

# ON DETECTING THE GRAVITOMAGNETIC FIELD OF THE EARTH BY MEANS OF ORBITING CLOCKS

H.I.M. Lichtenegger<sup>1</sup>, F. Gronwald<sup>2</sup> and B. Mashhoon<sup>3</sup>

<sup>1</sup>*Institut für Weltraumforschung, Österreichische Akademie der Wissenschaften, A-8010 Graz, Austria,*

<sup>2</sup>*Institut für Theoretische Physik, Universität zu Köln, D-50923 Köln, Germany,*

<sup>3</sup>*Department of Physics and Astronomy, University of Missouri, Columbia, Missouri 65211, USA*

## ABSTRACT

Based on the recent finding that the difference in proper time of two clocks in prograde and retrograde equatorial orbits about the Earth is of the order  $\sim 10^{-7}$  s per revolution, the possibility of detecting the terrestrial gravitomagnetic field by means of clocks carried by satellites is discussed. A mission taking advantage of this influence of the rotating Earth on the proper time is outlined and the conceptual difficulties are briefly examined.

## INTRODUCTION

General relativity was designed to be a field theory formally similar to electrodynamics, yet some basic differences remain. However, in the case of low velocities and weak fields, the formal analogy between gravity and electromagnetism gives rise to a number of similar phenomena, known as gravitoelectromagnetism. Due to the weakness of the gravitomagnetic effects, however, their existence has yet to be verified.

The proof whether rotating masses do indeed modify the gravitational field is important, both from a physical as well as a philosophical point of view, since it is intimately related to our view of space as being either relational or absolute. Hitherto discussed solely on a philosophical level, Newton raised the problem of the status of space on a physical basis by ascribing to absolute space a physical reality which becomes apparent via inertial forces. By criticising Newton's concept of absolute space, Mach stated that dynamics should not contain absolute elements since in his positivistic view observable effects should have observable causes. Because inertial forces are observed only to occur in reference frames accelerated with respect to the fixed stars, for Mach it was therefore obvious to suppose that the distribution of matter in the universe determines the local inertial frames and the reliance on absolute space becomes superfluous (Mach's principle). Although it is still a matter of debate to what extent general relativity complies with Mach's principle (see e.g. Mashhoon, 1988, and Barbour and Pfister, 1995), it is generally believed that certain gravitomagnetic effects are the most direct manifestations of Machian ideas in general relativity (Mashhoon, 1993). Hence the study of these phenomena can help to clarify one of the most puzzling questions in physics: Where does inertia come from?

Several space missions have been proposed to measure gravitomagnetism directly, among them Gravity Probe-B, LAGEOS III and the Superconducting Gravity Gradiometer Mission (see e.g. Ciufolini and Wheeler, 1995). The Gravity Probe-B satellite will carry four superconducting spherical quartz gyroscopes

to measure the precession of the gyroscopes relative to the distant stars. LAGEOS III could be launched in an orbit whose inclination is supplementary to that of LAGEOS I and to measure the precession of the line of nodes of these two satellites. The Superconducting Gravity Gradiometer Mission envisages orbiting an array of three mutually orthogonal superconducting gravity gradiometers around the Earth to measure directly the contribution of the gravitomagnetic field to the tidal gravitational force.

In contrast to the above proposals, in this paper we consider the possibility to detect gravitomagnetism by means of clocks, i.e. by measuring the difference in proper time displayed by two clocks orbiting in pro- and retrograde direction around the Earth.

## THE GRAVITOMAGNETIC CLOCK EFFECT

The gravitational field outside a rotating body of mass  $M$  and angular momentum  $J$  is given by the Kerr metric

$$ds^2 = \left(1 - \frac{R_s r}{\rho^2}\right) c^2 dt^2 + \frac{2R_s r}{\rho^2} a \sin^2 \theta c dt d\phi - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2 - \sin^2 \theta \left[ r^2 + a^2 + \frac{R_s r}{\rho^2} a^2 \sin^2 \theta \right] d\phi^2, \quad (1)$$

with

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - R_s r + a^2$$

and

$$a = \frac{J}{cM}, \quad R_s = \frac{2GM}{c^2}.$$

Restricting ourselves to a circular equatorial orbit, one of the geodesic equations of Eq. (1) becomes

$$\frac{d\phi}{dt} \equiv \Omega = \Omega_0 \left[ \pm 1 - \frac{a}{c} \Omega_0 + \mathcal{O}\left(\frac{a^2}{c^2}\right) \right] \cong \pm \Omega_0 + \Omega_{\text{LT}}, \quad (2)$$

where the gravitomagnetic correction in the equatorial plane to the Kepler period  $\Omega_0 = (GM/r^3)^{1/2}$  (as determined by static observers at infinity) is just the Lense-Thirring precession  $\Omega_{\text{LT}} = -GJ/c^2 r^3$ . Now we consider two clocks moving along pro- and retrograde circular equatorial orbits, respectively, about the Earth. According to Eq. (2), the difference in proper time for the two clocks after some fixed coordinate time  $t$ , say, one Kepler period  $t = T_0 = 2\pi/\Omega_0$  becomes ( $\tau_+$  and  $\tau_-$  denote the proper time along the pro- and retrograde orbits, respectively)

$$(\tau_+ - \tau_-)_{t=T_0} \cong 12\pi \frac{GJ}{c^4 r} \approx 3 \times 10^{-16} \text{ s}, \quad (3)$$

where we have inserted the values  $M = 6 \times 10^{24}$  kg,  $J = 6 \times 10^{33}$  kg m<sup>2</sup> s<sup>-1</sup> and  $r = 7000$  km. This is a very tiny effect which, even apart from all perturbations, is not detectable with today's atomic clock technology and which is probably the reason why clocks are barely considered as proper means to measure the gravitomagnetic contribution of the Earth's rotation. However, as recently pointed out by Cohen and Mashhoon (1993), this time difference is considerably enlarged when calculating the difference in proper time of two counter-orbiting clocks for some fixed angular interval rather than for some fixed time interval. Integration over, say,  $2\pi$  yields (see Gronwald *et al.*, 1997)

$$(\tau_+ - \tau_-)_{\phi=2\pi} \cong 4\pi \frac{J}{Mc^2} \approx 1 \times 10^{-7} \text{ s} \quad (4)$$

and it is interesting to note that this difference is independent of both  $G$  and  $r$ . The relatively large value in Eq. (4) could underlie an experiment to directly measure the terrestrial gravitomagnetic field (called Gravity Probe-C (GP-C) in Gronwald *et al.* (1997)). The above analysis can be extended to circular orbits with finite inclination (Theiss, 1985), showing that the time difference decreases with increasing inclination; for a polar orbit the effect vanishes as it is expected because of symmetry.

## ERROR SOURCES

Although the value in Eq. (4) is several orders of magnitude larger than that in Eq. (3) and well within the measurement capabilities of modern atomic clocks ( $(\tau_+ - \tau_-)/T_0 \sim 10^{-11}$  for a near-Earth orbit), it is not a simple task to separate the time difference induced by gravitomagnetism from all other effects resulting likewise in a time shift between the clocks. The main error sources that may affect the experiment can be divided into 4 categories and are listed below:

1. Errors from orbital injection errors
2. Errors from gravitational perturbations
  - Static odd zonal harmonic perturbations
  - Static non-zonal harmonic perturbations
  - Non-linear harmonic perturbations
  - Solid and ocean Earth tides
  - Sun, Moon and planetary tidal accelerations
  - Relativistic perturbations
3. Errors from non-gravitational perturbations
  - Direct solar radiation pressure
  - Terrestrial radiation pressure (albedo, infrared radiation)
  - Satellite eclipses
  - Anisotropic thermal radiation (Yarkovsky effect)
  - Thermal thrust (Yarkovsky-Schach effect)
  - Poynting-Robertson effect
  - Atmospheric drag
  - Drag from interplanetary dust
  - Earth's plasma environment (charged particle drag)
  - Earth's magnetic field (magnetic despin)
4. Errors from uncertainties in the determination of the orbital parameters

After launch, the orbital elements have to be adjusted by inflight corrections as close as possible to the values desired, i.e. to zero eccentricity and inclination in order to maximize the effect. Further, an accurate tracking of the satellites is required since uncertainties in the radial and azimuthal location of the clocks readily cover the gravitomagnetic effect. A simple error analysis yields  $\Delta r/r \sim \Delta\phi/\phi \sim \Omega_0 J/Mc^2 \sim 10^{-11}$ , i.e. one must keep track of the satellites with an accuracy of  $\Delta r \leq 0.1$  mm and  $\Delta\phi \leq 10^{-2}$  mas per revolution for orbits with  $\sim 7000$  km altitude (Gronwald *et al.*, 1997). Equivalently, accelerations down to  $\Delta r/T_0^2 \sim 10^{-11}$  g should be taken into account.

Gravitational perturbations due to the nonsphericity of the Earth will at least partially cancel out for two satellites following opposite orbits. While current models of the static and tidal Earth's gravitational field induce orbital errors in all three spatial directions of several mm (Ciufolini *et al.*, 1998), the gravitational influence of other celestial bodies on the satellites is essentially due to the Moon ( $10^{-7}$  g) and the Sun ( $10^{-8}$  g) and can be modeled with sufficient accuracy (Gronwald *et al.*, 1997). Finally, accelerations due to relativistic effects other than the relativistic Kerr corrections to the terrestrial field can be neglected (Mashhoon and Theiss, 1982, Ashby and Bertotti, 1984).

Several non-gravitational mechanisms which perturb the orbits of satellites have been modeled in some detail in the course of data analysis of laser-ranged satellites (e.g. Rubincam, 1982). The observed average drag on the LAGEOS satellite is  $\sim 10^{-12}$  m/s<sup>2</sup> and is due to Yarkovsky thermal, neutral and charged particle drag (Rubincam, 1990); thermal thrust induces a secular variation in inclination of LAGEOS of the order of  $\sim 1$  mas/yr (Farinella *et al.*, 1990). Although the perturbations will depend on inclination and are also controlled by the eclipses (the LAGEOS satellites are on nearly polar orbits), non-gravitational perturbations on LAGEOS-type satellites (i.e. spherically symmetric, small ratio of cross-sectional area to mass satellites) along equatorial orbits are not expected to differ significantly from those in polar orbits. Based on the experience of LAGEOS's orbital variations, it might thus be possible to correct for the non-gravitational perturbations to the desired order without the use of drag-free satellites; however, a conclusive answer cannot be given without a thorough analysis of these effects.

## SUMMARY AND CONCLUSION

Based on the fact that the difference in proper time of two counter-orbiting clocks along circular equatorial orbits after completing one revolution each is several orders of magnitude larger than the time difference after one Kepler period, a mission is proposed to directly measure the gravitomagnetic field of the Earth via spaceborne atomic clocks. While many of the effects causing deviations from an ideal circular Keplerian orbit can be incorporated into the analysis, the accurate tracking of the satellites and the correct modeling of the dynamical part of the Earth gravity field seem to be the most difficult tasks for a Gravity Probe-C mission. However, the reported clock effect is cumulative and after 100-1000 revolutions it should be detectable even with today's tracking techniques. Also, the next generation of static and time-dependent terrestrial gravity field models is expected to possess a precision sufficient for GP-C. Hence we consider GP-C as an alternative way to detect gravitomagnetism, which ideally completes missions like Gravity Probe-B and appears technically less demanding.

## REFERENCES

- Ashby, N. and B. Bertotti, Relativistic Perturbations of an Earth Satellite, *Phys. Rev. Lett.*, **52**, 485–488 (1984).
- Barbour, J. and H. Pfister, editors, *Mach's Principle: From Newton's Bucket to Quantum Gravity*, volume 6 of *Einstein Studies*, Birkhäuser (1995).
- Ciufolini, I. and J.A. Wheeler, *Gravitation and Inertia*, Princeton Series in Physics, Princeton (1995).
- Ciufolini, I., E. Pavlis, F. Chieppa, E. Fernandes-Vieira, and J. Perez-Mercader, Test of General Relativity and Measurement of the Lense-Thirring Effect with Two Earth Satellites, *Science*, **279**, 2100–2103 (1998).
- Cohen, J.M. and B. Mashhoon, Standard Clocks, Interferometry, and Gravitomagnetism, *Phys. Lett. A*, **181**, 353–358 (1993).
- Farinella, P., A.M. Nobili, F. Barlier, and F. Mignard, Effects of Thermal Thrust on the Node and Inclination of Lageos *Astron. Astrophys.*, **234**, 546–554 (1990).
- Gronwald, F., E. Gruber, H. Lichtenegger, and R.A. Puntigam, Gravity Probe Clock - Probing the Gravitomagnetic Field of the Earth by Means of a Clock Experiment, in *Fundamental Physics in Space*, pp. 29–37, ESA SP-420 (1997).
- Mashhoon, B. and D.S. Theiss, Relativistic Tidal Forces and the Possibility of Measuring Them, *Phys. Rev. Lett.*, **49**, 1542–1545 (1982).
- Mashhoon, B., Complementarity of Absolute and Relative Motion, *Phys. Lett. A*, **126**, 393–399 (1988).
- Mashhoon, B., On the Relativity of Rotation, in *Directions in General Relativity*, vol. 2, edited by B. L. Hu and T. A. Jacobson, pp. 182–194, Cambridge University Press, Cambridge (1993).
- Rubincam, D.P., On the Secular Decrease in the Semimajor Axis of LAGEOS's Orbit, *Celest. Mech.*, **26**, 361–382 (1982).
- Rubincam, D.P., Drag on the LAGEOS Satellite, *J. Geophys. Res.*, **95**, 4881–4886 (1990).
- Theiss, D.S., A General Relativistic Effect of a Rotating Spherical Mass and the Possibility of Measuring it in a Space Experiment, *Phys. Lett. A*, **109**, 19–22 (1985).