arXiv:gr-qc/0202054v1 14 Feb 2002

Acceleration-Induced Nonlocality: Uniqueness of the Kernel

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February 7, 2008

Abstract

We consider the problem of uniqueness of the kernel in the nonlocal theory of accelerated observers. In a recent work [14], we showed that the convolution kernel is ruled out as it can lead to divergences for nonuniform accelerated motion. Here we determine the general form of bounded continuous kernels and use observational data regarding spin-rotation coupling to argue that the kinetic kernel given by $K(\tau, \tau') = k(\tau')$ is the only physically acceptable solution.

PACS numbers: 03.30.+p, 11.10.Lm, 04.20.Cv; Keywords: relativity, nonlocality, accelerated observers.

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1 Introduction

The special theory of relativity deals with physics in Minkowski spacetime [1]. The test observers in this theory are in general noninertial; for the results of measurements of a noninertial observer, the theory asserts that such an observer is locally inertial ("Hypothesis of Locality"). In this way, Lorentz invariance may be applied in a pointwise manner to make physical predictions regarding what accelerated observers measure. Thus Lorentz invariance and the hypothesis of locality together constitute the pillars of the special theory of relativity.

The inhomogeneous Lorentz transformations (i.e. elements of the Poincaré group) connect the physical measurements of ideal inertial observers. These have a special significance: The fundamental laws of microphysics involve quantities that are ultimately measured by such ideal inertial observers. However, all actual observers are accelerated. It is therefore necessary to specify how the measurements of an accelerated observer can be connected with the basic laws of physics; that is, a connection is needed between the accelerated and inertial observers. The hypothesis of locality provides such a connection as it postulates that an accelerated observer is at each instant equivalent—in terms of measurements of physical quantities—to an otherwise identical hypothetical inertial observer that has the same state (i.e. position and velocity) as the accelerated observer. The approximate nature of this assumption, which is natural from the standpoint of Newtonian mechanics, was discussed by Lorentz in the specific context of his electron theory [2]. The hypothesis of locality extends to all measuring devices the assumption that the rods and clocks of the standard theory of relativity are locally inertial [1, p. 60].

If the duration of the basic phenomenon measured by the noninertial observer is such that its velocity vector and spatial reference frame do not change appreciably over this time interval, then the observer may be regarded as inertial and the hypothesis of locality is valid; that is, the acceleration of the observer is locally immaterial. To quantify this criterion, we note that there are certain invariant acceleration time scales $\mathcal{L}/c$, given typically by c/g and $1/\Omega$, associated with a noninertial observer. These are related to the magnitudes of the observer's translational acceleration $\mathbf{g}$ and the rotational frequency $\mathbf{\Omega}$ of its spatial frame, respectively [3, 4]. If λ/c is the intrinsic time scale of the phenomenon under observation, then the expected deviation from the hypothesis of locality is $\sim \lambda/\mathcal{L}$. Such a deviation turns out to be too small to be detectable in most physics experiments. To illustrate this point,

let us note that experiments are typically performed in a laboratory fixed on the rotating Earth, where $c/g_\oplus \sim 1$ year and $1/\Omega_\oplus \sim 4$ hours, while for a laser beam $\lambda/c \sim 10^{-15}$ second. In this way, one can account for the great success of the standard theory of relativity. As a matter of principle, however, it would be interesting to construct a viable theory of accelerated observers in Minkowski spacetime that goes beyond the hypothesis of locality. Such a nonlocal theory of accelerated systems is described in section 2. For the sake of concreteness, electromagnetic radiation fields are considered throughout this paper; however, the final results in their general form would be valid for any field.

The nonlocal theory of section 2 involves a kernel that needs to be determined on the basis of physical principles discussed in section 3. In this way, a class of bounded continuous kernels is identified. In section 4, the resulting nonlocal theory is confronted with observational data regarding the measurement of electromagnetic radiation fields by a uniformly rotating observer. Thereby a unique kernel is tentatively identified. The final section contains a discussion of our results.

2 Nonlocality of accelerated observers

We consider a global inertial frame in Minkowski spacetime with coordinates $x^\alpha = (ct, \mathbf{x})$. This is the only coordinate system that is needed here; in particular, we avoid the use of "accelerated coordinate systems" due to their fundamental limitations associated with the measurement of distance in such systems [3, 4].

The accelerated observer follows a worldline with tangent vector ${\lambda^\mu}_{(0)} = dx^\mu/d\tau$, where τ is the proper time along the path. The observer refers its measurements to an orthonormal tetrad frame ${\lambda^\mu}_{(\alpha)}$ defined along its worldline such that

$$\frac{d{\lambda^\mu}_{(\alpha)}}{d\tau} = {\phi_\alpha}^\beta {\lambda^\mu}_{(\beta)}. \tag{1}$$

Here the scalars $\phi_{\alpha\beta}$ form an antisymmetric acceleration tensor such that the "electric" and "magnetic" parts correspond to the acceleration $\mathbf{g}$ of the observer ($\phi_{0i} = g_i/c$) and the rotation frequency $\mathbf{\Omega}$ of its spatial frame ($\phi_{ij} = \epsilon_{ijk}\Omega_k$), respectively. We note that along its worldline the accelerated observer passes through a continuous infinity of hypothetical instantaneously comoving inertial observers each with the instantaneous tetrad frame

$\lambda^{\mu}{}_{(\alpha)}$. To avoid unphysical situations involving the expenditure of an infinite amount of energy in order to keep the observer accelerated, we assume that the acceleration is turned on at τ_0 and turned off at a later time τ_1.

Let $f_{\mu\nu}$ be an electromagnetic radiation field in Minkowski spacetime; in fact, $f_{\mu\nu}$ is the Faraday tensor as measured by the standard set of static inertial observers in the background global frame. Let $F_{\alpha\beta}$ be the corresponding radiation field as measured by the accelerated observer. The hypothesis of locality implies that the field as measured by the accelerated observer is given at each point along the worldline by the field as measured by the hypothetical momentarily comoving inertial observer. For such an observer the measured field is the projection of $f_{\mu\nu}$ upon its tetrad frame by Lorentz invariance, i.e.

$$\hat{f}_{\alpha\beta} = f_{\mu\nu}\lambda^{\mu}{}_{(\alpha)}\lambda^{\nu}{}_{(\beta)}. \tag{2}$$

The accelerated observer passes through an infinite sequence of such momentarily comoving inertial observers; therefore, the most general linear relationship between $F_{\alpha\beta}$ and $\hat{f}_{\alpha\beta}$ consistent with causality is [5]

$$F_{\alpha\beta}(\tau) = \hat{f}_{\alpha\beta}(\tau) + \int_{\tau_0}^{\tau} K_{\alpha\beta}{}^{\gamma\delta}(\tau,\tau')\hat{f}_{\gamma\delta}(\tau')d\tau'. \tag{3}$$

It is important to recognize that this ansatz only involves spacetime scalars; moreover, the kernel K must vanish for an inertial observer. Therefore, the nonlocal ansatz (3) is physically reasonable if K is related to the acceleration of the observer.

Equation (3) has the form of a Volterra integral equation of the second kind [6]. It follows from Volterra's theorem that in the space of continuous functions the relationship between $F_{\alpha\beta}$ and $f_{\mu\nu}$ is unique [6]. This uniqueness result has been extended to the Hilbert space of square-integrable functions by Tricomi [7]. In this sense, therefore, we assume boundedness as well as continuity throughout this paper. Further details regarding acceleration-induced nonlocality can be found in [5, 8].

3 Determination of the kernel

The hypothesis of locality has a consequence that goes against the spirit of relativity theory: a pure radiation field can stand completely still with respect to a uniformly rotating observer. This is most easily seen for the case

of an observer rotating uniformly with frequency $\Omega_0\mathbf{n}$ about the direction of propagation—characterized by the unit vector $\mathbf{n}$—of a plane electromagnetic wave of frequency ω. The Fourier analysis of $\hat{f}_{\alpha\beta}$ in this case reveals that $\hat{\omega} = \gamma(\omega \mp \Omega_0)$, where the upper (lower) sign refers to positive (negative) helicity incident radiation. This result differs from the transverse Doppler effect $\gamma\omega$ that involves the time dilation factor $\gamma = dt/d\tau$; moreover, the subtraction and addition of frequencies has a simple intuitive interpretation. The electromagnetic radiation field rotates about the direction of propagation with frequency ω $(-\omega)$ for a positive (negative) helicity wave; therefore, the rotating observer perceives radiation of definite helicity but with frequency $\omega - \Omega_0$ $(\omega + \Omega_0)$. The deviation of this result from the transverse Doppler effect provides an instance of the general phenomenon of spin-rotation coupling. Partial observational evidence for this general coupling is reviewed in [9]. In the case of electromagnetic radiation with $\omega \gg \Omega_0$, experimental results in favor of this coupling are available in the microwave and optical domains [9]; moreover, this effect has been observed for radio waves ($\nu \sim 1\,\mathrm{GHz}$) as a phase wrap-up in the GPS system as described in [10]. In all the experimentally viable cases at present $\Omega_0/\omega \ll 1$; in this regime, the spin-rotation coupling has therefore a solid observational basis for electromagnetic radiation [9, 10]. On the other hand, $\omega' = 0$ for positive helicity radiation of frequency $\omega = \Omega_0$, i.e. the wave stands completely still with respect to the observer. More generally, for oblique incidence $\hat{\omega} = \gamma(\omega - M\Omega_0)$, where $M = 0, \pm 1, \pm 2, \ldots$, is the multipole parameter such that $\hbar M$ is the component of the total angular momentum of the radiation field along the axis of rotation of the observer; as before, a multipole radiation field with $M = \omega/\Omega_0$ can stand completely still with respect to the rotating observer [8, 9]. In the case of inertial observers, this cannot occur since the speed of the observer is always less than the speed of light [11]. Thus in the formula for the Doppler effect $\omega' = \gamma\omega(1 - \mathbf{n}\cdot\mathbf{v}/c)$, $\omega' = 0$ implies that $\omega = 0$. We demand that the same should happen for the accelerated observer in the nonlocal theory. That is, we postulate that a fundamental radiation field can never stand completely still with respect to *any* observer. Thus if $F_{\alpha\beta}$ turns out to be constant in equation (3), then $f_{\mu\nu}$ should be constant as well. Expressing the Faraday tensor as a six-vector $f_{\mu\nu} \to (\mathbf{E}, \mathbf{B})$ and writing equation (2) in matrix notation as $\hat{f} = \Lambda f$, we note that equation (3) can be written as

$$F(\tau) = \Lambda(\tau) f(\tau) + \int_{\tau_0}^{\tau} K(\tau, \tau')\Lambda(\tau') f(\tau')\, d\tau', \tag{4}$$

where $f(\tau)$ is the restriction of the field measured by the standard static inertial observers to the worldline of the accelerated observer. Thus in equation (4) a constant F would imply a constant f only if

$$\Lambda_0 = \Lambda(\tau) + \int_{\tau_0}^{\tau} K(\tau, \tau')\Lambda(\tau')\, d\tau', \tag{5}$$

where $\Lambda_0 = \Lambda(\tau_0)$. Once the kernel is determined using equation (5), it follows from the Volterra-Tricomi uniqueness theorem that for any true radiation field $f_{\mu\nu}$, the accelerated observer will never measure a constant field (cf. [8, 9] and the references therein).

To determine the kernel $K(\tau, \tau')$, equation (5) must be solved under the requirements that (i) $K(\tau, \tau')$ exists due to the temporal variation of Λ, so that $K(\tau, \tau') = 0$ if Λ is constant, since the kernel must vanish for an inertial observer and (ii) the nonlocal contribution to the field in equation (4) is always bounded. This latter requirement turns out to be crucial. To see this, let us consider two possible solutions of equation (5) assuming that $K(\tau, \tau')$ is a function of only one variable: $K(\tau, \tau') = k(\tau')$ and $K(\tau, \tau') = \tilde{k}(\tau - \tau')$. In the first case, equation (5) is solved by simple differentiation and the result is [12, 13]

$$k(\tau) = -\frac{d\Lambda(\tau)}{d\tau}\Lambda^{-1}(\tau). \tag{6}$$

The second case involving a convolution kernel is more complicated; nevertheless, equation (5) is sufficient to determine $\tilde{k}(\tau - \tau')$ uniquely. Requirement (i) is satisfied in either case; moreover, they give the *same* kernel for *uniform* accelerated motion. However, for nonuniform acceleration the convolution kernel can lead to divergence [14] in contrast to equation (6). Thus the first case, where the kernel (6) is directly proportional to the acceleration of the observer gives an acceptable solution called the *kinetic kernel* [14].

Once an acceptable solution of equation (5) is available, such as the kinetic kernel $k(\tau)$, then the general solution may be written as

$$K(\tau, \tau') = k(\tau') + L(\tau, \tau')\Lambda^{-1}(\tau'), \tag{7}$$

where L is a 6×6 matrix that vanishes for constant Λ and involves bounded continuous functions such that

$$\int_{\tau_0}^{\tau} L(\tau, \tau')\, d\tau' = 0. \tag{8}$$

It follows from an application of the general theory of Fourier series that over the interval $[\tau_0, \tau]$, $L(\tau, \tau')$ is a linear superposition of functions of the form

$$a_k(\tau - \tau_0)\, e^{2\pi i k \frac{\tau' - \tau_0}{\tau - \tau_0}} \tag{9}$$

for any integer $k \neq 0$, where a_k are bounded continuous matrix-valued functions. In this way, equation (8) is satisfied and all that remains is to ensure that the time dependence of $a_k(\tau - \tau_0)$ is *solely* due to the variation of $\Lambda(\tau)$.

The requirement that $L(\tau, \tau')$ must vanish for a constant Λ is satisfied by expressing $a_k(\tau - \tau_0)$ as a double series

$$a_k = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C_{km}^n \left(\frac{d^m \Lambda}{d\tau^m} \right)^n + Q_k, \tag{10}$$

where C_{km}^n are constants such that the double series is absolutely convergent. Here $Q_k = Q_k(\Lambda)$ is a 6×6 matrix that is constant with respect to $\tau - \tau_0$ and such that $Q_k(\Lambda) = 0$ whenever Λ is constant. A class of such functions is given by

$$Q_k = \int_{\tau_0}^{\infty} \Phi_k(\tau') \mathcal{Q}_k(\Lambda(\tau'))\, d\tau', \tag{11}$$

where Φ_k and $\mathcal{Q}_k$ are bounded continuous functions and

$$\int_{\tau_0}^{\infty} \Phi_k(\tau')\, d\tau' = 0. \tag{12}$$

Let us note that C_{km}^n may also be proportional to constants of the form given by equation (11). Combining these results, we may therefore express $L(\tau, \tau')$ as a uniformly convergent series

$$L(\tau, \tau') = \mathrm{Re} \sum_{k \neq 0} a_k(\tau - \tau_0)\, e^{2\pi i k \frac{\tau' - \tau_0}{\tau - \tau_0}}, \tag{13}$$

where a_k is given by equations (10)–(12). Substituting equation (13) in equation (7), we find the general form of the kernel $K(\tau, \tau')$ that satisfies our physical requirements.

We must next consider the physical consequences of the general kernel for the nonlocal theory. The field measured by the accelerated observer is

obtained from the substitution of equation (7) in equation (4). We observe that in the calculation of $F(\tau)$ beyond the kinetic kernel, all terms would involve expressions of the form

$$f_k(\tau - \tau_0) = \int_{\tau_0}^{\tau} e^{2\pi i k \frac{\tau' - \tau_0}{\tau - \tau_0}} f(\tau')\, d\tau', \tag{14}$$

for $k \neq 0$. It follows that

$$F(\tau) = \hat{f}(\tau) + \int_{\tau_0}^{\tau} k(\tau') \hat{f}(\tau')\, d\tau' + \mathcal{F}(\tau - \tau_0), \tag{15}$$

where

$$\mathcal{F} = \mathrm{Re} \sum_{k \neq 0} a_k f_k. \tag{16}$$

Thus the measured field consists of a superposition of what would be expected on the basis of the kinetic kernel alone together with $\mathcal{F}$ that consists of the extra terms proportional to f_k. It would be interesting to examine the physical consequences of the presence of the extra terms in the field as measured by an accelerated observer. This is done in the next section for an observer rotating uniformly with $\Omega_0 \ll \omega$, since excellent observational data are available in this case for the coupling of the angular momentum of the electromagnetic radiation field to the rotation of the observer [9, 10].

4 Spin-rotation coupling

The observational results regarding spin-rotation coupling for electromagnetic radiation described in [9, 10] may be used as evidence against the presence of $\mathcal{F}$ in equation (15). To this end, we consider a plane monochromatic wave of frequency ω and definite helicity that is normally incident on an observer rotating uniformly with frequency $\Omega_0 \ll \omega$ on a circle of radius r in the (x, y)-plane. Experiments indicate that the measured frequency is $\hat{\omega} = \gamma(\omega \mp \Omega_0)$, where the upper (lower) sign refers to incident positive (negative) helicity radiation and γ is the Lorentz factor corresponding to $\beta = v/c$ with $v = r\Omega_0 \ll c$. We will compare and contrast this result with the predicted spectrum based on equations (15) and (16).

The uniformly rotating observer has been discussed in detail in [8, 14]. We assume that for $\tau < \tau_0$ the observer moves along a straight line with uniform speed ($\beta \ll 1$) such that $x = r$ and $y = \gamma v(\tau - \tau_0)$ and at $\tau = \tau_0$ begins uniform circular motion with $x = r\cos\varphi$ and $y = r\sin\varphi$, where $\varphi = \gamma\Omega_0(\tau - \tau_0)$. The natural orthonormal tetrad frame of the uniformly rotating observer is given by

$$\begin{aligned}
\lambda^{\mu}{}_{(0)} &= \gamma(1, -\beta\sin\varphi, \beta\cos\varphi, 0),\\
\lambda^{\mu}{}_{(1)} &= (0, \cos\varphi, \sin\varphi, 0),\\
\lambda^{\mu}{}_{(2)} &= \gamma(\beta, -\sin\varphi, \cos\varphi, 0),\\
\lambda^{\mu}{}_{(3)} &= (0, 0, 0, 1).
\end{aligned} \tag{17}$$

The acceleration tensor $\phi_{\alpha\beta}$ is given in this case by a centripetal acceleration $\mathbf{g} = -\gamma^2 v\Omega_0(1, 0, 0)$ and a rotation frequency $\mathbf{\Omega} = \gamma^2\Omega_0(0, 0, 1)$ with respect to the tetrad frame (17).

The incident field along the worldline of the observer may be expressed as

$$f(\tau) = \frac{1}{2} i\omega A \begin{bmatrix} \mathbf{e}_\pm \\ \mathbf{b}_\pm \end{bmatrix} e^{-i\,\gamma\omega(\tau-\tau_0)} + c.\ c., \tag{18}$$

where A is a constant complex amplitude, "c. c." indicates the corresponding complex conjugate term, $\mathbf{e}_\pm = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2}$, $\mathbf{b}_\pm = \mp i\mathbf{e}_\pm$ and the upper (lower) sign indicates positive (negative) helicity radiation. Here $\mathbf{e}_1$ and $\mathbf{e}_2$ indicate unit vectors along the positive x and y axes, respectively. The field as measured by the hypothetical comoving inertial observers is given by $\hat{f} = \Lambda f$, where

$$\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2 \\ -\Lambda_2 & \Lambda_1 \end{bmatrix} \tag{19}$$

and

$$\Lambda_1 = \begin{bmatrix} \gamma\cos\varphi & \gamma\sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & \gamma \end{bmatrix}, \quad \Lambda_2 = \beta\gamma \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -\cos\varphi & -\sin\varphi & 0 \end{bmatrix}. \tag{20}$$

The kinetic kernel, which can be worked out using equations (6), (19) and (20) turns out to be a constant matrix

$$k = \begin{bmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{bmatrix}, \tag{21}$$

where $k_1 = \Omega \cdot \mathbf{I} = \gamma^2 \Omega_0 I_3$ and $k_2 = -\mathbf{g} \cdot \mathbf{I}/c = \gamma^2 \beta \Omega_0 I_1$. Here $I_i, (I_i)_{jk} = -\epsilon_{ijk}$, is a 3×3 matrix proportional to the operator of infinitesimal rotations about the x^i-axis. If in equation (16) $a_k = 0$ for all integers $k \neq 0$, then $\mathcal{F} = 0$ and the observed field is

$$F = \frac{1}{2} i\gamma\omega A \begin{bmatrix} \hat{\mathbf{e}}_\pm \\ \hat{\mathbf{b}}_\pm \end{bmatrix} \frac{\omega e^{-i\hat{\omega}(\tau-\tau_0)} \mp \Omega_0}{\omega \mp \Omega_0} + c.\ c., \tag{22}$$

where $\hat{\mathbf{b}}_\pm = \mp i \hat{\mathbf{e}}_\pm$ and

$$\hat{\mathbf{e}}_\pm = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm i\gamma^{-1} \\ \pm i\beta \end{bmatrix}. \tag{23}$$

This field is only due to the kinetic kernel [8]. For $\Omega_0/\omega \ll 1$, F involves a small constant term with amplitude Ω_0/ω and a harmonic term of frequency $\hat{\omega}$, as expected. The predicted constant term is a direct result of nonlocality and has not yet been experimentally verified; however, it may be rather difficult to search for such a term of very small amplitude Ω_0/ω in the presence of noise.

To find the frequency spectrum of the extra field $\mathcal{F}$, let us note that the substitution of equation (18) in equation (14) results in an expression for $f_k(\tau - \tau_0)$ that exhibits transient as well as steady-state behaviors. The nature of the transients is illustrated in Figure 1. For observational purposes, we are only interested in the steady-state behavior of f_k. At late times $\tau - \tau_0 \gg 2\pi|k|/\omega$, f_k approaches a steady state given by

$$f_k(\tau - \tau_0) \sim \frac{1}{2}\gamma^{-1} A \begin{bmatrix} \mathbf{e}_\pm \\ \mathbf{b}_\pm \end{bmatrix} (1 - e^{-i\gamma\omega(\tau-\tau_0)}) + c.\ c.\,, \tag{24}$$

which is independent of k. Once the steady state is established, f_k is real and can be expressed as a constant term together with a harmonic term of frequency $\gamma\omega$. Let us next consider the frequency content of $a_k(\tau - \tau_0)$ given by equation (10). We note that

$$\Lambda \begin{bmatrix} \mathbf{e}_\pm \\ \mathbf{b}_\pm \end{bmatrix} = \gamma \begin{bmatrix} \hat{\mathbf{e}}_\pm \\ \hat{\mathbf{b}}_\pm \end{bmatrix} e^{\pm i\varphi}. \tag{25}$$

We can determine the frequency content of $a_k(\tau - \tau_0)$ by taking derivatives of equation (25) with respect to τ and using the fact that for $m > 1$

$$\frac{d^m \Lambda}{d\tau^m} \begin{bmatrix} \hat{\mathbf{e}}_\pm \\ \hat{\mathbf{b}}_\pm \end{bmatrix} = (\pm i\gamma\Omega_0)^m \Big\{ \begin{bmatrix} \hat{\mathbf{e}}_\pm \\ \hat{\mathbf{b}}_\pm \end{bmatrix} e^{\pm i\varphi} + O(\beta^2) \Big\}, \tag{26}$$

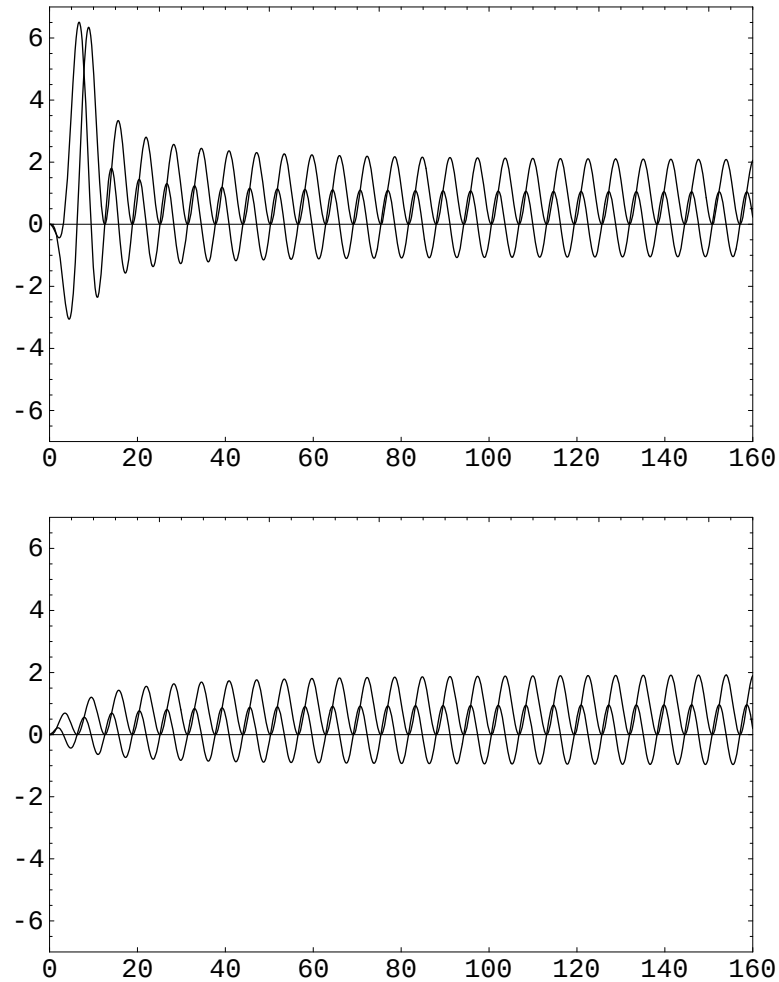

Figure 1: The function $f_k(\tau - \tau_0)$ defined by equations (14) and (18) is given, up to constant proportionality factors, by $\mathcal{R}_k(x) + i\mathcal{I}_k(x)$ for $x = \gamma\omega(\tau - \tau_0)$. Here k is a nonzero integer, $\mathcal{R}_k(x) = (1 - 2\pi k/x)^{-1}(1 - \cos x)$ and $\mathcal{I}_k(x) = (1 - 2\pi k/x)^{-1} \sin x$. These functions are plotted here versus x for $k = 1$ in the top panel and for $k = -1$ in the bottom panel. For $k > 0$ we have $\mathcal{R}_k(2\pi k) = 0$ and $\mathcal{I}_k(2\pi k) = 2\pi k$; therefore, the amplitude of the transient could be very large. The graphs illustrate the fact that the k-independent steady state is established for $x \gg 2\pi|k|$.

where $O(\beta^2)$ indicates terms proportional to either $\cos\varphi$ or $\sin\varphi$ with amplitudes that are smaller than the corresponding amplitude of the main term by a factor of $\sim \beta^2$. Combining these results, we find that the Fourier content of $\mathcal{F}$ is given in this case as follows: the term proportional to Q_k consists of a constant plus a harmonic term of frequency $\gamma\omega$, which disagrees with observation. The next term proportional to C^1_{km} consists of harmonic terms with frequencies $\hat{\omega}$ and $\gamma\Omega_0$; the latter term is contrary to observation. Moreover, the term proportional to C^n_{km} for $n > 1$ would contain principal harmonics $\gamma(\omega \mp n\Omega_0)$ and $n\gamma\Omega_0$ that would be in contradiction with experimental results as well as terms whose amplitudes would be smaller by a factor of $\sim \beta^2$. We can thus conclude that either $\mathcal{F} = 0$ or that its amplitude is so small as to have escaped detection thus far. We may therefore proceed with the tentative assumption that $\mathcal{F} = 0$ and the kinetic kernel (6) is unique, while keeping in mind the possibility that future experimental data may prompt us to take $\mathcal{F}$ into account as well. This eventuality appears highly unlikely, however, from a theoretical standpoint since $\hat{\omega} = \gamma(\omega \mp \Omega_0)$ for $\Omega_0 \ll \omega$ emerges from the simple kinematics of Maxwell's theory [9].

5 Discussion

In this paper we have looked for bounded continuous kernels within the framework of the nonlocal theory of accelerated observers that would be consistent with Lorentz invariance and satisfy the requirement that a basic radiation field would never stand completely still with respect to an accelerated observer. Concentrating on electrodynamics and taking into account observational data regarding spin-rotation coupling, we find that the kinetic kernel $K(\tau, \tau') = k(\tau')$ given by equation (6) is the only one consistent with the data thus far. We therefore adopt the kinetic kernel for the nonlocal theory in general, regardless of the nature of the field.

For the kinetic kernel, the nonlocal contribution to the field has the character of a weighted average such that the weight function is proportional to the acceleration of the noninertial observer. This result is consistent with the idea put forward by Bohr and Rosenfeld [15] that the measured field is an average over a spacetime region. For the case of an accelerated observer, the spacetime region reduces to the past worldline of the observer due to certain basic limitations on the measurement of distance discussed in [3, 4]. Furthermore, Bohr and Rosenfeld [15] considered only inertial observers for which

the weight function would be unity due to the homogeneity and isotropy of inertial frames of reference. However, the field measurements of a noninertial observer would be weighted according to its acceleration along its past worldline.

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