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GRADIOMETRY AND GRAVITOMAGNETIC FIELD DETECTION

M3775294

BAHRAM MASHHOON
Department of Physics and Astronomy
University of Missouri
Columbia, Missouri 65211

I. GRAVITATIONAL "MAGNETIC" FIELD

Gravitomagnetism was apparently first introduced into physics about 120 years ago when major developments in electrodynamics and the strong similarity between Coulomb's law of electricity and Newton's law of gravity led to the hypothesis that mass current generates a fundamental force of gravitational origin analogous to the magnetic force caused by charge current. Holzmüller (1870) and Tisserand (1872, 1890) showed that this novel interaction led to the precession of planetary perihelia. The ratio of this velocity-dependent force to the Newtonian force of attraction contained only the speed of propagation of gravity as a new parameter. This parameter was taken to be the speed of light. The excess motion of perihelia would disappear if the speed of propagation approached infinity. There were attempts to use this fact to account for the excess perihelion precession of Mercury (Whittaker 1951). However, Einstein's relativistic field theory of gravitation provided a natural explanation for the excess perihelion motion. It is due to a small relativistic correction to the "Newtonian" gravitoelectric field of the Sun. Furthermore, Hans Thirring showed, in 1918, that the rotation of a massive body does indeed generate a gravitational "magnetic" field according to general relativity. The general investigation of the excess motion of planets and the moons due to the gravitomagnetic field is due to Lense and Thirring (1918). The resulting perihelion precession turned out to be much smaller than, and in the opposite sense of, the excess motion of Mercury (Mashhoon *et al.* 1984).

According to general relativity, the rotation of a body leads to the dragging of the local inertial frames. In the weak-field approximation, the dragging frequency can be interpreted, up to a constant proportionality factor, as a gravitational "magnetic" field. There is, as yet, no direct evidence regarding the existence of such a field. This work is concerned with the possibility of detecting the gravitomagnetic field of the Earth by gravity gradiometry.

II. GRAVITY GRADIOMETRY IN GENERAL RELATIVITY

Imagine two neighboring particles falling freely in an external gravitational field characterized by a Newtonian potential ϕ. The relative motion of the particles can be described in Newtonian theory by

$$\frac{d^2\xi^i}{dt^2} + K_{ij}(t)\xi^j = a^i \tag{1}$$

to first order in the relative displacement ξ. Here a is the relative acceleration caused by nongravitational forces and $K_{ij} = \partial^2\phi/\partial x^i \partial x^j$ is the tidal matrix. The tidal matrix is symmetric, and its trace is proportional to the local density of matter with the proportionality constant determined by the Newtonian constant of gravitation. The situation in general relativity is remarkably similar, except that equation (1) holds in the local inertial frame with t replaced by the proper time τ along the path of the particles, and the tidal matrix is given by certain components of the Riemann curvature tensor as measured in the local frame carried along the path (Mashhoon 1977). Specifically, let $R_{\mu\nu\rho\sigma}$ be the spacetime curvature for the exterior field of a rotating mass such as the Earth. In Schwarzschild-like coordinates, the components of the Riemann tensor would contain contributions from the mass M, angular momentum J, and higher multipole moments of the Earth. Let $x^{\circ} = x^{\circ}(\tau)$, and $x^{i} = x^{i}(\tau)$ represent the orbit of a gradiometer freely falling in the Earth's field. An orthogonal parallel-propagated set of three local space-like directions ("gyroscopes"), $\lambda^{\mu}_{(i)}$, is necessary so that the orthonormal tetrad $\lambda^{\mu}_{(a)}$, with $\lambda^{\mu}_{(o)} = dx^{\mu}/d\tau$, could be used to define a local inertial frame. With respect to such a frame, equation (1) holds with Newtonian time replaced by τ and

$$K_{ij}(\tau) = R_{\mu\nu\rho\sigma}\lambda^{\mu}_{(o)}\lambda^{\nu}_{(i)}\lambda^{\rho}_{(o)}\lambda^{\sigma}_{(j)} \qquad (2)$$

The equation of motion with respect to any other local frame can be obtained from equation (1) by means of a transformation,

$$\xi'^{i} = M_{ij}(\tau)\xi^{j} \; . \qquad (3)$$

A gradiometer measures the relative acceleration $d^2\xi'/d\tau^2$, hence, effects of gravity gradients are mixed with terms arising from the deviation of actual gradiometer axes from the local inertial frame.

It is important to note that (K_{ij}) contains, besides the "electric" parts of the field, the Lense-Thirring orbital precession as reflected in the tangent to the worldline $\lambda^{\mu}_{(o)}$, the gravitomagnetic precession and nutation of gyro axes as reflected in $\lambda^{\mu}_{(i)}$, as well as the contribution of the gravitational "magnetic" field to the spacetime curvature. Consider, for instance, a gradiometer on a circular (equatorial or polar) orbit about the Earth (Braginsky and Polnarev 1980, Mashoon *et al* 1985). The components of the tidal matrix are simple when expressed in terms of the local polar coordinate system $(\hat{r}, \hat{\theta}, \hat{\varphi})$, which is essentially the Earth-pointing orientation. The tidal matrix consists of a diagonal Newtonian part of order $\omega_o^2 = GM/r^3 \simeq 10^{-6}\ \mathrm{sec}^{-2}$ for a near-Earth orbit, a diagonal relativistic "electric" part of magnitude $3(GM/c^2 r)\omega_o^2$, which is $\simeq 10^{-9}$ of the Newtonian part, and a "magnetic" part with amplitude of order $6(J\omega_o/c^2 M)\omega_o^2$, which is $\simeq 10^{-10}$ of the Newtonian part. The "magnetic" part is constant and diagonal for an equatorial orbit and off-diagonal for a polar orbit. These off-diagonal elements contain harmonic and mixed terms of frequency ω_o.

To detect the gravitomagnetic field of the Earth, it is therefore necessary to use a highly sensitive gravity gradiometer such as the low-temperature device developed by Paik (1985).

III. PAIK'S SUPERCONDUCTING GRAVITY GRADIOMETER

During the past year, Ho Jung Paik, Clifford Will, and the author have studied- in the context of a wide class of gravity theories (Will 1984) the feasibility of detecting the Earth's gravitomagnetic field using Paik's superconducting gravity gradiometer (Paik *et al.* 1987). The main conclusions of our rather preliminary investigation can be stated as follows.

(1) The gravitomagnetic effect must be separated from local frame effects; this requires that the local frame be defined at the same level of precision as in the Stanford gyro experiment (GP-B). Hence the orientation of the gradiometer must be controlled such that the pointing errors remain below $\sim 10^{-3}$ arc second $\mathrm{Hz}^{-1/2}$ at signal frequency $2\nu_0 \simeq 3.4 \times 10^{-4}$ Hz appropriate for local inertial orientation determined by gyroscopes. This general conclusion is expected to hold even if the orientation of the gradiometer is defined using telescopes. Hence GP-B's superconducting gyros or cryogenic telescopes are essential for such an experiment.

(2) The error due to the internal misalignment of gradiometer axes, *i.e.*, the deviation of the axes of gradiometer from perfect orthogonality, turns out to generate second-order effects for Paik's rigid three-axis gradiometer. The misalignment error must therefore be kept below $\sim 10^{-6}$.

(3) It is possible to separate the gravitomagnetic signal from the Newtonian and post-Newtonian gravitoelectric effects of the mass of the Earth by a signal differencing scheme. Consider, for instance, a Paik gradiometer in polar orbit. In the Earth-pointing orientation, the subtraction of gradiometer outputs in the $(\hat{r} + \hat{\varphi})$ and $(\hat{r} - \hat{\varphi})$ directions, or the $(\hat{\varphi} + \hat{\theta})$ and $(\hat{\varphi} - \hat{\theta})$ directions, would essentially eliminate gravitoelectric effects.

(4) This subtraction is complete if the two sensitive axes of the gradiometer are identical. There is, therefore, a strict scale factor stability requirement of 5×10^{-9} $\mathrm{Hz}^{-1/2}$ at the signal frequency of $2\nu_0$.

On the basis of requirements that follow from our preliminary analysis, it is projected that Paik's superconducting gravity gradiometer can resolve the expected gravitomagnetic signal in 1 year of data collection with a signal-to-noise ratio of 10^2. It thus appears, that the cryogenic, as well as drag-free technology, associated with GP-B can be combined with Paik's work to provide a novel method of detecting the gravitomagnetic field of the Earth.

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DISCUSSION

HELLINGS: What do you use as a reference for the pointing requirement for your gravimetric axes?

MASHHOON: Thus far we have used the local inertial frame as defined by ideal orthogonal gyroscopes. This system is related to the Earth-pointing orientation along a circular orbit by a rotation of frequency w_0. A reference system based on telescopes requires a separate investigation since aberration effects need to be taken into account.

CLAUSER: What have you assumed for the gradiometer sensitivity of Paik's experiment?

MASHHOON: 10^{-5} E $\mathrm{Hz}^{-1/2}$.

NORDVEDT: What do you exactly mean by 'gravitomagnetism has never directly been detected?' I know of quite a few phenomena for which the gravitomagnetic interaction is needed and to explain the observation.

MASHHOON: It is certainly true that information regarding the gravitomagnetic interaction has been obtained from observations within the framework of parameterized post-Newtonian approximation scheme. On the other hand, general relativity predicts that an isolated uniformly rotating mass would cause a dragging of the local inertial frames that is independent of the motion of the observer relative to the rotating mass. That is, the gravitomagnetic field is present even when the observer is at rest with the rotating mass. This is a fundamental proposition that deserves to be tested directly.

CIUFOLINI: It is interesting to observe that the first one to calculate one effect of the gravitomagnetic field, after general relativity was discovered, has been de Sitter in 1916. He calculated the tiny precession of the perihelion of Mercury due to the angular momentum of the Sun an effect, he found, much smaller that the Schwarzschild perihelion precession.

MASHHOON: In his (first) 1916 paper (published in Monthly Notices) on the astronomical consequences of general relativity, de Sitter considered, among other things, the perihelion precession of a planet in an equatorial orbit due to the axial rotation of the Sun. However, the general discussion of the gravitomagnetic field and its consequences for orbital motion is due to Thirring and Lense.

NIETO: A historical comment. It turns out that Maxwell also noted the similarity between Newton's Law and Coulomb's Law as contained in his theory of electromagnetism. In his great treaties on electromagnetism, Maxwell tried to develop a (vector) theory of gravity which would be similar to his electromagnetic theory. However, because he had to change the sign of the energy (like charges or masses had to attract), the system was a run-away. It did not conserve energy. This is an analogue of the non-conservation of energy in "anti-gravity" theories.