TOWARD A FRAMEWORK FOR SYSTEMATICALLY CATEGORIZING FUTURE UAS THREAT SPACE

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ABSTRACT

The development of unmanned aerial vehicles (UAVs) is occurring as fast or faster than any other innovation throughout the course of human history. Building an effective means of defending against threats posed by malicious applications of novel technology is imperative in the current global landscape. Gone are the days where the enemy and the threat it poses are well defined and understood. Defensive technologies have to be modular and able to adapt to a threat technology space which is likely to recycle several times over during the course of a single defense system acquisition cycle. This manuscript wrestles with understanding the unique threat posed by UAVs and related technologies. A thorough taxonomy of the problem is given including projections for how the defining characteristics of the problem are likely to change and grow in the near future. Next, a discussion of the importance of tactics related to the problem of a rapidly changing threat space is provided. A discussion of case studies related to lessons learned from military acquisition programs and pivotal technological innovations in the course of history are given. Multiple measures of success are proposed which are designed to allow for meaningful comparisons and honest evaluations of capabilities. These measures are designed to facilitate discussions by providing a common, and comprehensible language that accounts for the vast complexity of the problem space without getting bogged down by the details. Lastly, predictions for the future threat space comprising UAVs is given.

The contributions of this work are thus threefold. Firstly, an analytic framework is presented including a detailed parameterization of the problem as well as various solution techniques borrowed from a variety of fields. Secondly, measures of success are presented which attempt to compare the effectiveness of various systems by converting to expected values in terms of effective range, or extending the popular concept of kill chain and collapsing effectiveness into units of time. A novel technique for measuring effectiveness is presented whereby effectiveness is composed of various individual probabilities. Probabilities and associated distributions can be combined according to the rules of joint probabilities and distributions and allows performance against a probabilistic threat to be measured succinctly and effectively. The third contribution concerns predictions made with respect to the UAS threat space in the future. These predictions are designed to allow for defensive systems to be developed with a high expected effectiveness against current and future threats. Essentially this work comprises a first attempt toward developing a complete framework related to engagement and mission level modeling of a generic

defensive system (or combination of systems) in the face of current and future threats presented by UAS.

APPROVAL PAGE

The faculty listed below, appointed by the Dean of the School of Graduate Studies, have examined a dissertation titled "Toward a Framework for Systematically Categorizing Future UAS Threat Space," presented by Shawn Malachy Herrington, candidate for the Doctor of Philosophy degree, and hereby certify that in their opinion it is worthy of acceptance.

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CHAPTER 1

INTRODUCTION

1.1 Agile Threats

The development paradigms for military defense systems can be slow especially compared to developing high-tech industries such as the unmanned aerial systems (UAS) sector. While military defense systems development cycles can suffer from a number of historically documented problems, fast-moving technology sections like the UAS industry present particular difficulties in two specific ways.

The first way in which rapid proliferation of a never-before-seen technology can be disrupted can be referred to the *difference of degree*. In this case, the new technology itself can drive modifications to the expectations that would be associated with a traditional product development cycle. A bonafide technological breakthrough or exceptional consumer demand can lead to a much faster than expected maturation of a new technology. This represents the upside to the otherwise risky prospects of bringing a unproven technology to market.

The second way in which such technology can be disruptive can be referred to as the *difference of kind*. In this case, the emerging technology is so disruptive that the entire traditional development paradigm may be cast aside in favor of something new and unknown. Popularly, the development paradigm adopted by Tesla Motors in casting off many aspects of traditional automobile development cycles in favor of a rapid “ship it now, fix it later" mentality is an example of a so-called *difference of kind*. The nature of the *difference of kind* means that products can arrive to the market which were conventionally thought to be improbable or impossible prior to the shift in understanding. This type of paradigm shift in development cycles can be even more disruptive than in the case of *difference of degree* because it can create and/or eliminate objectives or even entire industries almost overnight.

Even under the most ideal circumstances, the development of military defense technologies resembles the arcade game "whack-a-mole" wherein the player attempts to strike a mechanical pop-up mole doll with a mallet only to have more moles continually appear until eventually the player is overwhelmed. The term "whack-a-mole" is often used colloquially to refer to any situation where enacted solutions to a problem prove to be superficial and result in a temporary, if any improvement [1]. Such a description is applicable to the process that inevitably ensues when long-term development cycles are enacted in response to current threat space only to have such systems finally come online long after the original threat has been replaced with new and better threats which may or may not be related to the original planned threat in any way. In the worst case scenario, by the time the development cycle is complete, there are no longer any moles left to defend against.

Consider next solutions to the quandary which has been discussed. In the first place, consider if there is a way to slow the rapid maturation cycles associated with high-technology consumer goods. The answer is obviously "no" for a variety of reasons. If barriers to military or government intervention in the dealings of private business were to be overcome such that the maker of a new innovative product could be asked to stop development or denied the right to sell their new technology, in many cases, another supplier would pop up to provide a similar product or service. Additionally, consider the benefit to society at large presented by many UAS use cases. Advocating for obstructing against these undeniable benefits in support of the "common good" would be an exceedingly difficult case to make.

The next best solution is to revolutionize the maturity cycle for defensive technologies. Unfortunately, part of what makes many defense systems effective is their reliability, robustness, and care in design. These traits are not typically associated with a rapid "ship it now, fix it later" type of mentality. Disrupting the development cycle for military defense technology is certainly an option, and there are improvements to be made; however, there is a fundamental justification for these cycles always being longer in nature compared to the cycles for the technology they are designed to protect against. For "the bad guys" to win, they only have to be successful once. For "the good guys" to be successful, they have to win every single engagement.

1.2 Fortune Telling

The next question then is how can defense systems be prepared for future technologies considering the development of defensive technologies cannot be sped up and that threat technology development cannot be slowed down? Ideally, the military development cycle would leap-frog the threat development cycle so as to allow for defense systems to be developed with the future threat space in mind. This can be accomplished in a number of ways which are presented here in order of increasing predictive power and simultaneously decreasing distance into the future.

The first method involves pure prediction disparagingly referred to here as "prophesy". Generally, military defense systems are expensive and important and thus development of such systems cannot begin in earnest without a reasonably certain picture of the threat space likely to be encountered. Predicting the future is more within the realm of science fiction authors than engineers and scientists. While it may be easy to foresee robots or dystopian futures in a general sense within the context of popular literature (and many such "predictions" may come true), there is a large gulf between the musings of science fiction writers and the realm of so-called actionable intelligence.

The second method of making educated guesses, is a little closer to the present, albeit with the cost of looking a shorter distance into the future compared to pure prediction. Consider trade studies, pattern matching, and brainstorming by subject matter experts all to fall within the realm of educated guess making. Educated guesses may be accomplished with reasonable certainty because they are made by persons in the best position to understand the near-term and mid-term outlook for a particular field. As an illustrative example consider that experts in aircraft design were writing academic papers about UAVs years before UAVs were widely commercially available. Such predictions would be useful for defense system planning even if the authors of such predictions failed to fully grasp the scope of the impact eventually caused by a particular technology.

The final way that the blue team can endeavor to anticipate the future technology needs is adaptation. Adaptation presents a two-pronged set of priorities. Firstly, modular and scaleable technology solutions should be universally preferred due to the possibility that such solutions may be more able to adapt to previously unknown threat spaces. Secondly, resources should be dedicated to studying ways in which existing technology can be adapted in novel ways to extend the lifetime and effectiveness of existing defense systems.

The next question to consider is related to the time scale differences between the different development paradigms. For example, the military defense system development paradigm has already been identified as long in duration compared to many other paradigms (especially for high-tech products and emerging industries). The longest prediction scale will be referred to as "epoch" time scale prediction. Such predictions are again the purview of science fiction writers. While predictions about the emergence of future robot overlords within the lifetime of the author's grandchildren may likely be highly "certain" such predictions are unlikely to be made with any meaningful level of detail. These predictions, while interesting, do not represent information which is useful during the decision making process.

The next time scale to be considered is predictions made on the scale of a working human life. Many subject matter experts are capable of providing a reasonable picture of the expected trajectory within the given window of time. Paradigm shifts within an industry may throw off such predictions, but generally the occurrence of paradigm shifts themselves are relatively predictable to persons with the requisite knowledge. Thus, predictions of this nature are expected to be relatively certain within an actionable time window; although, the details of such predictions can be problematic especially in the case of disruptive changes within an industry.

The next longest time scale to be considered is the military defense system development process. Developments accomplished on such a time scale account for highly certain emerging technologies identified during the design process but may fail to fully understand or account for the implications of the latest developments. In some cases, the late-breaking and unanticipated enhancements to existing threat spaces can result in substantial changes to the performance of defensive systems.

Finally, the smallest time scale to be considered is the high-technology development paradigm duration. Within this time scale, products which seem to be the result of science fiction seem to appear without warning. Technologies can go from academic curiosity to mass-market viability within as little as $5\,\mathrm{yr}$. Products being developed at this kind of speed are historically incredibly difficult to predict.

To formalize the concept of tactical “whack-a-mole” consider the typical development paradigm. The current threat space is evaluated, then modest predictions about the future are conducted, the developed system is adjusted to reflect the encountered threat space and eventually the process begins anew. “Evaluate/predict/adjust/repeat” is no better than “whack-a-mole”. In what should be considered more than just a turn of phrase, consider the implications of changing the “predict” step to the “anticipate” step. When anticipating the threat space, highly certain predictions are made resulting in detailed and actionable information. And all of this is accomplished before any development has begun in earnest. Prior to the selection of defense technologies, the near-term and mid-term future threat space should be understood with as much certainty as possible. Only once the future threat space has been thoroughly “anticipated” can the design and development process begin in earnest. The result of adopting such a paradigm is defensive technology which is better prepared for the threat space when that technology finally comes online.

1.3 Know Yourself

Additionally, a modification to existing defensive or military development paradigms rather than a proposal to anticipate future threat spaces is proposed. Engaging in a tough and honest investigation into the vulnerabilities of specific technologies (past, present and future) and understanding how an “optimal” enemy can be expected to exploit weaknesses can lead to better planning decisions early in the design process. Such investigations may identify needs for future defense systems; however, such investigations can ostensibly lead to an enhanced understanding of ways in which defensive technologies can be extended to new situations.

Consider a given problem space. If a specific and detailed taxonomy of the problem space can be developed, then precise discussions and possibly accurate predictions can be made based on the common language of the problem space. Asking a specific question about a particular characteristic such as “how much faster are enemy UAS likely to get compared to friendly UAS within the next 3 years?” may be substantially easier to answer with a good amount of certainty compared to generic questions like “how are we going to win”.

A thorough taxonomy also allows for comparative analysis to be conducted with respect to various input parameters. Consider a hypothetical scenario where blue UAS defeats all red UAS. If the red-team UAS maximum speed increases, answering how many more (slow) blue-team UAS are needed to ensure the same outcome (blue victory) represents a trade-off analysis. This type of comparison is important for its value in informal discussions and the ability to understand and grasp the problem in a tangible way.

Consider the worst-case-scenario where the development of defense systems has resulted in a system coming online which is not a good match to the current threat space. If careful planning has been undertaken during the development of that system then a road map already exists for expanding the capabilities in a way that helps to better match the reality of the threat space. Additionally, planning for the future throughout the development cycle allows decisions to be made based on which technologies have the best chance of meeting future threats or being adapted. A technology which is the most appropriate for the current threat space may be rejected in favor of a technology which offers imperfect coverage of the current threat but would be more easily adapted for future threats.

1.4 Contributions

The major contributions of this work are threefold and are detailed below. To the author's knowledge, no prior work has presented a complete framework for the consideration of the totality of the scenario comprising defending a region from the unique threat comprised of UAS as is presented in the manuscript to follow. The first contribution of this work includes the analytic framework used to discuss scenarios which is comprised of all the basic characteristics forming the paramaterization of the scenarios as well as various solution techniques borrowed variously from the fields of game theory, differential games, and pursuit evasion puzzles. The second contribution comprises proposed measures of effectiveness including relative strengths and weaknesses of methods related to units of length and units of time as well as a presentation of the novel method based on joint probability distributions formed by considering parameters which are not conventionally thought of as probabilistic. The third contribution consists of predictions of how the threat space is likely to evolve in the near-term and further into the future. General predictions are presented as well as sensitivity of the presented analytic framework with respect to areas of potential weakness as well as areas of potential strength.

1.5 Thesis Organization

This thesis is organized as follows. First a comprehensive literature review is presented. This first includes a review of forecasting and prediction methodology. Next a review of game theory, differential games, and pursuit-evasion games is presented. Historical case studies in the development of defense systems is next presented followed by several case studies related to high-profile defense acquisition programs. These case studies are presented to help understand the time required for development of defensive systems as well as to bring awareness to the reasons why programs fail specifically with respect to inadequate threat analysis of the threat space prior to endeavoring to develop requirements. Finally, sections on UAV technology development are presented including case studies on UAV performance as well as a review of literature on UAV futures.

Next, methods for developing an analytic framework for the modeling and simulation of generic defense scenarios versus a diverse threat are presented. This includes an introduction to game theory including the framing of the problem as a static game of complete information. Next, differential game techniques are applied to study various scenarios analytically with the goal of gaining insight into scenario sensitivities. A detailed taxonomy of the generic problem setup being considered is presented along with various numerical studies which are designed to shed light on specific relationships of interest.

Next, a chapter is dedicated to proposing and evaluating various models for determining the absolute and relative success for various scenarios. These include dimensional reduction techniques which result in comparisons being made in terms of duration (kill-chain) and distance (effective range). Additionally, a more generic framework is proposed whereby the performance of a given system is considered to be a joint probability distribution formed by considering its various components probabilistically.

Finally, predictions are made with respect to the near-term trajectory of threats composed of UAVs. This represents references to various futures documents as presented earlier in the manuscript as well as the synthesis of new information and predictions based on the subject matter expertise and opinion of the author. The prospective future events are viewed through the lens of the aforementioned analytic framework and measures of success.

CHAPTER 2

LITERATURE REVIEW

2.1 State of the Art in Prediction and Forecasting

This section has been a challenge because the search terms for related literature are either far too broad or far too specific. It took a considerable amount of time and effort to hone in on a way of approaching the problem that allowed room for related work to exist without opening up a combinatorial can-of-worms in terms of papers to review. A work by Kirkwood was specifically helpful in redefining the problem (and the key words) to be more about “strategic decision making” (with or without uncertainty) instead of making “predictions” [2]. Additionally, Kirkwood makes several resources available for a course taught from the cited book which have been specifically reviewed in the following section.

In the article denoted in [3, 4] the authors endeavor to understand the value of strategic decision making in the context of studying articles in a variety of journals to get an idea of how the topic is applied across multiple disciplines. Interestingly, of the 86 papers which fit the selection criteria for the study, only 8 are specifically related to military operations research. Thus, yet again, the problem of studying and preparing for a future threat space is a multidisciplinary one. Whether because the relevant military research is proprietary or because the problem space itself is so vast, a successful prognosticator looks to vast and varied subject areas in order to best prepare for the unknown threats of the future.

As can be expected, there is no step-by-step guide for determining how best to synthesize the available uncertain information (collectively referred to as "weak signals" [5]) in order to optimize outcomes in the future. Specifically the problem considered in this manuscript is related to defensive and offensive systems as related to UAV technology; however, limiting the historical study to UAVs specifically is not advisable because the technology is too new to have deep history. Focusing on the evolution of military threat space leaves a fair amount of literature to be studied but the timeliness of that information is limited to what can be released publicly. The most current estimates of the future threat environment by various governments around the world, is by its nature, proprietary and sensitive.

Looking again to the field of economics and the broader field of strategic decision making opens the flood gates in terms of potential opportunities for reviewing historical methods. Most work in the field of economics is freely available including information on the methods used to make predictions as well as works dedicated to reviewing the efficacy of predictions made in the past. Economic problems and many other specific problems which contain some element of human behavior may fall under the umbrella of game theory. Most problems which falling under the broad category of game theory, are openly published despite the fact that some game theory scenarios have been developed specifically as surrogates for scenarios which cannot be discussed openly.

Problems mentioned in [3, 4] run the gamut from determining the price a buyer would be willing to pay for a defunct power plant while facing large uncertainties, to developing systematic ways to manage the portfolio of a broker in the oil and gas industry, to selecting the best technology from a list of competitive alternatives to be used in the management of solid waste for a landfill project, to the formal applications of decision making tools to manage risk in terms of investments by a large bank. The possibilities for using the methods of strategic decision making are nearly without end. The essence of the problem being considered in general terms is that a decision is to be made regarding a course of action and that course of action will determine long-term outcomes. The information available to make a decision at the current instant is in some way insufficient. This could be due to high uncertainty, low signal-to-noise ratio, too much information, too little information, etc. The goal of the decision maker in this case is to use the information available at this instant to maximize the value of future outcomes.

The specific problem discussed in this manuscript is related to how to best make decisions related to the UAV threat space in terms of developing technologies related to using or defending against UAVS in order to best inform the decision making process. The information available is vast. There much academic literature and at least as much information available comprising commercial marketing materials, advertisements, press releases and news articles. Much of the academic literature is more concerned with laboratory demonstrations, and thus decoupled from what is possible using COTS technology right now, but the marketing literature is notoriously unreliable with respect to the veracity of manufacturers' claims. Neither source of information is as relevant to the problem at hand as could be desired. A variety of methods can be used to study trends which may be on the horizon including meta-analysis of the publication record, as well as meta-analysis of claims by manufacturers. Additionally, subject matter expertise can be leveraged through brain-storming sessions. Combining all of this information into a cohesive format, then effectively using this information to propose decision making infrastructure (including proposing a way of evaluating this decision making process periodically) constitutes an important and challenging problem.

The low signal-to-noise ratio (so-called “weak signals” [5]) may be more easily parsed by subject matter experts (SMEs) than by the general public. The same applies for marketing claims, news articles, speculation, etc. All of the written record with regard to a growth industry, like the UAV industry, presents challenges in terms of certainty. Technologies, companies, and whole sub-industries can appear and disappear almost without warning meaning that understanding what is a signal and what is noise can be challenging. The academic publication record is a lagging indicator to some extent but may be useful for studying trends related to expected timeliness of technologies nearest to market as well as for getting an idea of what kind of things might be possible in the more distant future.

In the 1960’s the Delphi method was introduced by researchers at the RAND Corporation with an eye toward keeping tabs on likely future events [6]. The Delphi method involves a number of steps including convening a panel of experts and carefully crafting a questionnaire related to a specific future topic. During the course of the execution of the method, the expert panel is asked to answer the questionnaire and then variously the results are compiled and provided back to the panel (with individual contributions kept anonymous). One of the key elements of the Delphi method is related to the redistribution of information about the questionnaire after the experts have been consulted [6].

Essentially, the members of the panel are informed of the consensus of the group. Experts holding opinions outside of some range compared to the consensus are asked to reconsider their answers or to justify why their response which falls outside of the consensus range is warranted. Several rounds of this process comprise the totality of the Delphi method, though the details are not important to this discussion. The Delphi method has considerable power in terms of reducing the inter-quartile range with respect to quantitative predictions and with building a consensus where a consensus may not have existed at the initial questionnaire phase [6].

The considerable time required in building a questionnaire and repeatedly submitting the questionnaire to a panel of experts, not to mention, convening and keeping tabs on a panels of experts comprise a substantial investment for the team conducting the futures analysis. Selection of experts on the panel must be conducted carefully or the results can be affected. Lack of care in crafting the questionnaire can also influence the results. Though the Delphi method has been demonstrated to lead to consensus in many cases, the evidence to suggest that this consensus is necessarily the correct consensus is less definitive. A casual article published by RAND gives a high-level view of predictions from the past decades and how they have panned out [7].

A very quick overview of predictions presented in the article indicates that some predictions have been accurate, such as those related to artificial organs and automatic language recognition technology. Other predictions have not been so successful. For example, the panel mentioned in the article anticipated a world population equal to approximately 8 billion people in the year 2100; whereas, the world population in the year 2021 is already 7.8 billion [7]. The Delphi method itself can be strongly influenced by the setup, the questionnaire, the selection of experts among other factors [6]. A quick case study of a particular application of the Delphi method and indicates as expected, that the results of various studies based on the Delphi method have been mixed overall [7].

In Europe especially, futures analysis is more widely known and used with some methods focusing as much on predictions as they are focused on shaping policy decisions in relation to the proposed future [8]. Foresight analysis as it is called, is strictly in-line with the original justification presented for the Delphi method but is more focused on policy-making compared to other types of futures analysis which are more common in the United States [8].

Forecasting is a scientific topic in its own right, which brings together experts from various fields along with pure mathematicians and is engaged in studying mathematical patterns which can be used to predict or forecast future events [9, 10]. In the pure forecasting profession, as well as in the field of economics (where much of the science of forecasting found its original home), much contemporary work is focused on the effect of combining predictions to increase the certainty and accuracy of the prediction [9, 11]. In addition to macro-economic predictions, the field of forecasting has special interest for portfolio managers who operate with the goal of making intelligent investment decisions and outperforming the market average growth rate year-after-year [12].

Economic predictions are often published and freely available; thus, comparisons exist related to prediction accuracy of so-called expert panels versus the prediction accuracy of individual subject matter experts [13]. In many cases, easily quantified economic projections, as made by individual experts and compiled mathematically, outperform the predictions from bodies set up specifically for the purpose of making predictions such as the IMF. This could be partly due to bias on the behalf of the expert panels considering that organizations like the International Monetary Fund may possess an overall agenda or policy direction which their predictions will necessarily tend to fit or favor [13]. This observation simply drives home the point that the choice of an expert panel and how best to combine predictions from disparate groups can have a large impact on the outcome of the prediction.

There are some easily recognizable cases which have been published and provide an opportunity to discuss the quality of predictions such as a document related to futures in the journalism and media fields [14]. Many predictions, such as those related to misinformation, the importance of digital subscriptions, investment in data, and the impact of voice assistants, have mostly come true [14]. Other predictions such as the rise and preeminence of visual search and the augmented reality eye wear (which seems to always be a year away) have not come true. However, this particular example of futures analysis is not an entirely perfect comparison since the futures analysis is conducted with a short horizon (about $1\,\mathrm{yr}$ to $3\,\mathrm{yr}$) and is conducted in part by interviewing major players in the industry who have the ability to shape the future they are predicting. Thus, futures analysis such as this does not truly represent the synthesis of uncertain information in order to gain a better understanding of the future so much as it represents various stakeholders making strategic statements about their plans for the near future and then either implementing those plans or changing them as needed during the specified time period.

2.1.1 Military Futures

Various publicly available publications discuss military futures specifically. The caveat remains that the most pertinent information is most likely the most sensitive and thus the timeliness, and veracity of publicly available information with respect to the United States military may not be as truthful or reliable as desired. However, for older publications which have been declassified, it is more likely that the published information represents the whole and accurate truth. Additionally, information on the past allows for the comparison of predictions with historical facts to understand how predictions may have been right or wrong.

To begin, a RAND Corp. report which was published in 1998, is considered. The author predicted that despite changes, the United States would remain a global actor. This is despite the fact that some prognosticators seem to be perpetually predicting the imminent fade of the U.S. as a great power [5]. Such a prediction still seems prescient in 2021, likely as much as it did in 1998. And though the global landscape has changed, many continue to predict the imminent fall of the United States a globally engaged actor to this day. The report predicts major shifts in the global balance of power, specifically the rise of Asian countries. The rise of major global players such as China is expected to be related to the growth of trading relations with the United States [15]. In fact, the rise of countries such as China and India, at the cost of stagnation in status of other former global powers (specifically in Europe), was more or less predicted with accuracy in the report.

The report predicts that countries in the Middle East will continue to demand U.S. military attention as they experience growth and work through historical issues. Additionally, the character of Russia was being formed in the wake of the fall of the U.S.S.R. and exactly what type of stability will emerge in Russia was expected to have broad reaching implications considering the size and influence of Russia [15]. The status of Russia and China as global powers is considered uncertain, with either country gaining or losing status according to a wide range of uncertain variables [15].

Importantly, the report predicts the $50\,\mathrm{yr}$ long conflict between North Korea and South Korea is likely to be resolved in the immediate future. This prediction of course has not come true. The report makes the prediction that the United State homeland, which has traditionally been impervious to attacks by foreign actors, will most likely become the target of attacks most probably by terrorists [15]. Of course, the events of September 11, 2001 proved this prediction to be true. The report also predicts that foreign actors could enter into the United States through the "porous borders" and use unconventional warfare to attack targets inside the U.S. [15].

Hearkening to the rapid rise of the personal computer and the cell phone, future technological breakthroughs are predicted. Importantly, the development is considered to occur most importantly in the direction of proliferation [15]. This prediction is fairly vague, but has come true, specifically with respect to the UAS industry. However, consider that the article predicts the preeminence and proliferation of space-based technology such as satellite imagery. And while satellite imagery has changed the world, the proliferation predicted by the article has not occurred in the sense that ordinary citizens do not have access to space technology in the way predicted. Still, the outcomes of the prediction, that space-based technology would influence daily lives, has come true.

Many predictions are made related to the widespread development and usage of nuclear, biological, and chemical (NBC) weapons. It was expected in 1998 that the future decades would see widespread usage of NBCs [15]. This prediction was partially correct, but fortunately, NBC weapons have not proliferated or been nearly as widely used as was predicted.

Variously, U.S. military involvement in humanitarian crises around the globe, such as the events of the Battle of Mogadishu (documented in the book as well as the film *Black Hawk Down*) was predicted [15]. These predictions seem almost perfectly accurate, but this is likely due to the vagueness with which they were made. No actionable information related to the locations or types of problems to be encountered was offered at the time, though the operational and political difficulties in responding to such crises was well understood at the time.

The most useful information from the article in hindsight is related to sections on what the most important factors will be to watch in the near term (both for their implications on the global power dynamics as well as for the sake of understanding the accuracy of the predictions). The findings are reproduced in the following list.

- “the fate of democratization and market reform in Russia and China” [15]
- “the manner in which the countries of central and eastern Europe are reintegrated into the continent’s political-economic structure, and how Russia responds to that process” [15]
- “the pace and extent of European unification” [15]
- “the internal dynamics of the greater Middle East, especially the outcome of the Arab-Israeli peace process” [15]
- “the evolution of Sino-American relations and Beijing’s choices about its role in Asia and the world’ [15]
- “the rate and extent of the spread of NBC weapons” [15]

Examining the items in this list gives an important insight into those factors which the author thought most important. The predictions as discussed were accurate in some ways, not as accurate in others but importantly, elements related to how best to use such predictions at the time they were made was not within the scope of the report. However, the author does present a call-to-action in the form of the list reproduced above. Several items in the list are nearly as important to consider as they were when published in 1998. The continued growth of the Chinese economy and thus the rise of the influence of China on the global stage, wars in the Middle East, the European unification (and troubles associated with Brexit) were not necessarily foreseen, but all of these areas were indicated as important to consider going forward. Continuing spats between Russia and Ukraine over disputed territories were considered likely.

In contrast, the prospect of peace between Israel and Arab countries has not been achieved within the time line predicted (and in some ways, the most recent "peace process" was not foreseen by the author in any meaningful way). Additionally, while the potential prevalence of NBC weapons was important to the U.S. war in Iraq, use of NBC weapons has fortunately not gone according to some grave predictions. Thus, the most useful information gained from the report is taken from the list which indicates those matters and events which should be considered hinge points. Importantly, the author does little to make recommendations or to provide quantifiable predictions which can make use and interpretation of such information difficult at the time the predictions are made.

Another report is interesting considering its title and the expected time line of the predictions given. The information in [16] is targeted to the future of Air Force capabilities in the year 2025. Additionally, this work was prepared with the intention that the information would be used by military planners and thus the furnished predictions are more specific and more closely tied to action compared to the predictions in [15]. Both works however, originate from roughly the same time period.

Specifically, the study indicated that in order for the U.S. Air Force to occupy "the high ground" in the envisioned future, the Air Force should focus on an ordered list of systems given below.

- Global information management system [16]

- Sanctuary base [16]

- Global surveillance, reconnaissance, and targeting system [16]
- Global area strike system [16]
- Uninhabited combat air vehicle [16]

Additionally, the development of space-based and solar powered high-energy laser (HEL) systems was indicated as important as well as several systems related to what would now be called UAVs [16]. Considering the possibility that the development of some current or past military technologies could be accomplished in secret, it may eventually come to light that some such technologies do currently exist but are not widely known. However, the importance of developing surveillance and reconnaissance systems and the interrelationship with UAVs cannot be understated. Additionally, the focus on information management represents a correct and important prediction about the proliferation of data.

The list of systems given is framed as a list of recommendations on where to place the focus. Thus, the veracity of predictions is more difficult to make considering the predictions themselves are not so much predictions, as recommendations. Importantly, the U.S. Air Force has continued to occupy the “high ground” as far as the author’s defined it, but not all of their recommended technologies have come to fruition (at least publicly). Additionally, considering the actionable nature of the report, how closely the recommendations were followed is easily confused with how accurate the predictions were. Thus, while such analysis provides an interesting window into the process of making recommendations about the future related to uncertain information, it is more difficult to make a judgment of the prediction quality in the work.

Later, in 2003, testimony was provided to the U.S. congress by the Director of the Defense Intelligence Agency (DIA) and is transcribed in [17], which is again, a document concerned with recommendations, but those recommendations are based on predictions related to the uncertain future. As expected, the first predictions were related to the ongoing Global War on Terror specifically targeting Al-Qaeda wherever they seek refuge. Predictions about increased prevalence of attacks in Europe have proved accurate; whereas, predictions about the widespread use of small "dirty" bombs have not been so accurate [17]. The testimony includes self-serving predictions about the behavior of Saddam Hussein which led to the beginning of the U.S. war in Iraq later in the year [17]. Predictions about the behavior of Hussein were largely accurate, but the accuracy of these predictions are in some ways the direct result of pressure on the Iraqi leader from the U.S. military.

Predictions related to the proliferation of nuclear weapons by the regime in North Korea has failed to come to fruition [17]. Again, such predictions may have failed to come true simply due to good policy decisions which were made as a result of the testimony. The pressures from uneven economic growth, particularly in the Middle East, as well as the wide-reaching effects of globalization that are apparent today as evidenced by the so-called "Arab Spring" starting in 2010, were foreseen to a great extent [17].

In his testimony, the director of the DIA concludes with a succinct statement about the transformation from threat analysis which focuses on a few select adversaries, to a threat analysis which includes consideration of a wide range of types of actors and technologies, which have not been traditionally the purview of military threat analysis [17].

This prediction has proven particularly cogent to the purposes outlined in this manuscript. Though again, the specific nature of the threat (in this case UAVs) or a specific time line was not provided. Fundamentally, the problem of developing accurate but actionable information about the future is (and most likely will remain), challenging. The "intelligence transformation process" indicated in the testimony, and of which in a general sense, the work outlined in this manuscript represents a small piece of, is indicated as the most important step moving forward [17].

Finally, the most contemporary document reviewed was published in 2020, and considers the implications of the COVID-19 pandemic on military futures. This document is concerned with predicting the future the U.S. Air Force should prepare for in the year 2035. Predictions made include the effects of loss of trust in institutions, and the rise of medical surveillance in terms of importance for force superiority among other topics [5]. The use of military forces to accomplish tasks such as food distribution and peace-keeping in the wake of potential global collapses, as a result of loss of faith in government, is predicted in one of the more dismal scenarios [5].

The report highlights several key areas of interest including competition in space and autonomous systems which are relevant to the research detailed in this manuscript [5]. Likewise, focus areas related to responding to coordinated disinformation campaigns and bolstering supply chains may not be directly related to the counter UAV problem, but may involve the development of enabling technologies which could be used directly.

2.2 State of the Art in Game Theory

The techniques of game theory find an obvious application with regard to military strategy. Much of the historical development of game theory has flowed directly from the desire to understand and improve military strategy. Indeed, militaries around the world still practice so-called "war gaming" in which a simulated conflict is played out between players representing allies and enemies in order to better understand the various inherent complexities which define global conflicts. The so-called "Colonel Blotto" game and its variations represent a classic application of game theory in the realm of military strategy. In the game, two (or more) sides must make choices on the allocation of limited resources across multiple battlefields. Victory on a given battlefield (and ultimately the game at-large) is determined by comparing the resources each player has allocated in a given space [18–22].

The Colonel Blotto game has been used extensively in the literature with various modifications to attempt to understand behavior and model complex real-world scenarios. Modified Blotto games may include relaxation of constraints on how resources are allocated such as removing the requirements that all resources will be consumed or lost during the game [23]. Additionally, much work has been done to study the effects of variously modifying the players and the game such that the players have asymmetrical resources and the conditions for victory may be substantially heterogeneous [24–26]. Additionally, the original simple game has been variously extended to include an arbitrary number of players [27] and modified to be more applicable to a whole general class of real-world problems [28].

To be more specific, game theory, and indeed the Blotto game has been applied to help understand the unique challenges present in wars fought against insurgents, terrorists, or those using widely asymmetrical tactics [29–34]. When large asymmetry is present between players in the Blotto game, a unique difficulty arises because the great-power must always win in order to be successful; whereas, the upstart or insurgent need only win once (or at least win less often) in order to achieve victory. An additional challenge arises in considering the asymmetry of information available to both sides in the Blotto game [29, 30]. For example, if one player knows a certain battlefield contains a highly valuable asset, and thus losing that battlefield could be substantially injurious, then that player may behave differently than in the case of the Blotto game with complete information (where both players know everything the other player knows).

A particularly powerful implication of game theory involves the consideration of outcomes with known probability distributions. Much work has been done to categorize human behavior with respect to uncertain payoff [20, 35]. Thus, it is possible to describe precisely in terms of quantifiable payoffs and probabilities the choices being faced by players in complex games. Considering that most real-world applications of game theory necessarily involve some level of uncertainty, the extension of game theory's language of tactics to include uncertain circumstances is important to modeling complex situations.

2.2.1 Applications of Game Theory to Battle Space Evaluation

Game theory has been applied to threat evaluation basically since its inception. Pioneers of game theory working at the behest of the U.S. Government for the RAND corporation were some of the first to lay out the framework for the modern theory of games. Of course, for centuries before, humans have been playing and attempting to win various types of games. War gaming specifically refers to a particular type of strategic "game" where rival military powers face-off in a simulated engagement. War gaming has been ongoing for virtually as long as wars have been being waged.

More recently, papers have explored the impact of war gaming with respect to modern threats. For example, the insurgency threat faced by U.S. forces in the Middle East is difficult to capture in war games as traditionally framed [36]. War gaming is often conducted at a force or campaign level of detail where rival military actors command large quantities of resources with low level of detail. War games structured in this way would fail to account for most insurgent forces altogether due to the small size of many insurgent units. Similarly, evaluating an insurgent threat in the same way as a hierarchical, top-down military organization may fail to account for some of the dynamic and adaptable nature of a small and agile force [36].

2.3 State of the Art in Differential Games

In the preface to that which would comprise an essential summary of work completed by Rufus Isaacs at the RAND corporation during the 1940's and 1950's, Isaacs states "Although combat problems were its original motive, this book has turned out to be far from a manual of military techniques." [37] The technical details of the work Isaacs completed at the RAND corporation during his tenure were virtually unknown due to the sensitivity of the information up to the time when the work was published in [37]. The

RAND corporation was formed in 1948 to "connect military planning with research and development decisions" [38]. The directive under which RAND was formed is summarized in a statement by then Commanding General of the Air Force H.H. "Hap" Arnold to the Secretary of War, where Arnold articulated the strong connection between the scientific research and development community and the technology needs of the U.S. Government [38]. Specifically Hap Arnold recognized that "Scientific planning must be years in advance of the actual research and development work" [39].

The potentially sensitive nature of the work undertaken by Isaacs and others at the RAND corporation led to the creation of surrogate scenarios which may have strange or unfamiliar names. Take for example, the "homicidal chauffeur" game as described by Isaacs which consists of a "car" with a high maximum rectilinear speed but limited turning radius, pursuing and attempting to run over a "pedestrian" which moves with no inertia but a maximum rectilinear speed lower than that of the "car" [37, 40]. In this game, the "car" and "pedestrian" Isaacs was most likely truly concerned with were a guided torpedo and a small ship attempting evasion [40]. In this way, many generic scenarios were created and discussed in such euphemistic terms, which included the aforementioned "homicidal chauffeur", as well as "the game of two cars", and the "isotropic rocket" [37]. Additionally, Isaacs considered the topic of traditional war games more explicitly with consideration of the "war of attrition and attack", and the "battle of Bunker hill" [37].

In the preface to the work, Isaacs states that the most important contribution may be the method of solutions rather than any particular solution itself and indicates that the work presents as a result, "a mathematical entity which fuses game theory, the calculus of variation, and control theory...” [37]. It is noted that if any game can be reduced to a single-player game, a substantial reduction in complexity is accomplished and the development of a solution may be greatly simplified. In many cases, single-player games can be solved entirely using methods from the calculus of variations [41]. The innovation by Isaacs was the recognition of the unique challenges presented by true two-player games and then developing a method of working through such problems in order to arrive at useful insights.

2.4 State of the Art in Pursuit-Evasion Games

Whereas game theory typically involves games with discrete steps, which can be thought of as individual moves, the topic of differential games extends many game theory concepts into the continuous time domain. Differential game theory allows interesting problems to be studied including many problems related to pursuit and evasion of the players. Within the pursuit evasion sub-category of differential game theory, there are a handful of major sections. The first of these to be mentioned here involves multiple pursuers and evaders who are engaged in a mutual chase.

Many problems have been posed in the academic literature both contemporarily and historically [42]. One popular historical problem is the so-called “n-bug” cyclic pursuit problem where an arbitrary number of bugs begins the pursuit at the vertices of a polygon with each bug chasing exactly one adjacent bug thus forming a cyclic pursuit. Historically, the n-bug problem has presented a number of challenges such as finding the capture conditions and curves of pursuit for arbitrary starting configuration (both regular and non-regular polygons) [43]. Whether or not the pursuit curves for a given set of initial conditions necessarily form a regular n-gon throughout the pursuit [44], and under exactly what starting conditions it is possible for a cyclic pursuit to end with a non-mutual capture [45] are also considered. Other works have focused on studying the geometric configurations which arise during the pursuit for various initial conditions [46, 47].

Periodic pursuit may seem an odd topic to mention here, but other than being an interesting specific problem within differential game theory, the question of cyclic pursuit is closely related to the problem of multiple radially spaced evaders headed toward a central goal being pursued by a single pursuer. Such a scenario is related to cyclic pursuit due to the simplicity of the trajectory of each evader leading to predictable geometric patterns in the pursuit paths for the pursuer specifically. Although, not exactly the same as cyclic pursuit, studying cyclic pursuit could lead to important insights about the related, if slightly different, problem of multiple similar pursuits.

A related class of pursuit-evasion problems is concerned with a scenario which contains a single evader, and multiple pursuers. Such problems are commonly encountered in robotics disciplines and often include the goal of simultaneously conducting the pursuit, coordinating multiple pursuers, and mapping the environment at the same time [48]. Studies on such problems include the effect of imperfect sensor information [49–54], and how overlapping sensing regions can mitigate the effects of imperfect sensors [51, 55, 56]. Some studies examine the effect of changing characteristics such as variable but bounded speeds on the agents [57]. And some multiple-pursuit papers consider the output of the research to be an optimal joint strategy for multiple pursuers [58, 59] rather than a specific pursuit path or any guarantees on capture [53, 54, 60, 61].

A wrinkle arises in general with pursuit-evasion problems with relation to the kinematic equations associated with various vehicles. Considering most robots are non-holonomic, this is an important step in connecting results from modeling and simulation with expected real world performance. Studies examining the effect of non-holonomic motion constraints exist for the cyclic pursuit problem [62] as well as for pursuit-evasion in general [63]. Within the realm of non-holonomic studies, often a surrogate vehicle type is used [63].

Many works are concerned with the intersection of differential and discrete such as [64, 65]. The concept of incomplete information effects behavior within the differential game scenario in potentially interesting ways as outlined in [64]. Specifically, the evader in a multiple-pursuer scenario, has the advantage that incomplete information is not as much of a detriment as it is for the team of pursuers. The pursuers rely on better information and sharing that information in order to be successful in executing a capture [64].

Information theory within the umbrella of game theory has the important caveat, that the other player can only be manipulated by incomplete information right up to the point where the player realizes that his opponent may be manipulating the information shared in order to gain a benefit [66, 67]. Beneath a certain threshold of distrust, rival agents may be able to manipulate one another by controlling the flow of information.

However, once the threshold of distrust has been exceeded, one or more parties will understand that their opponent is most likely sharing information intended to encourage them to make sub-optimal strategic choices. At this stage, distrusting agents may choose to do the opposite of that which would be predicted given the known information about the opponents actions. It can become very difficult to track and predict behavior at this point and the potential advantage of manipulating information is quickly swallowed by the potentially erratic behavior driven by the distrust.

Paramount to solving the problem of multiple-pursuit-evasion is the communication and cooperation among disparate pursuing agents. Up to this point, works discussed have mostly dealt with global strategies which would be calculated and transmitted to individual agents by a centralized planning node with access to the global information about the position of all agents. However, much work has been done specifically with regard to how best to communicate and coordinate among agents, specifically how pursuers should network and share information in order to expedite capture [52, 59, 68–74].

2.4.1 Historical Pursuit and Evasion

Historically, many problems have been posed which fall into the broad category of pursuit-evasion games. Many such problems have been solved using differential game techniques. Several problems predate the formal introduction of the theory of differential games and many were first posited as mathematical oddities. One such problem, regarded by many to be the “first” pursuit-evasion problem is concerned with finding the pursuit curve for a faster pirate ship overtaking a slower merchant ship [41]. The classic pirate ship pursuit-evasion problem was presented by Pierre Bouguer and thus the problem is referred to as the “Bouguer Pirate ship problem” and is mentioned in many papers related to differential games and pursuit-evasion problems [75–79].

Another classic problem is often referred to as the “Appolonius pursuit”, or “Appolonius circle” problem. The most common real-world application of this problem and its solutions is for finding the interception course for an unguided torpedo which is launched in such a way to intercept with a non-maneuvering (and relatively slow moving) ship which is the target of the torpedo. This problem and solution are mentioned in [41, 80–82]. The classic solution is notable for its reliance on geometry (no differential calculus required) [41]. A simple modification to such problems exists in cases where aim-ahead may be appropriate [83] such cases may arise any time a ballistic or unguided projectile is launched at a much slower (often non-maneuvering) target.

Pursuit-evasion problems and indeed the entirety of the field of differential games is notable for proposing (and usually solving) problems with substantial real-world applicability. Many historical problems are surrogates for problems of military interest and have been thus obfuscated to avoid issues with information security. The already discussed “homicidal chauffeur” problem was famously proposed (and partially solved) by Isaacs in [37]. This problem has obvious applications in terms of air-to-air combat, air-to-air missiles, surface-to-air missiles as well as many other applications.

The problem of the homicidal chauffeur has been discussed in various papers including works which sought to solve parts of the problem which were not solved when the problem was first introduced [84–86]. Various modifications to the original problem have been proposed including stochastic modifications which make the problem easier to solve [87]. The problem statement has been modified to include multiple pursuers [88]. Various papers have discussed solutions [85, 89, 90] with some contemporary papers choosing to focus on numerical solutions [91, 92]. Novel techniques such as fuzzy control have been applied to the classical problem as well [93]. The most classical approach treats the problem as a controls problem [94–96].

The related sister problem where the pedestrian is now trying to intercept the car is referred to as the "suicidal pedestrian" problem [97]. Generically the homicidal chauffeur problem and its derivatives are part of the family of two-car problems where a pursuer and evader are defined by their relative maximum speed and relative maneuverability.

In the simplest instance of the generic two-car problem, each agent can be treated as a Dubin's car which can either travel in a straight line or turn at a maximum rate (minimum radius) [98]. Other two-car problems assume simple kinematics where both pursuer and evader travel with maximum speed at all times, but an important and realistic modification allows for variable speed [99]. This greatly increases the problem complexity but correspondingly greatly increases the realism of the resulting solutions. The class of two-car problems has been solved comprehensively in the literature [100].

An important modification to the two-car problem and its derivatives is related to terminal constraints on the capture. This represents the real-world case where the termination of the game represents a "kill" by an air-to-air missile or a center-line mounted gun during an air-to-air combat scenario. Such problems have been variously considered and solved [86, 95, 100–105]. The two-car problem with terminal constraints is sometimes referred to as the “tail chase” game due to the nature of the terminal constraint where the pursuer must come within some distance threshold as well as be directly behind the tail of the evader in order to win the game. Such games have been studied specifically with respect to UAV trajectories [106–108] and ultimately the planar two-car game can be extended to include 3-dimensions and full-state inertial models of vehicle motion [109]. This of course increases the complexity of the problem and the techniques needed to arrive at a solution and correspondingly decreases the generality of the solutions found. However, realistic air-to-air combat is best represented in 3-dimensions and necessarily makes use of inertial models to avoid ignoring important characteristics of constituent trajectories.

In the classical two-car problem, or the homicidal chauffeur problem, the pursuer and evader are not trying to reach any specific goal other than to terminate the game. That is, the evader is simply trying to evade the pursuer, and wherever the two may travel globally during the pursuit is immaterial to the game. An important modification to these games involves subjecting the trajectories of the two agents to external constraints. One technique involves solving the classical differential game after exposing the agents to an external field which influences the trajectories of each player [110–113]. Likewise, the so-called “lifeline game” sees an evader tasked with breaching some type of protected region, and a pursuer tasked with defending said region [114]. Such modifications may complicate or simplify the solution to the basic problem considering that an evader who has a clear global goal may be substantially more predictable than an evader who is simply evading a pursuer wherever the two may travel.

A natural extension of the family of two-car games, involves considering an arbitrary number of pursuers and evaders [115, 116]. Such problem formulations may be collectively referred to as “multiple pursuit-evasion” and open up for consideration many new problems including the optimal number of pursuers or evaders given some physical characteristics of the agents or the region in which the game is to be played. The so-called “cops and robbers” game is a related problem which is often played in a discrete space, rather than a continuously differentiable space. The cops and robbers game is concerned with defending some region using “cops” who move from station-to-station but must be within some specified proximity to the other players (“robbers”) in order to protect against the “robbers” entering some predefined zone which is to be protected. These games are closely related to cyclic pursuit, and often are concerned with establishing the minimum number of agents required for one outcome or another [117, 118].

2.4.2 The Man and the Lion Puzzle

One additional interesting problem mentioned here, which shows up historically and contemporaneously within literature is the so-called “man and the lion” problem. The problem of the man and the lion is an old mathematical puzzle. The problem statement considers a man released into a circular arena being hunted by a lion. Neither the man nor the lion can leave the arena, and the lion is faster than the man. The question to be answered is whether or not it is possible for the man to evade the lion forever, and if not, to discover a relationship for the finite amount of time required for the lion to capture the man.

On its face, the solutions to the problem related to the trajectories of the man and the lion may be interesting with respect to the study of pursuit and evasion generically. However, the modern accepted solution is not particularly relevant to the scenarios presented here for reasons that will be discussed shortly. The problem is notable historically in that it was "solved", but it turned out to have not really been solved at all (which is fairly common; although, this problem is deceptively simple) [41, 119]. The mathematical solution to the problem is that the man can escape indefinitely. Again, this sounds like an interesting result, but this is a case where the perfect mathematical solution is almost entirely useless in terms of real world application.

There is a specific mathematical technique which is presented in the commonly accepted solution for the man's evasion strategy. This strategy relies on the man traveling in a perpendicular direction to the lion's current path, prior to turning perpendicular to his own path after some time has passed [119]. The mathematics interestingly give rise to an infinite series describing the path length for the man (and the lion). Thus, mathematics considers that the man can evade indefinitely [119].

However, the path length for the man after each successive "swerve" is reduced according to a harmonic series. The distance the man evades after each swerve is reduced and the duration of time after which the man must swerve gets infinitesimally small in short order. Obviously any practical extension to the problem would conclude that the man's strategy is not that effective and that since the lion need only be within some finite distance of the man, the total evasion path would actually be quite short since the lion would bite the man as soon as he could reach him [119].

This brief explanation is given in a way as a cautionary tale. The problem of the lady and the lake is far more interesting and relevant in terms of the greater scope of this work. However, consideration of evasion strategies and optimal behavior for both the pursuer and the evader (the lion and the man in this case) is a critical part of gaining insight into the types of scenarios being considered. However, in this particular case, the mathematically correct solution which states "the man can evade indefinitely" is totally impractical and is almost immediately violated in the case where even small real-world enhancements are added to the problem. That is not to say that the solution cannot be used to find the point of capture when some tolerance between the position of the lion and the man is given for the capture condition; however, the simplicity and finality of the mathematically perfect solution in this case is a gross mis-characterization of the results from a real-world problem.

2.4.3 Lady in the Lake

An additional case study in pursuit-evasion games (or in this case, mathematical puzzles) comes in the form of the so-called "lady in the lake" problem. This problem goes by many names sometimes with the "lady" and her "lake" supplanted by a "duck" and a "pond". Many contemporary sources trace the origin of the modern problem description to Martin Gardner's *Mathematical Carnival* which was first published in 1967 [120]. The problem setup as given by Gardner and others is given below.

A lady is stranded in the middle of a perfectly circular pond in a row boat. The perimeter of the lake is patrolled by a hungry monster who can run 4 times faster than the

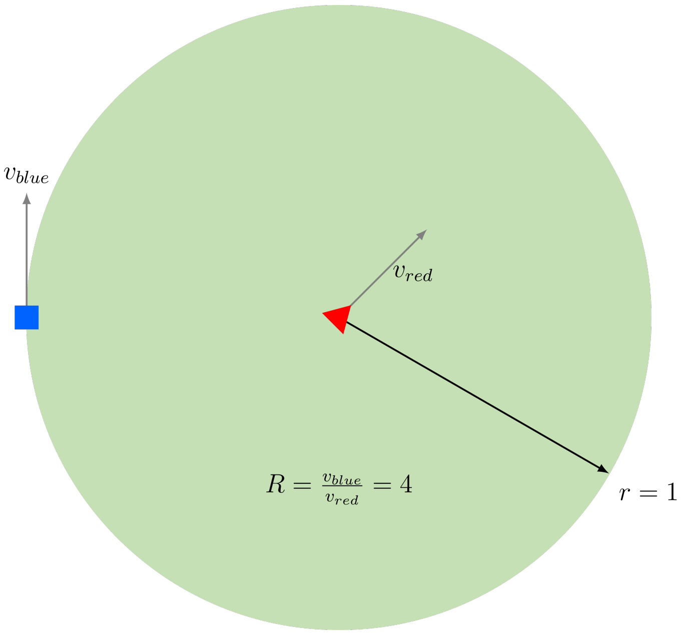


Figure 1: Schematic Diagram of the Problem of the Lady in the Lake

lady can row her boat. If the lady can reach the shore without coming in contact with the monster, she will depart from the rowboat and can easily escape. Part 1 of the original problem asks if it is possible for the lady to escape given the conditions provided. Part 2 of the problem asks from exactly how much faster of a monster will the lady be able to escape. A schematic representation of the basic problem setup is given in Figure 1. As is typically the case with mathematical puzzles like this, both the lady and the monster are considered to be experts in mathematics and act strictly rationally throughout the engagement.

To answer the first part of the problem, it is possible for the lady to escape if she adopts a clever strategy. Additionally, it is possible for the lady to escape from a monster that is more than 4 times faster than she is; although, proving just how fast the monster can be involves a more detailed explanation. The most interesting aspect of the solution in both cases is that the winning trajectory for the lady involves two stages. The first stage uses the angular velocity advantage the lady has on the monster up to a certain critical radius to position the lady and the monster as far way as possible prior to beginning the second stage. During the second stage, the lady makes a mad dash to the shore knowing that she can just barely escape given the proper initial conditions. The problem will be solved later in the manuscript as an introduction to and an illustration of the methods used.

2.5 Illuminatory Historical Case Studies

In the following subsections, historical case studies will be discussed. The relevance of these case studies varies. Some of the case studies comprise a point in history where the difference in outcomes may have been related to a single chance encounter or may have come down to the difference of a few months between competing entities. These case studies are presented for two reasons.

The first reason is obvious and that is to determine and discuss anecdotally if there is a pattern to be discovered related to historical happenstances which lead to better outcomes for the “good guys”. If such patterns can be identified, understanding the historical context could potentially lead to insights into how best to foster or recreate the conditions which could lead to those patterns repeating themselves. Of course, this is an aspirational goal and generally history is a difficult beast to tame, but the insights gained may be valuable nonetheless.

The second reason is less obvious and is related to the motivation for this work. In the course of history, there are many instances where the determination between who wins and who loses is based on the smallest of details. An inch here or there, or a month earlier or later can make a huge difference in terms of outcomes. A historian writing specifically about the case study related to the development of the atomic bomb wrote “events that change a time scale by only a few months can nevertheless change history” [121]. Thus the second purpose of these case studies is to drive home the importance of being one or more steps ahead of your adversary when it comes to matters tied up with global or societal consequences.

2.5.1 Frisch-Peierls Memorandum

In 1939, Albert Einstein had grown increasingly concerned about the state of nuclear weapons technology development. Einstein reasoned that if the Nazis possessed the means to develop an atomic weapon that they would be inclined to use such a weapon with devastating effect. Additionally, there was some credible intelligence at the time which suggested that the Nazis were in fact working on nuclear technology (it turns out the technology the Nazis were working on was the infamous V-2 rocket, not an atomic bomb, but more on that later). Einstein penned a letter to U.S. President Franklin Delano Roosevelt expressing his worry that the Nazis must not be the first to develop nuclear weapons and thus encouraging Roosevelt to invest resources toward research into the technology within the United States [122].

Later, in 1940, two expatriate German-Jewish physicists penned a memorandum now known as the Frisch-Peierls memorandum which laid out a compelling argument for the possibility of a nuclear bomb large enough to make a decisive difference in a global conflict, but could potentially be small enough to fit on an airplane. Though Frisch and Peierls got some of the details wrong, their calculations proved useful in shifting the conventional understanding, which at the time held that nuclear fission could be harnessed for destructive purposes, but that the mass of fissile material required and the rate at which the energy was dispersed would make any war-time use of such technology impractical [123].

Frisch and Peierls, being expatriate Germans, were not privy to the secret discussions in Great Britain or the United States where their memorandum would fatefully lead to the formation of the Tube Alloys and Manhattan project respectively [123]. Similarly, Einstein was not invited to work on the research that he suggested Roosevelt fund because of his similar expatriate status. Interestingly, the spark which set off the chain of events leading to the development of the atomic bomb, and the use of that weapon on Japanese cities, was set off by a handful of Germans displaced by the rise of the Nazi party.

The story of the Frisch and Peierls memorandum involves a fair amount of serendipity. The two men drafted the memorandum, and it was passed through a chain of British officials and circulation of the memorandum eventually led to the formation of the so-called “MAUD” committee. The MAUD committee was the British pre-cursor to the Manhattan Project in the United States. At the time the memorandum was circulating, a British committee set up to study the potential for nuclear fission as a method of power generation, was preparing to disband having found the technology to be impractical for the proposed use. The memorandum caused this action to be reversed and spurred renewed focus on destructive uses of nuclear fission. The information gleaned by the MAUD committee eventually made its way to the United States where it was similarly important in terms of convincing policy-makers of the urgency of developing an atomic bomb [123].

Referring to the circuitous and unlikely way in which the information was passed first from Frisch and Peierls to the British authorities and then from the MAUD committee to the United States, Leo Szilard wrote "if Congress knew the true history of the atomic energy project, I have no doubt but that it would create a special medal to be given to meddling foreigners for distinguished services, and that Dr. Oliphant would be the first to receive one." [124] Dr. Oliphant, referred to by Szilard, is generally credited as being the one who advocated that the Frisch-Peierls memorandum was delivered to and taken seriously by the highest officials in the British government. Similarly, he is credited with taking his knowledge of the progress made by the MAUD committee and sharing it with the United States before securing explicit permission to share all of the details. Dr. Oliphant as well as Frisch and Peierls no doubt played a substantial role in creating the world we live in today.

2.5.2 Father of Stealth: Pyotr Ufimtsev versus Denys Overholser

Consider next the case of Pyotr Ufimtsev who is inadvertently the godfather (or maybe the great grandfather) of stealth technology. Ufimtsev was the author of a little-known and seemingly unimportant paper which laid the groundwork for the advent of stealth technology. Ufimtsev authored the paper during the Cold War and the Soviet censors did not think the information contained within was critical enough to warrant export control. Thus, as part of the normal process of information gathering from within the Soviet bloc, agents in the United States acquired the paper and translated it for its potential scientific value.

The now famous Skunk Works arm of the Lockheed Corporation was working on various technologies related to defeating and overwhelming the U.S.S.R. Products of the Skunk Works department included the SR-71 Blackbird which could fly higher and faster than any Soviet defense systems as well as the U-2 Dragon Lady spy plane which overflew sensitive Soviet sites for years before its existence was even confirmed [125]. The folks at Skunk Works had a habit of developing technology that was just a little bit better than what the Soviets had.

In the case of stealth, the Soviets actually made the most important discovery related to stealth technology arriving in operational aircraft. The equations and methods proposed by Ufimtsev formed the basis for a primitive computer simulation developed by Denys Overholser an engineer at Lockheed. The computer simulation was later used by Overholser and other Lockheed engineers to design the highly faceted and unusual fuselage of the Have Blue demonstrator which was the precursor to the aircraft known as the F-117 Night Hawk. The F-117 Night Hawk introduced the world to stealth aviation [125]. In the years that followed, military planners and airplane designers in the United States would leverage stealth technology on many other aircraft including the F-22 Raptor, F-35 Lightning II, B-2 Bomber, and others. If the Soviets had not freely released the paper by Ufimtsev, then it is possible that Lockheed never would have developed the computer program which set off the stealth revolution.

In this case, failure to recognize the important or the potential implications of a seemingly boring or esoteric scientific finding and its dissemination allowed a critical piece of information to become available to engineers at Skunk Works. The driving force behind how that technology was used was twofold. Firstly, the Skunk Works department was always trying to demonstrate a competitive advantage over their rival defense contractors and no one else could deliver stealth technology. Secondly, the driving force behind defense development at large during the Cold War was chiefly related to defeating the Soviets. Ironically, the technology which would make Skunk Works competitive and help the U.S. defeat the Soviets finds its headwaters deep within the confines of the iron curtain. Popularly, Ben Rich, who was the head of the Skunk Works department, gets credited as "the father of stealth", when in fact, the true father of stealth could easily be considered to be the man responsible for the fundamental work upon which the Skunk Works stealth computer program was based.

2.5.3 German V-2 Rocket

Walter Dornberger was the chief of the German Army Board of Ordnance rocket development from its creation in 1931 [126]. After the war, Dornberger came to the United States and built a career as a technical consultant [126]. Dornberger makes the case that historically, when the time is right, individuals all over the civilized world will find themselves working on the same problems such as was the case for rocket technology in the 1930s. Dornberger lists several scientists from all over the globe who were involved with work similar to that which eventually led to the development of the V2 rocket [126].

Of course, the Germans were the first to bring rocket technology to bear and Dornberger endeavors to explain how this happened. The reason the Germans invested in the unproven rocket technology, it turns out, is the same basic reason for countless technological discoveries throughout history. The Germans were interested in developing a superior weapon systems by using rockets to deliver munitions with the added wrinkle that many kinds of technological development would be off-limits according to the Treaty of Versailles [126].

Contrary to popular contemporary understanding, Dornberger insists that the development of the A4 rocket (which would become the V2) was accomplished by a dedicated group of scientists, engineers and technicians working to realize the dream of rocket propulsion and the military applications of that technology where viewed simply as a funding source [126]. The famous name of the weapon system "Vengeance Weapon 2" was coined by Hitler, but Hitler's acknowledgement of the project and understanding of the potential uses of the rockets being developed by Dornberger's team didn't occur until 1943 [126]. Up to 1936, funding for the fledgling rocket research was funneled from various research programs by Major General Karl Becker, who in 1936, informed Dornberger "If you want more money, you have to prove that your rocket is of military value." [126]. Up to this point, most of the team's development work had been on rocket motors on static test stands with some success in developing subsonic sounding rockets.

It was after this fateful discussion that Dornberger, "an old long-range artillerist" (by his own description), laid out the mission parameters for the so-called "large rocket" as well as various other potential use cases for rocket technology including rocket-assisted-takeoff for aircraft, rocket propulsion for aircraft, and rocket propulsion for very heavy artillery shells [126]. Dornberger based his notional "large rocket" mission on his knowledge of artillery and made comparisons with the famous Parisian gun. In order for the rocket to have a true military use, Dornberger envisioned a weapon with a massive range (longer than the Parisian gun) but which could be much smaller and lighter and substantially more accurate [126]. Dornberger also indicates that some of the problems related to building the V2 required the engineering team to build solutions which defied conventional understanding at the time. For example, it was a "proved fact that an aerodynamically-controlled body could not fly stably at supersonic speed" yet the rocket envisioned would go on to prove this notion wholly incorrect [126].

When addressed with the prospect of assigning credit for inventing the V2, Dornberger insists that many individuals played a role and that the workmanlike, step-by-step approach of all involved, their faith in their work, combined with a good measure of luck were all essential in the development of the V2 [126]. Interestingly, Dornberger is of the opinion that a similar group of people, working with the same constraints and motivations, under the same conditions and with the same work ethic as his team, necessarily would have developed a similar solution [126].

Considering Dornberger's knowledge as an insider in the German Army, his rebuttal to conventional understanding of the forces behind the V2 rocket and his opinion on the trajectory of the war and its causes are of particular interest. Dornberger states that Hitler did not support the engineers and scientists working on the V2 rocket until

1943 when it was far too late for the proposed weapon system to have any impact on the war [126]. In fact, it was the volume of aircraft production in the United States which would eventually lead to air supremacy over Europe and Africa combined with the inability of the Germans to protect their fuel supply from allied attacks which set the stage for the German defeat [126]. Dornberger indicates that unfortunate predictions with respect to anti-aircraft technologies and high speed interceptors led to a $2\,\mathrm{yr}$ to $3\,\mathrm{yr}$ development delay for these technologies which could have changed the tide of the war in favor of the German Army [126]. Without a way to protect against the ubiquitous air assault from the allies, driven by the high volume of aircraft production in the United States, the Germans could only fall further and further behind.

Dornberger also reiterates that despite popular misunderstanding related to Hitler's name for the rocket as well as for the rockets use once the fate of the Germans was mostly sealed, the V2 rocket was developed to be like a long-range artillery gun [126]. The purpose was thus to deliver a shell over a vast distance, but in the case of the rocket, with substantially improved accuracy and smaller logistical footprint compared to a conventional gun [126]. Again, Dornberger reiterates that if the German Army had invested in anti-aircraft technology or invested more heavily in the V2 rockets allowing them to be used more effectively on the European continent, and if the devastating bombings of the German industrial base could have been prevented, then the trajectory of the war could have been much different [126].

The paper by Dornberger represents a case study on the development of the V2 rocket as well as on the trajectory of WWII especially the war in Europe. By Dornberger's estimation, failure to foresee the ability of the United States to deliver massive quantities of aircraft for the allied bombing efforts and thus a deprioritization of effective anti-aircraft technology led to an inability of the Germans to protect tactically important sties. Dornberger's asserts that the difference between victory and defeat could have been earlier development of the V2 by a few months. Additionally, the observations of Dornberger, seem to illustrate the importance of effective anticipation of enemy capabilities so that development of counter-measures can be undertaken in the most efficient way possible with no allowance for delays.

2.6 Military Acquisition Programs

The following sections will provide discussion of military acquisition programs by various agencies within the Department of Defense in the Unites States. Acquisition programs can be hot-button political issues, and in response to budget cuts and changing requirements, various programs have been canceled in hotly contested and drawn out battles over the years. Conversely, hundreds of successful programs have also been launched and completed without accruing infamy. Roughly, most acquisition programs follow a template such as the following.

- **Recognition of need:** the need can be technological or strategic, the need can exist now or in the future, prediction of the future battle space can be an integral part of this step.

- **Determination of program requirements:** at this point, force needs for the future (or current) battle space are translated into requirements documents in preparation for some kind of acquisition program (often a competitive bidding process, or a demonstrator (prototype) will be developed in response to this step).

- **Execution of the program:** this stage varies depending on the type of acquisition strategy being used, this stage can include various milestones for testing along the way but roughly covers the period beginning at the development of a solution and culminating with the full-rate production (or cancellation) of the specific solution.
- **Maintenance of the program:** this stage represents the long-term sustainment of a given program, often new requirements are added, often systems are adapted for battle space realities, substantial differences can exist between the program as maintained and the program as planned during prior stages.

Failure to correctly predict the future battle space accurately is a major cause of failure in terms of defense acquisition systems. Great risk is also present when transitioning programs from the prototype or need definition phase through to the full-rate production within the execution stage. Such risk can be mitigated by carefully designing acquisition programs and by continuously comparing the program being developed with the current estimation of needs. Once a program enters the maintenance phase, little risk remains of cancellation; although, many programs at this stage will essentially enter a new development cycle as an improvement program which seeks to increase capabilities without the substantial risk incurred by opening a proper new acquisition program.

Many of the case studies which are to be presented suffer from a similar problem related to the collapse of the Soviet Union during the 1990's. With the collapse of the U.S.S.R. and the subsequent switch to focus on dual wars in the Middle East which pitted U.S. forces against lower capability insurgent forces (as opposed to a sophisticated near-peer state actor in the U.S.S.R.), a drastic paradigm shift in all aspects technological and strategic occurred. Programs designed for the battle space prior to the collapse of the U.S.S.R. were largely inapplicable to the battle space reality presented in the Middle East.

There are many causes for the fall of the Soviet Union. Discussion of all of them and comparing and contrasting explanations is beyond the scope of this work. Despite the discussion that follows, the projection of power by the United States military and the United States defense industry certainly added pressure to the situation. Thus, despite the fact that many high profile acquisition programs were deemed failures after the need to acquire the technology dissolved along with the U.S.S.R., it is likely that the process of developing at least some of these programs contributed to defeating the Soviet Union by lending credibility to projections of U.S. power.

Any of the case studies presented should not be thought of as a critique of any of the specific programs or acquisition strategies. This manuscript is only concerned with the mechanics of acquisition program design insofar as they involve the intersection of the technological threat space and the technological defense system space. The case studies are presented to get a cross-section of the things that can go wrong when the future or current battle space is not properly understood as well as to get an idea of how long programs should be expected to take so predictions of the future battle space can be made on the appropriate time line to ensure programs in development will remain relevant in the future.

2.6.1 Mosaic Warfare

Mosaic warfare is a modern military concept whereby the appropriate force make-up for a given scenario is composed in near real-time from a collection of small interchangeable pieces (like the tiles which make up a mosaic). In a nutshell, the doctrine of mosaic warfare encourages modularity among component systems such that they can be combined arbitrarily in whatever configuration best suits the perceived threat at the time of need [127]. Recently a study utilized the tenets of game theory to analyze the basic premise of mosaic warfare using the Colonel Blotto game [128].

The results of the aforementioned study indicate that the applicability of mosaic warfare does not necessarily trump brute force when brute force is the most appropriate way to succeed in a given conflict [128]. However, as expected, for small, specialized, missions, the modularity of mosaic technologies can offer a substantial advantage. Additionally, resiliency can be greatly enhanced by applying mosaic concepts because the importance of the failure of any single component to the whole mission is greatly reduced [128]. Mosaic warfare, and modularity, according to this particular study, might not represent the true silver bullet in the sense that at the very least more research is needed to determine if mosaic warfare can supplant a larger, vaster or better equipped force in a variety of threat scenarios.

2.6.2 F-22 Raptor Program Termination

The F-22 Raptor provides an illustrative example of a program canceled as a result of the fall of the U.S.S.R. The requirements for the program were first conceived in the early 1980's and the aircraft was designed to compete with top-of-the-line fighters out of the U.S.S.R. at the time. Additionally, focus was placed on stealth considering that a flare-up of the Cold War into outright war would likely require deep penetration missions over parts of Europe and Asia. Obviously after the U.S.S.R. fell, the need for a deep penetration mission mostly vanished [129, 130].

Additionally, by this point the program was beset by budget overruns and delays. This is a common pattern in acquisition programs. The aircraft that the F-22 was designed to square off against, were no longer a threat and the expensive jet was ill-suited to any of the missions demanded by the wars in the Middle East. Thus in 2009, the F-22 program was terminated in order to purchase the favor required to secure funding for a new (as yet mostly secret) stealth bomber program [131, 132].

2.6.3 F-35 Joint Strike Fighter Program up to 2021

According to a report published by the Library of Congress Washington DC Congressional Research Service in 2012, "the [joint strike fighter] (JSF) began in the early to mid-1990's" [133]. Although a footnote indicates that the Joint Operational Requirements Document was not issued until 2000, the program which would eventually combine requirements from the U.S. Air Force, the U.S. Navy and the U.S. Marine Corps (as well as cost sharing international partners) was borne out of the cancellation of independent efforts and interest by the separate agencies to meet specific operational needs [133, 134].

Despite a variety of setbacks, the JSF program has delivered operationally capable jets for all three missions with demonstration flights of the most complex B variant (featuring short take-off and vertical landing capabilities) taking place in 2010, only a few months behind the original schedule [133–135]. Problems with the JSF program are representative of problems which are encountered by other military acquisition programs. These problems include: higher than expected total programs costs [135–137], lower than expected readiness [135, 138], unexpected technical issues [139–143], and skyrocketing sustainment costs [138, 144]. Such persistent problems have some concerned that the JSF program could be subject to cancellation like several high profile military programs from the past [136, 145] with some pundits going so far as to declare the program a "failure" [146, 147].

As this manuscript is being written, in the year 2021, the decision on whether or not the program will be approved for full rate production has been deferred [148]. Taking 1995 as the approximate year in which development began in earnest, the technology was demonstrated in 2010, after 15 yr. Now in 2021, 26 yr after the program formally began, and 21 yr after the official requirements for the program were issued, the JSF is still not approved for full rate production. There are a number of reasons for this long time line; although, an exhaustive discussion of such reasons is beyond the scope of this manuscript.

The selected history provided in this manuscript is not meant as a critique of the

JSF program nor is it meant to discuss in detail any of the technical or procurement challenges associated with the program. There are ample resources which do a fine job explaining the program in detail such as [135–137, 148, 149]. The history provided here is meant as a demonstrative example of what should be expected in terms of duration between the initial development of a requirements and the arrival of the solution in terms of a full-rate production, fielded solution. In this case, the JSF program has delivered on many promises of the program but has not achieved all mission readiness goals and is still not in full rate production around 26 yr after the initiation of the program [135, 148].

2.6.4 RAH-66 Comanche Development

The RAH-66 Comanche began as the Light Helicopter Experiment (LHX) program with a focus on developing an armed, stealth, scout and attack helicopter [130]. The program was originally conceived during a doctrinal shift in Army policy in response to a future battle space which was to be occupied by the U.S.S.R. (or other Warsaw Pact nations) [129]. Military planners postulated that winning a war over the European continent would require technologies which could allow deep penetration into contested terrain, thus stealth would be a critical part of the LHX program [129, 130].

During the course of the RAH-66 development, which began as the LHX in 1982 through the eventual cancellation of the program around the year 2004, several important changes were noted in the requirements dictated by the battle space. The dissolution of the U.S.S.R. saw the requirement for penetration over contested or hostile terrain (and thus the focus on both the armament and the stealth aspect of the RAH-66) become less important [129]. To keep the program alive, various restructurings were accomplished which saw the number of delivered aircraft reduced as well as multiple modifications to the technical requirements. Eventually, the RAH-66, which was to complement the AH-64 Apache, became a direct competitor with the AH-64 Apache in terms of resources for development and maintenance [150]. At one point, the original plans for an armed scout helicopter were modified to help keep the program relevant by placing a large focus on integrated intelligence, surveillance, and reconnaissance (ISR) technologies which essentially required a midstream redesign of many aspects of the aircraft [150].

The program lasted about $22\,\mathrm{yr}$, but one main difference with the RAH-66 cancellation versus many other case studies in defense acquisition of controversial programs, was that the RAH-66 program died without even a whimper and the cancellation was largely uncontested. There are several reasons for this. The most important reason being the shift in battle space, driving a change in requirements, and the direct competition between the dollars required for RAH-66 design and the dollars required for improvements which could save troops deployed in the Middle East [129, 130]. Another important reason the cancellation was allowed to move forward without substantial push-back is that the helicopter industrial base was healthily employed providing maintenance and upgrades to the existing fleet at the time of cancellation [150]. When the RAH-66 program originally came under fire, even though the requirements were no longer needed because the Warsaw Pact was no longer in existence, there were no other immediately obvious requirements, and the helicopter industrial base would have been left idle had the program been canceled. This forms a partial explanation for the initial energetic attempts to extend the capabilities of the RAH-66 program to ensure it would continue in development even as the battle space was obviously changing [150].

The RAH-66 program and its eventual cancellation are notable in a sense for the expert work of the program management team and their willingness to adapt to changing needs as well as to be reasonable in terms of adjusting program expectations [130, 150]. This allowed the program to exist long enough to sustain the industrial base during a time when helicopter production was low as well as to gracefully cancel the program at the point other priorities became more important [150]. Ultimately, the cancellation of the RAH-66 program had mostly to do with the dissolution of the U.S.S.R. which represented a substantial paradigm shift in terms of required tactics and technology for the current and future battle space. The wars in the Middle East drove home the point that the battle space had changed and forced a decision to be made in terms of allocating limited resources. The armed scout/attack helicopter was deemed less critical in 2004 than originally planned in the year 1982. It is unlikely technological predictions could have helped save the RAH-66 Comanche as the paradigm shift was driven by a political event rather than a technological watershed.

2.6.5 Seawolf-class Submarine Program

The Seawolf-class submarine program, beginning with SSN-21 Seawolf, began in 1982 after government studies revealed that the Los Angeles class submarine could no longer absorb planned improvements to meet the doctrinal and technological needs required to defend against the anti-submarine warfare (ASW) threat from the U.S.S.R. [151].

The primary mission for the planned submarine was to track down Soviet ballistic missile submarines [151]. Much like the RAH-66 program, the SSN-21 design placed a large focus on stealth (in this case, acoustic stealth) to avoid detection while operating deep within contested regions [151].

The requirements for SSN-21 ultimately involved several different high-risk technological developments including the capability to operate in deeper water (which required updated steel), hull strengthening (for operation under the Arctic ice-pack), a new quiet propulsion system, and various electronics systems [151, 152]. This type of development risk ran contrary to existing best-practices at the time which dictated that only one major technological innovation should be introduced for any acquisition program [151].

In the midst of the development of SSN-21, the battlefield requirements changed firstly in the form of changing Naval doctrine and later, as was the case with the RAH-66 and the F-22, because of the dissolution of the Soviet Union [151]. The changing battlefield requirements, coupled with the unusually high risk associated with the myriad required technological innovations, put the program in a bind. The acquisition program for SSN-21 was special because it was explicitly and publicly designed in part to prop up the nuclear submarine building industrial base [151]. This caused the Navy to make more unusual decisions such as starting building before the design process was mostly complete as well as splitting the design and building tasks between two vendors [151].

Ultimately, the Seawolf-class was canceled after the delivery of three ships out of a planned 29. The first ship was delivered in 1997 with design and building lasting 15 yr. The final ship in the reduced program was delivered in 2005, 23 yr after the programs inception. The unit costs and maintenance costs associated with the class skyrocketed and the problems appeared worse considering the number of units was reduced from 29 to 3 [151–153]. Ultimately, the cancellation was a political football and at one point the conventional wisdom in Washington D.C. was that canceling the program (the original 29 ship plan) would somehow cost more than paying for delivery of the remaining ships in the planned program [153]. Unlike the RAH-66 Comanche program, the reduction and finally the cancellation of the Seawolf-class was politically divisive. Stake holders on both sides fought to keep the program in-place in order to prop-up the industrial base and the eventual cancellation was only finally accomplished when the industrial base was properly supported by new submarine acquisition programs [151, 153].

In contrast to the RAH-66 cancellation, the SSN 21 cancellation was a hard-fought battle owing in-part to the fact that no new work had arrived to keep the industrial base busy in the meantime. If a new submarine need had arisen in the midst of the cancellation which could have allowed the program to die gracefully while relieving concerns that those employed in the industrial base would be laid off, then most likely the cancellation would have been less contentious.

2.7 Commercial Development Cycles

The term “drone” was first used to refer to a UAV (or using the parlance of the day remotely piloted vehicle, (RPV)) during the 1940’s [154]. For many people alive today, awareness and discussion of military UAVS or “drones” has always been and will always be a normal part of day-to-day life. While many UAV technologies are sold as ground-breaking or the stuff of science fiction, the reality is that UAVs were proven materially useful long before many of those currently alive were born. In fact, the de Havilland DH.82B Queen Bee is considered the first modern "drone" or UAV. Many of the first instances of UAVs were used exclusively for target practice as was the case for the Queen Bee and her immediate descendants [155]. In the 1970's and 1980's the military of Israel used UAVs to great effect during the Yom Kippur War as well as during the Lebanon War [156, 157]. More contemporaneously, the MQ-9 Reaper and the earlier MQ-1 Predator are what many people automatically associate with the term "drone" specifically with respect to military applications [158].

Usage of the term "drone" as well as the mission space for UAVs has expanded since those first target drones were developed a few years before WWII. The terms UAV and "drone" have become so diluted in recent years in-part due to the rapid proliferation of commercial and industrial as well as consumer UAV technology. Technologies once the purview of science fiction authors and military commanders with endless resources, are becoming available in accessible flying platforms which have almost limitless applications. The products are readily available, easy to fly, and inexpensive enough to be owned by just about anyone. Motivation for documenting and understanding the proliferation of such technologies is two-fold.

Firstly, the rapidity of the proliferation of technology within the UAV sector is not unique to the UAV sector. Understanding how this particular technology has advanced so far in such a short time offers side benefits related to understanding how other high-tech products might develop and disrupt existing paradigms. Secondly, the UAV sector is not done developing. In fact, the UAV sector has become sort of a test bed for a variety of science fiction ideas which are rapidly deployed and tested using inexpensive or commercially available ready-to-fly platforms. Being prepared for where this technology sector might go next is an important step in staying at least one step ahead of those who would use UAVs for evil.

2.7.1 Unmanned Aerial Vehicles

The rapid emergence and maturation of UAV technologies has been a disruptive force in many industries. Defense, military and related industries have not been immune to this trend. Various agencies within the Department of Defense (DOD) have conducted studies, developed uses or deployed UAVs with varying levels of effectiveness. A 2014 study investigated the use of UAVs to be used in a remotely piloted modes (not fully autonomous) [159]. The study was constructed based on a notional mission derived from existing missions flown by the MQ-9 Reaper and importantly did not consider jamming of the global navigation satellite system (GNSS) or command and control (C2) systems on-board the remotely piloted UAV [159]. The C2 data links between the UAV and the operator in many cases form a critical weak point. Thus, disruption of UAV missions could likely focus on interfering with communication. Of course, fully autonomous systems would be less vulnerable to such attacks.

In contrast, GNSS jamming and disruption can be harmful even to fully autonomous

UAV missions. While on-board or remote human pilots demonstrate resiliency, which allows navigation to be accomplished in GNSS denied environments, remotely piloted or autonomous UAVs are not typically as flexible in response to such challenges. Much of the rapid proliferation of UAV technologies, specifically in the consumer technology space, is directly related to the ability of small, inexpensive UAVs to essentially fly themselves with little-to-no pilot intervention. In many cases, such flight modes rely entirely on GNSS navigation for autonomous navigation and position hold functionality. There are exceptions, and some systems can use optical flow or other auxiliary sensors (such as RADAR, LIDAR, and SONAR) for low resolution positioning once GNSS communication is lost. However, typical UAV systems rely heavily on GNSS signal for navigation.

Several studies have been undertaken since UAV technology first began to emerge to help understand the variety of roles such technology could take on in a future war. Some research indicates several types of notional mission profiles for UAVs to include ISR, payload drop/delivery, and one-way destructive missions [160]. Of these missions types, the ISR mission profile simultaneously involves the lowest risk to the UAV, requires the most modest UAV capabilities, and offers the least immediate impact in terms of threat [160]. That is not to say that information gleaned from ISR missions is not important; however, the type of threat offered by a one-way mission for a UAV is much more immediate and easy to visualize.

Again, the vulnerability of C2 and GNSS systems is underscored with the caveat that C2 systems represent one area where technology may improve rapidly and unexpectedly in the near future [160]. The easiest to accomplish ISR mission profile is also most likely the most vulnerable to GNSS or C2 jamming, unless surveillance information is to be collected later, after the UAV has landed rather than downloaded live from a live data link [160]. One of the biggest challenges most UAV threat missions represent to traditional defense systems is that they can be nearly impossible to detect due to their small size [160]. Of course, larger UAVs can carry more dangerous payloads for one-way or payload drop missions; however, as UAV size increases, the ability to detect them early and engage them using more traditional defense systems (such as C-RAM or CIWS) becomes much more straightforward.

Over the last decade or more, as UAV technologies have come online and revolutionized various marketplaces (including military aviation, and consumer technology spaces), a heavy focus has been put on the idea of a fully autonomous so-called “swarm”. This verbiage typically implies a large group of UAVs acting together to accomplish a mission. The language “swarm” is rather imprecise because it is used differently within various industries and often describe substantially different behavior dependent on the context. In the interest of clarity, a swarm and its characteristics are defined here explicitly. Firstly, a swarm may be “cooperating” or “non-cooperating” which describes the degree of coordination among the swarm agents. Cooperating agents will use live data exchange to adapt and modify individual trajectories to ensure mission objectives are being met throughout the course of the mission. Non-cooperating swarms require little-to-no live data capability as individual mission contributions will be pre-programmed or require only basic, local information about nearby agents. Non-cooperating swarms are unable to adapt the mission profile intelligently to large changes in circumstance which may occur while conducting the mission.

Swarms may be “homogeneous” or “heterogeneous” which describes the make-up of the agents which comprise the swarm. In a homogeneous swarm, all agents are approximately similar in terms of capabilities and mission objective. A heterogeneous swarm could make use of distinct UAVs to accomplish various different mission profiles. For example, battle damage assessment, C2 relay, and ISR missions may use smaller, less-armed and less-capable UAVs whereas one-way delivery tasks within the mission could make use of much heavier, bomber-like UAVs. A heterogeneous swarm can make much more efficient use of UAVs to accomplish a complex mission profile.

Within the realm of cooperating swarms there are two paradigms: “centralized” and “de-centralized” planning. With centralized planning, a single source is responsible for coordination among agents. Severing the connection between the central planning station and any swarm agent can alter the outcome of the mission and may force individual agents to revert to non-cooperating behavior (as a result of lost communication with the coordinating agent). This can also be called the “mother ship” type swarm. Disabling the mother ship in this situation could severely limit the capabilities of the swarm by converting a coordinated mission to a non-coordinated one, and in some cases, could even cause the mission to be aborted or otherwise fail altogether. The de-centralized planning paradigm makes use of each agent in the swarm to solve the planning and coordination problem partially and cooperatively. Disabling any one agent cannot cause the entire swarm to fail or change behavior because the planning and coordination is distributed among the agents. Disrupting the coordination in such a case would involve attriting the swarm to the point that the force required is no longer sufficient to accomplish the prescribed mission.

The most potent threat which can be offered by a so-called “swarm” (and what many people think of when they hear the sci-fi-like term) involves a coordinated, heterogeneous, de-centralized swarm. This is the case where the mission can be adapted in real-time, and attacking any individual agent cannot change the behavior of the swarm substantially. Fortunately, the technology and knowledge for fully realizing such a threat is less readily available than the technology for accomplishing lesser “swarm” threats.

An interesting report for the U.S. Air Force investigates how contemporary technology might be used to leverage the benefits of a large force of UAVs without waiting for true coordinating swarm technology to fully come online [161]. The case study interestingly explores the logistical footprint required to deploy approximately $1{,}000$ UAVs to be used to develop a targeted surveillance curtain around an asset such as a Navy vessel passing through a narrow strait [161]. The important takeaway from the analysis presented is that even at the current technology level, and requiring no further sci-fi developments, inexpensive UAVs could be used to greatly enhance ISR or C2 capabilities in an adaptable and targeted manner. No coordination, no artificial intelligence, and no machine learning are required to realize the scenario envisioned in this report as current UAV technology is simply applied en-masse using traditional military techniques [161].

At this point, UAVs usage in warfare has been ongoing for a number of years and public documentation of some specific threats is available for study. The most primitive usage of UAVs documented essentially consists of employing small UAVs to carry payloads for one-way destructive missions. Such missions are often undertaken using GNSS guidance over a live C2 link and can use a remote human pilot or a pre-planned mission profile. Missions as described make use of existing improvised explosive devices (IEDs) which can be remotely detonated [162, 163]. Such IEDs may be constructed from raw materials or from existing explosive ordnance which have been adapted for remote detonation [162, 163]. The most easily adapted IEDs for UAV deployment will be lightweight, such that small, likely multi-rotor UAVs, will be capable of accomplishing the mission. There are cases of other types of vehicle threats including documentation of an autonomous boat outfitted with IEDs designed to explode after colliding with another vessel [164]. The explosive payload and remote-control or autonomous guidance systems are easily adapted to a variety of vehicles and it should be expected that remotely piloted or autonomous vehicles such as boats and ground vehicles will be used the same way aerial vehicles have been used.

One-way destructive missions have been used by the Islamic State [165]. Such threats are essentially conventional IEDs which have been affixed to UAVs [165]. Such use cases have also been documented being used by Houthi insurgent forces in Yemen [166]. Using IEDs in this way essentially restricts the total explosive payload according to the capabilities of the UAV. Small UAVs which may be better suited to ISR mission profiles can be adapted to such missions but they can only carry grenade-sized explosive payloads. Whereas, large gas engines allow much heavier explosive payloads to be carried and the flying IEDs are much more closely related to loitering munitions (or cruise missiles) than the inexpensive UAVs typical on the U.S. consumer market [165, 166].

UAVs are already being used by U.S. and other militaries around the world. The aforementioned MQ-9 and related programs such as the MQ-1 predator as well as RQ-4 Global Hawk represent ISR and attack mission capabilities available to U.S. Armed Forces. Conversely, flying IEDs are mostly a threat associated with non-state, or insurgent actors.

Near-peer adversaries have also been developing UAVs for various mission profiles, and one report specifically discusses development within the Chinese military to use UAVs to enhance Naval operations [167]. Considering the Chinese GNSS network is not as developed as GNSS systems in other parts of the globe, thus UAVs represent a particularly inexpensive opportunity for rapid expansion of precision global positioning capabilities without all the associated infrastructure and long timeline associated with traditional GNSS matriculation [167]. The report also puts a special emphasis on ISR usages to improve over-the-horizon targeting and C2 communication relay applications specifically for maritime missions. An interesting note in the report points to lack of export restrictions and globally competitive prices as factors that will cause any UAVs developed in China to find their way quickly into other parts of the world [167].

Although, slightly dated, having been published in 2010, a report by the RAND corporation is particularly interesting for the way in which UAV missions are categorized. The report cites the dangerous, dirty, dull, demanding, and different mission descriptions as a perfect fit for UAVs for various reasons [168]. The categorizations used in the report have been embodied in real-life. UAVs have used for long-term surveillance missions where the crew can be periodically swapped out (dull missions). UAVs have been used for infrastructure inspection in environments which are hazardous for humans (dangerous missions). At the time of its publication, the report did not perceive penetrating strike, or close air support as viable UAV missions, and likewise, pointed to C2 down link delays as reasons UAVs would continue to be inappropriate for air-to-air (dog fighting) for the foreseeable future [168]. The most concerning mission profile presented in the report is captured by the description "different" because such missions represent a paradigm shift inherently. UAVs may be appropriate for missions which have not been fully envisioned because the technology has not existed previously.

In an effort to understand the pathways taken by various transformative technologies, consider the following case studies.

2.7.2 Terrain Following for UAVs

Briefly consider the academic publication record related to terrain following UAVs. Frequently vision sensors are used, so the development of inexpensive optical sensors and optical flow algorithms was a prerequisite to this technology being demonstrated in the academic realm. A 1991 paper discusses challenges and techniques for so-called "nap-of-the-earth" flight specifically for manned rotorcraft [169, 170]. "Nap-of-the-earth" flight refers to flight close to terrain which makes it difficult or impossible for enemy radar systems to detect and identify aircraft. Flight paths may make use of natural terrain such as trees, mountains, canyons and rivers as natural cover and generally avoid being caught silhouetted against a clear sky. Detection of low-flying aircraft against a cluttered background is substantially more difficult compared to detecting aircraft against a clear sky.

Later works propose innovations in vision navigation including using less expensive sensors [171], and eventually many innovations were combined into a demonstration aircraft which was capable of nap-of-the-earth flights with minimal human intervention in 2002 [172]. Such technology is not uniquely suited to UAV usage. Any pilot augmentation for hovering aircraft can offer immediate benefit to rotorcraft pilots especially in low visibility environments and when landing. The development of vision-based obstacle detection and avoidance in real-time for UAVs was essentially accomplished at the level of a low technical readiness level (TRL) around the year 2002 [172]. Sometimes vision sensors have been used as a replacement for other absolute position determination methods like GNSS such as in 2009 [173]. Improvements to the initial demonstration of terrain following UAVs was made throughout the 2000's [174–182].

Next, consider the appearance of terrain following in the consumer UAV market. Many of the technologies which will be discussed in this section, and those sections that follow, were first seen on drones made by the company DJI. This is not a coincidence considering DJI is both the leader in market share in terms of commercial UAVs, as well as being a technology leader. There have been many other companies especially in the United States which have attempted to compete with DJI but have been unsuccessful. The Phantom 4 from DJI is the first widely available consumer-grade UAV which was capable of terrain following [183]. The aircraft was released in 2016 [184], and the terrain following available was unsophisticated and safety was prioritized over features. The earliest terrain following algorithms from DJI allowed altitude adjustment when ascending but were not capable of adjusting altitude downward to follow descending terrain [183]. Currently terrain following is available on a number of commercially available UAVs. Some systems using external sensors allowing for very precise positioning which facilitates flying with sensor packages attached such as ground penetrating radar which require precise and repeatable positioning above the ground [185].

The most succinct version of the history of terrain following traces a path from the first academic discussion of the challenges associated with nap-of-the-earth flying in the 1990's up to the first unmanned demonstrator of terrain following which appeared in the early 2000's. Finally, about 15 yr later the technology arrived in consumer-grade UAVs. Currently in 2021, several consumer-grade drones are available and the terrain following sensors and algorithms have become more sophisticated allowing for closer proximity flying to obstacles in more challenging environments. Approximately 20 yr after the first academic demonstration aircraft, the terrain following technology is mature enough to have an impact in threat scenarios.

2.7.3 Swarm Technology for UAVs

The origins of UAV swarm technology in the academic setting can be traced to trajectory planning and collision avoidance for generic multi-agent systems. Often such systems were comprised of multiple robotic arms in a single manufacturing cell or even multiple joints or effectors comprising a single robotic arm. Many innovations which would later be applied to UAVs were thus developed based on an industrial use-case.

Some of the earliest work related to the multi-agent problem are specifically cross-over works which seek to transfer lessons from the industrial collision avoidance problem over to the problem of mobile robots which first appeared around the year 1990 [186]. Recognizing some of the unique characteristics of the problem of multiple mobile robots (as opposed to constrained robotic arms or joints in 1993 [187] and 1996 [188]) led to further work on control laws designed specifically for mobile robots [189]. Many proposed control schemes involved some hybridized approach which mixed centralized planning and task allocation with some form of real-time lower-level planning on-board individual agents in the 1990's and early 2000's [186, 190]. Various other techniques were also proposed and some implemented [191–194]. Other papers focused on specific demonstration programs [195] or specific implementations and enabling technologies [196–200].

The proper beginning of swarm technology in the academic realm is around the year 1990 when multi-agent planning and collision avoidance technologies first made the switch over from robotic arm systems research into the realm of mobile robots. About $10\,\mathrm{yr}$ later, proper swarm technology was researched and demonstrated on-board unmanned spacecraft as well as various UAV systems starting in the early 2000's [201].

Around the year 2016, several companies demonstrated implementations of UAV swarm technology specifically for the creation of light shows using UAVs carrying light arrays flying against a dark sky as a canvas. The algorithm for planning such shows, and indeed the first demonstration of such technology, was seen in 2012 [202]. Intel was able to demonstrate a much more sophisticated light show system in 2016 [203]. The newest implementation by Intel greatly reduces the logistical footprint and uses artificial intelligence to lessen the pilot workload, both before during, and after, the show. Additionally, though the earliest system involved months of planning for simple shows [202], the updated Intel system allowed planning complex shows by a small team over a few days of time [203]. In the ensuing arms race, several major players including Intel have recently taken to setting and resetting the record for the largest number of drones to be simultaneously flown, with the most recent showing in support of a luxury car brand launch in the Chinese market setting the record at $3{,}281$ aircraft [204].

The first flying swarms were present in academia around the year 2000, though theories about such systems,s as well as some demonstrations, existed several years prior (around the mid 1990's). Consider the year 1998 to be the year the first large-scale flying demonstration was accomplished [195]. Then consider that in 2012 a company was able to demonstrate a turnkey drone light show system involving swarm technology. The technology was improved and mature by the year 2016, and now many companies offer similar products with similarly small logistical footprints considering the technological challenges associated with drone light shows. From the first appearance of a viable demonstrator at a low TRL to a commercially viable, mature solution took about $18\,\mathrm{yr}$ (1998-2016).

2.7.4 UAV Obstacle Avoidance

The next technological case study to be considered concerns UAVs using vision for navigation and specifically for obstacle avoidance for autonomous flying in cluttered environments. Like the terrain following example before, much of the work done on this topic stands on the shoulders of work done to enhance the quality of vision-enabled hardware and optical flow algorithms. Thus, the discussion begins with papers concerned with vision on UAVs including work to enable planar flight [205] in cluttered environments, as well as 3D flight in cluttered environments [206]. A specific focus is placed on solutions based on biomimicry as well as low-cost solutions with the first papers appearing around the year 2000 [205, 207, 208].

Some work focused on specific autonomous maneuvers, specifically landing [209–212], and then other works focused on specifically difficult applications where obstacle avoidance could have a large impact on productivity and outcomes such as power-line inspections [213]. Later work refined earlier ideas and laid out algorithms for obstacle avoidance as well as for using vision in multi-agent flights for relative aircraft position and orientation information in the mid 2000's [214–216]. Other works focused on tracking UAVs using vision systems either to be used for tracking by ground-based systems [217, 218] or for use by flying systems to maintain separation from other flying systems [219].

Again the ubiquitous DJI is generally regarded as being one of the first to bring an easy-to-use obstacle avoidance system to market with the technology first appearing on the Phantom 4 which was released in 2016 [184]. The obstacle avoidance systems present in 2016 have been surpassed in terms of quality, reliability, and effectiveness. Currently the systems offered by DJI on the Mavic Pro 2, as well as the systems on the U.S.-made Skydio 2, are much more capable compared to the initial offerings by DJI [220]. The Skydio 2 is generally regarded as having the best obstacle avoidance technology available on the market today and the aircraft represents the state-of-the-art in terms of set-it-and-forget-it, hands-off flying [221]. The technology on the Mavic Pro 2 is not quite as good as the Skydio 2 but still represents a high level of autonomy which is accessible to casual pilots [222]. The true arrival of high autonomy, easy to use and inexpensive, but also highly effective obstacle avoidance occurred between 2019 and 2020 with the arrival of both the Skydio 2 and the DJI Mavic Pro 2 [220–223].

From the first mentions of functional demonstrations of real time obstacle avoidance in the academic record which appeared in the early 2000's, to the arrival of the first fully-featured and accessible obstacle avoidance systems, approximately $15\,\mathrm{yr}$ to $20\,\mathrm{yr}$ elapsed. The range is given in this case due to an unclear distinction about when obstacle avoidance first arrived on the consumer market. Substantially less-sophisticated systems than are currently available were introduced earlier, but nonetheless fall under the umbrella "obstacle avoidance". Deciding whether the earlier or later technological offerings truly comprise the arrival of "obstacle avoidance" leads to the usage of a range on the technology timeline in this case.

2.7.5 Summary of Case Studies

In all the cases presented above, the time from the first mentions of a functional demonstrator in the academic literature to the time that a fully featured, game-, technology arrived on the consumer market is about $15\,\mathrm{yr}$ to $20\,\mathrm{yr}$. From this limited data set, conclusions might be drawn where predicting the future battle space is as simple as searching through the academic record and projecting out $20\,\mathrm{yr}$. However, the identified $15\,\mathrm{yr}$ to

20 yr maturity window comes with some important caveats.

The plot shown in Figure 2 shows the trends which were discussed in the preceding sections. The curves describe the total number of publications matching a curated selection of keywords for the year indicated. Unfortunately, the keyword search method is highly sensitive to small changes in keyword choice. Additionally, in this case, the total number of publications has been normalized for the three topics discussed due to the difference in overall prevalence of the keyword hits discovered. Obviously, normalization in this case disposes of important information which may be relevant to determining which new technologies are on the cusp of becoming inexpensive and widely available. This is partly true; however, the wide variation in total number of publications, the suitability of the keywords used, etc. go a long way to render most attempts at blindly predicting technological trends in such a way, a fruitless endeavor.

To predict, in this case, is not some kind of process to be accomplished by some automatic machine. The role of an unbiased subject matter expert in this case cannot be eliminated. Blindly parsing the publication record leads to two possible outcomes. In the first case, the search returns results which are specific, possibly biased, by the search terms or the person conducting the analysis, and any action taken based on those predictions will be flawed due to the unsuitability of the information upon which it is based. On the other hand, predictions are so broad and uncertain that they are not useful.

The most obvious prediction process going forward will make use of all available techniques but must make use of a subject matter expert. And in most cases, the prediction should focus on that which is known to the SME rather than beginning with a blind

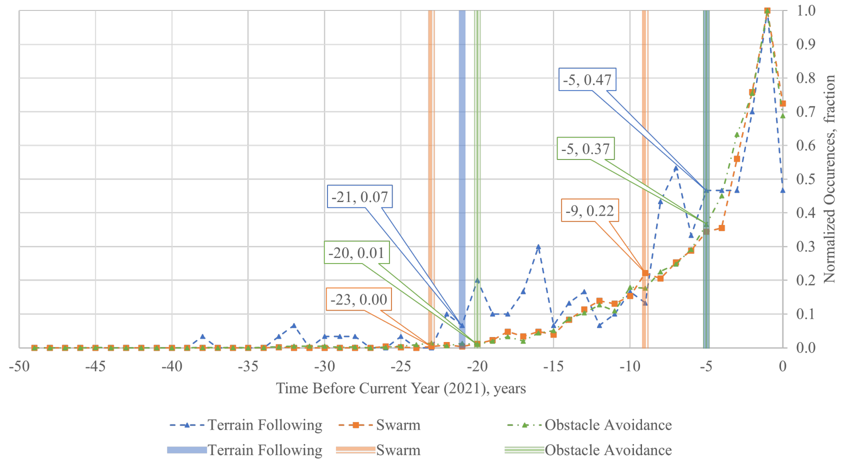


Figure 2: Plot Showing Publication Quantity by Keyword Search for Technology Case Studies.

guess or a survey of something such as the publication record. Obviously, some of the case studies mentioned in previous sections indicate that UAV technology first made an appearance in adjacent applications which may or may not be familiar and obvious even to an SME. For this reason, in the ideal case, a committee of SMEs would be convened as is done for some state-of-the-art forecasting techniques [6].

The methodology shown doesn't account for technologies which were written about in the academic record but did not make the transition into a commercial product. In fact, technologies could be making their way to market currently outside of the identified maturity window which would seemingly disrupt the notion of the $15\,\mathrm{yr}$ to $20\,\mathrm{yr}$ window. Additionally, some technologies may never make the transition into the commercial market because there simply isn't a commercial need. However, technologies which never see widespread commercial usage are not necessarily germane to the discussion taking place here.

Consider that this threat analysis is designed to capture the impact of commercial products specifically on the threat space. Military or proprietary innovations which are not part of the public academic record are also largely ignored by this type of analysis. Fortunately, government agencies in charge of intelligence gathering should be relied on for understanding what military threats are specifically being posed since this information is not expected to be widely available. The analysis presented here is not concerned with state-actor level technology developments which will necessarily be unsuitable for discussion in a public document.

In many ways the identified duration of $15\,\mathrm{yr}$ to $20\,\mathrm{yr}$ can be thought of as a minimum time from first-mention to maturity. It is possible for products to be slightly faster, but considering supply-chain difficulties and that a product must be widely available and easy to use, the estimate is most likely conservative. There are certainly technologies which can be identified from literature review but will never find commercial use, or find commercial use much later than predicted for a variety of reasons. Understanding the myriad market factors which allow innovative technology to morph into a profitable business is the purview of business experts and largely outside the scope of predictions considered here. Fortunately, much insight can be gleaned from well-documented use-cases for UAVs which may be obviously enhanced given certain technological improvements.

2.8 UAV Futures

A review of literature related to the evolution of the UAV space has been conducted. Literature on the topic represents the most closely related subject area to the topic considered in this manuscript. Some literature on expected threats does exist, but far more literature is available related to the most likely future direction of the UAV industry.

For example, a paper uses a key word search on a patent database in order to understand and predict UAV trends [224]. The study was conducted specifically on South Korean patents so the insights gained are not necessarily comprehensive and global; however, the findings are still interesting. Keywords like “data” and “camera”, and “real-time” were commonly encountered in the relevant patent literature [224]. Additionally, the study identified various categorical topic areas and then ranked these topics and considered whether the trends in growth for a given topic indicated that the topic was “hot” or “cold.

The study identified that patents related to communication technology are the most important and growing the fastest at the time of the research [224]. This topic area includes communication between UAVs, as well as communication between an operator terminal and an air system, as well as antenna designs and time lag considerations [224]. Power technologies including alternative propulsion such as fuel cells were also indicated as a growing topic area [224]. This topic area also includes various developments related to easy-to-swap power packs in the form of batteries [224].

Signal processing and flight control system subject areas were all indicated to be cold in terms of growth [224]. This could be due to the higher level of maturity in some technologies related to these fields. Flight control systems and signal processing are associated with a certain level or maturity considering that these technologies are already in-use by systems currently on the market. Thus the slowing rate of occurrence in patent literature (or academic literature for that matter) is to be expected.

Overall, the study finds that technology areas related to communication technology, navigation systems, commercial functions and applications, and UAV attachments are expected to grow [224]. This is consistent with the assumption that the UAV industry is on the cusp of a period of wide proliferation. Obviously the industry has already exhibited explosive growth, but as the availability exceeds some critical threshold, it is expected that the number of use-cases will also grow. As use-cases expand, the number of solutions available on the market will increase as various manufacturers seek to carve a niche out of the market. This will necessarily include various proprietary designs for aircraft fuselage and battery systems as well as attempts to patent entire novel use cases to maximize profits.

Another study discusses trends in scientific publication by performing meta-analysis of articles appearing in a particular journal [225]. The identified research areas include remote sensing, geology, chemistry, environmental science and agriculture [225]. Again, this supports the conclusion that applications will be a main driver of technological innovation as foundational UAV technologies become more mature. The number of publications in the engineering topic area has slowed, while the publications in instrumentation, remote sensing, and geosciences has increased according to the analysis [225].

The authors of the study suggest that as UAV technology matures, that an increased interest in attachments, and sensor improvements will be accomplished while fundamental engineering work associated with basic operation will become less and less prevalent [225]. Analysis of citations paints a similar picture. Focus on core engineering work is down year-over-year whereas citations related to applications, ancillary engineering (sensors and instrumentation) is up substantially [225]. The most cited papers for the most recent round of publications are related to remote sensing and survey applications [225]. The author indicates the year 2016 as the turning point when UAV technology development (as measured by scientific publications) transitioned from steady to exponential growth [225]. Interestingly, this agrees nicely with analysis indicating that many UAV technologies arrived on the market around the same year.

Finally, a review of UAV technology is presented with specific interest in the problem of UAVs used for cargo delivery [226]. This analysis indicates that the civil market is expected to grow at a faster pace compared to military UAVs in the near future [226]. Additionally, the gap between military UAVs and consumer UAVs, which is greatly diminished anyway, is expected to diminish even further as the boundary between UAVs intended for professional and consumer use is blurred further [226]. The study is careful to point out that some potential for innovation is stifled by regulatory obstacles and that the expected use-cases for aerial delivery (e-commerce and home delivery) have not yet delivered on promises to revolutionize the cargo UAV sector [226].

The analysis does foresee UAVs taking over existing unrelated markets as use cases expand and barriers to ownership and operation are eliminated (to include breaking regulatory barriers) [226]. Additionally, for the intended cargo use-case (as well as other use-cases), the prevalence of vertical take-off and landing (VTOL) is expected to be important [226]. Currently, VTOL is offered by multi-rotors, and while UAVs exist which are capable of VTOL as well as highly efficient, level-flight, this UAV technology sector is expected to grow. Specifically, aircraft have been developed which can transition from vertical to horizontal flight by manipulating the actuators or by using duplicate actuators; however, the focus on so-called tail-sitter aircraft is expected to grow [226].

The study indicates that improving launch logistics and fleet maintenance are expected to be important problems moving forward with many UAV use-cases [226]. Additionally, the prevalence of truly modular UAVs which can be configured to suit a variety of uses is expected to increase [226]. The study mentions that a company in the U.S. has announced plans to build a jet-powered UAV specifically for transatlantic delivery but very little specific information about the development of such a system is currently available [226]. A bevy of regulatory hurdles needs to be overcome before such a technology can be demonstrated.

2.9 UAV Technology Improvement Time Line

Consider next a *difference of degree* in an existing technology. Vision navigation, swarm communication, and obstacle avoidance can all be disruptive technologies, but this work is also concerned with more mundane changes in the technological atmosphere. What are the implications for example of doubling the speed or maneuverability capabilities of a given class of aircraft? How likely is the max speed to double? Is there a theoretical limit to for some characteristics which is unlikely to be exceeded?

Answering such questions can provide important insight into where resources might be invested in order to be ready for whatever evolution may occur in the threat space. Even developing scenarios to test out theories about the impact of changes in characteristics can be difficult without some understanding of the scale and scope of the problem. For example, some characteristics such as aircraft maximum speed might be likely to increase by one amount, say $50\,\%$; whereas, other characteristics such as aircraft maximum payload might be able to increase by $200\,\%$. Still other characteristics such as the level of autonomy of a swarm or the logistical footprint required for a swarm of a given size may change drastically and could be difficult to quantify.

Thus, in the following sections, an effort will be made to understand how likely certain UAV characteristics are to change in the short term given the paradigm of *difference of degrees*. This section will not be concerned with totally disruptive technologies, but rather with changes in known characteristics.

2.9.1 Improvements in UAV Performance

When the Drone Racing League (DRL) launched in 2016, each pilot competed by flying an identical Racer2 drone developed by DRL. The Racer2 was capable of a top speed of around $80\,\mathrm{mph}$ ($35\,\mathrm{m/s}$) [227]. For the new DRL season in 2017, the Racer3 was introduced which is capable of a top speed around $90\,\mathrm{mph}$ ($40\,\mathrm{m/s}$) [228–230]. The drones built by DRL differ from some other aerial vehicles previously discussed in this work in that they are built primarily for speed with little consideration for durability or other factors which help make UAVs marketable to the general public [230].

In July of 2017, a team from DRL hand-built a UAV which was used to set the Guinness World Record for the fastest ground speed by a battery-powered remote-controlled quadcopter with a speed of $163.5\,\mathrm{mph}$ ($73\,\mathrm{m/s}$) [231–233]. The UAV in question weighs only $1.76\,\mathrm{lbf}$ ($0.8\,\mathrm{kg}$) and there is no allowance for an external payload. Additionally, the UAV built by the DRL team was built using similar techniques and components as the UAV built for the DRL races; however, no expense was spared in the chase for the world record speed and reliability was sacrificed in the name of raw performance. Several reports of the world record attempt indicate that DRL-built UAVs would spontaneously burst into flames due to the sheer amount of electrical power being handled on board [231, 233]. The world record flight required two passes in opposite directions with the official top speed recorded as the average of the two runs to account for any potential advantage due to the prevailing wind direction and speed. Off the record, the world record holding UAV was able to reach speeds of nearly $180\,\mathrm{mph}$ ($80\,\mathrm{m/s}$). The increase in maximum horizontal speed for the world record setting UAV versus the DRL Racer2 is about $100\,\%$ in a little over 1 year.

The world record holding UAV does not necessarily represent a technological innovation versus the “production” UAVs which DRL builds for the racing events. However, the world record-holding UAV does represent the bleeding edge of capabilities (limited by the power through the electronics) which is likely to improve more slowly. The top speed that UAVs are capable of should be expected to increase, less so in terms of absolute maximum speed possible in the extreme case, but more so in terms of max speed expected on robust “production” type racing drones that would be more easy to build and sell. In other words, the world-record speed run is important to understand raw capabilities, but more so important in understanding the bar that has been set in terms of what is possible. It should be expected that manufacturers with a more commercial interest (compared to the DRL UAV builders) will be interested in closing the gap between the world record speed and the speed that products they are trying to sell are able to achieve.

In a more relevant case study, consider the capabilities of commercially available, consumer-grade UAVs. Consider the Phantom series from the manufacturer DJI. The first in the series known as the Phantom 1 was released in early 2013 and had a maximum horizontal speed of $22\,\mathrm{mph}$ ($10\,\mathrm{m/s}$) [234]. The Phantom 1 was the first largely popular, consumer-grade UAV which was affordable, easy to fly, and capable enough to pique the interest of consumers worldwide. Thus, these capabilities probably don't represent the true state of the art in terms of raw performance, but should be expected to be conservative for the sake of "taking-it-easy" with a first offering to the global market.

Consider then when the Phantom 2 launched later in 2013 (about 11 months after the introduction of the Phantom 1), DJI had upped the specifications. The Phantom 2 was capable of a maximum horizontal speed of $33\,\mathrm{mph}$ ($15\,\mathrm{m/s}$) [235] which represented no less than a $50\,\%$ improvement in capabilities over the Phantom 1. Again, this does not necessarily reflect a change in raw performance, rather it represents a change in allowed performance considering the maturity of the product and the comfort level of the manufacturer. The Phantom 2 similarly saw an improvement in flight time to $25\,\mathrm{min}$ versus $15\,\mathrm{min}$ or an improvement of about $67\,\%$ from the Phantom 1 [234, 235]. Other characteristics remained largely unchanged. It should be noted that the listed maximum descent speed and tilt angles for the Phantom 2 were actually reduced compared the Phantom 1. This supports the hypothesis that changes were made by the manufacturer to decrease the likelihood of conditions which could cause UAVs to crash.

Next, skip ahead to midyear in 2018 when the latest (and ostensibly final) version of the Phantom was released, called the Phantom 4 Pro V2.0. The maximum horizontal speed in the most aggressive flight mode is now listed as $45\,\mathrm{mph}$ $(20\,\mathrm{m/s})$ [236] which represents a $100\,\%$ increase over the original Phantom 1, in about $5.5\,\mathrm{yr}$ (or a $50\,\%$ increase compared to the Phantom 2 in about $4.5\,\mathrm{yr}$). The maximum flight time also is up to $30\,\mathrm{min}$ which is a smaller percentage change over the older models of about $33\,\%$ and $20\,\%$ for the Phantom 1 and Phantom 2 respectively. The smaller percentage change in the maximum flight time could indicate the technological limitations on this flight characteristics (most likely battery chemistry), and the trade-off between all-up-weight and endurance) is less flexible compared to the technological barriers to higher speed. Since the introduction of the Phantom 1, DJI has further segmented the UAV market and now offers several aircraft in a variety of weight classes designed to serve a variety of needs. The Phantom series has always been a sort of middle-of-the-road offering which is accessible to most UAV enthusiasts and thus represents a good average in terms of performance specifications.

A note here on other characteristics for DJI manufactured aircraft. Basically since the introduction of DJI UAVs to the consumer market, a software ceiling of $1{,}600\,\mathrm{ft}$ $(500\,\mathrm{m})$ above ground level has been enforced by the flight control system. This ceiling has not been modified or extended for later versions or for new aircraft and likely represents some nexus between regulatory requirements and maximum conservative operational altitude where performance can still be guaranteed. Thus, no information is available from DJI in terms of growth of maximum altitude capability.

In mid 2021, DJI released their FPV offering which includes a video system and goggles which allow the pilot to fly as if they were on-board the aircraft. This is how the DRL pilots operate UAVs for the purposes of racing. So called "FPV" or first-person-view flying has been around since the advent of the modern UAV. Hobby airplane pilots have been operating using some form of FPV camera system and video display for nearly as long as they have been flying, and in fact, remotely piloted aircraft used some form of FPV flight control as early as 1943 [155]. While the flying concept itself is not innovative necessarily, the delivery of FPV-style flying in a familiar and easy-to-use package from a well-known manufacturer has the potential to bring FPV-style flying and the advantages it offers to a much larger audience.

The DJI FPV UAV has a maximum horizontal speed of $87\,\mathrm{mph}$ $(39\,\mathrm{m/s})$ which is a $160\,\%$ and $95\,\%$ increase over the capabilities for the Phantom 1 and Phantom 2 respectively [237]. The maximum endurance is about $20\,\mathrm{min}$ and represents a decrease over Phantom models, but the FPV model is not meant to perform the same kind of flying as the Phantom series and thus a lower endurance is acceptable. The characteristics of the DJI FPV are generally more aggressive including a more dramatic allowed tilt angle as well as a higher level of possible wind penetration (for flying in more extreme conditions) as well as a flight mode offering unlimited (essentially power-off) descent speed. The DJI FPV has a diagonal dimension with propellers of about $15.8\,\mathrm{in}$ $(402\,\mathrm{mm})$ which is smaller than the Phantom series which has historically had a diagonal measurement close to 22.6 in (575 mm) [234–237].

Consider another offering from DJI known as the Inspire. When first introduced in late 2014, the Inspire 1 represented one of the first widely available “pro-sumer” level UAVs boasting an interchangeable camera and a large stable and capable flying platform [238]. The maximum horizontal speed for the Inspire 1 is advertised at 49 mph (22 m/s), and considering the UAV is larger and designed for notional commercial or professional uses, other parts of the flight envelope have also been expanded. The Inspire 1 is capable of operation up at up to 8,100 ft (2,500 m) or up to 14,000 ft (4,500 m by changing to a high-altitude propeller) [238]. Additionally the Inspire 1 has a diagonal dimension of approximately 35 in (890 mm) and an all-up-weight of 7.1 lbf (3.5 kg) [238]. The Inspire 1 and all other UAVs made by DJI are still subject to the maximum altitude above ground level compared to the takeoff location of 1,600 ft (500 m) as previously discussed. This does not mean that the UAV cannot operate above 1,600 ft (500 m) above sea level, but rather that the aircraft will not fly more than 1,600 ft (500 m) above the take off point.

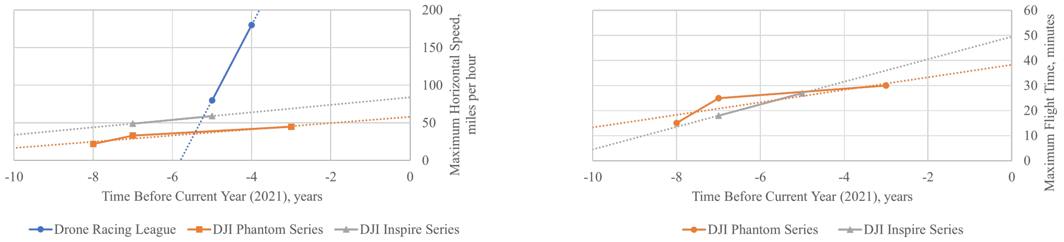
Consider the specifications of the Inspire 2 which was released near the end of 2016. The aircraft dimensions remained virtually the same compared to the Inspire 1 but the all-up-weight increased by 22 % to 9.4 lbf (4.25 kg), while the maximum horizontal speed increased by 18 % to 59 mph (27 m/s) [239]. The best-case flight time for the Inspire 2 increased to 27 min versus the 18 min possible on the Inspire 1 which represents a 50 % increase [238, 239].

The Inspire series does not continue past the Inspire 2 at this time; however, some of the function of the Inspire series (as well as other legacy DJI aircraft) are now filled by the full line-up of (in DJI's words) "enterprise" level UAVs. This line-up consists of UAVs from the Mavic series with a diagonal size of $21\,\mathrm{in}$ ($550\,\mathrm{mm}$) which is just slightly smaller than the Phantom series all the way up to hex-rotors with an all-up-weight of $35\,\mathrm{lbf}$ ($15.5\,\mathrm{kg}$) in the Matrice 600 Pro [240, 241].

Within the realm of commercial and enterprise level UAVs, the capabilities are generally not so focused on "raw performance" or capabilities which sound good on a marketing brochure, so much as the improvement of requirements is related to whatever enterprise and commercial customers need in order to work more efficiently. Thus, improvements include expansion of supported accessories including increasingly sophisticated camera systems beginning with the simple interchangeable-style cameras first offered on the Inspire 1 and now expanded to include hyper-spectral, infrared, interchangeable lens cinematic, and other camera-types. Additionally, many models intended for commercial use include the ability to interface with a real-time-kinetic (RTK) module which can improve position-hold flight performance substantially over position-hold with GNSS only [238, 240, 241]. The changes in such parameters are more difficult to track because thus far DJI has only released one version of the largest of the enterprise-level UAVs (the Matrice 600), and though the model has been updated and adapted, the basic performance characteristics have remained relatively constant.

Similarly, the Mavic series was first introduced within the consumer sector, but with the advent of the Mavic 2 Pro Enterprise and other similar models, the Mavic series carved out a niche within the enterprise UAV sector. The Mavic concept is highly portable (folding) UAV which can be discreetly transported and is easily deployed and stowed. The price point and flight envelope are comparable to the Phantom series in a smaller, less expensive, and more capable system. The inclusion of the Mavic series in the enterprise line-up represents high technology once reserved for the most expensive of commercial-application-only UAVs trickling down to smaller UAVs which are much less expensive to own. The Mavic series was first introduced around the end of 2016; whereas, the Mavic 2 Pro and the similar enterprise model were introduced 2 years later near the end of 2018. The RTK upgrade was introduced to the Phantom series around the same time (late 2018) [242] and thus the transition of the highly accurate positioning capabilities with RTK which first appeared as a do-it-yourself kit for the bare A3 flight controller in 2016 [243] was accomplished in about 1 year.

The plots shown in Figures 3a and 3b give a visual reference for some of the discussed trends in UAV performance improvement. For all curves, a linear trend line is projected, though this is not to say that linear trends for all performance characteristics are expected. In fact, linear trends are unlikely in almost all characteristics and it is far more likely that the performance traits will converge to maximum possible values asymptotically. Additionally, the raw performance at this point is far less important to a manufacturer like DJI compared to the continued market segmentation and proliferation of capabilities.



(a) Plot Showing Maximum Speed for Various UAV Models Including Linear Projections.

(b) Plot Showing Flight Time for Various UAV Models including Linear Trend Projections.

Figure 3: Plots showing trends in Important UAV Characteristics for DJI Phantom Series, DJI Inspire Series and World Record UAV Speed over Time.

2.9.2 A Note on the Longitudinal, versus Lateral Progression

When discussing threat technology relevant to a traditional conflict with a near-peer adversary, the progression of technological development, the so called “ragged edge” of what is possible, is chiefly relevant. However, considering threat technology relevant to an asymmetrical conflict, the benefits of understanding and predicting the progression of the “ragged edge” may be less useful. Consider a hypothetical world-beating technology which would enable a near-peer adversary to upset the balance of power. When considering a conflict with a non-state-actor or any adversary smaller than a near-peer, such a technology may not have the same impact because it is as of yet unattainable to an adversary who does not already posses the resources and industrial base to enable the use of such a hypothetical technology. In terms of the “ragged edge”, there is some truth to the adage that the early bird gets the worm. The largest disparate impact on outcomes will belong to the entity who gets the new technology first. Despite the proliferation of nuclear technology, the leverage in large part belonged to the country who first developed and used atomic weapons during war.

Consider the alternative scenario where the adversary has fewer resources, is generally less defined, less-organized, and lacking a dedicated research arm and industrial base. Thus, understanding what threats may arise boils down to understanding what threat technologies may be widely available. A technology such as RTK positioning augmentation could open up a new threat vector; however, if that technology (despite its existence) is not widely available, easy to implement, easy to buy, or easy to build, then it will not be available to many asymmetrical adversaries. That is not to say that the adversary cannot develop or acquire RTK, but the cost to attain it and thus the probability of such technology coming online without warning is low. Conventionally, in the asymmetrical battle, defensive systems are not tasked with defending against the state-of-the-art for a given technology but rather against the state-of-the-art in terms of market availability.

The obvious advantage is that information gathering related to the state-of-the-art available for threat technology to a near-peer adversary necessarily involves spying; whereas, understanding what is available may be as simple as looking through a catalog or reading a news release. Widely available technologies must be advertised or listed in some way in order to be distributed effectively and if distribution is key to their adoption as threat technology, then threat vectors might be understood by reading readily available advertisements. Obviously, the truth is more complex than this simple discussion would suggest; however, there is a difference in the predictive power and the risk associated with making predictions as they relate to proliferation rather than to longitudinal innovation.

In many ways, if the progression of the “ragged edge” for a particular technology represents a longitudinal (or “forward”) trajectory through time, then the proliferation (or wide adoption, distribution and availability) of a technology represents a lateral movement. Pursuant to the previous sections, one could ask “should we be more worried about high performance UAVs capable of higher maximum horizontal speed or be more worried about moderately equipped UAVs which are incredibly easy to buy, use, and maintain?”. In terms of the discussion in this manuscript, the lateral movements are both easier to study, easier to discuss, and hopefully easier to defend against. Much of the true information related to the “ragged edge” of development and the so-called longitudinal development is not easily discussed in a widely distributed and freely available document. Thus, partially out of necessity, the threat space discussed herein is mostly thought to be the result of lateral rather than longitudinal growth trajectories. To answer the question, we are concerned with predicting what might happen when once transformative technologies make the transition from expensive laboratory experiments to widely available, easy-to-use, market-ready product.

CHAPTER 3

WAR GAMING

3.1 Introduction to Game Theory

The topic of game theory is most often associated with the field of economics. Why then, the concern about such a topic here? To answer that question consider a simple example. Consider a simple coin toss game where the player is asked to predict whether a tossed coin will land with a specific side of the coin (designated as “heads” or “tails”) face up and the player wins the game when their prediction is correct, and loses the game otherwise. The mathematical fields of probability and statistics can provide some insight into the coin toss game. Namely, for a fair coin, the player is equally likely to win the game regardless of the prediction. That is, the probability of the coin landing with “tails” showing is equal to the probability of the coin landing with “heads” showing.

While the coin toss example is elementary, two important points arise as a result of the explanation. Firstly, a substantial amount of the informal, intuitive, and colloquial language used to describe the coin toss game comprises the formal language of game theory. In other words, we are already using the nomenclature of game theory and in fact, it will later be demonstrated that formalizing this nomenclature enables a richer discussion of the scenarios which form the focus of this work. Consequently, the second important point simply emphasizes that developing a fundamental knowledge of game theory enriches the scenarios which can be considered and facilitates a deeper understanding of results (especially results which appear counter intuitive or difficult to explain using other forms of analysis).

3.2 Basic Nomenclature

In the coin toss example presented above, several terms have already been introduced. The first term to consider is **game**. Formally **games** are defined as "...strategic situations in which player who interact understand their environment, how their action affect the outcomes that they and their counterparts will face, and how these outcomes are assessed by the other players."[244]. It is important to understand that the broad category of games includes such scenarios which are colloquially referred to as "games" such as card games, coin toss, and game show games, as well as many other scenarios (referred to as "decision problems" [244]) which sophisticated strategic games such as chess and "war games". Specifically this work will deal with "war games". However, it is important to grasp that the framework of game theory will allow generalization of conclusions such that conclusions about a particular war game may apply to a coin toss scenario or a game show and vice versa. This is a particularly powerful technique in situations where the specifics of a particular strategic decision constitute controlled or sensitive information.

The next term to discuss is **player** which is simply defined as a participant in a game. A game may have one or many players. Next we discuss three characteristics of players which form the basic structure of all games. First consider **actions** defined as "the alternatives from which the player can choose."[244] **Actions** can be understood mostly from the colloquial meaning of the word. Within the context of the game, **actions** represent any specific thing the player might choose to do. Related to the player's actions are the related **outcomes**. **Outcomes** are "the possible consequences" [244] related to a particular action. Outcomes will be defined for each possible action a player might take within the context of a given game. Next, **preferences** "describe how the player ranks the set of possible outcomes"[244]. The meaning of **preferences** is also closely related to the colloquial definition of the term. It is important to understand that players have **preferences** over **outcomes** not over **actions**.

Next the concept of a **rational preference relation** is introduced. The preference of a player over outcomes must possess two qualities in order to be considered a **rational preference relation**. Specifically, the preference relation must be complete meaning that there is a preference for each possible outcome within the complete set of outcomes. Additionally, the preference relation must be transitive, which means "for any three outcomes x, y, $z \in X$, ... $x \succsim y$ and $y \succsim z$ then $x \succsim s$"[244]. [1] This is similar to the definition of the transitive property which will be familiar from the field of mathematics. Completeness of a preference relation simply implies that each outcome is ranked by the player such that preference for any outcome may be compared to preference for any other outcome.

Next consider a formal proposition that simplifies analysis by directly connecting actions with outcomes. Specifically, the **payoff function** of a player maps the complete set of actions for that player to the complete set of outcomes for that player. In this way,

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[1] The symbol $\succsim$ is similar to the commonly used $\geq$ meaning "greater than or equal to". However, in this context the symbol should be read as follow: the expression $x \succsim y$ is read as, x is at least as good as y. Other variations on this symbol should be interpreted according to the analogy with the common mathematical symbols as would be expected. For example $\prec$ would be read as "is not better than".

actions and outcomes are no longer considered as separate entities as they are linked by a one-to-one relationship described by the payoff function. It is important to note that payoff functions have no inherent meaning except that they preserve the order of the associate preference relation. Payoff functions give payoff values which are "an *ordinal* construct" [244]. In order for a player to be considered rational, the player must always prefer actions which lead to better payoffs. This is a fundamental tenet of game theory analysis.

3.3 Classification of Games

The coin toss example presented above represents what is formally called a "static game of complete information". "Static" meaning the decision made by the player is once-and-for-all after which the game proceeds without interference. And "complete information" indicating that the player is fully aware of the specific way in which the game will proceed prior to making a decision. This does not imply that the player knows the outcome of the game prior to making a decision. In fact, refer to the coin toss example as a perfect example of this fact.

As has been hinted, games are characterized by two primary characteristics. One, how often will the players be allowed to make decisions (static games require a once-and-for-all approach, whereas dynamic games may include many individual "moves" by one or more players). Static games will also be referred to as "normal form" games, and dynamic games will be referred to as "extensive form" games. The second characteristic concerns what information is available to players at the specific time at which a decision is made. Simply the information may be “complete” or “incomplete”.

The combinations possible are static games of complete information (the simplest case), dynamic games of complete information, static games of incomplete information, and dynamic games of incomplete information (the most flexible and complex case). In the following sections, each of these categories will be considered. Static games of complete information are most easily understood and have been studied extensively but rely on assumptions which bake in a certain unreality to the conclusions. Conversely, while dynamic games of incomplete information are the most “realistic” in terms of not relying on limiting assumptions, such scenarios may be sensitive to initial conditions, or just difficult to analyze owing to the complexity of the problem presented. The necessity to balance model fidelity and real-world applicability is not unique to this topic, this manuscript, or the discipline of mechanical engineering. In any case, the language of game theory is well suited to describe the analysis contained within this manuscript.

3.4 Uncertainty in Game Theory

The simple coin toss example game presented in the introduction to this chapter contains an important element which we have so far ignored. Static games of complete information such as the coin toss example, may not be always “winnable” by the player. This may seem counter intuitive since by complete information it has been stipulated that the player has all the necessary information prior to making a choice. Specifically, complete information does not imply that the player knows the outcomes beforehand. In fact, through uncertainty in outcomes a type of strategy a player may adopt which is formally known as a “mixed” strategy is introduced. In contrast to a “pure” strategy where a player always chooses the same action, a “mixed” strategy has a player choosing among different strategies according to a probability distribution.

Consider another example where a goalie is asked to stop a penalty kick to decide a soccer match. If the goalie stops the ball from entering the net, the goalie’s team will win the match, otherwise the goalie’s team will lose. The concept of this game may not be altogether unfamiliar to the reader. Consider the further stipulation that in this particular case, the penalty kick is conducted in such a way that the goalie must choose whether to defend the left side or the right side prior to the kick occurring. Once the goalie is committed he cannot change his choice because there is no time to move his body to the other side of the net. Consider that the penalty kicker as another player in this game. The penalty kicker will be considered to be played by “nature”; specifically the choices made by “nature” are governed by a probability distribution. In this specific case nature chooses between shooting right or left with equal probability. A naive solution to this problem may suggest that since the goalie must choose a side based on uncertain information, that the goalie can be easily defeated. However, it can be shown that as with the coin toss game, a smart goalie who understand the game cannot do worse than to stop $50\,\%$ of penalty kicks (for repeated instances of the simple game).

With the complete information on the probability distribution used by nature when applying the mixed strategy which dictates whether the kick will be aimed to the right or left side, the goalie should apply the mixed strategy that maximizes the chances of stopping the penalty kick. In this case, the goalie can adopt three different strategies, two pure strategies defined as “always right”, and “always left”, or choose from the two pure strategies according to some distribution in a so-called mixed strategy. A rational goalie will be indifferent to choose between the two pure strategies and indeed the equilibrium solution to this game involves the goalie adopting a mixed strategy using a $50\,\%$ chance of choosing either strategy in a given instance of the game. This same logic can be used to show that in the coin toss game, the player should similarly adopt a mixed strategy which grants equal probability of choosing “heads” compared to choosing “tails”.

The concept of another player represented by nature can be substituted with the concept of a lottery. A lottery is a game which is described by a random payoff [244]. For example in the coin toss example, the lottery is described as the probability of the coin coming up “heads” or “tails”. Lotteries are conditional on the decision of the player, but in the case of the coin toss the condition is specifically that the outcome must not depend on the choice by the player (thus is strictly not conditional). In the case of the coin toss example, the player is choosing a best strategy in response to a lottery which will determine which side of the coin will land facing upward.

Next consider a concept first proposed in [245]. Thus far the outcomes of a particular game for a particular player have been governed by a payoff functions. Since uncertainty has now been introduced, the concept of payoff functions must be modified. The concept of expected utility theory posits that the payoff function for a particular player in a particular game may be replaced with a simple function which accounts for the randomness in the outcomes. Intuitively, the potential payoff for the player is multiplied by the probability of that payoff occurring. Consider a lottery in the traditional sense where a player chooses to buy a ticket. This ticket may carry a large payoff with a small probability of occurrence or a zero payoff with a large probability of occurrence. To illustrate the point consider that the probability of winning $\$10$ is given at 0.10 (or $10\,\%$). The probability of winning nothing is therefore 0.90 (or $90\,\%$). Thus the expected utility of playing the lottery is given by $0.10 \times \$10 = \$1$; whereas, the expected payoff for not playing the lottery is given by $0.90 \times \$0 = \$0$. Using expected utility theory, the player should always choose to play the lottery in this case. Note that the cost to buy a ticket for this lottery has not been included and thus this does not represent a realistic example.

The concept of a lottery provides a convenient framework to discuss another factor which may affect the behavior of players known as risk attitude. Players may be risk neutral, risk loving, or risk averse. The meaning of these terms within game theory is similar to their meaning in the colloquial sense. The formal definition of risk preference will be explained briefly. Note that this work gives an overview of theories about risk attitudes and their relationship to expected value theory. In the years since Morgenstern and Von Neumann proposed expected value theory, much work has been done related to characterizing and modeling behavior in cases where players seemingly do not act optimally [35]. For the purposes here, the finer points related to risk attitude and the paradox of problems where behavior varies from the ideal theory are not discussed. First consider a lottery as previously explained. A lottery is some process by which outcomes are determined probabilistically. A degenerate lottery is a lottery where the probability of a particular outcome is exactly equal to one. That is, a degenerate lottery may be considered to be not much of a lottery at all.

To illustrate risk attitudes, consider a game where the player chooses between two different lotteries. If the first lottery pays out $\$10$ with a probability of 0.10 and the second lottery pays out $\$5$ with a probability of 0.20. The expected utility of these two lotteries is exactly equivalent and given by $0.10 \times \$10 = 0.20 \times \$5 = \$1$. Thus, to a rational player, the outcomes are equivalent. A risk neutral player would choose to apply a mixed strategy whereby either lottery is played with equal probability. To be precise, a risk neutral player will exchange any lottery with a degenerate lottery with a payout equal to the expected utility of the original lottery.

Now consider a risk averse player. A risk averse player will choose a sure thing over a payout with a lower probability even if the expected payouts have the same value. Precisely, a risk averse player will choose to exchange a degenerate lottery for any non-degenerate lottery with an equivalent expected payoff. Finally, consider the case of risk loving. A risk loving player will strictly prefer any outcomes which carry risk to other outcomes for the same expected payoff. To be precise, a risk loving player will prefer any non-degenerate lottery to any degenerate lottery with the same expected payoff. This topic forms an important point of discussion in this work but also represents a relatively advanced topic within the topic of game theory.

3.5 Analysis Tools

Matrix representation for games will be used to simplify the explanation and analysis of the scenarios which are soon to be discussed. To understand this notation, an example is presented. The game to be considered is known at the prisoner's dilemma.

In this game, there are two players, both of whom have been arrested for a crime they were indeed involved in committing. Each player faces interrogation without knowledge of the decision or status of the other player. The decision to be made by each player is to either keep quiet about the crime (stay "mum") or to blab on the accomplice ("fink"). If a single player plays fink while the other plays mum, then the player who plays fink gets off easy (1 year in prison) while the other player does hard time ($5\,\mathrm{yr}$ in prison). If both players play mum, then both players serve a shortened prison sentence ($2\,\mathrm{yr}$). If both players choose fink then both players do hard time ($4\,\mathrm{yr}$). The matrix representation which describes this problem is given in Table 1.

Table 1: Prisoner's Dilemma Matrix Representation.

		Player 2	
		M	*F*
Player 1	*M*	-2, -2	-5, -1
	F	-1, -5	-4, -4

The table is read as follows. The columns represent the choices which can be made by **Player 1**, specifically these are designated by *M* for "mum" and *F* for "fink". The rows similarly represent the choices which can be made by **Player 2**. The payoffs for a given scenario are given in order within each cell. For example, for the game in which **Player 1** chooses *M* and **Player 2** chooses *F* (bottom left cell), the payoffs are given as $-1, -5$ which are read as "the payoff for **Player 1** is -1" and "the payoff for **Player 2** is -5". This is consistent with the description of this game provided above.

The next concept to be considered are equilibrium conditions. The prisoner's dilemma provides a good example to consider because there are two distinct equilibrium conditions which can be identified. One of these equilibrium conditions can be identified by inspection. This is the case where both players choose *M*. In this case the total expected payoff is maximized with a value of -4. This condition is known as a Pareto equilibrium and is defined specifically as that outcome which maximizes the overall payoff (considering both players at once).

The next equilibrium concept is simultaneously more important and far less intuitive. The Nash equilibrium for this game is represented by the condition where both players choose *F*. An exhaustive explanation of how to derive Nash equilibrium is beyond the scope of this work. However, in this case, it suffices to say that if at any point either player feels that they can gain an advantage by playing a particular strategy (regardless of the moves of other players), then the player should take that strategy. Thus both players will choose *F*, which leads to the non-Pareto equilibrium condition where the global expected payoff is actually minimized. Remember this is a static game so neither player knows the choice of the other player prior to making a decision.

3.6 Static Games of Complete Information

The rest of this section will proceed as an evolution of various scenarios which can be considered using the framework of game theory. These scenarios will start out quite simple and progress to more complex (and more useful scenarios). The goal of this exercise is to identify scenarios and the corresponding solutions. This section is a complement to analytic solutions. Solutions found through game theory methods represent the easiest solutions to find as well as the easiest solutions to support. On the other end of the spectrum, numerical solutions found via simulation represent potentially the most interesting and representative scenarios but will carry an inherently high level of uncertainty. It is through the application of game theory and other analytic methods by which the class of problems to be considered can be narrowed to include only the most essential scenarios which cannot be analyzed by other means.

The simplest scenario to be considered here is represented by a simple two player game. In contrast to the section of this work concerned with the analytic solution to the dynamics problem, this section will not be concerned with the geometric details (at least not yet). For the sake of simplicity we will name the players in this scenario. Player 1 will be referred to as *blue* while player 2 will be called *red*.

In the most basic scenario to be considered, each player simply decides whether or not to play the game. In order for either player to choose to play (Y) incurs a cost. If both *blue* and *red* choose to play the game, then *blue* will win. This scenario closely mirrors the scenario presented in the pursuit and evasion section for which the blue team is guaranteed to capture the red team. If both players choose to play then *blue* incurs a cost as well as *red*, but the costs are disproportionate. Namely, any move by *blue* is substantially more expensive than any move by *red*. If *blue* loses, the cost is relatively high compared to the cost for a *red* loss. If either player chooses not to play, then the other player automatically wins. There is disproportionate reward in that victory for *red* results in a high cost for *blue*; whereas, victory for *blue* is still very costly for blue. Consider the basic scenario presented in matrix form in Table 2.

A plausible explanation for this scenario goes as follows. *Blue* is defending an

Table 2: Basic Blue versus Red Scenario.

		Red	
		Y	*N*
Blue	*Y*	-100, -11	-100,0
	N	-10100, -1	0, 0

expensive prize using an expensive defense system. If the prize is lost, or the weapon is fired, *blue* incurs a high cost. The cost of firing the weapon is 100 and the cost of losing the prize is $100{,}000$. For *red*, the cost to play is 1; whereas, the cost to lose is 10. Analysis of this game leads to the conclusion that both players should choose not to play the game. This equilibrium solution satisfies both Nash and Pareto criteria. This should be a clue that the proposed scenario does not accurately capture the real situation in this case.

Now consider the following modification for the case where red chooses to play. *Red* is awarded a positive payoff anytime *blue* incurs a cost. This payoff for *red* has a nominal value of 20. The modified scenario is given in Table 3.

Table 3: Modified Basic Blue versus Red Scenario.

		Red	
		Y	*N*
Blue	*Y*	-100, 9	-100,0
	N	-10100, -1	0, 0

Analysis of this scenario shows that there is still a Pareto optimal solution where both players choose not to play the game. However, since *red* is able to improve outcomes by deciding to play the game, the player should be expected to do just that and thus the unique Nash equilibrium solution to this game involves both players choosing *Y* and agreeing to play. *Red* should always choose *Y* because for any scenario, *red* can gain an increased payoff by playing *Y*. Being a rational player, *blue* should anticipate this move and choose to play *Y* as well because the potential cost of choosing *N* is very high.

The presented modified basic scenario is not unlike scenarios in which a vast, technologically advanced apparatus such as the military of a nation state is pitted against a non-state actor. Characteristically, the nation state incurs costs for all actions explained simply as the costs associated with activating the large and complex apparatus of self defense. Conversely, the agile non-state sponsored actor is decentralized such that they may not be easily defeated, they do not incur a large penalty for a loss and also highly value ideological (moral) victories. This is a specific example of a classic scenario which is commonly referred to as "asymmetrical warfare". In this scenario, one player must choose to defend itself at great cost while the other player is free to conduct mischief and annoy the other player while waiting around to achieve a single victory (with a low probability).

Note that this scenario can also be framed in terms of risk attitudes. Specifically, *blue* is considered to be highly risk averse. In this situation, the risk aversion is reflected in the high cost of losing. Conversely, *red* is considered to be risk loving. That is, *red* is willing to take a chance on a long shot at doing a large amount of damage to *blue*. The risk aversion will not be considered further in this specific case. The payoff function is assumed to have accounted for the risk attitudes of the players in this scenario. Again, the specifics of theories which exhaustively consider risk attitudes such those in [35] is beyond the scope of this work.

The next scenario to be considered can be considered the follow-up game to the scenario just presented. In this case, consider that both players have decided to play and now the payoffs for such an action are to be refined. In this scenario, *blue* is not granted a choice. In fact *blue* is now bound to defend itself; however, the success of a given defensive action is given by a probability distribution. In the matrix representation, *blue's* choices will be given as *S* for successful defense and *U* for unsuccessful defense. Otherwise, the game shown in Table 4 is similar to the game shown in Table 3.

Table 4: Modified Basic Blue versus Red Scenario with Different Pure Strategies for Blue.

		Red	
		Y	*N*
Blue	*S*	-100, 9	-100,0
	U	-10100, -1	0, 0

While the matrix form looks the same, the analysis of this form provides interesting insights. Namely, now that *blue's* actions are cast as *S* or *U* for successful or unsuccessful respectively, a probability distribution governing the likelihood of these events can be developed. Since *red* now has the only real choice in this game, it is possible to determine if any probability of successful vs. unsuccessful defense is enough to cause *red* to choose to not play the game. This would represent the best possible outcome for *blue*. If *red* can be convinced to choose not to play the game, then *blue* will not incur expensive costs associated with self defense.

Consider the expected payoff function for *red* if *red* chooses to play is given by the probability of a failed engagement (p) and has the form $9p$ and $-1(1-p)$ depending on the success or non success of *blue*. If *red* chooses not play, the payoff is simply equal to 0. In order for *red* to choose *N* the probability of success for *blue* would need to be high enough to cause the expected payoff from the choice Y to become less than zero. Consider the inequality $9p - 1(1 - p) = E(p)$ where $E(p)$ is the expected payoff for a given probability p. Substituting $E(p) < 0$ and solving gives Eq. (3.3).

$$0 > 9p - 1(1 - p) \tag{3.1}$$

$$0 > 9p - 1 + p \tag{3.2}$$

$$p < 1/10 \tag{3.3}$$

$$\tag{3.4}$$

This result indicates that, for any probability of failure less than 0.10 or $10\,\%$ then *red* should be motivated to choose *N* and not play the game at all. That is *red* chooses not to play whenever the chance of *red* winning is less than $10\,\%$. Stated another way, whenever the *blue* defense system is more than $90\,\%$ effective, red is expected to choose not to play.

Now comes the all important question. How reasonable are the assumptions on the costs and payoffs for the two players? Increasing the payoff for *red* for a *red* victory or decreasing the cost of a *red* loss will drive the required probability of a successful *blue* engagement increasingly higher. Consider the effect of increasing the *red* payoff from a *red* victory from 9 to 101. The inequality on which *red* is to base its strategy is now given by $p < 1/100$ or a probability of 0.01 or $1\,\%$. This small change in the payoff functions imposes a much more stringent burden on the success rate of the *blue* defense system. This analysis does not consider the potential risk loving nature of *red* which may drive seemingly irrational behavior in order to seek out risky outcomes. For the purposes of this work, the risk loving nature is assumed to be contained within the payoff function in the form of a possibly larger than would be expected value for the payoff itself.

Consider a notional scenario for a *blue* defense system with a model success rate of $95\,\%$ ($p = \frac{1-95}{100} = 0.05$). Experience dictates that this scenario leads to repeated *red* engagements, thus we can back solve for the payoff function in order to check assumptions previously made about the value of payoffs. In this case, the previously given equations must be solved for an unknown value of payoff (given by R) for a fixed value of probability of unsuccessful engagement (p) the cost of losing will remain constant out of convenience.

$$0 > 0.05R - 1(1 - 0.05) \tag{3.5}$$

$$0 > 0.05R - 0.95 \tag{3.6}$$

$$0.05R < 0.95 \tag{3.7}$$

$$R < \frac{0.95}{0.05} \tag{3.8}$$

$$R < 19 \tag{3.9}$$

The solution in Eq. (3.9) shows that in this notional scenario, the reward for *red* victory need only increase to 19 in order to guarantee that *red* will choose *Y* to play the game when the *blue* defense system is effective $95\,\%$ of the time. Notice, based on the two examples given, that doubling the payoff value for a *red* victory requires approximately a halving of the failure rate in order to ensure that *red* will still choose not to play. Recall, that this is a game of complete information and a key assumption is that *red* is fully aware of the probability of success of *blue* prior to choosing whether or not to play. This assumption is not realistic and the real scenario involves incomplete information. Furthermore, even if *blue* possessed capabilities which would eschew *red* from playing at all, it would be difficult for *blue* to convince *red* to believe this fact. Even without considering *red* to be risk-loving, it is possible that *red* would play simply for lack of trusting *blue's* word in terms of probability of success.

The point to be made here, is that *red* is unlikely to choose not to play the game regardless of how successfully the *blue* defense system is found to operate. In fact, *blue* being risk averse should accept nothing but a $100\,\%$ success rate; this however, is unrealistic. Thus, the most realistic scenario at this stage involves transforming the payoff amounts such that *red* chooses to play for any non-zero probability of *blue* defense system failure.

3.7 Taxonomy of the Problem

In this section, the parameterization for the problem is presented. This parameterization differs from the problem discussed in the previous section in the sense that further detail is given about the details. The minimum detail required for the matrix representation of the game was provided previously, but in this section, analysis will require a more detailed accounting of the scenario.

The area to be defended is roughly defined as a stationary (or slow moving) area.

The size of this area is potentially as small as the deck of a large ship or as large as a vast military installation. This range is roughly defined as $4 \times 10^4\,\mathrm{ft}^2$ to $4 \times 10^8\,\mathrm{ft}^2$ ($0.002\,\mathrm{km}^2$ to $10.4\,\mathrm{km}^2$. In the obvious case that the area to be defended comprises an installation then it need not be stated that the maximum speed that the area to be defended will move is equal to 0. However, in the case of a large ship or the less obvious case where a mobile convoy of some kind is to be protected, then the speed is not expected to exceed $30\,\mathrm{mph}$ ($13.4\,\mathrm{m/s}$). Thus, the range of speeds possible are described as $0\,\mathrm{mph}$ to $30\,\mathrm{mph}$ ($0\,\mathrm{m/s}$ to $13.4\,\mathrm{m/s}$).

The agents being defended against are comprised of a threat necessarily containing UAS; though, other agents may be present. These UAS are defined roughly in terms of the parameters given describing the area to be defended. The UAS being considered here (in terms of planform area) will be $1\,\mathrm{ft}^2$ to $50\,\mathrm{ft}^2$ ($0.09\,\mathrm{m/s}$ to $4.6\,\mathrm{m/s}$). The enemy agents are thus 4×10^4 to 8×10^6 smaller than the area to be defending. Considering the capabilities of off-the-shelf UAS technology, the agents to be defended against are expected to have a maximum rectilinear speed of $30\,\mathrm{mph}$ to $300\,\mathrm{mph}$ ($13\,\mathrm{m/s}$ to $134\,\mathrm{m/s}$). This means that the enemy UAS are up to 10 times faster than the area to be defended in the case where the area is mobile.

3.7.1 Basic Parameterization of Capabilities

The specific traits within the wide range presented in the preceding section are next discussed. This list is the product of several rounds of refinement to capture only the most critical parameters to avoid diluting the problem space with redundant or unnecessary quantities. The reduction to the most essential parameters facilitates a more clear-headed evaluation of capabilities compared to more details parameterizations. In this case, the characteristics are also broken down by category.

3.7.2 Dynamics

For a dynamic agent (specifically mobile evaders are considered, but this characterization also applies to pursuers or fixed-defense systems), the effect of motion in three-space can be fully modeled according to two simple parameters. These characteristics are specified as *maximum rectilinear speed* and *maximum angular speed*. Maximum rectilinear speed gives a measure or how fast an agent can move and in the case of a mobile agent the angular speed gives the minimum turning radius or maximum rate turn. For the case of a fixed-defense system, the angular speed gives the slew rate and the maximum rectilinear speed would be zero.

3.7.3 Strategy

Characteristics related to strategy are those which relate to the game setup, or which may be controlled but generally are more of a pre-game decision, or at least are made on a longer time scale compared to decision made with respect to instantaneous trajectories. All these characteristics apply equally to red and blue agents. The first characteristics considered is the *number of agents*. Of course, either player can choose to change the number of agents involved in the game in the middle of the game; however, the optimal tactics in many case will dictate that both players make the best choice related to number of agents possible prior to starting an engagement or risk losing badly.

Next is the *type of entities* to be used. Either team may select entities which perform specific tasks in various numbers in order to accomplish the overall mission. Entities could be specifically suited to the ISR mission for example, while other entities are tasked with delivering a destructive payload. Similarly, for the blue agents, consider some entities may be fixed-defense entities which are stationed near the goal point as a last line of defense; whereas other agents fulfill the role of scout by venturing far away from the area to be protected. How many agents to use and the exact make-up of those agents comprise important pre-engagement decisions.

3.7.4 Physical

Consider next the physical characteristics. The rough bounding conditions on the sizes of agents is given above; however, the size in this case should be given with more specificity. The *size*, and *weight* of an agent can be important in determining what kind of threat an unknown agent might pose and could determine how easily an agent is detected or identified by radar. For blue, knowing these traits is important during the detection, identification and tracking phase but consider that red agents may deploy countermeasures which rely on detecting incoming blue interceptors, thus the size and weight and radar cross section of the blue agents can be just as important to consider.

3.7.5 Effector

The characteristics in this section are specific to directed energy defense systems which can be employed on mobile agents or as fixed-defenders. However, many of these characteristics have approximately equivalent meaning with respect to other types of systems such as missiles and artillery. These characteristics include *beam width*, *range*, *magazine*, and *time to effect*. In many cases, the beam width and beam range will be sufficient for characterizing a region where the probability of kill is sufficiently high that any agent within this zone will be considered neutralized.

The magazine trait refers to the ability of a directed energy (or other) systems to fire subsequent shots in rapid succession. A system with limited magazine will be unable to fire many subsequent shots in rapid succession and may even require and extended cool-down period after an engagement. A system with an unlimited magazine can be considered approximately equivalent to a system which is always on. The decision when to begin engagements in the case of a limited magazine forms an interesting wrinkle in some problems.

3.7.6 Logistical

In the case of both red and blue agents, the logistical footprint of a particular agent can be a critical consideration. Consider the *cost* of a particular type of agent, as well as the *support required* and *power required* to comprise the logistical concerns. Obviously, agents which are more capable will be preferred intuitively; however, consideration of logistical traits forms the basis of imposing a penalty for systems which are expensive to operate or difficult to move from place-to-place. Additionally, consider a threat technology which is very dangerous but requires a predictably large and identifiable footprint of equipment operators and support staff. Such a threat may pose a case where it is easier to detect the logistical footprint and disable the operation altogether rather than to face down the formidable characteristics of the technology itself.

3.7.7 Tracking

Lastly, consider the tracking category of characteristics. As mentioned previously this typically refers to the tracking and identification of threats; however, if the threats are able to deploy countermeasures, it could be equally important for threats to track defense systems and thus these characteristics apply to both red and blue agents. The first step in identifying an incoming threat is simply to recognize an unidentified object of interest. This is referred to as detection and thus the first characteristic is *time to detect*. Before any follow-up action can occur, the unidentified agent needs to be identified and thus the second characteristics *time to identify*. Agents which are difficult to detect at all pose the most obvious threat; however, agents which are difficult to distinguish from natural phenomena such as birds or clouds can also represent a potent threat.

3.7.8 Summary of Taxonomy

In this section the characteristics are presented in the form of a list for clarity.

- Dynamics (blue and red team)
    - Maximum rectilinear speed
    - Maximum angular speed
- Strategy (blue and red team)

  - Number of agents
  - Mission, type of agent
- Physical (blue and red team)
  - Size
  - Weight
- Logistics (blue and red team)
  - Cost
  - Power required
  - Support required
- Effector (blue team only)
  - Beam width
  - Beam range
  - Probability of kill
  - Time to effect
  - Magazine depth, maximum firing rate
- Radar, Tracking, Identification (mostly blue team, could be used by red for countermeasures)
  - Time to detect

  - Time to identify
  - Tracking error
- Configuration Traits (settled before the game begins)
  - Size and shape of area to be defenended
  - Size and shape of interior denial area (used to defined success/failure of scenario)
  - Allowed leakage rate

3.7.9 Game Configuration

The last category of characteristics are those which can be controlled mainly by the blue team, but must be changed well prior to any engagements. These characteristics include the *size* and *shape* of the area to be defended. It should be apparent that electing to defend an arbitrarily large area could be unnecessarily expensive; whereas, defending a very small area could allow too many agents to get too close to that which is being protected.

The *denial area*, meaning the sub-area inside the area to be defended past which no enemy agents are allowed can also have a big impact on the outcomes for an engagement. The size of the denial area relative to the larger area to be defended is also critical considering that the distance between the two areas comprises the total time red must "stay alive" in order to be successful in an individual engagement.

Finally consider that if the blue team elects to allow a certain number of red agents to get arbitrarily close to the goal point, or even explicitly allows a non-zero red agent *leakage rate*, then scenarios which once represented red victories can be transformed into blue victories. Allowing leakers can be supported in two obvious ways. First, the goal area may be passively hardened against enemy actions such that whatever is to be protected has a high likelihood of surviving most attacks. Secondly, leakers which make it into the denial area may be otherwise unable to complete their mission depending on the size of the denial area and the complexity of the mission. Just because a red agent "makes it" to his goal does not necessarily mean that the red agent has been able to successfully execute the intended mission.

3.8 Trajectories and Tactics

3.8.1 Notes on Maneuverability

In the following sections, engagements are discussed in detail with the goal of characterizing the problem as it relates to agents moving within the configuration space. An issue central to the usefulness of the work presented herein is related to whether or not the models proposed are sufficiently accurate to capture detail about the behavior being studied. Specifically, most of the work considers holonomic vehicle models. In the field of robotics, holonomic vehicles are those vehicles for which the controllable number of degrees of freedom (DOF) is exactly equal to the total number of degrees of freedom for the vehicle. For a planar, point model, where the vehicle position is fully described by two DOF such as two coordinates (often thought of as x and y coordinates), this means the vehicle is fully in control of both position coordinates without any consideration of heading angle, turn rate limitations, acceleration effects, or other effects. Real-world vehicles are not holonomic by definition. The most high-fidelity models used to describe vehicle motion are thus non-holonomic models.

The vehicle models used in the succeeding sections are holonomic and thus decidedly not-realistic; and while this results in a necessarily less accurate model, the level of detail required in this case makes the use of a lower-fidelity model appropriate. Consider a three DOF model which includes two positional coordinates as well as a single coordinate describing vehicle orientation, specifically the heading, or pointing (also called yaw) angle. Motion is constrained to in-the-plane and the only additional piece of information over the two DOF model is related to which direction the vehicle is facing. This accurately describes the simplified models which are used extensively in the sections that follow.

Consider a representative scenario where the motion of a single agent is to be modeled. In this scenario, the agent is represented by three DOF as described before (two positional coordinates and a yaw angle). The single agent is tasked with traversing from a designated starting point to a designated goal point subject to a minimum turning radius (according to the vehicle turn rate limitations at the speed currently being traveled). Since this model is only three DOF, the minimum turn radius is fixed because there is no coordinate (DOF) available to describe the vehicle speed. The vehicle heading is unconstrained at the start of the engagement (the vehicle can point wherever it needs to point prior to starting). However, the terminal heading (at the goal point) is constrained and in this case is stipulated as being normal to the straight line connecting the starting and goal point. For the purpose of demonstration this heading will be “straight-up”; although, the conditions for “straight-up” and “straight-down” are symmetrical.

For the case of a holonomic vehicle, the vehicle can traverse the straight line distance and arrive at the goal point before instantaneously reorienting its heading to point in the prescribed direction. In the case of the scenario described thus far, a non-holonomic vehicle is being discussed. The non-holonomic constraint relates the vehicle heading to the direction of travel, specifically by ascribing a minimum turning radius according to a maximum possible turning rate at a given vehicle speed (assumed to be fixed throughout the engagement).

Consider next another parameter, referred to as the basic modeling dimension which is defined here as the minimum length which can be discerned within a simplified model. The most realistic model techniques would utilize continuous dimensional space, but when discrete modeling space is used, the smaller the basic dimension, the more closely the model approximate continuous (real-world) effects. In this case, a discrete model is being discussed and thus the smallest dimension in the model provides a starting point for understanding the baked-in inaccuracy arising from resolution given by the chosen minimum dimension.

The basic modeling dimension will be called d_{model} for this discussion. The magnitude of the turn rate limitation is $r_{vehicle}$ and the straight-line path length is $l_{straight}$. Consider situations where the modeling dimension is substantially smaller in magnitude compared to the size of the vehicle turning radius $d_{simulation} << r_{vehicle}$. If the primary output of the model is the distance along-the-path traveled by the agent from the start to the goal point, then the accuracy or inaccuracy of the model may be judged by considering the limitations on the accuracy of the path length measurements according to known limitations on the model. In the scenario described, the model dimension is sufficiently small that the path length would be expected to be captured accurately by the simplified model. Next consider the error arising from using the estimated (straight-line) path length versus the exact path length which accounts for the curvature. If the exact path length is modeled sufficiently accurately by using a small modeling dimension, then it is obvious that large differences between the estimated and exact path lengths could be detected by such a model.

Next consider a similar scenario but now the modeling dimension is much larger than the minimum turning radius $d_{model} >> r_{vehicle}$. In such a model, almost no information about the curvature of the path can be captured by the model and thus is omitted or ignored from the results when considering the path length as the model output. To understand under what circumstances information about the path curvature is unlikely to have a large impact on the results we next consider the relationship between the turn rate limitation and the estimated (straight-line) distance.

For this discussion, as before, assume that the only output of the model being considered is the exact distance along-the-path required for the agent to traverse from the starting point to the goal point with the prescribed terminal heading. Assume that the modeling dimension is sufficiently small to capture information about the path curvature $d_{model} << r_{vehicle}$. Next consider the scenario where the vehicle turning radius is much smaller than the estimated straight-line path length. In the limiting case, the vehicle turning radius is infinitesimally small compared to the estimated path length $r_{vehicle} << l_{straight}$ and the effect of considering the curvature would be expected to be very small. Without solving for the path length including the curvature which will be done below, a crude estimate is proposed wherein the estimated path length is given by $l_{straight}$ and the exact path length is given by the the estimated (straight-line) length, minus the minimum turning radius plus some portion of the circumference of a circle described by the minimum turning radius $l_{exact} = l_{straight} - r_{vehicle} + k(2\pi r_{vehicle})$. In this formula k is a constant of proportionality where $k < 1$. For this discussion, we need not know the value of k exactly (this will be explored in more detail in the following section).

$$\frac{l_{straight}}{l_{exact}} = \frac{l_{straight}}{l_{straight} - r_{vehicle} + k(2\pi r_{vehicle})} \tag{3.10}$$

For comparison purposes the ratio between the estimated path length and the exact path length is formulated as given in Eq. (3.10). For the limiting case, consider the limit as given in Eq. (3.11) in which the limit as the vehicle turning radius approaches zero is considered. Such a limit is equivalent to considering the case where the estimated (straight-line) distance approaches infinity given in Eq. (3.12).

$$\lim_{r_{vehicle} \to 0} \frac{l_{straight}}{l_{straight} - r_{vehicle} + k(2\pi r_{vehicle})} = \frac{l_{straight}}{l_{straight}} = 1 \tag{3.11}$$

$$\lim_{l_{straight} \to \infty} \frac{l_{straight}}{l_{straight} - r_{vehicle} + k(2\pi r_{vehicle})} = \frac{l_{straight}}{l_{straight}} = 1 \tag{3.12}$$

The ratio as given in Eq. (3.10) describes the agreement between the estimated and exact path lengths given by the straight-line approximation compared to a simplified expression for the exact path length found by accounting for the curvature. When solving for the limit in both cases, the ratio is found to have a value equal to one indicating perfect agreement between the estimated and exact path lengths in the limiting case. This indicates that for the limiting case of infinitely large estimated (straight-line) distance, or infinitesimally small minimum turning radius, the estimated path length is exactly equal to the exact path length. Of course, the path with curvature has been approximated with a simple model here, and although this approach is probably acceptable in terms of solving mathematical limits (where slight differences in specific magnitude of the terms may not matter when some terms are dominating) further justification is provided below.

This justification includes an actual formulation for the curved path as well as granular consideration of cases other than the limiting cases to understand how the relationship between the estimated distance and the minimum turning radius affects the accuracy of path length measurements resulting from the model.

3.8.2 Numerical Results on Maneuverability

Consider a scenario similar to those described in the previous section. A single agent with three DOF (two position coordinates plus a vehicle heading) travels from a prescribed starting point to a prescribed goal point while complying with a minimum turn radius constraint. A numerical study was conducted by varying the magnitude of the limited minimum turning radius against a normalized straight line distance between the start point and the goal point. That is, the estimated (straight-line) path length has a fixed magnitude equal to 1, $l_{straight} = 1$ and the minimum turning radius varies in proportion to the estimated path length according to Eq. (3.13).

$$0 < r_{vehicle} < \frac{l_{straight}}{2} \tag{3.13}$$

The lower limit in this case, $0 < r_{vehicle}$ is given considering that the vehicle turning radius must not be negative, but a value of 0 corresponds to an unrestricted turning radius (which corresponds with a holonomic vehicle). The upper limit $r_{vehicle} < \frac{l_{straight}}{2}$ is required by the geometry of the problem. It is possible for the minimum turning radius to exceed this limitation but in that case the trajectory to reach the goal location would necessarily include a phase where the agent would move away from the goal location in order to get in position to arrive at the goal location while account for the turn rate limitation. This type of trajectory is not considered in this case.

An equation was found to solve for the intersection point from the prescribed origin point to a point tangent to the minimum turning circle constructed such that the agent arrives at the goal point with the prescribed heading. Finding this equations relies on formulating an equation for a generic intersection point on the circumference of a circle with a prescribed radius. Then formulating an equation for a generic intersection with a line and simultaneously solving both equations subject to the constraint that the line also intersects the starting location. Additionally the slope at the intersection point must be tangent to the circle. The resulting equation for the $x_{intersection}$ and $y_{intersection}$ coordinates of the intersection point are given in Eqs. (3.14) and (3.15). The starting point is given by x_0 and y_0 and the minimum turning radius is given as $r_{vehicle}$.

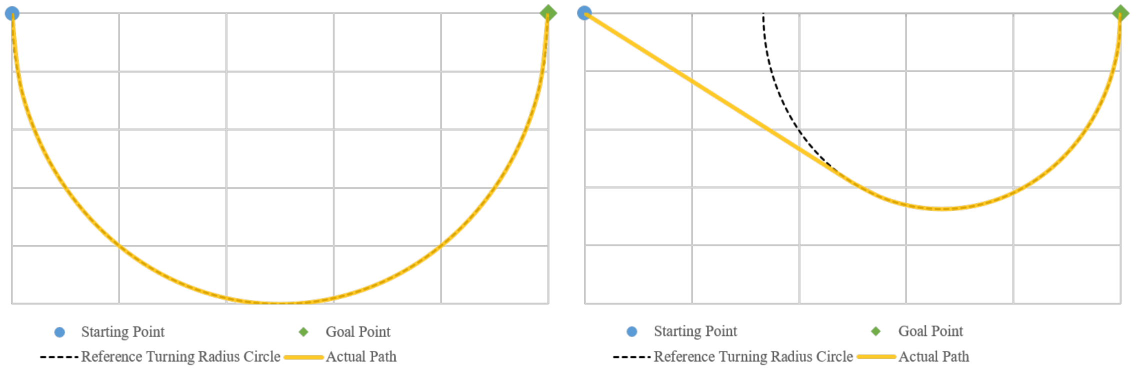
$$x_{intersection} = (x_0 \frac{r^2_{vehicle}}{x_0^2 + y_0^2} + \frac{r}{(x_0^2 + y_0^2)\sqrt{(x_0^2 + y_0^2) - r^2_{vehicle}}}) - y_0 \tag{3.14}$$

$$y_{intersection} = (y_0 \frac{r^2_{vehicle}}{x_0^2 + y_0^2} + \frac{r}{(x_0^2 + y_0^2)\sqrt{(x_0^2 + y_0^2) - r^2_{vehicle}}}) x_0 \tag{3.15}$$

The solution to the intersection equations is the solution to a quadratic equation. It should be apparent that there are always two solutions to the proposed circle intersection problem which are symmetrical about a line which connects the chosen starting point with the center of the circle. For the purposes of this study, only one of these intersection points is needed and thus the solved equations omit the customary ± notation and instead give the solution for one of the symmetrical intersection points according to the dictates of convenience.

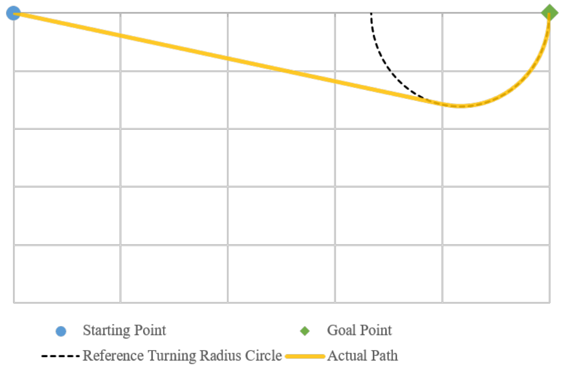
Since the estimated (straight-line) distance $l_{straight}$ is held constant (normalized) for this study, the value of $l_{straight}$ does not appear in the equations in Eq. (3.14). However, the parameterization is presented in terms of the ratio between the minimum turning radius and the estimated (straight-line) distance $\frac{r_{vehicle}}{\frac{l_{straight}}{2}}$. Various levels of this ratio are covered from 1 to 0.0001. The resulting output curves are shown in Figure 5.

The left-axis on this plot and the traces shown in green represent the ratio of the exact distance (computed accounting for the curvature) to the straight line distance and expressed in percentage. The worst-case condition is marked with the dotted green line with a value of $64\,\%$. This can be interpreted as the estimated distance found according to the straight-line approximation is only $64\,\%$ of the exact distance found according to the curvature. As expected, as the ratio $\frac{r_{vehicle}}{\frac{l_{straight}}{2}}$ approaches 0 the estimates and exact path



(a) Illustration of Exact Trajectory for Straight Line Distance Equal to Two Times the Minimum Turn Radius.

(b) Illustration of Exact Trajectory for Straight Line Distance Equal to Four Times the Minimum Turn Radius.



(c) Illustration of Exact Trajectory for Straight Line Distance Equal to Ten Times the Minimum Turn Radius.

Figure 4: Representative Illustrations for Model Describing 3 DOF Agents with Planar Motion with Turn Rate Limitations.

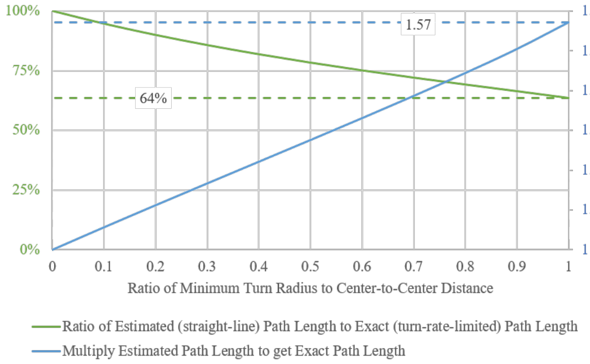


Figure 5: Plot Showing Two Ways of Comparing Estimated (straight-line) Path Length to the Exact (curved) Path Length.

lengths appear to converge.

The right-axis on the plot in Figure 5 and the traces shown in blue represent the same relationship except now the ratio between the estimated (straight-line) path length and the exact path length is represented by the ratio $\frac{l_{exact}}{l_{straight}}$. The same worst-case scenario is now presented by the dotted line with a value of 1.57, which can be interpreted as the exact path length being 1.57 times greater than than the estimated distance.

The most important takeaway from the plot in Figure 5 is that as the ratio of the minimum turn radius to the estimated path length approaches zero, the error in using the estimated path length compared to the exact path length also approaches zero. Practically this can be interpreted to provide context related to the importance of studying maneuverability in relation to the overall size of the engagement to be studied. Consider a scenario where the entire engagement is to take place over a region which is many times larger than the minimum turning radius. In this case, the effect of considering the turning radius is expected to be minimal compared to using the simple straight-line estimate for path length measurements.

Table 5: Exact Values Corresponding to the Effect of Turn Rate on Path Length.

$\frac{r_{vehicle}}{l_{straight}/2}$ (*ratio*)	$\frac{l_{straight}}{l_{exact}}$ (%)	$\frac{l_{exact}}{l_{straight}}$ (*ratio*)
1	63.6	1.57
0.9	66.5	1.50
0.8	69.3	1.44
0.7	72.1	1.39
0.6	75.2	1.33
0.5	78.4	1.28
0.4	81.9	1.22
0.3	85.7	1.17
0.2	89.9	1.11
0.1	94.7	1.06
0.01	99.4	1.01
0.001	99.9	1.00
0.0001	100.0	1.00

If the goal point is moving (trying to intercept an evading agent) then the effect of considering maneuverability on overall path lengths is expected to be larger especially if the evader is reacting to the pursuit. However, the general result here is that the effect of maneuverability only becomes a substantial effect when the grid size of the model (or the overall dimension describing the engagement) is reasonably close in magnitude to the the magnitude of the turn rate limitation. For the purposes of computing time-to-transit, the overall dimension of each engagement considered is by definition substantially larger than the minimum turning radius for the agents and thus maneuverability is not expected to be a substantial factor and is mostly ignored. Conversely, once inside of a certain neighborhood region of a goal location (especially for pursuit-evasion scenarios with evading agents) the effect of maneuverability will be a substantial factor and must be considered. This region of the pursuit is to be distinguished from the region considered for time-to-transit according to the phases of pursuit.

Whenever time-to-transit is being considered, the engagement is assumed to be such that maneuverability is not a factor. However, once the pursuit enters the so-called “endgame” phase, maneuverability must be considered. In this work, the two regions are treated separately and the bulk of the remaining work in this section is specifically related to understanding big-picture time-to-transit engagements while spending very little time discussing maneuverability. Considering system performance to be related to the sum of individual contributions due to certain characteristics or considerations allows for the time associated with chasing a maneuvering target during endgame to be considered separately, while focusing on other parts of the pursuit trajectory here.

3.8.3 Presentation of the Problem of a Single Effector versus a Single Aggressor

The simplest problem to be solved in the context of pursuit-evasion is as follows. A single effector (including tracking, identification, and threat elimination mechanisms) is tasked with engaging, and eliminating one target at a time. In the simplest case the aggressor motion is constrained into a single dimension, meaning an engagement comprises a straight-line-run at the effector with maximum possible speed. Justification for this type of aggressor path will be presented later. In order to further clarify this scenario effector boundaries must be established.

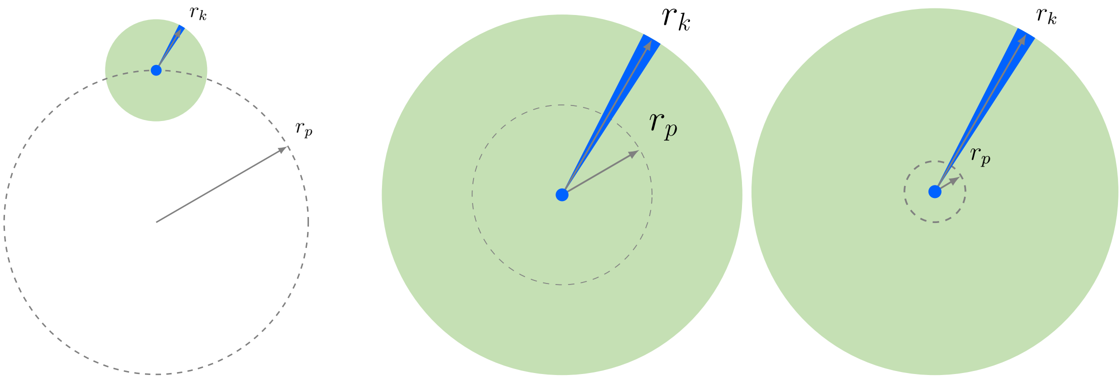
Effectors in this case are modeled as a point source projecting a “kill area” which can pivot about the point with some maximum angular velocity called $\dot{\Psi}_{max}$. Any mutual contact between the aggressor and the "kill area" at any time during the engagement results in the immediate elimination of the aggressor from the engagement.

Next, let us consider classifications of game space by size ratio in rough orders of magnitude. Specifically, the radius of the "kill area" (defined as the kill radius ,r_k) must be contained within the protection area (defined by the protection radius, r_p). In the first case, the kill radius is smaller than the protection radius by roughly an order of magnitude $r_k \ll r_p$. In this case, effective defense will require the use of multiple effectors. This is consistent with a coastal defense or base perimeter installment consisting of several effectors. The next scenario consists of situations in which the kill radius is close to the same size as the protection radius $k_r \approx r_p$ (within the same order of magnitude). This case is actually more complex than the other cases to be discussed here, and thus investigation of this scenario is deferred to later in the chapter. The last scenario to consider involves a kill radius which is much smaller than the protection radius $r_k \gg r_p$. In the final case, the size and shape of the protection area can be ignored and considered to be a point during scenario generation.

Next we must describe the orientation of the space occupied by the effector and the aggressor including defining the conditions under which each party is considered to have achieved victory. In the first effector configuration, the effector is positioned along a perimeter and responsible for defending the area on one side of this perimeter. The effector coverage area in this case is represented by an arc swept from the perimeter boundary on one side of the effector to the perimeter boundary on the other side (180° for the linear boundary case). The effector is necessarily positioned in the middle of the boundary. The effector will be successful in any engagements ending with the aggressor eliminated prior to breaching the defined boundary. Conversely, the aggressor will be successful (a *leaker*) in any engagements where the aggressor passes or contacts the perimeter. It is important to note in this case that the effector is easily capable of covering the area immediately behind the perimeter as well; however, any aggressor which breaches the boundary will be considered a leaker even if the effector could (and possibly would) be able to eliminate the target after the boundary has been breached.

The next obvious orientation arises when the effector "kill area" is considered to be substantially larger than the area to be protected. In this case, the effector is centered in the middle of a circular field of play. The "kill area" is defined as before but now the effector necessarily will cover the entire circular perimeter by sweeping through $360°$. In this case, effector victory occurs whenever an aggressor is eliminated prior to intersecting the origin point of the effector. Conversely, any aggressor which arrives at the origin point of the effector will be considered a *leaker*.

Next consider the first scenario in the development of the mathematical foundation upon which guaranteed victory conditions can be developed. First we must define quantities. The maximum angular velocity of the effector has already been mentioned and will be denoted by $\dot{\Psi}_{max}$ with units of degrees per second. The maximum translational velocity of the aggressor will be denoted by v_{max} with units of feet per second. The maximum range at which the effector can engage a target is given as d with units of feet and the maximum sweep angle of the effector is given by Ψ_{sweep} with units of degrees. Beam width w_b (or beam angle) with units of degrees is another important characteristics.

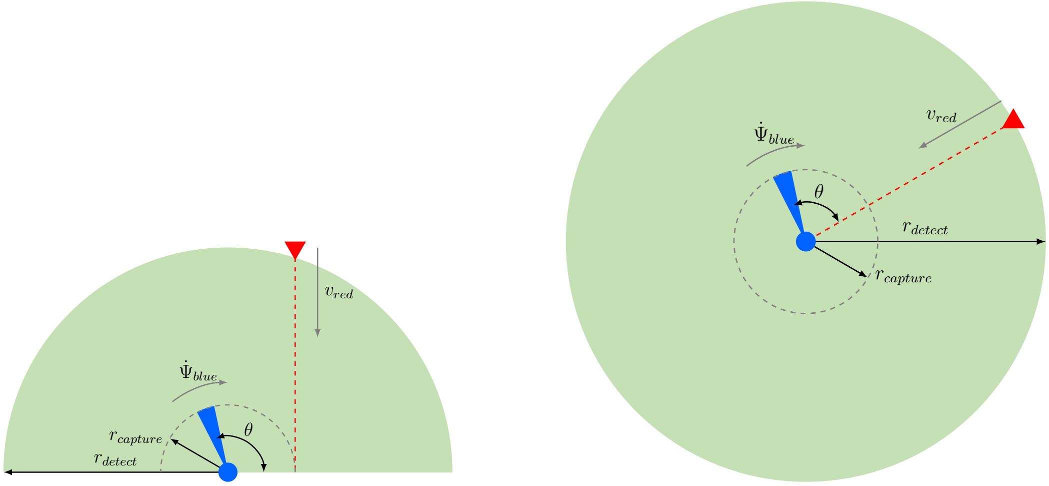


(a) Kill Radius much smaller than Protection Radius ($r_k \ll r_p$).

(b) Kill Radius and Protection Radius of Similar Magnitude ($r_k \approx r_p$).

(c) Kill Radius much larger than Protection Radius ($r_k \gg r_p$).

Figure 6: Scenarios Classified by Relative Size of Effector versus Area to be Protected.



(a) 180° Scenario.

(b) 360° Scenario.

Figure 7: Schematic Diagram of Two Protection Scenarios.

Consider an effector scenario where the sweep angle and the beam width are the same $w_b = \Psi_{sweep}$. In this scenario, the effector can engage the entire kill area without slewing. Also consider an effector which is capable of engaging n-targets simultaneously and instantaneously. Time to detect, track, identify, engage, and kill are all ignored and the effector aiming characteristics are also ignored. The effector in this case is able to engage the entire kill area at the same time. We consider this type of effector to be omnidirectional and instantaneous.

The omnidirectional and instantaneous effector is not a practical case but rather an intentionally impractical case which provides a basis for comparison. Such a basis is needed when simultaneous engagements ("swarm" type engagements) are to be considered. This need arises because the $100\,\%$ victory conditions as established below begin to break down when an arbitrary number of engagements are allowed to occur simultaneously.

The conditions for a perfectly matched aggressor and effector are found as follows.

$$\frac{\dot{\Psi}_{max}}{\Psi_{sweep}} = \frac{d}{v_{max}} \tag{3.16}$$

Eq. (3.16) is simply a statement of equality between the distance covered by each of the parties. For a perfectly matched game, the pursuer and evader (in this case, the effector and the aggressor) must cover the maximum distance required in the same amount of time. Indeed, solving the equation leaves units of time on both sides of the equation. Since the main concern is with ensuring effector victory, the expression can be cast as an inequality can be used to find the minimum effector characteristics.

For the effector installed on a linear perimeter responsible for $180°$ of defense, the minimum angular speed required to guarantee success for all one-to-one engagements is given as $180°$ times the distance to be covered; the quantity then divided by the velocity of the aggressor given in Eq. (3.19). This expression effectively means that the effector must be faster to sweep from one extreme of travel to the other than the aggressor is to cover the distance to the boundary once inside the detection area. An important point arises here in that the detection area and the kill area have been considered to have the same range (represented by d). In fact, this likely represents a worse case than real-life since most systems will have a detection radius much greater than the effective kill radius.

$$\dot{\Psi}_{max} \geq \frac{\Psi_{sweep} d}{v} \tag{3.17}$$

$$\dot{\Psi}_{max} \geq \frac{180° d}{v} \tag{3.18}$$

$$\dot{\Psi}_{max} \geq \frac{\pi d}{v} \tag{3.19}$$

Naively, it could be postulated that the effector which is responsible for twice as much coverage area must be twice as fast. However, consider the circular protection area corresponding to a sweep angle of $360°$. No matter the geometric configuration, there is no scenario where the aggressor can cause the effector to sweep more than $180°$ during a single engagement. At this stage consider that the aggressor is still restricted to a straight line path heading straight for the goal area. There are paths which may "confuse" the effector which rely on a sort of zig-zag motion; however, these scenarios will be considered later in this manuscript.

Considering that the worst-case-scenario still only involves a $180°$ movement of the effector, the condition for guaranteed effector victory is the same regardless of the sweep area considers. This means that at this stage, there is a solitary solution for both problems as given in Eq. (3.19). It should also be noted that the scenarios presented back-to-back here are not equivalent; although, they both result in the same solution. In fact, the first scenario where the effector is only responsible for $180°$ is considered a short range scenario where the effector protection area is approximately equal to the kill or detection range of the effector. In the $360°$ scenario, the effector is covering an area substantially larger than the protection area. In fact, the protection area may be considered to be a point which is infinitesimally small rather than a shape with measurable area.

$$\dot{\Psi}_{max} \geq \frac{360° d}{v} \tag{3.20}$$

$$\dot{\Psi}_{max} \geq \frac{2\pi d}{v} \tag{3.21}$$

Armed with this knowledge about minimum required effector angular speed, the next step is to consider more complex scenarios and consider how the effector requirements change or how the effectiveness of an effector with specific characteristics will vary. The most simple enhancement available is to add a second aggressor to the engagement. In the simplest case, the second aggressor appears instantly after the first aggressor is either defeated or successfully penetrates the protection area. This arrangement is hereto referred to a “sequential” engagement. That is, only a single aggressor is ever on the field of play at a single instant.

Consider the case of a $180°$ effector. If a single aggressor starts on a straight path toward the boundary from a location as far as possible from the center of the protection area, an effector which is barely fast enough will intercept the aggressor exactly at the instant prior to the aggressor intercepting the boundary. The second aggressor would then be most likely to succeed by starting on a straight line path which puts the new aggressor as far away as possible from the current effector position. Interestingly, because the minimum effector speed was derived conservatively, that is derived for aggressors at the maximum possible distance (worse-case scenario for the effector), the minimum effector speed for a guaranteed victory with a single aggressor is exactly equivalent to the minimum effector speed required for guaranteed victory with sequential single aggressors.

The same does not hold true if aggressors are introduced simultaneously. If twin aggressors are introduced at exactly the same instant, both making straight line paths for opposite extremes sides of the effector then the minimum effector speed would double. Strangely, for a third aggressor, tripling of the speed is not required. The reason this occurs is again related to the limits of exactly what angle separates two maximally separated entities. For either the $180°$ or $360°$ effector, the introduction of a third simultaneous aggressor does not change the minimum effector speed requirement. If two simultaneous aggressors are able to occupy the extreme points of the effector protection area then adding a third aggressor does not necessarily result in any further effort on the part of the effector.

In the case of the $180°$ effector, no scenario can be proposed where the additional aggressor is not automatically defeated as part of the motion to move to and defeat the two aggressors which have been previously discussed (considering that the first two aggressors are placed at the points of maximal separation to begin with). With the $360°$ effector case, the situation is more interesting. If the first two aggressors are spaced such that they have separation of $120°$ rather than the maximal separation of $180\,\mathrm{degree}$ then a third aggressor can be added in such a way to "trick" the effector into taking the longer path ($240°$, rather than $120°$) to get to the second of the first two aggressors. For the two aggressor case, this would not be possible since the effector would necessarily take the shorter path. In order to guarantee victory in this case, the effector speed would need to be responsible for covering $240°$ instead of the usual $180°$ and thus the speed would need to increase by a factor equal to $\frac{240}{180} = 1.33$ versus the two aggressor case. This represents the beginning of a trend of diminishing returns in terms of increasing simultaneous aggressors in engagements with an effector responsible for $360°$.

The $240°$ scenario presented above, is a slight simplification. In order to trick the effector into taking a longer path, information is needed about how the effector decides where to point and when. The doctrine of the effector is comprised of the set of rules which dictates at which targets the effector will be pointing (or attempting to be pointing) and at what time. So far, simple effector doctrine has been considered such that the effector will be pointed wherever needed in order for an effector to be perfectly successful for a given engagement. In practice, the doctrine of the effector is non-trivial. Consider for example a problem of two simultaneous aggressors whereby the aggressors know exactly the kill radius of the effector and thus are able to "trick" the effector by flirting with the kill radius while making a slow approach toward the protection boundary. By forcing the effector to constantly switch between extreme positions on either side of the protection boundary, both aggressors could theoretically cross the protection boundary while the effector remains worthlessly pointed straight ahead. Of course, this scenario represents an extreme example using a doctrine that would be quickly rejected. However, the exploitation of doctrine must not be ignored.

Up to now, simultaneous multiple-on-one engagements have been considered. The next step in the increasing complexity of scenarios considered does not involve adding more aggressors. Instead, consider now the time at which aggressors are "released" (straight line paths will still be considered exclusively for the sake of simplicity). For any generic (non-simultaneous) three aggressor engagement, the outcomes of the engagement are highly dependent on doctrine. If the aggressors understand that the effector can only chase one aggressor as a time and understand how long a given engagement will be likely to last, then the third aggressor can be sent in behind the first aggressor with a delay equal to just less than the time required for the effector to switch from successfully engaging the first aggressor, sweeping the entire kill area and then engaging and killing the second aggressor. In this case, the third aggressor would always be effective. Consider effector doctrine which tracks the closest target but places a cost on switching targets in the middle of an engagement and possibly also a cost associated with pointing away from the middle of the kill area. If the effector speed is increased to say 2.5 times the minimum required for a single aggressor, it is possible that this effector (depending on the tuning of the doctrine cost function) may prove to be guaranteed effective against most non-simultaneous three aggressor engagements.

There are some unrealistic consequences of considering effector doctrine in this way. For fast enough aggressors, it becomes impossible for the effector to successfully track any aggressor paths from the point of first contact to a successful kill. In such a situation, a $180°$ effector may be best occupied by simply pointing sideways along the protection perimeter and acting as a sort of force field which stops about $50\,\%$ of incoming aggressors by nature of protecting half of the protection boundary. This is an extreme example where appropriate effector doctrine can actually lead to a much higher than predicted rate of success. For $360°$ effectors, the situation is not quite as simple; although, scenarios may be postulated where the effector is best served by spinning madly around in continuous circles randomly scooping up aggressors with the sweeping beam at random.

Changing the discussion from one of mathematical purity and certainty into one where terms such as "most" or "many" or "enough" are to be used involves an important change in solution tactics. Once generic engagements are to be considered, the number of variables to be considered increases dramatically. Each engagement may be represented by a curvilinear path which can be represented as a series of hundreds of discrete points (depending on the grid size), each aggressor can be considered to start at any time during the engagement, the number of aggressors may be any number between one and some reasonable limit (for swarms this "reasonable" number could be as high as 100 or even $1{,}000$). Effector doctrine adds many potential layers of complexity depending on how complex the doctrine law is and how many parameters is is based on.

3.8.4 Angular versus Linear Velocity Considerations

For the fixed-effector scenario, the red evader will continually progress toward the goal point until the engagement is ended. The engagement may end in one of two ways. First, the engagement may end when the red evader reaches the goal point and is thus deemed a "leaker" or a loss for the blue team. An engagement may also end when the blue team eliminates the red evader prior to the red evader reaching the goal point. Engagements as discussed in the previous section consist of $180°$ fence defense and $360\,\mathrm{degree}$ point defense scenarios. For this section, the focus will shift to the $360°$ for ease of explanation.

Up to this point, red evaders which attempt to evade capture have been ignored. The primary goal for the red team in the scenarios thus far considered is simply to arrive at the goal point. Each evader is an individual agent which cannot communicate with other agents and thus each agent simply tries to get to the goal as quickly as possible. Of course, if the red evaders are able to observe or detect blue team action or communicate with one another, then it could benefit the red team to take evasive action and possibly to communicate in order to inform other agents on the current whereabouts of blue team assets. In the $360°$ point defense scenario, the fastest path for the red evaders to get from the circumference of the circle marking the limits of the blue team effector capabilities up to the goal point located at the center of the circle is simply a line segment comprising a radius of the aforementioned circle.

For reasonably large effector effective radii, it may be difficult for red evaders to travel or maneuver quickly enough to escape blue capture. Once the red team gets close enough, it should be apparent there exists a radius where the advantage will shift from red to blue and once inside that radius, the red team will be able to evade the blue team infinitely since the red team rectilinear speed capabilities will outmatch the blue team angular speed. The governing equation for this relationship solved for this critical radius is given in Eq. (3.22).

$$r_{critical} = \frac{v_{red}}{\dot{\Psi}} \tag{3.22}$$

The equation in Eq. (3.22) at first glance may appear to suffer from mixed units but applying simple dimensions analysis on the quantities when the angular speed $\dot{\Psi}$ is expressed in $\mathrm{rad/s}$ and the speed of the red evader v_{red} in $\mathrm{m/s}$. The expression on the right-hand-side will have units of $\mathrm{m/s}$ divided by $\mathrm{rad/s}$ which simplifies to $\mathrm{m/rad}$. Considering that units of rad are essentially unit-less the units of the output can be further simplified into m or equivalently whatever base unit of length is being used for the scenario considered.

For the sake of convenience, an alternative method of parameterizing the red team speed is given such that the given red team speed is normalized against the distance between the evader starting point and the goal point. Several levels of red team speed are given in the following analysis from 0.0167 to 1 which represents red team speed corresponding to a total time from start to goal of $60\,\mathrm{s}$ in the slowest case and $1\,\mathrm{s}$ in the fastest case. For convenience in this discussion the blue team speed is given in terms of $^{\circ}/\mathrm{s}$ but when doing numeric calculations the values are necessarily converted to $\mathrm{rad/s}$.

The blue team effector speeds are given in levels from $45\,^{\circ}/\mathrm{s}$ to $360\,^{\circ}/\mathrm{s}$. This

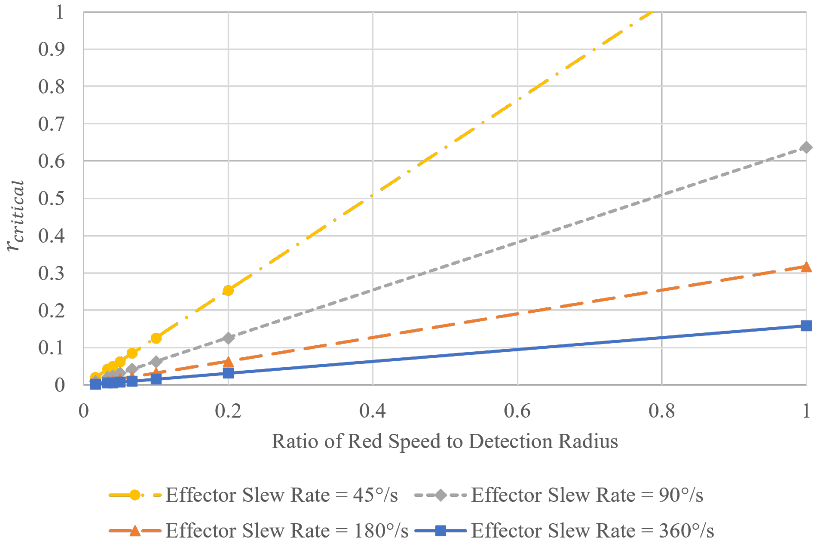


Figure 8: Plot Showing Normalized Critical Radius for Different Levels of Evader and Effector Speed.

means that the fastest blue effector traverses the entirety of the $360°$ defense zone in $8\,\mathrm{s}$ in the slowest case and $1\,\mathrm{s}$ in the fastest case. The given quantities for red and blue speed are intended to bracket the possibilities for real-world cases.

The results of a numerical study are shown in Figure 8. In this plot, the red speed is given as discussed above and is shown on the x-axis. The blue effector speed is given in $°/\mathrm{s}$ and is represented by different style lines as designated in the legend. The y-axis represents the critical radius as determined by Eq. (3.22) expressed as a fraction of the detection radius. The detection radius in this case is equal to 1 and is denoted as the farthest distance at which the blue effector can begin interacting with red evaders. In this simplified case $r_{detection} = r_{effective}$.

The relationships shown are linear according to Eq. (3.22). Thus, there is no obvious optimal point indicated on the plot in Figure 8. The implications of the results shown in the plot are that once the red team achieves the critical radius, theory supports that if red has complete information about the scenario, that if red acts optimally, blue cannot defeat red no matter what action blue takes. The values on the y-axis represent how close this critical radius is occurring in relationship to the goal point based on a normalized outer radius with a value of 1.

These results can be interpreted such that if the actual range of the first interaction is $1{,}000\,\mathrm{m}$ and the blue effector is slow with a maximum angular speed of $45\,{}^{\circ}/\mathrm{s}$ then for moderately fast evader with normalized speed equal to 0.4, then blue should expect red to be able to evade and win all engagements once one evader is inside of about $500\,\mathrm{m}$ or $50\,\%$ of the maximum range. Obviously, for faster effector speed, the effect of faster red evader speed is diminished in terms of determining where the critical radius occurs. Importantly, for a fast effector ($\dot{\Psi}$ =$360\,{}^{\circ}/\mathrm{s}$) and a fast evader ($v_{red} = 0.1$ which means red can traverse the entire field is about $10\,\mathrm{s}$) there is still an erosion of the effective blue defense radius according to $r_{critical} = 0.15$. Interpreting this result means that blue effectors should favor generous denial radii and strive to eliminate red evaders before they get close to the goal location. Once red evaders have achieved the critical radius, a change in blue doctrine may be required to avoid tactical exploits by the red team resulting in cascades of leakers.

3.8.5 Hunter-Killer Scenario

In this section, the scenario is considered where a goal region is to be defended against incoming agents (the red team) by a team of pursuers (blue team). Blue team victory occurs whenever all red team agents are captured up to and including a perfect

Table 6: Table Showing Exact Values Corresponding to the Critical Radius for Red Evasion.

v_{red} (*normalized*)	$\dot{\Psi}$ =360 °/s $r_{critical}$ (*ratio*)	$\dot{\Psi}$ =180 °/s $r_{critical}$ (*ratio*)	$\dot{\Psi}$ =90 °/s $r_{critical}$ (*ratio*)	$\dot{\Psi}$ =45 °/s $r_{critical}$ (*ratio*)
1	0.159	0.318	0.636	1.273
0.2	0.032	0.064	0.127	0.255
0.1	0.016	0.032	0.064	0.127
0.07	0.011	0.021	0.042	0.085
0.05	0.008	0.016	0.032	0.064
0.04	0.006	0.013	0.026	0.051
0.03	0.005	0.011	0.021	0.042
0.02	0.003	0.005	0.011	0.021

intersection with some predefined boundary (the denial radius) $\text{red}_{\text{distance to goal}} \geq r_{denial}$. The prescient questions are to find solutions to the problems given physical characteristics of the agents involved in the game including finding the minimum capture radius (closest evader) for various initial conditions and bounds on motion. Such problem setups can also be used to solve for the minimum number of blue agents required to defeat all red agents under a specific set of circumstances.

A unique question arises with respect to the hunter-killer scenario related to the initial position of the blue and red agents. In the case of a fixed-position base defense effector, the effector may be oriented in a variety of positions are the beginning of the engagement which are defined by the angular position. In the case of a mobile defender, the relative initial position is now two-dimensional. Both the relative angular position and the distance from the center of the protection region are now specified as initial conditions. A unique question arises with respect to the best position of blue agent(s) with respect to angular separation and distance from the goal. Moving blue agents farther from the center of the protection region allows capture of slow moving red targets with much greater capture margin in terms of larger distance-to-goal at the instant of capture. However, for closely matched red and blue agents, moving blue agents farther from the capture region could allow red leakers in the case where blue agents are overwhelmed by speed, maneuverability, or number of red agents.

Furthermore, it is desired to understand conditions under which multiple blue agents may be installed on a cyclic patrol pattern and how that will affect margins of victory versus the case when multiple blue agents are launched from the center of the capture region at the moment a red agent is first detected. Blue agents could also be deployed from satellite launch locations; although, consideration of such a situation adds substantial complexity to the basic scenario. In the case of patrolling blue agents, situations may arise where the blue agents best course of action is not to fly out to incoming red agents but rather to sit and wait for (potentially faster) red agents to come closer before initiating a capture. This may seem counter-intuitive at first, especially if one considers that margin of victory is to be maximized in all conditions. However, considering that chasing one red agent to maximize margin of victory for a single engagement could open the possibility of a leaking red agent in the case of multiple-on-multiple engagements.

Next consider the bounding conditions for the base protection by a group of mobile pursuers. In this scenario, exactly what constitutes the best and worst case scenario must first be considered. In the case of fixed effectors defending a base, the worst case red team configuration involves the maximal angular spacing between the current position of the effector beam and the position of the red agents at the instant they are detected. The corresponding scenario for the case of mobile protection agents involves maximal angular spacing and many of the same oddities arise.

For example, the maximal angular spacing for a one-on-one engagement is simply 180°. For a two-on-one engagement, the maximal radial spacing is 120° for the first evader and 120° for the second evader. Using this construction, the blue team always gets the last angular segment "for free" meaning that blue need not account for protecting the full 360° of the engagement zone for any single engagement. In the case of infinitely many evaders (in which case the angular spacing for each evader is infinitesimally small) the blue pursuer could theoretically be forced to cover nearly the entire 360° of the perimeter where the blue angular position at the first engagement would be immediately adjacent to the blue angular position at the last engagement.

In such scenarios it is certainly possible to allow for more than one pursuer in each engagement. Importantly, now two-on-two, and two-on-three, and arbitrarily many-on-many engagements must be considered. For the case of multiple blue agents, the basic construction and conclusions from the above section still hold true. Each blue agent gets some portion of the angular region "for free" but the amount each agent gets will depend on the total number of blue agents in the scenario. Some bounding conditions for the base protection by mobile effectors are given in Eq. (3.23).

$$\Psi_{max} = \frac{360}{n_{blue}} \tag{3.23}$$

$$\Psi_{free} = \frac{360}{n_{blue} + n_{red}} \tag{3.24}$$

The next order of business is to consider what path blue agents should take in order to ensure that red agents can be captured with maximum efficiency. Blue should capture red agents with the goal of minimizing the capture time for each engagement. This will allow for blue to capture the maximum number of red agents for a given scenario. Aiding in the development of a solution to the capture path problem is the fact that the red agents have a predictable path. At this stage, consider only scenarios where blue agents have equal or slightly greater speed than red agents $v_{blue} \geq v_{red}$. In this scenario, when considering the game of kind (where red strives to reach the goal and any engagement that ends with red capture is blue victory and any red leakers comprise a blue victory) red must not deviate from a straight line path toward the goal in order to act optimally. There are other possible strategies which arise when considering the game of degree (perhaps red is trying to maximize the amount of time blue spends pursuing) or consider communicating red agents which may strive to distract blue agents for as long as possible. Those strategies are not considered at this time.

Red agents will always move directly toward the goal once inside the detection radius. Thus the position and velocity of red agents is known at all times. Parameterizing the red velocity into polar coordinates results in a non-zero constant value for the derivative of the radial position with respect to time $\frac{dr}{dt} = v_{red}$. The derivative of the angular position is simply zero $\frac{d\theta}{dt} = 0$. Thus, one possible pursuit path involves blue agents (which can control both radial and angular velocities, simply setting their radial velocity equal to the radial velocity of the evading red agents and using the excess velocity to traverse in the angular direction. The described scenario is given by equations in Eq. (3.25).

$$\vec{v}_{red} = \frac{d\theta}{dt}\vec{\theta} + \frac{dr}{dt}\vec{r} \tag{3.25}$$

$$\vec{v}_{red} = \frac{dr}{dt}\vec{r} \tag{3.26}$$

For the case of a countable number of red agents, the intercept paths for blue agents are comprised of straight lines. However, moving to the infinite evaders case, with infinitesimally small spacing between said agents, results in continuous intercept paths. Such smooth paths would waste time for the case of a countable number of evaders; however, straight line intercept paths necessarily require an instantaneous direction change when transitioning from one intercept to the next. The corresponding smooth path does not suffer from this limitation.

Thus, studying the smooth intercept path (for infinitely many evaders) is a useful exercise for understanding how blue agents may best structure pursuits. Considering that the radial velocity is already given by simply matching the radial velocity of the red evaders, all that remains is to apportion the remaining blue velocity toward moving in the radial direction. At this stage consider that blue agents are attempting to capture red agents as quickly as possible (thus ruling out any less direct intercept paths).

The required "smooth" path for intercepts is comprised of a log spiral where the log spiral parameters are determined wholly by the speed ratio between red and blue agents.

Consider a typical hunter-killer scenario as depicted in Figure 9. For this scenario, the scenario begins with the blue and red agents located on the perimeter of the circle described by the detection radius which has a value of 1 ($r_{detection} = 1$). The speed of the

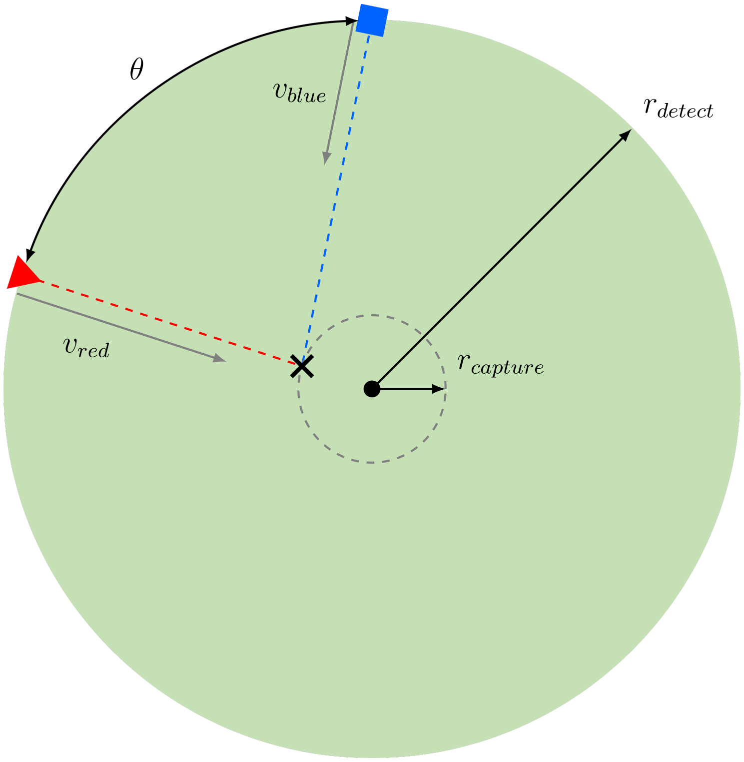


Figure 9: Diagram of Generic Hunter-Killer Scenario.

red and blue agents are given by v_{red} and v_{blue} and the ratio between them $R = \frac{v_{blue}}{v_{red}}$ is known as the speed ratio. The angle which separates the agents is given by Θ. The speed ratio (R), the number of red agents (n_{red}), and the number of blue agents (n_{blue}) are the only parameters required to completely define the scenario using this setup.

The number of agents can be used to determine the angle of separation between agents according to $\Theta = \frac{360}{n_{red}+n_{blue}}$. For the scenario depicted in Figure 10, the maximum angle Θ is found to be $180°$ according to $180 = \frac{360}{1+1}$. If the blue agent is responsible for defending the entire perimeter of the circle defined by the protection radius, then in general for the family of simultaneous scenarios, the blue agent gets the last segment "for free" as can be seen in Figure 10. The blue agent need not defend the right-half of

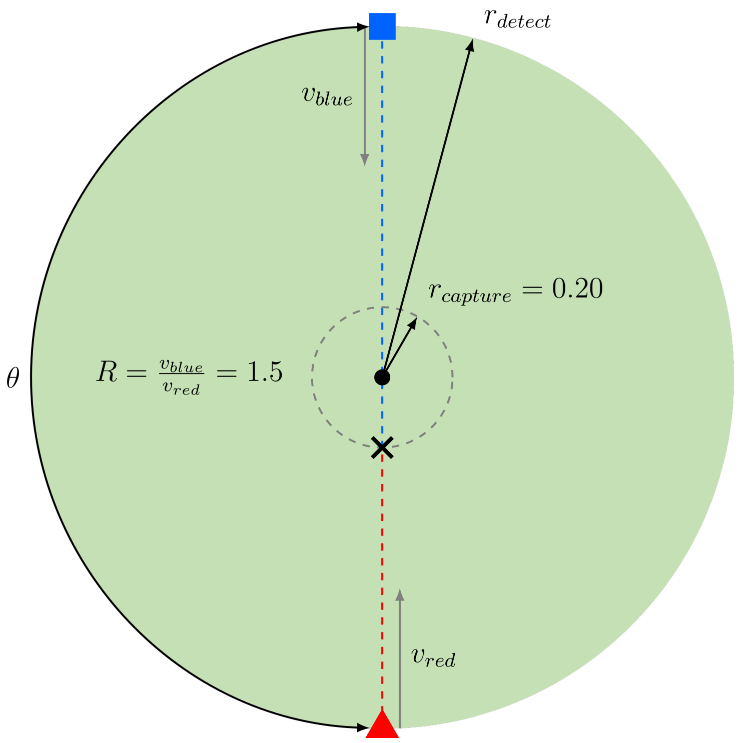


Figure 10: Diagram of 1 Red Agent versus 1 Blue Agent Hunter-Killer Scenario.

the protection circle and the total possible angular displacement less the specific angular separation for a given scenario (Θ) gives the angle that blue gets "for free".

Consider next, finding the speed ratio (R) required to allow for captures to occur with a specified capture radius ($r_{capture}$) (thus leaving the center of the protection region unadulterated by aggressors). The governing equation for this relationship is found by determining the distance traveled for each agent in order for a capture to occur exactly at the capture radius as shown in Eqs. (3.27) and (3.28). The time required for each agent to cover this distance can then be found according to Eq. (3.29). Recognizing that the times must be equal for the agents to arrive at the same time allows for the expressions to be equated as shown in Eq. (3.30) and solved for speed ratio as shown in Eq. (3.31). and is given in Eqs. (3.27) and (3.28). And finally the equation can be solved to determine the required speed ratio as shown in Eq. (3.32). The expression is Eq. (3.32) is shown solved using the parameters shown in Figure 10 but the expression holds true in general for the one versus one scenario for all values of capture radius.

$$d_{red} = 1 - r_{capture} \tag{3.27}$$

$$d_{red} = 1 + r_{capture} \tag{3.28}$$

$$t = \frac{d_{red}}{v_{red}} \tag{3.29}$$

$$\frac{d_{red}}{v_{red}} = \frac{d_{blue}}{v_{blue}} \tag{3.30}$$

$$R = \frac{v_{blue}}{v_{red}} = \frac{d_{blue}}{d_{red}} \tag{3.31}$$

$$R = \frac{d_{blue}}{d_{red}} = \frac{1 + r_{capture}}{1 - r_{capture}} = \frac{1 + 0.2}{1 - 0.2} = \frac{1.2}{0.8} = 1.5 \tag{3.32}$$

Consider the scenario depicted in Figure 11. The angular separation is now given by $\frac{360}{1+2} = 60$ and the blue agent will get the final $60°$ slice "for free". In this case, the trivial equations shown for the one versus one scenario will not provide a solution since the blue agent must take a more complicated path to defeat the red agents. The methodology however, remains the same. Specifically, the methodology involves determining the location of the closest red agent at the point of capture. By definition, the red agent which is captured closest to the capture radius will be the last red agent captured. Since the red agents are always following a straight line path according to this treatment, the position of the red agent and the distance traveled at the moment of capture can be determined easily.

Consider the setup shown in Figure 12. A number of details have been annotated

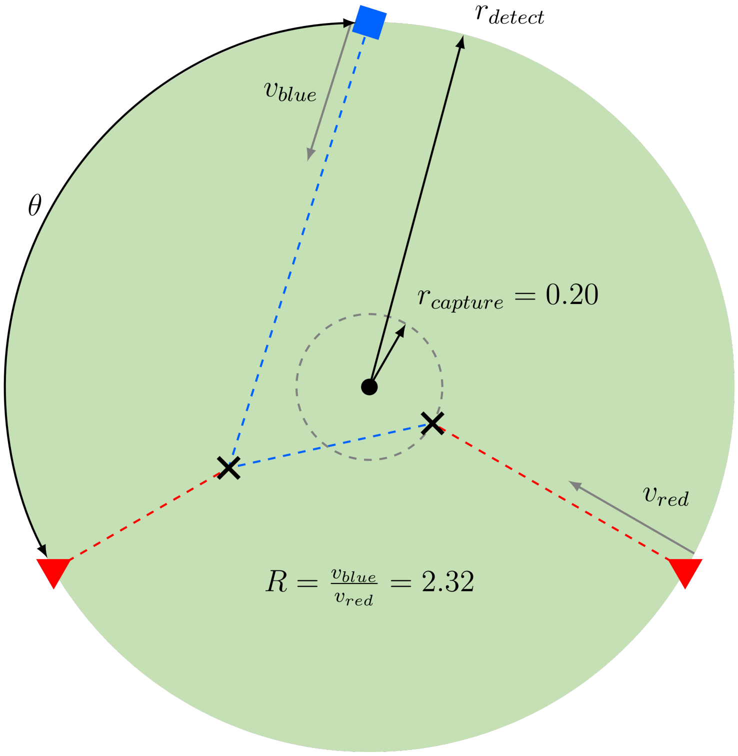


Figure 11: Diagram of 2 Red Agents versus 1 Blue Agent Hunter-Killer Scenario.

which will be important in the solution which is to be presented. First, the number of blue and red agents must be known. Consistent with previous treatments, it is assumed that if the number of red and blue agents are known, then the starting points for each blue and red agent is determined. Additionally, it is assumed that the red agents will follow a straight path toward the goal.

The position of the red agent is denoted by coordinates and is shown in Figure 12 as (x_{r_0}, y_{r_0}). The position of the blue agent is denoted by (x_{b_0}, y_{b_0}). The angle between the agents can be found according to the total number of agents according to considering that under worst-case assumptions, (for the blue agents to need to cover the maximum

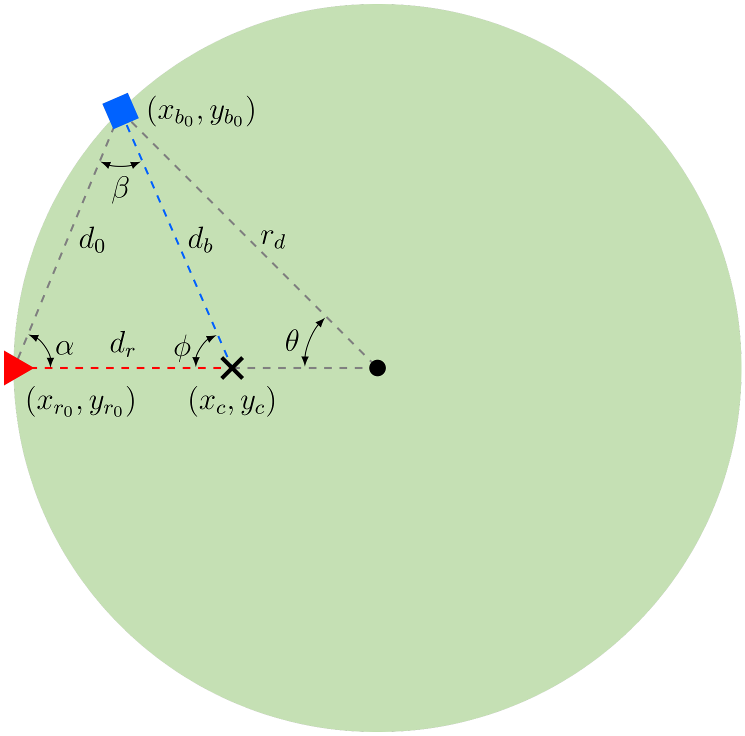


Figure 12: Diagram of 1 Blue Agent versus 1 Red Agent with Important Geometric Details Annotated.

distance for a given scenario) the red and blue agents will be equally spaced along the circumference of the circle (equal angular separation).

The angle shown as θ can be found by dividing the total angle which is to be protected (for the entire circle, this is 2π) divided by the number of agents for a given scenario as shown in Eq. (3.33). This relationship holds in general even for multiple blue agents or a reduced angular range in the protection area. The detection radius r_d will have a magnitude of one for the sake of convenience as has thus far been the convention. Assuming $r_d = 1$ and constraining the position of the red agent to always occur on the x-axis $(x_{r_0}, y_{r_0}) = (1, 0)$ allows the position of the blue agent to be found according to Eqs. (3.34) and (3.35).

$$\theta = \frac{2\pi}{n_b + n_r} \tag{3.33}$$

$$x_{b_0} = r_d \sin\theta \tag{3.34}$$

$$y_{b_0} = -r_d \cos\theta \tag{3.35}$$

The next step in forming the solution involves recognizing that there is an isosceles triangle formed by $\overline{d_0 r_d d_0}$. This triangle provides a convenient way to solve for α in terms of the known angle θ. Specifically the angle α is related to the angle θ which can be found using only the number of agents and the size of the protection area according to Eq. (3.36).

$$\alpha = \frac{\pi - \Theta}{2} \tag{3.36}$$

Since the position of the red agent has been fixed along the x-axis, finding the solution to the location of the capture is now a matter of finding an expression for a single quantity denoted as d_r which is the distance the red agent travels (along the straight-line to goal path). The capture point itself occurs at a yet unknown point denoted by (x_c, y_c). The next step in forming the solution involves writing the expressions for the law of sines for the triangle $\overline{d_r d_b d_0}$ as shown in Eq. (3.37).

$$\frac{d_b}{\sin(\alpha)} = \frac{d_r}{\sin(\beta)} = \frac{d_0}{\sin(\phi)} \tag{3.37}$$

Specifically the law of sines formulation for $\frac{d_r}{\sin(\beta)}$ is used to form Eq. (3.38). The expression $\frac{d_r}{d_b}$ can be substituted for $\frac{1}{R}$ since the speed ratio R is equivalent to $\frac{v_b}{v_r}$ which is equivalent to $\frac{d_b}{d_r}$ according to the basic kinematic expression for velocity $d_b = v_b t$ as shown in Eq. (3.39). Applying the inverse sine operation to both sides of the expression yields Eq. (3.40).

$$\sin(\beta) = \sin(\alpha)\,\frac{d_r}{d_b} \tag{3.38}$$

$$\sin(\beta) = \sin(\alpha)\,\frac{1}{R} \tag{3.39}$$

$$\beta = \arcsin\left(\sin(\alpha)\,\frac{1}{R}\right) \tag{3.40}$$

Next consider another relationship from the law of sines solved for d_r as shown in Eq. (3.41). A right-triangle (not pictured) can be formed by extending a perpendicular leg upward from d_r which is coincident with (x_{b_0}, y_{b_0}). The hypotenuse of this constructed triangle will be d_0. The right-triangle can be used to find the length of d_0 according to the Pythagorean theorem as shown in Eq. (3.42).

$$d_r = \frac{\sin(\beta)}{\sin(\phi)} d_0 \tag{3.41}$$

$$d_0 = \sqrt{\left(y_{r_0} - y_{b_0}\right)^2 + \left(x_{r_0} - x_{b_0}\right)^2} \tag{3.42}$$

Substituting long expressions for some variables yields the equations shown in Eq. (3.43). Note that ϕ can be found as the difference between the sum of the angles in a triangle, π and the other two angles denoted by α and β. Further substitution gives the equation shown in Eq. (3.44).

$$d_r = \frac{\sin\left(\arcsin\left(\sin\left(\alpha\right)\frac{1}{R}\right)\right)}{\sin\left(\pi-\left(\alpha+\beta\right)\right)} \tag{3.43}$$

$$d_r = \frac{\sin\left(\alpha\right)\frac{1}{R}\sqrt{\left(0-y_{b_0}\right)^2+\left(-1-x_{b_0}\right)^2}}{\sin\left(\pi-\left(\frac{\pi-\Theta}{2}+\arcsin\left(\sin\left(\alpha\frac{1}{R}\right)\right)\right)\right)} \tag{3.44}$$

The form shown in Eq. (3.44) is sufficient for finding d_r; however, this formulation does not drive home the point that d_r depends only on the number of agents and the speed ratio ($d_r = f\left(n_b, n_r, R\right)$). Thus the less comprehensible formulation presented in Eq. (3.45) represents an expression for d_r which relies only on the minimum quantities necessary (no intermediate quantities) to find arrive at a solution. Recall that $r_d = 1$ according to the normalization scheme adopted up to this point. Thus r_d has been omitted from Eq. (3.45).

$$d_r = \frac{\sin\left(\frac{2\pi}{n_{blue}+n_{red}}\right)\csc\left(\frac{1}{2}\left(\frac{2\pi}{n_{blue}+n_{red}}-\pi\right)-\arcsin\left(\frac{\sin\left(\frac{1}{2}\left(\pi-\frac{2\pi}{n_{blue}+n_{red}}\right)\right)}{R}\right)\right)}{R} \tag{3.45}$$

Thus far, a solution for a single capture has been presented. In many cases, there will be follow-up engagements for multiple red agents that blue must complete prior to "winning" the scenario. Given the formulation presented in Eq. (3.45) a simple method can be used to find all the required follow-up engagement capture locations. Since the detection radius is normalized with a value of 1 in the solution ($r_d = 1$), the value found for d_r actually represents a fraction of the initial radius at which the engagement started. Since each follow-up engagement will have the same angular characteristics (according to the equally spaced assumption), then the triangles which represent each engagement in a given scenario, will be similar to those already presented except that the initial radius r_d will have a different magnitude according to the location of the previous capture location.

For example, if a given set of characteristics results in a value such as $d_r = 0.10$, then the subsequent follow-up engagement will begin at $r_d = 1 - d_r = 1.00 - 0.10 = 0.90$. To find the value of d_r for the follow-up engagement, the new value of the detection radius simply needs to be multiplied by the normalized value of d_r which was previously found. Thus the second capture will occur at $d_{r_2} = 0.90 * 0.10 = 0.09$ from the first capture. The capture point in terms of the radius from the goal point is thus given by $1 - 0.10 - 0.09 = 0.81$. The third capture (if there were that many red agents) would start at $r_d = 0.81$, which multiplied by d_r gives the capture location of $r_{d_0} = 0.81 * 0.10 = 0.081$. If there were a fourth capture it would start at $r_d = 0.81 - 0.081 = 0.729$ and so on.

An expression arises which hints at the exponential nature of the solution. Since d_r is normalized against $r_d = 1$, then each subsequent capture occurs with d_r specifying the constant relationship between the starting radius for the engagement and the radius a red agent will have traveled when captured. Each subsequent capture has its starting radius (d_0 for clarity) equal to the radius from the goal point at the conclusion of the previous engagement. The radius at which the capture occurs for each engagement is thus given according to Eq. (3.46) where the first capture occurs at a location according to Eq. (3.47) since $d_0 = 1$ for the first capture.

$$r_{c_n} = (d_0 - d_{r_n}) \tag{3.46}$$

$$r_{c_1} = (1 - d_{r_1}) \tag{3.47}$$

Consider that the capture radius for any capture forms the starting radius (d_{0_n}) for the subsequent capture, formally Eq. (3.48). Since $d_0 = r_d = 1$ for the first capture and d_r was solved as a fraction of d_0 then the fraction formed by $1 - d_r$ will give the radius remaining for the red agent to reach the goal at the instant of the capture. The distance remaining for any red agent to reach the goal will be denoted as $r_{\text{remaining}}$. Since the distance remaining to reach the goal at the instant of capture is an important quantity in terms of judging the success of blue agents during any engagements, $r_{\text{remaining}}$ forms an important quantity for any scenario.

$$d_{0_i} = r_{\text{capture}_{i-1}} = \left(r_{d_{i-1}} - r_{d_{i-1}} \times d_r\right) \tag{3.48}$$

The quantity $r_{\text{remaining}}$ has been introduced to help illustrate the exponential nature of the problem. For many subsequent captures, the distance remaining to the goal for the red agent at the moment of capture can be found according to the equation shown in Eq. (3.49). Using this equation, scenario designers can specify a minimum required radius from goal at the moment of capture as needed for a given scenario. For example, if the blue agents must complete all captures before any red agents arrive within 0.1 of the goal location then the required remaining distance for the final capture would be 0.1 ($r_{\text{remaining}_n} = 0.1$). By specifying the total number of agents to be defeated, a value of

d_r can be found which satisfies the conditions. At this point, the number of red and blue agents, as well as the minimum distance to goal ($r_{\text{remaining}}$) at the final capture have been specified. Thus, the speed ratio between the agents given by R is the only remaining unknown. Thus, R can be found according to the scenario requirements.

$$r_{\text{remaining}_i} = d_0 \left(- (1 - d_r)^i \right) \tag{3.49}$$

The total distance traveled by the blue agent to execute successive captures can be found using a similar process. Although the distance remaining to the goal for any arbitrary red agent is found using a relatively simple formula, the distance traveled by a single blue agent will necessarily comprise a summation. Solving for the required speed ratio is therefore not as simple as solving a single equation. To find the distance capturing agents would need to cover for a given engagement, an expansion of the summation formula would need to be found according to the number of agents in the scenario. The statement of the summation formula is given in Eq. (3.50).

$$\sum_{i=1}^{n} \sqrt{d_0 \left(d_r^2 - 2d_r + 2\right) \left(d_r - 1\right)^{(i-1)}} \tag{3.50}$$

Thus for the sake of solving practical problems an alternative method has been adopted which relies in a smooth trajectory for the blue agent (for which the arc length formula is readily available). This approach will be detailed in the next section and relies on the exponential decaying nature of the distance remaining to goal at the instant of capture.

3.8.6 Logarithmic Spirals

At this point, the next logical consideration is to determine what happens when considering the boundary scenario. In this case, the boundary scenario is an engagement which there are infinite red agents arriving simultaneously at the detection radius with equal infinitesimally small angular spacing. As suggested by the equation in Eq. (3.49) the resulting solution involves an exponential decay in terms of the radius remaining to the goal for any capture throughout the scenario. Note here that the engagement scenario presented is a so-called "simultaneous" engagement which means that red agents arrive at the same time and thus the total angle protected by blue cannot exceed 2π for a single scenario.

In the bounding case where the number of red agents approaches infinity, the capture path forms a logarithmic spiral. The general equations for a log spiral are given in Eqs. (3.51) to (3.53) in polar and Cartesian coordinates respectively. In the degenerate case where the spiral does not decay Eq. (3.51) can be written as $r = a$ since the log decay parameter is set to zero ($k = 0$). Thus it can be seen that the parameter a in the polar equation is simply a scaling factor on the radius of the spiral. In the specific case considered here (recall $r_d = 1$) the radius parameter will simply be set to 1 ($a = 1$).

$$r = ae^{k\phi} \tag{3.51}$$

$$x = ae^{k\phi}\cos\phi \tag{3.52}$$

$$y = ae^{k\phi}\sin\phi \tag{3.53}$$

The governing equation for the log spiral which is specific to the scenario as described is given in Eq. (3.54). The spiral decay parameter k can be found using a simple transformation of the governing equation shown in Eqs. (3.55) and (3.56). A logarithmic spiral has the property of self-similarity meaning that despite the size of the spiral varying with ϕ, the shape of the spiral never changes. Thus, the slope through any quantity of angular displacement is constant by definition. Thus, to find k, any convenient choice of value for ϕ can be used.

$$r = e^{k\phi} \tag{3.54}$$

$$\ln(r) = \ln(k\phi) \tag{3.55}$$

$$k = \frac{\ln(r)}{\phi} \tag{3.56}$$

A value for ϕ can be chosen consistent with the non-infinite governing equation as given in Eq. (3.45) for a given speed ratio (in this case ϕ is given by the separation between agents θ and k is equivalent to d_r. Also, a value of ϕ can be selected according the the required remaining radius ($r_{\text{remaining}}$) at the final capture which necessarily occurs at the final angular position equal to 2π. In this specific case, the value for k is equivalent to the radius remaining for the final agent captured ($r_{\text{remaining}_{\text{final}}}$). Two ways of solving the spiral parameters are shown in Eqs. (3.57) and (3.58).

$$k = \frac{\ln\left(r_{\text{remaining}_1}\right)}{\theta} \tag{3.57}$$

$$k = \frac{\ln\left(r_{\text{remaining}_{final}}\right)}{2\pi} \tag{3.58}$$

The log spiral decay parameter k is also known as the polar slope. The polar slope represents a relationship between the change in the radial coordinate ($\frac{dr}{dt}$) versus a change in the angular coordinate ($\frac{d\phi}{dt}$). Each red agent in the scenario described is only advancing toward the goal along the radial coordinate ($\frac{dr}{dt} = c$, where c is constant) and thus the change in angular position is zero ($\frac{d\phi}{dt} = 0$). In order to capture successive red agents a blue agent must follow a path which necessarily includes components along the radial and the angular coordinate (except in the case where the red and blue agents are co-linear or in the case where the blue agents is tasked with defending a goal point while located exactly at the goal point. Neither of these cases is considered here.

Thus the problem of finding a path for the blue agent to intercept red agents thus forms an optimization between the change in the radial coordinate ($\frac{d\phi}{dt}$) and the change in the angular coordinate ($\frac{d\phi}{dt}$). For non-infinite (discontinuous line-segment pursuits) and for infinite (log spiral pursuits) the blue agent must divide motion between traversing inward toward the goal point and traversing around the perimeter of the circle toward the evader which will be captured last.

For a specific number of red agents (infinite or non-infinite) and a specific number of blue agents, the required speed ratio can be calculated from the log-spiral governing equation in Eqs. (3.57) and (3.58) or the equation in Eq. (3.45) in order to meet the minimum required capture radius for the final capture. The speed ratio found represents the minimum characteristic requirements on the blue agent (relative to the red agents) to meet the protection radius requirement. The blue agents can exceed this requirement which would allow for a shallower pursuit trajectory resulting in a larger margin on the protection radius at the final capture.

The pursuit path for a blue agent which exceeds requirements could be found by solving for the path according to the requisite equation or the path for a slower evader could simply be used without gaining the advantage in terms of protection radius. In the second case, the advantage of having the faster blue agent does not result in an improved margin in protection radius but does result in an improved margin on blue capabilities. A blue agent which is not required to run at $100\,\%$ of capabilities at all time could see an improvement in terms of maintenance and operations cost. Additionally, a blue agent which is capable of faster speed, may be able to attain an advantage in terms of maneuverability if that agent is allowed to travel at a slower speed than the maximum rate maneuver speed.

The main differences between the non-infinite and the infinite case for the pursuit paths is that the non-infinite case comprises a discontinuous path while the spiral path is continuous. Since the straight-line path is comprised of a series of line segments, the total distance traveled along a straight-line path for a given scenario will necessarily be shorter than the total distance traveled along a spiral path. Of course, the spiral path arises as a result of considering infinite red agents and thus the spiral path is not "optional" in this case. Also, the method for finding the straight-line pursuit paths results in the same result as the spiral path when the length of each line segment approaches zero and the number of red evaders approaches infinity.

A specific case is illustrating in Figure 13 for both the straight-line path and the spiral path for the same circumstances. The speed ratio has been found as $R = 2.32$ according to the requirements in Eq. (3.45) for a scenario comprising one blue agent

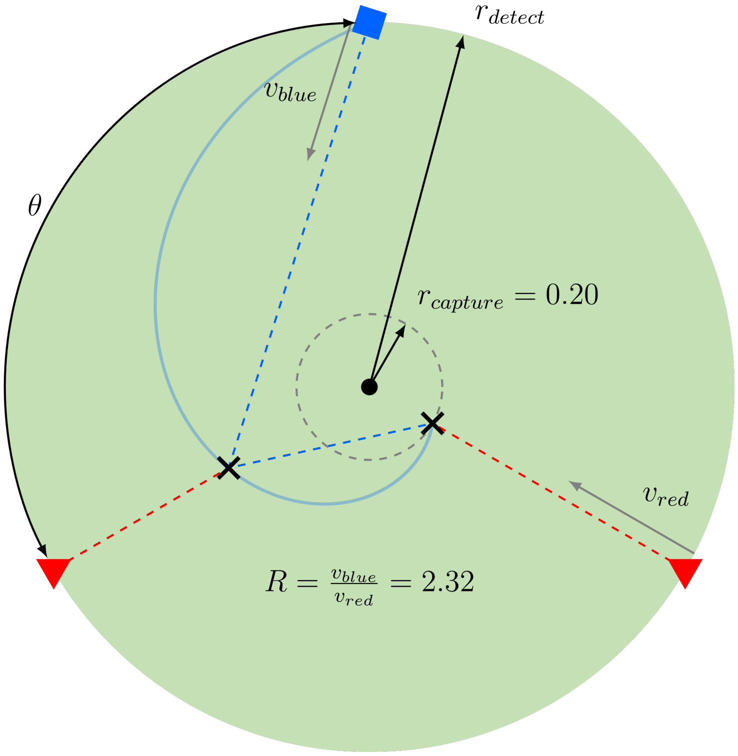


Figure 13: Diagram of Pursuit Paths showing Straight-Line (non-infinite) and Spiral (infinite) Engagement Paths for the Same Speed Ratio.

chasing two red agents with a radius remaining at the final capture equal to 0.20. Using the same speed ratio, the straight line-path is plotted as a dotted blue line versus the smooth spiral path which is shown in solid, but lighter blue.

In this case the path length for the pursuit path composed of straight-line segments is 1.8520 found by summing the length of each blue path (note that the second blue path is not shown). The first blue path is found by multiplying the speed ratio by the length of the red path for the first agent at the instant of capture. In this case $d_r = 0.5531$ thus $r_{\text{remaining}} = 1 - 0.5531 = 0.4469$ and the length of the path for the blue agent associated with the first capture is found according to $Rd_r = 2.32 \times 0.5531 = 1.2832$.

The second path length is found according by applying the same method to the second capture. First the path traveled by the red agent is found according to $d_r r_{d_2} = 0.5531 \times 0.4469 = 0.2472$ and then the blue path length can be found by applying the speed ratio to the displacement of the red agent $R \times 0.2472 = 2.32 \times 0.2472 = 0.5735$. Thus the total path length found as the sum of the components is found to be 1.8520

For this same set of conditions, using the log spiral path (but not a log spiral that ends at 2π but rather a log spiral that terminates at the same end locations for the non-infinite scenario) the path length is found to be 2.2295 and the smooth spiral path is about $17\,\%$ longer in this case. Finding the arc length of the spiral path is founding using three parameters. First, the radius at the beginning of the engagement which is always equal to one ($r_d = 1$). Next the radius of the final capture and the spiral decay parameter are needed. The arc length is given by $\frac{r_{\theta_{final}} - r_{\theta_{\text{start}}}}{\sin(\arctan(k))}$ where k is the spiral decay parameter.

The equation for finding the arc length of the spiral path is substantially simpler than the derivation required for the straight-line path in most cases. Despite the the spiral path will always be an overestimate of the total distance required the blue agents for a given scenario (except the case of infinite red agents), the spiral path may prove a useful estimator of blue requirements when the exactitude (but tediousness) of the expression in Eq. (3.45) is not required. The choice of whether or not the spiral path will comprise the ideal path for a given set of conditions hinges more on aircraft maneuverability requirements and what the end-game for a scenario actually looks like rather than path length. As the number of red agents grows relative to the number of blue agents, the difference between the path length for the blue agent following a spiral path and the blue

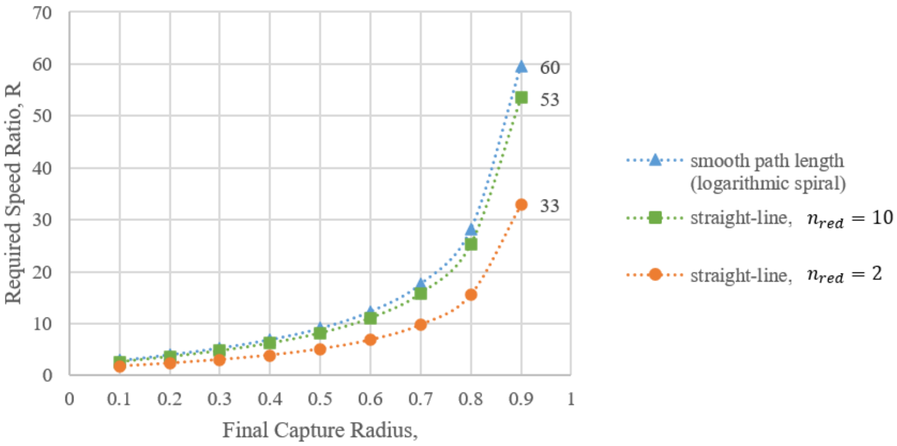


Figure 14: Plot Showing Comparison between Straight-Line Path Lengths and Spiral (continuous) Path Lengths in Terms of Speed Ratio Required to Capture All Agents in a Given Scenario.

agent following a straight-line path will approach zero.

The plot shown in Figure 14 shows a comparison in terms of the output variable, in this case the speed ratio R, for various cases considering the path length according to the straight-line pursuit paths and the continuous log spiral path. For engagement scenarios with few red agents, the advantage of using the straight-line path length in terms of the required speed ratio for blue to maintain victory is substantial. This advantage fades as the number of red agents for a given scenario gets large with the calculated values for the requires speed ratio beginning to converge for the various methods around $n_{red} = 10$.

This finding is consistent with the discussion in the preceding paragraphs. The maximum values for required speed ratio are annotated in Figure 14 near the right side of the plot. Though the values shown are unrealistically large in magnitude (a speed ratio equal to 60 is probably not possible for most scenarios), these values are shown to help

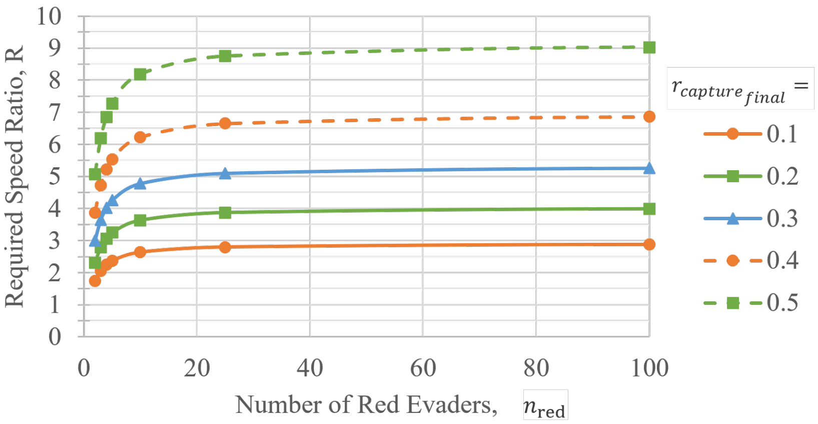


Figure 15: Plot Showing Required Speed Ratio for Capture to Occur at a Specified Ratio as a Fraction of the Detection Radius.

illustrate the difference between the curves. The curves are most dramatically different out toward the right side of the x-axis. The difference for more reasonable values of final capture radius are shown toward the left side of the plot and at these locations, there is even less of a percentage difference between the results for the different methods.

The plot shown in Figure 15 attempts to represent the relationship between the required speed ratio and the specified detection radius. Consider that speed ratio is bounded by the physics of the agents. That is to say, that for a given scenario, the speed advantage of the blue agents versus the red agents may be bounded. For a given set of scenario characteristics, the maximum speed and maneuverability of the agents is not simply bounded by the technology available in the moment. This is true to an extent but in terms of long-term capabilities the physics governing the scenario may be more important. Is it reasonable to specify a capture radius which requires a speed ratio equal to 10 ($R = 10$) for a given scenario?

The answer to questions like this is at the heart of the purpose of this work. If the scenario as-presented requires a certain speed ratio, then either that speed ratio must be met or the rules of the game should be changed to suit a more realistic set of characteristics. For example, the information in Figure 15 demonstrates that if the speed ratio cannot be met given the other constraints on the capabilities of agents, then the same task might be accomplished with a lesser speed ratio against the same number of red agents by a blue agent when the capture radius requirement is relaxed. That is, by allowing the red agents to get closer to the goal location prior to requiring their capture, a blue agent with poorer-than-specified characteristics may still be able to accomplish the mission.

Another point that arises when viewing Figure 15 is that the detection radius (r_d) which is possible for a given scenario may play a role in the speed ratio required. The information in Figure 15 is normalized against a detection radius of one ($r_d = 1$). Thus the capture radius is given as a fraction related to the detection radius. Take for example a specified capture radius with some magnitude with units of length, $100\,\mathrm{m}$ for example. If the detection radius is such that the capture radius represents $90\,\%$ of this magnitude, which is the case for a detection radius of $111\,\mathrm{m}$ then this represents a scenario characterized by $r_{\text{capture}_{\text{final}}} = 0.90$. If the detection radius for the exact same set of circumstances is increased to $1{,}000\,\mathrm{m}$ then this represents a scenario characterized by $r_{\text{capture}_{\text{final}}} = 0.10$ and thus the speed ratio requirement is substantially relaxed.

Regardless of other characteristics which define the scenario, prioritizing early detection and identification of red agents must be a priority. It can be seen from the data presented in Figure 15 that relaxing the specified capture radius and increasing the detection radius in absolute terms can have a profound impact on the required physical capabilities of the blue agent for a given workup. If the blue agent has already been selected for a scenario, or an existing type of agent is to be used to match a new threat, then such general conclusions may be helpful in understanding how all the factors interact which result in a successful engagement for blue agents.

Consider the results presented in Figure 16. The plots here shows the results considering multiple blue agents. The analysis is restricted to consider only cases where the number of red agents is evenly divisible by the number of blue agents. This is obviously not a real-world requirement but it simplifies the calculations if each blue agent is responsible for exactly the same number of red agents. For cases where this requirement is violated, the most conservative approach would be to round the number of red agents up to the next highest quantity which is a multiple of the number of blue agents. In such a case, one or more blue agents might be responsible for fewer captures than other agents and could stop the pursuit prior to chasing these nonexistent evaders.

Consider the previous discussion related to speed ratio. If speed ratio is to be bounded by the types of vehicles or other “hard” restrictions on characteristics, then the task of enhancing effectiveness of blue agents comes down to finding ways to improve performance without waiting around for a technological innovation or paradigm shift which might allow for a persistent and massive advantage. Consider the effect of adding multiple blue agents for a given set of conditions. The results in Figure 16 are partially misleading because the x-axis is given in terms of the ratio of red agents to blue agents. Thus, for a single point on any of the curves shown in Figure 16 where the x coordinate is equal to 25,

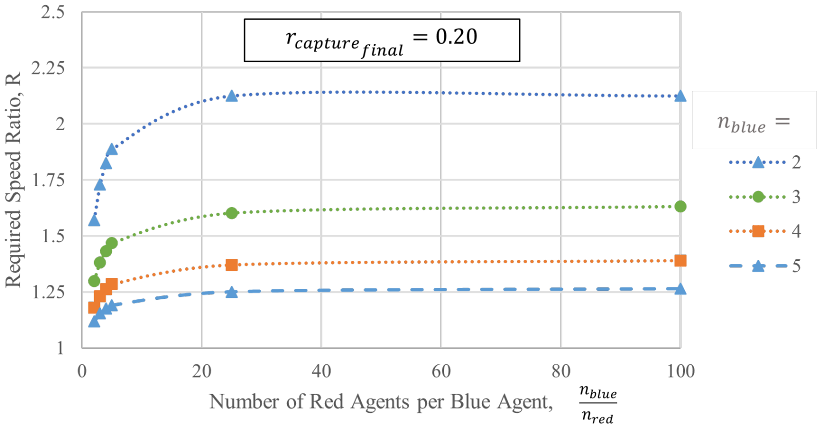


Figure 16: Plot Showing Required Speed Ratio for Capture to Occur at a 0.20 times the Detection Radius Including the Case of Multiple Blue Agents.

the actual number of red agents involved in the scenario can only be found by multiplying by the number of red agents.

For example, in the case of two blue agents (blue dotted line with triangular markers) the required speed ratio corresponding to 25 red agents per blue agent is about 2.1. This corresponds to 50 red agents (25 red agents each for each of 2 blue agents). Checking the speed ratio for the case where there are 5 red agents per blue agent indicates that a speed ratio of around 1.25 is required. In this case, the total number of red agents (for the same x-axis location) is actually much higher at 125 since there are 25 red agents per each one of the 5 blue agents. In this way, the plots shown in Figure 16 is actually conservative since it gives a comparison based on number of red agents per blue agent, the total number of red agents is under-counted by the x-axis coordinate alone and the plot understates the relative advantage of using multiple blue agents.

3.9 What are potential future threats that might be coming online during the windows of interest

- Swarming?
- 4G, 5G LTE, networking technologies
- Cooperative planning
- Decentralized swarm planning
- Failure tolerant decentralized planning
- Autonomous mapping/surveying
- Indirect detection, AI, NN
- Self-healing

CHAPTER 4

MEASURES OF SUCCESS

This chapter is still a work in progress. Expect revisions to style and grammar and content additions to the probability section specifically.

With the goal of developing an analytic framework to consider generic defense scenarios, there arises a difficulty when it comes time to give some meaningful measure of performance. Consider the previous chapter and despite all of the presented mathematics to define the scenarios and the so-called bounding conditions, developing a meaningful and consistent way to compare various scenarios remains to be accomplished. In the absence of analytic solutions, numerical simulations can be used to develop measures of success but there are necessarily wed to the experimental program used to derive them. That is, numerical results might give rise to highly detailed performance metrics but the ability of such results to generalize is in question.

Thus the task remains to propose measures of success which may give meaningful information within the analytic framework presented. These metrics should be as general as the modeling techniques thus far proposed but must also be able to capture the required level of detail as it related to system performance. In this case, simplifications which propose to throw away useful information are not to be tolerated. Three techniques have been proposed and are demonstrated with varying levels of success in the remainder of this chapter.

4.1 Expected Range for Omni-Directional Effector

Consider a non-mobile defense system protecting some goal point from enemy agents. The system will have some fundamental traits one of which is the maximum range at which targets can be engaged. Consider next that the maximum range at which engagements are possible is not likely to be equivalent to the range at which targets are expected to be defeated with high certainty.

Many factors contribute to this uncertainty including uncertainty associated with the effector technology itself, as well as simple observations such as that the effector cannot be pointed in two places at once and if multiple targets are detected simultaneously, some will be neutralized inside of the maximum range. Consider if all the uncertainty associated with target neutralization is removed and the effector is able to engage with the entire protection instantaneously and simultaneously. Thus, one proposal for reducing the dimensions of the problem to help understand trade-offs between different technologies and traits will be to convert characteristics into an equivalent effective range.

The expected range for omnidirectional effector (EROE) is the range at which all targets are expected to be neutralized. This formulation trades other forms of uncertainty for a decrease in range at which targets can be expected to be neutralized. A system which can engage at a very long range but misses 2 out of 3 shots may be considered less effective than a system with lesser range but a higher kill percentage.

The field of statistics provides the concept of expected value and this concept has already been used in prior chapters. Consider a similar formulation but the expected value is formulated to output the range at which kills might occur with some specified probability. By specifying this level of certainty that kills have occurred within a specified range, different technologies can be compared.

Consider the example briefly introduced above and recreated in Eq. (4.1). The task is to determine which effector will be preferred. The EROE for effector$_1$ can be found simply as the product of the maximum range and the probability that a kill will happen (ostensibly measured at the maximum range). Of course more complex functions for probability of kill are possible but this is a simple example.

$$\text{effector}_1 : r_{max} = 100, P_k = 0.33 \tag{4.1}$$

$$\text{effector}_2 : r_{max} = 50, P_k = 0.75 \tag{4.2}$$

The expected range formulated in the same way as the expected value from statistics will have units of length as desired when formulated as the product of the probability and the maximum range (with units of length). In Eq. (4.3) shows that the second effector will actually be preferred according to comparison of the EROE.

$$\text{effector}_1 : E(r_{max}, P_k) = E(100, 0.33) = 33 \tag{4.3}$$

$$\text{effector}_2 : E(r_{max}, P_k) = E(50, 0.75) = 37.5 \tag{4.4}$$

The formulation presented is most likely conservative in many scenarios. For many defense situations, a reduced chance that the enemy can be eliminated at a much longer distance from the area being protected is usually preferred unless the number of shots or the delay between shots is otherwise constraining the problem. Additionally, this formulation does not account for effectors which are mobile. The concept of a "mobile" effector need not include simply effectors which are physically able to move, this can also be considered a measure of agility in terms of other characteristics. Consider effectors which may have modular capabilities which allow for selection of levels of lethality. In many situations, such agility in terms of being able to adapt to mission requirements could be preferred but will not be reflected in the EROE.

Consider a similar scenario to that which was presented earlier and is shown in Eq. (4.5). In this case the effectors have the same probability function but they have different gimbal characteristics. Specifically, one effector is equipped with a gimbal 3 times faster than the other. Obviously the effector with the faster gimbal should be preferred; however, how does EROE capture this preference?

$$\text{effector}_1 : r_{max} = 50, P_k = 0.75, \omega_{max} = 1 \tag{4.5}$$

$$\text{effector}_2 : r_{max} = 50, P_k = 0.75, \omega_{max} = 3 \tag{4.6}$$

There exists a critical radius for the second effector where the slower gimbal will be able to track an equally fast target; albeit this range is closer to the area being protected. The fastest rectilinear speed each effector is capable of tracking at the maximum range can be found according to $\omega \times r_{max}$ as shown in Eq. (4.7).

$$\text{effector}_1 : r_{max} = 50, P_k = 0.75, \omega_{max} = 1\pi, v_{max_1} = r_{max}\omega_{max} = 50 \tag{4.7}$$

$$\text{effector}_2 : r_{max} = 50, P_k = 0.75, \omega_{max} = 3\pi, v_{max_2} = r_{max}\omega_{max} = 150 \tag{4.8}$$

To convert the capability advantage for Effector 2 into a quantity with units of range, the equivalent radius for the second effector to be able to match the rectilinear speed or an evader can be found simply by dividing the two velocities as found in Eq. (4.7). Thus $\frac{v_{max_1}}{v_{max_3}}$, the faster effector has an advantage in terms of EROE equal to the angular speed advantage which is simply a 3 times advantage. In this case, the EROE methodology seems particularly conservative. That is, the factor 3 reduction seems overly harsh.

The next task is to integrate the probability as was done in the previous step which gives the results shown in Eq. (4.9) which illustrates an important problem. There is no obvious relationship between the angular velocity and the effective range and thus the first pass at developing this measure of effectiveness gives an effective range which is longer than the maximum stated range of the effector. Obviously this is okay for the sake of relative comparison, but it is not okay for absolute measures of effectiveness.

$$\text{effector}_1 : E(r_{max}, P_k, \omega_{max}) = 37.5 \tag{4.9}$$

$$\text{effector}_2 : E(r_{max}, P_k, \omega_{max}) = 112.5 \tag{4.10}$$

The alternate formulation in Eq. (4.11) simply divides the expected range for the slower effector by the penalty amount equal to 3 times the range. The same problems arise as discussed before; however, now the maximum range specified can never be larger than the actual maximum range. The central problem is still that for characteristics for which an obvious relationship with distance does not already exist, creating a conversion which preserves relative rank in addition to absolute meaning is challenging. Obviously a faster effector would be preferred under most circumstances and stating this is not controversial.

Determining exactly how much faster of an effector is preferred is another matter entirely.

$$\text{effector}_1 : E(r_{max}, P_k, \omega_{max}) = 12.5 \tag{4.11}$$

$$\text{effector}_2 : E(r_{max}, P_k, \omega_{max}) = 37.5 \tag{4.12}$$

To generalize the concept of expected radius for omnidirectional effector, consider that to preserve absolute meaning in the presented measurement, the relationship to the quantity of range (with units of length) needs to be direct and obvious. The most obvious choices for parameters are as given originally in Eqs. (4.1) and (4.3) and include a measure of probability of effectiveness combined with the maximum range of the device in question. Again, the statistical construct of the expected value can be easily manipulated to allow for more complex distribution shapes according to the probability of effectiveness combined with more granular information about range. Though, in this case, the goal is still to reduce to a single parameter with units of length.

The most basic generic formulation is then given in Eq. (4.11). Enhancements to the effectiveness measurement can be constructed so as not to “ruin” the natural relationship with range (units of length) by insisting that no weighting factors which may get incorporated increase the baseline effectiveness measure above the nominal value given by Eq. (4.13). Thus any weighting factors would represent a decrease in range rather than an increase. While this prevents non-intuitive behavior like ranges increasing beyond the maximum effective range, it still does not provide an absolute measure of effectiveness which is based on any obvious rule or relationship.

$$E(r, P) = r \times P \tag{4.13}$$

A summary of scenario characteristics is provided in Table 7 with characteristics for which a natural relationship to EROE exists. The other parameters listed thus have no obvious or natural relationship to anything with units of length.

Table 7: Scenario Parameters and Coverage for Expected Range of Ominidirectional Effector.

blue	**red**	**configuration**
maximum rectilinear speed	maximum rectilinear speed	size of protection area
maximum angular speed	maximum angular speed	shape of protection area
number of agents	number of agents	size of denial area
mission/type	mission/type	shape of denial area
size	size	allowed leakers
weight	weight	
cost	cost	
power requirements	power requirements	
support requirements	support requirements	
beam width		
beam range		
probability of effectiveness		
time to effect		
magazine		
time to detect		
time to identify		
tracking error		

The primary limitation of any reduction of parameters which attempts to collapse factors into units of length will necessarily encounter this problem. If the parameter does not have units of length itself or otherwise possess some obvious relationship to units of length, then incorporating that measurement could lead to loss of absolute meaning (though relative meaning would be retained by a consistent application of a heuristic). Thus, EROE is an imperfect measure of success.

4.2 Kill Chain (durations)

After considering EROE as a proposed measure of success which functions by converting characteristics with varying dimensions into a range (units of length), the next obvious dimensional reduction involves converting various characteristics to durations of time. This is a logical extension to the concept of "kill-chain" as it is traditionally used in a military defense context. Additionally, usage of time as the dimension of interest provides a pathway for including most of the same parameters as were captured for EROE considering that length and time may be related whenever a velocity is known according to $d = vt$.

Kill-chain can be thought of as the total duration of time required between the moment an enemy agent is first detected to the instant the threat has been neutralized. Typically the kill chain will include the time required to detect and identify a bogey as a bandit, then the time to dispatch resources to neutralize the bandit, and finally the time required by the defensive system to neutralize the bandit and confirm the kill. In this way, the uniform units (time durations) of the pieces of the kill chain provides a convenient means for evaluating the effects various pieces have on the whole. If the time to detect a bogey and identify it as a bandit is substantially larger than the time required to dispatch resources and neutralize the threat, then resources should be expended to shorten the kill chain items associated with the longest delay. Likewise, if detection and identification are not dominating factors in the overall kill chain, then resources can be directed to wherever they are needed and can have the largest impact.

The language of the kill chain concept is easily understood by strategic planners and engineers alike. While the concepts of probability of effect and maximum range are only slightly technical in nature, the extra layer of abstraction provided by considering durations of time can be valuable in simplifying complex scenarios and in discussing scenarios with various stakeholders who may have unique design languages within their technical field.

An analogous new concept known as the “leak chain” is introduced. In this case “leak chain” will be used to represent the total elapsed time from the first time blue makes contact with a hostile red UAV to the time at which the red UAV has accomplished its mission. Simply stated, victory for the blue team occurs when the “kill chain” duration is shorter than the “leak chain”. The red team can accomplish a successful mission in two ways. The first way involves lengthening the duration of the “kill chain” by various methods including confusing tracking, identification, and targeting systems and armoring the red UAV against blue systems. These methods can either force the “kill chain” to start later in a given engagement or simply lengthen one or more components within the “kill chain” such that the total duration is increased above that of the duration of the “leak chain”. The second way red can accomplish a successful mission involves shortening the “leak chain” to be distinguished from lengthening the “kill chain” simply by the fact that changes in the “leak chain” will be accomplished by changing the red UAV without regard for the blue team (in other words, such changes are not exploits of the blue team).

The “kill chain” is comprised of any number of components but for the sake of simplicity, those components will be grouped according to the list below.

time to detect (ttd)/time to identify (tti): the duration from the instant first contact is made to the time at which the UAV has been identified as hostile

time to arm/aim (tta)/time in transit (ttt): the duration from the instant the decision is made to eliminate a threatening UAV to the time at which the weapon system is available to neutralize the target (this includes time-in-transit as well as many other parameters)

time to effect tte): the duration from the instant when the weapon system is activated to the desired effect being observed on the hostile UAV

The “leak chain” is comprised of any number of components but in this work will be simplified to the following:

time to traverse (ttt): the duration from the instant the first contact is made by the blue team to the time at which the UAV has completed its mission

How many of the scenario parameters will be covered by the “kill chain” or “leak chain” according to these categories? Consider the additional cells indicated in Table 8. All of the cells associated with EROE can be represented using durations (as discussed considering that the distance and the time for many actions are related by the speed of the agents). In addition, several characteristics which are explicitly given in terms of duration are now captured. Magazine characteristics can be converted via a binomial distribution into the expected number of shots which can be expressed in terms of the delay between shots times the expected number of shots to be taken.

Table 8: Scenario Parameters and Coverage for “Kill Chain” and “Leak Chain”.

blue	**red**	**configuration**
maximum rectilinear speed	maximum rectilinear speed	size of protection area
maximum angular speed	maximum angular speed	shape of protection area
number of agents	number of agents	size of denial area
mission/type	mission/type	shape of denial area
size	size	allowed leakers
weight	weight	
cost	cost	
power requirements	power requirements	
support requirements	support requirements	
beam width		
beam range		
probability of effectiveness		
time to effect		
magazine		
time to detect		
time to identify		
tracking error		

$$t_{detect} + t_{aim} + t_{effect} = t_{traverse} \tag{4.14}$$

$$t_{aim} = \frac{\Psi sweep}{\dot{\Psi}} \tag{4.15}$$

$$t_{traverse} = \frac{v}{r_{detection} - r_{protection}} \tag{4.16}$$

time to effect (tte)**:** blue wants to decrease this, this is mostly a decision of the kill mechanism and it is decided before the scenario can take place. Some technologies will have a shorter tte inherently and it seems like modifying easily changed parameters (power density, range to target when firing) are only likely to change this by degrees. On the other hand, red can probably accomplish substantial changes compared to what blue is expecting with some inexpensive or improvised hardening measures (to some extent this hardening would require some amount of knowledge about the type of effector being employed)

time to detect (*ttd*, *tti*): blue wants to decrease this which can be accomplished with more powerful radar (longer range) and/or better radar (increase resolution) which allows first contact at a larger standoff and more certain identification at a greater standoff respectively. Red wants to increase this which involves delaying first contact and increasing the time required for identification. This can be accomplished by disguising UAVs (think conventional camouflage, or stealth technology). Traditional exploits against radar also apply here.

time to aim (tta) or time in transit (ttt): blue wants to decrease this and the best way to accomplish that will be by increasing the velocity of the effector gimbal or the maximum speed of the mobile agent. Additionally, decreasing the angle or area of responsibility will also have a large impact while increasing the beam width will have an effect but not as direct of an effect as the changing the speed or area of responsibility. To increase the time to aim for blue, red needs to improve speed and maneuverability. Increasing blue time to aim must be weighed against increasing red's time to traverse. For example, doing circles at the periphery of the blue defense zone may be useful for some red mission (C2) but not useful for other red missions (destruction) and thus having infinite time to traverse can be useful in some situations but meaningless in others (equivalent to a red loss).

Binomial distributions and expected values can be used to quantify the effect of a limited number of shots or a finite time between repeat shots. This allows a number of statistical parameters to be condensed into a single quantity that we can denote as X to mean required number of shots to achieve the desired outcome. This formulation will be considered to be conservative as the required certainty for a kill on an individual engagement can be tuned to be consistent with a risk-averse blue team. The risk-aversion will also be encoded (and tuneable) within the formulation which is to be presented.

The probability of causing the desired effect (conventionally the probability of kill, P_k) is the probability that the blue system will kill the red system on this shot. Probability of kill can be considered a fixed value (assuming blue is only engaging within a strictly defined engagement zone where the values for P_k are narrowly defined). Or probability of kill can be extended to include non-ideal zones of fire which may be tactically advantageous to use under certain situations. Consider the minimum formulation to be $P_k = f(d, w)$ where distance d is the distance from the effector to the target and w is the angle (or distance) from the effector beam center line to the target.

To quantify the concept of a limited duty-cyle of magazine, consider the following scenario. A weapon system can be fired no more often than once every T units of time. Firing faster than this may be possible but will necessitate running a cool-down cycle which necessitates a longer than minimum off-time between successive shots. This formulation stipulates that the most efficient method of operation (the most shots downrange in a given duration of time) is accomplished by firing the weapon every T units of time exactly. Firing more slowly than this simply wastes time and firing more quickly than this also wastes time according to the extra time required for cooling after the "burst" has been completed. Consider that a "burst" may make tactical sense under certain circumstances but will always leave the system overexposed immediately after the "burst" is completed.

The success or failure of a given shot X is a random variable given by the binomial distribution according to $X\ B(n, P_k)$ where $X = 1$ when the shot is a success and $X = 0$ when the shot is unsuccessful. For scenarios another probability is introduced representing the minimum certainty with which the target must be eliminated in order to consider the shot a success according to the formula $P(X \geq 1) \geq P_{specificed}$. Conservatism dictates that we use a value for $P_{specified}$ which gives a small probability of having an accidental leaker q such that $P = (1 - q) = (1 - 1/1000) = 0.999$.

$$P(X \geq 1) \geq P_{specified} \tag{4.17}$$

$$P(X \geq 1) = 1 - P(X = 0) \tag{4.18}$$

$$P(X = 0) = (1 - P_k)^n \tag{4.19}$$

$$1 - (1 - P_k)^n \geq P_{specified} \tag{4.20}$$

$$1 - P_{specified} \geq (1 - P_k)^n \tag{4.21}$$

$$1 \geq \frac{(1 - P_k)^n}{(1 - P_{specified})} \tag{4.22}$$

$$\frac{\ln(1 - P_k)}{\ln(1 - P_{specified})} \leq n \tag{4.23}$$

$$n \geq \frac{\ln(1 - P_{specified})}{\ln(1 - P_k)} \tag{4.24}$$

Anytime the $P_{specified}$ is less than or equal to the P_k, then the number of required shots will be less than unity. Anytime $P_{specified}$ has a larger value than P_k for a given set of conditions then the decision whether or not to take the shot will need to take into account the likelihood that multiple shots are expected to be required.

For risk-averse blue with a limited magazine (unless the time-to-recharge is sufficiently low), the best approach is almost always going to be to wait until the engagement can take place in a region with higher P_k rather than firing within an uncertain region and planning on taking a follow-on shot. In fact, risk aversion alone dictates that uncertain scenarios (firing more than once) are less preferable than certain scenarios (firing once). This formulation, essentially supplants the concept of risk with the concept of expected number of shots. The certainty is handled by the certainty of kill $P_{specified}$ and thus the number of shots is considered certain at the specified level which is acceptable to blue. Thus, risk averse blue should always plan on having to take the specified number of shots (even though the kill may occur sooner with some probability) as this is the most conservative risk-averse strategy.

4.3 Probability Methods

Thus far the methods discussed for measuring the performance of various systems for both the red and the blue team essentially involve trying to force scenario characteristics into some common system of units. Be it the range at which neutralization of targets will occur or the time it takes for targets to be neutralized, the methods prevented so far have been useful but form an imperfect solution. There are two primary problems with usage of range or time as a catch-all performance metric.

The first problem, is that scenario characteristics will necessarily have their own units. In some cases a natural relationship exists between scenario characteristics and either time or range but this is not always the case. Furthermore, models or heuristics which attempt to wed scenario characteristics with disparate output units are doomed to fail. Firstly, any models proposed for making such a conversion will represent a simplification versus the reality and thus are likely to under or over-value certain characteristics. This makes any simplifying models less useful considering the purpose of this process is to allow for direct comparisons. Secondly, any extra abstraction from a specific scenario characteristic to the measure of performance makes the process of understanding the trade-offs between technologies more cumbersome, rather than less.

This leads to the second main problem which is that there is a natural output characteristic which forms the gold standard in terms of measures of success. Simply, the measure of success in this case is and always will be whether or not the engagement is successful for one side or another. For threat systems this involves whether or not the threat mission is accomplished and for defense systems this is concerned with whether the enemy was neutralized prior to accomplishing the threat mission. The best comparative characteristics for modeling purposes should strive to connect as meaningful and directly as possible to the scenario output characteristics which are of primary interest.

The statements in the last paragraph should be obvious. Of course the purpose is to study the effectiveness, and it is exactly the difficulty with condensing a number of complex and interconnected characteristics with disparate units which comprises the chief difficulty in this section of the work. If comparing technologies for use was as simple as looking up the relative effectiveness index in a catalog, then there would exist no purpose for the analysis presented herein. Thus, the question remains, what is the best way to simplify, quantify and compare effectiveness of various systems?

To start, methods which relate directly to overall effectiveness will be preferred. To be clear, overall effectiveness takes into account the whole scenario and gives as its output a simple metric which determines who will win given the initial conditions. It is perfectly natural to break up overall system effectiveness into the effectiveness of individual parts of the whole as is often done with the “kill-chain” in common usage where the engagement is split into various time periods representing topics such as target identification and tracking, defense system deployment, and confirming target neutralization. The difficulty thus far has been that though there are myriad ways to segment out the threat and defense system process, there are far fewer obvious and accepted ways to collapse and combine individual system effectiveness into a single measurements.

Thus, the ideal solution will include the ability to arbitrarily segment overall measures of success into sub-categories but must include an obvious and accepted way to combine these into a single measure for overall effectiveness. Additionally, the gold-standard performance metric will be the overall scenario effectiveness. One of the chief difficulties with the time-based and range-based methods as presented earlier in this chapter is that they are being evaluated according to a static metric against a probabilistic threat. That is to say, that numerical simulations may be needed to fully understand the effect of various parameter changes on defensive systems with respect to target. Additionally, it is possible that changing tactics to take advantage of scenario characteristics could greatly reduce the accuracy of simulation results which indicate effectiveness against more generic or other specific characteristics and tactics. This is consistent with previous statements about tactics designed to exploit the scenario in favor of either side.

The obvious answer to a probabilistic threat space is to consider the defense space to also be probabilistic. This avoids difficulties with studying boundary conditions where the trade-off analysis often considers worst-case for both sides of an engagement and thus draws bright lines between $100\,\%$ success for one side or the other with no gray area in between. Such predictions are useful for their mathematical precision, and in many cases may represent the truth in terms of the ideal mathematical solution, but the real-world is far more interesting than such models would predict. Thus, considering defense system to be probabilistic to some extent and considering more of the gray area (which is almost entirely where the real-world engagements occur) allows much more realistic and useful information to be gleaned from models.

Probabalistic measures of success for subsystems are naturally derived considering that all characteristics must be fundamentally tied to over system effectiveness. Additionally considering probabalistic engagements means that each subsystem will be defined by a single-point probability or a probability distribution. The rules for combinations of probabilities and distributions are well understood and natural according to elementary statistics knowledge. Adapting existing methods of probability combination for the specific use-case here is simply a matter of applying methods borrowed directly from statistical analysis or other seemingly unrelated fields to the specific problem being discussed here.

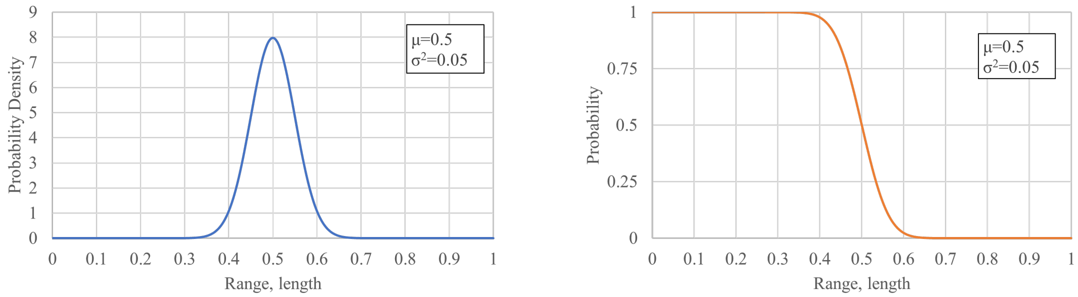
4.3.1 What is Probability of Effectiveness?

In a typical characterization for a war game-type simulation or model, a generic weapon system can be characterized largely by a single number which gives the system probability of effectiveness when encountering threats under specified conditions. This probability of effectiveness is only valid under the specified circumstances and may or may not include uncertainty related to other factors such as difficulty in tracking and identifying targets. Additionally, a characteristics known as probability of kill (P_k) is often encountered for some types of systems and is frequently given as a single number.

There are several pitfalls when probability of effectiveness has been reduced to a single parameter. Firstly, simplifying to a single number such as probability of effectiveness allows for direct comparison between systems or technigues and allows modeling and simulation efforts to move forward efficiently and without unnecessary detail. The reasons for the simplification should be apparent and the purpose here is not to advocate for using highly detailed models which may be difficult to understand or compute. Rather, the purpose of this discussion is to shed light on ways that probability of effectiveness measures may give intuitive but incorrect impressions about the capabilities of a given system.

The first pitfall associated with probability of effectiveness measures is that the probability of effectiveness is almost always related to range from the weapon system in some way. To reduce the dimensionality at the cost of this functional relationship with range leaves out important details. There are substantial implications to how range information can be interpreted (even if range is given, it is often stated as the maximum effective range). Certain weapon technology may be capable of operating beyond the maximum effective range with reduced probability of effectiveness; whereas other technologies may be unable to operate beyond the maximum effective range at all. If scaling is to be conducted for the region within (or outside) of the maximum effective range, the designer of the wargame is forced to make a guess as to what type of scaling should be used. For some systems, inverse square law may be a logical choice for how to reduce effectiveness but for others, a linear relationship may be more realistic.

More to the point, when P_k is given, it is often given in the form of a plot with a sigmoid shape. The sigmoid shape can be thought of as the result of finding the cumulative distribution function from the probability density function for a normal distribution. This implies that the weapon system has been characterized by taking repeated measurements of the range from the system at which some event happened which has been identified as the threshold for what should be considered as causing an effect. A hypothetical experiment of this type presents a handful of challenges by itself.



(a) Plot of PDF Curve.

(b) Plot of CDF Curve.

Figure 17: Plots of Probability Density Function (PDF) and Cumulative Distribution Function (CDF) for a Notional Normal Distribution with $\mu = 0.5$ and $\sigma^2 = 0.05$.

For a notional experiment, data would be collected with an defense system to establish the maximum effective range which would be give in the PDF plot such as in Figure 17a. This experiment would consist of measuring the effectiveness of the weapon system and recording the maximum range (minimum power) at which the system is no longer effective. The resulting data is expected to form roughly normally distributed output data like that shown in Figure 17a. Then the resulting CDF is found by taking the integral of the PDF.

In this case, the CDF may be an overestimate of system capability considering that as the x-coordinate moves toward the left of the plot, the probability approaches (and achieves) a value equal to 1.0 indicating that the threat would be neutralized at this range with perfect certainty. The justification for such a curve shapes may derive from the idea that certain defense systems exhibit a relationship between range and capability for neutralizing which may be characterized by the inverse square law (or an even more dramatic function). This means that a threshold of effectiveness exists at some range but at ranges closer than this threshold, there exists a substantial surplus of measured capability which is expected to be easily sufficient for neutralizing the target.

In all cases, the assumed probability equal to 1.0 for any threats inside of a specific range is optimistic from the standpoint that no weapon system is perfect. And while the capability of such a system may improve dramatically with decreasing range, it does not stand to reason that the probability of getting a kill given that capability is therefore perfectly equal to 1.0. Under ideal circumstances, the probability of effect being caused by the mechanism which depends on range, may indeed become effectively equal to 1.0;

however, stating that the probability for the entire weapon system is 1.0 could be a dangerous overstatement of capabilities.

4.3.2 Shortfalls for Experimental Programs

Part of the initial motivation behind the creation of this work was related to the observation of an ongoing experimental program to categorize and evaluate various defense systems. While this effort used state-of-the-art threat technology and best-possible tactics given the constraints on the experiment, the probability of effectiveness measures as a result of the collected data were inherently optimistic. For one, the focus on categorizing relatively low TRL technologies meant that the scenarios being considered were largely best-case scenarios for the defensive systems being considered. It was a level playing field in the sense that all the defense systems were pitted against approximately the same threats; however, those threats were not a realistic representation of the current or future threat technology space.

Obviously the converse of a scenario designed around the best-case-scenario for the defense systems, would comprise an experiment that studies exclusively the best-case-scenario for the threats (the worst-case-scenario for the defense systems). However, subject matters experts on UAV threat technology and tactics could probably have designed scenarios for which the probability of the defense system winning any engagements was very small. Thus the output from such an experiment would suffer from exactly the same difficulties. Namely, that the probability of effectiveness output from the experiment would be represent an apples-to-apples comparison for the systems relative to one another, but that the absolute magnitude of such measures would most likely represent a gross underestimate of defense system capability. In short, comparing defense technologies with reported probabilities of effectiveness very near to $0\,\%$ is no more realistic or useful than comparing systems with probabilities of effectiveness very near to $100\,\%$. Aspirationally, defense systems with probability of effectiveness near $100\,\%$ are preferred but creating such perfect systems which exist in the real-world is not accomplished easily, and artificially rating systems higher than their actual expected performance is simply a marketing strategy which should not be considered particularly useful to the modeling and simulation community.

There are also issues with common experimental methodologies which are used to develop such curves. Consider a hypothetical defense system which neutralizes threats through the use of a kinetic projectile. Assume that the shot grouping for such a system follows a normal distribution and assume that the distance to the target is the only factor related to whether or not the enemy agent is neutralized during a given engagement. The most direct way of obtaining a normal distribution for the maximum range such as that shown in Figure 17a would involve enemy agents approaching the defense system from a long range (outside of the expected maximum range of effectiveness) and the defense system issuing successive volleys of fire until the enemy agent is neutralized. The range at which the enemy agent was neutralized would then be recorded for that single trial.

In the alternative case, an individual trial would consist of an enemy agent starting at the origin of the defense system and traveling outward being subjected to repeated shots and recording the last range at which the shots were effective. Obviously this latter arrangment would be difficult for defense systems which render the enemy agent inoperable as part of neutralization. In some cases, a surrogate or on-board instrumentation may be used to provide a high-quality estimate of whether or not a kill has occured but the gold standard of testing which would involve the defense system of interest and a live-target in many cases would necessarily require several agents to be used to collect data for a single trial. In this type of arrangement, each trial would comprise as many engagements (and replacement agents) as needed until a region where the enemy agent is not neutralized can be found.

In reality, such an experimental arrangement may be incredibly expensive or difficult to accomplish. For one, the number of enemy agents required could be roughly proportional to the resolution being studied in the independent variable (range in this example). The data collected would also be inextricably tied to the starting point for the enemy agents. An experiment designed to save resources by beginning engagements at some reasonable range would necessarily collect no data on the certainty of effectiveness for engagements inside of this range. For the alternative case where the range for the enemy agent is coming toward the origin of the defense system, there is similarly no data collected for the ranges inside of the threshold kill range. In both case, the range inside of the demonstrated threshold value is more-or-less just assumed to be characterized by a very high probability of causing an effect. However, the assumption that this probability is equal to 1.0 is likely unrealistic and ignores other factors which could lower the probability.

4.3.3 Differences between Discrete and Continuous Domains

Another difficulty arises in that many weapon systems are represented by a binomial distribution in terms of their function. That is, a weapon system is activated a discrete number of times, and each instance of activation possesses some probability of causing an effect (neutralizing the target in most cases). Thus, basing probability on some continuous input parameter such as range can leave out important details associated with the discrete nature of most weapon systems. If the system is indeed capable of continuous (or pseudo-continuous) operation, then that advantage will become apparent during the study of discrete effects.

However, the complexity for situations in which a discrete firing event must take place can be substantial. Consider a hypothetical weapon system which fires with some rate, where the firing event is approximately instantaneous, but the weapon system must "re-load" in between shots thus creating a minimum duration between each shot and effectively creating a fixed firing rate. If the threat is moving toward the goal location (considered to be co-located with the origin of the weapon system for this example) then each unsuccessful firing event allows the threat to get ever closer to the goal location according to the speed of the threat multiplied by the delay associated with reloading. Consider that some weapon systems are expected to have a steep curve associated withe the probability of causing an effect. In this case, firing an early uncertain shot, may actually result in a threat which is allowed to get closer to the goal location compared to allowing the target to get closer on first engagement and expecting a higher chance that a single shot is effective in neutralizing the threat.

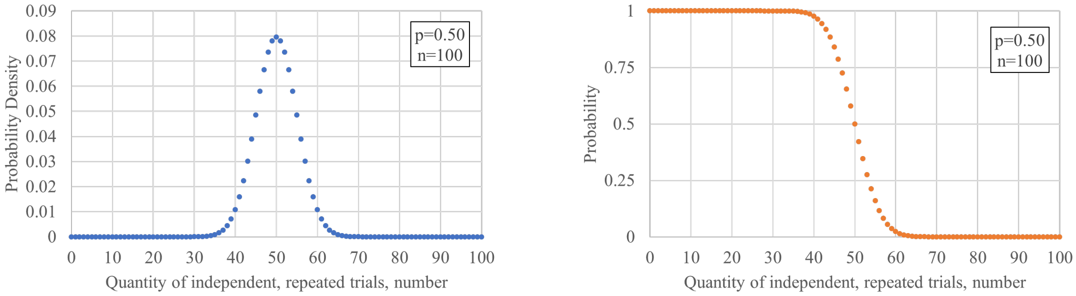
In a typical parameterization, the probability that a particular blue agent will be able to defeat a red agent can be given by the binomial distribution according to Eq. (4.25). In this case, the discrete random variable X is specifies whether a given trial is either successful or unsuccessful. The number of repeated independent trials for the experiment is given by n, and p is the probability that any independent trial will result in a successful outcome. By convention, the probability of success is given by p and thus the probability of failure is given by $1 - p$. Success in this case will be the case where the blue agent successfully neutralizes the red agent.

$$X \sim B(n, p), \text{for } n \in \mathbb{N} \text{ and } p \in [0, 1] \tag{4.25}$$

Consider next the probability mass function (since the output is a discrete random variable, instead of a continuous random variable) given in Eq. (4.26). Where $\Pr$ gives the probability of getting exactly k successes in n independent trials where the probability of success in any trial is given by p.

$$\Pr(k; n, p) = \frac{n!}{k!\,(n-k)!} p^k (1-p)^{n-k} \tag{4.26}$$

Notional plots of a binomial distribution PMF and CDF are shown in Figures 18a and 18b. These plots have a similar shape to plots shown for the notional normal distribution. However, the importance of the discrete nature cannot be understates. In this case, the input and output space are not continuous as is implied in the case of a normal distribution. If a shot is taken which results in a “miss”, then the next opportunity to attempt to neutralize targets represents another discrete attempt. The delay associated with taking successive shots and the resulting progress toward the goal made by red agents during that time can be substantial.



(a) Plot of PMF Curve.

(b) Plot of CDF Curve.

Figure 18: Plots of Probability Mass Function (PMF) and Cumulative Distribution Function (CDF) for a Notional Binomial Distribution with $p = 0.5$ and $n = 100$.

4.3.4 Other Distribution Shapes

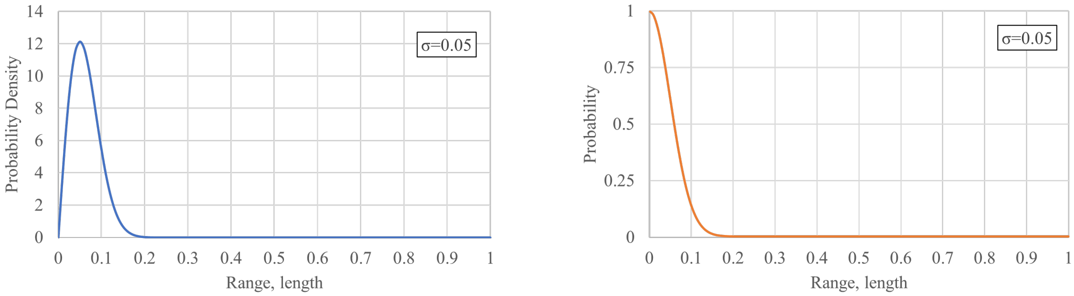
The Rayleigh distribution is a less commonly known and used distribution with a single parameter describing the distribution shape. The Rayleigh distribution gives random variables which are necessarily positive, thus the domain of $X \sim R(\sigma)$ is $[0, \inf)$. The Rayleigh distribution is frequently encountered as the result of the joint probability distribution created from two-dimensional Cartesian vectors with each component drawn from an independent normal distribution which have been reduced to polar coordinates (angle and magnitude). The magnitude of any such vector is necessarily positive and it can be demonstrated that the joint probability arising from two independent normal distributions converted into magnitude will follow the Rayleigh distribution.

The PDF and CDF for a Rayleigh distribution is given in Eq. (4.27) and notional plots for the PDF and CDF are given in Figure 19a. For example, if a target shooter wishes to analyze the shot groupings from a shooting session, fitting the miss distance from the center of the target with a Rayleigh distribution could be a useful bit of analysis. Since the Rayleigh distribution is the result of a simple joint probability arising from two perfectly mundane normally distributed independent variables, the Rayleigh distribution itself is not often discussed in elementary statistics courses. Though the distribution applies in a large variety of scenarios wherever repeated attempts are made at striking a target located in planar space as well as countless other cases where two-dimensional vectors are encountered. For example, the author's first exposure to Rayleigh distribution came when studying a database of two-dimensional wind velocities recorded during experiments in support of characterizing a robotic parachute system. When the two components of the wind data were converted into a magnitude, the newly created distribution was fit very closely by the Rayleigh distribution.

$$\phi(x) = \frac{x}{\sigma^2} \exp \frac{-x^2}{2\sigma^2}, x \geq 0 \tag{4.27}$$

$$\Phi(x) = 1 - \exp \frac{-x^2}{2\sigma^2} \tag{4.28}$$

The shape of the CDF for the Rayleigh distribution suffers from the same problem as the CDF for the normal distribution. Namely, probability of kill curves typically specify a curve which attains a very high probability (perfectly equal to 1.0) sharply and then that probability continues as the range decreases all the way to zero. As discussed, this may be unrealistic behavior for certain systems and thus a distribution which gives some type of falloff as the range decreases could be preferred. Additionally, considering that the Rayleigh distribution is specified by a single parameter, it could be difficult to get the appropriate probability and crossover location considering the loss of a degree of freedom.



(a) Plot of PDF Curve.

(b) Plot of CDF Curve.

Figure 19: Plots of Probability Density Function (PDF) and Cumulative Distribution Function (CDF) for a Notional Rayleigh Distribution with $\sigma = 0.05$.

4.3.5 Arbitrary Joint Probability Distributions

Since the Rayleigh distribution arises as a result of a simple combination of normal distributions, it may be worth considering forming arbitrary joint probability distributions to progressively arrive at approximately the desired shape for a given system under evaluation. The characteristics desired are explicitly stated below.

- Steep reduction of probability as range $\rightarrow \inf$
- Partial reduction of probability as range $\rightarrow 0$
- Reduction in maximum Pr from $100\,\%$

The first item reflects the consideration that once a threshold range is reached, that the effectiveness of the central mechanism for a given system is expected to be highly certain. This is not to say that probabilities equal to 1.0 should be considered realistic, but rather that once a threshold range (or intensity) is met, that a surplus amount of capability exists for causing the effect desired.

The first term must also reflect casual observation of various experimental programs which seems to indicate particularly severe uncertainty for various defense systems near the maximum effective range. This could be the result of the low TRL status of many of the considered system, which is expected to diminish some with time. However, it could also reflect overstated range by manufacturers attempting to sell technology by advertising the maximum capabilities possible. If the latter is true, then it will almost always be good practice to down-rate capabilities slightly based on what is advertised by the boosters of such systems.

The second item reflects the reality that as target arrive arbitrarily close to systems designed to neutralize them, that the differential speed between the two system may briefly approach infinity. Any system with a gimbal can experience gimbal lock under such situations and mobile defense systems could overshoot the target and have to perform dramatic turning maneuvers to arrive back in position to attempt to neutralize the targets. Thus, the overall probability of effectiveness, despite a perceived increase in the surplus destructive power to accomplish the task should be specified. Considered any system which relies on an explosive burst which may be barred from firing when targets are too close considering the possibility of collateral damage or self-harm.

The final criteria reflects the reality that the only protective systems which are $100\,\%$ effective are aspirational. Even if such a system were developed and tested and proven in a laboratory environment, the harsh reality of field testing or deployment would most certainly lower the overall effectiveness considering system maintenance, weather, logistics, power outages, etc. It is not possible for real-world systems to operate at $100\,\%$ effectiveness when exposed to real-world condition even if everything goes perfectly considering the uncertain and adverse nature of the threat environment. The enemy agents are always motivated to defeat the defense systems and thus even if some local system is deemed perfect, future attacks may focus on other more vulnerable avenues of attack.

4.3.6 The Rules of Joint Probability Distributions

For the purposes of creating arbitrary distribution shapes, most of the underlying distributions will be considered. That is, if the dynamic characteristics of the red and blue agents in a given scenario make it $50\,\%$ likely that the blue agent will not get sufficiently close to the red agent for a given set of circumnstances, then the $99\,\%$ likelihood that the weapon system of choice will be effective once the red agent is within range will be considered independently. Assuming independent events allows that the probabilities can be combined in a very straightforward way.

For the sake of simplicity, the joint probability from such a scenario as described above would be found as the multiplicative combination of the two component probabilities. Thus the probability of effectiveness for the blue team in this case would be $49.5\,\%$ as found by $0.99 \times 0.50 = 0.495$. Consider that if an overall scenario puts several of such a system in a layered configuration, such that in order for the threat to “win” the threat will need to defeat several independent blue systems, then the overall probability of effectiveness X of the "system" would then be described by a binomial probability with $X \sim B(n, 0.495)$ where n is the number of layers. The overall probability is however not given by the PDF but rather by the CDF because the probability of interest is that within the maximum of 3 trials what the chances of having at least one success are. Thus the overall effectiveness is about $88\,\%$ even though the individual probabilities for each layer are much lower.

Am alternate formulation ignores the binomial methods explicitly in favor of considering the joint probability directly. In this case an answer can be found by considering just the probability of effectiveness for a single system. Or specifically, the complement to the probability of effectiveness for a single system. If the probability p of a blue system winning a one-on-one engagement is equal to 0.495 then the probability that the system fails in that engagement (called the complement) is given by $1 - p$ or 0.505. In the case where the threat must successfully "defeat" multiple serially arranged defense systems, the combined probability associated with this event can be found according to $(1-p)^n$ where n is the number of layers. In this case the combined probability is found to be $12.8\,\%$ but recall, this is the probability that the red threat is able to successfully "win" the engagement. Thus the probability of overall blue success is given as the complement or about $88\,\%$. These formulations are thus seen to be equivalent.

Such a seemingly additive effect occurs whenever the constituent probabilities are related in a series rather than a parallel configuration. Consider instead if three instances of such a defense system are installed but in such a way that any threat need only encounter one in the course of arriving at the goal location. In such cases the effect of having multiple systems involved will be destructive in the sense that the overall probability of the combined make-up will be lower. In this case, the probabality of any single blue system defeating the one red system is given by 0.495 and thus the overall probability of the red threat being defeated if it only needs to encounter one such system is given by p^n where n is again the number of systems. Consider then that the overall probability in such a case is found as $12.2\,\%$. This seemingly destructive combination is the result of the fact that for the blue team to be unsuccessful only a single blue system needs to be unsuccessful on its own regardless of the behavior of the other two systems.

4.3.7 An Example Defensive Scenario in 1-Dimension

Consider that a region is to be defended. As has been the custom thus far, the region to be defended will be described by a normalized radius. That is, the radius at which any engagement can begin will have a value of 1.0 and all other scenario characteristics will be defined based on the normalized range as needed. Thus, the defense area size and shape will be defined relative to the normalized range. For the sake of simplicity, both regions will be considered to be circular in shape and share the same center point. Thus, the makeup of the scenario is fully-defined given the radius of the denial area with respect to the normalized outer radius.

Consider then, the plot in Figure 20. The blue trace in this case, represents the specified cumulative distribution which is desired for the specified level of protection. In actual use, such a curve could be drawn by someone with little to no formal experience with probability distributions. This is by design. The choice of the region to be defended and the boundaries defined by it, including the probability of enemy agents being neutralized within that region, will necessarily involve input from many different disciplines. The probability curve as shown, should prove to be a straightforward way to talk about the problem that can be understood and discussed both by scientific and strategic experts alike.

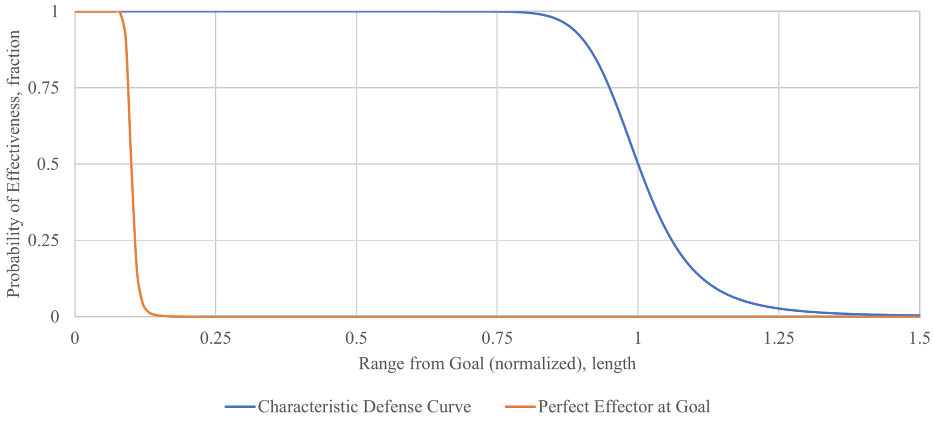


Figure 20: Plot showing Desired Probability of Effectiveness versus Normalized Range for a Hypothetical Defense Scenario.

Consider next, the orange trace which gives the notional cumulative distribution function describing the probability of neutralizing enemy agents for some generic defense system. The hypothetical defense system sigmoid curve shares an x-axis with the overall specified function for the protection area. In this case, the protection area is larger than the area which can be covered by a single instance of this particular defensive system. The question then, is how best to cover the area specified by the desired (blue) curve, using systems of the type which are modeled roughly by the hypothetical system (represented by the orange trace).

To begin to understand how to meaningful protect the specified region, the first consideration will be to create a meaningful and useful curve for each component of the defensive system. This will be accomplished simply by creating a CDF from a PDF using methods previously discussed. Namely, the CDF is found by taking the numerical integral of the PDF (in this case, the math is accomplished such that the probability increases with decreasing range). Secondly, the CDF will be re-cast. Instead of using absolute range along the x-axis, the range will now be represented as the distance from the design range. This accounts for challenges previously discussed where the effectiveness of a given system is expected to be maximum given some design range, but otherwise the effectiveness decreases.

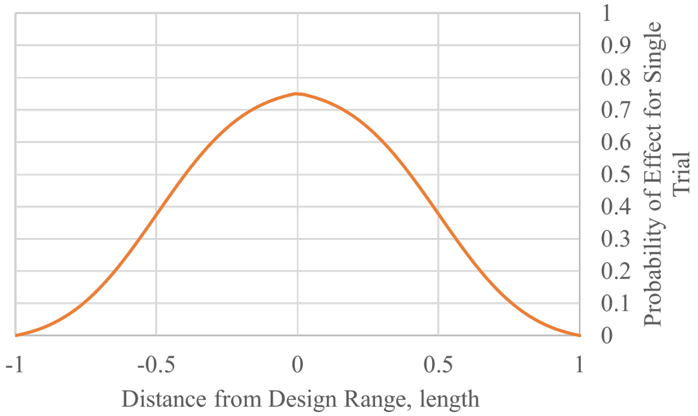


Figure 21: Plot showing Penalized, Mirrored, and Alternatively Characterized CDF for Notional Hypothetical Defense System.

Next, the CDF will be penalized such that the maximum probability of effectiveness is less than perfect. It was previously discussed that the probability of effectiveness for any real-world system is necessarily not perfectly equal to 1.0 due to a wide variety of complicating factors. Finally, the resulting modified CDF curve is mirrored against the new x-axis and gives the effectiveness of neutralizing agents incoming and outgoing compared to the design range of the notional defensive system. The result is notionally shown in Figure 21. In this case, the penalty is given as $75\,\%$ which indicates that considering all factors, the defensive system is only expected to neutralize targets with a certainty of about $75\,\%$ even when well inside of the range at which the energy available for neutralization is expected to be more than sufficient.

The resulting curve has been constructed using a notional PDF generated from a hypothetical experiment to determine laboratory probability of effectiveness according to methods discussed earlier in this chapter. The steepness of the fall-off is governed by the standard deviation resulting from the data collected during a hypothetical experiment. Due to the inverse-square or similar governing nature of many types of defense systems, this steepness is expected to be insensitive to changes in range from the perspective of the laboratory effectiveness rating of many systems.

However, the steepness of such a curve can be manipulated for the purposes of considering other effects which are unrelated to the laboratory effectiveness of a notional system. Consider that the x-axis uses units of normalized length. Thus, such a curve can be easily modified to account for uncertainty in position of agents for the case of mobile defensive systems. This is expected to correspond roughly to a widening and flattening of the notional curve, though at this stage, the total numerical area under the curve is no longer connected to the PDF, or the CDF. This means that the area under the curve is not governed or limited by any known rules of probability and to some extent can be manipulated at-will.

Though, it should be obvious, that if the total uncertainty on the position of a system in space is increased, then the point probability of such a system being effective at any specific location in space should necessarily decrease. One potential choice for governing the inclusion of multiple statistical distribution shapes into the single modified CDF curve, follows from the definition of joint normal probability distributions where the the joint standard deviation is given as the sum of squares of the individual standard deviations. For the sake of simplicity, most probabilistic modifications to the basic modified CDF curve will not be considered to have an effect on the location of the mean value (though this is certainly possible).

However, a popular and easy to understand modification will be applied to modify the magnitude of the curves with respect to the y-axis (probability). As previously discussed, the joint probabilities for a given event can be found according to the layout of the scenario. Namely, the probability of effectiveness for independent opportunities for neutralization are combined such that the overall chances of not neutralizing a given enemy agent decreases. And conversely, the probability of parallel opportunities for neutralization are combined such that the overall chances of not neutralizing a given enemy agent increases. This can be thought of as the result of the enemy agent in the latter case only needing to get lucky once; whereas, in the former case, the enemy agent would need to get lucky several times in a row.

Thus, the magnitude of a component CDF curve can be modified to give a different resultant probability of effectiveness for a given point under the following conditions. In the case that the area to be covered by a given agent is also covered by another agent, then the probability of effectiveness in that region will decrease according to the combination of the individual independent probabilities.

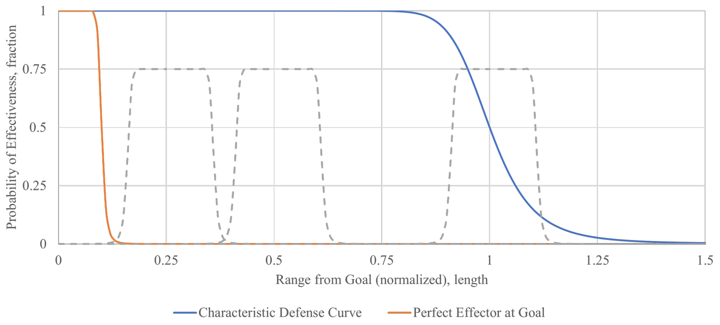


Figure 22: Plot Hypothetical Defense Scenario with Specified Effectiveness Curve along with Notional Defensive System CDF as well as Scenario Layup for the Case of 3 Independent Hunter-Killer Agents with Penalized CDFs.

Consider the plot shown in Figure 20. In this case, the area under the specified curve is being occupied by three individual agents each possessing a component effectiveness modeled according to the notional CDF presented in Figure 21. In this case, the position of each agent has not been modified with any additional uncertainty which would be expected to flatten and shorten the curve shapes. Additionally, none of the agents is overlapping in any way and thus the individual probability of effectiveness at any range is not expected to be larger than the individual maximum probability for any agent.

With the same underlying notional effectiveness CDF for a given agent, and the same number of overall agents, and the same defense scenario specification, consider how the area under the curve shown in Figure 23 is different. In this case, the agents are all assigned to occupy the same region in space and thus the individual probability of neutralizing an agent in that given area is much higher according to the joint probability $p = 1 - (1 - p^3_{\text{individua}} = 0.98$. Of course, arbitrary numbers of agents can be added to the

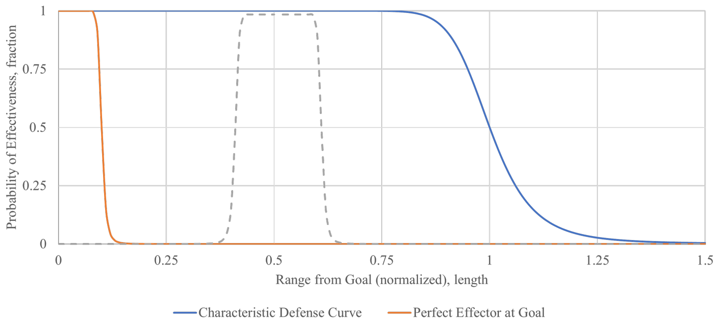


Figure 23: Plot Hypothetical Defense Scenario with Specified Effectiveness Curve along with Notional Defensive System CDF as well as Scenario Layup for the Case of 3 Independent Hunter-Killer Agents with Penalized CDFs Sharing the Same Location in Space.

scenario, and the positions can be dictated such that the probabilities in any region match the expectation set forth by the specified effectiveness curve.

Additionally, the penalized CDFs can be modified to account for various changes. Joint probabilities give a natural method increasing or decreasing the y-axis magnitude, although, this can also be accomplished by changing the penalty associated with the modified, mirrored CDF. Additionally, probabilistic positions can modify the expectation associated with the width of any notional CDF. In reality, this can be done in such a way that the point probabilities are reduced in any given region, or it can be done with no regard for keeping the area under the curve constant.

Finally, consider the plot in Figure 24. In this case, a single defensive system with a high maximum probability of effectiveness is stationed closest to the goal location and is represented by the nearest gray trace. This defense system is slow-moving because order to achieve the high certainty associated with a given point probability this system has a large energy storage system which reduces the maximum possible flight speed. In the

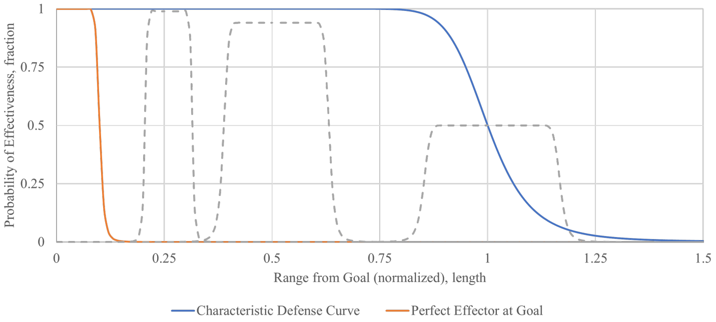


Figure 24: Plot Hypothetical Defense Scenario with Specified Effectiveness Curve along with Notional Defensive System CDF as well as Scenario Layup for the Case of Several Heterogeneous Independent Hunter-Killer Agents with Uniquely Penalized CDFs.

middle of the range from the goal, two agents are positioned such that they share territory and the probability of each individual system which is given by $p = 0.75$ is combined such that the overall probability of effectiveness in this region is given by $p = 0.94$. Finally, at the outermost region, a single agent which is fast moving (high uncertainty associated with position and thus a wide CDF curve). This agent can be thought of as a long-range mobile defender; whereas, the agents closer to the goal location can be considered to fulfill the goalie role.

These examples make obvious the fact that such graphs can be arbitrarily constructed to relate to a wide variety of scenarios. The use of a single variable on the x-axis which is given in terms of length is an intuitive way to understand where and how many enemy agents might be allowed within a certain proximity to the goal location. The width, position, and height of any individual CDF can me modified according to the specifications for a given system which may be manipulated differently compared to the CDF for any other agents in the same scenario.

4.3.8 Limitations and the Path Forward

The most obvious question that arises given the plots shown in the previous section is what happens when the problem is extended to multiple dimensions. Obviously, real-world scenarios would need to be described in at least 2-dimensions. In this case, the range would need to be replaced by two parameters such as range and angle (for polar coordinates) or x and y coordinate (cor Cartesian positions). The methodology holds up notionally when multiple dimensions are added arbitrarily to some extent. However, consider that a single agent which occupied the entire region under a curve as given in Figure 24 occupies only a single region (described in 2-dimensional or 3-dimensional space). The total coverage implied by the simple single-dimensional case is not expected to be an all-encompassing and accurate representation of agents in higher dimensional space.

Specifically, multiple agents may be required to be stationed along a circular perimeter defending a ring shaped area in order to provide the type of defense which is implied in plots like Figure 24. The extension into multiple dimensions raises questions related to the area and volume covered by an individual agent and how these quantities might be related to overall effectiveness. It is possible that area and volume can be simply considered in the way probabilistic positions were considered in the single-dimensional case. It is also possible that coverage for area or volume may be modeled with a more complex relationship. At this time, consideration of these questions in detail, including evaluations of the effectiveness of various techniques is outside the scope of this work.

Going forward, investigations into the extension into multiple dimensions should be considered. Additionally, the visual representation of the situation in multiple dimensions will need to be modified but is expected to generally be easy to understand and valuable, similar to the single-dimensional case. Additionally, specific methods for consideration of probabilistic terms and how to combine those terms (such a positional uncertainty with respect to maneuverability and how to combine that with a modified CDF), is an important topic going forward. Various methods should be considered and explored, though the simple representations given here should not detract from the potential usefulness of the proposed techniques.

CHAPTER 5

CONCLUSIONS

In the conclusion to this manuscript, first several predictions for the future of the UAV threat space are proposed. These predictions are made considering lessons from the review of capabilities presented in earlier sections. Sensitivities of the scenarios as presented with respect to the predictions being made are considered. Recommendations on how best to prepare for future threats as well as generic suggestions for the development of defensive systems are furnished.

5.1 UAV Futures

Commercial-off-the-shelf UAVs will continue to become more capable and less expensive. The wide proliferation of such technologies will continue at a rate greater than or equal to that which has been seen in the ten years prior to the publication of this manuscript. Large-scale industrial pressure by big players such as Intel and the continuing arms race to deliver larger and more complex UAV light shows will drive innovation in several important areas.

The logistical footprint of UAV operations, especially operations involving multiple aircraft using existing technology can be substantial. Basic UAV flight control systems and ground stations have no provisions for scaleability. That is the footprint for each additional system in operation is roughly equivalent to the footprint required for any single system on its own. As techniques such as artificial intelligence (AI) and machine learning (ML) become ever more mainstream, this technology is expected to find its way into commercial UAV operations. Specifically, operator workload for operating an arbitrarily large number of remotely piloted vehicles is expected to decrease to the point where the operational and logistic footprint is such that a single flight crew may be able to operate many UAVs.

Specifically for UAV light shows, technologies have been developed which allow for some level of self-sustainment by the UAVs. Currently this includes selection of the most appropriate UAVs for a given task based on battery life and starting position. Wireless charging for UAVs has already been demonstrated and UAVs are expected to gain functions in terms of self-management to include autonomous charging using wireless charging landing pads. Additionally, expect technology related to predictive maintenance to find its way into the UAV sector. For deploying large swarms of UAVs predictive maintenance and self-sustainment will be instrumental in allowing large numbers of UAVs to be operating with logistical and support requirements far below that which would be expected today.

The networked nature of the UAVs used for the light shows is also expected to be a factor. As the development of 5G mobile networks continues mainly in the mobile phone sector, expect more device to be connected in surprising ways. For example, the high bandwidth provided by an always-on 5G connection allows a much larger quantity of information to be quickly shared among swarm agents. This is expected to allow the precision and scale of drone operations to continue to grow. Additionally, expect UAV swarms to consist of UAVs as well as other connected devices. UAVs will make use of internet-of-things (IOT) devices, mobile phones, laptops, etc. to determine relative orientation in GNSS denied environments or indoors where GNSS may be unavailable or inaccurate.

Expect the proliferation of simultaneous mapping techniques to allow communicating UAV agents to quickly and efficiently develop real-time, intricate maps of the battle space. Such capabilities will rely on many different types of connected devices as well as various sensor packages. UAV sensor packages will continue exponential growth which will cause a large reduction in the cost of technologies such as multi-spectral and infrared imaging. With the incorporation of disparate types of connected devices expect information management systems to experience a revolution in terms of application of AI and ML technologies.

Though UAVs are currently used in a wide variety of use-cases which serve a variety of industries, the costs of collecting and cataloging information gained from various remote sensing systems can be high. Thus, expect systems to be developed which allow for autonomous cataloging of information nearly in real-time. This goes along with simultaneous mapping techniques in the sense that the information available from UAVs will be consumed almost immediately and converted into actionable information in real-time rather than in post processing.

Consider the implications for the ISR threat in light of these predictions. Though the immediate impacts of ISR threat missions are less dramatic compared to one-way destructive or destructive payload drop operations, the ISR threat is still potent. Standard operating procedure in many aspect of military operation favors obfuscating important operational details which could be more easily observed considering innovations in the way that ISR missions are conducted.

Currently, ISR threats mostly make use of post-processed data which must be downloaded and processed prior to use. This means that so long as hostile UAVs are neutralized prior to returning back to base, that the impact of collected data can be minimized. Some data can be downloaded in real-time; however, bandwidth is a limiting factor. Going forward, bandwidth limitations will be relaxed by the use of higher bandwidth networks as well as the leveraging of connected devices as communications relays. Thus, the information captures for ISR missions will be expected to be immediately available for use by adversaries.

Currently, the vast amount of information collected for a brief ISR mission must be sorted and cataloged in post-processing mostly manually. With the proliferation of AI and ML allowing for easy automation of mundane tasks including automated image and video processing expect the accuracy and timeliness of information obtained from hostile ISR missions to be greatly increased. More or less persistent surveillance missions accomplished by autonomous swarm agents can collect around-the-clock information on operations at various military installations. If the data recorded by such UAVs is then processed autonomously using novel AI and ML techniques, then adversaries may gain an upper hand in terms of situational awareness even without the use top-of-the-line satellite imagery or GNSS systems.

The proliferation of technology in the UAV sector means that while raw performance characteristics are expected to improve, the lateral movement of technology may prove equally as dangerous of a threat. The widespread usage of UAVs in commercial and industrial applications is driving innovation in terms of high quality sensor packages. This includes sensors related to remote sensing as these are typically related to the most prolific use-cases; however, UAVs are also used for various other missions. Enabling technologies for aerial delivery missions could be adapted to allow the delivery of destructive payloads. Expect a rise in the quality and quantity of sensor packages which are designed for use on UAVs specifically.

Continued developments of LIDAR, SONAR, optical flow and other localization related technologies will see the lines between indoor and outdoor UAV operations blurred. Systems which are designed for GNSS-denied operation will be able to operate indoors as well as outdoors as the localization sensor quality rises and the costs of such sensors goes down. High quality obstacle detection and avoidance as well as high resolution local mapping will be made by possible by algorithms as well as by sensor package development.

UAVs will continue to find new and exciting use-cases which did not exist before. The so-called "different" missions for UAVS represent the most potent and difficult to predict threat. Many use-cases are predictable and the sensor packages and tactics associated with those use-cases may be considered and extrapolated to threat scenarios. However, tasks UAVs will accomplish which represent an unforeseen paradigm shift could cause large disruptions in threat scenarios.

Usage and development of UAVs by near-peer adversaries is expected to rise. The technology being used by such systems is expected to be more potent compared to that which is commercially available. Of course, the barriers to usage by near-peer adversaries are different from those for a non-state actor. Still, the usage of UAVs in threats is expected to rise. Already, several high profile destructive missions have been accomplished using larger military-specific UAVs but usage of COTS UAVs by state actors could offer a degree of plausible deniability.

Consider the implications on the capability space as well as the regulatory framework posed by developments in air taxis. Additionaly, continued acceptance of high levels of autonomy in transportation in the form of autonomous cars is expected to drive innovation and acceptance of autonomous machines. The arrival of air taxi enabling technologies is related to the development of autonomous delivery platforms. The threat space could be transformed by UAVs with massive payload capacity which are designed for delivery missions. Such UAVs could fill the role of dedicated bomber within an heterogeneous swarm. Air taxis technology and the regulatory framework associated with large scale use and adoption of autonomous flying vehicles large enough to carry human beings is most like still decades away.

As military technology places a focus on maneuverability and ability to land in unprepared environments combined with the ability for high-speed cruising flight, expect a similar focus in the UAV sector. Multicopters have a number of advantage but as the UAV space matures, expect various other types of VTOL aircraft to become more popular for the combination of VTOL with high-speed cruise or VTOL with heavy lift capabilities. Specifically tail-sitters and other technologies which are software enabled rather than requiring technically complex novel actuators will be preferred. In addition to VTOL by design, expect high autonomy aircraft to further ease the difficulties associated with launch and recovery of UAVs. Expect catapult launch and net recovery systems to be supplanted by simple autonomous take-off and landing handled by the aircraft without external equipment.

5.2 Sensitivities

In terms of the parameterization given of the problem, the relationship between likely future outcomes and the parameters can be discussed. This provides an opportunity to consider not only the possible future scenarios but the effects of those future scenarios.

Maximum speed characteristics are not expected to be subject to unexpected paradigm shifts. UAVs which make use of novel propulsion technology or structural design which allows for much higher speed or maneuverability compared to what is expected could be disruptive. However, the opportunity for such technology to be developed by non-state actors without warning is unlikely. Additionally, once UAVs are large enough, they become vulnerable to traditional defense systems designed to protect against cruise missiles to some extent. Thus, defense systems should be designed considering the possibility of jet-powered or rocket-powered UAVs but such threats would most likely need to be defeated by a dedicated system or be handled by existing systems designed to handle loitering munitions.

The type of the mission performed by individual agents in a given scenario presents a number of important challenges. For one, dedicated types of defensive agents could enhance effectiveness but also present challenges in terms of task allocation and system acquisition. Determining the best force make-up for a given scenario presents an imporant step and if accomplished poorly could result in lower than expected effectiveness. In terms of threats, specialized agents could require specialized identification and specialized neutralization schemes. Dedicated ISR aircraft may not post a destructive threat but can also accomplish their mission with high stand-off distance and thus would need to be engaged quickly and at range. Conversely, bomber-type UAVs would expected to be slow and easy to detect and defeat. However, the penalty for failure to neutralize a bomber-type UAV could be high.

The power and support requirements for both blue and red represent one of the chief unknowns. Currently the logistical footprint for UAV operations is fairly large especially if multiple aircraft are to be used. However, as discussed, improvements in swarm technology is expected to drive down logistical and support requirements to the point that remote operation of thousands of UAVs may be accomplished by a very small team of operators from a safe location.

The size and shape of the protection and denial areas along with the concept of allowed leakage represent important considerations. While these requirements are not specific to the UAV threat, they may need to be amended with respect to expectations for other types of threat spaces. The doctrine appropriate for defense against cruise missiles may differ substantially from the doctrine for defending against UAVs. Scenarios presented in prior chapters demonstrated the importance of minimizing the denial area and maximizing the early warning capabilities. Arbitrarily expanding the denial area or losing the ability to detect UAVs as easly as possible could lead to a sharp decrease in overall system performance.

The strategic implications of allowable leakage may seem counter-intuitive, but it is possible in certain scenarios that hardening high-value assets against attack could be less expensive compared to outfitting an active defense system which is capable of higher effectiveness. Of course, passive defense is always a part of military defense systems; however, in the case of UAVs (particularly small UAVs with a limited destructive potential) it is possible that a high amount of allowed leakage would be suitable for certain circumstances. The strategic exploit discussed in this work related to red agents occupying blue defensive resources as part of a diversion is a real threat and doctrine must be designed to protect against such exploits.

Table 9: Table Showing Scenario Parameters.

blue	**red**	**configuration**
maximum rectilinear speed	maximum rectilinear speed	size of protection area
maximum angular speed	maximum angular speed	shape of protection area
number of agents	number of agents	size of denial area
mission/type	mission/type	shape of denial area
size	size	allowed leakers
weight	weight	
cost	cost	
power requirements	power requirements	
support requirements	support requirements	
beam width		
beam range		
probability of effectiveness		
time to effect		
magazine		
time to detect		
time to identify		
tracking error		

5.3 Contributions Wrap-Up

This work is essentially comprised of three contributions which are detailed below.

Firstly, the parameterization presented is considering an essential list of characteristics comprising the generic defensive scenario. That is not to say that this list is exhaustive, or that this list cannot change but rather that this list facilitates the appropriate level of detail for engagement-level modeling. Addition or subtraction of terms is possible but should be conducted with utmost care and in most cases, the effect of proposed extra terms could easily be accommodated in existing terms within the parameterization or that adding terms is unlikely to substantially effect the accuracy of the model output considering the specified level of detail. Importantly, the analytic solution techniques borrowed from game theory, differential games and pursuit evasion puzzles are presented as a critical first step in considering the complexity of the problem. Though the generic defensive scenario is likely to be modeled at some point using numerical simulations, the application of analytic techniques in an attempt to decrease the size of the required numerical solution domain should be considered a necessary first step.

Second, various measures of success which can be used to compare system effectiveness in the generic defensive system scenario are presented. Naturally, many defensive systems can be simplified to a single term related to to the effective range at which targets are neutralized. This measure of success is based on the statistical concept of expected values and is easy to formulate and understand. Unfortunately, not all scenario characteristics can be naturally converted to range. Second, the concept of kill-chain which is classically considering to comprise the time required for all the steps from first detection of a target to the confirmed neutralization of that target is extended such that relationships between scenario characteristics are converted to an associated quantity of time and the overall system effectiveness is expressed as an overall duration with units of time. Again, not all characteristics are easily converted to time and thus this method falls short of effectively providing an exhaustive framework for comparison. A novel technique is presented which is based on the fundamental statistical concepts related to the construction of combining probability distribution functions, the creation of cumulative distributions functions and the idea of joint probability for independent random variables. This technique allows for an analytic estimate of effectiveness with respect to probabilistic threats. Importantly, since overall effectiveness of systems and subsystems is the bottom-line output for engagement and mission level modeling efforts, directly estimating this effectiveness allows for an exhaustive treatment of the scenario paramterization considering all characteristics must be related to overall effectiveness in some way.

The third contribution involves the study of trends in the UAV sector related to the development of future threats. Predictions are presented based on a synthesis of available information including UAV futures predictions from various sources, a paper survey for the UAS industry, as well as subject matter expertise of the author related to the design, construction and operation of UAS including fleet operations. These predictions are also presented with respect to the scenario paramterization including the identification of sensitivities in terms of areas of weakness which could leave defensive systems dangerously exposed. Additionally, sensitive areas with respect to increased effectiveness are also identified which provide a suggestion for focus areas on how to improve defensive systems to be more effective against a changing threat environment.

Overall the work presented herein, makes substantially progress toward a comprehensive framework as well as a collection of methods for analytic consideration of generic defensive scenarios with respect to the unique threat presented by UAS. This framework includes a baseline parameterization as well as a consideration of what parameters can be important to consider with respect to the desired level of detail of the analysis in addition to various solution techniques borrowed from various subject areas. Additionally progress is made toward suggesting the best currency with which to discuss and compare effectiveness of various systems with respect to their performance against current and future UAS threats. A novel technique is presented which is founded on fundamental probabilistic techniques and conveniently allows for analytic measures of success to be developed with respect to probabilistic threats without resorting to numerical simulations which may require very large input domains for experimental design. And finally, predictions are made which rely on synthesis of the best available information along with the expertise of the author. Additionally, the sensitivity of various scenarios is considered with respect to the parameterization and solution techniques presented. The effort presents the most complete and cohesive consideration of the various complexities related to the formulation and consideration of generic defensive scenarios with respect to unique threats comprised of UAS.

Appendices

.1 Lady in the Lake Solution

The problem of the lady in the lake forms an important example considering the methods used for much of this work. Additionally the problem of the lady in the lake is an example of what happens when the scenario is poorly or inadequately defined. In the formal statement of the problem as classically given, several important details are left out which dictate the applicability of the solution to the second part of the original problem.

To solve the problem, the stages of the escape will be considered separately. The first stage of the escape requires the lady in the rowboat to take advantage of her advantage in terms of angular advantage to attain maximum angular separation from the monster. Simultaneously the lady wants to arrive at a point closer to the shore compared to her starting point at the center of the lake. There exists a critical radius which represents the maximum radial distance from the center of the pond that the lady can get to before the monster will be able to outpace and reduce the angular (and by extension euclidean) separation between the two. This critical radius is found by equating the angular velocity expressions for the lady and the man and solving for the critical radius according to Eqs. (1) and (3).

$$\omega_{lady} = \frac{v_{lady}}{r_{critical}} \tag{1}$$

$$\omega_{monster} = \frac{v_{monster}}{r} = \frac{4v_{lady}}{r} \tag{2}$$

$$\frac{v_{lady}}{r_{critical}} = \frac{4v_{lady}}{r} \tag{3}$$

Solving for the critical radius ($r_{critical}$), canceling the velocity of the lady (v_{lady}), and setting the overall radius (r) equal to one for the sake of convenience leads to the result in Eq. (4) which stipulates that the critical radius is equal to the reciprocal of the monster's speed advantage. The monster's speed advantage, also known as the speed ratio, is given by the symbol R and is given here because it will be needed later.

$$r_{critical} = \frac{1}{4} = \frac{1}{R} \tag{4}$$

Consider that if the lady rows along some arbitrary path to just less than the critical radius, she can manipulate her angular position relative to the monster. This maneuvering up to the critical radius and attaining the maximal angular separation from the monster comprises the first stage of the solution. The next stage of the solution involves a dash toward the short.

Considering the positioning of the lady relative to the monster, the maximal angular separation is given by π or $180°$ according to the geometry. Thus the time required for the monster to close that angular separation can be found by dividing half the circumference of shore of the lake by the monster's speed as given in Eq. (5).

$$t_{monster} = \frac{\pi}{v_{monster}} = \frac{\pi}{4v_{lady}} = \frac{\pi}{Rv_{lady}} \tag{5}$$

All that remains is to determine if the lady can escape. Consider the most obvious strategy is to minimize the distance the lady needs to cover which necessarily involves an escape path which is parallel to a radial from the center of the lake. Since the lady is at the critical radius, the time she would need to cover is given by solving for the distance from the critical radius to the perimeter of the lake and dividing by the lady's speed as in Eq. (6).

$$t_{lady} = \frac{r - r_{critical}}{v_{lady}} = \frac{1 - r_{critical}}{v_{lady}} = \frac{1 - \frac{1}{R}}{v_{lady}} \tag{6}$$

To determine if the lady can escape the time required for both the lady and the monster to arrive at the designate point on the shore can be substituted into the equality Eq. (7) which stipulates that the time required for the lady to arrive at the point must be less than the time for the monster in order for the lady to escape.

$$t_{lady} < t_{monster} \tag{7}$$

$$\frac{1 - \frac{1}{R}}{v_{lady}} < \frac{\pi}{R v_{lady}} \tag{8}$$

The solution is found by solving for the speed ratio as in Eq. (9) and the specific solution for the solution to the first part of the puzzle question is given by substituting the known speed ratio with value 4 as in Eq. (10). The general form of the solution allows for an easy solution to the second part of the problem (or so it seems). Solving for the inequality in Eq. (9) leads to the inequality $R < 4.14$ which is the condition under which the lady can escape from the lake using the strategy thus far described.

A schematic representation of the solution to the first part of the problem is given in Figure 25. The notations such as t_1 represent the position of the lady and monster at various times denoted by the subscript to help understand the escape path by the lady and the pursuit path by the monster. The subscript 1 denotes the initial position of both the

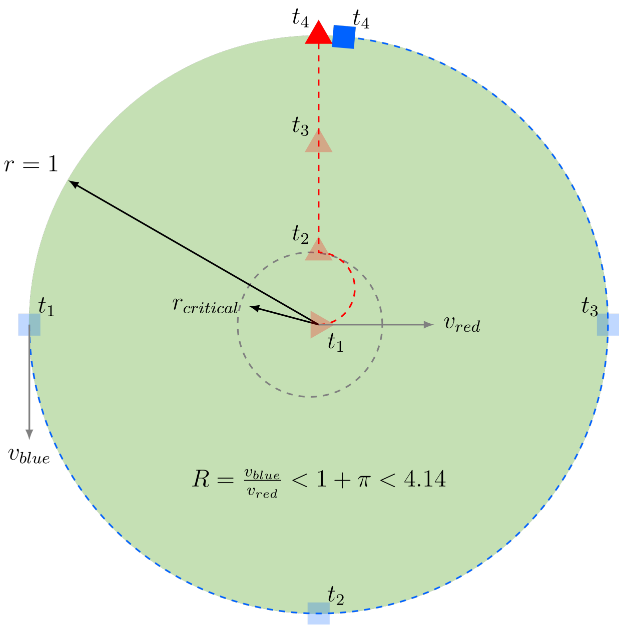


Figure 25: Schematic Diagram of the Solution to Part 1 of the Problem of the Lady in the Lake

lady and the monster, while the subscript 4 represents the final position (the final position also has the most opaque position indicators for the lady and the monster. The subscripts 2 and 3 necessarily represent some intermediate times between the initial and final position. The mark t_3 denotes the position of both the lady and the monster at the same instant during the pursuit.

$$R < 1 + \pi \tag{9}$$

$$4 < 4.14 \tag{10}$$

The limitation on speed ratio as given in the solution from part 1 is conservative compared to what is possible with a slight modification to the two-stage escape strategy.

The initial portion of the escape strategy is still the same and still involves the lady maneuvering such to maximize the angular separation between her and the monster. However, recall the critical radius as given earlier in Eq. (4). To most efficiently position herself as far from the monster as possible prior to beginning the second stage of the escape the lady will choose to row in a sermi-circular path starting at her position at the center of the lake and arriving exactly tangent to the circle with radius equal to the critical radius as given in Eq. (4). The radius of her stage 1 path consisting of a semi-circle will necessarily have a radius equal to half the radius the critical radius. This semi-circular stage 1 path was used in the part 1 solution even though it was not explained at that time.

In order to justify the alternative solution, the behavior of the monster needs to be specified in more detail. The monster will never change course if such a course change will necessarily increase his distance from the lady. Additionally, the monster will not sit still if moving in a particular direction can cause the distance between him the lady to decrease (or simply to increase less quickly). Additionally, the monster can be expected to behave as if he is unaware that the lady can escape in the case where the lady is able to escape.

This latter assumption is part of the given solution to part 1 considering that if the monster is in fact a math expert as described then he would be unlikely to participate in a contest he knows he cannot win. In the event that the lady can escape, the monster delays her escape by adopting his best possible strategy to pursue her. If the monster simply sat still, then the lady would be able to escape more efficiently, but as discussed, the monster prefers not to behave this way. If he cannot win the game, he chooses to inconvenience the lady as much as possible.

The main difference in the second stage of the escape strategy concerns the direction the lady will flee and thus the maximum amount of distance that both the lady and the monster will need to traverse in order to arrive near the same point on short. Previously, the lady fled along the most direct route possible to minimize her time to shore. In this case, the lady will choose to take a longer path to arrive at shore but will simultaneously force the monster to take a longer way around as well. This is why the extra stipulations on monster's behavior are critical to this solution because the boundary on the monsters pursuit is given by the geometry of the problem. Understanding how the monster can be forced to traverse more than half the perimeter of the lake (with angular displacement equal to π) is essential to the proof for this solution.

The lady will choose to row on a path tangent to her initial semi-circular path. Arguing for this path versus the straight-line escape path is paramount to allowing the lady to escape from a slightly faster monster. If the monster is less than 4.14 times faster than the lady then she should just row straight to the shore along a radial path and go on her way. However, if the monster is faster than the lady by 4.14 or more, then the lady will need to adopt an alternate strategy in order to escape.

Consider that at the moment the lady chooses to row along a tangent path to her stage 1 semicircular path, the distance she needs to cover to escape can be found as the difference of her current position less the intersection of her projected path with the perimeter of the lake. The lady's starting coordinate is given in Eq. (11). The lady's starting point for the second stage is taken to be located on one of the Cartesian axes to simplify the mathematics. In this case the lady starts closest to the south side of the lake in agreement with the diagram.

$$p_{lady_i} = (0, -r_{critical}) \tag{11}$$

The final position for the lady is located at the intersection of the line tangent to the critical circle and the perimeter of the pond as given in Eq. (12). The x-coordinate of the intersection can be found by recognizing that the pond perimeter is described by a circle with radius r. The y-coordinate of the intersection point is given by $-r_{critical}$ and thus the x-coordinate can be found by solving for the angle given the y-coordinate and then subsequently using the angle to find the x-coordinate according to Eq. (13).

$$p_{lady_f} = (x_{intersect}, -r_{critical}) = (x_{intersect}, -\frac{1}{R}) \tag{12}$$

$$x_{intersect} = r\cos\left(\theta\right) \tag{13}$$

$$y = r\sin\left(\theta\right) \tag{14}$$

$$\theta = \arcsin\left(\frac{y}{r}\right) \tag{15}$$

$$x_{intersect} = r\cos\left(\arcsin\left(\frac{y}{r}\right)\right) \tag{16}$$

$$x_{intersect} = r\sqrt{1-\left(\frac{y}{r}\right)^2} \tag{17}$$

$$x_{intersect} = 1\sqrt{1-\left(\frac{r_{critical}}{1}\right)^2} \tag{18}$$

$$x_{intersect} = \sqrt{1-\left(\frac{1}{r}\right)^2} \tag{19}$$

$$x_{intersect} = \sqrt{1-\frac{1}{R^2}} \tag{20}$$

Thus the distance that the lady needs to cover is given by $p_{lady_f} - p_{lady_i}$. Computing this distance allows for the repeated y-coordinate to be eliminated and thus the distance is given purely by the intersection coordinate as found in Eq. (13). The distance the lady will need to cover is given in Eq. (21).

$$d_{lady} = \sqrt{1-\frac{1}{F^2}} \tag{21}$$

Next it remains to find the distance the monster must travel. More importantly justification for causing the monster to follow a path which comprises more than half the circle will be necessary. Consider the solution shown in Figure 26. At any point during the engagement the monster could choose to change his direction; however, this would cause him to give up distance from the lady which he prefers not to do. Recall, the solution as

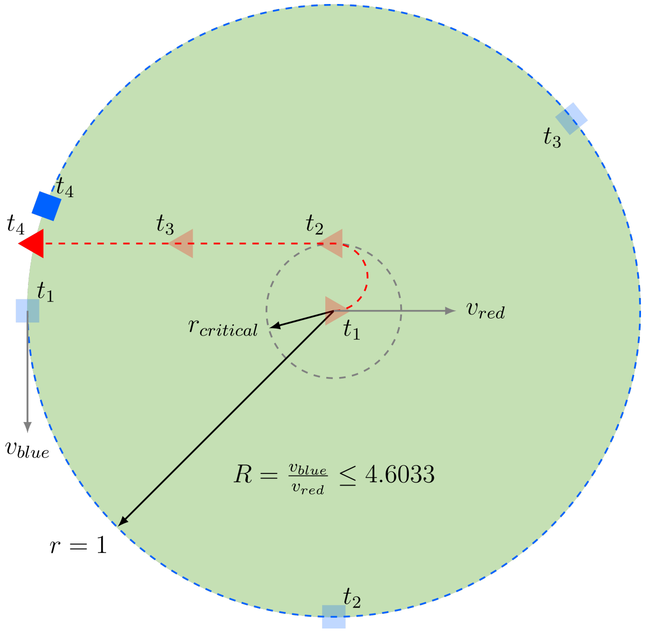


Figure 26: Schematic Diagram of the Solution to Part 2 of the Problem of the Lady in the Lake

provided does not allow the monster to win in cases where the speed ratio is sufficiently slow for the lady to escape; however, in these cases the monster wishes to get as close as possible to catching the lady. If he chooses to follow a different strategy then the lady will still escape but it may not take as much effort on her part and the monster wants the lady to work as hard as possible.

Consider the plot in Figure 27 in which the orange trace represents the distance between the lady and the monster. This plot has been constructed using the maximum speed ratio possible which still allows the lady to escape. The yellow trace describes the derivative of the distance between the lady and the monster. When the monster and the lady are locked in their chosen pursuit and evasion paths, they are locked into shape of the distance function as given. This means that if the monster decides to reverse direction

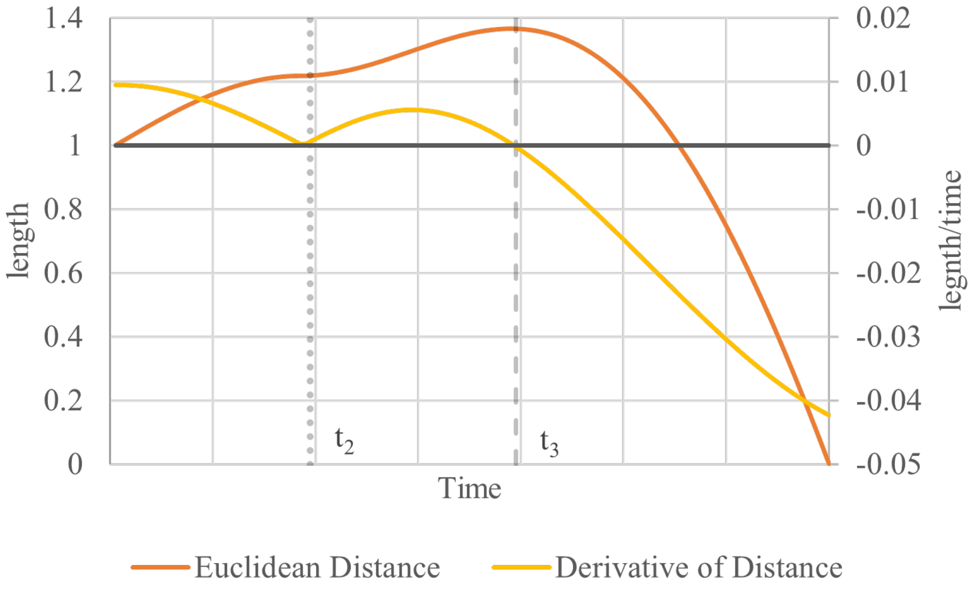


Figure 27: Plot Showing the Distance between the Lady and the Monster in Support of Part 2 of the Problem of the Lady in the Lake

around t_2 for example, that he would be moving the wrong way down the distance function and would be, by his action, accelerating the rate at which the lady is increasing distance between herself and the monster.

There are two inflection points visible in the orange trace showing the distance function and both of these correspond with zero-crossings on the derivative trace shown in yellow. The inflection point at t_2 is not truly a zero crossing since the distance function is simply in a local maximum here. This is consistent with the lady and the monster needed to choose which direction to continue when at t_2. Either direction is equally valid and since both the lady and the monster can turn instantaneously there is no reason for the lady or the monster to prefer either direction. However, once the lady chooses a direction and the monster follows, the monster will not choose to change direction since he will again be helping the lady get farther away faster.

The final critical point occurs near t_3. This point is truly a zero-crossing in terms of the derivative function and thus after this point, the lady is totally committed to fleeing toward the shoreline and the monster will of course not choose to change course because he is finally getting closer to the lady. The native slope of the distance curve indicates that after t_3 the lady is allowing the monster to get closer and closer with the knowledge that she will be able to escape by the time she finally reaches the shore.

Since the point at which the lady reaches the shore is known, the point that the monster would need to reach to capture her can also be found considering that the monster's position at t_2 is known. The monster will need to travel an angular distance equal to half the circle, plus the angle between the vertical axis and the intersection point for the lady on the shoreline. This distance is shown in Eq. (22).

$$d_{monster} = \pi + \arctan\left(\frac{\sqrt{1-\frac{1}{R^2}}}{\frac{1}{R}}\right) \tag{22}$$

In this chapter the characteristics of the defense problem being discussed will be laid out explicitly. Characteristics for the area to be defended, traits of the agents available to aid in the defense as well as the traits of the enemy agents will be laid out. The defense problem is characterized in terms of rough orders of magnitude related to the area to be defended which reflects that this work is primarily concerned with what is being defended against, rather than what is actually being defended.

The principal characteristic of the threat scenarios described herein, is that UAS are involved in some way. Though the original intention was to consider UAS as the threat vector, in the course of conducting research the purpose of the work has evolved to include using UAS to defend against UAS threats. The specific class of UAS to be considered is more of less fully described by the phrase "consumer-grade". Large military-grade UAS certainly comprise a viable threat and important work is being conducted related to defending against the threats posed by these types of UAS; however, that is not the chief purpose of the work described here.

The threat being considered here is envisioned to be accomplished using widely available, inexpensive and easy-to-use UAS which are available off-the-shelf. Defining the class of UAS being considered in this manner allows predictions more easily considering the nature of information related to consumer products. Additionally, the agility of the asymmetrical threat posed by isolated terrorists cells or individuals presents an acutely difficult problem which requires careful consideration and possibly transformative defense technologies and methodologies.

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VITA

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