

GRAVITATION AND NONLOCALITY

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The physical basis of the standard theory of general relativity is examined and a nonlocal theory of accelerated observers is described that involves a natural generalization of the hypothesis of locality. The nonlocal theory is confronted with experiment via an indirect approach. The implications of the results for gravitation are briefly discussed.

1 Introduction

Einstein's general theory of relativity [1] is a successful classical theory of gravitation inasmuch as it agrees with all experimental data available at present [2]. To develop a microphysical gravitation theory, it may prove useful to investigate the extent to which the basic physical tenets of general relativity conform with the fundamental principles of the quantum theory. This is the general approach adopted in this work.

The basic physical assumptions underlying general relativity can be determined by investigating the measurement problem in the theory of relativity. In this way one can characterize the observational assumptions that are necessary in order to provide a physically consistent interpretation of all relativistic formulas [3]. The resulting four basic pillars of the theory are the following:

- (i) Lorentz Invariance,
- (ii) Hypothesis of Locality,
- (iii) Einstein's Principle of Equivalence,
- (iv) Correspondence with Newtonian Gravitation and the Gravitational Field Equations.

Briefly, the inhomogeneous Lorentz transformations in (i) relate the measurements of physical quantities by ideal inertial observers in Minkowski spacetime. However, observers in Minkowski spacetime are generally accelerated, at least along a portion of their worldlines. To deal with realistic (i.e. accelerated) observers, the standard theory of relativity contains the assumption that an accelerated observer is at each instant momentarily equivalent to a hy-

pothetical comoving inertial observer; that is, the acceleration of the observer is considered *locally* immaterial (“Hypothesis of Locality”). Moreover, an observer in a gravitational field is presumed to be *locally* equivalent to a certain accelerated observer in Minkowski spacetime according to Einstein’s heuristic principle of equivalence. Assumptions (ii) and (iii) taken together imply that observers in a gravitational field are all locally inertial. The simplest way to connect these local inertial frames is via Riemannian geometry of curved spacetime, where the curvature is identified with the gravitational field and free test particles and null rays follow geodesics of the spacetime manifold. Finally, in the geometric framework of general relativity, Einstein’s field equations are the simplest equations that provide a natural and consistent generalization of Poisson’s equation of Newtonian gravity.

Dirac’s generalization of Schrödinger’s equation and the subsequent achievements of relativistic quantum field theory indicate that (i) can be integrated into the general framework of quantum theory. Therefore, the main part of this paper is concerned with the status of (ii) vis-a-vis quantum mechanics. Section 2 examines the physical basis of (ii) and section 3 describes a nonlocal theory of accelerated observers. The observational aspects of this nonlocal theory are examined in section 4. In conclusion, some of the implications of these ideas for the theory of gravitation are briefly described in section 5.

2 Hypothesis of Locality

The ultimate physical basis of the hypothesis of locality is Newtonian mechanics, where the *state* of a particle is determined by its position and velocity. Thus the accelerated observer and the otherwise identical hypothetical instantaneously comoving inertial observer are equivalent, since they both share the same state.

In terms of realistic measurements of the accelerated observer, the hypothesis of locality would hold if these measurements are essentially pointwise and instantaneous, so that the influence of inertial effects can be neglected over length and time scales that are characteristic of elementary local measurements. To state this criterion in a quantitative way, let us first note that an accelerated observer can be characterized by certain acceleration lengths \mathcal{L} that involve the speed of light c and certain scalars formed from the translational and rotational accelerations of the observer. If λ is the intrinsic length scale of the phenomenon under observation, then λ/\mathcal{L} characterizes the expected deviation from the hypothesis of locality. For instance, in a laboratory fixed on the rotating Earth, the typical acceleration lengths would be $c^2/g_{\oplus} \simeq 1$ lyr and $c/\Omega_{\oplus} \simeq 28$ AU; therefore, for most experimental situations λ/\mathcal{L} would

be negligibly small. It follows that the hypothesis of locality is approximately valid under most current observational situations. Measuring devices that, like the rods and clocks of classical relativity theory (cf. [1], p. 60), obey the hypothesis of locality are called “standard”; hence, a standard clock measures proper time along its worldline.

To delve deeper into the various limitations of the hypothesis of locality, we consider a thought experiment involving the reception of a normally incident plane electromagnetic wave by a rotating observer. Let us choose a global inertial system in which the observer moves with constant frequency Ω in the (x, y) -plane on a circle of radius R about the origin such that $x = R \cos \varphi$ and $y = R \sin \varphi$, where $\varphi = \Omega t$. Moreover, the monochromatic plane wave propagates along the z -axis with frequency ω .

It follows from the hypothesis of locality that the natural orthonormal tetrad frame of the rotating observer can be given with respect to the inertial frame by [3]

$$\lambda^\mu_{(0)} = \gamma(1, -\beta \sin \varphi, \beta \cos \varphi, 0) , \quad (1)$$

$$\lambda^\mu_{(1)} = (0, \cos \varphi, \sin \varphi, 0) , \quad (2)$$

$$\lambda^\mu_{(2)} = \gamma(\beta, -\sin \varphi, \cos \varphi, 0) , \quad (3)$$

$$\lambda^\mu_{(3)} = (0, 0, 0, 1) , \quad (4)$$

where γ is the Lorentz factor, $\gamma = (1 - \beta^2)^{-1/2}$, and $\beta = v/c = R\Omega/c$. The motion of the frame along the worldline can in general be characterized by six scalar quantities that form an antisymmetric tensor $\phi_{\alpha\beta}$ defined by $d\lambda^\mu_{(\alpha)}/d\tau = \phi_\alpha^\beta \lambda^\mu_{(\beta)}$. Here τ is the proper time, $d\tau = \gamma^{-1} dt$, the “electric” part of $\phi_{\alpha\beta}$ corresponds to translational acceleration $\tilde{g}_i = \phi_{0i}$ and the “magnetic” part $\tilde{\Omega}_i = \frac{1}{2}\epsilon_{ijk}\phi^{jk}$ corresponds to the rotational frequency of the spatial frame. For the tetrad (1) - (4), we find that the scalars $\tilde{g}_1 = -\gamma^2 R\Omega^2$ and $\tilde{\Omega}_3 = \gamma^2 \Omega$ are the only nonzero components of $\tilde{\mathbf{g}}$ and $\tilde{\mathbf{\Omega}}$ corresponding respectively to a centripetal acceleration of magnitude $\gamma^2 v^2/R$ and a rotation of the spatial frame about the z -axis of frequency $\gamma^2 \Omega$. It should be noted that *proper* acceleration scales can be constructed from the invariants $\frac{1}{2}\phi_{\alpha\beta}\phi^{\alpha\beta}$ and $\frac{1}{2}\phi_{\alpha\beta}^*\phi^{\alpha\beta}$, where $\phi_{\alpha\beta}^*$ is the dual of $\phi_{\alpha\beta}$; in the case of uniform rotation, these are respectively $\gamma^2 \Omega^2$ and zero. It follows that $\mathcal{L} = c/(\gamma\Omega)$, where $\gamma\Omega = d\varphi/d\tau$ is the proper rotation frequency of the observer [3].

Regarding the reception of the wave by the observer, we note that an electromagnetic field may be represented in terms of the components of the Faraday tensor $f_{\mu\nu}$ as measured by the standard set of ideal inertial observers at rest in the underlying global frame. The field measured by an arbitrary accelerated observer is then the projection of $f_{\mu\nu}$ on the orthonormal tetrad of the observer

$$\hat{f}_{\alpha\beta} = f_{\mu\nu} \lambda_{(\alpha)}^\mu \lambda_{(\beta)}^\nu . \quad (5)$$

It is possible to express (5) using the six-vector representation of the Faraday tensor, i.e. in terms of the electric and magnetic fields — $f_{\alpha\beta} \rightarrow (\mathbf{E}, \mathbf{B})$, as

$$\hat{f} = \Lambda f . \quad (6)$$

The incident electromagnetic radiation field under consideration may therefore be expressed as the real part of

$$f = i\omega A \begin{bmatrix} \mathbf{e}_\pm \\ \mathbf{b}_\pm \end{bmatrix} e^{-i\omega(t-z/c)} , \quad (7)$$

where A is a complex amplitude, $\mathbf{e}_\pm = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2}$, $\mathbf{b}_\pm = \mp i\mathbf{e}_\pm$ and the upper (lower) sign indicates radiation of positive (negative) helicity. Here \mathbf{e}_1 and \mathbf{e}_2 are unit vectors along the positive x and y directions, respectively. Using equations (1) - (7), we find that the field measured by the rotating observer is given by the real part of

$$\hat{f} = i\gamma\omega A \begin{bmatrix} \hat{\mathbf{e}}_\pm \\ \hat{\mathbf{b}}_\pm \end{bmatrix} e^{-i\hat{\omega}\tau} , \quad (8)$$

where $\hat{\mathbf{b}}_\pm = \mp i\hat{\mathbf{e}}_\pm$ and

$$\hat{\mathbf{e}}_\pm = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm i\gamma^{-1} \\ \pm i\beta \end{bmatrix} \quad (9)$$

involve unit vectors with respect to the tetrad axes, $\gamma\tau = t$ and

$$\hat{\omega} = \gamma(\omega \mp \Omega) . \quad (10)$$

To interpret these results, let us first note that a simple application of the hypothesis of locality would connect the instantaneous inertial frame of the rotating observer with the global inertial frame. In terms of the propagation four-vector of the wave, the result is the transverse Doppler effect; namely, the

accelerated observer measures a frequency $\omega' = \gamma\omega$, where the Lorentz factor accounts for time dilation. However, if the hypothesis of locality is applied to the field and the result is Fourier analyzed — a nonlocal operation in proper time — then, the result is equation (10), which goes beyond $\gamma\omega$ by terms of the form $\Omega/\omega = \lambda'/\mathcal{L}$. Therefore, $\hat{\omega} \rightarrow \omega'$ in the JWKB limit $\Omega/\omega \rightarrow 0$.

Equations (8) - (10) have a simple intuitive interpretation : For positive (negative) helicity radiation, the electric and magnetic fields rotate with frequency ω ($-\omega$) about the direction of propagation of the wave and the rotating observer perceives positive (negative) helicity radiation but with frequency $\omega - \Omega$ ($\omega + \Omega$) augmented with the time dilation factor γ . Partial experimental evidence is presented in [4] for this phenomenon; it is an example of the general spin-rotation coupling (see [5 - 9] for discussions and reviews). On the other hand, equation (10) has a remarkable consequence for which there is no observational evidence: for incident positive helicity radiation of frequency $\omega = \Omega$, the radiation field becomes static, i.e. the wave stands completely still, for the whole class of observers rotating uniformly with frequency Ω about the z -axis. More generally, for an obliquely incident radiation field $\hat{\omega} = \gamma(\omega - M\Omega)$, where M is the multipole parameter associated with the z -component of the *total* angular momentum of the field; therefore, the radiation can stand completely still with respect to the rotating observers for $\omega = M\Omega$.

Another general consequence of the hypothesis of locality reflected in equations (5) - (10) is the following: If the incident radiation is a linear superposition of the two possible helicity states, then the radiation as seen by the rotating observer has the *same* amplitudes in terms of the transformed basis (9) but different frequencies. It would also be interesting to confront this prediction of the hypothesis of locality regarding *helicity amplitudes* with observation.

3 Nonlocal Theory of Accelerated Observers

Consider anew the reception of electromagnetic radiation by an accelerated observer. How is the class of momentarily comoving inertial observers that measure $\hat{f}_{\alpha\beta}(\tau)$ related to the accelerated observer that measures $F_{\alpha\beta}(\tau)$ while passing through the continuous infinity of local inertial systems? The hypothesis of locality postulates that $F_{\alpha\beta}$ is equal to $\hat{f}_{\alpha\beta}$ at each instant of proper time τ . However, the most general linear and causal relationship between $\hat{f}_{\alpha\beta}(\tau)$ and the field $F_{\alpha\beta}(\tau)$ that is actually measured by the accelerated observer is given by a Volterra integral equation

$$F_{\alpha\beta}(\tau) = \hat{f}_{\alpha\beta}(\tau) + \int_{\tau_0}^{\tau} K_{\alpha\beta}{}^{\gamma\delta}(\tau, \tau') \hat{f}_{\gamma\delta}(\tau') d\tau' \quad , \quad (11)$$

where τ_0 is the instant at which the acceleration is turned on. It follows from Volterra's theorem that in the space of continuous functions the relationship between $f = \Lambda^{-1}\hat{f}$ and F given by equation (11) is unique; this uniqueness result has been extended to square-integrable functions by Tricomi [10]. Equation (11) is manifestly Lorentz covariant as it deals only with scalar quantities. It remains to determine the kernel $K_{\alpha\beta\gamma\delta}$ in terms of the acceleration of the observer.

Let us note that if the kernel K is simply proportional to the acceleration, then the magnitude of the nonlocal part in equation (11) would generally be of the form λ'/\mathcal{L} , as expected. The basic approach to the determination of the kernel in this theory is to exclude the possibility that an incident radiation field could stand completely still with respect to an accelerated observer (cf. section 2). In the case of ideal inertial observers, this comes about because no observer can move at the speed of light; therefore, it follows from the Doppler formula $\omega' = \gamma\omega(1 - \boldsymbol{\beta} \cdot \mathbf{n})$ for radiation propagating along a unit vector \mathbf{n} that if $\omega' = 0$, then $\omega = 0$. We demand a similar outcome for all observers; that is, if in equation (11) $F_{\alpha\beta}$ is constant, then the incident field $f_{\alpha\beta}$ must be constant. Imposing this requirement, equation (11) in six-vector notation reduces to

$$\Lambda_0 = \Lambda(\tau) + \int_{\tau_0}^{\tau} K(\tau, \tau')\Lambda(\tau')d\tau' \ , \quad (12)$$

where $\Lambda_0 = \Lambda(\tau_0)$ is a constant 6×6 matrix. The Volterra-Tricomi uniqueness theorem now ensures that true incident radiation fields will never be found to stand completely still by any observer [11].

Equation (12) is not sufficient to determine a unique kernel K ; other simplifying assumptions are necessary. To this end, let us suppose that $K(\tau, \tau')$ is only a function of one variable. Two possible situations [12 - 13] are of interest $K(\tau, \tau') = k(\tau')$ or $\tilde{k}(\tau - \tau')$. A detailed analysis reveals that of these two possibilities only the former ("kinetic") kernel is acceptable, since the latter convolution kernel can lead to divergences for nonuniform acceleration [14]. It follows from equation (12) that the kinetic kernel is directly proportional to the acceleration of the observer and is given by

$$k(\tau) = -\frac{d\Lambda(\tau)}{d\tau} \Lambda^{-1}(\tau) \ . \quad (13)$$

In this case, equation (11) can be written as

$$F = \hat{f} + \int_{\tau_0}^{\tau} k(\tau')\hat{f}(\tau') d\tau' \ , \quad (14)$$

so that the nonlocal contribution to the field is a weighted average over the past history of the accelerated observer. This circumstance is consistent with the observation of Bohr and Rosenfeld that the electromagnetic field cannot be measured at one spacetime point; an averaging process is necessary [15]. From this standpoint, the kinetic kernel k appears to be unique [16].

The nonlocal theory of accelerated systems is consistent with the observed absence of an elementary scalar (or pseudoscalar) particle in nature. For a scalar field $\Lambda = 1$, hence $k = 0$ and the theory is local. Thus it would in general be possible for an observer to stay completely at rest with a pure scalar radiation field, a possibility that is excluded by our physical postulate. Hence the theory predicts that any scalar (or pseudoscalar) particle would have to be a composite.

To illustrate the nonlocal theory, we return to the thought experiment discussed in detail in section 2. Let us suppose that for $t < 0$ the observer moves uniformly in the (x, y) -plane such that $x = R$ and $y = R\Omega t$ and at $t = \tau = 0$ begins the rotational motion discussed before. Then the kinetic kernel k given by equation (13) turns out to be a constant for the case of uniform rotation

$$k = \begin{bmatrix} k_r & k_t \\ -k_t & k_r \end{bmatrix}, \quad (15)$$

where k_r and k_t are 3×3 matrices given by $k_r = \tilde{\mathbf{\Omega}} \cdot \mathbf{I} = \gamma^2 \Omega I_3$ and $k_t = -\tilde{\mathbf{g}} \cdot \mathbf{I} = \gamma^2 \beta \Omega I_1$. Here I_i , $(I_i)_{jk} = -\epsilon_{ijk}$, is a matrix proportional to the operator of infinitesimal rotations about the x^i -axis. The radiation field according to the rotating observer is given by the real part of

$$F = \hat{f} \frac{\omega \mp \Omega e^{i\hat{\omega}\tau}}{\omega \mp \Omega}, \quad (16)$$

where \hat{f} is given by equation (8). Two consequences of nonlocality should be noted here: For positive helicity radiation with $\omega = \Omega$, the result (16) has the character of resonance and F turns out to be a linear function of proper time τ . We note that this linear growth (and eventual divergence) of the field with time would be absent for any finite incident *wave packet*. Moreover, as a direct result of the fact that k is constant and the nonlocal part of the field in equation (14) involves an integration over time, the frequencies $\omega \mp \Omega$ appear in the denominator resulting in a larger (smaller) measured amplitude for the positive (negative) helicity radiation as in equation (16).

Let us recall from the results of section 2 that according to the hypothesis of locality incident waves of frequency ω with opposite helicities and equal

amplitudes will be measured by the rotating observer to have equal amplitudes but different frequencies $\hat{\omega} = \gamma(\omega \mp \Omega)$. Thus for the *relative* amplitudes of the two helicity states, the radiation field is not affected by the rotation of the observer according to the standard theory of relativity. The nonlocal theory predicts, however, that the field strength will be higher (lower) when the electromagnetic field rotates in the same (opposite) sense as the rotation of the observer by the factor $1 + \Omega/\omega$ ($1 - \Omega/\omega$) for $\Omega/\omega \ll 1$. For instance, $\Omega/\omega \sim 10^{-7}$ for radio waves with $\lambda \sim 1$ cm incident on a system rotating at a rate of 500 rounds per second.

This helicity dependence of the amplitude of the radiation field is a definite signature of nonlocality and the next section is devoted to a discussion of this effect.

4 Confrontation with Experiment

The whole observational basis of the theory of relativity involves experiments performed in accelerated systems of reference; however, the acceleration scales are typically very large compared to the intrinsic scales that are relevant in such experiments and hence rather high levels of observational accuracy would be needed in order to detect nonlocal phenomena. In planning such high-sensitivity experiments, a new problem would be encountered: one must consider the influence of acceleration on the accelerated measuring devices as well. In view of these difficulties, it is interesting to explore a novel approach based on the correspondence principle suggested by Steven Chu [17]: Under appropriate circumstances, the electrons in atoms may be viewed as “accelerated observers”; the predictions of the classical theory could then be compared with quantum mechanics.

This idea can be developed in connection with the thought experiment concerning the measurement of the electromagnetic field of a normally incident wave by a uniformly rotating observer as follows: Let us imagine the photoionization process involving the absorption of an incident photon by an electron bound by a potential and the subsequent ejection of the electron from the system. To simulate our thought experiment (cf. section 2) in the quantum domain, we consider the nonrelativistic motion of the electron on a “circular orbit” in the hydrogen atom, so that the stationary state of the electron is specified by the quantum numbers $n > 1$, $l = n - 1$ and $m = l$; initially, the photon is incident on this state along the z -direction. The electron spin is neglected. It is interesting to note that the *impulse approximation*, originally suggested by Fermi in treating certain problems in quantum scattering theory [18], is the quantum analogue of the hypothesis of locality in this case. Just as in

the hypothesis of locality the accelerated observer is at each moment replaced by an otherwise identical free inertial observer, the impulse approximation in effect replaces the bound electron by a free electron of definite momentum [19]. It then follows that the cross section for photoionization in this approximation is independent of the helicity of the incident radiation. However, the helicity dependence enters the calculation once the Coulomb interaction is properly taken into account in the final state. A detailed treatment reveals that the quantum results and the predictions of the nonlocal theory are in *qualitative* agreement [20]. For instance, for $mc^2 \gg \hbar\omega > E_n$, where m is the electron mass, $-e$ is its electric charge and $-E_n = -me^4/(2\hbar^2n^2)$ is the electron energy in its initial “circular orbit,” the total cross sections σ_+ (positive helicity) and σ_- (negative helicity) for photoionization in the dipole approximation are such that

$$\frac{\sigma_-}{\sigma_+} = \frac{3(n-1) + 2n\eta}{2n[2(n-1)^2 + n(2n-1)\eta]} \quad , \quad (17)$$

where $\eta = \Omega_n/\omega$ and $\Omega_n = 2E_n/(\hbar n)$ is the Bohr frequency of the electron in the circular orbit with Bohr radius $r_n = \hbar^2n^2/(me^2)$. The dipole approximation requires that $\omega r_n \ll c$, hence $\eta \gg (137n)^{-1}$; moreover, we note that $\hbar\omega > E_n$ implies that $\eta < 2/n$. For $n = 1$, the ground state is spherically symmetric and hence $\sigma_- = \sigma_+$; however for $n > 1$ the electron following the “circular orbit” with $m = l = n - 1 > 0$ tends to move on average like the observer in our thought experiment and equation (17) implies that $\sigma_- < \sigma_+$, as expected.

Further corroboration of the predictions of the nonlocal theory may be obtained from the consideration of “circular orbits” of electrons in a uniform magnetic field B along the z -axis. In fact, on the basis of Larmor’s celebrated theorem it would be natural to consider such electronic states in connection with our thought experiment involving an observer in a rotating frame of reference. Ignoring any motion along the z -axis, such a “circular orbit” would be characterized by the quantum numbers N and M , where N denotes the energy states $\hbar\Omega_c(N + \frac{1}{2})$ and $\hbar M$ is the z -component of the angular momentum of the electron. Here $\Omega_c = eB/(mc)$ is the cyclotron frequency and we assume that $\hbar\Omega_c \ll mc^2$. The correspondence with the classical cyclotron motion of the electron with orbital frequency Ω_c can be established for $N \sim M \gg 1$. In this case, the nonrelativistic calculation of electric dipole transitions due to a normally incident radiation of frequency ω , $\hbar\omega \ll mc^2$, and definite helicity reveals that transition is possible only for $(N, M) \rightarrow (N+1, M+1)$ with $\omega = \Omega_c$ and positive helicity incident radiation, while for incident negative helicity radiation of $\omega = \Omega_c$ the transition $(N, M) \rightarrow (N+1, M-1)$ is forbidden. This is

in qualitative agreement with the result of the nonlocal theory, equation (16), for the case of resonance $\omega = \Omega$: At resonance, the field amplitude for positive helicity radiation diverges linearly with proper time, while the amplitude for negative helicity radiation remains finite. Thus in terms of the ratios of helicity amplitudes the result is qualitatively the same as in quantum mechanics.

5 Gravitation

It appears from the results of the previous section that the acceleration-induced nonlocality must be taken seriously, since its predictions are closer to reality (as defined by the quantum theory) than the standard theory of accelerated systems. This circumstance raises the question of whether the nonlocality extends to gravitation as would be intuitively expected from Einstein's heuristic principle of equivalence.

Gravitation is a universal interaction that is qualitatively different from other interactions and so it may not be surprising if it could be described in terms of a nonlocal classical field in Minkowski spacetime such that in a suitable eikonal limit this nonlocal field would have an interpretation in terms of the local curvature of a certain spacetime manifold as in general relativity.

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