

ON THE SPIN-ROTATION-GRAVITY COUPLING*

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ABSTRACT

The inertial and gravitational properties of intrinsic spin are discussed and some of the recent work in this area is briefly reviewed. The extension of relativistic wave equations to accelerated systems and gravitational fields is critically examined. A nonlocal theory of accelerated observers is presented and its predictions are compared with observation.

KEYWORDS: Inertia of intrinsic spin; nonlocality

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The inertial and gravitational couplings of intrinsic spin have recently received attention as experimental evidence for spin-rotation coupling has become available. Friedrich W. Hehl has made significant contributions to this important topic and it is therefore a great pleasure for me to dedicate this paper to him on the occasion of his sixtieth birthday.

1. INTRODUCTION

The spin-rotation-gravity coupling has appeared in the work of many authors who have been mainly interested in the study of wave equations in accelerated systems and gravitational fields [1]. Indeed, the coupling under consideration here directly involves wave effects that pertain to the physical foundations of general relativity. Classically, motion occurs via particles as well as electromagnetic waves. The basic geometric structure of Einstein's theory of gravitation accords a special status to the motion of classical test particles and null rays, since these idealized physical systems follow geodesic paths that are intrinsic to the geometry of the spacetime manifold [2]. In contrast, the motion of a wave packet in general relativity does not pertain to intrinsic geometric properties of the spacetime. Can one provide a purely geometric description of *diffraction* phenomena, for instance? To illustrate the problem, let us consider the following thought experiment: Imagine a ray of light that has frequency ω according to observer O and the class of observers boosted with respect to O at the same event along the direction of propagation of the ray. The frequency measured by any such observer is $\omega' = \gamma\omega(1 - \beta)$ in accordance with the Doppler effect. It follows that the wavelength of the radiation can become extremely large or extremely small according to the boosted observers; however, the respective limiting values of infinity and zero are excluded since $|\beta| < 1$. On the other hand, it can be shown that the effective radius of curvature of spacetime as measured by the boosted observers is generally Lorentz contracted [3]. According to all observers, however, the worldline of the ray is a null geodesic even when the measured wavelength far exceeds the measured radius of curvature. The only physical conclusion that one can draw from this analysis is that the wavelength of the radiation must be zero for all observers in order that the complete absence of diffraction can be satisfactorily explained. Thus null geodesics would carry infinite energy in the quantum theory; hence, the standard axiomatic formulations of general relativity in terms of clocks and light

rays are physically unrealistic.

Einstein formulated general relativity as a theory of pointlike coincidences [2]; therefore, the theory is most consistent when wave phenomena, which generally require extended intervals of space and time for their characterization, are treated in the eikonal limit. In general, wave phenomena in a gravitational field depend upon the observer; moreover, a completely covariant analysis is not possible since an observer can set up an admissible coordinate system in its neighborhood only within a spatial region of radius $R \ll \mathcal{L}$, where \mathcal{L} is an acceleration length, and only wavelengths $\lambda < R$ can then be determined by the observer.

Consider, for the sake of simplicity and the exclusion of matter-related effects, the scattering of electromagnetic radiation from a black hole in terms of the standard set of inertial observers in the asymptotically flat region of the spacetime. It turns out that for a Schwarzschild black hole the amplitudes for the scattering of right circularly polarized (RCP) and left circularly polarized (LCP) waves are equal and hence the spherical symmetry of this field preserves the polarization of the incident radiation in the scattered waves. However, for a Kerr black hole RCP and LCP radiations are scattered differently. This can be traced back to the influence of a gravitational coupling between the intrinsic spin of the radiation field and the rotation of the source. In this way, the deflection of the radiation by a rotating mass becomes polarization dependent [4]. Imagine a rotating body with mass M and angular momentum $\mathbf{J} = J\hat{\mathbf{z}}$ with its center of mass at the origin of coordinates and a beam of radiation propagating above the body nearly parallel to the x-axis with impact parameter D . The Einstein deflection angle for the beam is $\Delta = 4GM/c^2D$; however, RCP radiation is essentially deflected by an angle $\Delta - \delta$ and LCP radiation by $\Delta + \delta$, where $\delta = 4\lambda GJ/c^3D^3$. In the JWKB limit, $\delta \rightarrow 0$ and the principle of equivalence is recovered. The differential deflection of polarized radiation is very small; e.g., it is of order one milliarc-second for radio waves with $\lambda \sim 1$ cm passing just over the poles of a rapidly rotating neutron star. Upper limits on the deviation from the principle of equivalence for polarized radio waves deflected by the Sun have been placed by Harwit *et al.* [5]. Astrophysical implications of this effect have been considered by a number of authors [6]; in particular, it may become interesting in connection with microlensing with polarized radiation [7].

The differential deflection of polarized radiation is a consequence of the coupling of photon helicity with the gravitomagnetic field of a rotating mass

$\mathbf{B}_g = c\boldsymbol{\Omega}_P$, where

$$\boldsymbol{\Omega}_P = \frac{GJ}{c^2 r^3} [3(\hat{\mathbf{J}} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \hat{\mathbf{J}}] \quad (1)$$

is the precession frequency of a free test gyroscope at position \mathbf{r} . According to the gravitational Larmor theorem [4], a gravitomagnetic field can be locally replaced by a frame rotating at frequency $\boldsymbol{\Omega}_L = -\boldsymbol{\Omega}_P$. It follows that similar spin-rotation coupling effects are expected in a rotating frame of reference. This may be illustrated with a thought experiment: Consider an inertial reference frame \mathcal{S} and an observer rotating in the positive sense about the direction of propagation of a plane monochromatic electromagnetic wave of frequency ω . We are interested in the frequency of the radiation as measured by the rotating observer. Special relativity is based on Poincaré invariance and the *hypothesis of locality*. The latter states that an accelerated observer in Minkowski spacetime is at each event equivalent to a momentarily comoving inertial observer. Thus the rotating observer is instantaneously inertial and the transformation between this local inertial frame \mathcal{S}' and \mathcal{S} results in the transverse Doppler formula, $\omega' = \gamma\omega$, for the frequency of the radiation. On the other hand, the observer needs to measure at least several oscillations of the wave before an estimate for ω' could be computed from the data. It follows from this line of argument that the transverse Doppler formula must be valid in the eikonal limit. It is more reasonable to assume that the hypothesis of locality applies to the field at each event; then,

$$F_{(\alpha)(\beta)}(\tau) = F_{\mu\nu}\lambda_{(\alpha)}^\mu\lambda_{(\beta)}^\nu \quad , \quad (2)$$

which is the projection of the Faraday tensor on the tetrad frame of the observer, is Fourier analyzed over the proper time τ of the accelerated observer to determine its frequency content. This is the extended hypothesis of locality for wave phenomena and provides the physical basis for the extension of relativistic wave equations to accelerated frames and gravitational fields (“minimal coupling”). For the thought experiment under consideration, we find in this way that $\omega' = \gamma(\omega \mp \Omega)$, where the upper (lower) sign refers to RCP (LCP) incident radiation. This result has a simple physical interpretation: The electric and magnetic fields rotate in the positive sense

with frequency ω about the direction of propagation in a plane RCP wave; therefore, from the viewpoint of the rotating observer the radiation is also RCP but with frequency $\omega' = \gamma(\omega - \Omega)$. Here the Lorentz factor takes due account of time dilation. A similar argument for the LCP radiation leads to the addition of frequencies and $\omega' = \gamma(\omega + \Omega)$. In terms of the photon energy $E' = \gamma(E \mp \hbar\Omega)$, so that the helicity of the radiation couples to rotation producing an effect that goes beyond the eikonal limit. That is $\omega' = \gamma\omega(1 \mp \lambda/\mathcal{L})$, where $\mathcal{L} = c/\Omega$ is the acceleration length of the observer. It is important to point out that experimental evidence for such wave effects due to helicity-rotation coupling with $\lambda \ll \mathcal{L}$ already exists for microwaves as well as light and will be described elsewhere [8].

It is possible to show that for an arbitrary direction of incidence

$$\omega' = \gamma(\omega - m\Omega) \quad , \quad (3)$$

where m is a parameter characterizing the component of the total angular momentum of the radiation field along the direction of rotation (“magnetic quantum number”). For a scalar or a vector field, $m = 0, \pm 1, \pm 2, \dots$, while for a Dirac field $m \mp \frac{1}{2} = 0, \pm 1, \pm 2, \dots$. Thus ω' could be negative, zero or positive. In the case of a linearized gravitational radiation field, the helicity-rotation coupling has interesting consequences for celestial mechanics [9].

The observational consequences of spin-rotation coupling for neutron interferometry in a rotating frame of reference have been explored in connection with the assumptions that underlie the physical interpretation of wave equations in an arbitrary frame of reference [10]. In general, the spin-rotation phase shift is smaller than the Sagnac shift [11] by roughly the ratio of the wavelength to the dimension of the interferometer.

A proper theoretical treatment of the inertial properties of a Dirac particle is due to Hehl and his collaborators [12]. This treatment has been extended in several important directions by a number of investigators [13-16]. The significance of spin-rotation coupling for atomic physics has been pointed out by Silverman [17]. Moreover, the astrophysical consequences of the helicity flip of massive neutrinos as a consequence of spin-rotation coupling have been investigated by Papini and his collaborators [18]. Bell and Leinaas [19] attempted to explain certain depolarization phenomena in circular accelerators in terms of a thermal bath caused by the centripetal acceleration of

the (polarized) particles involved; however, Papini *et al.* [20] have shown that the data should be interpreted instead in favor of spin-rotation coupling. In fact, there is no experimental evidence for an acceleration-induced thermal ambience at present; moreover, it does not come about in the theoretical structures discussed in this paper. To appreciate this point, imagine the energy-momentum tensor of the field as measured by an accelerated observer $T_{(\alpha)(\beta)} = T_{\mu\nu}\lambda_{(\alpha)}^\mu\lambda_{(\beta)}^\nu$; once the field is absent in the inertial frame, the energy-momentum measured by any standard device vanishes. A similar result involving the vacuum expectation value of the energy-momentum tensor is expected to hold in the quantum theory.

Direct evidence for the coupling of intrinsic spin to the rotation of the Earth has recently become available [21, 22]. In fact, according to the natural extension of general relativity under consideration here, every spin- $\frac{1}{2}$ particle in the laboratory has an additional interaction Hamiltonian

$$H \simeq -\boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_\oplus + \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_P \quad , \quad (4)$$

where $\hbar\Omega_\oplus \sim 10^{-19}$ eV and $\hbar\Omega_P \sim 10^{-29}$ eV for the gravitomagnetic field of the Earth. The observation of the extremely small gravitomagnetic Stern-Gerlach force $-\nabla(\boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_P)$ would be of basic interest since it would demonstrate that the spin part of the gravitational acceleration is not universal: particles in different spin states fall differently in the gravitational field of the Earth. This quantum gravitational force has a classical analog in the Mathisson-Papapetrou force.

2. CAN LIGHT STAND STILL?

An important consequence of the general formula (3) for ω' is that ω' can be negative or zero. Since rotation is absolute and there is therefore an absolute distinction between the rotating observers and the inertial observers, negative ω' cannot be excluded. A comment is in order here regarding the formal possibility of reinterpreting radiation with negative ω' as positive frequency radiation propagating in the opposite direction. This would imply that the causal sequence of events would depend upon the motion of the observer; moreover, to keep ω' positive in all cases one has to assume that the observer-dependent causal sequence is also dependent upon the details of the physical process under consideration. This is hardly acceptable phys-

ically and it appears more consistent to simply admit to the possibility of existence of negative energy states according to *noninertial* observers.

Let us next consider the possibility that $\omega' = 0$ for $\omega = m\Omega$ in equation (3); that is, the radiation can stand still for a rotating observer. For instance, in the thought experiment involving the uniformly rotating observer, a positive helicity wave of frequency $\omega = \Omega$ would stand completely still due to a mere rotation of the observer. There is no experimental evidence in support of this circumstance.

It is possible to interpret the classical theory of Lorentz invariance in terms of the relative motion of the inertial particles and the absolute motion of electromagnetic waves. The motion of radiation is absolute in the sense that it is independent of any inertial observer. This basic consequence of Lorentz invariance can be generalized to all observers and raised to the status of a physical principle that would then exclude the possibility that a fundamental radiation field could stand completely still with respect to an accelerated observer [23]. It is important to describe briefly how such a physical principle would fit in with the foundations of the theory of relativity. The idea of *relativity* has to do with the possibility of changing one's standpoint for the purpose of observation. This is kinematically permissible with classical point particles, since an observer can stay at rest with a classical particle. In fact, Minkowski elevated this circumstance to the status of an axiom [24]. On the other hand, Lorentz invariance implies that an inertial observer can never stay at rest with respect to a classical electromagnetic wave. In this sense, the motion of the wave is nonrelative, i.e. *absolute*. These issues are related to an important observation due to Mach [25]: The intrinsic state of a Newtonian point particle, i.e. its mass, is not directly related to its extrinsic state (\mathbf{x}, \mathbf{v}) in absolute space and time. Let us note that this extrinsic state could therefore be shared by any observer, say, that would momentarily stay at rest with the particle. Extending Mach's observation to the case of an electromagnetic wave, we note that the intrinsic properties of a wave, i.e. its frequency, wavelength, amplitude and polarization, are directly related to its extrinsic state in (absolute) time and space $\psi(t, \mathbf{x})$. Our basic assumption then implies that this state of the wave cannot be "shared" by a local observer in the sense that regardless of its motion the observer can never stay at rest with the electromagnetic wave. The duality of classical particles and waves can thus be extended to their motion as well and our basic postulate may be stated in terms of the principle of complementarity of absolute and

relative motion [23].

To implement this physical principle, it is necessary to take a more general view of the relationship between accelerated and inertial observers. The basic laws of physics have been formulated with respect to inertial systems; therefore, accelerated observers must be linked to inertial observers and the hypothesis of locality provides the first step in this process. A more general treatment leads to the nonlocal theory of accelerated observers.

3. ACCELERATED OBSERVERS AND NONLOCALITY

Let us suppose that a pulse of electromagnetic radiation is incident on an accelerated observer in Minkowski spacetime. The observer determines the field amplitude to be $\mathcal{F}_{\alpha\beta}(\tau)$. Let $F'_{\alpha\beta}(\tau) = F_{(\alpha)(\beta)}(\tau)$ be the field amplitude instantaneously measured by the momentarily comoving inertial observers. The accelerated observer passes through a continuous infinity of momentarily comoving inertial observers; therefore, the most general linear relationship between $\mathcal{F}_{\alpha\beta}$ and $F'_{\alpha\beta}$ consistent with causality is

$$\mathcal{F}_{\alpha\beta}(\tau) = F'_{\alpha\beta}(\tau) + \int_{\tau_0}^{\tau} \mathcal{K}_{\alpha\beta}{}^{\gamma\delta}(\tau, \tau') F'_{\gamma\delta}(\tau') d\tau' \quad , \quad (5)$$

where τ_0 is the initial instant of accelerated motion. It is expected that the kernel \mathcal{K} would be directly related to the acceleration of the observer and so the nonlocal part would in general be of order λ/\mathcal{L} , so that the hypothesis of locality would be recovered in the eikonal limit $\lambda/\mathcal{L} \rightarrow 0$. It is a general property of the Volterra system (5) that for continuous functions there is a unique relationship between $\mathcal{F}_{\alpha\beta}$ and $F'_{\mu\nu}$. The acceleration is usually assumed to be turned on at some initial time and then turned off after a finite duration of proper time in order to avoid unphysical situations such as the infinite energy required to keep a hyperbolic observer of uniform acceleration g in motion for all time. Once the acceleration is turned off, the observer measures a constant additional field that is the residue of past acceleration; in fact, such a constant memory field is always allowed since Maxwell's equations are linear partial differential equations and any solution is determined up to a constant field. For a laboratory device, the residue is canceled once the device is reset.

To determine the kernel \mathcal{K} , it is natural to assume that \mathcal{K} is a convolution-type kernel depending only upon $\tau - \tau'$. We have seen that it is possible for

$F'_{\alpha\beta}$ to become a constant under certain circumstances. According to the principle developed in the previous section, the measured field $\mathcal{F}_{\alpha\beta}$ should never become a constant for an incident radiation field $F_{\mu\nu}$. To implement this idea, we recall that for inertial observers the Doppler effect implies that $\omega' = 0$ only when ω vanishes so that once the radiation field is constant according to one observer, then it must be constant according to all observers. Generalizing this circumstance to arbitrary accelerated observers, we conclude that if $\mathcal{F}_{\alpha\beta}$ is constant, then $F_{\mu\nu}$ must be constant. Following this line of thought, we write equation (2) as $F' = \Lambda F$ and equation (5) as

$$\mathcal{F}(\tau) = F'(\tau) + \int_{\tau_0}^{\tau} \mathcal{K}(\tau - \tau') F'(\tau') d\tau' \quad , \quad (6)$$

and we find the following integral equation for the kernel \mathcal{K} in terms of $\Lambda(\tau)$,

$$\Lambda(\tau) + \int_{\tau_0}^{\tau} \mathcal{K}(\tau - \tau') \Lambda(\tau') d\tau' = \Lambda(\tau_0) \quad . \quad (7)$$

This equation can be solved in terms of the *resolvent kernel* \mathcal{R} ,

$$\Lambda(\tau_0) + \int_{\tau_0}^{\tau} \mathcal{R}(\tau - \tau') \Lambda(\tau_0) d\tau' = \Lambda(\tau) \quad , \quad (8)$$

which implies that

$$\mathcal{R}(\theta) = \frac{d\Lambda(\tau_0 + \theta)}{d\theta} \Lambda^{-1}(\tau_0) \quad . \quad (9)$$

Thus the resolvent kernel is proportional to the acceleration of the observer. The kernel \mathcal{K} can be expressed in general in terms of an infinite series in the resolvent kernel \mathcal{R} ; equivalently, \mathcal{K} can be determined via \mathcal{R} by means of Laplace transforms. If the observer is inertial, $\mathcal{R} = 0$ and hence $\mathcal{K} = 0$ and the standard theory of Lorentz invariance is recovered.

Our treatment (6) - (9) is valid for any field F , though we have considered electromagnetism for the sake of concreteness. Moreover, the kernel \mathcal{K} is in general nonzero except for constant Λ which is the case for a scalar (or a pseudoscalar) field. Thus a scalar field is local according to this theory.

Hence a fundamental scalar field can stand completely still with respect to an accelerated observer. This is contrary to the principle formulated in the previous section; therefore, a basic scalar field is excluded by the nonlocal theory [26]. It thus follows from the nonlocal theory that any scalar field found in nature must be a composite.

It is important to subject the nonlocal theory to direct experimental test. The current status of this problem is considered in the next section.

4. DISCUSSION

In the thought experiment employed in section 1 to illustrate spin-rotation coupling for radiation received by a uniformly rotating observer, the nonlocal contribution to the amplitude of the measured radiation constitutes a direct test of the nonlocal theory. It turns out that for the experimentally viable case of $\omega \gg \Omega$, for example, there is a relative increase (decrease) in the measured amplitude by Ω/ω as a consequence of nonlocality for incident RCP (LCP) waves [26]. In the JWKB limit, however, $\Omega/\omega = \lambda/\mathcal{L} \rightarrow 0$ and the result of the standard theory is recovered, as expected. This effect may be searched for – in the rotating frame – in order to test the nonlocal theory; however, the influence of rotation on the measuring device must then be taken into account. The problems associated with the standard electrodynamics of accelerated *media* are quite nontrivial. The assumptions that are usually employed in the design of electrical equipment have been reviewed by Van Bladel [27]. It therefore appears that the proposed search for nonlocality of order $\Omega/\omega \sim 10^{-8}$ in the rotating system would have to involve rather delicate experiments [26]. To circumvent such problems, Shoemaker [28] has proposed a test of nonlocal electrodynamics in the laboratory (i.e. inertial) frame.

In view of the above remarks, let us therefore consider the problem of testing the nonlocal theory in a different context: instead of an observer in a rotating system, let us imagine an electron in a Rydberg state of high angular momentum. In the correspondence limit, the interaction of the incident radiation field with the electron would be expected to reflect the nonlocal effect under consideration here. It is therefore interesting to search for evidence in connection with the nonlocal theory in the standard quantum treatment of atomic transitions such as the photoelectric effect. The polarization dependence of the photoelectric effect has recently received attention in connection

with the angular distribution of the electrons that are ejected as a result of the interaction of atoms with x-rays from synchrotron light sources [29]. To test the nonlocal theory, it appears necessary to study the explicit form of the total cross section for the photo-effect in the case of incident circularly polarized radiation. In this regard, it is interesting to note that the *impulse approximation* of quantum scattering theory [30] is physically equivalent to the hypothesis of locality. Therefore, it is in general necessary to go beyond the impulse approximation and include the influence of the Coulomb interaction explicitly. These issues require further investigation.

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