EXTENSION THEOREMS
IN VECTOR SPACES OVER FINITE FIELDS

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ABSTRACT

We study the $L^p - L^r$ boundedness of the extension operator associated with algebraic varieties such as nondegenerate quadratic surfaces, paraboloids, and cones in vector spaces over finite fields.

We obtain the best possible result for the extension theorems related to nondegenerate quadratic curves in two dimensional vector spaces over finite fields. In higher even dimensions, we improve upon the Tomas-Stein exponents which were obtained by Mockenhaupt and Tao by studying extension theorems for paraboloids in the finite field setting. We also study extension theorems for cones in vector spaces over finite fields. We give an alternative proof of the best possible result for the extension theorems for cones in three dimensions, which originally is due to Mockenhaupt and Tao. Moreover, our method enables us to obtain the sharp $L^2 - L^r$ estimate of the extension operator for cones in higher dimensions. In addition, we study the relation between extension theorems for spheres and the Erdös-Falconer distance problems in the finite field setting. Using the sharp extension theorem for circles, we improve upon the best known result, due to A. Iosevich and M. Rudnev, for the Erdös-Falconer distance problems in two dimensional vector spaces over finite fields.

Discrete Fourier analytic machinery, arithmetic considerations, and classical exponential sums play an important role in the proofs.