Neutron Interferometry in a Rotating Frame of Reference

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The physical basis for the description of phenomena by a rotating observer is investigated. A simple, yet tentative, extension of the hypothesis of locality is used to determine the interference phase shift induced by the rotation of a neutron interferometer. The result consists of the Sagnac term, which is due to the coupling of the orbital angular momentum of the neutron with the rotation of the frame, and a new term which arises from a similar coupling because of the neutron spin. The latter effect is generally smaller than the Sagnac phase shift by the ratio of de Broglie wavelength of the neutron to the dimension of the interferometer. The possibility of detecting the new effect is briefly discussed.

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The standard description of physical phenomena according to accelerated observers is based on a hypothesis of locality which states that an accelerated observer at each instant along its worldline is equivalent to a hypothetical inertial observer at the same event and with the same velocity as the noninertial observer. This assumption forms the basis for the extension of the Poincaré-invariant theory of relativity to general frames of reference as well as gravitational fields.[1,2] The results of measurements performed by the accelerated observer are therefore identical to those of the inertial observer at the same spacetime point. The hypothesis of locality thus restricts the range of *elementary* measurements that can be performed by the accelerated observer to those that are pointwise. That is, the validity of the standard theory is limited to the prediction of the results of basic measurements that an accelerated observer can perform over negligible intervals of time and space.

Consider, for instance, a particle of energy E and momentum $\mathbf{P}$ with respect to an inertial frame F_0 (with coordinates $t,\mathbf{x}$) and a frame F' (with coordinates $t'=t,\mathbf{x}'$) that is related to F_0 by a uniform rotation of angular frequency $\mathbf{\Omega}$. An observer at rest in F' with velocity $\mathbf{v}(t)$ with respect to F_0 measures the energy of the particle. According to the hypothesis of locality, the result is

$$E'=\gamma(E-\mathbf{v}\cdot\mathbf{P}), \tag{1}$$

where $\gamma=(1-\beta^2)^{-1/2}$ and $\beta=v(t)/c$. This equation can be rewritten as

$$E'=\gamma(E-\mathbf{\Omega}\cdot\mathbf{L}), \tag{2}$$

since $\mathbf{v}=\mathbf{\Omega}\times\mathbf{x}$. Here $\mathbf{L}=\mathbf{x}\times\mathbf{P}$ is the orbital angular momentum of the particle with respect to the common origin of F_0 and F'. For a system of relativistic particles interacting via central forces, the Hamiltonians in F' and F_0 are related by

$$H'=H-\mathbf{\Omega}\cdot\mathbf{L}, \tag{3}$$

which is the relativistic generalization of the standard Newtonian formula.[3] This result is consistent with Eq. (2) since H' is the generator of the variation of the system in coordinate time t', while E' refers to the energy measured by the rotating observer with proper time τ related to t' by $d\tau=dt'/\gamma$. It is important to note that Eqs. (1)-(3) hold even when $\mathbf{\Omega}$ is not constant in magnitude or direction. Moreover, the linear and angular momenta are independent of the rotation of the frame of reference. These results for classical particles have counterparts for classical waves as well, but only in the geometric optics limit. The corresponding relations for the frequency in terms of the propagation vector follow from the *invariance of the phase of a ray* in the eikonal approximation.

The hypothesis of locality is thus valid for phenomena involving classical particles and rays since their physical characteristics can be measured at a point in spacetime. However, wave properties such as period and wavelength require extended intervals of time and space, respectively, for their determination. It is therefore necessary to develop a prescription for the measurement of wave characteristics by accelerated observers in such a way that the hypothesis of locality is recovered in the eikonal limit. A natural, though tentative, generalization of the hypothesis of locality can be stated—for the present purposes—as follows[2]: Let the wave function $\psi(t,\mathbf{x})$ be the spacetime representation of the quantum state of a physical system according to static observers in F_0. At any time t, the wave function ψ' according to the uniformly rotating observers is the same—up to a constant phase factor—as the wave function according to inertial observers at rest in a system F_0' which differs from F_0 only by a (passive) rotation of angle Ωt. The rotating frame F' thus passes through an infinite sequence of inertial frames F_0'. It follows that

$$\psi'=\hat{U}\psi, \tag{4}$$

where $\hat{U}$ is a unitary operator given by

$$\hat{U}=\exp(it\mathbf{\Omega}\cdot\hat{\mathbf{J}}/\hbar) \tag{5}$$

up to a constant phase factor and $\hat{\mathbf{J}}$ is the *total* angular momentum operator of the system. The wave function ψ satisfies the Schrödinger equation $\hat{H}\psi = i\hbar\,\partial\psi/\partial t$, so that ψ' satisfies the Schrödinger equation $\hat{H}'\psi' = i\hbar\,\partial\psi'/\partial t'$ in the rotating system, where

$$\hat{H}' = \hat{U}\hat{H}\hat{U}^{-1} - \mathbf{\Omega}\cdot\hat{\mathbf{J}}. \tag{6}$$

The total angular momentum operator is the sum of the orbital and spin contributions, i.e.,

$$\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}}. \tag{7}$$

The Hamiltonian $\hat{H}'$—together with other observables which correspond to "invariants," i.e., their transformation from the inertial frame to the rotating frame can be collectively described via $\hat{O}' = \hat{U}\hat{O}\hat{U}^{-1}$—provides a description of physical phenomena according to uniformly rotating observers. In particular, vector operators such as position $\hat{\mathbf{x}}'$, momentum $\hat{\mathbf{P}}'$, and angular momentum $\hat{\mathbf{J}}'$ represent the uniform rotation of the corresponding vectors in F_0. This can be simply illustrated for the components of an arbitrary vector $\hat{\mathbf{V}}'$ with respect to the *inertial* frame F_0:

$$\hat{V}_1' = \cos(\Omega t)\hat{V}_1 - \sin(\Omega t)\hat{V}_2, \tag{8}$$

$$\hat{V}_2' = \sin(\Omega t)\hat{V}_1 + \cos(\Omega t)\hat{V}_2, \tag{9}$$

and $\hat{V}_3' = \hat{V}_3$, provided the coordinate axes are oriented such that the angular frequency vector $\mathbf{\Omega}$ is along the x_3 axis. Moreover, it can be shown that Eq. (6) holds even when the rotation is nonuniform, $\mathbf{\Omega} \rightarrow \mathbf{\Omega}(t)$, in which case $\hat{U}$ is the product of operators of the form of Eq. (5) over time intervals (obtained from ordered partition of $0 \rightarrow t$) short enough such that $\mathbf{\Omega}$ may be considered uniform over each interval. These results appear to provide a natural quantum-mechanical extension of the classical treatment of rotating observers.

A detailed comparison of Eq. (6) with its classical analog, Eq. (3), reveals, however, the existence of a new effect associated with the coupling of intrinsic spin with rotation and given by the Hamiltonian,

$$\delta\hat{H}_{\mathrm{SR}}' = -\gamma\mathbf{\Omega}(t)\cdot\hat{\mathbf{S}}, \tag{10}$$

where the relativistic factor γ has been introduced to indicate the strength of the "interaction" as determined by the rotating observer. The theory presented here thus leads to a complete quantum-mechanical analog of the classical Larmor theorem[4,5]; furthermore, the spin-rotation coupling has an interesting classical analog in this context. Consider the motion of a particle of mass M, charge q, and magnetic moment μ in a uniform external magnetic field $\mathcal{B}$. To first order in β and the strength of the magnetic field, the Hamiltonian for the particle in the presence of the field is given by

$$H_{\mathcal{B}} = H_0 - \frac{q}{Mc}\mathbf{P}\cdot\mathcal{A} - \boldsymbol{\mu}\cdot\mathcal{B}, \tag{11}$$

where $\mathcal{A} = \frac{1}{2}\mathcal{B}\times\mathbf{x}$ is the vector potential. A generalization of the Larmor theorem consistent with Eq. (6) is obtained if $q\mathcal{B}/2Mc = \mathbf{\Omega}$ is identified with the Larmor frequency and the gyromagnetic ratio for the particle is assumed to be unity, i.e., $\boldsymbol{\mu} = q\mathbf{S}/2Mc$, as expected for a particle with classical "intrinsic" spin.

It is interesting to investigate the observational consequences of the new effect given by Eq. (10) in the context of interferometry with polarized neutrons. Imagine the interference of neutrons in a rotating frame as depicted in Fig. 1. Neutrons from a source are split into identical semicircular beams which are then made to interfere before being detected. Along the counterclockwise (clockwise) path, the neutron spin is assumed to be polarized parallel (antiparallel) to the direction of rotation of the apparatus. The superposition principle implies that the wave function at the detector (D) is given by

$$\psi_{\mathrm{D}}' = \psi_{\mathrm{I}}' + \psi_{\mathrm{II}}', \tag{12}$$

where "I" and "II" refer to the counterclockwise and clockwise neutron paths, respectively, originating from the source (S). *Thermal* neutrons are generally employed in neutron interferometry experiments; therefore, for the purposes of present discussion the quasiclassical approximation holds and the interference phase shift is given by $\delta\phi' = \phi_{\mathrm{I}}' - \phi_{\mathrm{II}}'$, where

$$\hbar\phi' = \int_{\mathrm{S}}^{\mathrm{D}} (\mathbf{P}'\cdot d\mathbf{x}' - H'dt'). \tag{13}$$

The neutron beam is split coherently at the source; ϕ' is the difference between the phase of the neutron wave at the detector at time t_{D}' and the phase at the source at time t_{S}', $t_{\mathrm{D}}' - t_{\mathrm{S}}' = \pi R/v_n$. Here v_n is the neutron group speed in the inertial frame. Therefore, the principal contribution to $\delta\phi'$ is caused by the difference in the eigenvalues of the Hamiltonian of the neutron along the separate paths. It follows from Eqs. (6) and (7) that $\delta\phi'$

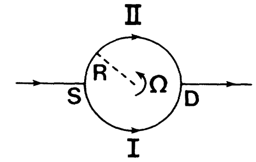

FIG. 1. Interference of initially monochromatic thermal neutrons in a rotating frame of reference. For thermal neutrons of wavelength $\lambda \simeq 1.8$ Å in a typical interferometer of radius $R \simeq 5$ cm rotating rapidly at a rate of 10^2 rounds per second, $\beta = R\Omega/c \simeq 10^{-7}$ so that $\beta/\beta_n \simeq 10^{-2}$. The inequality $\beta < \beta_n$ is assumed throughout this paper. In practice, the incident beam is polarized parallel to the rotation axis and rf coils are used along the clockwise path to invert the polarization of the neutrons immediately after splitting and before recombination.

consists of two parts: $\delta\phi' = \delta\phi'_{\rm Sagnac} + \delta\phi'_{\rm SR}$, given by

$$\delta\phi'_{\rm Sagnac} = \frac{2m}{\hbar}\mathbf{\Omega}\cdot\mathbf{A}\,, \qquad (14)$$

where m is the neutron mass and $A = \pi R^2$ is the area of the interferometer (cf. Fig. 1), and

$$\delta\phi'_{\rm SR} = \pi R\,\Omega/v_n\,. \qquad (15)$$

The Sagnac term has already been observed in experiments at the University of Missouri-Columbia[6] and MIT.[7] It corresponds to the classical orbital-angular-momentum-rotation coupling that is evident in Eq. (2), and is a consequence of the invariance of the phase under coordinate transformations as confirmed by the theoretical studies of Page (prediction in Ref. 8 of $\delta\times\phi'_{\rm Sagnac}$ in the rotating frame) and of Dresden and Yang (prediction in Ref. 9 of $\delta\phi_{\rm Sagnac}$ in the inertial frame). The Sagnac term is thus the invariant phase shift in the eikonal limit. On the contrary, the spin-rotation coupling term, $\delta\phi'_{\rm SR}$, is a wave effect which arises from the intrinsic spin of the neutron ($\hbar/2$). This effect, which comes about because the total angular momentum (i.e., orbital plus *spin*) is the generator for spatial rotations, is smaller than the Sagnac effect by the ratio of the (reduced) de Broglie wavelength of the neutron to the diameter of the interferometer,

$$\delta\phi'_{\rm SR}/\delta\phi'_{\rm Sagnac} = \lambda\!\!\!^{-}/2R\,, \qquad (16)$$

where $\hbar/\lambda\!\!\!^{-} = P \approx m v_n$ is the neutron momentum. The Sagnac effect is proportional to the *area* of the interferometer, whereas the spin-rotation coupling phase shift is proportional to the *length* of the separate neutron paths.

The rotation-induced neutron phase shifts given by Eqs. (14) and (15) are valid for thermal neutrons with $\beta_n = v_n/c \sim 10^{-5}$. In fact, these formulas represent the nonrelativistic limit of $\delta\phi'$. It follows from Eqs. (6) and (13) that the (invariant) relativistic Sagnac phase shift is larger than the low-energy limit, Eq. (14), by a factor of $\gamma_n = (1-\beta_n^2)^{-1/2}$. The relativistic form of the spin-rotation phase shift is more complicated, however, since a relativistic description of the interferometric experiment should include, in general, the effect of "mirrors," etc., that guide the neutrons on the semicircular paths in the interferometer. Thus, in addition to any contribution from the interaction of the neutrons with the mirrors, an effective $\delta\hat{H}_{\rm Thomas} = -\beta^{-1}\beta_n(\gamma_n - 1)\mathbf{\Omega}\cdot\hat{\mathbf{S}}$ must be added to the inertial Hamiltonian to take due account of Thomas precession. This would imply, via Eq. (6), that

$$\Delta\hat{H}'_{\rm SR} = \gamma(\delta\hat{H}_{\rm Thomas} - \mathbf{\Omega}\cdot\hat{\mathbf{S}}) \qquad (17)$$

represents the effective spin-rotation coupling according to the rotating observer. The net kinematic phase shift due to the spin-rotation coupling is thus given by $\pi(\gamma_n - 1 + \beta/\beta_n)$.

In a typical neutron interferometer,[10,11] the influence of the spin-rotation coupling on the neutron phase shift is very small and to separate it from the much larger Sagnac and other (e.g., gravitational) effects would require a very sensitive interferometer. To overcome this difficulty, Werner[12] has proposed an experiment using a new type of interferometer which ideally would be insensitive to Sagnac and gravity[13] effects and could be used, in principle, to search for the new effect. Figure 2 illustrates Werner's "null interferometer" concept. The construction of large-scale interferometers necessary for such experiments is now under active development.

The basic significance of the coupling of spin with rotation is that the measured energy of the neutron is affected not only by its velocity relative to the observer but by the absolute rotational acceleration of the observer as well. This circumstance is a direct consequence of the extended form of the hypothesis of locality for *wave* phenomena, which has been the starting point of this analysis. Thus the treatment of neutron inter-

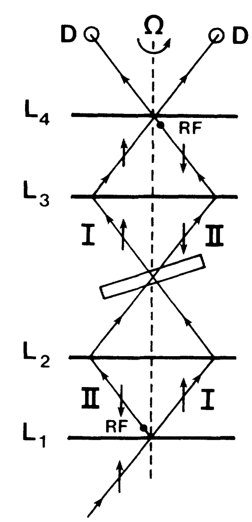

FIG. 2. Werner's proposal for a large-scale null interferometer to search for the influence of neutron spin-rotation coupling on the neutron phase. Thermal neutrons polarized along the direction of rotation of the whole apparatus (interferometer plus the detectors) are coherently split at L_1 by Bragg reflection. The crystal slabs L_1, L_2, L_3, and L_4 are ideally parallel to each other and the separation between L_1 and L_2 is half the separation between L_2 and L_3 and is equal to the L_3-L_4 distance. The beams I and II pass through the phase rotator (i.e., a slab of material that can be adjusted to shift the phase of one beam relative to the other) in the middle of the interferometer and are recombined in L_4. The rf coils are employed to invert the polarization of beam II after splitting and before recombination. The interference of the two beams is reflected in neutron counting rates at the detectors D. The phase shift due to neutron spin-rotation coupling is given by $\Omega l/v_n$, where l is the path length along beam I or II in the interferometer. This phase shift amounts to approximately 1° for polarized neutrons of wavelength $\lambda \simeq 1.8$ Å with a path length of $l \simeq 10^2$ cm in an interferometer rotating at six rounds per second.

ferometry in a rotating frame of reference presented in this paper differs from the standard approach[14]—based implicitly on the hypothesis of locality—by terms that are linear in the wavelength of neutron radiation [cf. Eq. (16)]. It has been argued[2] that this extension of the hypothesis of locality to wave phenomena provides only a first-order correction beyond the eikonal limit. To illustrate this proposition in the present context, imagine a neutron at rest in the inertial frame. According to the accelerated observer, the neutron has energy eigenvalues

$$E'_{\pm} = \gamma(mc^2 \mp \tfrac{1}{2}\hbar\Omega), \qquad (18)$$

where E'_+ (E'_-) represents the energy of the neutron with spin polarized parallel (antiparallel) to the direction of the rotation of the observer. Thus the rotation of the observer splits the ground state of a free neutron such that the level splitting relative to the average (i.e., unsplit) energy is given by the ratio of the Compton wavelength of the neutron to the acceleration length c/Ω. The simple connection between the rest mass of a particle and its energy as measured by an ideal inertial observer must therefore be generalized to include the spin of the particle as well since actual observers in general rotate. For observers on the Earth ($c/\Omega_\oplus \simeq 28$ a.u.), the splitting is negligibly small ($\sim 10^{-29}$). The same conclusion holds even for observers on a rapidly rotating neutron star. Nevertheless, either for a hypothetical spinning particle of very small mass (cf. Ref. 2) or for an observer with extremely high angular frequency this ratio may, in principle, become larger than unity such that E'_+ becomes negative. A basic theoretical resolution of this difficulty is not available at present.

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