sensitizing theories and how they might fit in the theoretical framework that underpin this study, as depicted in the findings of Chapter 4.
I also color-coded each piece of data. Red stood for data that came from those participants who were categorized as having low self-efficacy in mathematics throughout their K-12 experience; yellow stood for those participants who were categorized as having mixed self-efficacy around mathematics throughout their K-12 experience; and green stood for those participants who were categorized as having high self-efficacy in the area of mathematics throughout their K-12 experience. I did this because the intent of this research is to develop theory around efficacious practices of elementary teachers, so I needed a way to see how the responses of participants in each of these categories varied—or whether they did. After all data had been coded, I noted the frequency with which each of the initial categories was present throughout the data. I also determined which percentage of the responses came from each of the efficacy categories. In several cases, this provided quite an intriguing perspective, or lens, by which to view the qualitative data that constituted the stories and experiences of the teacher participants.

Memo writing was utilized throughout the coding process to assist me in questioning the data, organizing it into analytic categories, and reorganizing when compared with new data sources. In my memos, I took note of times my codes defined another view of a process, action, or belief other than that which was spoken to be held by a participant, as these ideas “may rest on covert meanings and actions that have not entirely surfaced yet”; “providing other ideas to check” (Charmaz, 2014, p. 132). As I closely studied the data through active coding, I interacted with the data again and again, asking different questions of them. Close study of the data led me back to interactions with participants and led me to cross-analyze conversations during interviews, content in their reflective documents, and observed phenomena in their classrooms. This at times led me to dig deeper into certain categories or
constructs with participants and at other times led me to check fresh ideas with new participants (Charmaz, 2014, p. 115). Charmaz (2014) also suggested checking your categories or constructs or with other data sources, which was seen in this study with the variety of data sources (mixed survey responses, interviews, observations, reflective documents, and focus group data). I additionally used constant comparative methods (Glaser & Strauss, 1967), comparing data with data to find similarities and differences. In grounded theory, the researcher acts on their data, and these actions sustain the researcher’s involvement with the data (Charmaz, 2015, p. 115). My ongoing experience working with teacher participants in math PD and having had previously participated in the PD processes they reflected over through my position as a participant researcher provided me the perfect setting.

Participant data were coded either red (low efficacy), yellow (mixed efficacy) or green (high efficacy) to seek variances in descriptive data based upon K-12 efficacy category. Charmaz (2014) cautioned researchers not to code for types of people, as she stated:

> Assigning types to people casts them with static labels. In effect, you make them one-dimensional although their behavior on which you based the label may represent only a small part of who they are and what they do. Both may change as events change. Thus, such coding freezes people in time and space and also erases or minimizes defining variation in the studied phenomenon. (p. 117)

It is important to note that for the purposes of this research study, the efficacy categories of low, mixed, or high participants were assigned based on the coding of their responses to open-ended questions in the participant selection survey, only meant to be representative of most of their K-12 experiences. It is noted several times throughout this research study that multiple realities were often present in each of these participants’ data and that many of them
changed in their math efficacy since graduating from high school. Part of what this research sought to inquire into is how the K-12 experiences and efficacy might change the PD process and pedagogies of practicing elementary teachers, so I chose to continue with these categories to see if trends could be found in the data based on efficacy categories.

After initial analysis of the mixed survey that utilized Likert scale items and open-ended questions, interviews, observations, reflective writing documents, the initial findings were shared with the participants through a focus group for member checking (see Appendix J: Teacher Participant Focus Group-Initial Findings Protocol). The focus group was also recorded and transcribed and coded using selective coding to be compared to initial results. The findings were also taken to a focus group that consisted of three facilitators of math professional development for feedback (see Appendix K: Facilitator Focus Group Interview Protocol). Each focus group, both the participant as well as facilitator, was 60 minutes in length. The participant focus group was held approximately three weeks after initial interview and observation data collection began, with the facilitator focus group being held soon thereafter in the fourth week after initial data collection.

Following, in the final section of this chapter, is a discussion on the limitations of this study, including validity, reliability, and ethical considerations that were addressed in relation to my constructivist grounded theory research study. To accurately capture the experiences of the participants in this study, I had to make my own biases explicit to ensure authenticity of the data. Ethical considerations for researchers must take into account the Institutional Review Board (IRB) guidelines to ensure the rights of human subjects are protected.
Limitations Including Validity, Reliability, and Ethical Considerations

There were a few limitations to this research study. Limitations are potential weaknesses of a qualitative research study (Creswell, 2013). One possible limitation was my personal involvement in the setting, as well as dynamics of power regarding trust and honesty of participants when responding and reflecting on the PD opportunities. As I serve in the district of the research study in the role of Principal, the nature of my role requires me to evaluate teachers, as well as collaborate with the PD facilitators on a regular basis. This could have impacted the level of vulnerability and transparency teacher participants would offer. However, I built trusting relationships with teachers, and given that the nature of PD is to support teachers in improving their pedagogical practices, there was an aspect of collaboration embedded into each of our roles that benefited this research process. Teachers understood that by helping me understand and reflect over the mathematics PD they had experienced, it would only help me further design more effective supports to help them become increasingly effective at their jobs (the relationship and dynamic was symbiotic because of the nature of my relationship as participant observer).

To minimize dynamics of power inherent in my role, I purposely did not conduct the research study in the building where I served as principal. I had never served in an evaluator role to any of the participants who chose to participate in this study. Participation was optional, and it was communicated up front they had the option to discontinue the study if at any time they felt discomfort (see Appendix L, Participant Recruitment Email Script). I also made sure they knew the purpose of my classroom observation was not evaluative in nature; that I was taking off my “principal hat” for the purposes of this research study. I also communicated to teacher participants, both in the informed consent form as well as during
their interview, that all information observed and communicated to me throughout the study was kept confidential, not to be shared with their building principal, district administrators, or PD facilitators.

Another limitation is the setting. While the research site provides a theory-focused sample, possibly the results of this research study might be different in a school district where the contextual elements (Sun, 2015) were different; for example, one where the district and building-level professional development experiences for teachers were not aligned, or where the commitment to professional development for teachers in mathematics was not sustained with a common focus over time. Given that all participants in this study are from the same district and have participated in the same types of ongoing PD experiences, further research would need to be conducted to determine whether results from this study would be consistent with PD experiences that differed from those offered in this setting.

Another potential limitation could be that participants opted to participate in this study. By the nature of being willing to complete an optional survey about mathematics, there could be other teachers who had such negative background experiences about mathematics or low math efficacy that they chose not to even complete the participant survey. Additionally, all nine of these participants had already participated in on-going PD around mathematics, which likely already had played a role in shaping their beliefs about the nature of mathematics, math efficacy, and pedagogy. Had participants been surveyed prior to participation in each type of PD and then immediately following, it may give a different perspective as to how their experiences have been changed by each individual PD experience. Participants in this study have had a variety of math PD experiences across the past five years. While this can also give a wider breadth of experiences and perspectives,
future research may follow up on each individual PD programs to determine the impact of each rather than the collective.

Another limitation is sometimes dialogue/questions could not be heard or accurately transcribed during classroom observations where students were actively collaborating. For this reason, most of the classroom observational data that was transcribed and used for this study was group discussion or a focused part of the lesson where the teacher was leading students through a discussion, sharing student work, or modeling a strategy—whole class work. Some data were included from teachers’ work with small groups or individual conferences, but not all were discernible due to the nature of several students talking at the same time. One low efficacy observation was very limited in the data that was discernible for this reason—the volume of student talk/collaboration was high.

**Validity and Reliability**

To increase validity and reliability in this study, purposeful sampling was utilized for participant selection. Theory-based sampling was utilized and supported by data initially collected in the original participant responses to the open-ended questions on the mixed selection survey. Reliability in a qualitative research study can be enhanced by the researcher employing quality recording procedures and transcription of recorded interview data, transcribing to indicate crucial pauses and overlaps (Creswell, 2013, p. 253). Creswell (2013) additionally recommended that computer programs can additionally be utilized to assist in recording and analyzing data. Both recommendations were utilized in this research study. Additionally, Strauss and Corbin’s (1990) criteria to consider when developing a grounded theory study were utilized in developing this research study methodology. Charmaz’s (2006,
2014) evaluative questions for grounded theorists were also utilized during data analysis to test generated theory.

Validity Concepts

Lincoln and Guba (1985) stated that to establish “trustworthiness” of a qualitative research study, constructivist criteria should be considered regarding credibility, authenticity, transferability, dependability, and confirmability. They stated these are the “naturalist’s equivalents” for internal validation, external validation, reliability, and objectivity (p. 300). The following considerations regarding validity and reliability were utilized for this research study.

Credibility

Multiple data types (Creswell, 2013) were used to support or contradict interpretations and proposed theory. When making sense of the data in this research study, theories from existing literature were combined with developing theory and continually tested with the addition of each new data set (Maxwell, 2013). Participants were asked to review and verify their statements for accuracy and completeness during the data collection process. Additionally, as the researcher, I was cautious not to assert my own preconceived theoretical assumptions or bias up front when communicating with participants. Trustworthiness was established between participants and the researcher as I, the researcher, am a participant observer and was able to affirm their experiences and/or connect with ideas and concepts they shared during the data collection process. Trustworthiness was also established between me and the participants because I had spent a lot of time at the data collection sites (Patton, 2015), so was able to understand the experiences and contextual factors teacher participants mentioned, both with the participants during this research
process, as well as at various schools and PD experiences throughout the district during my time working in various roles over the years. The process of line-by-line coding that was utilized for data analysis also freed me as the researcher from “becoming so immersed in the research participants’ views that they were accepted without question” (Charmaz, 2014, p. 127).

**Authenticity**

Participants had opportunities for authentic dialogue with me throughout the research process. Member checking was utilized throughout the process, as I shared pieces of updated data analyses findings with participants during interviews as applicable as well as during the teacher participant focus group, and asked them to respond to the data, providing insight into areas where they agreed or disagreed with findings.

**Transferability**

Findings from this research study may or may not be transferrable to other settings. It is likely the theory generated regarding elementary mathematics professional development could be applied in future program design, but other contextual factors will need to be considered to determine whether the same effects will occur with a different group of participants in another setting. For example, the PD facilitator may need to determine the background stories of participants or learn more about participants’ K-12 self-efficacy to decide which portions of the professional development may best fit their participants. This information is not readily available and likely requires an element that determines participant background be built into the initial phases of a PD program to increase transferability of PD strategies to a different setting. Additionally, factors that may have prepared participants for the elementary mathematics PD in this school and district; namely, previous professional
development experiences, may have prepared teacher participants and increased their readiness or motivation for each corresponding PD experience. Seven years before this research study was conducted, this district implemented classroom Number Talks K-5 (Parrish, 2010, 2014). The PD that occurred during this implementation was not included in this research study, but likely laid a strong foundation for corresponding PD experiences that took place. Had participants in another setting not been through that PD process, their results may vary from the participants’ perspectives in this study (although not all participants in this study were a part of that initial training). Another setting may need to consider additional PD experiences in elementary mathematics similar to those the district utilized in this research study to increase transferability. Other settings may have not had such a sustained long-term effort with PD. District administration and expectations in the district chosen for this research study support teaching mathematics in a multidimensional way. If this study was conducted in a district with a different philosophy or with different expectations, it may have different results. Thick description is utilized to ensure transferability between participants and researcher in this study, as well as when proposing theory to increase opportunity for transferability to other settings.

**Dependability**

Dependability of data reporting was established through an auditing of the research process (Creswell, 2013). This research process was audited by describing research steps taken from the start of the study to the development and reporting of findings. Research steps, process, and findings were continually audited through my ongoing collaboration with those on my research committee. To ensure reflexivity, I maintained a detailed and critically reflected written record of my experiences and subjected myself to “regular periods of
debriefing” with colleagues and members of my supervisory committee (Grbich, 2013, p. 17).

**Confirmability**

A confirmability audit was conducted along with the dependability audit. Confirmability is “the degree to which findings are the product of the focus of its inquiry and not the biases of the researcher” (Lincoln & Guba, 1985, p. 290). With this in mind, I reviewed data findings and made my own biases explicit, as well as analyzed findings for bias or distortion. Through collaboration with an auditor and through member checking, I checked findings and conclusions to ensure they were explicitly supported by data sources.

**Crystallization**

Crystallization, the practice of determining the multidimensional connections between themes that are identified in data, provides “a deepened complex, thoroughly partial, understanding of the topic” (Richardson, 2000, p. 934). Constructed codes based on what is learned while studying data were “developed into categories that crystallize participants’ experiences” (Charmaz, 2014, p. 133). Crystallization of identified categories was utilized during data analysis in this constructivist grounded theory research study to propose a multidimensional theoretical framework through analysis and making meaning of multiple data sources.

**Ethical Considerations**

As the primary researcher in this study, it was my ethical responsibility to protect the rights of participants and ensure that the research did not have any unnecessary negative impacts on them. Permission to conduct this research was obtained from the University of Missouri-Kansas City (UMKC) Institutional Review Board (IRB). The IRB is an
administrative body within UMKC, established by appointment by the Provost of the University to protect the rights and welfare of human participants in research who are asked to participate in research activities conducted by UMKC students. The Belmont Report is a report created by the National Commission for the Protection of Human Subjects of Biomedical and Behavioral Research. The Belmont Report (U.S. Department of Health, Education, and Welfare, 1978) established three key principles that serve as guidelines for all research proposals:

1) **Respect for Persons**: Each individual has a right to be treated as an autonomous agent, and those with diminished autonomy are entitled to protection. Informed consent is defined by the individual’s ability to make fully informed decisions.

2) **Beneficence**: Each individual has a right to be treated in an ethical manner not only by respecting their decisions and protecting them from harm, but also by making efforts to secure their well-being. Researchers have an obligation to ensure two general rules apply to participants of research studies, including (1) do no harm, and (2) maximize possible benefits and minimize possible harms.

3) **Justice**: Five formulations apply to equal treatment of participants in a research study, including: (1) to each person an equal share, (2) to each person according to individual need, (3) to each person according to individual effort, (4) to each person according to societal contribution, and (5) to each person according to merit. In other words, those most likely to benefit from the research should bear the burden of any potential risks involved.

To ensure alignment with the Belmont Report, participants selected for this research study were given a letter of informed consent that outlined all known risks or benefits to
participants in this study. Additionally, participants were made aware that their participation in this study was confidential, anonymous, and strictly voluntary. This research has the potential to benefit the participants as well as the greater educational community, both of this school, district, and other educational organizations when designing future professional development programming. The participants, by consenting to participate, were sharing their stories and contributing to generating theory that could potentially benefit their colleagues and students.

**Data Organization and Management**

Precautions were taken to safeguard the participants’ confidentiality. Pseudonyms were used to protect the names of participants, facilities, districts, and organizations involved. Interview transcripts, observation transcripts, and reflective documents were coded to ensure anonymity of participants. All electronically stored files were password protected. Upon completion of the study, all data will be securely stored in a locked file for potential future use. Information will be kept for a minimum of seven years following the collection of the data.

The next chapter provides a detailed overview of my findings as it relates to the research questions that guided this constructivist grounded theory study with the primary unit of analysis being teacher participants’ theories related to their self-efficacy in mathematics.
CHAPTER 4

FINDINGS

Most teachers strongly agree that the role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics (National Council of Teachers of Mathematics, 2014). However, this principle is often easier to believe than to put into action. High-quality, effective professional development for teachers in elementary mathematics is needed. Specifically, a need exists to determine which aspects of elementary mathematics PD are most powerful for teachers (Boston & Smith, 2011; Elliott et al., 2009; Polly et al., 2014).

The purpose of this constructivist grounded theory qualitative research study was to develop a theoretical framework of nine teacher participants’ sense of mathematics efficacy including K-12 schooling and throughout professional development. My pragmatic goal was to develop a framework for implementing efficacious professional development. Constructivist grounded theory was selected because “it serves as both a way to learn about the worlds we study and a method for developing theories to understand them” (Charmaz, 2017, p. 17). In this research, it was used to learn about how teacher participants’ sense of efficacy was shaped during their K-12 experiences and by participation in professional development, and then develop theories to understand this process. As Charmaz (2014) stated:

In the original grounded theory texts, Glaser and Strauss talk about discovering theory as emerging from the data separate from the scientific observer. Unlike their position, I assume that neither data nor theories are discovered either as given in the data or the analysis. Rather we are part of the world we study, the data we collect, and the analyses we produce. We construct our grounded theories through our past and present involvements and interactions with people, perspectives, and research practices. (p. 17)
My involvement as participant as observer also made constructivist grounded theory a perfect fit for this research study, as I had been part of the world of elementary professional development and math PD, and it is these experiences that have helped me construct this grounded theory.

**Research Questions**

The following research questions were designed to generate theory about the design of mathematics professional development experiences to support efficacious strategies:

RQ1: What is the theory that explains the process for enhancing the efficacies of teachers in a large suburban school district?

- Which parts of the mathematics professional development process do participants attribute to their theories regarding efficacy in math?
- What theories do they generate about the nature of mathematics because of the PD process?

RQ2: What professional development theories contribute to shifting mathematics pedagogical practices?

- What motivates teachers to engage in professional development?
- In what ways do their pedagogical practices change?

RQ3: What do findings suggest about leadership theories to support the facilitation of mathematics professional development?

- How do leaders facilitate safe environments for enhancing the efficacies of teachers?
- What support do participants need for trying out theory-based pedagogical practices in classrooms?
This research study utilized five sources of quantitative and qualitative data for generating theory; however, interviews that offered the voices of teachers, in addition to their reflective writing documents, were the major data sources. Other sources included the mixed survey that utilized Likert scale items, observations, and focus group interviews. A mixed survey was sent out to teachers at six suburban elementary schools that included a mix of Likert-scale survey questions about teachers’ beliefs about the nature of mathematics and mindsets, as well as open-ended questions regarding their earliest math experiences. The quantitative section of the survey was only used to select nine elementary teacher participants in each of three efficacy levels (high, mixed, or low) for participation in this study. Level of participation in PD, as shown in Table 4, was determined as follows: (1) participated in one ongoing PD experience; (2) participated in two different ongoing PD experiences; (3) participated in 3 different PD experiences; (4) participated in 4 or more ongoing PD experiences.

Table 4

Teacher Participant Efficacy and Professional Development Participation Levels

<table>
<thead>
<tr>
<th>Participant Pseudonym</th>
<th>Number of Years Teaching</th>
<th>Efficacy Level*</th>
<th>Level of Participation in PD</th>
<th>School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tabitha Clayton</td>
<td>12</td>
<td>high</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Elle Taylor</td>
<td>3</td>
<td>low</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Rebecca Williams</td>
<td>17</td>
<td>low</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Leah Adams</td>
<td>16</td>
<td>mixed</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Marie Cahill</td>
<td>13</td>
<td>mixed</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Ella Simmons</td>
<td>8</td>
<td>high</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Maggie Mills</td>
<td>13</td>
<td>low</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Mario Mason</td>
<td>8</td>
<td>high</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Lucy Ingram</td>
<td>24</td>
<td>mixed</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

*as determined by sampling survey data
Process for Determining Efficacy Categories of Teacher Participants

While I was able to utilize the participant’s initial selection survey narratives to identify in a general sense their background experiences and math efficacy throughout their childhood through becoming a teacher, my initial desire was to find participants who not only clearly fit into the “lines” of each of these descriptors, but also whose beliefs about the nature of mathematics would “fit” their efficacies. For example, I had wrongly assumed that someone who had low self-efficacy might hold more one-dimensional beliefs about the nature of mathematics (Boaler, 2016) and a fixed mindset about their own ability to do mathematics and that one with high-self efficacy might have more multidimensional beliefs about the nature of mathematics, and/or a growth mindset about their own abilities and that of their students.

What I found instead was that in most cases, there were variances between one’s own self-efficacy regarding mathematics as a child, when becoming a teacher, and in regard to how their beliefs about the nature of mathematics and/or mindsets about themselves and others had been modified throughout their lives. When I was a teacher, we utilized the Teacher’s College Reading Units of Study (Calkins & Tolan, 2010). I keenly remember a unit that focused on teaching our youngest readers how people, just like characters in books, are “not just one way” (p. 41). Similar to how mathematics can be multidimensional, the participants in this study were also multidimensional. The more I learned about each of their stories throughout this process, the more I learned there were a lot of blurred lines. For this reason, while a single person’s journey with efficacy around mathematics likely ebbed and flowed, moving through peaks and valleys regarding the efficacy they held for themselves, I
ended up settling my description of each of the three efficacy levels based on the teacher participants’ overall K-12 childhood experience.

The participants in the “high efficacy” category generally always felt positive about math in their K-12 experience. There may have been a few challenges in high school or college they spoke of, but while these experiences challenged their self-efficacy around mathematics, they did not define it. They either blamed their own lack of effort for negative outcomes (which is still aligned with an incremental, or growth mindset rather than an entity, or fixed mindset); or possibly acknowledged challenges but never found themselves overcome by the challenges—they still had enough agency, or self-efficacy, to utilize strategies to overcome these challenges.

Those who fell in the mixed efficacy category had some positive experiences in K-12 mathematics, but also had some negative early memories of math. For two of the three participants, this meant positive experiences early on, that were challenged beginning in junior high and continued to remain a challenge through high school and potentially into college. For the third of the mixed efficacy participants, math began as a struggle early on, until she had a teacher who “challenged her and took an interest in her learning”; a turning point that “sparked an interest…to have a love for learning math even though it was not [her] easiest subject” (Cahill, initial reflective document).

For the participants I identified as low efficacy, they pretty consistently accepted for themselves throughout their K-12 education (and often, beyond) that they were not “math people.” Consistent among these three participants were “deeply rooted memories of being in” what Mills from School 3 referred to as “the ‘struggling math group’ in elementary school and feeling like (they) were different from others, when all [they] really wanted to do
was blend in” (Mills, initial reflective document). For her, after avoiding math “at all costs” (Mills, initial reflective document) throughout her K-12 experience, a college math class began to spark a shift in self-efficacy for mathematics. For the other two low efficacy participants, it wasn’t until much later—either while completing a teaching degree thirty years after beginning it in undergrad, or during participation in the math PD process as an in-service teacher—that these participants began to see themselves in a different light regarding mathematics.

**Data Sources**

After selection, participants were interviewed to follow up on their written responses from the open-ended survey questions which constituted an initial reflective writing document. These interviews also included questions about the mathematics PD processes in which they had participated, and were voice recorded and transcribed. Each teacher participant was then observed teaching a math workshop in their classroom. These observations were 45-60 minutes long, voice-recorded and transcribed for analysis. Documents were also analyzed as part of the data collection process. These documents included the initial reflective document from the survey, as well as reflective documents written within 24 hours following the classroom observation. Prompts for these reflective responses asked participants to share what parts were observed in their lessons that demonstrate evidence of their background experiences, efficacy, or strategies gained from participation in the PD process. Most of the data (participant selection survey, interviews, observations, reflective documents) were collected within three weeks’ time. After these three weeks, during the fourth and fifth weeks, I was immersed in data analysis. There was some interaction between myself and the participants at this time while I shared transcripts to
be proofed for accuracy, through member checking, and collected demographic data from them, as well as to schedule our focus group for theoretical sampling of initial findings. The focus group was held within the sixth week after the data collection of the field phase began. I spent about two and a half hours with each of the nine participants across this period. The total time of their involvement in the research study was likely about three to three and a half hours.

**Reflections on Building Trust**

When participants first received my email recruiting participants, I am sure many who I did not yet know viewed me primarily as the principal next door. Some viewed me as their former colleague and/or friend, as I had worked in two of the six buildings at some point earlier in my career. Early in the process, I emphasized I had taken off my “principal hat” for the purposes of this research process. As appropriate, during the interview process, (particularly when affirming vulnerable pieces of stories teachers shared from their own journeys), I shared bits and pieces of my own journey to finding more efficacy as a mathematician—one that included being sparked as a Teaching and Learning Coach (TLC) to help teachers who were much like me—those who had low self-efficacy in mathematics.

It was through my own participation in PD as a TLC geared towards helping the teachers I served, research for this literature review, attending a math conference and taking an online course, that I learned it was not too late for me; that I did not have to continue to perpetuate misconceptions of people not having a “math brain” in my work with students (and teachers). One of the first college level math courses where I felt success was in doctoral level statistics coursework. While I only shared bits and pieces of this journey as appropriate with participants, I believe it helped strengthen the rapport we built. For those
with low or mixed efficacy, it increased the level of trust in what they felt they could share with me as the researcher (who is also an administrator in their district). For those with high efficacy, they were willing to share challenges and areas that they felt they could continue to grow in, or what they really thought about the PD process (a portion of the coded data were categorized as “confessions”). Trust was built, and after each interview I felt I really knew the participants whom I had not known well prior to this process.

Joining teacher participants in their classrooms, then, did not feel as out of the ordinary, at least for me, as it could have otherwise. I acknowledged there are always some level of nerves anytime you have “an outsider” join you in your classroom, particularly one who serves as an administrator in your district; but again I reassured the teacher participants I was not there to evaluate. Some agreed being observed could be uncomfortable for them, while others simply explained beforehand what I might see and how it may be consistent or differ from the ordinary in their math workshop.

Particularly, the setting for the interviews and observations, which began in March 2021—after we had been teaching a year into the COVID-19 Coronavirus pandemic—made for a refreshing change of landscape to set the stage for us to talk about the background stories that each individual teacher brought to their professional learning process and the nuts and bolts of teaching elementary mathematics. The teachers also simply acknowledged how what I would observe may be the same or different from what they would normally do if not teaching in COVID times. For the most part, since our schools had been face-to-face learning again since July 2020, things felt more routine by the time this research occurred. For the last year, while I fought to maintain normalcy for my own school, these teachers had done the same while balancing teaching face-to-face and virtual students. It had been at times difficult
to pause and join one another in a collaborative conversation while managing COVID-19 protocols. This research gave us an opportunity to do just that, and it was refreshing.

I have briefly reiterated the problem, purpose, and methods for the study, and placed an emphasis on developing trust; my overall reflection about the process follows.

**Validity and Reliability**

To address threats of bias, after interviews, observations, and focus groups were voice recorded and transcribed, I sent the transcriptions to participants to check for accuracy before data analysis. Memo writing was utilized to (a) engage with my data and emerging comparative analyses, (b) help me identify analytic gaps, (c) provide material for sections of papers and chapters and (d) record and develop ideas at each stage of the research project (Charmaz & Bryant, 2008).

I also utilized the teacher participant and facilitator participant focus groups as an opportunity for member checking (Charmaz, 2014) and authenticity. Initial findings were shared with participants for them to interact with and continue to help me formulate the theory to describe their sense of math efficacy throughout their K-12 experiences, into teaching, then throughout professional development. There were some parts of the data participants were surprised by, as sometimes beliefs they held for themselves were not made overt through the data from their interview and classroom observation transcripts. Other times, their beliefs were represented more highly in an efficacy category other than the one with which they most associated. I did not make known to them in the focus group to which efficacy “category” they were assigned, as I did not want to cause discomfort to any member of the participant group that could lead to shame. Had I shared the efficacy category with them, that too may have been a helpful way to check the data; whether they agreed or
disagreed with their K-12 “label” for the purposes of this study. However, the efficacy categories were more for the purpose of checking data against each other (Maxwell, 2013) during the theory generation process; to ensure credibility (Lincoln & Guba, 1985). I think from the responses I was hearing from participants in the focus group, they were able to assume accurately which efficacy group I had selected for them based on their K-12 experiences, as many were giving explanations for their efficacy category or mentioning surprises that were aligned with their assumptions. There were also times the findings by efficacy category could not be generalized to all members of that efficacy category. For example, one member of the efficacy category could mention a concept several times, which would increase the frequency count and percentage for that efficacy category, but not necessarily accurately represent the thinking of all members of that category (as is the case for most group think). There were also times teacher participants would affirm they really connected to pieces of data being true that had originated from their own data sources.

The opportunity to share initial findings with facilitator participants was also quite helpful, as I was able to share the initial findings and the participants’ reactions to them and gain input on what pieces they found most valuable based on their plethora of experiences facilitating PD for elementary teachers in a variety of settings (both within this school district and others). The facilitators connected with several pieces of data but helped me in narrowing the focus for which pieces may be most important to include in theory generation based on their experiences. In this way, authenticity and conformability (Lincoln & Guba, 1985) were achieved. I also met regularly with members of my research committee throughout the analysis process as a strategy for auditing the research process to ensure validity. Lincoln and Guba (1985) also confirmed that a check for validity is a check for
reliability; “since there can be no validity without reliability (and thus no credibility without dependability), a demonstration of the former is sufficient to establish the latter” (Lincoln & Guba, 1985, p. 316).

Crystallization, a form of validity, involves the practice of determining the multidimensional connections between themes that are identified in data (Richardson, 2000), and was achieved by using multiple data sources and multiple cycles of coding and analysis to develop interconnected categories based on participants’ experiences.

In the next section, I report on the process for generating theory, which incorporates Charmaz’s (2014) method of grounded theory.

**Process for Generating Theory**

Charmaz’s (2014) method of grounded theory is used to report on the findings with an emphasis on her two phases for grounded theory coding: (1) the initial phase “naming each word, line, or segment of data;” and (2) “the focused, selective phase that uses the most significant or frequent initial codes to sort, synthesize, and integrate and organize large amounts of data” (p. 113). Theoretic sampling was utilized through analysis of initial participant selection survey data to determine participants from each of the efficacy categories. During Charmaz’s (2014) initial phase, I used line-by-line coding to name words, lines, or segments of data, often with focused codes. Charmaz’s (2014) line-by-line coding was utilized to identify focused codes.

While coding, I asked questions about what was happening in the data (Charmaz, 2015; Glaser, 1978), and then sought answers through cross-referencing other data, or during follow-up interviews. Focused codes were then analyzed and grouped into summaries to create a smaller number of categories, themes, or constructs. The focused codes were then
further analyzed by testing them with the data, comparing them to one another, and considering what kinds of theoretical categories these codes indicated. Memo writing was utilized throughout to allow exploration of ideas, improve codes, make conjectures, examine assumptions, and express doubts (Charmaz, 2015). Comparisons of categories, and the process of writing memos helped me move from description of an experience into a conceptual analysis of it (Charmaz, 2015).

I still had several categories/themes identified in the data and was having difficulty taking these categories and using them to generate theory. I looked then back to the frequency count of each initial focused code, and this time, conducted more focused coding based only on those initial codes that were mentioned the most frequently. This helped me narrow the scope a bit, while still considering the overarching categories that had been previously identified. I also looked back to my research questions to determine which codes and categories might help answer each. Strauss and Corbin (1998) stated, “At the heart of theorizing lies the interplay of making inductions (deriving concepts, their properties, and dimensions from the data) and deductions (hypothesizing about the relationships between concepts)” (p. 22), and I found this to be evident. I then utilized Charmaz’s (2014) reflective questions for “pulling the pieces together” when writing your draft (p. 291) to help me study memos and codes, sorting and diagramming and clustering them in ways that demonstrated relationships between them, then using this to write a first draft of findings. I also used Charmaz’s (2014) suggestions for constructing and evaluating grounded theory, which included lists of several more reflective questions or “criteria for grounded theory studies” (pp. 337–338) to help me consider my major categories and drafts of theoretical concept maps through different lenses.
As I analyzed the codes and categories, I also examined “the data/findings in terms of theory-derived sensitizing concepts and applying theoretical frameworks applied by others” (Patton, 2015, p. 543). I went back to the empirical literature, testing each potential generated theory map against what already exists in the world to generate and add new contributions to the field. It was through this process I generated the theory of math efficacy and motivation for teachers of elementary mathematics.

**Theory of Math Efficacy and Motivation for Teachers of Elementary Mathematics**

After identifying the focused codes for categories that had the most frequent number of occurrences in the data, I organized these codes by research question. I began to identify a few major categories. The core categories identified for the Theory of Math Motivation for Teachers of Elementary Mathematics are as follows: (1) Experiences Shape Math Efficacy, (2) Experiences Shape Beliefs about the Nature of Mathematics, (3) Math Efficacy Shapes Motivation, and (4) Math Efficacy Shapes Pedagogy.
To generate theory for supporting the math efficacy of teachers, all data were analyzed collectively including reflective writing documents, interviews, observations, and focus group interviews. This means I collapsed all data sources and used Charmaz’s (2014) constructivist grounded theory process to identify sub-categories and categories or themes in the data. The major categories or themes, depicted in Figure 1, are discussed in the following sections.
Theory 1: Experiences Shape Math Efficacy

Throughout the data analyzed, participants told stories of how specific experiences in their lives shaped the efficacy they held about themselves related to mathematics. While the efficacy categories for the purposes of this study focused on participants’ K-12 mathematics experiences, embedded in the stories they told were also memories of experiences as college students or during PD that continued to shape the math efficacy they held for themselves.

Data aligned to the category of Efficacy was identified through analyses of the following sub-categories (1) Math Efficacy of Elementary Teacher Participants; (2) The Role of Experiences in Shaping Math Efficacy; and (3) Participant Theories About Their Success/Failure in Math. These categories were identified by first identifying several initial codes, then grouping the initial codes into focused codes, the focused codes into sub-categories, and then identifying major categories or themes among the sub-categories.

Math Efficacy of Elementary Teacher Participants

Data were analyzed according to comments that were made regarding each teacher participant’s self-efficacy in mathematics, both in childhood (defined as throughout their K-12 mathematics learning experiences) as well as at the beginning of their teaching careers. Data that indicated changes in efficacy, whether that was regarding efficacy being challenged or growing, were also included in development of this category. Sub-categories identified included (1) Math Efficacy-Childhood, (2) Math Efficacy-Beginning Teaching Career, and (3) Changes in Efficacy.

Math Efficacy-Childhood. For the sub-category Math Efficacy-Childhood, three focused codes were identified: (1) high efficacy; (2) mixed efficacy; and (3) low/negative efficacy. The low/negative efficacy sub-category encompassed two initial codes, low self-
efficacy in math as a child and negative stereotypes/societal influence on efficacy–gender roles. This data, consistent with the rest of the data analyzed for this research study, were still color-coded red/yellow/green by each participant efficacy category. This helps account for times when people are not just one way; where they do not fit exactly into the childhood K-12 efficacy category they have been assigned. However, likely because the participants’ initial survey data was utilized to categorize them into efficacy categories, these categories remained true to the percentage of responses that were seen in each (see Table 5).

Table 5

Math Childhood Efficacy Data: K-12 Efficacy, by Participant Category

<table>
<thead>
<tr>
<th>Efficacy category identified</th>
<th>High</th>
<th>Mixed</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High efficacy</td>
<td>10 / 77%</td>
<td>0 / 0%</td>
<td>0 / 0%</td>
</tr>
<tr>
<td>Mixed efficacy</td>
<td>2 / 15%</td>
<td>3 / 60%</td>
<td>5 / 38%</td>
</tr>
<tr>
<td>Low efficacy</td>
<td>1 / 8%</td>
<td>2 / 40%</td>
<td>8 / 62%</td>
</tr>
</tbody>
</table>

Note. Data are based on participant selection survey open-ended question responses. Reported by frequency count / percentage of total data from this efficacy category.

High Math Efficacy. Most of the data fitting this initial code came from participants who were in the high efficacy category. Some came from those participants in the mixed efficacy category, and one count came from a participant in the low efficacy category. Much of the data coded in this category were of experiences or memories where teacher participants spoke of being good at math as a child. Many participants made mention of feeling confident in their math abilities because they were able to memorize strategies, algorithms, or math facts, performing well on timed tests: “I felt like I was good at math
because I was in the upper part of my class for memorizing those facts and getting done pretty quickly with the tests” (Ingram, initial reflective document). “I would say that I was able to memorize the strategy taught and would call myself a good math student” (Simmons, initial reflective document). “It was definitely all about memorization, but that was okay with me because I was good at memorizing” (Clayton, initial reflective document).

I was always a “good math kid,” I rarely struggled, at least in elementary and middle school; in fact, I would often get bored and find my attention fading in and out of lessons and into trouble....As a young mathematician, I had drive to be “successful” in Math by being fast and accurate. (Mason, initial reflective document)

There were also two counts of mixed efficacy data included here, as well as one count of low efficacy data. The mixed efficacy data were due to positive memories about math early on, that changed in middle school:

As a child in math, I feel like I did pretty good. Most of my memories of math in elementary school where of fast fact timed tests. I remember just wanting to get 100% on 10 so I could move on to the next operation. I felt like I was good at math because I was in the upper part of my class for memorizing those facts and getting done pretty quickly with the tests. (Ingram, initial reflective document)

There was also one mention of a high efficacy moment in early childhood by a low efficacy category participant: “My early memories of math began with workbooks my parents would buy for me from stores. I played school and graded my own work. I even remember giving myself feedback” (Taylor, initial reflective document). However, this participant was still categorized as low efficacy in childhood, as she continued to explain how soon thereafter, and for the next 40 years of her life, she felt low confidence in math: “Eventually, the love for worksheet and workbooks stopped and numbers became something I feared. I remember the fear settled in around the fourth grade when my peers seemed to understand what was being taught and I faked knowing” (Taylor, initial reflective document).
Mixed Math Efficacy. For the initial code of mixed math efficacy, most of the data were from mixed efficacy category participants, and some of the data came from low efficacy category participants, all describing the complexity of their K-12 experience with mathematics. Often, mixed efficacy participants had positive math memories early on, which changed during middle school: “When I was in junior high, all of this changed. Integers and equations were so difficult. Multiple step problems were the death of me” (Adams, initial reflective document). Another explained:

In junior high and high school my math memories were of taking copious notes daily and trying to memorize formulas so that I could pass the test. Oh, and hope that the questions looked like the examples because if they didn’t, I was in trouble. I don’t think at that time in my life that I thought I was particularly good at math. (Ingram, initial reflective document)

One mixed efficacy participant explained her experience was the opposite; having trouble early on until later when a teacher intervened and changed this trajectory: “Math was not always my easiest subject in school; however, a high school teacher challenged me and took interest in my learning” (Cahill, initial reflective document).

The data included in the mixed efficacy initial code from a low efficacy category participant was interesting because while this participant overall named several challenges and lack of confidence in math throughout her participant survey narrative and interview, she also mentions an indifference towards it early on, which is why this piece of data was included in the mixed efficacy results during initial coding:

From the earliest memories as a student, I never believed I was good at math. I never really believed I was bad at math either. I certainly knew there were people who were way smarter than me, but it didn’t bother me. At the time, I thought it was just who I was....I loved math and then I didn’t. And I don’t know the exact time. I know as I went through middle school it got more difficult. (Williams, initial reflective document)
**Low Math Efficacy.** For the focused code of low/negative math efficacy, two initial codes were identified in the data: *low self-efficacy in math as a child*, and *stereotypes/societal influence on efficacy–gender roles*. None of data from high efficacy category participants was identified in either of these initial codes, while some of the data in each of these initial codes came from mixed efficacy category participants, and most of the data in these initial coded categories came from low efficacy category participants.

Participants could vividly recall experiences from their early K-12 experiences where math became a challenge, did not make sense: “When I first started in school, I really actually struggled with almost all subjects. When I first started, I was not successful at all. Math is kind of very black and white, and I just really didn’t understand it” (Cahill, interview).

Well, I never saw myself as a mathematician. I felt like I knew enough to get by and that’s it. For me, with math, it was like the Charlie Brown, like when people start talking numbers, I hear, “Wah, wah, wah.” I’m like, “Wait a minute. I got to write it all down. I got to see it.” That just was my experience, my educational experience. And even in college, it was like my sorority sisters, “Help me. Tutor me on this test. I got to pass it.” (Ingram, interview)

As a student myself, I was not a fan of math. I felt like math was hard and so I avoided it at all costs. I didn’t understand the process behind mathematical concepts and didn’t see the worth in it....Struggling with math continued through high school. I wanted to do the bare minimum to get by and be done with my math courses to graduate. (Mills, initial reflective document)

Participants often accepted this as part of their identity—it was just not who they were, not something that was attainable to them: “I really felt like, there were math people and there weren’t math people, and I wasn’t a math person” (Taylor, interview). Participants recalled how they got by through their K-12 math experiences:
I copied assignments from friends and failed tests throughout school….My math teacher was my softball coach, and I ner got called out for cheating. And I hate to even admit that, still as an adult, but it was my only way through. (Taylor, interview)

They recalled how math only became harder as they went from elementary, to middle, to high school: “Once I reached high school, I was completely lost when it came to math” (Adams, initial reflective document).

It wasn’t until high school, when enrolled in geometry as a sophomore, that it became painfully obvious, I was terrible at math. I cried often. At the time, I thought the only career I could ever have was an elementary school teacher; I would never be smart enough to do anything else. (Williams, initial reflective document)

**Math Efficacy–Beginning Teacher Career.** The second sub-category under the category of Math Efficacy of Elementary Teacher Participants identified was Math Efficacy–Beginning Teacher Career. Because I had quickly learned that teacher participants could not be put into a neat little efficacy category for the entirety of their lives, thus categorizing them based on most of their K-12 experience for the purposes of this study, I wondered how their K-12 experience, and the efficacy they held upon graduating high school, then affected their efficacy in their early teaching career. How many teachers were changed by their teacher education preparation programs? Did their efficacy grow, or did they just continue to get by, doing whatever it took to graduate? After analyzing data, most of the data from high efficacy teachers indicated they had *high math efficacy as beginning teachers*, while some of mixed efficacy teachers made comments that indicated they had *high math efficacy as beginning teachers*, and only a small amount of the *high math efficacy as a beginning teacher* data came from the low childhood efficacy category participants (see Table 6).
### Table 6

*Math Efficacy—Beginning Teacher Career, by Participant Efficacy Category*

<table>
<thead>
<tr>
<th>Efficacy Category Identified</th>
<th>High</th>
<th>Mixed</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High efficacy</td>
<td>4 / 57%</td>
<td>0 / 0%</td>
<td>2 / 33%</td>
</tr>
<tr>
<td>Mixed efficacy</td>
<td>2 / 29%</td>
<td>2 / 100%</td>
<td>0 / 0%</td>
</tr>
<tr>
<td>Low efficacy</td>
<td>1 / 14%</td>
<td>0 / 0%</td>
<td>4 / 67%</td>
</tr>
</tbody>
</table>

*Note. Efficacy category based on participant selection survey open-ended question response. Frequency/percentage of total data from this efficacy category.*

Similarly, all of the data coded as *mixed math self-efficacy as a beginning teacher* was from mixed-efficacy category participants. Many of the participants in the high-efficacy K-12 math category also had *high math efficacy as beginning teachers*. Even when they encountered challenges that all teachers may face, they still had enough efficacy to believe that they could influence their own growth as math teachers:

> When I became a teacher, I may have lacked some confidence in the specific skills that I would be teaching, but once I brushed up on the algorithms, I was able to teach from a position of knowledge and could lead students further from memorization. I have always felt confident in my ability to teach students effectively. (Mason, initial reflective document)

> So, I started with (PD facilitator) in college, but then I….Especially the first couple of years, I took her PGC courses through the district. And then was fortunate to be here where we’ve worked with her on and off throughout the years up until last year even. And even when I think I hear everything she’s had to say, and I remember everything, I don’t. I pull back notebooks from the first time and I may have done it at that point, but just constantly that reinforcement, I think in hearing it again….Because I had that strong foundation in college, I don’t think my methods have changed dramatically over the years, but every time I sit down with (PD facilitators) I am reminded of those great strategies and why I chose to do this….I’ve taught quite a few grade levels. I’m pretty confident in most of them. As I’ve gotten the higher grade levels, I’ve definitely had to do a little work to get that confidence. (Clayton, interview)

Regarding participants who were categorized as mixed math efficacy throughout their K-12 experience, the data also supported having *mixed efficacy* about one’s mathematics abilities,
and mathematics teaching abilities, at the beginning of their teaching career, stating: “As a teacher my early years of teaching math were just going by the books provided by the district. I felt comfortable teaching math because, after all, I teach kindergarten” (Ingram, initial reflective document)….“I can handle that math” (Ingram, interview).

Interestingly, a third of the low self-efficacy as a beginning teacher data came from a high-efficacy category participant (with none coming from mixed, and most coming from low efficacy childhood category participants). This participant was an outlier but mentions that her college math education courses presented things differently (more multidimensional) than she was accustomed to (one-dimensional), and this challenged her high self-efficacy upon leaving college and stepping into the classroom. Her coursework seemed to do enough to open a Pandora’s Box, if you will, of other ways of thinking about and teaching mathematics; however, she did not feel confident she knew enough about it to effectively teach that way:

I feel like I left college thinking, “I don’t know how I’m going to teach math.” And even a couple years in to teaching I still felt that way. (Simmons, interview)

I was terrified to teach math my first year or two. I followed the designated curriculum day by day and didn’t really introduce any outside strategies. I was in a math survival mode. (Simmons, initial reflective document)

More of the data in the low math efficacy as a beginning teacher initial code came from low efficacy participants, who recounted their fears upon stepping out of college and into their elementary classroom to teach math: “Being asked to departmentalize and teach fifth grade math became very ironic. I stumbled through the first two years” (Taylor, initial reflective document). “That was scary for the world. I thought, I’m the best person to teach math?” (Taylor, interview). These data indicate that much of the high math efficacy that was present
throughout the K-12 experience continued with these teachers, and much of the low math efficacy that was present in participants of that category in the K-12 experience continued into the classroom with them as they stepped into their classrooms and began teaching students.

Changes in Efficacy

The next sub-category in the category Math Efficacy of Elementary Teacher Participants identified was Changes in Efficacy. The initial codes identified in this sub-category were (1) efficacy being challenged, (2) efficacy ebbing/flowing as a teacher-flashbacks, (3) strategies used as an adult to remind self of capabilities, (4) growth in efficacy, (5) real-world relevance/experiences-impact on efficacy, and (6) changes in efficacy leading to changes in teaching. These initial codes were analyzed to create the focused code of changes in efficacy.

Regarding efficacy being challenged as seen in the data, this was most mentioned by high-efficacy category participants who recalled times their math efficacy was challenged throughout their lives. Usually, although they felt efficacious about math for most of their lives, they could keenly articulate a time where they felt their math efficacy being challenged:

In high school Algebra, I used to always use the back of the book for the odd answers to make sure I was following the algorithm correctly....I remember struggling with “proofs” and really feeling for the first time that I need help. (Clayton, initial reflective document)

I took a whole year of elementary math with (Taibi) in college, and that kind of rocked my world because I could do the more complex problems, but then here she’s given me like simple problems but asking me why. And I’m like, well, I don’t know. Because it works, because it works. I don’t know why. I can multiply and make it work like that. But I can, and that works, and I have the right answer. (Clayton, interview)
And as math became more compartmentalized and more just turn the textbook and do the odds and whatever it was in high school, that’s where I just fell out of it....To be honest, here’s some vulnerability for you, I failed college Algebra twice. (Mason, interview)

In college I remember learning all the new methods and strategies that were coming out and I felt overwhelmed. This wasn’t the way I was taught, so how was I going to be able to teach these to my students? (Simmons, initial reflective document)

I found it interesting that often in the reflective narratives or interview data, participants talked about specific moments they recall their *efficacy growing* or being *challenged*, as well as this ebbing and flowing still even as a teacher. Teacher participants talked about having what could be described as flashbacks to a time where they did not feel as efficacious as a mathematician, or teacher of mathematics, as they might now, and how this can play out even during their instruction with students, such as this high-efficacy teacher participant sharing how teaching geometry can trigger challenges to her efficacy, as it did previously in high school: “Maybe it’s because it wasn’t something that just followed an algorithm. Ironically, we are working on geometry right now in fourth grade and I still get that feeling, even at this much more basic level!” (Clayton, initial reflective document). One low-efficacy category participant discussed strategies she uses to remind herself of her capability as a mathematician; that she is not that person anymore:

When we worked together and I was asked to pull the high math learners, I often did not know how to teach them, and I went back to that space. I realized that while I was planning, I didn’t do the work of reminding myself that I am successful and knowledgeable and have worked really hard to feel this way. It is a process I have to come back to often....My confidence returned when I was one on one with my small group. My method of teaching is interactive and centered around all students feeling successful. (Taylor, interview)
This same participant vulnerably shared with me that even the process of being observed for this research project challenged her efficacy. She explains her experience of how this occurred:

I reflected on why my instruction felt like it did not flow naturally and that my lesson felt boring, I realized part of my reluctance was feeling nervous and my old [young] math self was surfacing again. I was worried that I may do or say something wrong in front of you. (Taylor, interview)

The implications of how the act of being observed, whether it is by an evaluator, coach, or by peers as part of a PD process can trigger flashbacks to times of low self-efficacy for participants, is important to note. PD that acknowledges that all teachers will be in the same position provides a safe environment in which teachers can be vulnerable. Observing others was also one of the experiences noted in the data that led to growth in efficacy for teacher participants: “It also gave me the confidence, once I watched other teachers. It just gave me the confidence I needed to facilitate those types of lessons” (Adams, interview).

For the initial code growth in efficacy, most of the data came from mixed and low efficacy category participants. It seems for those who had low efficacy, or mixed (which would indicate at some point they had low efficacy about their math abilities in their K-12 experience), the moments of growth in efficacy are experiences that are particularly meaningful, as for these participants they marked turning points in the way they viewed their own math abilities. For the high efficacy participant, her math efficacy was only challenged during college elementary math courses when she was introduced to strategies and tools (a more multidimensional approach to learning math) that varied from the way she had been taught (a more one-dimensional approach), so she reflected: “With time I felt more confident and now I love it. It’s one of my favorite subjects to teach” (Simmons, interview). For the
low and mixed efficacy participants, the data that were identified and coded in this category were often aligned to a person who had shown interest in them to helped influence a turning point, or reflections about how exciting this time of their life was since it was something that had been a challenge for so long:

And I remember one night laying in bed thinking when this is over, I’m gonna be really sad. How am I going to keep doing math? Because I was so excited that I got it, and it was something that I’d been battling forever. (Taylor, interview)

Some of the data in this section included metacognitive/reflective thoughts rather than turning points, that demonstrate a participant who was categorized as low efficacy for their K-12 experiences, but their comments reflected that since then, they have changed the way they view themselves: “My journey to loving, maybe appreciating and valuing would be most accurate, math has been one of my greatest accomplishments in my 50 years of life” (Taylor, initial reflective document).

As I’ve gotten older and as I kind of learned more and learned more about myself, I realized that it’s not so much....You’re not born with it. It’s a process, but you have to learn strategies and figure out what works for you. (Mills, interview)

There was also evidence that as participants’ math efficacy changed, it aligned with changes in their pedagogy: “So that’s kind of how I changed over time, and certainly has changed my instruction” (Williams, interview).

And saying I feel confident, I feel like I’m able to give those kinds of experiences that we’re talking about. Those are the ones that I feel like lend themselves more to being able to explore first and being more....Those for me, are easier to figure out how I’m going to make it hands-on for the kids and more concrete. (Ingram)

There was also data in support of the idea that real-world relevant experiences could contribute to building efficacy: “As well as using real world examples, I remember how it
helped me” (Clayton, initial reflective document). Real-world experiences seemed to build efficacy of participants even if in hindsight, through metacognitive reflections:

> I ended up working at Quik Trip, waiting tables and bartending through college, and I can count money faster than someone can say their ABCs. And you don’t realize, oh my gosh, I’m counting on. Or I just did removal, or I just did compensation….You don’t realize, man maybe I really am good at math because I was able to apply it in a real-life situation, that I never had to do before. (Williams, interview)

**Theory 2: Experiences Shape Beliefs about the Nature of Mathematics**

While analyzing the data, it became apparent that experiences teachers had, from their earliest childhood experiences, throughout their K-12 experience, college, and as practicing teachers shape their beliefs about the nature of mathematics. For the first category, Beliefs About the Nature of Mathematics, three sub-categories were identified through data analysis. They include (1) One-dimensional Mathematics, (2) Somewhere along the Progression between 1D and “5D,” (Mason, interview) and (3) Multidimensional Beliefs about the Nature of Mathematics. Table 7 shows an overview of the data from participants of each K-12 efficacy category regarding frequency as well as what percentage of the total data were reflected for each of the three categories (high, mixed, and low K-12 efficacy).

**Table 7**

*Theory of Math Motivation for Teachers of Elementary Mathematics*

<table>
<thead>
<tr>
<th>Experiences Shape…</th>
<th>High Efficacy</th>
<th>Mixed Efficacy</th>
<th>Low Efficacy</th>
<th>Facilitators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math efficacy</td>
<td>Y 35/30%</td>
<td>Y 32/37%</td>
<td>Y 51/43%</td>
<td>Y</td>
</tr>
<tr>
<td>Beliefs about the nature of mathematics</td>
<td>Y 63/41%</td>
<td>Y 49/32%</td>
<td>Y 41/27%</td>
<td>Y</td>
</tr>
</tbody>
</table>

*Note.* Y indicates “Yes” was seen in data. First number listed = frequency / Second number = percentage of total data in from this efficacy category.
While the majority of initial participant survey responses (see Table 3) indicates most teacher participants believe they should teach mathematics in a way that is Multidimensional, mentions of One-dimensional (1D) Mathematics were made more frequently by high-efficacy participants, often when describing the way they were taught, and found success with, as a child. Boaler and Greeno (2000) found that the primary focus of traditional, or one-dimensional mathematics classrooms, is on one form of mathematical work and that often means only a narrow range of students, for whom this work is appealing, are successful. While there were 36 mentions of 1D Views of the Nature of Mathematics by all participants, most of these mentions were made by high efficacy participants. High-efficacy category participants also made several mentions regarding Multidimensional Beliefs about the Nature of Mathematics. The name of the Somewhere along the Progression Between 1D and “5D” sub-category was identified through one high-efficacy participant’s data in support of multidimensional mathematics, where, after asked in his interview how his beliefs about the nature of mathematics had been changed through the PD process, he replied: “So yeah, I’m all in on the 4D, 5D, 10D, whatever” (Mason, interview). In contrast, most of the mentions of Multidimensional mathematics came from low and mixed efficacy participants, who often held strong beliefs that learning should be Multidimensional because that is not the way they learned it, and they remember negative math experiences and attribute this to their low K-12 efficacy as a result, as heard here:

Kids need to talk, they need time to think, they need time to try lots of strategies (even the ones that don’t fit in the box), they need to analyze others’ work, they need effective ways to communicate mathematical thinking, they need practices that are universal to all mathematical standards, they need lots of resources, they need to be told their ideas are valuable, they need to have the freedom to make mistakes without shame. (Williams, initial reflective document)
One mixed efficacy participant, during her interview, spoke of how when she learned that math could be Multidimensional, during PD, this raised her own math efficacy:

Then also in that training and some of the other math things that I’ve gone to, I feel like in my education, it was never presented as there’s more than one way to get to the answer. It was always, “This is how you do it.” And so going to this training and figuring out that made me feel like, “Okay, maybe I can be a mathematician because I don’t have to do it like everybody else. But if I get to the same answers, I’m right.” And then I was just thinking, “Man, I wish I knew this earlier.” (Ingram, interview)

High efficacy participants would also highlight the importance of Multidimensional learning experiences and how they’ve grown in this area because of PD, but also place value on some elements that could connect with One-dimensional classrooms:

(PD) changed my perspective on things. Before I was very, like I said, procedural.... But (PD) really pushed it to that number sense. I mean, I will die on the hill of number sense....I’ll be honest. I’m kind of...my relationship with Number Talks sometimes ebbs and flows with the needs of my class. If I feel like more direct instruction or more actual practice is needed and that mental isn’t going to benefit as much, then I kind of stray away, I stray away from it maybe sometimes. (Mason, interview)

So being able to go through a lot of problems and letting them see it and see it over and over again has also really helped. Because I think at least the repetition in math, you think, oh, fact recalling and things like that. But I think even getting a little bit of that automaticity in solving lots of problems or seeing lots of problems solved quickly has helped them as well....I’m not going to get out and go base-10 for two-digit multiplication. That doesn’t...maybe once, just to show them, but then it’s not efficient anymore. His are more efficient. (Clayton, interview)

Similar values were upheld by another low-efficacy participant as well, who stated: “I do still think that kids really need to just memorize multiplication facts. You can’t do fifth grade math if you don’t just know them like that, or you struggle” (Taylor, interview). It seemed moving between Multidimensional and One-dimensional values (Somewhere along the Progression between 1D and “5D”) extended beyond efficacy category but was seen more by teachers of intermediate grade levels who see the gaps in a child’s math skills or lack of
automaticity as holding students back from problem-solving at high levels, in which case they revert to the value of role memorization or automaticity for some concepts. One high efficacy teacher shared the cognitive dissonance between teaching in One-dimensional or Multidimensional ways she identifies in herself in the interview:

Growing up, I think it just came easy for me. Numbers just make sense. There’s a right and a wrong answer. I would always say I was a good math student. I was able to memorize the way that they taught me. But I can’t really go beyond that and that’s what I’m trying to have to teach myself; that there’s more than one right answer or one way to get the right answer. I think that’s how I’ve had to shift my thinking; becoming a teacher and having to think of it that way. I remember having a conversation with one of my friends who taught in a different district and she struggled with math growing up and she said that makes her a good math teacher, she can relate to the kids more. I really had to step back and be like, “Okay, math was easy for me but it’s not going to be easy for everyone so how do I change my thinking about math and how do I explain the different ways to get answers to my students who are struggling?” That’s how I’ve evolved, I would say. …I don’t think thinking about the one way is wrong. I think blending them together. That there are multiple ways but there are more efficient ways to get the answer. (Simmons, interview)

**Theory 3: Math Efficacy and Beliefs Shape Math Motivation**

In this section, I share findings on how teacher participants explain how their experiences, either K-12, college, or during PD, shaped their motivation to engage in PD and/or motivated them to try out new pedagogical practices in their classrooms. When in the theory identification stage of analyzing data, two sub-categories were synthesized to identify the category Math Efficacy and Beliefs Shape Math Motivation. These sub-categories, which were identified through analysis of focused codes and frequency included: (1) Motivated by Students and (2) Past Negatives Giving Purpose for Students.

**Motivated by Students**

There were 16 mentions of various reasons given by participants at all math efficacy levels related to being Motivated by Students to participate in or try pedagogy learned in the
math PD to help their students finding success. Teachers mentioned being motivated by their students’ struggles, the current district adopted resource not working for every student, and poor performance data on assessments:

Just seeing that my kids were stuck...how I was teaching it and that they bombed the first test, that was really eye-opening....Okay, we are following (the district adopted math resource) like we should, or we thought we needed to. But that wasn’t working for our kids, and it wasn’t working for us, so we needed to teach things. (Simmons, interview)

Teachers spoke of realizing that the way they learned math may not work for all of their students, and that being engaged as a participant motivated them to try new strategies that would be engaging for their own students: “…when it really engaged me. If I felt like I was engaged with it, then I thought it was something my students would also be engaged with, so I wanted to take that back to try it” (Mills, interview). Consistently, teacher participants, particularly those in the high efficacy group, spoke of their students being the prime motivator for their professional learning or interest in change. They realized that while they may have found success in their K-12 math educational experiences, the same ways they learned may not be working for their students, and that as the numbers become increasingly larger, students need new, more efficient strategies:

I think, to find something that works. Knowing that not the one same strategy is going to work for all these kids, but also knowing that I need them to know at least a couple of strategies, and some, specifically. Like what we’re doing....Division has lots of strategies. You can use repeated subtraction. But then where we are now, that’s no longer efficient. And the strategies we’re going to do today and tomorrow lead up to the bigger digits. So, it’s like, yes, is this two-digit by one-digit? Yes. I know you can skip count. Yes. I know you can....You know, subtract a whole bunch of times, and maybe hopefully get it. But tomorrow when I give you a four by one digit, you can’t do that anymore. (Clayton, interview)
Teacher participants also spoke of being continually motivated by their students’ learning:

I learn so much from the kids, too. Even after eight years, I’m learning from the kids—different strategies that I never thought of or different ways to solve problems that I never would have expected. But now I like it, so I add it to my tool kit (Snyder, interview)....I am continuously learning new ways of thinking about math and math problems from my students and I love it! (Simmons, initial reflective document)

**Past Negatives Motivating Teachers’ Purpose for Professional Learning**

Similarly, several teacher participants from the mixed or low math efficacy categories were motivated to learn more and do better by their students because of the negative experiences they recalled from their pasts: “I stumbled through the first two years and finally realized that I was actually a really good math teacher. Because, not in spite of, my journey” (Taylor, initial reflective document). Similarly, a mixed efficacy participant shared how she is motivated to show students she cares and is invested in their thinking because she never felt this way in school:

Since grade school, I didn’t have a teacher that cared if I passed. It was like, “Eh, you’re going to go onto the next grade level anyway. Eh, whatever.” And so, in junior high, I was able to get C’s. …I hated junior high. It was horrible, worst two years of my life. And so, I’m always available. So, I’m always walking around. I’m always checking in. I’m very in tune with who’s doing what....It’s just more of a you’re not alone. You’re not by yourself. And that’s how many math classes felt. (Adams, interview)

Low-efficacy participant Mills reflected that the lack of multidimensional teaching in her childhood math experiences made her appreciate making sense of numbers in PD and be motivated to teach in this way even more:

My real change began when we started breaking numbers apart and understanding the WHY behind it. It made me a better mathematician because of the fact that I wasn’t taught that along the way, just that I had to do it that way. Period. Many well-meaning teachers were a part of that journey, but for me, it was frustrating. (Mills, initial reflective document)
Table 8 summarizes the percentage and frequency from the most frequently mentioned motivators for participating in PD and changing pedagogy as seen in the data and analyzed to develop the theory of math motivation for teachers of elementary mathematics.

Table 8

Theory of Math Motivation for Teachers of Elementary Mathematics: How Math Efficacy and Beliefs Shape Math Motivation

<table>
<thead>
<tr>
<th>Facilitators</th>
<th>High Efficacy</th>
<th>Mixed Efficacy</th>
<th>Low Efficacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motivated by students</td>
<td>Y 9/82%</td>
<td>0/0%</td>
<td>Y 2/18%</td>
</tr>
<tr>
<td>Past negatives motivating purpose for students</td>
<td>Y 1/6%</td>
<td>Y 11/65%</td>
<td>Y 5/29%</td>
</tr>
</tbody>
</table>

Note. Y indicates “Yes” was seen in data. First number listed = frequency / Second number= percentage of total data in from this efficacy category

Attribution of Motivation Theories Put Forth by Participants

It is also worth mentioning that sometimes, as participants were sharing how their experiences have changed their pedagogy and adult learning process, they put forth potential attribution theories regarding their own success or failure in math. For one high efficacy participant, when his math efficacy was challenged, he mostly attributed this to either external characteristics or an acknowledged lack of effort on his part. He had not internalized his challenge in efficacy to be permanent, or fixed:

Math got more complicated for me, kind of the further my connection to school got, I guess. I wasn’t a great high school student by any stretch. And most of that was just effort. As a kid, I took math as a challenge....As I got into middle school, we moved in middle school, so maybe that had an effect. I’m not sure. We moved out of state and then came back two years later....I failed college algebra twice…certainly at least one of those was completely related to effort. The other one, I do feel like I tried, but I think I was so far removed from the application of things and I don’t feel like the instruction was the best. (Mason, interview)
For a low efficacy participant who made suggestions of what her lack of success could be attributed to, she reflected:

I just remember thinking I don’t understand this, and just crying and crying and crying. And I never could get it….I guess maybe along the way, there was no really, thinking about math....it was just telling me how to do it, so I can feel comfortable. And then when I don’t know how to do it, I’m screwed. (Williams, interview)

For this participant, in hindsight, now that she knows the value of Multidimensional Mathematics, she attributes the lack of being taught in a way that emphasized making sense of math as contributing to her challenges. However, also consistent in her data is that throughout her K-12 experience, she internalized her low math efficacy, believing her lack of math ability was fixed. This was the case up until participating in the recent math PD:

“Well gosh, maybe I am good at math. I’m only 41, and I’m figuring this out.” So, …it really got me to thinking more about have I been good this whole time, but never had the chance to do it in the way that I’m not supposed to. Because I didn’t fit this box. And is math really a box? So that’s kind of how I changed over time, and certainly has changed my instruction. (Williams, interview)

In the facilitator focus group, when I shared initial findings, one of the facilitators of math PD connected with the idea of experiences changing beliefs, efficacy, motivation, and pedagogy of teachers, stating:

…one of the mantras I’ve had recently, and it’s from doing lots of different projects, and one of them is working with [an outside agency]. I’m a math coach for some of the districts in [another state]. And there’s a systems coach, and the systems coach is this professor at [a college]. And she said, “The way that we get people to change their beliefs is by changing their experience.” So I’ve just had this....This has kind of been my mantra. I’ve got to help change people’s experiences so that they can then change their beliefs about themselves, about what kids can do. To me, that speaks to that. (Michaels, focus group interview)

While much has already been seen in the data regarding changes teachers made to their pedagogy as their beliefs about the nature of mathematics and/or efficacy were changed by experiences, the next section looks more explicitly at shifts teacher participants made
because of their efficacy and beliefs having been changed by their life and/or PD experiences.

Theory 4: Math Efficacy and Beliefs Shape Math Pedagogy

The category Math Efficacy and Beliefs Shape Math Pedagogy was identified during the theory generation phase of data analysis and was comprised of several sub-categories identified in the data of pedagogical strategies gained by participants through participation in PD, including: (1) Use of Questioning; (2) Student-centered, Efficacy-building Practices; (3) Contextual Elements: District/Standards–aligned; (4) Five Practices (Smith et al., 2009); and (5) Concrete to Abstract Continuum (Kauchak & Eggen, 1980).

Use of Questioning

The first sub-category, Use of Questioning, was most frequently seen in the data—in fact, 220 questions were pulled from teacher observations that could be described as used to engage students in thinking/facilitating discourse. Initially, I pulled these questions in groups sometimes, as often the teacher would ask multiple questions right after another, in clusters. However, I then began to wonder if the efficacy categories were accurately represented in that data if each question was not analyzed on an individual level and moved to separating them out individually. This sub-category code also included questions that could be categorized as “Talk Moves” (Chapin et al., 2013, p. 11); for example, “Can I revoice what you just said?” or “(Student), can you add on to that?” (Williams, observation). Interestingly, there were far more questions asked in each category by low and mixed efficacy participants than high. I did not further analyze these questions for depth of knowledge or quality, as previous studies have already identified some questions are more useful than others at eliciting higher level thinking from students.
Student-centered, Efficacy-building Strategies

The second sub-category, Student-centered, Efficacy-building Strategies, was identified through analysis of the two initial codes most frequently mentioned in the data, which included (1) noticing and naming-agency (Johnston, 2004); (2) affirmations of student thinking/praising process.

Data from several participants across all three efficacy categories showed the teachers would intentionally notice and name (Johnston, 2004) practices, processes, or strategies students used that helped them so that they would be able to replicate them again more intentionally in the future with success. “Once we start noticing certain things, it is difficult not to notice them again; the knowledge actually influences our perceptual systems” (Harre & Gillet, 1994). Noticing and naming was seen most in mixed efficacy participants from the data in this study; however, one of the mixed efficacy participants did this a lot, and it was also seen in high and low efficacy participants. Examples of this practice include low efficacy participant Williams, who also used it as a form of affirmation during her classroom observation: “She used really good math communication,” and “[Student] was working really hard at making sense of all those parts in our story, right?” High efficacy participant Simmons used this practice during her classroom observation while walking around monitoring collaborative student groups, commenting as she shared the groups’ work via mirrored iPad on the projector: “Every table sorted them into the colors. That was step one, good job. And then this table, they spread them out then they laid them out and you can kind of see one is longer than the other.” It was common that when teachers were noticing and naming process strategies for students, these processes often aligned with the Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council
of Chief State School Officers, 2010) or Math Mindset Norms (Boaler, 2016) that had been modeled/taught to teachers through the PD process.

Sometimes, this practice was transferred to students, and they were asked to notice and name what other students did during a classroom discussion. Another way this was seen is when mixed efficacy teacher Cahill would refer to students as “mathematicians” (classroom observation), which is an agency-building practice (as is noticing and naming). Johnston (2004), in his discussion around building student agency, shared:

children bring with them to school well-learned cultural narratives acquired in cooperatively retelling family stories from a very young age. Those narratives hold models of the possible forms narratives can take, who is allowed to take which roles, and so forth. (Pontecorvo & Sterponi, 2002)

Given that so many societal norms and stereotypes in America allow student to accept they are not “math people,” even something as simple as referring to young learners as “mathematicians” can help them build their identity, and agency; particularly if we show they how to do the things mathematicians do (by noticing and naming skills, strategies, and practices they can then later replicate independently more intentionally). Johnston (2004) stated it this way:

Our job is to change these narratives so that both boys and girls have a productive sense of the implications of the choices they may and the strategies they choose. We do this by foregrounding these in the agentive narratives through which we help them reconstruct the events. (p. 40)

Mixed-efficacy participant Ingram, while moving around the room to provide feedback to kindergarten students who were working in partnerships playing differentiated games, used noticing and naming, as well as questioning, to secretly teach these young mathematicians:

Oh, so plus means we’re adding, we’re getting more….Okay, so we’re both kind of trying to do it efficiently and fast? Okay, here we go….Teacher: So, it’s more
efficient to have some already on there instead of starting with zero every time?
Student: Yeah. The middle takes longer. (Ingram, classroom observation)

**Affirmations of student thinking/praising process** was seen frequently in the data (41 times).

While this occurred more with the high and mixed-efficacy participants, it was present in all three efficacy categories. This is also a practice that can help reinforce a growth mindset culture in the classroom and build student agency and efficacy. This was seen when low-efficacy participant Williams reinforced productive practices students used during the portion of the math workshop where students were sharing their thinking:

Thanks for coming up with a really great “I think,” looking at someone else’s math. …thank you for letting us learn from you today! Thanks for using courage!….I believe in you so much that I will absolutely wait for you to see something. (classroom observation)

Mixed efficacy participants made comments presuming positive intent of students before worktime: “I love how eager you are to get started” (Cahill, classroom observation) or to praise thinking when facilitating a classroom discussion: “Okay. Very interesting. I want you to keep that in your head as we’re talking through today. That’s a big turnaround fact” (Cahill, classroom observation). This language used by a mixed efficacy teacher to facilitate classroom discussion also is affirming of students’ willingness to share their thinking with others, supporting the practice of being vulnerable in the safe math learning environment “Somebody want to be brave?…Is anybody brave enough to share what they’ve got on that second line?” (Cahill, classroom observation)

**Contextual Elements – District/Standards-aligned**

The third sub-category identified from the data for the pedagogy/strategies gained from participating in PD (as named by participants) theme is Contextual Elements–District/Standards Aligned. Sun (2020) referred to contextual elements as being forces...
outside the classroom that either support or challenge teachers’ ability to align their beliefs with teaching behaviors. In their research, Talbert and McLaughlin (1992) highlighted that “the attitudes and practices of those in any one level of the system”—for example, the “classroom, school, or district”—are “conditioned by the activities and attitudes of actors in other parts of the system” (p. 33).

In the case of this research, most of the Contextual Elements identified as codes were seen in a positive light by participants; they appreciated learning strategies in PD that aligned with what their district or state asks of them, and this alignment made the PD experience more relevant, positive, and productive for them. This is in line with previous literature (Hill et al., 2020; Sun, 2015). There was evidence in the data from all levels of efficacy of use of the Standards for Mathematical Practice (SMPs), which were initially introduced as part of the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Several teachers had the SMPs posted visually in student-friendly terms as references on their classroom walls and used these to help facilitate discussions or reinforce efficacious mathematical practices by students, as heard during by low-efficacy participant: “one of those really important math practices, is that we choose the tools that are effective for the job that we’re doing. A back-up for your kind of thinking, your kind of math” (Williams, classroom observation). Again later, she references the SMPs while modeling: “I want to do this by creating a representation” (Williams, classroom observation). Here, she combines two to help students see how they can be intentional in the tools they use to make sense of math:

When we are working on learning math and making sense of math, one of the best ways to make sense of math is, make sure that the tools you pick [are] appropriate.
We need to make sure it’s appropriate for the problem, but we also need to make sure that it’s appropriate for our level of understanding. (Williams, classroom observation)

This high-efficacy participant also, while noticing and naming what collaborative groups of students did during worktime, refers to SMP #6, Attend to Precision:

They had to be really careful to make theirs nice and neat so that you can easily tell which one, if they were kind of messy with it and they were kind of crooked, you wouldn’t be able to tell. (Simmons, classroom observation)

Mixed efficacy participant Ingram also referred to SMP #5, Use appropriate tools strategically, here: “We know you can talk about how tools sometimes help us get smarter at math. That’s okay, this one fell off [referring to the visual previously hung on the wall], but we still know” (classroom observation). Use of the SMPs, when used intentionally by teachers, can also help give students a relevant context for why they use math tools or strategies.

Elements of the district math instructional you do/we do/I do model were mentioned by participants from all efficacy levels. This practice or structure of introducing math tasks first, allowing students to productively struggle through it (you do), before some collaborative discourse and sharing of strategies (we do), then possibly some explicit modeling by the teacher (I do) to clear up any misconceptions, is constructivist in nature (aligned with the WIG model mentioned earlier by Perkins, 2013) and is modeled in one of the mathematics PD cohort experiences many of these participants attended. After working with a consulting company who introduced this model, the district where teacher participants teach adopted similar structures, so both the PD as well as district context support use of these practices. Teachers made mention of how they found this style of instruction effective for students: “The components of you do, I do, we do allows every stakeholder to have a
voice” (mixed efficacy participant Adams, observation reflective document). About this practice, another mixed efficacy participant stated:

Instead of me saying how to do it, you’re asking them questions and pushing them past that point...I think it kind of made me reflect on what’s going to benefit them the most, and I think it’s allowing them to continually talk and work through the problems themselves instead of me just showing them to start, they’re working through it themselves and already thinking about it before I even teach. (Cahill, interview)

As reference to specific parts of this model, participants mentioned modeling, or the “I do” portion of this structure, and how sometimes it is appropriate to support students through misconceptions by explicitly modeling—but that would not be the primary focus of their mathematics classrooms. Many times, these references were made during the participants’ observation reflective narratives, reflecting over whether the portion of time they spent modeling was appropriate, or too much: “I followed that up by making sure my ‘teacher share’ was short and concise and did not take up the math block with direct instruction” (Williams, observation reflective document); or, “I feel like I talked a lot, it was not very inquiry based today” (Clayton, observation reflective document). This is indicative of a multidimensional use of what could be a more traditional or one-dimensional pedagogical practice. High and mixed efficacy participants also more frequently modeled based on an identified student need in the moment to scaffold learning for students. High-efficacy participant Clayton mentioned this in her reflective narrative: “I do think that inquiry and questioning is important but based on how the past few days had gone, I felt like they needed a little direction instruction to clear up some misconceptions.”
**Five Practices Model**

Participants also spoke of adding elements of the Five Practices model (Smith et al., 2009) to their pedagogical practices after participating in PD. The Five Practices model had been introduced during PD sessions with the outside consulting firm, as well as often by two of the other facilitators that had worked with teachers who participated in this study. It is comprised of the following Five Practices Model:

1. anticipating student responses to challenging mathematical tasks
2. monitoring students’ work on and engagement with the tasks
3. selecting particular students to present their mathematical work
4. sequencing the student responses that will be displayed in a specific order, and
5. connecting different students’ responses and connecting the responses to key mathematical ideas. (Smith et al., 2009, p. 550)

Occurrences in the data that were analyzed as aligning to the use of the Five Practices, as mentioned by teachers, in their pedagogy because of participating in PD, included initial codes identified as “intentional sharing of student work—sequencing and selecting, to facilitate discourse”; “using misconceptions to teach students”; “students sharing their work”; and “connecting or comparing/contrasting concepts/ideas.” There were 58 occurrences of this type of work analyzed in the teacher participant data, with most coming from high efficacy participants. These occurrences came from both interview and document data as well as transcripts of classroom observations. Teachers talked about how the way they plan for their lessons has changed because of the PD and using these practices, stating that now they “Plan for possible dialogue between children” (Williams, observation reflective document), because of the experience she had during PD:
I thought it was very interesting that he went through the thought process of how he chose which work to show. So, going from concrete mathematical strategies to semi-concrete, to semi-abstract and abstract, and how we can project what might happen and what strategies might present themselves during problems. (Williams, interview)

Another high-efficacy participant talked about how now he includes “stacking work, by range of complexity, I’ve used that a ton of times, especially with new concepts” (Mason, interview), and that this pedagogical practice came from their experience during PD when the facilitator “came around and stacked our work up based on misconception, you know what I mean?” (Mason, interview). Similarly, mixed-efficacy participant Ingram uses the Five Practices Model now in her instruction to facilitate discussion/discourse through sharing student work:

I also chose to project the work of different students so the class could have some discussion about what we were doing. Discussions from PD and listening to the thinking of others was important for me as a learner. I could see that if I added their thinking to what I knew about what we were working on I could be more efficient in the work we did. Providing those discussions in my math lessons. (observation reflective document)

**The Concrete to Abstract Continuum**

The fifth sub-category identified in the theory generation of Math Efficacy and Beliefs Shape Pedagogy identified in the data was the Concrete to Abstract Continuum (Kauchak & Eggen, 1980). The Concrete to Abstract Continuum, when applied to mathematics, focuses on (1) a concrete stage, where students use manipulatives and/or real-world objects; then (2) a semi-concrete stage, where students draw the manipulatives or objects while working with them; (3) a semi-abstract stage, where students use pictures or drawings provided to them; and (4) an abstract stage, where students use and understand symbolic representations. This continuum, which integrates concepts from Piaget (1950), Bruner (1960), and Dienes (1971), who all encouraged use of multiple representations in
math, was summarized by Kauchak and Eggen (1980), and supported by others (Haistings, 2009; Peck & Jencks, 1987). Recent research has shown the idea that students must go through a certain developmental stage, be a certain age or have obtained a certain emotional maturity before they can learn mathematics is not true; students are as ready as the experiences they have had, and if students are not ready, they can easily become so with the right experiences (Boaler, 2016, p. 8). However, there is still widespread agreement concrete experiences, no matter the age or past experience, can be helpful in developing conceptual understanding of mathematics.

Particularly, one of the facilitators of PD in which several of these teachers participated very strongly encourages its use when approaching classroom mathematics pedagogy. A high-efficacy participant who, until working with this facilitator, did not feel she knew “the why” behind math concepts, shared the changes this made for her pedagogy:

She kind-of made us just back way, way up to the beginning and the Base-10 and why it’s Base-10 then really understanding the principles of mathematics and then using that for every other operation. Through multiplication, division, fractions, and starting at the beginning, and then showing how you can use that understanding to make sense of even the more complex problems and really focusing on that concrete, pictorial, abstract in every concept that you do....knowing the why something works, which to me would always go back to that concrete model, like even two-digit by two-digit multiplication, you can make that with Base-10 and you can see how it all comes together and works....And I think also just sharing new strategies and strategies that can be shown in a way that is at that concrete level. Like if it’s something that I’m struggling with, that’s what I need to see. I think making them do it, I think showing a concrete and then how that gets to the abstract, writing it, drawing it, and then solving it with the algorithm. I think that whole progression. (Clayton, interview)

Another mixed efficacy participant shared that understanding the concrete to abstract continuum really helped build her efficacy, which in turn, also helped shape her pedagogy:

When she came and really explained and talked about, like, the concrete, you know—you got to start with the concrete and move to the representational. And I just felt like that was just key for me as like, “Oh, that is where I need to be and to be always
intentional about that.”….I think I’m just more intentional about making sure that I try to provide those concrete opportunities, especially the importance of when you start something, if it’s a new concept. (Ingram, interview)

A low efficacy participant talked about that learning this concept changed the way she plans lessons, as she now includes “Looking for concrete to abstract strategies” (Williams, observation reflective document) as one consideration. The Concrete to Abstract Continuum was mentioned most by high efficacy participants. For all three efficacy levels, there was evidence learning about this concept helped them better conceptually understand the math they were responsible for teaching, which built their efficacy, in turn, changing their pedagogy. Table 9 summarizes the pedagogical practices named by teachers because of participation in PD.

Table 9

Theory of Math Motivation for Teachers of Elementary Mathematics: Math Efficacy and Beliefs Shape Pedagogy

<table>
<thead>
<tr>
<th></th>
<th>High Efficacy</th>
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<td>Y 45/20%</td>
<td>Y 95/43%</td>
<td>Y 80/36%</td>
<td>Y</td>
</tr>
<tr>
<td>Student-centered, Efficacy-building</td>
<td>Y 25/34%</td>
<td>Y 28/ 38%</td>
<td>Y 20/27%</td>
<td></td>
</tr>
<tr>
<td>Contextual Elements-district/standards</td>
<td>Y 28/38%</td>
<td>Y 26/35%</td>
<td>Y 20/27%</td>
<td>Y</td>
</tr>
<tr>
<td>Five Practices Model</td>
<td>Y 36/58%</td>
<td>Y 7/11%</td>
<td>Y 19/31%</td>
<td>Y</td>
</tr>
<tr>
<td>Concrete to Abstract Continuum</td>
<td>Y 9/50%</td>
<td>Y 2/11%</td>
<td>Y 7/39%</td>
<td>Y</td>
</tr>
</tbody>
</table>

Note. Y indicates “Yes” was seen in data. First number listed = frequency / Second number= percentage of total data in from this efficacy category
Framework of Math Professional Development for Efficacious Behaviors

My pragmatic goal for this study was to develop a framework of theories of teacher participants to support efficacious behaviors. The four theories of math efficacy and motivation for teachers of elementary mathematics (a) experiences shape math efficacy; (b) experiences shape beliefs about the nature of mathematics; (c) math efficacy shapes motivation; and (d) math efficacy shapes pedagogy) are integrated with this framework (see Figure 2), as if this framework is used to design professional development for elementary teachers of mathematics, it is likely to provide experiences that can shape teacher participants’ efficacy and beliefs about the nature of mathematics in ways that support and motivate efficacious behavior and multidimensional beliefs/pedagogy in their classrooms. The categories below were identified to develop a theoretical framework for mathematics professional development through the analysis of most frequently mentioned categories by teacher participants. The math professional development for efficacious behaviors theoretical framework (see Figure 3), is framed by the following categories:

1. Real-world Relevance
2. Hands on/Do it Yourself (DIY)
3. Growth Mindset Culture/Safe Environment
4. Observation of New Practices Modeled (facilitator, peers)
5. Elements of Productive Struggle (cognitive dissonance, discourse)
6. Collaboration—time to talk/reflect with colleagues
7. Aligned to Contextual Factors
8. Immediate Application of Learning
Figure 2

Connecting the Math Professional Development for Efficacious Behaviors Theoretical Framework with the Theory of Math Efficacy and Motivation for Teachers of Elementary Mathematics
Frequency of times each category was seen in the data, as well as percentage by teacher participant K-12 efficacy category (high/mixed/low) is reported in Table 10.
Table 10

*Math Professional Development for Efficacious Behaviors Theoretical Framework*

<table>
<thead>
<tr>
<th></th>
<th>High Efficacy</th>
<th>Mixed Efficacy</th>
<th>Low Efficacy</th>
<th>Facilitators</th>
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<tr>
<td>Real-world Relevance</td>
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<td>Y 16/39%</td>
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<tr>
<td>Hands on/DIY</td>
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<td>Y 15/28%</td>
<td>Y 9/23%</td>
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<td>Growth Mindset</td>
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<td></td>
<td></td>
<td></td>
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<td>Culture/Safe Environment</td>
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</tr>
<tr>
<td>Observation of New Practices Modeled</td>
<td>Y 5/13%</td>
<td>Y 8/21%</td>
<td>Y 25/66%</td>
<td>Y</td>
</tr>
<tr>
<td>Elements of Productive Struggle</td>
<td>Y 8/29%</td>
<td>Y 9/32%</td>
<td>Y 11/39%</td>
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</tr>
<tr>
<td>Collaboration</td>
<td>Y 3/14%</td>
<td>Y 8/38%</td>
<td>Y 10/48%</td>
<td>Y</td>
</tr>
<tr>
<td>Aligned to Contextual Elements</td>
<td>Y 24/40%</td>
<td>Y 18/30%</td>
<td>Y 18/30%</td>
<td></td>
</tr>
<tr>
<td>Immediate Application of Learning</td>
<td>Y 5/56%</td>
<td>Y 2/22%</td>
<td>Y 2/22%</td>
<td>Y</td>
</tr>
</tbody>
</table>

*Note.* Y indicates “Yes” was seen in data. First number listed = frequency / Second number= percentage of total data in from this efficacy category

**Real-world Relevance**

Real-world Relevant experiences connect with theory (1) experiences shape math efficacy, and (3) experiences shape beliefs about the nature of mathematics, of the theory math efficacy and motivation for teachers of elementary mathematics. as seen in participant data, real-world relevant experiences helped teachers gain a sense of self confidence and helped them see math more multidimensionally when used, either in life experiences or professional development. There were 41 mentions of Real-world Relevance in the data, with
high and mixed efficacy participants making the most mentions of the value of relevance in building their efficacious behaviors, either through their early mathematics experiences, PD, or evidence of them utilizing in their classrooms after participating in PD. Some participants, when recalling their earliest childhood math memories, could most recall the few and far between times math had been made relevant in a real-world context for them, as seen in high efficacy participant Clayton:

I also have a vague memory of fractions, but it wasn’t at school, it was actually at home. I had to have been a third or fourth grader. I wasn’t understanding fractions, but unrelated to school I was helping my mom in the kitchen and she had me measure something and showed me how three thirds equal a whole and the light bulb clicked. It was so simple, but I needed a real-world example for that to make sense. (initial reflective document)

Many references were made by teacher participants about parts of the PD experience that were particularly meaningful for them in growing efficacious behaviors in math regarding the use of Real-world Relevance by facilitators, or showing them the importance and how to do this with students:

I just remember what stuck with me from that was anytime a child draws a fraction, they should be drawing a picture of that fraction. And I still do that today, because [Taibi’s] point was basically that fractions are so abstract in the real-world sense. And you know, every fraction is a pizza, but in the real world, not every part of something or partial whole is a pizza. (High efficacy participant Mason, interview)

So, I believe that her [referring to her observation of a Teaching and Learning Coach], the levels and the dimension to what she does is the realistic things...but what can kids use, and how can they use their brain? Because if you’re doing fractions, or if you’re doing shapes, you’re going to need to look around and see what you could use. And that’s bigger than me handing you a packet of polygons, anything. (Low efficacy participant Taylor, interview)

Participants who had participated in these ongoing math PD experiences also made references to applying relevance when introducing concepts and in their everyday classroom pedagogy because of participation in PD: “I’ve changed lessons to put songs in them. I
changed lessons to bring in candy. I did a fraction lesson with pickles, because this kid and I loved pickles” (Williams, low efficacy participant, interview). This was evidenced in classroom observations, as well. One example was seen in high-efficacy participant Simmons’ classroom observation, where she framed a second grade learning activity around displaying data through the lens of what would be the easiest for their cafeteria manager to read when giving her class data around ice cream choices:

If I sent this picture to Miss. Janet, do you think she could easily tell what this class liked?....If I sent her these pictures with, she was going to easily order it, right? And is this better than if I would’ve sent her these pictures, do you think that Miss Janet could order the ice cream easier with the ones that look like a pictograph? (Synder, classroom observation)

High-efficacy participant Mason gave the math task of the day a Real-world Relevant context by framing it as Fantasy Football data for fifth graders:

So, if you’ve never heard of, seen, tried, played Fantasy Football, let me just give you a little background knowledge to help you here, okay? Fantasy Football is a game for nerds like me, okay? Where you pretend to draft football players to your own team. You compete against your friends, whoever, who have done the same thing, but they can’t have the same players as you, usually….This question, how many more points would the loser have needed to win the championship? So somebody’s going to have less points than the other one, I want you to figure out how many more points would they have needed to win, okay? Now the last one, what is the difference between Rashad’s highest scorer and Lauren’s highest scorer? I mean individual score, not team, okay? (Mason, classroom observation)

Observation reflections also included references to how teacher participants, in hindsight, may have made the math concepts of that day’s lesson increasingly relevant for kids. For example, low-efficacy participant Taylor commented: “The concept of pyramids could have been introduced as a mini clip of pyramids in Egypt to gain attention” (observation reflective document).
**Hands on/Do It Yourself (DIY)**

Hands on/Do-it-Yourself (DIY) experiences connect with Theories 1, 2, and 4 of the theory of math motivation for teachers of elementary mathematics, as participant data showed when teacher participants did the activity themselves, it built their efficacies about the math concepts, often changed their beliefs about the nature of mathematics from one-dimensional to multidimensional and changed their pedagogical practices for teaching these math concepts to become more multidimensional, as well. Hands on/Do-it-Yourself (DIY) was referenced more than any other of the initial codes in the sub-category of Helpful PD Strategies (as named by teacher participants). Initial codes analyzed in the determination of this theoretical sub-category included DIY, use of manipulatives and/or games, and parallel practices in PD. Particularly, participants with high K-12 efficacy mentioned doing their own assignments in PD as helpful. They mentioned playing the games themselves that they would use to teach students math concepts, as well as solving math tasks—either tasks like those they would give students, or possibly a bit above where their students are conceptually so that they as adults had to do more of the problem-solving a bit outside of what might be their teaching/content comfort zone:

> It wasn’t until I had the opportunity to be on a [PD] team and work through the math with fellow colleagues to help me understand the math concepts fully (Simmons, initial reflective document)….do the work ourselves…they would give us a problem and it would be something that we could use with our students, but it was at our level…It was something that I had to think through myself as an adult to get the answer but I could apply the strategies that I used as an adult for my students. (Simmons, interview)

This was frequently mentioned by all three mixed efficacy teacher participants, as well: “…where we were doing the work kind of a demonstration of like when they were going through what you would be doing if you were a facilitator” (Cahill, interview); or “I
learn by doing” (Cahill, interview); “When we were able to actually do the activities ourself, when they gave us a problem and they asked to do it ourselves, was helpful” (Ingram, interview). This type of work was also mentioned by two of the low-efficacy participants as helpful: “Trying the work” (Williams, observation reflective document); “I’m a hands-on learner, so the more I can do and talk with people is better than sitting and getting...” (Mills, interview).

Teacher participants in each of the efficacy categories equally valued the PD experience including parallel practices to what they could go back and teach in their classroom. Sometimes, this work was connected to the DIY work or use of manipulatives and/or games. High efficacy participant Simmons commented: “I could apply the strategies that I used as an adult for my students”; while mixed efficacy participant Ingram particularly liked when the content was aligned to precisely what she could teach at the kindergarten level:

But for me, the most helpful is when it was like things I would do with the kids. And I say that for me because I feel a lot of times, for kindergarten, we’re like, “Okay, this what we’re going to do,” but we have to make it fit for us. And I feel like in the training, there was sometimes it was geared for us. (Ingram, interview)

Low efficacy participant Williams also valued parallel practices when used in PD:

So, he walked through a lesson, that I could use in the classroom, and he just gave us a problem. And then we just solved it, however we wanted to do it. And then he had talked about going as he walked around and watched everything and didn’t give advice, the things that I do now. (Williams, interview)

**Growth Mindset Culture/Safe Environment**

Growth Mindset Culture/Safe Environment connects with all four theories in the theory of math motivation for teachers of elementary mathematics. One sub-category that was frequently seen in participant data, both regarding reflecting over what was powerful for
them at PD as well as pedagogical practices they took away to utilize in their own
classrooms, was Math Mindset. Two initial codes were analyzed to develop the focused code
of Math Mindset: *growth mindset beliefs* and *fixed mindset beliefs*. Research has shown
adults holding a *fixed mindset* are more oriented toward diagnosing people’s stable traits,
often from preliminary information, whereas those holding *growth mindset* tend to be more
open to information about change over time (Butler, 2000; Heslin et al., 2005; Plaks et al.,
2001). Research has also shown that students’ implicit theories of ability affect their
motivation, learning, and achievement outcomes. Those holding a *fixed mindset* are more
likely to draw conclusions about their ability (versus effort) from setbacks and to give up
more quickly when faced with challenges, as compared with those holding a *growth mindset*
(Blackwell et al., 2007; Dweck, 1999; Heinie et al., 2001). Dweck (2014) and colleagues
researched a diverse sample of seven Fortune 500 companies to determine what kind of
mindset their company held, and what characteristics employees identified aligned to the
company’s mindset. They found employees in a *growth mindset* company are

47% likelier to say their colleagues are trustworthy, 34% likelier to feel a strong sense
of ownership and commitment to the company, 65% likelier to say that the company
supports risk taking, and 49% likelier to say that the company fosters innovation.
(p. 28)

In the same sense, participant responses in this research suggest a Growth Mindset Culture in
PD for teachers that includes strategies to support a Safe Environment through *setting norms*
that value risk taking, mistake-making, making sense and holding belief for one another that
all can achieve can also cultivate an environment that can enhance the efficacious behaviors
of teacher participants. This category was determined through analysis of four initial codes
identified in the data: *growth mindset beliefs, fixed mindset beliefs, math norms*, and the
importance of/how to set up a safe environment. Interestingly, data coded as aligning with a growth mindset, or incremental mindset, was more evident in the data from low-efficacy category participants as compared with mixed and high-efficacy participants. Data coded as aligning with a fixed mindset, or entity mindset was seen more frequently in high efficacy participants.

Several mentions were made in the data of beliefs that align with a growth mindset approach to mathematics instruction. Some of these practices include creating a culture of risk-taking; an emphasis on the idea that fast learning is not always the deepest or best learning, that sometimes students who take longer may understand things at a deeper level; and having students set goals around what they would like to improve; emphasizing challenge, not success; giving students a sense of progress; praising the process; and grading for growth (Dweck, 2010). The PD that many participants in this study had been a part of often began with reviewing Boaler’s (2016) “Setting up Positive Norms in Math Class” to set up a Safe Environment for learning for teacher participants, which include:

1. Everyone can learn math to the highest levels.
2. Mistakes are valuable.
3. Questions are really important.
4. Math is about creativity and making sense.
5. Math is about connections and communicating.
6. Math class is about learning not performing.
7. Depth is more important than speed. (pp. 269–277)

These norms were explored and upheld during the duration of the participants’ time together at PD. During the interview process, one participant who had been identified as low efficacy throughout her K-12 experience attributed growth in her own math efficacy to the Growth Mindset Culture that was present during PD:
Me: What was it in that (PD) turning point that helped you grow your efficacy?

Participant: I think not being wrong. And not having to fit into this, “oh my God. I didn’t remember the algorithm.” I didn’t remember how to do it, but I came up with a way that made sense to me. And it didn’t matter really, because I was making sense of math. (Williams, interview)

Another low efficacy participant, in her reflective observation response, shared that her self-efficacy and background life experiences have helped her value that Safe Environment and change her own pedagogy to align with what was modeled in PD in this way:

I try to make sure all my students feel that their voices are valued and also that their voice is safe in our space. Telling someone they did something “wrong,” or making fun of others for a strategy or answer is not something that I allow. Even though one student didn’t create a line plot, but wanted to share her strategy, no one said anything to her about it. I wanted her (and others who weren’t brave enough to share) to be able to see how it was supposed to be while still feeling valued. Some students also chose to use a whiteboard for the task or warmup because they didn’t feel they had enough room on their iPad. I encourage them to use whatever tool works best for them. (Mills, observation reflective document)

A mixed efficacy participant also described how the PD influenced her pedagogy this way:

I am able to create an atmosphere where students feel comfortable to share their thinking and it is ok if they are not correct. I know the importance of creating an atmosphere where students are comfortable with changing their thinking or adding to their own thinking of a math problem. (Ingram, observation reflective document)

In the teacher participant focus group interview, teachers talked about the learning that took place when they had to teach new practices in front of one another, often with teachers from another building in their same grade level in the district they had never met before. When I asked what made the environment safe enough that they felt they could be vulnerable with each other, high-efficacy participant Simmons said “Just knowing that we were all going to watch each other. I’m going to go in Tina’s classroom, she’s going to come into mine. I think that was just beneficial, just…all being vulnerable at some point.” Other participants agreed;
the idea of being put in equally vulnerable positions through the PD process when immediately trying out new strategies learned build trust within the Safe Environment.

In the focus group, participants also alluded to the facilitator slowing down and giving them time to process as characteristic of a Safe Environment for PD:

The best piece of advice I got this year was just to slow down...with the process, and to take time to sit in each step is so incredibly important. When they’re teaching us that, the same thing, they give us time to go off and work, and then the immediate feedback makes sense for us. (Taylor, low-efficacy participant)

Mixed-efficacy participant Adams echoed this sentiment: “We need time to think about why we’re doing what we’re doing, just like they need time to process or share what they’ve learned and how they’ve learned that” (focus group interview).

While participants who were identified as having high efficacy in their K-12 mathematics experience made fewer references to these beliefs and practices than participants who were categorized as having low self-efficacy in their childhood math experiences, there was still evidence they too believed there is value in facilitating a growth mindset approach to mathematics learning:

I think, like I try to tell the kids, we learn more from our mistakes than we do the right answers. As long as we understand why we made the mistake....It’s okay to make mistakes in math. You should make mistakes in math. (Mason, interview)

This same participant referred to the mindsets students have about learning mathematics, which was included in the growth mindset data as it implies the student has the ability, and power, to work to change the areas of mathematics that may be difficult for them (this lack of ability is not fixed):

Well, it’s not that you’re bad at multiplication, it’s that you don’t understand how standard algorithm works and that’s okay. But if you’re not asking for help or you’re not trying something different, it’s not just going to magically make sense to you. Because that’s just not how anything in math works. (Mason, interview)
Mixed efficacy participants had the least amount of data that could be tied to a growth mindset stance, but did refer to behaviors that would foster a Growth Mindset Culture in their classroom instruction when facilitating student learning: “Sometimes we know, oh, if we persevere, we get better at math. We don’t give up when it’s hard. We just keep trying, right?” (Ingram, classroom observation). Low efficacy participants, as compared to the high and mixed efficacy participants, most frequently referred to characteristics that connect to fostering a growth mindset in their mathematics learning environment:

Math is about taking chances and being willing to “fail.”....They say that to each other. “It’s okay. Come on, just try, it’s okay.” “Oh, that was a good mistake.” I’ve even heard that the other day. And I thought that was awesome. (Taylor, interview)

…as a kid, things were harder, and so I just didn’t view myself as a mathematician. As I’ve gotten older and as I kind of learned more and learned more about myself, I realized that it’s not so much....You’re not born with it. It’s a process, but you have to learn strategies and figure out what works for you. That in turn has passed on to my students too. I feel like that’s very important. (Mills, interview)

Sometimes this was seen in the facilitation of the math workshop instruction during lesson observation, such as when setting up expectations or norms for the day’s work:

…remember, when we are showing our screen, we’re looking at the math of every person in our room, right? And a mathematician. Remember that we are not wrong, in math. Okay? We talked about that earlier. We’ve talked about here, that mistakes are valuable. That’s one of the beliefs that we have in second grade, and I want you to carry that belief right through with you! If we make a mistake, that means our brain just learned something new. (Williams, classroom observation)

Growth mindset characteristics were also communicated by teacher participants during observations when they ask students to reflect over their assumed progress, asking: “I want you to go back and look at your numbers you’ve given yourself, and decide, ‘Am I still there?’ ‘Have I grown?....You’ve grown a lot, right?’” (Taylor, classroom observation) Or: “…Okay, tell me two ways you got smarter by sitting with me today.” (Taylor, classroom
observation). Other times, this was modeled by the teachers themselves: “I can see your Class Kick, your work on Class Kick. That’s why I love Class Kick, because now I can go home and study your work and see how I need to get smarter today, too” (Taylor, classroom observation); or, “Yes, I would. Here’s my first try. That didn’t go so well for me” (Taylor, classroom observation). Growth Mindset Culture could also be seen through the way teachers framed affirming feedback to students by praising their process: “I love that. That was last week, but I’m smarter this week” (Taylor, classroom observation); “I know it’s hard work, but we are doing something that is difficult” (Mills, classroom observation). Conversely, this low-efficacy participant mentioned their belief in the power of a Growth Mindset Culture and the implications for those who do not hold one for themselves when learning math: “If you do not have an open mind ready to receive the lessons, you cannot learn” (Taylor, interview).

Previously, I discussed how cognitive dissonance between beliefs about the nature of mathematics could be seen in the data. One-dimensional and multidimensional mathematics pedagogical practices can be on viewed on a progression, with teachers learning how to move beyond the 1D into multidimensional practices in PD, but still at times reverting to 1D beliefs or practices. Similarly, participants would discuss the value of growth mindset for their own learning and that in their classroom; yet sometimes unknowingly still revert to fixed mindset language. When discussing a task, she had recently given to students, one low-efficacy teacher participant reflected, “Some of my smart kids were like, “I don’t know what I’m supposed to do” (Mills). This was in reference to finding the right amount of scaffolding when facilitating math instruction with students to ensure the struggle is productive. This same participant communicated her mindset about math ability has shifted from when she was young, that she knows it is not something you are born with but can grow. Yet,
unknowingly, the language she uses to describe student learners could still suggest a *fixed mindset* about their abilities (Are they always low or smart? Or is everyone smart in different ways? Did the “low” students just not reach proficiency yet?). The cognitive dissonance between knowing and doing, between teachers’ new beliefs and their old behaviors (fixed language) continues. Sometimes, this comes in the form of teachers holding conflicting beliefs simultaneously. This was seen in another low-efficacy participant who had several *growth mindset*-aligned mentions in the data, who acknowledged a *fixed mindset* can be detrimental to math learning: “If you say I can’t, when you’re working with math, your brain really will shut down,” (Taylor, interview), but also holds some belief that math understanding does innately come easier for some than others, suggestive of a *fixed mindset*: “My highest learner in math, who it comes natural, and I do believe that. I believe it comes natural to some, and not natural to others…” (Taylor, interview).

**Observation of New Practices Modeled**

Observation of New Practices Modeled connects with theory 1 and theory 3 of the math efficacy and motivation for teachers of elementary mathematics theory, as teachers share how observing others builds their efficacy for replicating new pedagogical practices and motivates them to try new practices in their classrooms. The category Observation of New Practices Modeled was identified through analyzing four initial codes were identified in the data: (1) *modeling in PD*; (2) *observation of colleagues*; (3) *think alouds by facilitator/model teacher*; and (4) *parallel practices in PD*. In these initial codes, participants mentioned observation of colleagues the most (20 times) when discussing the most beneficial parts of the PD process. Particularly, across this sub-category, the mixed efficacy participants seemed to value the *modeling in PD* and *observation of colleagues* more than the low and
high efficacy participants. Along these lines, mixed efficacy participant Adams mentioned both valuing observation of the “expert” in PD: “The most powerful thing was the observations and watching the master teacher, so to speak, kind of show us their craft. Like, how did you put the new model? How did you facilitate the new model in your classroom?”

(interview), as well as observation of peers:

I think having teachers demonstrate what’s worked for them. Like, teachers showing other teachers strategies that worked for them and then providing, either providing the tools that they’ve used, whether it be...anytime teachers are teaching teachers, teachers would rather learn from other teachers than any facilitator on the planet, I would put money on it....we have a lot of respect for people that are doing it, are in the trenches right now. (Adams, interview)

*Introducing a new way of structuring instruction and think alouds by the facilitator/model teacher*, codes under the subcategory of Observation of New Practices, were mentioned by low efficacy participants, but not the mixed or high. As low efficacy participant Williams stated:

I think as a teacher, it was beneficial to watch him pose the way we teach math, this math model, that I had never done before. And then thinking about, “Wow, there’s other models out there that could be powerful.” And so, then you’re going to pose this problem first and not give a kid any help?....when I go to a PD I’d like to hear that background, or I like to hear the, “Hold on kids, let me explain.” So I can think about myself as a professional. And is that something I can add to my class? (interview)

Mixed efficacy participant Adams echoed the value she put on the peer observation component of the PD experience in helping build her efficacy:

Well, adults and kids, yeah, they learn differently. But some of those elements of learning apply to both. We need modeling, they need modeling. … But I think the modeling piece, just like Ella and I spent time in the cohort. We got to watch each other teach. Then we talked to each other about what our strengths and our weaknesses were. We were all just very transparent, like, “Help me. I’m learning this new model. Please tell me anything you can think of.” So just that time to do that. (Teacher participant focus group)
Parallel practices in PD refer to times where the specific way the instruction was leading teacher participants through the PD process was mirroring a model they could implement with their students. Participants of all three efficacy categories mentioned this equally, noting “I could apply the strategies that I used as an adult for my students” (Simmons, interview).

Low efficacy participant Williams shared her memory of participating in practices during PD parallel to what she now implements:

So he walked through a lesson, that I could use in the classroom, and he just gave us a problem. And then we just solved it, however we wanted to do it. And then he had talked about going as he walked around and watched everything and didn’t give advice, the things that I do now. (interview)

This was also something that was brought up by facilitators in their focus group interview as important:

I think the other thing I always try to do as well is when I’m in a workshop, I want the teachers to see me...model what I would want them to do with children. If I’m going to stand up and lecture about how not to lecture, then what the heck am I doing in that room? So, I think the facilitator has to...Like we hear Micheals (other facilitator) say he asked all those prompting questions. He’s modeling what he hopes teachers will then do in their classrooms. Modeling for them good teaching practices is important in PD for educators. (Taibi)

**Elements of Productive Struggle**

Including Elements of Productive Struggle in PD experiences for elementary math teachers connects with all theories that make up the theory of math efficacy and motivation for teachers of elementary mathematics, as will help to provide mastery experiences (Bandura, 1977, 1986) that build teacher’s efficacy and motivation for mathematics, as well as model how teachers can do this in their classroom (vicarious experiences; Bandura, 1977, 1986). Hiebert and Grouws (2007) used the word “struggle” to refer to the effort made by students to make sense of mathematics or figure out something that is not immediately
apparent (p. 387). To determine the category Elements of Productive Struggle when analyzing the data, initial codes of productive struggle for both kids and adult learners were identified. Teacher participants mentioned having to productively struggle through math tasks as part of the PD process, whether those like they would ask of their students, or some that may be a bit above their grade level content, causing them to be a bit beyond their conceptual understanding regarding Mathematical Knowledge for Teaching they may have with their own grade level content. There was more mention of productive struggle for adult learning mentioned by participants with low or mixed childhood efficacy. Regarding providing students a productive struggle, this was still mentioned the most by low efficacy participants. However, high and high efficacy participants also noted the importance of productive struggle for students.

High efficacy participant Mason, who had held primarily high efficacy for himself throughout his K-12 experience, recalls this efficacy being challenged due to productive struggle during a PD session:

…one of my first years they just gave us, whoever was at the table, “Here’s a fractions problem.” It was complex. It’s not even one that we would have done in fifth grade and they’re like “Figure it out.” And I got it terribly wrong. I was totally twisted and confused. Again, that’s how I saw the power of just giving people things that they’re not necessarily ready for or have it….I mean, there may not have been a direct instruction on and just see how they do. Because I think, like I try to tell the kids, we learn more from our mistakes than we do the right answers. As long as we understand why we made the mistake. And so that activity for me was like, “Well, what did I do to screw up here?” So then I started noticing everybody else’s drawings and what the other people in my group were even saying, as we were disagreeing on what the answer might be, or even trying to figure it out. (interview)

While this experience initially challenged his efficacy, it also helped him see math through a new light, helping him see the value in teaching math more multidimensionally with students. During the facilitator focus group, the facilitators also agreed that while it can be an
uncomfortable experience for teachers, there is value in putting adults in a situation during PD where they *productively struggle*. One facilitator commented: “Even if they struggle...and I’m sure Michaels has seen this too...even when they’re struggling to understand it and do it, it’s still going to help their efficacy. What do you think, Michaels?” (Taibi, facilitator focus group interview). To this, the other facilitator shared a great example of how a teacher participant worked through this *productive struggle*, and it grew her efficacy:

I had a teacher....I think I had done this right before I did it in here. I did this activity with the jug of milk problem, if you remember that. And I had a teacher in [another city] where she called me over and she couldn’t figure it out and she wanted me to tell her. And I was asking her questions and trying to lead her, and she got really upset with me. I was like, “Oh, she’s giving me a bad evaluation,” because I wouldn’t tell her. She’s like, “Just tell me!” And I said, “Well, I have some questions. I’m trying to help you to make sense of it.” And it just got to the point where I had to walk away from her, because she was just getting so [mad]. Her evaluation, I could tell which one was hers. She basically said, “I was really upset, and you still wouldn’t tell me. I talked with colleagues, and I was able to figure it out. And so, thank you for not stealing my learning.” That really stuck with me. I’m like, “Okay, maybe it would have been so much easier if I had just told her or held her hand and dragged her through it.” That would have been really easy, right? It was actually hard to walk away saying, “If I stay here, I’m going to do that, so I’d better walk away and trust the process. Trust the productive struggle.” (Michaels, facilitator focus group interview)

Another facilitator highlighted the importance of *productive struggle* for teachers during PD, connecting it to helping them understand the perspectives of their students:

But don’t you feel like, Michaels, I mean, with any of those situations, we have to put the teachers the best we can in the students’ shoes to make them experience some of those things for them to understand the relevance in their own classroom? For example, if I have the time and I’m going to be teaching place value with teachers and I’m given an ample amount of time, I often take teachers back to learn in a different base first. They’ve never learned a base three or base five, and they’re struggling, and they hate it, and they don’t understand it, and they need their manipulatives to solve the problems. That’s that situation where they have to do it themselves to understand why it’s going to work for kids. That’s the same thing (he’s) talking about. We had to let her do productive struggle, or she wouldn’t have understood the purpose of productive struggle in her own classroom. (Taibi, facilitator focus group interview)
Collaboration

Providing collaborative experiences in PD connects with theory 1: experiences shape math efficacy from the theory of math efficacy and motivation for teachers of elementary mathematics, as collaboration can provide social persuasions (Bandura, 1977, 1986) to encourage one’s own teaching or math abilities. These social persuasions can also increase motivation, connecting with theory 3: math efficacy shapes motivation. Collaboration, as Adams shared above, was also mentioned throughout the participant data as a key component to helping build teacher efficacy throughout the PD process. Initial codes were identified when analyzing data where teacher participants referred to learning through their work as part of a curriculum writing collaborative team, discourse/discussions that were held as part of the PD process, appreciating being given time in general to collaboratively generate ideas and/or strategies, and valuing being given time for collaborative reflection about pedagogy. About this, mixed efficacy participant Cahill shared:

open dialogue with whoever’s presenting is huge because there’s so many questions that come up during watching somebody else. And if you can’t take the time to have a conversation about it, sometimes people lose that skill because they didn’t get to use it or they didn’t get to actually have an in-depth conversation about it or have an understanding of why you’re doing this. If you can’t tell me why the growth was there, then I don’t know that I will take the time to actually put it in my class because I don’t know if it’s going to work. (interview)

Low efficacy participant Taylor described a strong collaborative relationship she holds with a colleague who is a friend, and a math demonstration teacher in the district:

“She’s the one I can call if I’m just stuck, and she’ll talk me through it very quickly. But I think the collaboration just with people who have been in it, and she’s a demo teacher” (interview). Another mixed efficacy participant referred to the high value she places on a
collaborative relationship with a colleague who also serves as a math demonstration teacher in the district as having been helpful to building her efficacy. One of the facilitators, during the facilitator focus group, also agreed that time to talk about what they are learning is particularly important for teachers:

But lots of time in PD to talk with each other, to process, to reflect, to ask questions. All of those kinds of things, that’s important for teachers. That may not be important for businessmen or women when they’re in some kind of PD, but for teachers, you cannot….It’s hard to always reel them back in, but they need that. (Taibi)

Collaboration, overall, was mentioned more by low and mixed efficacy participants than by high efficacy participants, causing one to wonder: Do teacher participants who have always held high efficacy for themselves as mathematicians not feel they need other colleagues as much as those who formerly had poor self-efficacy in this area?

**Aligned to Contextual Elements**

Contextual Elements have been previously defined as forces within a classroom, school, or district that can support or challenge teachers’ ability to align their teaching beliefs with behaviors (Sun, 2015; Talbert & McLaughlin, 1993). Alignment of PD experiences with Contextual Elements supports theory 1 from the math efficacy and motivation for teachers of elementary mathematics theory, as teachers’ efficacies can be built if they are able to successfully implement expectations placed upon them by their district and state as a result of PD. Theory 3, math efficacy shapes motivation, also connects with Aligned to Contextual Elements as the data demonstrates teachers are more motivated to participate in PD when it helps them do what is expected of them or aligns with the resources they have been given. The initial codes that were identified in the data and analyzed to create the sub-category of aligned to contextual elements included: (1) *vertical alignment/work*;
Vertical alignment work was mentioned by participants of all efficacy categories, most frequently by high and low efficacy participants, as valuable as part of the PD or professional learning process. Low efficacy participant Taylor stated: “I think that it would be very beneficial to have fourth grade teachers there saying, ‘This is what we did last year’” (interview). High efficacy participant Clayton, when reflecting over going back to what she learned in PD several times over the years, stated: “It’s helpful because I’ve taught second through fifth, so some of those strategies are applicable wherever you are, but some of them are not” (interview). The majority of data from participants showed they appreciated when the PD was aligned with their grade level standards or the Standards for Mathematical Practice (SMPs), as again, this helps them be successful in the context of what is expected of them.

Participants also referenced feeling more efficacious because of PD when the PD incorporated use of the district-adopted math resource program, explored the newly introduced district math workshop instructional model, focused on their grade level standards, or focused on pedagogical strategies they are expected to enact in the classroom. When asked about ideal future PD experiences, one high efficacy participant shared that ideally: “If I’m thinking just specific to our district, it would … incorporate the resource we’re using” (Simmons, interview). Vertical work with the state grade-level standards was mentioned by participants as beneficial, stating:

I’m big into vertical work. I think it gets left behind a lot. I think we get too inundated in our own IAGs and our own standards and our own pacing and we forget what they come in with, or we don’t even know what they come in with. And we may not pay
attention to what they need to go out with. I think vertical work’s important… So, something that brings in all of the, I mean, I guess it could be district specific, but maybe we just paint a broader picture of, “Here’s where it goes from here. Here’s what comes to you depending on what grade level you’re at.” (Mason, high efficacy participant, interview)

**Immediate Application of Learning**

Providing participants the experience of immediately applying what is learned in PD connects with theory 4 from the theory of math efficacy and motivation for teachers of elementary mathematics, math efficacy shapes pedagogy. When participants try out the new pedagogical practice right away, it is more likely they will have a mastery experience (Bandura, 1977, 1986) which builds their efficacy and has also changed their pedagogy. Mentions were made by participants of all efficacy levels that would indicate they value PD that includes practices they can immediately apply into practice in the classroom; with most of these mentions coming from high efficacy participants. Sometimes, this was the next day, but often, this was a component within the PD, as high-efficacy participant Mason shared: “…the application of whatever the knowledge, whatever it was we were learning and working on that day. We got to go into the classroom that day and put it into practice” (interview). Mixed efficacy participant Adams echoed this sentiment, stating:

…the best thing for me is to watch someone and then immediately go to my room and put that into practice because then I’m kind of, I’m obviously taking their tools and their tips and then applying that immediately, so it sticks with me. (interview)

This was also notably important to low efficacy participant Williams, who shared “I think that as a professional development, as a teacher, I also want it to be something that I can change tomorrow” (interview).
Summary

Overall, the findings from this study generated new theory for how teachers’ math experiences change their math efficacy and beliefs about the nature of mathematics, and how their efficacy and beliefs about math then led to changes in their motivation and pedagogy. Teacher participants’ K-12 experiences with mathematics often shaped their self-efficacy throughout K-12 schooling, which often continued through college and into the beginning of their teaching careers. Often, it was PD experiences that helped positively change their efficacy for the better. Many teachers were motivated to participate in PD because they felt a duty to students to provide them better experiences than the ones they had in their own K-12 experiences, or to find strategies that went beyond one-dimensional/traditional teaching pedagogy to reach struggling students. While participants varied in which professional development strategies were most efficacious based on their K-12 background experiences, there was some consensus about which strategies were included in their elementary PD experiences for efficacious behaviors. These findings also generated new theory for strategies that are recommended to be included in professional development for teachers of elementary mathematics for efficacious behaviors, which are as follows: (1) Real-world Relevance; (2) Hands on/Do it Yourself (DIY); (3) Growth Mindset Culture/Safe Environment; (4) Observation of New Practices Modeled; (5) Elements of Productive Struggle; (6) Collaboration; (7) Aligned to Contextual Factors; and (8) Immediate Application of Learning.

In Chapter 5, I connect the theory of math motivation for teachers of elementary mathematics through the mathematics professional development for efficacious behaviors framework, answer the research questions, report on the findings and recommendations, and
provide recommendations for future research. I close with reflections regarding the research process and what I learned about myself as a leader.
CHAPTER 5
IMPLICATIONS OF FINDINGS AND RECOMMENDATIONS

This constructivist grounded theory study was conducted to develop a theoretical framework of teacher participants’ sense of mathematics efficacy for a mathematics professional development program. Many of today’s teachers were taught in a one-dimensional, traditional way characterized by procedural, algorithmic, step-by-step instruction, often causing a disconnect for students between skills and conceptual understanding (Fosnot, 1992). Sun (2015) found that if teachers have closed (Boaler, 1998) or one-dimensional views of math and instructed in one-dimensional ways, they are more likely to communicate fixed messages regarding math ability to students. Because many of today’s teachers were instructed in this way, there is often a lack of conceptual understanding of mathematics, making it difficult for teachers to facilitate mathematics learning that is multidimensional in their classrooms. Often, elementary teachers hold low levels of self-efficacy regarding mathematics ability during their K-12 math learning experiences, which can stay with them throughout college and into their professional careers. In this study, I examined how the experiences of nine elementary teacher participants in their K-12 schooling shaped their math efficacy and professional development process.

This study adds to the literature on how professional development can be designed to create experiences that support efficacious behaviors of elementary teachers, specifically when considering the level of self-efficacy they held for themselves throughout much of their lives. The results from this study generated new theory for sense of efficacy for elementary teacher participants of a mathematics professional development program. The efficacy and motivation for teachers of elementary mathematics theory includes four theories:
1. Experiences shape math efficacy
2. Experiences shape beliefs about the nature of mathematics
3. Math efficacy shapes motivation, and
4. Math efficacy shapes pedagogy

These theories provide a lens that help bring to light the theories regarding the sense of efficacy these teacher participants held about themselves, and how their experiences continue to shape their efficacious behaviors. The math professional development for efficacious behaviors theoretical framework includes eight core categories, in order of frequency as identified in the data:

1. Real-world Relevance
2. Hands on/Do it Yourself (DIY)
3. Growth Mindset Culture/Safe Environment
4. Observation of New Practices Modeled (facilitator, peers)
5. Elements of Productive Struggle (cognitive dissonance, discourse)
6. Collaboration—time to talk/reflect with colleagues
7. Aligned to Contextual Factors
8. Immediate Application of Learning

These categories provide a framework of strategies that can be utilized with practicing elementary teachers to provide the experiences that support efficacious behaviors, multidimensional beliefs about the nature of mathematics, increase math motivation, and motivate multidimensional pedagogical changes (see Figure 3).

This chapter answers the research questions, reports on the implications of the findings, provides recommendations for future research, and concludes with final reflections regarding my research journey.
Answering the Research Questions

The research questions were addressed through a mixed survey that utilized Likert scale items and open-ended questions, interviews, observations, reflective writing documents, and focus group interviews for generating theory. Using qualitative and quantitative methods, I analyzed the data to generate new theory to explain the theories nine teacher participants of elementary professional development held about how their experiences from early education through professional development shaped their sense of efficacy. The research questions guided the research inquiry process and were helpful in the theory generation process as well as when identifying areas for future research. A summary of these findings, aligned with central questions and sub-questions, is included in this section.

References to the data analyzed include all written documents by teacher participants (two reflective narratives for each of nine participants, nine interview transcriptions, nine transcriptions of participant classroom observations, and one teacher participant focus group interview for each of the nine participants), in which line-by-line coding (Charmaz, 2014) was utilized. Additionally, I reviewed the transcripts from the facilitator focus group to emphasize categories that were found in the data that were also supported as important by facilitators of elementary mathematics professional development. While the facilitators were not the focus of this study, their focus group was utilized as member checking and to further illuminate how elementary teachers’ efficacious behaviors can be changed through their experiences in the professional development process. In the next section, I answer each of the research questions and accompanying sub-questions.
Research Question 1: What is the Theory that Explains the Process for Enhancing the Efficacies of Teachers in a Large Suburban School District?

Sub-question One: Which Parts of the Mathematics Professional Development Process do Participants Attribute to their Theories Regarding Efficacy in Math?

This question is best answered using the theory of math motivation for teachers of elementary mathematics in combination with the math pd for efficacious behaviors theoretical framework. Participant data from this study was analyzed to generate the theory of math motivation for teachers of elementary mathematics explain the process for teacher participants of how experiences, whether from K-12 schooling, undergraduate coursework, or professionally as a practicing teacher can change the math efficacy and beliefs about the nature of mathematics of teachers. As teachers’ math efficacies and beliefs about the nature of mathematics are changed, this in turn changes their math motivation and pedagogy. Pragmatically, the math PD for efficacious behaviors theoretical framework lifts the veil on theories of just what specifically is suggested by elementary teacher participants to be included in these experiences. Eight major categories were determined from the data around strategies elementary math teachers theorized most helped support their efficacious behaviors when teaching mathematics.

The first core category, Real-world Relevance, indicated teachers became more efficacious about mathematics content and deepened their conceptual understanding when math concepts/tasks were introduced to them within a Real-world Relevant context. Making the connection between algorithms or one-dimensional ways of viewing mathematics as they had learned it and a more meaningful, relevant way these can be used in the world was valued by participants, and they theorized it built their efficacious behavior as teachers.
The second category, Hands On/Do It Yourself, was identified through teacher participants’ speaking of how using games, math manipulatives, solving math tasks themselves, or being led through an instructional structure/pedagogical strategy by the facilitator that they could use in their own classroom helped build their efficacy for teaching math concepts. Teacher participants referred to “being able to play the games or do the work ourselves…they would give us a problem and it would be something that we could use with our students, but it was at our level” (Simmons, interview) as helping them become more efficacious during PD. This was also echoed by facilitators in the facilitator focus group:

If you’re looking at the kind of PD that you normally do where you’ve got a group of teachers in a big area, the traditional PD....I think the strategies that have helped, I just think they have to be hands-on, and they have to do it. They have to try it. They cannot just listen. It cannot be a sit and get ever. (Taibi)

The third category, Observation of New Practices Modeled, encompassed the value teachers placed on observing others teaching mathematics in a multidimensional way or in a way that aligned with the expectations placed on them during PD to help build their efficacious behaviors. This was in reference to observation of the facilitators, either while leading them through DIY learning activities, or with students, as well as peer observations. Observation was noted as beneficial for increasing efficacious behaviors by all three efficacy categories and was mentioned more frequently by mixed and low efficacy participants. The process of observing someone in building efficacy would be described by Bandura (1977, 1986) as a vicarious experience—one where participants see someone else perform a task, then formulate the belief they can also accomplish the task.

The fourth category, Elements of Productive Struggle, was identified through analyzing all mentions of productive struggle, whether by adults or students, that were shared
by participants. Participants found mistake-making and productively struggling through new learning until individuals or collaborative groups were able to make sense of the math and develop a new understanding for the concept valuable in the learning process. Often, this process was laden with cognitive dissonance and discourse that may have been resolved by facilitation of a discussion where participants are encouraged to respectfully agree or disagree with one another’s thinking, or where many different strategies were sequenced and shared in a purposeful way to illuminate new understandings of math concepts. Teachers of all efficacy categories theorized this process built efficacious behaviors, both in themselves as participants of PD, as well as with their students when they had tried it out in their classrooms.

The fifth category identified as meaningful in the math professional development for efficacious behaviors theoretical framework was Collaboration—Time to Talk/Reflect with Colleagues. While teachers of all efficacy categories made mention of collaboration as helping support their efficacious behaviors as math teachers, this was mentioned most by the low efficacy participants. In the participant focus group, mixed efficacy participant Adams theorized the collaborative aspect after peer observations was very helpful for her in the process of building efficacy: “…getting that reflection and feedback, having that time to go, “Okay, what could I have done differently?…that time to apply, time to reflect, time to get feedback, and then tweaking, it’s almost like that’s super, super important.”

The sixth core category, Aligned to Contextual Factors, indicated that teachers appreciate when the PD experience will help them be successful in their professional job as teachers. When there is alignment between the professional development and the standards set for teachers by pressures outside of them (such as state standards, district curriculum,
Performance-Based Teacher Evaluation expectations, the expectations of their administrators, district instructional structure expectations, and so forth) the PD experience helps build their efficacy as it gives them a place to safely try out practices with the support of colleagues and have mastery experiences (Bandura, 1977, 1986) that build their efficacy for successfully teaching these practices once back in their classrooms alone. In the focus groups, teachers talked about how these outside pressures impact their teaching and emotional stress levels. Low-efficacy participant Gates—who was categorized as a low-efficacy participant based on her K-12 experiences, but has since grown a lot in her efficacy as a result of participating in PD experiences that have changed her beliefs and mindset about math—shared she still feels these pressures as a demonstration teacher: “There’s this underlying pressure, of I have to do what the models tells me to do” (interview). Another mixed efficacy participant shared how outside pressures impact her teaching:

I mean, at times, I mean we talk about kids need time, slowing down, kids need time to process. At one point I think I was in tears because I’m like, “But my warmup was great, but it was eight minutes instead of ten, or eight minutes instead of five.” Because you have pressure from the top and then there’s pressure you put on yourself. And then there’s pressure that we apply with the time element and then so it’s pressure, pressure, pressure. (Adams, mixed efficacy participant, focus group interview)

As such, several of the core data analyzed and identified as Contextual Factors that support teachers’ efficacious behaviors were those that Aligned with the Contextual Factors, or pressures, they encounter in their daily work. Teachers appreciated time in PD to focus on grade level state standards and vertical work (referring to time looking at grade level standards, including those below and above their grade level content), incorporating the district-adopted resource, focusing on pedagogy, and exploring a newly introduced district
math workshop model, as they felt these activities would support their efficacious behaviors in the classroom.

The seventh category participants theorized build their efficacious behaviors was identified as a Growth Mindset Culture/Safe Environment. While these categories are mentioned in order of the most frequently mentioned in the data, I also propose it is likely this must be in place for many of the other PD categories/strategies to successfully support efficacious behaviors. Even when teacher participants spoke of turning points regarding positive changes in their efficacy that happened prior to PD (in high school or college experiences), there was often mention of trust with a person who made the difference for them (be it a teacher, tutor, trusted colleague, or friend). Similarly, teachers mentioned the culture or environment of the PD experience was influenced by the facilitator and the norms, or expectations, they set for the PD experience:

Michaels was one of our Math Solutions teachers, but then when I went again, it wasn’t Michaels, and it didn’t feel as good. So, I think even though I didn’t have a relationship with Michaels when he was there, I felt like I did. And so being able to, I don’t know, gosh darn maybe you’re just a teacher, you just have to be able to walk into the room and own the room. And I don’t know how to teach that to a presenter, but I think having norms set up is really important. And I think that is something because I’ve been to, you know, 4,000 PDs—we say that at the beginning of every PD and we typically kind of adhere to it, but do we really believe that it’s okay to make mistakes? Do we really believe it’s okay to disagree with someone across the table? And so, I think that makes this hard, because I might be in a room full of people I’ve never met, and you maintain this sense of collegiality and professionalism with strangers. But it’s okay to disagree with them and so having those norms and having fewer, in lieu of twenty of them, and just from teaching and having the norms in my classroom, it doesn’t happen overnight. (Williams, interview)

The Growth Mindset Culture/Safe Environment that were intentionally included in the PD experience cultivated a rich environment for teachers’ efficacious behaviors to grow. This
changed teachers’ beliefs about the nature of mathematics, and teachers often replicated this in their math classrooms.

The eighth category identified to frame the math professional development for efficacious behaviors theoretical framework is Immediate Application of Learning. Participants theorized that learning new pedagogical practices in PD that could immediately be applied (either during the PD session, collaboratively trying them out with colleagues, or by going back to their classroom the next day and putting new learning into practice) was beneficial to building their efficacious behaviors. Bandura (1977, 1986) theorized mastery experiences—successful completion of tasks—led people to be more likely to attempt, persevere, and put effort forth in that type of task or those like it in the future. Although they felt uncomfortable at times, participants particularly valued the opportunity to try the newly introduced pedagogical strategies out in the presence of colleagues, including coaches and facilitators, then reflecting together over how to improve for next time when they are on their own: “being able to put it into practice and try it out and see how it works and how you need to tweak it” (Simmons, high efficacy participant, interview).

These core categories were consistent with the literature previously reviewed regarding constructivist approaches to math instruction, motivation, conceptual understanding, and mathematics professional development, and delved deeper into specifying precisely how previous recommendations might be put into practice through specific strategies by educational leaders designing PD for elementary mathematics teachers.
**Sub-question Two: What Theories do they Generate about the Nature of Mathematics because of the PD Process?**

Teacher participants, all of whom spoke about being taught mathematics in a one-dimensional way in their K-12 experience, theorized the use of these strategies in the PD process helped shift their beliefs about the nature of mathematics from one-dimensional to multidimensional. While some were fortunate to have college professors begin to unveil more multidimensional approaches for teaching elementary mathematics to them, several did not have this opportunity until participating in PD as practicing elementary math teachers. The new research that was shared with them as part of the PD process (including Positive Math Norms; Boaler, 2016) was integral in helping them change their beliefs about the nature of mathematics and increase efficacious behaviors. Rather than using a teacher-directed strategy or algorithm to solve math problems, teachers theorized that productive struggle, both for adult learners and children, is beneficial in helping people make sense of mathematics concepts, supporting multidimensional views of mathematics. Several participants shared how participation in PD shifted their beliefs about the nature of mathematics, stating “there’s more than one way to skin a cat, so to speak, or there’s six different ways to get it, as long as you get there” (Adams, interview), who later elaborated:

one of the things I strived for and starting to do is realize it’s not one way. It can’t be one way. And especially with, you know, learning more about learning styles and learning about different ways to teach things just kind of revolutionized the way, and it should have! If it didn’t happen that way for most teachers, it’s sad—because all new ways of doing things just kind of opened the pathways for kids to think about math differently instead of just this black and white version of math. So I, myself, through PD, through my own desire, through my own experience with just the...do it the way the teacher did it, now, it’s—it’s—math has become really a beautiful subject because kids are allowed to think and get there. And after we give them different ways to do it, they pick what works for them. So, it might be the standard algorithm for that very black and white thinking child. Like, “Just show me how to get the answer.” Whereas the other one’s like, “No, I want to draw a picture,” or “No, I need
tools.” So, it’s just opened, all the training, technology, everything has just changed the way that I teach. It’s changed the way I look at math. And I understand it better. I understand math now way better than I did, even as fresh, as say, a college freshman. I understand math in ways I never did. (Adams, mixed efficacy participant, interview)

One of the facilitators, who has worked with many of the schools at the district site for years, stated this about the shift in beliefs about the nature of mathematics for this school district based on the PD, after reviewing initial results by efficacy category:

One of the big things...that’s why I was asking when this was given is I do think the PD that’s been happening in this district with these teachers is changing mindsets. That’s what I do think, and I see that in multiple questions. I think if you would have done this survey 10, 12 years ago, some of these things would not have scored the same way at all. Teachers would not have said that students need to struggle, that they need tasks. Some of these things would not have been said, so I do see that this shift that’s happening is happening for everyone. (Taibi, facilitator focus group interview)

I will next answer the sub-questions that address the second central research question: What professional development theories contribute to shifting mathematics pedagogical practices?

**Research Question 2: What Professional Development Theories Contribute to Shifting Mathematics Pedagogical Practices?**

*Sub-question One: What Motivates Teachers to Engage in Professional Development?*

For teacher participants in this study, experiences shaped their self-efficacy and beliefs about the nature of mathematics, and their self-efficacy and beliefs shaped their motivation and pedagogical changes. This is seen in the theory of math efficacy and motivation for teachers of elementary mathematics (see Figure 1). The two main sub-categories identified through analysis of the data were (1) Motivated by Students and (2) Past Negatives Giving Purpose for Students. Several mentions were made of participants being motivated to attend or engage in PD to help their students. Participants also shared that often, their negative K-12 experiences gave them a drive to learn more about Multidimensional Mathematics so they could provide their students better experiences than what they
remember. Once they realized the difference between traditional, or One-dimensional Views of Mathematics, and Multidimensional Mathematics, their beliefs began to shift about the importance of teaching in a Multidimensional way, and many realized they lacked the conceptual understanding of math concepts and needed to learn more to provide students better learning experiences. Regarding efficacy category, more high-efficacy participants were motivated by their students, whereas more mixed and low-efficacy participants were motivated to learn more because of their past negative experiences.

**Sub-question Two: In What Ways Do Their Pedagogical Practices Change?**

After participating in PD, teacher participants’ pedagogical practices changed by becoming less traditional and more Multidimensional. Many participants used Questioning to Engage Students in Thinking or Facilitate Discourse and discussions in their math workshop, including *Talk Moves* (Chapin et al., 2013). More participants utilized Student-centered, Agency-building practices, such as *noticing and naming* and *affirmations of student thinking/praising the process*. Participants’ pedagogy aligned more closely with Contextual Elements, using the *Standards for Mathematical Practice* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) or the use of the site district’s math instructional model, as well as characteristics that were more closely aligned with the values of the school district, including responsive teaching and scaffolding. Several participants utilized the Five Practices Model (Smith et al., 2009) because of participating in PD, as well as the Concrete to Abstract Continuum (Bruner, 1960; Dienes, 1970; Haistings, 2009; Kauchak & Eggen, 1980; Peck & Jencks, 1987; Piaget, 1950).
In the next section, I answer the sub-questions that address the third central research question: What do findings suggest about leadership theories to support the facilitation of mathematics professional development?

**Research Question 3: What do Findings Suggest about Leadership Theories to Support the Facilitation of Mathematics Professional Development?**

**Sub-question one: How do Leaders Facilitate Safe Environments for Enhancing the Efficacies of Teachers?**

In the data, the category of Growth Mindset Culture/Safe Environment was identified and included as part of the Math Professional Development for Efficacious Behaviors Theoretical Framework. One high efficacy participant spoke of learning how to use the Five Practices Model (Smith et al., 2009) model to facilitate safe, respectful discourse in this way: “I thought it was a good way to kind of exploit errors and individuals’ repeated mistakes without saying, ‘You do this every time.’ Something like that, belittle them, if you will” (Mason, interview). The facilitator had used this practice in PD, at a time he had been Productively Struggling to solve a task, to help participants compare their problem-solving strategies until they reached a consensus of which strategies were most accurate and efficient. Another mixed efficacy participant shared: Through the PD I have received I am able to be more intentional with the problems I choose to pose in math. I am able to create an atmosphere where students feel comfortable to share their thinking and it is ok if they are not correct. I know the importance of creating an atmosphere where students are comfortable with changing their thinking or adding to their own thinking of a math problem. (Ingram, observation reflection document)

Another participant categorized as having had low math efficacy in her K-12 experience talked about a college professor who made an impact. Similarly, this professor had created a
Safe Environment in her math methods course, and this experience helped change her efficacy and pedagogy:

He never made anyone feel less for any ideas they had, and I felt respected by him as a member of our math community. He is one of the big reasons I am the teacher I am today. He gave me the confidence and skills to feel like I had some tools in my bag coming out of college to teach math. He was the one who showed me what it felt like to feel like I was respected in a community of math learners and made me want to be that kind of person for my students as well. (Mills, initial reflective document)

Another low-efficacy participant spoke of not completing her teaching degree initially due to low math efficacy, and then revisiting it 30 years later as she returned to complete her undergraduate teaching degree. She attributes the turning point in her efficacious behaviors to the way her math tutor created a Growth Mindset Culture/Safe Environment:

First of all, that he made me feel it was okay to make mistakes. We were going to make plenty of them together. I remember him saying that. There was no judging. There was...I mean, every time too, he’d look at me and I know he’d be thinking, we’ve done this 100 times, so let’s roll. And he was just willing to put in the work with me. I knew he was as invested in me, as I needed him to be. And that really has carried into teaching too. The work is for you also, and for you to be a better mathematician, for you to be a better scholar, for you to be more confident all around. So, confidence is a huge part of it, patience, and just strategies that he gave me. (Taylor, interview)

Participant data in this research study support the idea that setting up norms for respectful math discourse (Chapin et al. 2013; Boaler, 2016) is one way to create a Growth Mindset Culture/Safe Environment for not only students, but also, teacher participants during PD. Strategies for this include sharing or incorporating the research on growth/mathematical mindsets (Boaler, 2016) within norms shared with teachers and set for the PD experience, and then modeling and upholding the math norms consistently throughout the PD session, will help teacher participants build trust with the facilitator as well as other participants. Teacher participants theorized another way to set up a Growth Mindset Culture/Safe Environment
was when facilitators provided opportunities for shared vulnerability, building trust among participants. In the experiences they spoke of, all teachers were going to be put in the same position to teach in front of one another; they were all in this experience together, which made them feel more comfortable taking risks and trying out new practices—even if it was through productive struggle or failing forward. Slowing down the pace to allow time for collaborative talk and reflection was another strategy for decreasing stress/pressure for teacher participants in this research study.

Sub-question Two: What Support Do Participants Need for Trying Out Theory-based Pedagogical Practices in Classrooms?

A Growth Mindset Culture/Safe Environment. Teachers need to be encouraged by their administrators to take risks, productively struggle, and fail forward—all characteristic of a Growth Mindset Culture, whether in a classroom or for the school as an organization. Dweck (2014) and colleagues found:

employees at companies with a fixed mindset often said that just a small handful of “star” workers were highly valued. The employees who reported this were less committed than employees at growth-mindset companies and didn’t think the company had their back. They worried about failing and so pursued fewer innovative projects, regularly kept secrets, cut corners, and cheated to get ahead. (p. 28)

Many of these same characteristics mirror the experiences participants in this study from the K-12 low efficacy category shared—they did not feel they were math people, as only some select people could be math people; worried about failing and even sometimes, felt they had to cheat to get by. If we expect teachers to facilitate Growth Mindset Culture/Safe Environments in their classrooms for students to learn mathematics, educational leaders must facilitate Growth Mindset Culture/Safe Environments for them to take risks and make
mistakes for them to learn the value of this experience, and to learn how they can support students in doing so in their classrooms.

**Opportunities for Immediate Application of Learning with Efficacy-Building**

**Feedback.** Teachers in this study appreciated the opportunities they had, either built into the PD process, or immediately thereafter upon returning to their classrooms the next day, to immediately try out new practices. They spoke of the value of doing this either with their peers who could give constructive, strengths-based feedback, or in the presence of someone who is already considered an expert in the area they are working on—be it a teacher, tutor, PD facilitator, teaching and learning coach, principal, demonstration teacher, or colleague. This aligns with the idea of a more experienced other brought forth by Vygotsky (1978) and emphasized by Chval et al. (2015). If done in a supportive and non-judgmental way, potentially using questions that cause participants to reflect and construct their own learning either individually or through collaborative conversations (Shapira-Lishchinsky, 2015), it is likely to enhance the learning conditions of a Growth Mindset Culture/Safe Environment, enhancing PD for educators. Aligned with the findings of Rattan et al. (2011), educational leaders who give strategy (focusing on concrete suggestions) or control (containing two statements of caring) feedback to teacher participants versus comfort feedback are likely to increase the math motivation and efficacy of participants. Hattie et al. (2020) stated, “Motivation is a function of the feedback learners receive as they work on a task; specifically, as they make progress (or not)” (p. 2). Schunk and DiBenedetto (2020) agreed: “As learners work on tasks they acquire self-feedback and feedback from others on their learning goal progress. The belief that they are making progress substantiates their self-efficacy, which enhances motivational outcomes” (p. 3). To grow their pedagogical practices,
teachers need on-going cycles of new learning that include Immediate Application in a Safe Environment with feedback from someone with more experience.

**Time to Learn, Collaborate, and Reflect.** When discussing Contextual Factors that create stress, teacher participants referred to the benefit in PD of slowing down and having the time to reflect with colleagues collaboratively. With Contextual Factors such as instructional pacing calendars, assessment plans, state standards and assessments, and Performance-Based Teacher Evaluation cycles putting pressure on today’s educators (not to mention Contextual Factors brought on during the data collection phase of the setting of this study, the COVID-19 pandemic), teachers value educational leadership that prioritizes their time to learn, and doing so with a pace that is not rushed, supporting a Safe Environment.

**Findings and Recommendations**

**Include Categories from the Mathematics Professional Development for Efficacious Behaviors Theoretical Framework in Professional Development**

Simply put, I recommend mathematics professional development for elementary teachers include the type of experiences included in the mathematics professional development for efficacious behaviors theoretical framework. It is likely inclusion of experiences of this nature will support teachers’ efficacious behaviors and growth of multidimensional beliefs, further supporting their math motivation and pedagogy. Following are leadership theories and recommendations that would support successful implementation of this framework as strategies in PD.

**Constructivist Leadership**

Constructivist leadership, defined as “the reciprocal processes that enable participants in an educational community to construct meanings that lead toward a shared purpose of
schooling” (Lambert, 2002, p. 36) is recommended to support the growth of elementary teachers in mathematics. Lambert (2002) emphasized that “individuals and organizations bring past experiences and beliefs…into the process of learning; all of these influence how we interact with and interpret our encounters with new ideas and events” (p. xvi.). Given the baggage many carry into their classrooms from negative, or one-dimensional math experiences from their own childhood, it is critical adult learners can unpack their bags of math trauma within a Growth Mindset Culture/Safe Environment and construct new meanings together through the context of professional development experiences. Lambert (2002) recommended that constructivist leaders must cultivate trusting relationships among participants, which is a result of shared experience over time. She recommended program designers engage participants in learning communities that:

- Meet people where they currently live in their heads and hearts
- Challenge and support self-disclosure, risk taking, and reflection
- Sustain trusting relationships throughout the development and learning process.

(p. 212)

From what has been learned through this study, strategies similar to those found to be successful in the classroom by MacCallum and Morcom (2008) are likely to be helpful in cultivating constructivist learning experiences for teacher educators, including: developing common agreements/norms among participants, the use of social groups to build social connections and trust among group members, giving teachers collaborative opportunities to try out new instructional practices with equal risk, and collaborative opportunities scaffolded by a more experienced other (Vygotsky, 1978). Simon (1995) found that central to a model that supported constructivist approaches to mathematics instruction was “the creative tension between the teachers’ goals with regard to student learning and his responsibility to be
sensitive and responsive to the mathematical thinking of the students” (p. 114). The same was found to be true in this study, that educational leaders, too, must work within the tension of desiring highly effective teachers of mathematics for our youngest learners, and being sensitive and responsive the goal of building the efficacious teachers hold upon entry to mathematics PD experiences due to often negative past experiences. This may be a large contributing factor to the lack of educational progress in mathematics nationwide for the last forty years (educational leaders, in an attempt to quickly improve academic outcomes due to Contextual Factors such as accountability, implement quick fixes rather than committing to sustained change that shifts beliefs into behavior).

Rosenholtz (1989) observed that teacher uncertainty (or low sense of efficacy) and threats to self-esteem are recurring themes in teaching. Fullan (2016) stated that when improvement in teaching is a collective rather than individual process, “teachers are more likely to trust, value, and legitimize sharing expertise, seeking advice, and giving help both inside and outside of the school” (p. 109). By providing teachers constructivist leadership in which to collaborate and built trust, it is likely their collective sense of efficacy will be built, increasing motivation to do even better pedagogically and impact student achievement.

**Constructivist Leadership: Know Your Audience**

Along the lines of building and sustaining trust, I recommend educational leaders of elementary mathematics PD have a strategy to get to know the background experiences and efficacy levels of their audience—the teacher participants—prior to beginning PD. When I asked the facilitators in the focus group interview what they thought of this idea, one shared that in undergraduate college courses, she uses an activity along these lines to learn about her students’ backgrounds:
I give them a question: if math were a color, what would it be? Connect it to a color. Connect that color to an emotion. So, if math were a color, for me it would be purple. I love purple. It’s always been my favorite color. My mom did loads of laundry that were purple. It’s just been the love of my life, and I just continue to find this passion for it. Somebody else might see it as black, a dark hole. I get into math, I’m completely lost. And it does help. It helps me reach students. (Taibi)

Like the use of social circles and class meetings recommended by MacCallum and Morcom (2008) towards developing motivation through participation in collaborative opportunities, organizing groups of teachers to participate in learning activities that require some level of vulnerability can build trust, as well as give the facilitator insight into how to frame feedback, provide scaffolding, and support each participant more effectively. Another participant, during the participant focus group interview, talked about participating in a popular equity exercise, The Privilege Walk, and how it was a negative experience for her because she was behind everyone else (Williams). This participant had a lot of negative experiences K-12 and was assigned to the category of low-efficacy participants. While to her the equity Privilege Walk was a DIY experience, she did not prefer it because it publicly made her feel poorly about herself. This made me wonder if a facilitator might seek to learn more while maintaining privacy, possibly by administering a survey like the one I had sent to participants here, but before PD. I am sure there could be challenges to honesty and completion if the participants did not know the facilitator ahead of time, and there could be some benefit in developing the Safe Environment/Growth Mindset Culture as a group by having participants discuss/share out like the ideas in the focus group. Regardless, I recommend facilitators utilize a strategy to learn about their participants’ efficacy and backgrounds in mathematics before planning and engaging in the PD to best personalize the learning experience for the individual or collective group.
Strategic Planning to Align Contextual Elements

It is helpful when PD aligns with Contextual Factors—the standards, evaluation requirements, and district/school expectations of teachers—so that teacher participants feel the PD is meaningful and will help them become more efficacious towards the goals of outside pressures. This aligns with the research on motivation—that person factors (expectations, efficacy), social factors (modeling, comparisons) task values, goals, and perceived costs and benefits all influence a person’s motivation (Hattie et al., 2020). These recommendations require school administrators prioritize teachers’ mathematics professional development through allocation of resources such as time, financial support, as well as strategic planning to ensure all the various pressures on teachers are aligned so as not to create further stress for teachers, allowing their physiological response to be ready to learn when they come to PD. When facilitating a strategic planning process that includes shifting the long-held efficacies and beliefs of the teacher participants, educational leaders must first build strong, trusting relationships with teachers (Fullan, 2016; Gruenert & Whitaker, 2015). Trusting relationships provide the conditions needed for the leader to discuss “nondiscussables” (Barth, 2013, p. 198) with stakeholders and facilitate fierce conversations (Scott, 2004) as they work together to develop shared vision for the future of mathematics at the school or district. These trusting relationships will also empower the teachers to take risks in their classrooms without fear of failure. The paradoxical irony of the recommended strategic planning process for change is that it must include what Fullan (2016) referred to as a “focused-innovation, or letting-go, phase” that “does not mean anything goes, but is more a recognition that people should try new things and learn from them” (p. 80). Because, as stated by a facilitator participant in this study during the focus group, “Clarity is not easily
achieved” (Russell), a plan will always have limitations, and it is most important that the organization continue to maintain a learning orientation or Growth Mindset Culture throughout the process. Fullan (2016) recommended locating a situation or setting where change seems to be working, where things have been deliberately transformed from a previous state to a new one to find out how that change occurred. As stated by another facilitator participant in this study:

the PD that’s been happening in this district with these teachers is changing mindsets. That’s what I do think, and I see that in multiple questions. I think if you would have done this survey 10, 12 years ago, some of these things would not have scored the same way at all. Teachers would not have said that students need to struggle, that they need tasks. Some of these things would not have been said, so I do see that this shift that’s happening is happening for everyone….But then I look and I see some blips in this data that says it’s still not totally 100% their belief, like you said. Some of this is not solid, but it’s definitely making a difference. I think by working with teachers as long as I have, I think that’s what I can say about the data overall. (Taibi)

This study has provided a theoretical framework to understand how teachers’ efficacious behaviors have been changed because of participating in ongoing mathematics professional development. While these changes in efficacious behavior were happening for participants from each of the K-12 efficacy categories (low, mixed, and high), this data also indicates that the definition of ongoing professional development, as used in this study, is likely not enough time—that this commitment must continue for teachers’ efficacious behaviors to continue to be built, particularly at system-wide capacity for a large suburban school district.

Strategic Planning: Resources

Considerations for financial support include compensation for the facilitators, substitute teachers so teachers have additional time to learn within their contractual day without feeling rushed/pressed for time, as well as for materials/resources to teach math in
multidimensional ways, which may include concrete manipulatives and games. The strategic planning of administrators must also consider providing extended time (repeated sessions aligned by a common goal/focus) across a school year or years, as while participants credited even one year involved in on-going math PD as defined by this study as shifting their efficacy, beliefs, and pedagogy, many who have participated in several of these experiences (Level 4 participants) said they still can always learn more. All participants were able to name math content that is still difficult for them to teach in a multidimensional way that leads to deep conceptual understanding.

Teachers who are trying out the theory-based pedagogical practices recommended by this research study in their classrooms need to be given a real-world relevant context for the new learning, and time to figure out how they can facilitate it with students in ways that are relevant to the real world. Teachers need focused time to try out new practices together, and time to collaboratively reflect over these practices—what worked, what didn’t, and generate solutions for how to improve the practice in the future. This is most beneficial if done with a facilitator who has experience and an understanding of how mathematics can be taught in a multidimensional way, be this a consultant, professor, coach, or administrator—a “more experienced other” (Chval et al., 2015, p. 122; Vygotsky, 1978). Teachers need time to try out the practices themselves, in a DIY, hands-on way, and time to discuss the new learning. For constructivist leadership, Lambert (2002) recommended a year-long cohort with 20-25 participants—with the greatest possible diversity among members—be utilized to give time for trust to be built and to nurture relationships that will build a rich history of shared experience. Similarly, here, we saw that the more time participants spent together, across the
period of a year, the more trust was built through experiences that promoted shared and equal vulnerability.

**Suggestions for Future Research**

I have several suggestions for future research. Even with the breadth of this research study utilizing multiple data types; as the old saying goes, the more you know, the more you feel you don’t know.

- I recommend future research be conducted on a larger scale to determine if K-12 efficacy category correlates to a preference for certain PD strategies over others. I suggest information be collected prior to beginning PD about the participants’ efficacy and beliefs to help the PD facilitator personalize the PD experience based on the participants efficacy and beliefs, and then again after to determine how use of personalized strategies impacted each teacher’s efficacy and beliefs about mathematics.

- Future research could seek to measure or explore further the idea of a Growth Mindset Culture/Safe Environment during PD and seek to correlate its effect on the PD’s effectiveness.

- It would be interesting to explore the relationship of teacher efficacy with student achievement, and how this changes because of PD. Since this district had been conducting mathematics PD that could be characterized as multidimensional for at least seven years, it would have been interesting to give the surveys in this research study prior to the start of PD, then continue to compare results after two, four, and six years to see how efficacy, beliefs, and mindsets continued to change.
throughout the process, and try to correlate which specific types of PD led to each change.

- It would be interesting to find out how long this change takes to help educational leaders in their strategic planning for how much time should be allotted to PD, and whether that number differs if a beginning teacher comes out of college with high efficacious behaviors in math that lend to teaching in a multidimensional way.

- One high-efficacy participant theorized that math had come naturally:

  As I’ve grown up now, I realized that I deal more in reason and logic than maybe some, most, whatever you want to say. I don’t know. My brain deals primarily in those things though. I don’t do a whole lot of fantasizing or things like that. I’m very procedural. (Mason, research interview)

  This comment reflects more of a fixed mindset or entity theory about the nature of mathematics and would indicate that while Mason credits PD as changing his beliefs and mindset about mathematics, it also indicates some cognitive dissonance (Festinger, 1962) that he still also holds belief there are fixed traits about him that lent to his success in math. This leads me to believe it could be helpful to determine if/how one’s learning styles or personality traits could lend more to success or failure in math; to somehow combine this research with that of mindset or efficacy research, even if only to further dispel the math myths that some teachers struggle to let go of while concurrently believing math ability is incremental.

- I would also be interested if someone conducted this same study in another district, or with a larger group of participants and facilitators, whether the results
would be similar or they would have different findings, because the contextual elements may be different, or the style of math PD may vary from what has been available in this district.

- Future research might also seek to identify more clearly a framework of constructivist leadership. While I theorize some of the categories suggested in the math professional development for efficacious behaviors theoretical framework could be included in constructivist leadership regardless of the subject to impact teacher learning, the literature prior to this study was limited and not always specific as to how, specially, constructivist leaders can support adult learners.

- Finally, future research could test the categories suggested in the math professional development for efficacious behaviors theoretical framework to determine their effectiveness or correlation regarding adult learning as well as the impact on student achievement.

**Reflections on the Process**

I would be interested, if I could go back and re-do some parts of this research study, in finding out how participants would categorize themselves regarding K-12 efficacy category. I had wrongly assumed this would come up naturally in the teacher participant focus group; that they would want to know. I think because many of the participants had not met one another up until this point, I was hesitant to bring it up for fear I would be triggering shame in them, making them feel like they were not a math person, or that I was supporting fixed mindset traits, such as assigning them an intelligence category. They did not ask, so I am not sure whether they would have agreed with the assignments.
I would also have liked to take the theoretical frameworks back to the participants and facilitators during the focus group interviews. In our time together, I shared initial findings, which included data that was coded into initial codes, focused codes, sub-categories, and categories or themes, as well as survey results (with the facilitator participants) demonstrated in descriptive statistics. Both focus groups communicated they wished they had more time to look over the data. I, too, felt the data were rich with possibility for analysis. It might be used again in the future to further extend or elaborate on how participants efficacy is changed throughout their K-12 or PD experiences, as there were interesting pieces that were not seen as frequently in the data that I chose not to focus on here because other categories were represented more frequently. The content of this study itself—how the experiences of teachers change their efficacy and beliefs about math, and therefore, their motivation and pedagogy, is complex and interconnected. Efficacy is difficult to define in a teacher at a fixed point in time, as it is a construct that is ever-changing, and one that they are rarely asked to be metacognitively reflective about regarding their own efficacious behavior.

I know this all too well, as someone whose math efficacy has continued to ebb and flow across the course of my K-12 experience, college, and professional life. Even with all I have learned because of this process about incremental and entity theories of mathematics, I still must remind myself that I, too, can be a math person when challenged in this area. The best grade I have received in a math class since my own K-12 experience was while taking doctoral level statistics to complete this degree—after I learned about the math mindset research.

As I reflect over my own leadership, there are areas I believe have reflected the categories in the theory generated by this study, including providing Collaborative
opportunities to learn and reflect; including Elements of Productive Struggle; giving the teachers opportunities to work Hands on, Doing their own Assignments; Observe one another; and teaching them explicitly, then modeling, a Growth Mindset Culture and Safe Environment. However, when it comes to time and pace—I know I have also at times succumbed to the Contextual Factors, and/or pressures, inherent in educational leadership. When I recall times in the past when I directly influenced these categories with teacher participants, I often recall my time serving as a Teaching and Learning Coach. As a principal, my role in providing these opportunities to teachers has been at times, more indirect—through providing resources, or by strategic planning and organizing teams within our school so that teacher participants have access to resources, facilitators, and opportunities to collaborate.

This likely feels even more the case due to our current contextual factor—the COVID-19 pandemic, which hit during my first year as a building principal. Over the last year, I joined my educational leadership colleagues in managing our basic needs—the safety of our stakeholders—before anything else. While there have been some benefits to technology helping us continue professional learning and collaboration, even when virtually and distanced, some of the trust and connections that are inherent in face-to-face interactions (unmasked) cannot be replaced. I am excited for a time, hopefully in the near future, we can move back into these more authentic learning experiences together. As I reflect on the beginning of the pandemic, much of our online collaboration was based on the emotions and well-being of stakeholders and a focus on creating a safe environment, whether physically or virtually. I am hopeful the forced slowing down brought on by the pandemic and the relationships we have built through managing crisis together have helped build a strong
foundation for vulnerability, trust, and a safe learning environment as we continue to learn together.

In the past I have shared the mindset research with staff, as well as how my own math journey was impacted by this new research, and how it has built my efficacious behavior. My staff survey last year, pre-pandemic, showed staff efficacy as an area for growth. Our motto has been “We Got This,” and I would like to think working together through the global pandemic has built the efficacious behaviors of my staff as I continually notice and name the ways they have successfully blended face-to-face and virtual learning, even amidst new safety protocols, to make great things happen for students. I am excited to follow my own recommendations next year in hopes of continuing to grow my staff efficacy. I will continue to support the process of growing the efficacious behaviors, beliefs, and pedagogies of teachers for the benefit of today’s and tomorrow’s students, like my own children, so that they will have mathematics learning experiences that differ from those many of us experienced; instead, participating in multidimensional experiences that build their efficacies and mindsets for mathematics.
### APPENDIX A

**THEORIES OF MATH MOTIVATION**

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APPENDIX B

CONSENT TO PARTICIPATE IN RESEARCH

Study Title:

Motivating Mathematicians: A Theoretical Framework of Professional Development for Teachers of Elementary Mathematics

Authorized Study Personnel

Principal Investigator: Dr. Donna M. Davis, Ph. D. Office: (816) 235-5956
Secondary Investigator: Margaret Helwig M.S. Office (816) 321-4797; Cell (816) 392-0807

KEY INFORMATION
You are being asked to take part in this research study because you are an elementary mathematics teacher who has participated in mathematics professional development (at least six hours over the course of one school year) within the last five years or a facilitator of ongoing professional development. Research studies are voluntary and only include people who choose to take part. The purpose of this constructivist grounded theory study is to develop a theoretical framework for the design of mathematics professional development experiences to support efficacious strategies. For teachers, the initial survey is expected to take 15-20 minutes. If selected as one of nine participants, the total amount of time you would be in this study is 3-4 hours over the course of this semester. During full participation you will be involved in taking a survey, writing one reflective journal entry, one interview with the researcher, one classroom observation, and one focus group with other participants. The observation will not be an evaluation. Facilitators will be asked to participate in one 45-60-minute focus group. Your participation in this study will be confidential; pseudonyms will be used when data/results are shared. Taking part in this research involves the following risks or discomforts: discussing past experiences with mathematics, revealing information about yourself to the researcher, who is a principal at a school in your district, and opening your classroom to an outside visitor for observation. Taking part in this study includes the following benefits: an opportunity to collaborate with other participants as well as the researcher, as well as potentially participating in improved future professional development experiences in the area of mathematics. You have the alternative of not taking part in this study.

Please read this consent form carefully and take your time making your decision. As the researcher(s) discusses this consent form with you, please ask her to explain any words or information you do not clearly understand. Please talk with your family and friends before you decide to take part in this research study. The nature of the study, risks, inconveniences, discomforts, and other important information about the study are listed below.

Margaret Helwig will conduct the study.

WHY IS THIS STUDY BEING DONE?
The purpose of this study is develop a theoretical framework for the design of mathematics professional development experiences to support efficacious strategies.

You are being asked to be in this study because you are an elementary teacher (grades K-5) in (a large suburban school district) who has participated in ongoing (at least six hours over the course of one year) mathematics professional development within the last five years or because you are a facilitator for ongoing professional development with (a large suburban school district).

HOW MANY PEOPLE WILL TAKE PART IN THIS STUDY?
Approximately 12 people will serve as participants in this study at UMKC (9 teacher participants, 3 facilitator participants). It is expected that at least 25 teachers will complete the participant sampling survey.

WHAT IS INVOLVED IN THE STUDY?
Teacher participants will be asked to complete:

- One survey to be utilized for purposeful participant selection. This survey will include a series of questions regarding your beliefs about mathematics, as well as a reflective narrative describing your past experiences with mathematics. (15-20 minutes)
- One individual interview conducted either in person or via Microsoft Teams (30 minutes)
- One classroom observation of math instruction (45-60 minutes)
- One survey with reflective journal entry to be completed within 24 hours of classroom observation (15-20 minutes)
- One focus group with all participants for the purpose of member checking of data, to be held either in person or via Microsoft Teams (30-60 minutes)

Facilitator participants will be asked to complete:
- One interview OR focus group focused on initial study results (45-60 minutes)

HOW LONG WILL I BE IN THIS STUDY?
This study will be conducted over the course of the spring 2021 semester (February 1–May 28). There is no intent to collect follow-up information after May 28, 2021.

WHAT ARE THE RISKS OF THE STUDY?
There is minimal risk involved in participation of this study.

There are no physical risks associated with this study. There is, however, the potential risk of loss of confidentiality. Every effort will be made to keep your information confidential; however, this cannot be guaranteed.

Some of the questions asked as part of this study may make you feel uncomfortable. The researcher conducting this study is an elementary school principal in your district and will have access to all information you disclose. You may refuse to answer any of the questions, and you may take a break at any time during the study. You may stop your participation in this study at any time.

This research presents risk of loss of confidentiality, emotional and/or psychological distress because the surveys involve sensitive questions about your background experiences with mathematics.

ARE THERE BENEFITS TO TAKING PART IN THE STUDY?
Benefits of participation in this study include collaborative, reflective professional opportunities with other participants as well as the researcher. However, you may not get any benefit from being in this research study.

The benefits to education and society include the development of a theoretical framework for the design of elementary mathematics professional development experiences to support efficacious strategies.

WILL MY INFORMATION BE KEPT CONFIDENTIAL?
The University of Missouri System, Authorization No. 00-018 requires research data to be retained for 7 years after the final report.

Reasonable steps will be taken to protect your privacy and the confidentiality of your study data.

The data will be stored electronically through a secure server on a password protected computer and will only be seen by the research team during the study and for 7 years after the study is complete. Pseudonyms will be utilized to protect anonymity. Interviews will be recorded using a digital recording device and would remain secure and confidential. Transcripts of interviews and classroom observations will be written and sent to participants for checking before data analysis. Any student work submitted will not have student identifiers included.

The only persons who will have access to your research records are the study personnel, the Institutional Review Board (IRB), and any other person, agency, or sponsor as required by law. The information from this study may be published in scientific journals or presented at scientific meetings but the data will be reported as group or summarized data and your identity will be kept strictly confidential.

WHAT ARE THE COSTS TO YOU?
There is no cost to you to be in this research study.

WHAT ABOUT COMPENSATION?
Those who complete only the initial sampling survey will be entered in a drawing. 4 participants who complete the initial survey will be randomly selected to earn a $25.00 gift card (your choice; either to Starbucks, or via Visa Giftcard).
The 9 teacher participants selected for full participation will receive $75.00 for participation upon completion of this study. These gift cards will be distributed according to the following schedule:

- $25 after the completion of the first interview
- $25 once the researcher completes the classroom observation and receives the reflective narrative
- $25 after the conclusion of the focus group

The 3 facilitator participants will each receive a $25 gift card for participation in this study.

WHAT SHOULD YOU DO IF YOU HAVE A PROBLEM DURING THIS RESEARCH STUDY?
Your well-being is a concern of every member of the research team. If you have a problem as a direct result of being in this study, you should immediately contact one of the people listed at the beginning of this consent form.

WHAT ABOUT MY RIGHTS TO DECLINE PARTICIPATION OR WITHDRAW FROM THE STUDY?
You can choose to stop participating at any time without penalty or loss of any benefits to which you are entitled.

You can decide not to be in this research study, or you can stop being in this research study (“withdraw’) at any time before, during, or after the research begins for any reason. Deciding not to be in this research study or deciding to withdraw will not affect your relationship with the researcher(s) or with the University of Missouri Kansas City.

You will not lose any benefits to which you are entitled.

WHOM DO I CALL IF I HAVE QUESTIONS OR PROBLEMS?
You may ask any questions concerning this research and have those questions answered before agreeing to participate in or during the study.

For study related questions, please contact the researcher(s) listed at the beginning of this form.

For questions about your rights as a research participant, or to discuss problems, concerns or suggestions related to your participation in the research, or to obtain information about research participant’s rights, contact the UMKC Institutional Review Board (IRB) Office

- Phone: (816) 235-5927
- Email: umkcirb@umkc.edu

STATEMENT OF CONSENT
The purpose of this study, procedures to be followed, risks and benefits have been explained to me. I have been allowed to ask questions, and my questions have been answered to my satisfaction. I have been told whom to contact if I have questions, to discuss problems, concerns, or suggestions related to the research, or to obtain information. I have read or had read to me this consent form and agree to be in this study, with the understanding that I may withdraw at any time. I have been told that I will be given a copy of this consent form.
APPENDIX C

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APPENDIX D
PARTICIPANT SURVEY SELECTION TOOL

Q1 Name (Last name, first name): __________________________________________________

Q2 Email address: _______________________________________________________________

Q3 Phone number: ______________________________________________________________

Q4 School where you currently teach:

- School 1
- School 2
- School 3
- School 4
- School 5
- School 6

Q5 Grade level you currently teach:

- Kindergarten
- 1st Grade
- 2nd Grade
- 3rd Grade
- 4th Grade
- 5th Grade
Q6 Choose any and all ongoing elementary mathematics professional development opportunities in which you have participated within the last five years.

(ONGOING PROFESSIONAL DEVELOPMENT, FOR THIS STUDY’S PURPOSES, IS DEFINED AS SIX OR MORE HOURS OVER THE COURSE OF ONE SCHOOL YEAR.)

☐ NKC Schools’ (outside consulting firm partnership) cohort (one year)

☐ NKC Schools’ (outside consulting firm partnership) cohort (multiple years)

☐ (Local elementary mathematics college professor/consultant) building-based workshops

☐ Other

Q7 If other, please describe: ________________________________________________________________
Q8 Teaching and Learning Beliefs Survey:

Please select the answer that most accurately describes your Teaching and Learning Beliefs in regard to elementary mathematics. Be honest; the intent of this survey is not to look for “right answers” or what one “should believe”; but instead, to assist in selecting a variety of participants.


<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree (1)</th>
<th>Disagree (2)</th>
<th>Agree (3)</th>
<th>Strongly Agree (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Mathematics learning should focus on practicing procedures and memorizing basic number combinations.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Students can learn to apply mathematics only after they have mastered the basic skills.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Students can learn mathematics through exploring and solving contextual and mathematical problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
11. The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others. (11)

12. Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems. (12)

Q9 Please consider your own mathematics abilities when answering the following:


<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I can learn new things, but I can’t really change how intelligent I am when it comes to mathematics.</td>
<td>(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. No matter how much intelligence I have in mathematics, I can always change it quite a bit.</td>
<td>(3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Q10 Please select the choice that most describes your opinions and/or beliefs:

<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. People either are born “math people,” or they’re not.</td>
<td>(1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Males are generally better at mathematics than females.</td>
<td>(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. I prefer teaching primary grades to avoid intermediate mathematics.</td>
<td>(3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. I chose to be an elementary teacher to avoid higher level mathematics.</td>
<td>(4)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Q11 Do you wish to continue the survey? Study participants will be selected from those who complete the survey in its entirety. If you choose yes, you will be asked to complete one more narrative response question. If you choose no, the survey will end.

- Yes (1)
- No (2)

Q12 Mathematics Personal Narrative
Think back to your earliest experiences with math, from childhood up until becoming a teacher.

Write a reflective narrative describing your memories and experiences.

In your narrative, please include responses to the following questions:
Why were each of the chosen memories meaningful to you?
Upon becoming a teacher, how did you feel about your own ability to do mathematics? What about your ability to teach math effectively to your students?
What did you believe was the best way to teach math when you first started teaching? What about now?
# Teaching and Learning Beliefs Participant Selection Survey Results

<table>
<thead>
<tr>
<th>Survey Questions</th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics learning should focus on practicing procedures and memorizing basic number combinations.</td>
<td>0% 0% 0%</td>
<td>66% 33% 100%</td>
<td>0% 66% 0%</td>
<td>0% 0% 0%</td>
</tr>
<tr>
<td>The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.</td>
<td>33% 0% 33%</td>
<td>66% 100% 66%</td>
<td>33% 0% 0%</td>
<td>0% 0% 0%</td>
</tr>
<tr>
<td>All students need to have a range of strategies and approaches form which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures</td>
<td>0% 0% 0%</td>
<td>0% 0% 0%</td>
<td>0% 0% 33%</td>
<td>0% 100% 66%</td>
</tr>
<tr>
<td>The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.</td>
<td>0% 0% 0%</td>
<td>0% 0% 0%</td>
<td>0% 0% 0%</td>
<td>100% 100% 100%</td>
</tr>
<tr>
<td>Mathematics learning should focus on developing understanding of concepts and procedures through problem solving.</td>
<td>0% 0% 0%</td>
<td>0% 0% 0%</td>
<td>0% 33% 0%</td>
<td>100% 66% 100%</td>
</tr>
</tbody>
</table>
reasoning, and discourse.

| Reasoning and Discourse | An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused. | Students can learn to apply mathematics only after they have mastered the basic skills. | Students can learn mathematics through exploring and solving contextual and mathematical problems. | An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics. | The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests. | The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the | | 100% 0% 0% 0% 66% 100% 0% 33% 0% 100% 0% 0% | 0% 0% 33% 66% 66% 33% 0% 33% 33% 0% 0% 0% | 0% 0% 0% 0% 0% 33% 0% 33% 0% 33% 33% 100% | 0% 0% 0% 0% 0% 33% 0% 33% 0% 66% 66% 100% | 0% 0% 0% 0% 0% 66% 0% 0% 0% 100% 0% 0% | 0% 0% 0% 0% 0% 66% 0% 0% 0% 0% 100% 100% |
reasoning of others.

Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

<table>
<thead>
<tr>
<th>100%</th>
<th>0%</th>
<th>100%</th>
<th>0%</th>
<th>100%</th>
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<th>0%</th>
<th>0%</th>
</tr>
</thead>
</table>

*Results show here are only for the 9 teacher participants who were selected for full participation in the research study.*
APPENDIX F
MATH MINDSET PARTICIPANT SELECTION SURVEY RESULTS

<table>
<thead>
<tr>
<th>Please consider your own mathematics abilities when answering the following:</th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
<td>Mixed</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>I can learn new things, but I can’t really change how intelligent I am when it comes to mathematics</td>
<td>66%</td>
<td>0%</td>
<td>66%</td>
<td>33%</td>
</tr>
<tr>
<td>No matter how much intelligence I have in mathematics, I can always change it quite a bit.</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>People either are born “math people,” or they’re not.</td>
<td>66%</td>
<td>33%</td>
<td>66%</td>
<td>33%</td>
</tr>
<tr>
<td>Males are generally better at mathematics than females.</td>
<td>100%</td>
<td>33%</td>
<td>66%</td>
<td>0%</td>
</tr>
<tr>
<td>I prefer teaching primary grades to avoid intermediate mathematics.</td>
<td>66%</td>
<td>33%</td>
<td>33%</td>
<td>0%</td>
</tr>
<tr>
<td>I chose to be an elementary teacher to avoid higher level mathematics.</td>
<td>66%</td>
<td>66%</td>
<td>33%</td>
<td>0%</td>
</tr>
</tbody>
</table>

*Results show here are only for the 9 teacher participants who were selected for full participation in the research study.*
APPENDIX G

INTERVIEW PROTOCOL – ELEMENTARY MATHEMATICS PROFESSIONAL DEVELOPMENT TEACHER PERCEPTIONS

1. How has your efficacy regarding mathematics changed throughout your life?
2. Which parts of the professional development process do you attribute to your efficacy in math?
3. What were the most beneficial/helpful parts of the math PD for you?
   a. Was there anything specific the facilitator did or said that made it beneficial?
   b. How did it change your pedagogy?
   c. What part motivated you to try something different/new in your classroom?
4. How were your beliefs about the nature of mathematics modified as a result of participation in the PD?
5. What elementary math concepts do you feel particularly knowledgeable about?
6. Which concepts do you feel like you lack understanding/could use more PD?
7. What might the facilitator do to help you/your colleagues learn more about that concept?
APPENDIX H

CLASSROOM OBSERVATION PROTOCOL

When I come to observe your math workshop lesson, I will audio record data to be transcribed. Additionally, I will take general notes of what I observe; including my wonderings, patterns I see that align with our interview or your sampling survey responses, etc. I may take photos of unidentifiable student work, anchor charts, or other evidence of learning. I may ask students to tell me about their problem-solving/thinking during independent work time. It will be my aim to blend in; to cause no/minimal disruption; mostly serving as a silent observer and/or a part of your classroom for the day.

After the lesson, I will send you transcription notes. Please let me know if there is something that does not reflect accurately what occurred. Within 24 hours of the observation, please complete the reflective journal entry with the link provided. If there are student work samples that demonstrate how the professional development changed your pedagogy and/or align with your reflective journal entry, please scan and send to me without student names/identifiers included.

Thanks again for your participation!
APPENDIX I

OBSERVATION REFLECTIVE DOCUMENT PROTOCOL

1. How have your self-efficacy and background life experiences in the area of mathematics impacted your instructional decisions/pedagogy as seen in today’s lesson?
2. How did the mathematics professional development impact your pedagogy as seen today?
3. What went well?
4. What would you change in the future?
5. What future mathematics PD would help you and your students continue to grow as mathematicians?
APPENDIX J

TEACHER PARTICIPANT FOCUS GROUP–INITIAL FINDINGS PROTOCOL

1. What about these findings is particularly meaningful to you? What rings true?
2. What surprises you?
3. What are the most important strategies or elements of professional development that should be included to help increase your self-efficacy?
4. What do you wish the facilitators knew that would help support your learning as an adult learner?
APPENDIX K

FACILITATOR FOCUS GROUP INTERVIEW PROTOCOL

1. What about these findings is particularly meaningful to you? What rings true?
2. What surprises you?
3. Describe how you see this playing out when facilitating mathematics professional development with adult learners.
4. How have you seen this while observing teachers’ pedagogy, or while assisting them in instructional decision making/planning?
5. Which parts of PD/what strategies do you think enhance efficacious behaviors of teachers?
APPENDIX L

PARTICIPANT RECRUITMENT EMAIL SCRIPT

Good morning colleagues,

To those of you whom I haven’t yet met, my name is Margaret Helwig, and I am currently a doctoral student at the University of Missouri–Kansas City. I also serve as the Principal of (an elementary school in a large suburban school district). The reason for my email today is I would like to invite you to participate in my research study titled “Motivating Mathematicians: A Theoretical Framework of Professional Development for Teachers of Elementary Mathematics.”

My research study will focus on K-5 teachers who have participated in ongoing elementary mathematics professional development in the last five years. On-going, for the purposes of this study, is defined as 6 or more hours within one school year. Likely, this would have occurred through participation in a district math cohort with a national consultant, or a series of building-based workshops with a local consultant (however; you may have participated in 6 or more hours of math PD within one school year through another avenue).

If this describes you, please review the attached informed consent and complete the brief survey (linked below) that will be used to select 9 participants for the study.

Research will take place in March 2021, both at your site in person and/or via Microsoft Teams, and will be scheduled at your convenience.

The purpose of this study is to develop a theoretical framework for the design of elementary mathematics professional development experiences to support efficacious strategies.

If you would like to participate, please complete the survey no later than Friday, March 5th. Should you have questions or need additional information, please email me at Margaret.Helwig@nkcschools.org or call (816) 321-4797.

Thank you in advance,

Margaret Helwig

*You may open the survey in your web browser by clicking the link below:
https://qfreeaccountssjc1.az1.qualtrics.com/jfe/form/SV_25LexbNuruco1yq
Password: Helwig2021


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National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.


Regional Assistance, Regional Education Laboratory Southwest.
http://ies.ed.gov/ncee/edlabs


VITA

Margaret L. Helwig (Ploch) was born on May 7, 1984, in Saint Louis, Missouri. After receiving her high school diploma, she attended the University of Missouri-Columbia and received her Bachelor of Science in Education in 2006. Mrs. Helwig received her Master of Science in Education degree in Educational Leadership: Elementary from Northwest Missouri State University in 2008. She currently serves as an elementary school Principal in a large suburban school district.