Mashhoon Replies: Several years ago, I predicted a novel effect due to spin-rotation coupling in connection with neutron interferometry experiments in a rotating frame of Ref. [1]. Recently Anandan [2] has discussed this effect using the same Schrödinger equation as I employed and has criticized certain aspects of my paper [1]. I show in the following, however, that Anandan's criticisms are incorrect.

The theoretical basis for interpreting the results of measurements by accelerated observers is the hypothesis of locality, namely, the instantaneous equivalence of an accelerated observer with a comoving inertial observer. This is valid for classical point particles and rays; for classical waves, the hypothesis needs to be extended [3]. The extended hypothesis of locality amounts to a physical description of wave phenomena in accordance with the principle of minimal coupling. On this basis Hehl and Ni [4] have discussed the behavior of a Dirac particle in a rotating frame and have provided an independent and rigorous verification of my main result [1]. A heuristic discussion of the hypothesis of locality and its limitations provided the impetus for my work. Imagine a wave function $\Psi(t,\mathbf{x})$ representing the nonrelativistic state of a spinless system according to static inertial observers. A uniformly rotating observer with velocity $\mathbf{V}=\mathbf{\Omega}\times\mathbf{x}$ is instantaneously equivalent to a comoving inertial observer; therefore, the wave function Ψ' for the rotating observer is related at each instant to Ψ by a (Galilean) *boost*. Thus $\Psi'=U\Psi$, where $U=\exp(it\mathbf{V}\cdot\mathbf{P})=\exp(it\mathbf{\Omega}\cdot\mathbf{L})$. The wave function in the rotating system is in effect related to that in the inertial system by a *rotation* of angle Ωt. This can be generalized in a natural way to include systems with spin. The generator for rotation is the *total* angular momentum operator; thus for a system with spin $U\rightarrow\exp(it\mathbf{\Omega}\cdot\mathbf{J})$. The treatment of the Schrödinger equation under a Galilean boost presented here differs from the standard approach due to Bargmann [5] and used by Anandan [2]. In my approach [6], $H'=H-\mathbf{V}\cdot\mathbf{P}$ under a Galilean boost in agreement with the nonrelativistic limit of the relativistic transformation formula for energy while in the standard approach the Schrödinger equation is assumed to be form invariant. The physical results, however, must be the same. This can be seen from the fact that the classical Lagrangians in the two cases are related by $\mathcal{L}'_M=\mathcal{L}'_B+dF/dt$, where $F=m\mathbf{x}'\cdot\mathbf{V}+\frac{1}{2}mV^2t$. In quantum theory, the invariant wave function in my approach will be related to Bargmann's by a phase factor of $\exp(iF)$, as can be most clearly seen from the path integral approach to quantum mechanics.

There exist different, but equally valid, ways to compute the phase shift associated with the new effect. The Sagnac effect generally refers to the fringe shift in an interferometer (as in Sagnac's classic experiment) as well as the frequency difference resulting in a beat in an interferometer or resonant cavity (as in a ring laser gyroscope). These possibilities (for either photons or neutrons) can also be realized with regard to the new effect. For the neutron spin states in the first case the energy E' is the same and hence the wavelengths are different, while in the second case E and the wavelengths are the same but E' is different. The latter case can generally be realized in an interferometer by placing the source in the inertial frame; however, no external source is needed for a resonant cavity.

Finally, consider a Dirac wave function $\psi(x)$ according to static inertial observers, and its representation according to uniformly rotating observers $\psi'(x')$; then $\psi'(x')=T\psi(x)$, where T has been determined in this case [7]. Using standard conventions, T is given by a diagonal matrix with elements $(\lambda,\lambda^{-1},\lambda,\lambda^{-1})$, where $\lambda=\exp(i\Omega t/2)$, as expected for a rotation of angle Ωt. Imagine a free positive-energy Dirac particle represented by $\psi=a(\mathbf{p})\times\exp(-iEt+i\mathbf{p}\cdot\mathbf{x})$. It follows from the expansion of a plane wave in terms of spherical waves that $\psi'(x')$ is a superposition of wave functions each with time dependence of the form $\exp(-iE'\tau)$, where $\tau=t/\gamma$ is the proper time and $E'=\gamma(E-M\Omega)$. Here $M=\mu\pm\frac{1}{2}$ is the component of the total angular momentum of the particle about the direction of rotation and μ represents the orbital part, $\mu=0,\pm1,\pm2,\ldots$. Thus E' can be negative; in particular, the frequency of rotation can be so chosen as to make $E'=0$. A vivid illustration of this situation is the possibility that a plane electromagnetic wave can stand still with respect to a rotating observer.

I am grateful to F. W. Hehl for helpful discussions.

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Received 23 January 1992
PACS numbers: 03.65.Bz, 04.20.Cv

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