HYBRID TRUCK-DRONE SYSTEMS FOR LAST-MILE DELIVERY LOGISTICS

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Abstract

With growing consumer demand and expectations, companies are attempting to achieve cost-efficient and faster delivery operations. The integration of autonomous vehicles, such as aerial and ground drones, in the last-mile network design, could curtail many operational challenges and provide a competitive advantage. This dissertation deals with the problem of delivering orders to a set of customer locations using multiple drones that operate in conjunction with a single truck. Four variants of the problem are considered. The first variant takes advantage of the drone fleet by parallelizing the delivery tasks via concurrently dispatching the drones from a truck parked at a focal point (ideal drone launch location) to the nearby customer locations. The key decisions to be optimized are the partitioning of delivery locations into small clusters, identifying a focal point per cluster, and routing the truck through all focal points such that the customer orders in each cluster are fulfilled either by a drone or truck. The first problem variant restricts each focal point to one of the customer locations, while the second addresses the same delivery problem when allowing truck stops to be anywhere in the delivery area (i.e., a customer or non-customer location). Mathematical programming models are developed to jointly optimize both the clustering and routing decisions.

The third variant suggests allowing the usage of non-customer locations (referred to as flexible sites) as truck stops for drone launch and recovery operations (LARO). This relaxes a common constraint in the literature restricting the drone LARO to customer locations. The proposed variant also accounts for three key decisions - (i) assignment of each customer location to a vehicle, (ii) routing of truck and UAVs, and (iii) scheduling drone LARO and truck operator activities at each stop, which are always not simultaneously considered in the literature. A mixed integer linear programming (MILP) model is formulated to jointly optimize the three decisions. Furthermore, to handle large problem instances, we develop an optimization-enabled two-phase search algorithm by hybridizing simulated annealing and variable neighborhood search. The fourth variant in this dissertation proposes utilizing a network of docking stations and repositioning of drones to enhance the efficacy of delivery operations. In particular, such stations are used for drone docking before and after delivery operations to avoid both loading all required drones to the truck at the depot and waiting of the truck on its route to recover drones. A MILP model is formulated to optimize the management of this setting of facilities and delivery operations. Finally, extensive computational experiments are conducted in this dissertation for the four variants to obtain several insights aiding the logistics practitioners in decision making.

Chapter 1

Introduction

In the era of online shopping, several components of e-commerce experience, such as marketing and transaction processing, have dramatically transformed. However, the dependency on delivery vans or trucks for the last-mile operation has remained unchanged for decades. This traditional process of delivering goods is not efficient in the long-run due to increasing traffic congestion and growing customer expectations [1, 2]. Over 70% of the consumers across the globe are price sensitive and are reluctant to pay for faster delivery [3]. Yet, a majority expects reliable as well as quick delivery of goods. As a result, companies are competing to achieve cost-efficient and faster last-mile delivery operations. The integration of emerging technology, such as unmanned aerial and ground vehicles or drones, in the last-mile network design could overcome these challenges and provide a competitive advantage.

1.1 Industrial background and challenges

The last-mile delivery market is projected to have a compounded annual growth rate of 16% during the next five years, primarily due to the proliferation of e-commerce transactions [4]. Consequently, the traffic congestion and greenhouse gas emissions are expected to increase by 21% and 32%, respectively, in the absence of an effective last-mile intervention strategy [5]. On the other hand, companies are also striving to meet the faster delivery expectations of the consumer and gain a competitive advantage. A potential approach to transform the last-mile delivery is to adopt emerging logistics technologies, namely drones, as they can circumvent ground-based traffic congestion and parallelize delivery operations [6]. In particular, an aerial drone can surmount its limited payload capacity and flying range by collaborating with a truck that can serve as a moving depot and recharging (or battery-swap) platform [7, 8, 9, 10, 11]. Similarly, a ground vehicle can collaborate with a truck to perform the last-mile delivery tasks, especially at congested areas [12, 13, 14, 15, 16].

Owing to their relatively low operating cost and congestion-free aerial and sidewalk routes, drones are an attractive alternative for performing last-mile delivery operations [17]. Besides, drones provide an environmentally friendly mode of delivery if low-carbon electricity sources are used to charge batteries [18, 19]. Due to rapid growth in Business-to-Consumer (B2C) transactions, where packages are smaller, drones are viable to deliver most customer orders [20]. Recently, the application of drones in delivery logistics has emerged prominently, especially after the initiatives of big corporations such as Amazon [21], Google [22], and DHL [23]. Figure 1.1 exhibits some models of aerial and ground drones used for last-mile delivery operations.

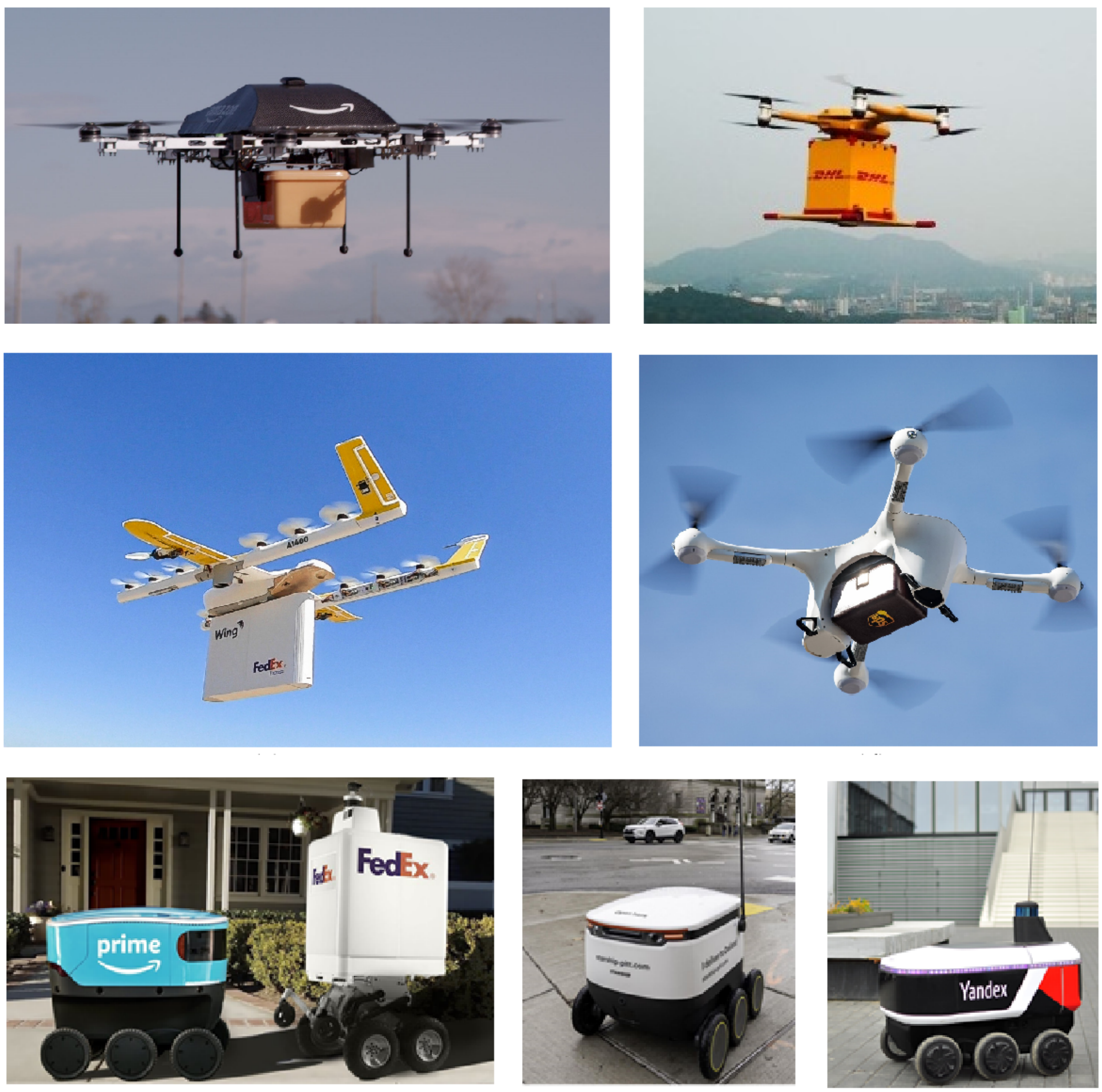

Figure 1.1: Some drone models of big companies working in last-mile delivery operations. [Source: [24, 25, 26, 27]]

On the regulatory aspect, several countries across the globe are relaxing rules to adopt commercial drones in the low altitude airspace. For instance, the Federal Aviation Administration (FAA) in the United States is now allowing small-sized drones to fly over people and operate beyond visual line of sight (BVLOS), even at night, thereby removing significant barriers to their widespread use [28]. Similarly, the European Union Aviation Safety Agency (EASA) has standardized drone regulations for all its member states, and provides authorization for BVLOS operations [29].

Despite the potential benefits, drones have two major technical restrictions - (i) limited flight/travel range or distance and (ii) restricted payload capacity [30]. These two drawbacks impede the possibility of adopting a stand-alone drone shipment as an alternative to traditional truck delivery. Nevertheless, a drone-truck combination could be used for efficient delivery, where a truck carries a set of drones and customer orders to a stop that is within the flight range. In that way, drones can be employed to transport multiple low-weight orders [31, 32]. This would allow drones to ship most e-commerce orders as they are typically within the payload capacity. For instance, 86% of packages shipped by the e-commerce retailer, Amazon, weigh less than 3 kg (6.5 pounds) [20], which is less than the drone's common payload capacity [30]. Thus, the necessity of using drones in delivery logistics is evident from different perspectives.

However, most attempts by companies and research studies focus on using just a single drone in tandem with a truck [9, 33, 34]. Hence, the current challenge is to effectively operate multiple drones and a truck to achieve cheaper and quicker delivery.

Figure 1.2: Models of hybrid truck-drone vehicles [Source: [5, 35]]

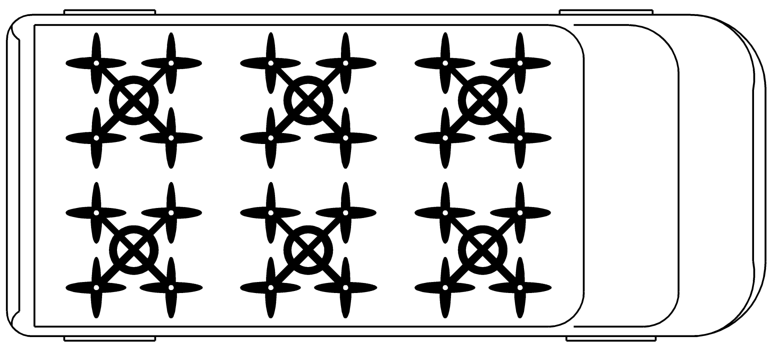

Figure 1.3: A top-view of a truck carrying six aerial drones on its roof for last-mile delivery tasks

Realizing the potential of hybrid truck-drone systems spurred several companies to develop proof-of-concept technologies for embracing the idea (e.g., Mercedes Vision Van with automated cargo space management and drone deployment [35]). Also, pilot experiments of truck-drone deliveries have been conducted [36, 37]. For example, Workhorse, a delivery company in Ohio, tested residential deliveries using a hybrid truck-drone system [37]. Figure 1.2 exhibits three models of hybrid trucks working in tandem with drones and Figure 1.3 illustrates the top-view of a specially designed truck equipped with six aerial drones on its roof for last-mile delivery tasks.

1.2 Research objective

This dissertation considers the problem of delivering orders to a set of customer locations using multiple drones that operate in tandem with a single truck. Four variants of the problem is considered as shown in Fig. 1.4. The first variant takes advantage of the drone fleet by parallelizing the delivery tasks via concurrently dispatching the drones from a truck parked at a focal point (ideal drone launch location) to the nearby customer locations. The focal points are restricted to customer locations in consistent with the most previous research in the literature. The following research objectives pertaining to the first variant are considered.

- Optimize the following key decisions of delivery operations by drones - (i) total drones to be employed per truck, (ii) number of focal points (or truck stops) required and their locations, (iii) assignment of delivery locations to a focal point and (iv) truck route that covers all the stops.
- Develop an efficient solution method for the policy of restricting the drone-dispatch locations to the customer locations (or depot).
- Develop a holistic joint optimization model for the clustering af customer locations and hybrid drone-truck routing to minimize total operational cost and delivery completion time.
- Formulate a multi-criteria model to achieve the best trade-off between total cost and delivery completion time.

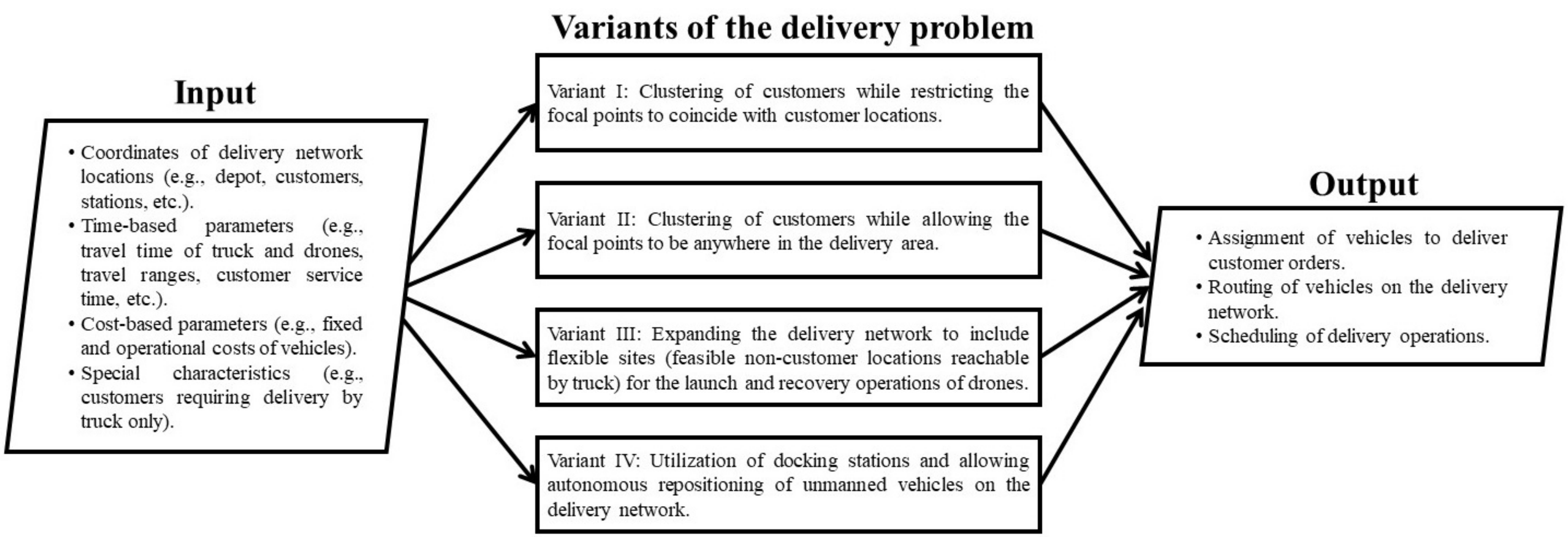

Figure 1.4: Overview of the delivery problem variants in this dissertation

- Integrate several characteristics necessary for the real-life implementation of truck-drone delivery, such as payload-dependent flight range, allowing the depot to be a potential drone dispatch point, and situations requiring delivery only by a truck.

The second variant allows the focal points to be anywhere in the delivery area (i.e., a customer or non-customer location). It shares the same research objectives of the first variant while considering the respective policy of locating focal points. The third variant suggests allowing the usage of feasible non-customer locations (referred to as flexible sites) as truck stops for drone launch and recovery operations (LARO). This relaxes a common constraint in the literature restricting the drone LARO to customer locations. The research objectives of the third variant are as follows.

- Quantify and demonstrate the impact of using flexible sites rather than restricting the drone LARO to customer locations.
- Consider three key decisions - (i) assignment of each customer location to a vehicle, (ii) routing of truck and UAVs, and (iii) scheduling drone LARO and truck operator activities at each stop, which are always not simultaneously considered in the literature.
- Formulate a mathematical programming model to jointly optimize the three decisions.
- Develop an optimization-based heuristic to efficiently solve problem instances at

scale.

The fourth variant in this dissertation proposes utilizing a network of docking stations and repositioning of drones to enhance the efficacy of delivery operations by truck-drone tandems. In particular, such stations are used for drone docking before and after delivery operations to avoid both loading all required drones to the truck at the depot and waiting of the truck on its route to recover drones. The fourth variant's research objectives are as follows.

- Quantify and demonstrate the impact of operating a network of docking stations stations in enhancing the performance of last-mile delivery tasks.
- Formulate a mathematical model to optimize the delivery operations while utilizing a network of docking stations.
- Consider the tradeoff, in delivery by truck and drones, between operating traditional trucks versus hybrid trucks accommodating drones.
- Provide insights on how to select the locations of docking stations and the characteristics of optimal solutions.

In general, this dissertation aims at bridging the knowledge gaps of optimizing operations pertaining to the emerging technology of using drones in delivery operations.

1.3 Dissertation outline

The remainder of this dissertation is organized as follows. Chapter 2 describes different problem configurations of last-mile delivery by drones and provides a detailed review of the relevant literature and research gaps. In Chapter 3, an integer programming (IP) model is formulated for the policy of restricting truck-drone stops to be customer locations. Two objectives are considered in the IP model, minimizing total cost and delivery completion time. Next, an extensive set of test instances is generated and solved to demonstrate the proposed model. Similar to Chapter 3, a mathematical programming model is developed in Chapter 4 but for the policy of allowing the truck-drone stops to be anywhere in the delivery area (i.e., a customer or non-customer location). Due to the complexity of relaxing the locations of truck-drone stops, the model is formulated initially as a mixed integer non-linear programming (MINLP) model and then a linearization procedure is presented. Further, new strategies to accelerate the solution time of the computationally expensive optimization model are presented.

In Chapter 5, a MILP programming model is formulated to optimize delivery operations while using flexible sites for drone LARO. Furthermore, an optimization-enabled two-phase search algorithm is developed, to handle large problem instances, by hybridizing simulated annealing and variable neighborhood search. Next, rigorous computational experiments are presented to demonstrate the impact of using flexible sites and the effect of critical parameters on the delivery operations performance. Chapter 6 presents a MILP model for the delivery tasks while utilizing a set of docking stations. In addition, it provides several insights for the utilization of such a delivery system setting. Finally, Chapter 7 presents conclusions and directions for future research.

Chapter 2

Literature Review

2.1 Motivation

Drones, or unmanned aerial/ground vehicles, can be employed for a variety of civilian applications, such as security and surveillance [38], disaster management [39], agriculture [40], photography [41]. Among these applications, last-mile delivery by drones is promising in replacing (or substantially collaborating with) the traditional trucks in the delivery operations. Owing to the aerial/sidewalk travel and relative low cost, drones are viable for such operations from both research and industrial perspectives. In practice, numerous companies around the globe have initiated projects to incorporate drones in their delivery operations, such as Amazon [21], DHL [23], UPS [42], FedEx [43], and Alibaba [44]. However, there are two technical issues inhibit the usage of stand-alone drones in delivery tasks. First, there is a limited flight/travel range of drones as opposed to trucks. Second, drones are characterized by limited payload capacity. Nevertheless, employing trucks to work in tandem with drones can solve the first issue, where a truck can transport both packages and drones until the customer location becomes within the drone flight/travel range. Many models of hybrid truck-drone systems are developed, such as the Vision Van by Mercedes-Benz [35]. Furthermore, the vast majority of packages in the era of business-to-consumer transactions are under the typical capacity of drones. For instance, 86% of Amazon's orders weigh less than 3 kg [20], which is less than the common payload capacities of drones [30].

Although the prospect of using drones in delivery operations were sporadically suggested throughout the last two decades, the Amazon's announcement in 2013 of using drones for delivering their packages increased the attention in both industrial and research arenas [33]. The aim of this review is to survey the growing literature in delivery by drones, specifically in the area of routing and scheduling delivery operations. The problem of last-mile delivery by drones (LMDD) have different configurations based on five characteristics:

- **Vehicles involved and heterogeneity:** Stand-alone drones can be employed for delivery logistics or other vehicles, such as trucks, are involved. Furthermore, the fleet of vehicles can be identical or heterogeneous.

- **Relative motion of vehicles and interaction level:** If other vehicles than drones are used, they all may move simultaneously or not. Moreover, drones may rendezvous other vehicles to enable wider coverage of delivery area or not.
- **Stationary facilities and network connectivity:** The network under consideration may involve a single or multiple hubs for providing supplies and fueling. Further, some locations, such as islands and mountains, can be non-reachable by some kinds of vehicles.
- **Capacity of vehicles:** Drones can carry a single or multiple units and with respect to a limited payload weight. Similarly, truck capacity may be relatively limited or it can be sufficient to carry all designated orders.
- **Drone launching and landing locations:** Based on technical and operational considerations, drones can be allowed to launch and land at specific locations.

Next, the state of the art in terms of solution methodologies developed for the LMDD problem are reported. Finally, the literature is analyzed and research gaps are listed.

2.2 Review methodology

This literature review is specialized in the LMDD problem, which distinguishes it from other literature reviews handling diverse civilian applications of drones. For readers with interests in such general overview, we refer them to Otto et al. [30] and

Chung et al. [45], who included topics such as area coverage, search operations, and task assignment to employ drones in different applications, such as agriculture, disaster management, environmental monitoring, security, and entertainment. Our review methodology starts with a material collection process, and then a descriptive analysis is conducted. Next, in Sections 2.3 and 2.4, we classify the content of the selected material based on the problem configuration and solution methodology, respectively. Finally, literature analysis and gaps are presented in Section 2.5.

2.2.1 Material collection

To guarantee a comprehensive coverage of relevant articles in the LMDD problem, a structured search is conducted on Scopus. The search was not restricted to specific year bounds, but the resulting articles are expected to be published in recent years. The required material is collected based on the following inclusion and exclusion criteria.

2.2.1.1 Inclusion criteria

To quantify the literature size, we started the search by "drone", "robot", "ground vehicle", "unmanned aerial vehicle", or "UAV" in the title and by "delivery" or "logistics" in the abstract. Note that we allowed other words that share the same root throughout the material collection process; for example, searching by "logistics" implies also “logistic” and “logistical”. This stage results in 5,522 papers. The resulting articles were further refined by considering one of the following keywords to be in the abstract as well, “customer”, “package”, “parcel”, “courier”, “truck”, “last mile”, “route”, or “schedule”. By adding these keywords, the Scopus search included 1,241 papers.

2.2.1.2 Exclusion criteria

The 1,241 papers are filtered on three consecutive stages to exclude irrelevant material - (i) subject area (e.g., physics or chemistry), (ii) source title (i.e., title of journal or conference), and (iii) paper title. Table 2.1 exhibits the description of the three criteria and the remaining papers after each stage. In summary, the material after the filtration is 712 papers. However, among them, 72 relevant papers are selected for review based on manual scrutiny.

2.2.2 Descriptive analysis

The majority of the selected papers were published in or after 2015 with an exponential increase, as shown in Figure 2.1, indicating the growing interest in this topic. Also, it is noticed that many of these papers have been published in high-quality and peer-reviewed journals as shown in Table 2.2. Furthermore, Table 2.3 illustrates that the United states dominates the highest proportion of papers, where many big corpo-

Table 2.1: Exclusion stages of irrelevant papers

Exclusion criteria	Description	Remaining papers after exclusion
Subject area	“Physics and Astronomy”, “Material science”, “Earth and Planetary Sciences”, “Immunology and Microbiology”, “Agricultural and Biological Sciences”, “Chemical Engineering”, “Chemistry”, and “Biochemistry, Genetics and Molecular Biology”	1,054
Source title	“military”, “defence”, “aerospace”, “aerodynamic”, “aeronautic”, “aviation”, “robot”, “electric”, “electronic”, “micro”, “dynamic”, “digital”, “telematics”, “cyber”, “mechanic”, “mechatronic”, “wireless”, “radio”, “signal”, and “sensor”	798
Paper title	“communicate”, “wireless”, “navigate”, “detect”, “sensor”, “trajectory”, “risk”, “disaster”, “agriculture”, and “traffic”	712

rations such as Amazon and FedEx are headquartered and demanding this research for their operations. In addition, the table shows that research in delivery operations by drones is of interest to many researchers in more than different 60 countries.

Table 2.2: Number of papers per journal

Journal	Number of papers
Transportation Research part C	18
Networks	14
European Journal Of Operational Research	10
Computers and Operations Research	9
Transportation Research part E	7
Transportation Research part D	6
Others (57 journals)	≤ 5

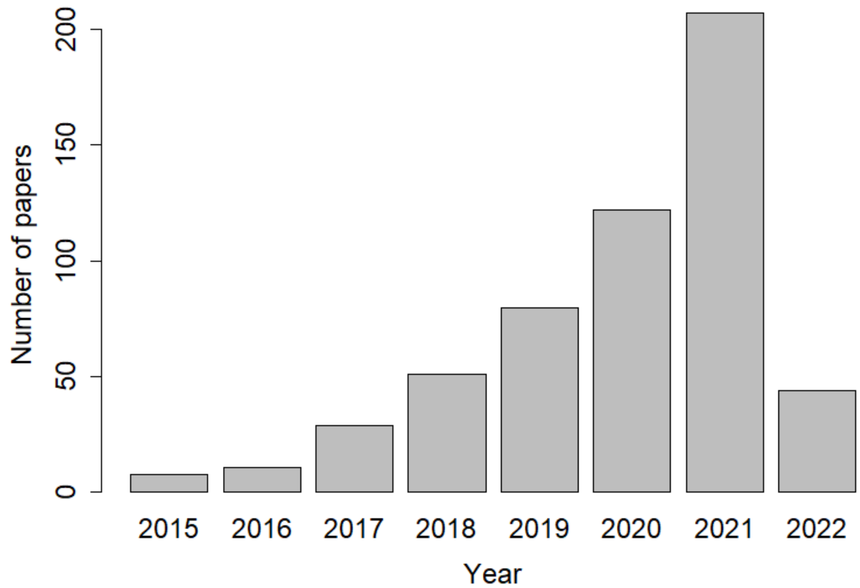

Figure 2.1: Number of papers per year starting 2015

2.3 Problem configuration of last-mile delivery by drones

The LMDD problem have different configurations based on five characteristics. Table 2.4 categorizes alternative characteristics that can form different problem configurations, and then Table 2.5 lists the problem configuration in some surveyed articles.

2.3.1 Vehicles involved and heterogeneity

Many research studies considered employing a fleet of drones simultaneously for delivery operations [7, 32, 33, 46, 56]. Due to the travel range restriction, most of these

Table 2.3: Number of papers per country

Country	Number of papers
United States	194
China	110
Germany	52
South Korea	43
United Kingdom	35
India	31
Italy	30
Australia	28
France	28
Canada	26
Others (52 countries)	≤25

Table 2.4: Abbreviations of different problem characteristics

Category	Abbreviation	Details
Vehicles involved and heterogeneity	SD	Single drone
	MD	Multiple drones
	NT	No-trucks
	ST	Single truck
	MT	Multiple trucks
	ID	Identical drones
	HD	Heterogeneous drones
	IT	Identical trucks
	HT	Heterogeneous trucks
Relative motion of vehicles and interaction level	EO	Either drones or trucks can be operated at a time
	BO	Both vehicles can be operated simultaneously
	NR	Drones do not rendezvous trucks
	DR	Drones rendezvous trucks
Stationary facilities and network connectivity	SH	Single hub
	MH	Multiple hubs (or stationary facilities)
	RN	Reachable nodes by any vehicle (i.e., drone or truck)
	NN	Some nodes are non-reachable by a specific vehicle type (i.e., drone or truck)
Capacity of vehicles	SU	Single unit capacity of drones
	MU	Multiple units capacity of drones
	LC	Limited capacity of trucks
	UC	Unlimited capacity of trucks
Drone launching and landing	AC	Launching only from customer locations or hubs
	AS	Launching only from stationary stations or hubs
	AA	Launching from anywhere
	ZC	Landing only from customer locations or hubs
	ZS	Landing only from stationary stations or hubs
	ZA	Landing from anywhere

Table 2.5: Summary of problem configuration of surveyed papers

Paper	Vehicles involved and heterogeneity	Relative motion of vehicles and interaction level	Stationary facilities and network connectivity	Capacity of vehicles	Drone launching and landing locations
Coelho et al. (2017) [46]	MD-NT-HD	-	MH	MU	AS-ZS
Hong et al. (2018) [7]	MD-NT-ID	-	MH	-	AS-ZS
Murray and Chu (2015) [33]					
variant 1	SD-ST	BO-DR	SH-RN	SU-UC	AC-ZC
variant 2	MD-ST-ID	BO-NR	SH-NN	SU-UC	AS-ZS
Agatz and Bouman (2018) [47]	SD-ST	BO-DR	SH-RN	SU-UC	AC-ZC
Ha et al. (2018) [34]	SD-ST	BO-DR	SH-RN	SU-UC	AC-ZC
Yurek and Oz-mutlu (2018) [48]	SD-ST	BO-DR	SH-RN	SU-UC	AC-ZC
Gonzalez-R et al. (2020) [49]	SD-ST	BO-DR	SH-RN	MU-UC	AC-ZC
Dell'Amico et al. (2019) [50]	SD-ST	BO-DR	SH-RN	SU-UC	AC-ZC
Dell'Amico et al. (2020) [51]	MD-ST-ID	BO-NR	SH-NN	SU-UC	AS-ZS
Ferrandez et al. (2016) [31]	MD-ST-ID	EO-DR	SH-NN	SU-UC	AA-ZA
Chang and Lee (2018) [32]	MD-ST-ID	EO-DR	SH-NN	SU-UC	AA-ZA
Salama and Srini-vas (2020) [11]					
variant 1	MD-ST-ID	EO-DR	SH-RN	SU-UC	AC-ZC
variant 2	MD-ST-ID	EO-DR	SH-RN	SU-UC	AA-ZA
Karak and Abdel-ghany (2019) [7]	MD-ST-ID	EO-DR	MH-NN	MU-UC	AS-ZS
Murray and Raj (2019) [52]	MD-ST-HD	BO-DR	SH-RN	SU-UC	AC-ZC
Poikonen and Golden (2020) [53]	MD-ST-HD	BO-DR	SH-RN	MU-UC	AS-ZS
Ham (2018) [54]	MD-MT-HD-HT	BO-NR	MH-NN	SU-UC	AS-ZS
Wang and Sheu (2019) [55]	MD-MT-ID-IT	BO-NR	MH-RN	MU-UC	AC-ZS
Kitjacharoenchai et al. (2019) [56]	MD-MT-ID-IT	BO-DR	SH-RN	SU-UC	AC-ZC
Sacramento et al. (2019) [57]	MD-MT-ID-IT	BO-DR	SH-RN	SU-LC	AC-ZC

studies proposed establishing a set of stations for charging drone batteries [7, 46]. The charging stations can be stationary or mobile based on the spatiotemporal dynamics of delivery orders. Furthermore, the fleet of drones can be identical [32, 33, 56], or heterogeneous [46, 52, 53, 54]. For instance, some drones are equipped with avoidance sensors to suit urban areas, as opposed to less costly drones for rural areas [33].

To avoid the cost of establishing stations, numerous studies investigated the use of trucks to work in tandem with drones [32, 33, 47, 56]. It is not only to overcome the travel range restriction but also to deliver the orders with weights exceeding the drone payload capacity. The fleet of trucks can be also identical or heterogeneous based on capacity, speed, and accessibility to specific locations. Furthermore, since the delivery by drones is recently introduced, the proportion of truck fleet that can launch and receive drones would be gradually growing.

2.3.2 Relative motion of vehicles and interaction level

There are two common approaches for employing drones in tandem with trucks. A conservative approach prevents the truck to move from the drone launching locations until all drones are recollected [8, 11, 31, 32]. This is justified by the fear of road traffic uncertainty that may cause a different location of truck in a designated time. The second approach is more common, which suggests that both drones and trucks can move simultaneously [33, 47, 49]. In that way, drones can be launched from a truck at a specific location and then the truck move to deliver packages to customers at different location(s) before rendezvousing the drone again. In case of employing multiple trucks, some studies assumed that launching and receiving each subset of drones is assigned to a specific truck [57]. Another practical perspective, yet complex, is allowing drones to be dispatched from a truck and received by another [56].

2.3.3 Stationary facilities and network connectivity

A typical network for delivery operations may contain a single depot [32, 33, 47, 57] or multiple hubs [54, 55]. Although similar forms of networks are observed for the LMDD problem, the function of stationary stations may be limited to charging or swapping batteries to extend the drone flight range [7]. The stationary facilities can be established for the sake of incorporating drones in delivery operations [7]. Alternatively, many studies consider adapting the existing network of stores in a city for that function [46].

A common assumption in generic routing problems is that all network nodes are reachable by the used vehicles. Some research in the LMDD problem adopted the same network attribute for both drones and trucks [33, 47, 57]. In contrast, others determined either a specific set of nodes to be visited only by trucks due to the limited drone flight range [51], or visited only by drones due to the geographical barriers such as the locations in islands or on mountains [58]. Lastly, a few studies assumed that packages can be delivered only by drones [8, 31, 32].

2.3.4 Capacity of vehicles

Most developed models consider that drones are capable of carrying a single package [32, 33, 34, 47, 48, 52, 56]. Nevertheless, a few models adopt the case of drones carrying multiple packages [8, 53, 55]. The capacity of trucks are categorized into a proportion for packages and another for drones. Typically, they are separated as aerial drones are mounted on the truck roof while ground drones are docked in the bottom half of the truck [8, 35]. However, if drones share the same space inside the truck, the capacity of packages is considered simultaneously with the number of allowable drones on the truck [55].

2.3.5 Drone launching and landing locations

Technically, most commercial drones can be launched from anywhere. However, some studies assumed that the launching should be conducted from the depot or stationary stations [7, 8, 51, 54], while others assumed that the launching are from customer locations [47, 48, 52]. On the other hand, a few articles considered that drones can be launched from anywhere in the delivery area [32], or along the truck route [45]. Similarly, three cases of drone landing are considered in the literature.

First, drones should land only in the depot or stationary location [7, 8, 51, 54, 55]. Second, the landing can be on truck waiting at customer locations [47, 48, 52]. Finally, the drone can rendezvous the truck anywhere in the delivery area [32].

2.4 Solution methodologies

In this section, a detailed review of the LMDD models are presented. Research on employing drones in delivery operations has gained a lot of attention in recent years. Some studies considered direct drone delivery from distribution centers by using a network of recharging stations to overcome the travel range limitation [7]. To circumvent the cost of establishing such stations, a truck collaborating with drones is considered in many other studies, in which a truck carries both drone and order until the customer location is within the drone travel range [32, 33, 34]. Hence, the truck functions as a *moving depot,* as stated by Hong et al. [7]. The literature on coordinating the logistics operations using a truck-drone combination focused predominantly on extending classical routing problems - a variant of traveling salesman problem with drones (TSP-D) [9, 33, 34, 47, 48, 59, 60], and a generalization of vehicle routing problems to include drones (VRP-D) [55, 57, 61, 62, 63].

2.4.1 Traveling salesman problem with drones (TSP-D)

In TSP-D, a set of customer locations are served either by a drone or truck. The truck carrying a single drone starts from, and returns to, a depot exactly once. Besides, a drone can only be launched from, and recovered at, customer locations. The truck can continue delivery after dispatch, as the drone may return to a site that is different from the original dispatch location. Prior research has adopted different approaches to solving the TSP-D. One of the earliest studies on the truck-drone routing was presented by Murray and Chu [33], who developed a mixed integer linear programming (MILP) model along with simple heuristics to minimize the overall delivery completion time. They found the MILP model to be computationally expensive as it could not achieve optimality even after a 30-min runtime, whereas their heuristics were able to produce better solutions in less time. To overcome the drawback of not achieving optimality using the MILP model, subsequent works adopted different approaches for the TSP-D. Agatz et al. [47] developed an integer programming model as well as several fast route-first cluster-second heuristics to solve the TSP-D. They achieved optimality for 12 customer instances within a two-hour run time, while their heuristics yielded near-optimal solutions quickly. Besides, their experimental analysis indicated a substantial reduction in delivery completion time when using truck-drone in tandem as opposed to truck-only delivery. However, they assumed drones to follow the road network, which diverges from previous works that consider Euclidean drone travel. To further expedite the time to optimality, Yurek and Ozmutlu [48] proposed an iterative algorithm based on a decomposition approach for the TSP-D. Their model was able to achieve the optimal solution for 12 customer instances with an average solution time of 15 min. While all the above-mentioned research on TSP-D aimed at minimizing completion time, Ha et al. [34] developed MILP formulation and two heuristics to minimize total operational costs. The authors found that minimizing cost has a significant improvement in drone utilization, but also increases the overall delivery completion time. Although this research above is similar to our work in the aspect of using truck and drone in tandem for delivery, there are many stark differences. In particular, all the aforementioned literature considers only a single drone, while our problem involves the use of multiple drones. In a recent study, Campbell et al. [64] demonstrated the effectiveness of using multiple drones per truck in minimizing the operational cost. While previous models on TSP-D aim to minimize either delivery completion time or operational costs, the current research considers both the objectives to unveil their independent impact and best trade-offs.

2.4.2 Vehicle routing problem with drones (VRP-D)

The VRP-D is an extension of the TSP-D, where a fleet of homogeneous trucks, each carrying a fixed number of drones, is used in package delivery to a set of customers [57, 61, 63]. Wang et al. [63] introduced the VRP-D to minimize completion time and presented a theoretical study to compare the optimal objective value with and without using drones. They considered different settings and concluded that integrating drones in the delivery process has substantial time savings. Particularly, they found two factors that impact the completion time - number of drones per truck and the drone speed (relative to the truck speed). However, the authors assumed rectilinear travel paths for both trucks and drones and unlimited drone battery-life. Poikonen et al. [61] extended their work by relaxing these two assumptions. In addition, they related the VRP-D to the close-enough VRP (CEVRP), where a truck need not visit the actual location but achieve close proximity. In contrast to the previous two articles on VRP-D, Sacramento et al. [57] presented a MILP model to minimize the total operational costs, while considering capacity and completion time constraints. The authors compared their formulation with the truck-only approach and concluded that using drones can achieve up to 30% cost savings. However, they assume that each truck is equipped with only a single drone. Wang and Sheu [55] dealt with this limitation by formulating an arc-based model for VRP-D to minimize total fixed and operational costs. Their results show a 20% reduction in total cost, besides reducing the average delivery time by five minutes per customer. Nonetheless, they assume an unlimited supply of drones and restrict them to land only at a service hub or depot, but not at a customer location. The main commonality between our work and the VRP-D literature is the use of multiple drones for last-mile delivery operations. However, unlike this paper, all the previous work on VRP-D uses only a specific set of locations for launching and recovering drones.

2.4.3 Clustering of customer locations and routing of truck and drones

Another category of hybrid drone-truck routing for optimizing last-mile delivery involves the coordination of multiple drones working in tandem with a single truck [8, 31, 32]. This can be conducted by clustering of customer locations and routing of truck and drones. It is therefore analogous to the traditional truck and trailer routing problem (TTRP). In TTRP, some customers must be served only by a truck, while the remaining customers can be visited by a single truck or a truck-trailer combination [65, 66, 67]. Akin to TTRP, we route two types of vehicles (a truck and drone instead of a truck and trailer) to different sets of customers (those who require truck-only delivery, and others who can be served either by a truck or drone). Nonetheless, our problem allows the truck and drone to operate independently, while a trailer cannot move without a truck in TTRP. On the other hand, our solution strategy is vaguely similar to the ring star problem (RSP), which considers a set of nodes (retailer locations) and a central depot with the goal of identifying a subset of retailers that can also act as small depots. The selected small depots are interconnected (ring structure) and supplied by the central depot, whereas the remaining retailers are served by the closest small depot (star topology) [68]. The RSP is similar to our work in the sense that both problems require the identification of focal points that serves neighboring locations. But, the RSP establishes several small depots in the network, while our problem locates truck stops. However, in contrast to our research, the RSP restricts each small depot to be one of the existing nodes (retailers). Besides, RSP focuses only on constructing a route connecting the focal nodes (small depots) and assigning the remaining nodes to exactly one of the focal nodes. Our research accomplishes the same, but additionally considers the delivery of orders from the focal point to its assigned nodes. Also, RSP assumes a single-vehicle, whereas our problem involves multiple drones working in tandem with a truck.

Recently, Karak and Abdelghany [8] considered such an arrangement for minimizing the total operational cost of pick-up and drop-off services, where a set of customers are visited (for order pick-up and/or delivery) by drones that are deployed from a truck halted at a station. Their research shares the following aspects with our work in chapters 3 and 4. A single truck carrying multiple drones is departed from, and returned, to a depot. Along its route, it visits a number of truck stops (defined as stations in their study and cluster focal points in our work) and waits to dispatch and recollect drones. Yet, the truck stops are assumed to be known apriori by Karak and Abdelghany [8], while our approach seeks to establish their optimal locations. Furthermore, we consider both truck and drone to be capable of serving a customer, whereas Karak and Abdelghany [8] do not use the truck for delivering customer orders. On the other hand, we are aware of only two works that adopt a clustering-based method to minimize the completion time of last-mile deliveries with one truck and several identical drones [31, 32]. Both papers proposed a similar multi-phase sequential approach. First, the delivery locations are grouped into non-overlapping clusters. Subsequently, a truck stop per cluster is located for launching and recovering multiple drones that simultaneously serve the customers in that cluster. Finally, the best truck (carrying multiple drones and customer orders) route is established such that the truck starts from the depot, visit each truck stop exactly once, and then returns to the depot. Such an approach would also lower the operating cost compared to the truck-only delivery since the cost of using drones is significantly less than trucks [55]. In particular, Ferrandez et al. [31] used an unsupervised machine learning approach, k-means algorithm, to partition the network into clusters, where the centroid of a cluster would serve as a truck stop for drone dispatch. Consequently, the authors developed a genetic algorithm to determine the best truck route covering all the cluster centroids. They compared the truck-drone operation with a truck-only delivery and concluded that a significant reduction in delivery completion time is possible only when employing two or more drones per truck. Chang and Lee [32] improvised their approach by using the classical TSP model to find the shortest truck route through all the cluster stops. Besides, the authors shifted the cluster centroids (truck stops) towards the depot using a computationally intensive non-linear mathematical model to further reduce the delivery completion time.

2.5 Literature analysis and gaps

Employing drones in last-mile deliveries is recently introduced in the literature and practice, and therefore there are many future research directions. The relevant literature is limited since the research and commercial attention is driven by the recent Amazon's announcement in 2013 to use drones in delivering their packages. Based on the problem configurations of surveyed papers in Table 2.5, many research gaps can be identified by creating different combinations of attributes. In this dissertation, the following research gaps are addressed.

- The clustering of customer locations and routing of truck and drones is tackled in the literature using sequential heuristics. Such an approach allows solving realistic problem instances but only achieves sub-optimal solutions as the decisions are interrelated. Therefore, an efficient joint optimization approach should be able to handle realistic instances and produce better solutions.
- All surveyed articles in optimization of hybrid truck-drone systems consider the truck stop to be a customer or a predefined location. This critical restriction can be avoided in practice, which potentially allows better delivery schedules. However, limiting truck stops to customer locations can be a managerial policy. Therefore, two operational policies should be offered for decision makers − allowing the truck stops to be a customer or non-customer locations.
- The number of drones per truck is fixed irrespective of the input data of customer

locations. Such an approach would affect drone utilization and also increase the operational cost. Hence, the number of drone mounted on each truck should be optimized.

- The compromise between minimizing delivery completion time and operational costs was not investigated in the literature of delivery by truck-drone hybrid systems. Commercial companies aim at achieving a balance between customer satisfaction by minimizing delivery time and the operational cost.
- Each drone is assumed to perform a single sortie in each cluster of customer locations. If multiple sorties are allowed, the fleet of drones can be utilized better which results in less operational costs.
- Drones are assumed to be loaded on truck at the depot, and the truck must recover them before returning back to the depot. This results in less utilization of the available fleet of drones and slower delivery operations, as the truck must frequently wait to recover the drones. Using a network of stations for docking of drones may potentially enhance the efficacy of delivery operations.

Chapter 3

Joint optimization of customer location clustering and truck-drone routing with restricted stops

3.1 Problem description

This chapter considers the delivery of goods from a depot (l_0) to N customer locations by using a truck and fleet of identical drones, where $\mathcal{L} = \{l_0, l_1, l_2, \ldots l_N\}$ denotes the set of delivery network vertices. The truck can dock up to G drones, where each drone has a restricted payload capacity. As a result, some customer orders require a truck-only delivery. Thus, a subset of customer locations can be served either by a drone or truck ($\mathcal{L}^D \subset \mathcal{L}$), whereas the remaining locations must be visited only by a truck ($\mathcal{L}^T = \mathcal{L} - \mathcal{L}^D$). The maximum drone travel range (F_l) is negatively associated with the outbound shipping weight, and therefore dependent on the delivery location [9, 58, 69]. Besides, F_l is set to null for orders exceeding drone payload capacity (i.e., $F_l = 0, \forall l \in \mathcal{L}^T$).

The delivery problem necessitates the customer locations to be partitioned into non-overlapping clusters, where each cluster has a focal point (or truck stop) defined by two-dimensional coordinates. To perform the delivery operations, the truck (carrying the drones), traveling at a speed of V^T, must start from the depot, visit every cluster stop exactly once to fulfill the orders within each cluster, and finally return to the same depot. If a cluster has more than one location assigned to it, then multiple drones are dispatched from the truck stop to parallelize shipping operations. The drone travels with a velocity V^D, unloads the package at the customer location in S_l time units, and then returns to the truck waiting at the same focal point.

Theoretically, the number of clusters established ranges from 1 to N. A single cluster indicates that the drones are directly dispatched from, and returned to, the depot, and therefore does not require a truck. On the other hand, a network with N clusters, where each cluster contains exactly one location, corresponds to a truck visiting each delivery location - a classical TSP which does not require any drones

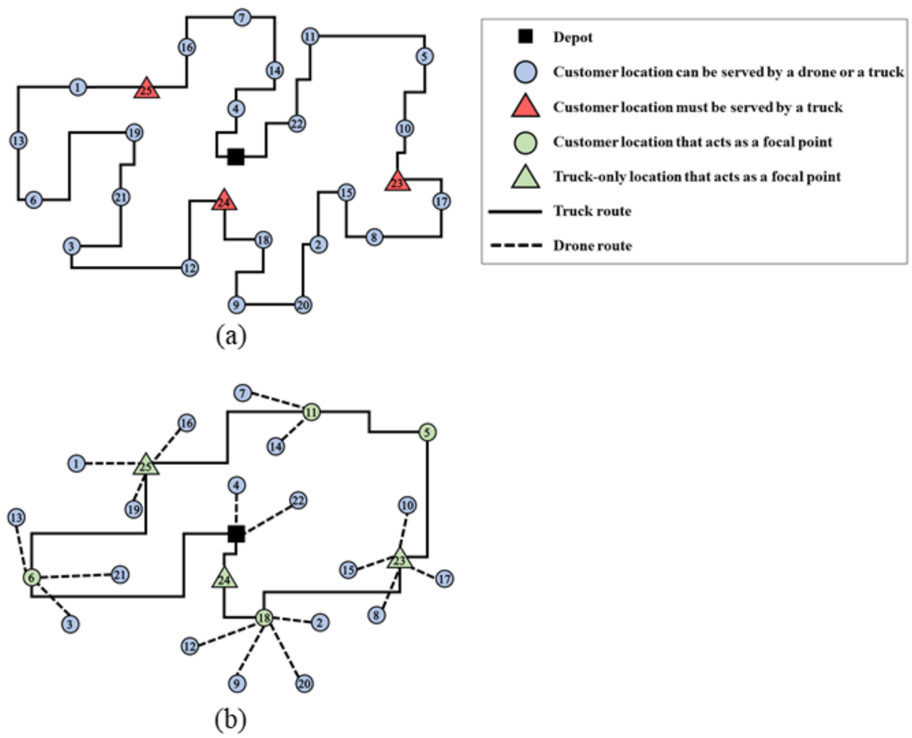

Figure 3.1: Illustrative example of delivery using: (a) truck only; (b) truck with drones dispatched from customer locations

(Figure 3.1(a)). Finally, if the number of clusters is between 1 and N, then it would require both truck and drones to cover all delivery locations (Figures 3.1(b)). In this research, we consider the maximum number of allowable clusters ($\widehat{K}$) as a parameter so that it can be controlled by the decision-maker based on operational needs. However, among the $\widehat{K}$ possible clusters, a cluster $k \in \mathcal{K}$ is formed if and only if it contains at least one delivery location.

Thus, given the set of delivery locations ($\mathcal{L} = \mathcal{L}^D \cup \mathcal{L}^T$), vehicles (a truck and up to G drones) along with their velocities (V^T and V^D) and maximum allowable clusters ($\widehat{K}$), the objectives of our problem are to (i) independently minimize the total operational cost and completion time required to deliver all customer orders either by a drone or truck and (ii) obtain efficient trade-off solutions by simultaneously considering both the objectives. The proposed approach will optimize the objective by concurrently determining the following decisions - (i) total drones to deploy (g) given a capacity restriction of G drones, (ii) number of clusters to establish and their locations, (iii) delivery locations assigned to each truck stop, and (iv) the truck route. In this chapter, we consider the policy of restricting truck stops to be customer locations (Figure 3.1(b)).

To formulate the optimization models for the problem under consideration, the following operating conditions are assumed.

- The drones are fully charged before leaving the truck.
- The drone launch sequence from a truck stop follows the farthest travel distance first policy within a cluster.
- The velocity of drone is independent of its payload.
- Drones carry only one delivery package at a time.
- Each drone makes at most one delivery per cluster.
- Drones travel between a cluster focal point (truck stop) and delivery location based on the Euclidean metric, while truck travels from one focal point to another on a rectilinear metric.
- The truck has sufficient capacity to accommodate all the customer orders. In

the B2C delivery operations, where the vast majority of orders (more than 85%) weigh less than 5 lb, the trucks seldom run out of capacity as indicated by Sacramento et al. [57].

To address the problem presented above, we propose an integer programming (IP) model to jointly optimize delivery locations clustering and truck-drone routing (JOCR) with the objective of minimizing the total cost, namely, fixed cost of drones and travel costs of truck and drones. In this chapter, the cluster focal points are restricted to coincide with a delivery location (min-cost JOCR-R). An overview of the research methodology is provided in Figure 3.2. A collection of input parameters are used to model the JOCR-R policy as an integer program (IP). Besides, two variants are proposed to control the model characteristics - using an alternative objective function and dealing with two conflicting objectives.

3.2 Integer programming model

The min-cost JOCR-R model is formulated using the following notation (indices and sets, parameters, and decision variables).

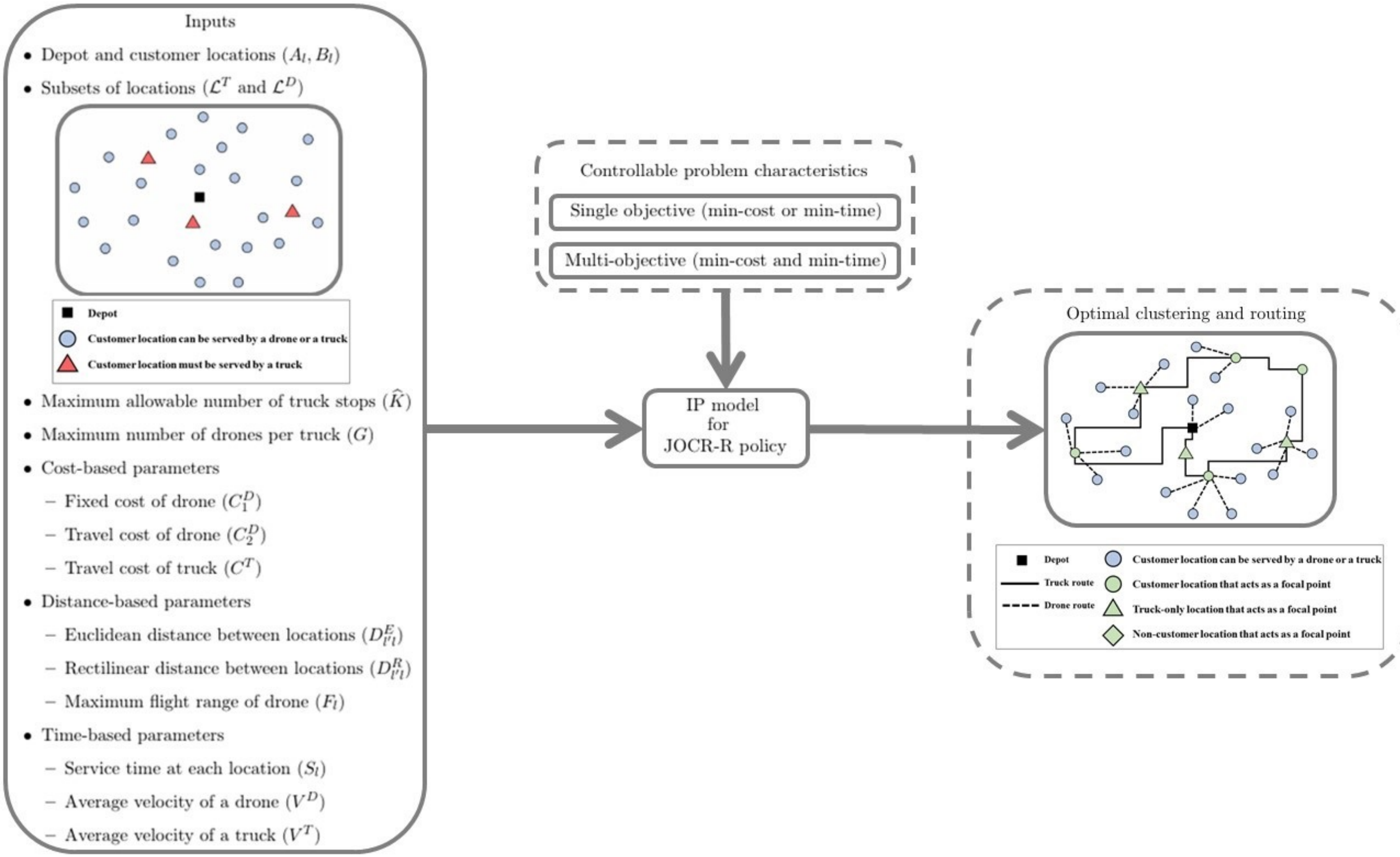

Figure 3.2: Overview of research methodology for the JOCR-R policy

Indices and Sets

$l, l' \in \mathcal{L}$	set of depot and customer (or delivery) locations, $\mathcal{L} = \{l_0, l_1, l_2, l_3, \ldots l_N\}$, where l_0 denotes the depot location
$l \in \mathcal{L}^D$	subset of delivery locations that can be served by drones or truck, $\mathcal{L}^D \subset L$
$l \in \mathcal{L}^T$	subset of delivery locations that must be served only by a truck, $\mathcal{L}^T \subset L$

Parameters

N	number of customer locations
$\widehat{K}$	maximum allowable number of clusters
G	maximum number of drones that can be docked on the truck
C_1^D	fixed cost for employing a drone (in \$/drone)
C_2^D	travel cost of drone (in \$/mile)
C^T	travel cost of truck (in \$/mile)
(A_l, B_l)	coordinates of delivery location $l \in \mathcal{L}$
$D_{l'l}^E$	Euclidean distance (in miles) between delivery locations $l' \in \mathcal{L}$ and $l \in \mathcal{L}$, where $D_{l'l}^E = \sqrt{(A_{l'} - A_l)^2 + (B_{l'} - B_l)^2}$
$D_{l'l}^R$	rectilinear distance (in miles) between delivery locations $l' \in \mathcal{L}$ and $l \in \mathcal{L}$, where $D_{l'l}^R = \lvert A_{l'} - A_l\rvert + \lvert B_{l'} - B_l\rvert$

F_l maximum travel range of a drone (in miles) serving delivery location $l \in \mathcal{L}$

Decision variables

g number of drones carried by the truck

x_{ll} 1 if a delivery location $l \in \mathcal{L}$ acts as a focal point for its cluster, 0 otherwise

$x_{l'l}$ 1 if a delivery location $l' \in \mathcal{L}$ is assigned to a focal point $l \in \mathcal{L}$, 0 otherwise

$y_{l'l}$ 1 if truck travels from cluster focal point $l' \in \mathcal{L}$ to another focal point $l \in \mathcal{L}$, 0 otherwise

u_l order in which cluster focal point $l \in \mathcal{L}$ is visited by the truck

$\mathcal{C}^R_{JOCR}$ total cost for the JOCR-R policy

The JOCR-R model is mathematically formulated as follows.

$$Minimize \quad \mathcal{C}^R_{JOCR} = C^D_1 g + C^D_2 \sum_{l' \in \mathcal{L}} \sum_{l \in \mathcal{L}} x_{l'l} \times (2D^E_{l'l}) + C^T \sum_{l' \in \mathcal{L}} \sum_{l \in \mathcal{L}} y_{l'l} D^R_{l'l} \tag{3.1}$$

$S.t.$

$$\sum_{l \in \mathcal{L}} x_{ll} \leq \widehat{K} \tag{3.2}$$

$$x_{l'l} \leq x_{ll} \qquad \forall l', l \in \mathcal{L} \tag{3.3}$$

$$\sum_{l \in \mathcal{L}} x_{l'l} = 1 \qquad \forall l' \in \mathcal{L} \tag{3.4}$$

$$x_{ll} = 1 \qquad \forall l \in \mathcal{L} \ni l = \{l_0\} \tag{3.5}$$

$$\sum_{l' \in \mathcal{L}} x_{l'l} \leq g + 1 \qquad \forall l \in \mathcal{L} \tag{3.6}$$

$$g \leq G \tag{3.7}$$

$$x_{l'l} D^{E}_{l'l} \leq F_{l'} \qquad \forall l', l \in \mathcal{L} \tag{3.8}$$

$$\sum_{l' \in \mathcal{L}, l' \neq l} y_{l'l} = x_{ll} \qquad \forall l \in \mathcal{L} \tag{3.9}$$

$$\sum_{l \in \mathcal{L}, l \neq l'} y_{l'l} = x_{l'l'} \qquad \forall l' \in \mathcal{L} \tag{3.10}$$

$$u_l - u_{l'} + (\widehat{K} - 1) y_{ll'} + (\widehat{K} - 3) y_{l'l} \leq (\widehat{K} - 2) \qquad \forall l, l' \in \mathcal{L} \backslash \{l_o\}, l' \neq l \tag{3.11}$$

$$x_{l'l}, y_{l'l} \in \{0, 1\} \qquad \forall l', l \in \mathcal{L} \tag{3.12}$$

The objective function (3.1) minimizes the total cost of operating the truck and set of drones for the JOCR-R policy. Constraint (3.2) restricts the total truck stops to be capped by the maximum allowable clusters. Constraint (3.3) allows a delivery location $l' \in \mathcal{L}$ to be assigned to another location $l \in \mathcal{L}$ only if the latter serves as a focal point. Further, constraint (3.4) ensures that every delivery location $l' \in \mathcal{L}$ is assigned to exactly one cluster focal point location $l \in \mathcal{L}$. Constraint (3.5) forces the depot ($l_o \in \mathcal{L}$) to be a focal point so that it can dispatch drones to nearby locations. In addition, this constraint also guarantees a truck visit to the depot (i.e., the final stop). Since a drone is assumed to make a single trip per cluster, constraint (3.6) ensures that the number of drone-supplied locations in each cluster cannot exceed the total drones carried by the truck (g). The truck has a capacity restriction of G drones on its roof, and this condition is guaranteed using constraint (3.7). Constraint (3.8) stipulates that a location served by a drone is assigned to a cluster focal point only if the distance between them is within the travel range. Constraints (3.9) and (3.10) specify the truck route by confining its stops to cluster focal points (i.e., when $x_{ll} = 1$) and limiting the number of truck visits to each focal point to one. In addition, constraint (3.11) eliminates sub-tours to ensure a single trip of the truck to visit all focal points before returning to the depot. Finally, the binary restriction on decision variables $x_{l'l}$ and $y_{l'l}$ are specified by constraint (3.12).

3.2.1 Alternative objective function

While minimizing total operational cost is an important objective from a company's perspective, faster delivery is seen as a key priority to customers. Therefore, the objective of minimizing the delivery completion time is commonly adopted in the literature of last-mile deliveries [9, 33, 48, 63]. The proposed min-cost JOCR-R model can be easily adapted to minimize the delivery completion time. We introduce the following additional notation to represent the alternative objective function.

S_l	service time at a delivery location $l \in \mathcal{L}$
V^D	average velocity (in mph) of the drone
V^T	average velocity (in mph) of the truck
$\mathcal{T}^R_{JOCR}$	delivery completion time for the JOCR-R policy

The min-time JOCR-R model can be formulated by replacing objective function (3.1) with (3.13).

$$Minimize \quad \mathcal{T}^R_{JOCR} \;=\; \sum_{l\in\mathcal{L}} \max_{l'\in\mathcal{L}}(x_{l'l} \times (2D^E_{l'l}/V^D + S_{l'})) \;+\; \sum_{l'\in\mathcal{L}}\sum_{l\in\mathcal{L}} y_{l'l} \times D^R_{l'l}/V^T \tag{3.13}$$

However, objective function (3.13) requires the minimization of the maximum drone flight time within each cluster, thereby leading to a non-linear term. An exact linearization of this minmax term can be achieved by introducing a new variable t_l to represent the maximum drone completion time per cluster, and adding constraint (3.14). Thus, the objective function (3.13) can be rewritten as in function (3.15).

$$t_l \geq x_{l'l} \times (2D^E_{l'l}/V^D + S_{l'}) \qquad \forall l', l \in \mathcal{L} \tag{3.14}$$

$$Minimize \quad \mathcal{T}^R_{JOCR} = \sum_{l \in \mathcal{L}} t_l + \sum_{l' \in \mathcal{L}} \sum_{l \in \mathcal{L}} y_{l'l} \times D^R_{l'l}/V^T \tag{3.15}$$

3.2.2 Dealing with multiple conflicting objectives

Minimizing total cost and delivery completion time are two conflicting objectives for the problem under study. For instance, if minimizing delivery completion time is the sole objective, the JOCR model may utilize all available drones to exploit simultaneous order fulfillment. While this may shorten the overall completion time, it increases the fixed cost of using drones. Thus, given the conflicting nature, it is essential to obtain the set of best trade-off or Pareto optimal solutions, in which an improvement in one objective is not possible without degrading the other. In this section, we use the ϵ-constraint method, one of the best-known techniques to handle multi-objective problems, to obtain the Pareto-optimal solutions [70]. The method is based on optimizing one of the objectives (e.g., minimizing total cost), while the other is bounded from above by an additional constraint. A straightforward application can be the minimization of total cost while maintaining a desired completion time (e.g., shift hours of driver). The following model solves the min-cost JOCR-R model after adding constraint (3.16), where ϵ is the upper bound on the delivery completion time.

$$\begin{aligned} &Minimize \quad \mathcal{C}^R_{JOCR} \\ &S.t. \quad \text{constraints } (3.2 - 3.12) \\ &\qquad \mathcal{T}^R_{JOCR} \leq \epsilon \end{aligned} \tag{3.16}$$

3.3 Computational experiments

In this section, we conduct extensive numerical analysis to evaluate the effectiveness of the proposed JOCR-R models. Numerous test instances of different sizes are generated to evaluate the proposed models and compare their performance with a sequential heuristic method proposed in the literature [32]. However, the sequential heuristic assumes unrestricted truck-drone stops. Therefore, we intend to compare the optimal solution for the restricted case as opposed to near-optimal solutions for the unrestricted case by the heuristic. Also, we illustrate the possibility of obtaining Pareto-optimal solutions (cost and time trade-off) in the case of conflicting objectives. The proposed IP model has been developed using the General Algebraic Modeling System (GAMS 24.5.6) and solved using CPLEX 12.8 optimizer, while the heuristic algorithms were coded and solved using Python programming language. Further, all the computational instances were executed on a computer with Intel Core-i7 @ 3.9 GHz processor and 8 GB RAM.

3.3.1 Experimental setup of test instances

We create test instances by varying the number of customer locations (N) from 20 to 35 in increments of 5. To ensure a robust evaluation, ten replications are generated for each test instance. Besides, each instance requires 10% of the total customers to be served only by a truck ($\xi = 10\%$). Therefore, if N is divisible by 5 and not 10, then 50% of the replications will have $(N + 5) \times \xi$ truck-only locations and the remaining replications will require $(N - 5) \times \xi$ truck-only locations to ensure ξ to be 10% on average across all replications. The coordinates of the delivery locations are considered to be randomly distributed within an area of 30×30 miles^2. Consistent with the parameters in recent literature, the average velocity of both drone (V^D) and truck (V^T) is set as 25 mph [34, 55], and the maximum drone flight range (F_l) is restricted to 10 miles (for all delivery locations that can be served by drones) [57, 71]. The truck can accommodate up to six drones on its roof (i.e., $G = 6$), and every drone carried by truck is assumed to incur a fixed cost (C_1^D) of \$3. On the other hand, the operating cost of drone (C_2^D) is set at \$0.15 per mile, while the truck operating cost (C^T) is set at \$1.25 per mile [64]. The average service time at each location (S_l) is assumed to be one minute.

3.3.2 Evaluation of test instances

To evaluate the test instances and understand the impact of each objective, we solve the JOCR-R model by considering a single objective (i.e., independently minimizing cost and completion time). In addition, to benchmark our JOCR-R model, we consider an approach developed by Chang and Lee [32] for a similar problem. Their method involves the use of a sequential heuristic for clustering and routing decisions, but with unrestricted focal point locations (referred to as SHCR-U in this dissertation). This heuristic includes three sequential steps - (i) partitioning delivery locations that can be served by drones into non-overlapping clusters using a k-means clustering algorithm, (ii) optimizing cluster focal points by shifting the centroids of clusters, obtained from the first step, towards the depot, and (iii) routing the truck via all cluster focal points by using the standard TSP model. Since we force 10% of the customers to be served by a truck, their locations are included in the truck routing step of the heuristic.

Table 3.1 provides a comparison of the average total cost and delivery completion time across 10 replications for the JOCR-R and SHCR-U models. In addition, the table also summarizes the percentage difference in cost ($\mathcal{C}^{gap(R-U)}_{JOCR-SHCR}=\frac{\mathcal{C}^R_{JOCR}-\mathcal{C}^U_{SHCR}}{\mathcal{C}^U_{SHCR}} \times 100\%$) and time ($\mathcal{T}^{gap(R-U)}_{JOCR-SHCR}=\frac{\mathcal{T}^R_{JOCR}-\mathcal{T}^U_{SHCR}}{\mathcal{T}^U_{SHCR}} \times 100\%$) between the joint optimization approach for the restricted truck-drone stops policy and sequential heuristic for the unrestricted policy that is proposed in the literature. The results show that the gap between employing the proposed JOCR-R models and the SHCR-U heuristic is positive for $N = 20$. However, as N increases, the gap decreases for the total cost and reversed (i.e., negative gap) for the delivery completion time. Thus, the efficiency of the proposed JOCR-R models improves versus the SHCR-U heuristic when larger number of customer locations is involved. The JOCR-R model is fast since the running time is always in the order of seconds for all the evaluated instances. Specifically, it takes, on average, up to 163 and 138 seconds for the objective of minimizing cost and delivery completion time, respectively.

Table 3.1: Average minimum total cost and minimum total completion time of test instances

	Total cost			Delivery completion time		
N	$\mathcal{C}^U_{SHCR}$	$\mathcal{C}^R_{JOCR}$	$\mathcal{C}^{gap(R-U)}_{JOCR-SHCR}$	$\mathcal{T}^U_{SHCR}$	$\mathcal{T}^R_{JOCR}$	$\mathcal{T}^{gap(R-U)}_{JOCR-SHCR}$
20	122.2	135.3	10.7%	5.4	5.8	7.4%
25	138.6	143.4	3.5%	6.6	6.5	-1.5%
30	159.4	161.6	1.4%	7.3	7.2	-1.4%
35	169.2	170.5	0.8%	7.8	7.5	-3.8%

3.3.3 Influence of objective functions on clustering and routing decisions

In this section, we visually illustrate the influence of objective functions on clustering and routing decisions. A test case with 25 locations is chosen, as shown in Figure 3.3, in which the last three customer locations must be served only by a truck, while the remaining 22 locations can be served either by a drone or a truck. Figures 3.3(a) and 3.3(b) represent the best way to cluster the locations and route the truck for the min-cost and min-time JOCR-R models, respectively. The min-cost JOCR-R model

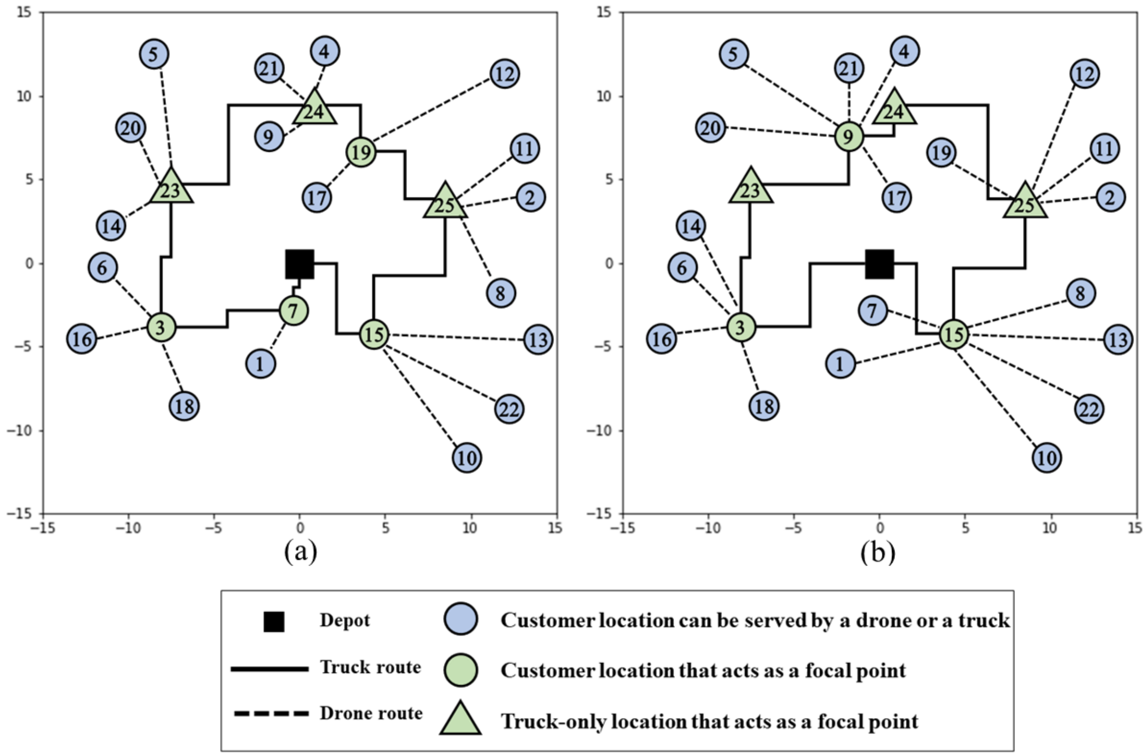

Figure 3.3: Solutions of an example using the proposed models: (a) min-cost JOCR-R ($\mathcal{C}^R_{JOCR}$=127.4 and $\mathcal{T}^R_{JOCR}$=6.7); (b) min-time JOCR-R ($\mathcal{C}^R_{JOCR}$=143.8 and $\mathcal{T}^R_{JOCR}$=5.7)

yields a total cost of \$127.4, and the corresponding delivery completion time for this solution is 6.7 hours. On the other hand, the min-time JOCR-R completes the delivery in 5.7 hours but results in a higher total cost of \$143.8. Thus, solely optimizing one objective may worsen the performance of the other criterion. If the decision-maker is interested in considering more than one objective, then a multi-criteria optimization approach is beneficial to obtain the best trade-off solution.

The objective function plays a key role in deciding the number of drones carried by truck. The solutions of min-time objective (Figure 3.3(b)) utilizes the entire fleet of drones and deploys them only at certain truck stops to achieve faster delivery completion time. Conversely, the solutions obtained using min-cost objective (Figure 3.3(a)) uses fewer drones and dispatches them at every truck stop, and therefore completes the delivery operation with lower cost but longer completion time. In addition, the min-cost objective results in shorter drone travel distance compared to the min-time objective. To achieve this, one of the key strategies adopted by the min-cost model is to serve more locations by a truck compared to the min-time model.

3.3.4 Obtaining Pareto optimal solutions

In this section, we illustrate the ability of our proposed models to handle multiple objectives and obtain Pareto-optimal solutions. We choose the same instance illustrated in Figure 3.3 as it clearly depicts the conflicting nature of the two objectives. For example, the JOCR-R model can achieve the lowest cost ($\mathcal{C}^*$) of \$127.4 for the instance depicted in Figure 3.3, but only at the expense of increasing the delivery duration by an hour from the fastest achievable time ($\mathcal{T}^*$) of 5.7 hours. However, lowering the completion time would increase the total costs. Therefore, we use the ϵ-constraint method to obtain a set of best trade-off solutions. Figure 3.4 presents the best compromise solutions for the chosen example, where the min-cost JOCR-R model is solved repeatedly after incorporating the completion time constraint and changing its limit from 5.7 to 6.7 in increments of 0.25 hours.

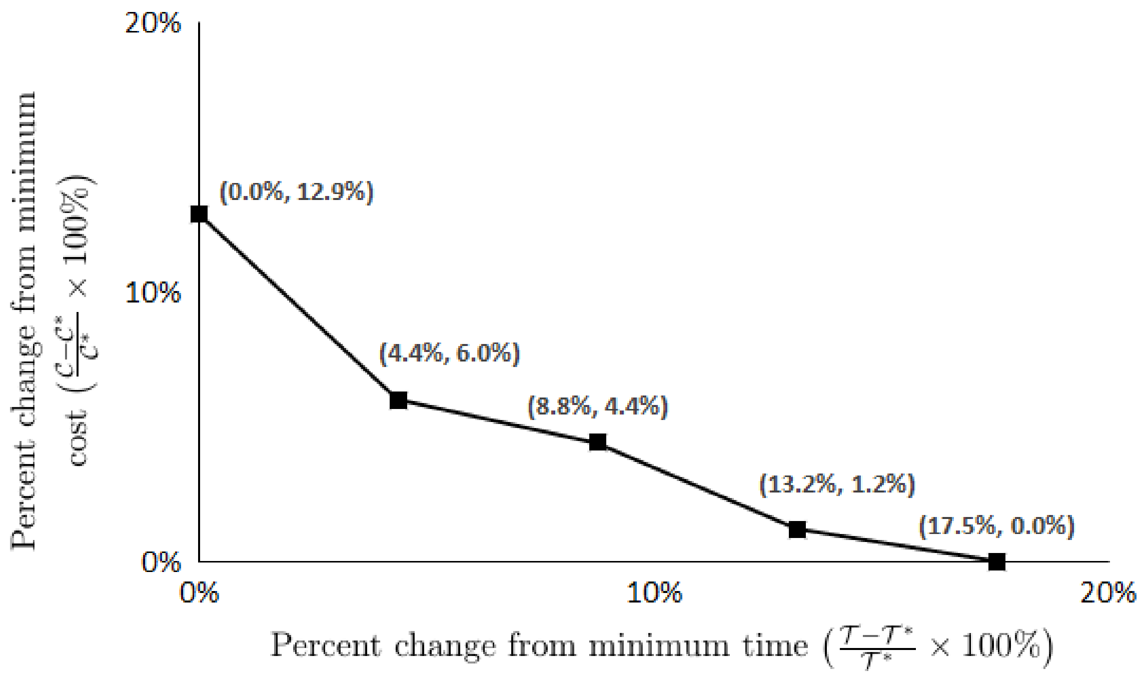

Figure 3.4: Conflicting min-cost and min-time objectives of an example for JOCR-R

The results demonstrate that focusing solely on minimizing total cost increases the corresponding completion time by 17.5% from $\mathcal{T}^*$. On the other hand, focusing only on reducing completion time worsens the total cost by 12.9% when compared to $\mathcal{C}^*$. According to the decision maker's priorities, a single objective can be adopted while sacrificing the potential reduction in the other, or a trade-off can be considered between the two objectives. For example, it can be observed from Figure 3.4 that investing 6.0% more than $\mathcal{C}^*$ would help us complete the delivery 13.1% faster than the worst-case situation or only 4.4% slower than the best-case result of 5.7 hours. Thus, this approach would enable practitioners to deal with two objectives simultaneously and facilitate decision-making.

3.4 Conclusion

In this chapter, the policy of restricting truck-drone stops to coincide with customer locations is considered. Joint optimization models are proposed for the clustering of customer locations and routing of truck and multiple drones. The most two common objectives in the literature are adopted in this chapter − minimizing total cost and delivery completion time. The proposed models can obtain the optimal solutions for the policy under consideration.

This chapter contributes to the literature in the following manner.

- The clustering and routing decisions is jointly optimized rather than using a sequential approach. Since these decisions are interrelated, a multi-phase method may not achieve the best possible outcome.
- The best truck stop location for each cluster is identified instead of choosing it to be the cluster's centroid [31] or shifting the centroid towards the depot [32].
- This research optimizes the drones required per truck, while previous similar work assumed the trucks to carry a fixed set of drones. This information would be useful for planning purposes, especially with the strategic decision on the number of drones to purchase.
- The depot is allowed to be a potential drone dispatch point.
- The customer locations can be visited by both truck and drones in our approach

rather than restricting them only to drones.

- Instead of assuming the drones to be capable of serving all customer locations irrespective of the order weight, certain locations are allowed to be designated as truck-only delivery.
- As drone's travel range is payload-dependent in practice [9, 58, 69], it is incorporated as an attribute in the proposed models instead of assuming constant travel range.
- Two different objective functions (operational cost and completion time) are considered independently and simultaneously instead of optimizing just the delivery completion time.

A comparison is conducted with the best known approach (refereed to as SHCR) in the literature handling a similar problem. Although the SHCR heuristic assumes that truck-drone stops can be anywhere in the delivery area (i.e., not just customer locations), our proposed optimization models could either outperform it or achieve a gap less than 4% for N greater than 25. Moreover, the gap decreases significantly as more customer locations are involved.

Chapter 4

Joint optimization of customer location clustering and truck-drone routing with unrestricted stops

4.1 Problem description

There is a potential to obtain better solutions if the restriction on cluster focal points is relaxed. In contrast to the min-cost JOCR-R model in Chapter 3, a min-cost JOCR-U model is formulated in this chapter to allow cluster focal points to be anywhere in the delivery area (i.e., at a customer or a non-customer location).

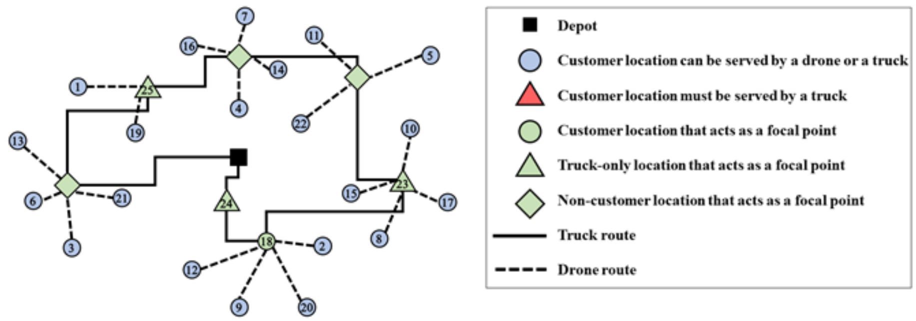

Figure 4.1: Illustrative example of delivery using truck with a fleet of drones can be dispatched from anywhere

Otherwise, both models share the same characteristics. Figure 4.1 illustrates with an example the JOCR-U policy.

An overview of the research methodology to obtain the optimal clustering of customer locations and routing of vehicles is presented in Figure 4.2. The min-cost JOCR-U model does not enforce the focal point to be a customer location. Hence, the problem is initially formulated as a mixed integer nonlinear program (MINLP). Since this kind of models is computationally intractable, it is approximated to a MILP model using some linearization procedures. In addition, novel acceleration techniques are used to reduce the search space and expedite the solution time of the MILP model.

4.2 Nonlinear programming model

The following new notation are considered to formulate the min-cost JOCR-U model.

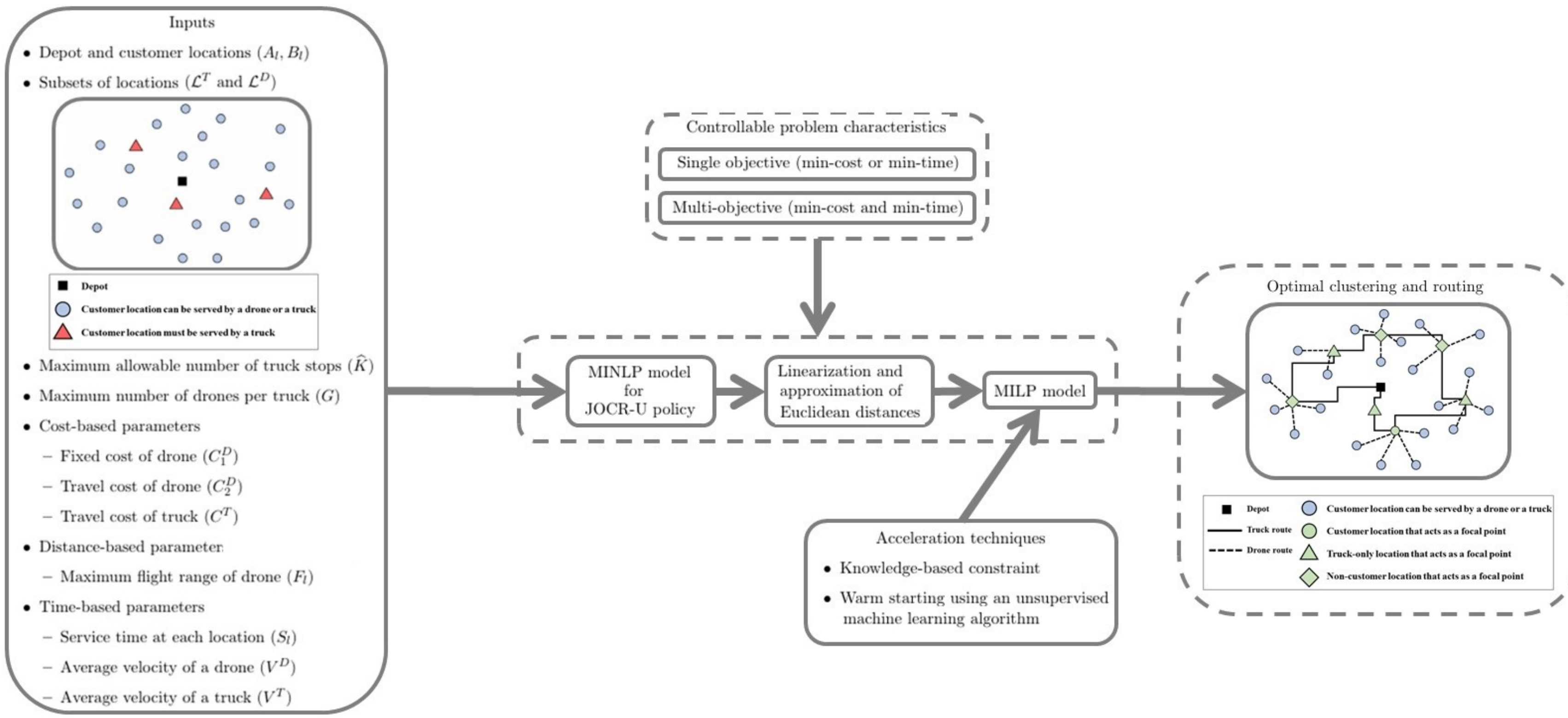

Figure 4.2: Overview of research methodology for the JOCR-U policy

Indices and Set

$k, k' \in \mathcal{K}$ set of possible clusters, $\mathcal{K} = \{k_o, k_1, k_2, k_3, \ldots k_{\widehat{K}}\}$, where k_o denotes the cluster with depot as its focal point

Decision Variables

(a_k, b_k) coordinates of truck stop or cluster focal point $k \in \mathcal{K}$

d^E_{lk} Euclidean distance (in miles) between delivery location $l \in \mathcal{L}$ and cluster focal point $k \in \mathcal{K}$

$d^R_{kk'}$ rectilinear distance (in miles) between cluster focal points $k \in \mathcal{K}$ and $k' \in \mathcal{K}$

x_{lk} 1 if a delivery location $l \in \mathcal{L}$ is assigned to cluster $k \in \mathcal{K}$, 0 otherwise

q_{lk} 1 if a delivery location $l \in \mathcal{L}$ is assigned to cluster $k \in \mathcal{K}$ and served by a drone, 0 otherwise

$y_{kk'}$ 1 if truck travels from cluster focal point $k \in \mathcal{K}$ to another focal point $k' \in \mathcal{K}$, 0 otherwise

$\mathcal{C}^U_{JOCR}$ total cost for the JOCR-U policy

The MINLP model for min-cost JOCR-U is mathematically formulated as follows.

$$Minimize \quad \mathcal{C}^U_{JOCR} = C^D_1 g + C^D_2 \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} x_{lk} \times (2d^E_{lk}) + C^T \sum_{k \in \mathcal{K}} \sum_{k' \in \mathcal{K}} y_{kk'} d^R_{kk'} \quad (4.1)$$

S.t.

$$\sum_{k\in\mathcal{K}} x_{lk} = 1 \qquad \forall l \in \mathcal{L} \tag{4.2}$$

$$d^E_{lk} = \sqrt{(A_l - a_k)^2 + (B_l - b_k)^2} \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.3}$$

$$x_{lk}\, d^E_{lk} \leq F_l\, q_{lk} \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.4}$$

$$\sum_{l\in\mathcal{L}} q_{lk} \leq g \qquad \forall k \in \mathcal{K} \tag{4.5}$$

$$g \leq G \tag{4.6}$$

$$\sum_{k'\in\mathcal{K}, k\neq k'} y_{kk'} = 1 \qquad \forall k \in \mathcal{K} \tag{4.7}$$

$$\sum_{k\in\mathcal{K}, k\neq k'} y_{kk'} = 1 \qquad \forall k' \in \mathcal{K} \tag{4.8}$$

$$u_k - u_{k'} + (\widehat{K} - 1)y_{kk'} + (\widehat{K} - 3)y_{k'k} \leq (\widehat{K} - 2) \qquad \forall k, k' \in \mathcal{K}\backslash\{k_o\}, k \neq k' \tag{4.9}$$

$$d^R_{kk'} = |a_k - a_{k'}| + |b_k - b_{k'}| \qquad \forall k, k' \in \mathcal{K} \tag{4.10}$$

$$(a_k, b_k) = (A_{l_o}, B_{l_o}) \qquad \forall k \in \mathcal{K} \ni k = \{k_o\} \tag{4.11}$$

$$x_{lk} \in \{0, 1\} \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.12}$$

$$y_{kk'} \in \{0, 1\} \qquad \forall k, k' \in \mathcal{K} \tag{4.13}$$

The objective function (4.1) minimizes the total cost for the JOCR-U policy. Constraint (4.2) ensures that every delivery location $l \in \mathcal{L}$ is assigned to one and only one cluster focal point $k \in \mathcal{K}$. Equation (4.3) determines the drone travel distance between a delivery location $l \in \mathcal{L}$ and its focal point $k \in \mathcal{K}$. Further, constraint (4.4) ensures this flight distance to be within the maximum drone flight range for location $l \in \mathcal{L}$. Also, constraint (4.4) forces delivery location $l \in \mathcal{L}$ to overlap with its cluster focal point $k \in \mathcal{K}$ (i.e., $d^E_{lk} = 0$) only when that location is not served by a drone ($q_{lk} = 0$). Besides guaranteeing truck visits to locations $l \in \mathcal{L}^T$, our formulation also allows them to act as cluster focal points for drone dispatch to potential nearby locations (based on constraints (4.3) and (4.4)). Constraint (4.5) limits the drone-served locations per cluster to the total drones carried by the truck (g), which is, in turn, governed by the capacity constraint (4.6).

With respect to truck routing, constraints (4.7) and (4.8) stipulate that each cluster focal point $k \in \mathcal{K}$ has exactly one inbound and one outbound visit by the truck. Constraint (4.9) is for sub-tour elimination, which ensures that the truck returns to the depot only after visiting all the truck stops. Equation (4.10) computes the rectilinear travel distance for a truck between two cluster focal points k and k'. In particular, the coordinates of cluster focal points (truck stops/drone dispatch points) are determined concurrently by equations (4.3) and (4.10), which yield the optimum combination of distances to be traveled by drones and truck, such that it leads to the minimum cost. Equation (4.11) assigns the cluster focal point $\{k_o\} \in \mathcal{K}$ to the depot with coordinates (A_{l_o}, B_{l_o}) to ensure a truck visit to the depot, in addition to allowing delivery locations to be assigned to the depot like any other cluster focal point. Finally, the binary restrictions on the decision variables are specified using constraints (4.12) and (4.13). Thus far, we have a MINLP model that is computationally complex to solve as the objective function (4.1) and constraints (4.3), (4.4), and (4.10) are non-linear. To achieve optimal solutions in a reasonable time, we linearize the model later in this chapter.

4.2.1 Alternative objective function

The objective of minimizing delivery completion time can be adopted by replacing function (4.1) with (4.14), where $\mathcal{T}^{U}_{JOCR}$ is the delivery completion time for the JOCR-U policy. The new adapted model is hereafter called min-time JOCR-U model.

$$Minimize \quad \mathcal{T}^{U}_{JOCR} = \sum_{k\in\mathcal{K}} \max_{l\in\mathcal{L}}(x_{lk} \times (2d^{E}_{lk}/V^{D} + S_l)) + \sum_{k\in\mathcal{K}} \sum_{k'\in\mathcal{K}} y_{kk'} \times d^{R}_{kk'}/V^{T} \tag{4.14}$$

The minimization of the maximum drone flight time within each cluster in the first term of objective function (4.14) can be linearized as follows. First, a new variable t_k is introduced to denote the maximum drone completion time per cluster. Then, constraint (4.15) is added to quantify that duration. Therefore, the objective function (4.14) can be rewritten as in function (4.16). However, the multiplication of decision variables x_{lk} and d^{E}_{lk} in constraint (4.15) and decision variables $y_{kk'}$ and $d^{R}_{kk'}$ in objective function (4.16) still lead to a non-linear model. This is handled latter in this chapter while linearizing the min-cost JOCR-U model, as these multiplications are existing in objective function (4.1) as well.

$$t_k \geq x_{lk} \times (2d^E_{lk}/V^D + S_l) \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.15}$$

$$Minimize \quad \mathcal{T}^U_{JOCR} = \sum_{k\in\mathcal{K}} t_k + \sum_{k\in\mathcal{K}} \sum_{k'\in\mathcal{K}} y_{kk'} \times d^R_{kk'}/V^T \tag{4.16}$$

4.2.2 Dealing with multiple conflicting objectives

The ϵ-constraint method can be used to obtain the Pareto-optimal solutions of minimizing total cost and delivery completion time [70]. In this section, the min-cost JOCR-U model is reformulated to incorporate the objective of delivery completion time as in the following model, where ϵ is the upper bound on the delivery completion time.

$$Minimize \qquad \mathcal{C}^U_{JOCR}$$

$$S.t. \qquad \text{constraints } (4.2 - 4.13)$$

$$\mathcal{T}^U_{JOCR} \leq \epsilon \tag{4.17}$$

Similarly, the min-time JOCR-U model can also be used to deal with the conflicting objectives by determining an upper bound for the total cost. It is to be noted that every solution obtained using the ϵ-constraint method is a Pareto-optimal. Thus, the model can be solved for diverse values of ϵ to provide the practitioner with a broad set of Pareto-optimal solutions.

4.3 Model linearization and acceleration

In this section, we linearize the objective function and non-linear constraints of the proposed MINLP model. The following additional notation are introduced to avoid non-linearity and reformulate the problem under study as a MILP model.

Parameters

P	number of planes for finding approximate Euclidean distance
θ	rotation angle between two consecutive planes
M	large positive number

Decision variables

$d_{lk}^{x\text{-}E}$	horizontal axis (or x-axis) length of Euclidean distance between delivery location $l \in \mathcal{L}$ and cluster focal point $k \in \mathcal{K}$ (i.e., $d_{lk}^{x\text{-}E} = \lvert A_l - a_k \rvert$)
$d_{lk}^{y\text{-}E}$	vertical axis (or y-axis) length of Euclidean distance between delivery location $l \in \mathcal{L}$ and cluster focal point $k \in \mathcal{K}$ (i.e., $d_{lk}^{y\text{-}E} = \lvert B_l - b_k \rvert$)

$d_{kk'}^{x\_R}$ horizontal axis (or x-axis) length of rectilinear distance between cluster focal points k and $k' \in \mathcal{K}$ (i.e., $d_{kk'}^{x\_R} = |a_k - a_{k'}|$)

$d_{kk'}^{y\_R}$ vertical axis (or y-axis) length of rectilinear distance between cluster focal points k and $k' \in \mathcal{K}$ (i.e., $d_{kk'}^{y\_R} = |b_k - b_{k'}|$)

There are many endeavors in the literature to find a good linear approximation to the Euclidean distance [72, 73]. In this research, we adopt the recent linearization technique developed by Xie et al. [74] to linearize constraint (4.3) as it has shown to produce a highly accurate approximation of the Euclidean distance metric (i.e., an error of less than -0.01% can be achieved). Their technique computes the Euclidean distance between two locations using x-axis and y-axis components and slope angle between the two locations. While the x-axis and y-axis components can be depicted using linear constraints in a mathematical model, the slope angle is challenging to represent linearly. Therefore, different values of slope angles, between 0 and $\frac{\pi}{2}$, are examined. Each of the examined slope angles represents one of the predefined P planes, while a fixed rotation angle (θ) is imposed between two consecutive planes. In this research, we choose six planes ($P = 6$) and a rotation angle (θ) of 0.2831 radian to achieve $< 1\%$ error in approximating Euclidean distance [74].

Thus, the non-linearity in equation (4.3) can be avoided by adopting a two-step reformulation. First, the horizontal and vertical axis components of the Euclidean distance is represented with a set of equivalent linear constraints (4.18)$-$(4.21). Then, the Euclidean distance is redefined in terms of the x-axis and y-axis components in constraint (4.22).

$$d_{lk}^{x\_E} \geq A_l - a_k \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.18}$$

$$d_{lk}^{x\_E} \geq a_k - A_l \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.19}$$

$$d_{lk}^{y\_E} \geq B_l - b_k \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.20}$$

$$d_{lk}^{y\_E} \geq b_k - B_l \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.21}$$

$$d_{lk}^{E} \geq d_{lk}^{x\_E} cos(p\theta) + d_{lk}^{y\_E} sin(p\theta) \qquad \forall l \in \mathcal{L}, k \in \mathcal{K}, p = 0, 1, 2, \ldots P-1 \tag{4.22}$$

Likewise, the equivalent linearization of equation (4.10) is achieved by linear constraints (4.23)−(4.26).

$$d_{kk'}^{x\_R} \geq a_k - a_{k'} \qquad \forall k, k' \in \mathcal{K} \tag{4.23}$$

$$d_{kk'}^{x\_R} \geq a_{k'} - a_k \qquad \forall k, k' \in \mathcal{K} \tag{4.24}$$

$$d_{kk'}^{y\_R} \geq b_k - b_{k'} \qquad \forall k, k' \in \mathcal{K} \tag{4.25}$$

$$d_{kk'}^{y\_R} \geq b_{k'} - b_k \qquad \forall k, k' \in \mathcal{K} \tag{4.26}$$

Constraint (4.4) is also non-linear as it involves the multiplication of two decision variables ($x_{lk} \times d_{lk}^{D}$). The purpose of constraint (4.4) is to ensure that the left hand side becomes zero if the delivery location $l \in \mathcal{L}$ is not assigned to the focal point $k \in \mathcal{K}$, and exactly equal to the distance between them if it is assigned. The same characteristics can be achieved without non-linearity by modifying constraint (4.22) as constraint (4.27). As a result, non-linear constraint (4.4) can now be rewritten as linear constraint (4.28). Similarly, the second term of the objective function can be rewritten as $(C_2^D \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} 2d_{lk}^E)$.

$$d_{lk}^E \geq d_{lk}^{x\_E} cos(p\theta) + d_{lk}^{y\_E} sin(p\theta) - M(1 - x_{lk}) \quad \forall l \in \mathcal{L}, k \in \mathcal{K}, p = 0, 1, 2, \dots P - 1 \tag{4.27}$$

$$d_{lk}^E \leq F_l \, q_{lk} \qquad \forall l \in \mathcal{L}, k \in \mathcal{K} \tag{4.28}$$

The third term of the objective function (4.1) is analogous to the objective function of the standard TSP model. However, in our approach, both $y_{kk'}$ and $d_{kk'}^R$ are decision variables. The non-linearity due to the multiplication of these two variables could be circumvented by fixing the binary variable $y_{kk'}$. Since the locations of cluster focal points (i.e., their coordinates) are not determined yet, any feasible truck route sequence could be chosen without affecting the quality of results. For example, equation (4.29) can be used to provide a feasible truck route, which gives a route: $k_o \rightarrow k_1 \rightarrow k_2 \rightarrow \cdots \rightarrow k_{|\mathcal{K}|} \rightarrow k_o$. In addition to overcoming non-linearity, using a fixed sequence of the truck route as an input leads to the exclusion of constraints

(4.7), (4.8), and (4.9), thereby decreasing the problem complexity.

$$y_{kk'} = 1 \qquad \forall k, k' \in \mathcal{K}\colon (k = k' - 1) \vee (k = |\mathcal{K}| \wedge k' = 0) \tag{4.29}$$

4.3.1 Accelerating solution time for min-cost and min-time JOCR-U

Allowing focal points to be anywhere on the delivery plane increases the search space substantially. Therefore, large problem instances may become computationally intractable. In this section, we present two problem-specific strategies to accelerate the solution time for JOCR-U.

4.3.1.1 Adding a knowledge-based constraint (KBC) to reduce search space

The drone travel range, coordinates of all the delivery locations, and the distances between them are known apriori. Therefore, based on this information, it is possible to establish the area (or all feasible coordinates) in which a potential cluster focal point will be located for delivery location l. Further, if the area of the prospective cluster focal points that can serve location l does not overlap with the area of potential focal points that can serve location l', then l and l' cannot be assigned to the same cluster. We can inject this prior knowledge in the MILP model and accelerate the solution time by adding constraint (4.30), where parameter $Q_{ll'}$ is 1 if delivery locations l and

l' can be served together within the same cluster and 0 otherwise.

$$(1 - Q_{ll'})(x_{lk} + x_{l'k}) \leq 1 \qquad \forall l, l' \in \mathcal{L}, k \in \mathcal{K}, l < l' \tag{4.30}$$

Note that the value of the binary parameter $Q_{ll'}$ should be set to one if the condition in inequality (4.31) is satisfied and 0 otherwise, where $D^E_{ll'} = \sqrt{(A_l - A_{l'})^2 + (B_l - B_{l'})^2}$.

$$F_l + F_{l'} \geq D^E_{ll'} \qquad \forall l, l' \in \mathcal{L} \tag{4.31}$$

4.3.1.2 Warm starting (WS) using a heuristic algorithm

Another strategy to accelerate the MILP solution time is by using a warm-start (WS) procedure, where we initiate the optimization problem with a good feasible solution. Prior research that considered a similar problem used an unsupervised machine learning algorithm, iterative k-means clustering, to effectively cluster the delivery locations and obtain the focal point of each cluster [31, 32]. However, unlike our study, their heuristics assumed all the delivery locations to be accessible by drones. Therefore, we propose a heuristic that modifies the clusters to accommodate all types of customer locations and then uses it as a feasible solution to warm-start the MILP model. The proposed heuristic is detailed in Algorithm 1 and adopts a three-step procedure. First, the locations that can be served by drones ($\mathcal{L}^D$) are clustered, and their focal points are obtained by using the iterative k-means clustering. In the second phase (lines 3-23 in Algorithm 1), the algorithm aims to move each cluster focal point to the nearest delivery location that is served only by a truck ($\mathcal{L}^T$), while taking into account the flight range restrictions of drones. Finally, the optimal truck route via the modified focal points is obtained using the standard TSP model.

Algorithm 1 Unsupervised Machine Learning Heuristic for Warm-starting

1: **Inputs:** set of delivery locations that can be served by a drone/truck ($\mathcal{L}^D$), locations that must be served by a truck ($\mathcal{L}^T$), delivery coordinates (A_l, B_l) of all locations, drone flight range (F_l, $\forall l \in \mathcal{L}$), total drones can be carried by a truck (G)
2: Obtain the cluster focal points for locations that can be served by truck or drone and the locations assigned to each cluster focal point using the iterative k-means algorithm
3: **for** each location $l \in \mathcal{L}^{\mathcal{T}}$ **do**
4:     **for** each cluster focal point $k \in \mathcal{K}$ **do**
5:         Compute the distance between cluster focal point k and location l (δ_{lk})
$$\delta_{lk} = \sqrt{(A_l - a_k)^2 + (B_l - b_k)^2}$$
6:     **end for**
7: **end for**
8: Establish a set consisting of distances between every cluster focal point and truck-only location
9: $$\mathcal{D} = \{\delta_{lk} \mid l \in \mathcal{L}^{\mathcal{T}},\ k \in \mathcal{K}\}$$
10: **while** $\mathcal{D} \neq \emptyset$ **do**
11:     Determine $(\hat{l}, \hat{k}) = \underset{l \in \mathcal{L}^{\mathcal{T}}, k \in \mathcal{K}}{\operatorname{argmin}} \{\delta_{lk}\}$
12:     **for** each location $l \in \mathcal{L}^{\mathcal{D}}$ **do**
13:         **if** location l is assigned to cluster $\hat{k}$ (i.e., $x_{l\hat{k}} = 1$) **then**
14:             Compute distance between $\hat{l}$ and location l ($\Delta_{\hat{l},l}$)
15: $$\Delta_{\hat{l},l} = \sqrt{(A_{\hat{l}} - A_l)^2 + (B_{\hat{l}} - B_l)^2}$$
16:         **end if**
17:     **end for**
18:     **if** $\Delta_{\hat{l},l} \leq R_l\ \{\forall\ l | l \in \mathcal{L}^{\mathcal{D}}, x_{l,\hat{k}} = 1\}$ **then**
19:         Set coordinates of focal point $\hat{k}$ as the coordinates $\hat{l}$, $(a_{\hat{k}}, b_{\hat{k}}) \leftarrow (A_{\hat{l}}, B_{\hat{l}})$
20:         Remove $\hat{l}$ from being considered as cluster focal point, $\mathcal{L}^T \leftarrow \mathcal{L}^T \backslash \{\hat{l}\}$
21:     **end if**
22:     Remove δ_{lk} from set $\mathcal{D}$ (i.e., $\mathcal{D} \leftarrow \mathcal{D} \backslash \{\delta_{lk}\}$)
23: **end while**
24: Compute the rectilinear distance among all the cluster focal points established
25: Obtain the optimal truck route covering all the focal points by using the rectilinear distance as inputs and solving the standard TSP model
26: **return** focal points of clusters (a_k, b_k), optimal truck route between them, and assignments of delivery locations to clusters ($x_{lk}\ \forall l \in \mathcal{L}, k \in \mathcal{K}$)

4.4 Computational experiments

The same set of instances and parameter setting that were used for the JOCR-R models (in Chapter 3) are employed here to evaluate the JOCR-U models. Further, a two-fold benchmarking approach is adopted in this section to assess the JOCR-U models. First, the results of the JOCR-R models are utilized in this chapter to compare the unrestricted truck-drone stops policy versus the restricted stops policy. Second, the performance of JOCR-U models is benchmarked against the SHCR heuristic in the literature [32]. The procedure of obtaining Pareto-optimal solutions (cost and time trade-off) in the case of conflicting objectives is illustrated. Finally, a sensitivity analysis is conducted to ascertain the influence of critical parameters on the performance measures.

4.4.1 Evaluation of test instances

A comparison of the average total cost and delivery completion time across 10 replications for the JOCR-R, JOCR-U and SHCR-U models is reported in Table 4.1. In addition, the table also summarizes the percentage difference in cost ($\mathcal{C}_{JOCR}^{gap(U-R)} = \frac{\mathcal{C}_{JOCR}^{U} - \mathcal{C}_{JOCR}^{R}}{\mathcal{C}_{JOCR}^{R}} \times 100\%$) and time ($\mathcal{T}_{JOCR}^{gap(U-R)} = \frac{\mathcal{T}_{JOCR}^{U} - \mathcal{T}_{JOCR}^{R}}{\mathcal{T}_{JOCR}^{R}} \times 100\%$) between the two focal point policies - restricted and unrestricted. Likewise, it also gives the percentage change in cost ($\mathcal{C}_{JOCR-SHCR}^{gap(U)} = \frac{\mathcal{C}_{JOCR}^{U} - \mathcal{C}_{SHCR}^{U}}{\mathcal{C}_{SHCR}^{U}} \times 100\%$) and time ($\mathcal{T}_{JOCR-SHCR}^{gap(U)} =$

$\frac{\mathcal{T}^U_{JOCR}-\mathcal{T}^U_{SHCR}}{\mathcal{T}^U_{SHCR}} \times 100\%$) between the joint optimization approach and sequential heuristic proposed in the literature.

Table 4.1: Average minimum total cost and minimum total completion time of test instances

	Total cost					Delivery completion time				
N	$\mathcal{C}^R_{JOCR}$	$\mathcal{C}^U_{SHCR}$	$\mathcal{C}^U_{JOCR}$	$\mathcal{C}^{gap(U)}_{JOCR-SHCR}$	$\mathcal{C}^{gap(U-R)}_{JOCR}$	$\mathcal{T}^R_{JOCR}$	$\mathcal{T}^U_{SHCR}$	$\mathcal{T}^U_{JOCR}$	$\mathcal{T}^{gap(U)}_{JOCR-SHCR}$	$\mathcal{T}^{gap(U-R)}_{JOCR}$
20	135.3	122.2	106.2	-13.1%	-21.5%	5.8	5.4	4.6	-14.8%	-20.7%
25	143.4	138.6	120.0	-13.4%	-16.3%	6.5	6.6	5.5	-15.4%	-15.4%
30	161.6	159.4	142.7	-10.5%	-11.7%	7.2	7.3	6.3	-12.5%	-12.5%
35	170.5	169.2	151.0	-10.8%	-11.4%	7.5	7.8	6.6	-14.3%	-12.0%

It is evident from Table 4.1 that the joint optimization approach outperforms the sequential heuristic method for all the test instances evaluated. The proposed JOCR-U model achieves an average cost reduction of 10%-13% and time savings of 12%-15% over the SHCR-U approach. Besides, allowing unrestricted focal points would achieve substantial savings with respect to cost ($\mathcal{C}^{gap}_{R-U}$) and time ($\mathcal{T}^{gap}_{R-U}$) when compared to restricted truck stop locations. Thus, the results demonstrate the dominance of the JOCR-U model, thereby highlighting the importance of adopting unrestricted focal points and joint optimization approach. For the computational performance, up to 1% gap to best known solutions can be obtained in less than 200 seconds for $N \leq 30$, while up to 679 seconds are required on average for $N = 35$.

Moreover, the benefit of adopting the unrestricted focal point policy is higher when the number of delivery locations per trip is fewer. This may be because of low customer density for smaller N, where the locations are likely to be spatially dispersed, which, in turn, makes a customer location an unattractive (or inefficient) focal point. Conversely, as N increases, the customer density also escalates and subsequently improves the likelihood of coinciding the best focal point coordinate with a delivery location. To validate our findings, the impact of customer density is examined by solving the min-cost and min-time models for four different densities, which are generated by varying the delivery area and fixing N to 20. Table 4.2 presents the average values of minimum cost and time across 10 replications for different customer densities and focal point policies. It is evident that both $\mathcal{C}^{gap}_{R-U}$ and $\mathcal{T}^{gap}_{R-U}$ decreases with increasing customer density, and is less than 5% when the delivery area is 15×15 miles^2 with 20 customer locations. Thus, as the spatial density of delivery locations increases, the solutions obtained by both JOCR models become more comparable.

Table 4.2: Average cost and time gaps between focal point polices for N=20 and different delivery areas

	Total cost			Delivery completion time		
Delivery area	$\mathcal{C}^R_{JOCR}$	$\mathcal{C}^U_{JOCR}$	$\mathcal{C}^{gap(U-R)}_{JOCR}$	$\mathcal{T}^R_{JOCR}$	$\mathcal{T}^U_{JOCR}$	$\mathcal{T}^{gap(U-R)}_{JOCR}$
30×30	135.3	106.2	-21.5%	5.8	4.6	-20.7%
25×25	110.7	93.8	-15.3%	4.8	4.1	-14.6%
20×20	85.9	77.8	-9.4%	3.8	3.4	-10.5%
15×15	63.9	61.3	-4.1%	2.7	2.6	-3.7%

4.4.2 Influence of objective functions on clustering and routing decisions

The same instance that was shown in Section 3.3.3 is considered here again but for the JOCR-U models. The test case includes 25 locations, as shown in Figure 4.3, in which the last three customer locations must be served only by a truck, while the remaining 22 locations can be served either by a drone or a truck. Figures 4.3(a) and

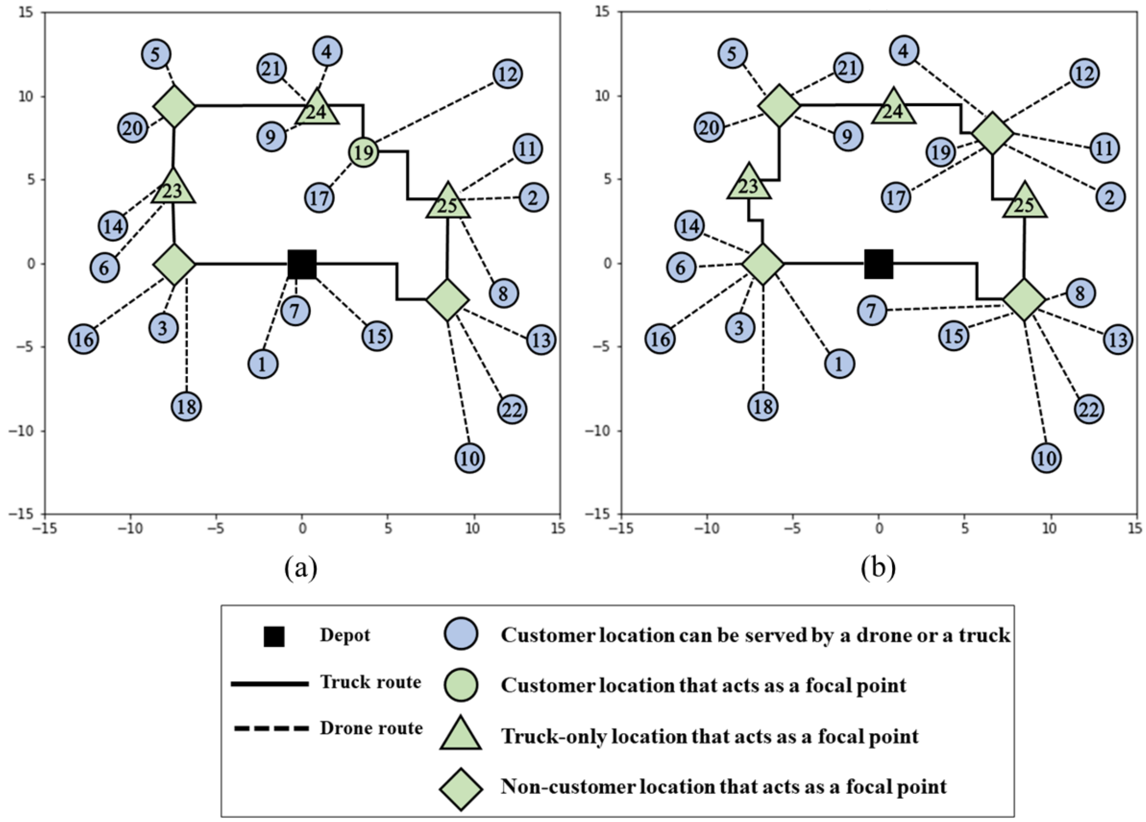

Figure 4.3: Solutions of an example using the proposed models: (a) min-cost JOCR-U ($\mathcal{C}_{JOCR}^{U}$=108.6 and $\mathcal{T}_{JOCR}^{U}$=6.9); (b) min-time JOCR-U ($\mathcal{C}_{JOCR}^{U}$=127.1 and $\mathcal{T}_{JOCR}^{U}$=4.9)

4.3(b) illustrate the best plan for minimizing total cost and completion time in the JOCR-U models, respectively. The min-cost JOCR-U model yields a total cost of \$108.6, and the corresponding delivery completion time for this solution is 6.9 hours. On the other hand, the min-time JOCR-U completes the delivery in 4.9 hours but results in a higher total cost of \$127.1. In other words, the min-cost solution results in lower cost and higher completion time when compared to the min-time solution, and vice versa. Thus, the best trade-off between the two objective may be of interest of practitioners.

It can be noticed that the solution of min-time objective (Figure 4.3(b)) utilizes the entire fleet of drones and deploys all of them in three out of four drone launching locations. Hence, a faster delivery completion time can be achieved. Conversely, the solution of min-cost objective (Figure 4.3(a)) uses only half number of available drones (i.e., three out of the six available drones), and therefore completes the delivery operation with lower cost but longer completion time.

In addition, the min-cost objective results in shorter drone travel distance compared to the min-time objective. Besides routing, the focal point location also plays a key role in minimizing the drone travel distance. For example, consider locations 12 and 17 in Figures 4.3(a) and 4.3(b). The minimum drone distance for the min-cost model is achieved by making these locations almost along a straight line with their focal point (location 19). On the other hand, the min-time model (in Figure 4.3 (b)) minimizes the maximum drone flight distance by locating a focal point to be approximately halfway between these two locations. Finally, both the objectives aim to lower the truck travel distance, and this is also influenced by the location of truck stops. For instance, in Figure 4.3 (a), the non-customer focal points to the north and the south of location 23 are optimized to share the same truck path, as moving them any further towards the left would increase the distance traveled by truck.

Aside from these differences, Figure 4.3 also highlights some of the unique features of the proposed models. For example, the depot is also used as a focal point if it is beneficial (e.g., locations 1, 7, and 15 in Figure 4.3 (a)). Furthermore, the JOCR-U models use the customer location as focal points when appropriate and choose a non-customer location only when it improves the objective value. All of these demonstrate the holistic approach of the proposed JOCR models to achieve the best clustering and routing decisions and also provide insights for the development of novel heuristics in future research.

4.4.3 Obtaining Pareto optimal solutions

The illustrated instance in Figure 4.3 is used to exhibit the procedure of handling multiple objectives and obtaining Pareto-optimal solutions. The ϵ-constraint method is employed to achieve a set of best trade-off solutions. Figure 4.4 presents the best compromise solutions for the chosen instance, where the min-cost JOCR-U model is solved repeatedly after incorporating the completion time constraint and changing its limit from 4.9 to 6.9 in increments of 0.25 hours.

Figure 4.4 shows that focusing solely on minimizing total cost increases the corresponding completion time by 41% from $\mathcal{T}^*$. In contrast, focusing only on reducing completion time worsens the total cost by 17% when compared to $\mathcal{C}^*$. Hence, a trade-off can be considered between the two objectives based on the decision maker's priorities. For example, it can be observed from Figure 4.4 that investing 9% more than $\mathcal{C}^*$ would help us complete the delivery 36% faster than the worst-case situation

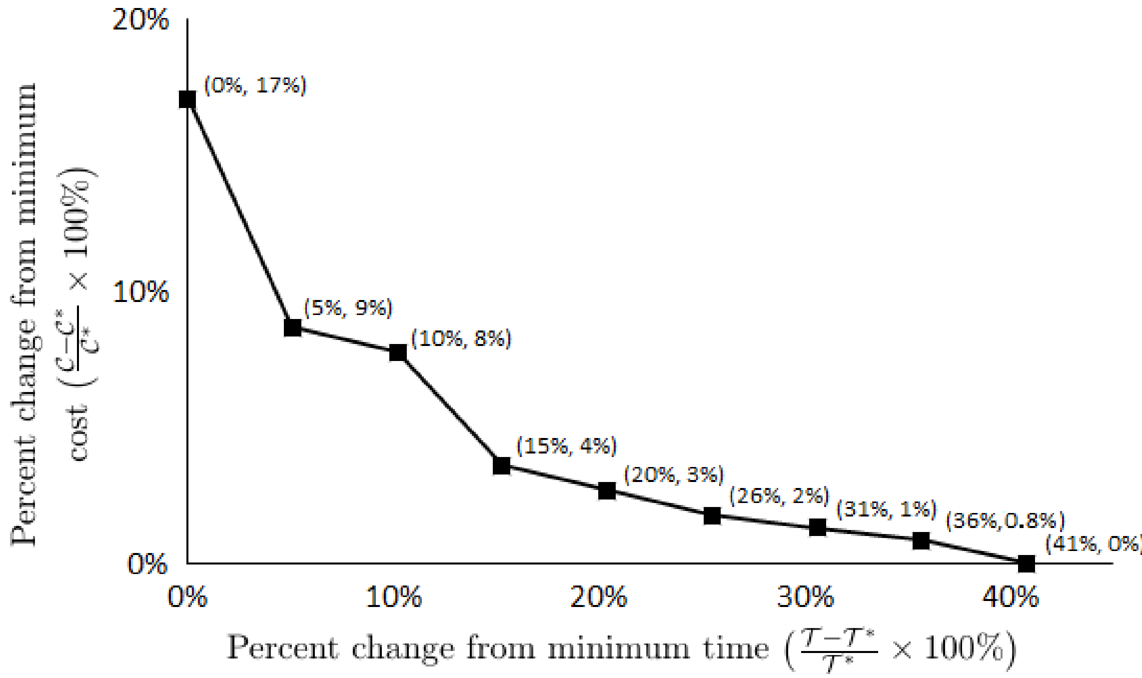

Figure 4.4: Conflicting min-cost and min-time objectives of an example for JOCR-R and JOCR-U

or only 5% slower than the best-case result of 4.9 hours.

4.4.4 Sensitivity analysis

In order to test the performance of the proposed JOCR models beyond the existing instances and identify the impact/sensitivity of the key parameters, we evaluate different cases by exploring additional scenarios. The analysis of previous test instances indicates that both min-cost and min-time objectives are substantially affected by the number of customer locations (N). In addition, the proportion of truck-only locations (ξ) and drone characteristics, such as drone velocity (V^D) and flight range (F_l), can also affect the system performance measures. To understand the influence of these four parameters, we consider two levels for each of them (low and high values) as shown in Table 4.3, and generate a total of 16 different scenarios (i.e., 2^4 possible combinations using the two levels of the four parameters). Besides, we assume each scenario to be independent of each other. For example, one case may assess the use of short-range low-velocity drones to serve a set of locations, while another scenario may investigate the effect of long-range high-speed drones on the performance measures.

Table 4.3: Experimental factor settings

Experimental Factors	Symbol	Levels	Settings
Number of delivery locations	N	2	20, 50
Percent of delivery locations that must be served by a truck	ξ	2	0%, 20%
Velocity of drone (in mph)	V^D	2	15, 35
Flight range of drone (in miles)	F_l	2	7.5, 12.5

For each scenario, ten replications are generated and solved using the proposed JOCR models and the SHCR approach resulting in a total of 960 cases (16 scenarios $\times$ 10 replications $\times$ 2 objectives $\times$ 3 models). Table 4.4 summarizes the results of the 16 scenarios for all models under consideration. Also, similar to Table 4.1, the impact of using a joint optimization approach over sequential heuristic ($\mathcal{C}^{gap}_{SHCR-JOCR}$ and $\mathcal{T}^{gap}_{SHCR-JOCR}$), and the savings achieved by unrestricted focal point policy over restricted truck stops ($\mathcal{C}^{gap}_{R-U}$ and $\mathcal{T}^{gap}_{R-U}$) are reported.

For all the scenarios under consideration, the JOCR-U outperforms both the SHCR-U and JOCR-R. On average, using the JOCR-U approach rather than the

Table 4.4: Average min-cost and min-time of 10 replications for 16 scenarios

Scenario	N	ξ	V^D	F_l	Minimum total cost					Minimum delivery completion time				
					$\mathcal{C}^R_{JOCR}$	$\mathcal{C}^U_{SHCR}$	$\mathcal{C}^U_{JOCR}$	$\mathcal{C}^{gap}_{SHCR-JOCR}$	$\mathcal{C}^{gap}_{R-U}$	$\mathcal{T}^R_{JOCR}$	$\mathcal{T}^U_{SHCR}$	$\mathcal{T}^U_{JOCR}$	$\mathcal{T}^{gap}_{SHCR-JOCR}$	$\mathcal{T}^{gap}_{R-U}$
1	20	0%	15	7.5	140.9	122.9	109.1	-11.2%	-22.6%	7.3	7.0	6.2	-11.4%	-15.1%
2	20	0%	15	12.5	117.0	98.3	84.4	-14.1%	-27.9%	6.4	6.9	5.8	-15.9%	-9.4%
3	20	0%	35	7.5	140.9	122.9	109.1	-11.2%	-22.6%	5.4	4.9	4.3	-12.2%	-20.4%
4	20	0%	35	12.5	117.0	98.3	84.4	-14.1%	-27.9%	4.4	4.5	3.3	-26.7%	-25.0%
5	20	20%	15	7.5	155.6	152.2	141.5	-7.0%	-9.1%	8.0	7.4	7.3	-1.4%	-8.8%
6	20	20%	15	12.5	143.6	149.0	132.8	-10.9%	-7.5%	7.3	7.3	6.9	-5.5%	-5.5%
7	20	20%	35	7.5	155.6	152.2	141.5	-7.0%	-9.1%	5.9	5.7	5.3	-7.0%	-10.2%
8	20	20%	35	12.5	143.6	149.0	132.8	-10.9%	-7.5%	5.3	5.6	5.0	-10.7%	-5.7%
9	50	0%	15	7.5	179.9	176.5	151.6	-14.1%	-15.7%	10.1	11.0	8.9	-19.1%	-11.9%
10	50	0%	15	12.5	161.5	171.1	134.2	-21.6%	-16.9%	9.1	10.9	8.3	-23.9%	-8.8%
11	50	0%	35	7.5	179.9	176.5	151.6	-14.1%	-15.7%	7.0	7.9	5.9	-25.3%	-15.7%
12	50	0%	35	12.5	161.5	171.1	134.2	-21.6%	-16.9%	6.3	7.8	5.3	-32.1%	-15.9%
13	50	20%	15	7.5	217.3	219.8	205.4	-6.6%	-5.5%	11.9	11.8	11.1	-5.9%	-6.7%
14	50	20%	15	12.5	211.5	223.5	199.6	-10.7%	-5.6%	11.5	11.7	10.9	-6.8%	-5.2%
15	50	20%	35	7.5	217.3	219.8	205.4	-6.6%	-5.5%	8.3	9.1	7.9	-13.2%	-4.8%
16	50	20%	35	12.5	211.5	223.5	199.6	-10.7%	-5.6%	8.1	9.0	7.8	-13.3%	-3.7%

sequential heuristic, saves about 12% and 15% in total costs and delivery completion time, respectively. Further, unrestricted focal point policy yields an average reduction of 14% in min-cost and 11% in min-time as opposed to restricting the focal points to customer-only locations. Besides, the best performance is achieved when all packages can be delivered by drones ($\xi = 0\%$) that have long-range ($F_l = 12.5$) and high-speed ($V^D = 35$). For serving 50 locations with such a setting, adopting our JOCR-U approach instead of the SHCR-U method provides an average reduction of 21.6% and 32.1% in min-cost and min-time, respectively.

Table 4.5 summarizes the 16 scenarios based on the low and high levels of each of the four factors. The following insights can be derived from the impact of each factor on the performance measures.

- ***Number of locations (N):*** As expected, the total cost and delivery completion time become higher if more customers (N) are involved. However, it is to note that when N is increased by 150% (from 20 to 50 locations), the delivery completion time increased only by about 47% for both JOCR-R and JOCR-U polices. Similarly, the best total costs for restricted and unrestricted JOCR increased approximately by 38% and 48%, respectively. In other words, cost and time do not change proportionally with N, but increase at a slower rate. Hence, practitioners should include many orders per truck trip whenever possible. Another advantage is that the performance (total cost and completion time) of JOCR-U relative to SHCR-U is further enhanced for larger values of N.

- ***Proportion of truck-only locations (ξ):*** Requiring 20% of the delivery locations to be served by trucks instead of 0% has a greater negative impact on the JOCR-U policy (41% and 29% increase in cost and time, respectively) as opposed to JOCR-R policy (21% and 18% increase in cost and time, respectively). The JOCR-R policy is less affected because it already applies the restriction of customer-only focal points by default. Although the JOCR-U policy has superior performance over JOCR-R and SHCR-U in all the cases, the improvement achieved is relatively lower when ξ is increased to 20%. Thus, to achieve faster and cheaper delivery, it is ideal to limit the proportion of truck-only locations to a smaller value. This may be achieved by investing in drones with higher payload capacity so that it can deliver a heavier package instead of a truck.
- ***Velocity of drone (V^D):*** Increasing the drone velocity reduces the delivery completion time by about 30% for both JOCR-R and JOCR-U polices. Besides,

the min-time JOCR-U model achieves greater improvement over the other two models with faster drones. Therefore, if the company is keen on achieving faster delivery, then investing in high-speed drones is recommended.

- ***Flight range of drone (F_l):*** Operating drones with longer flight range (12.5 miles instead of 7.5 miles) reduces the total cost and completion time by 9% and 7%, respectively, for both JOCR-R and JOCR-U models. Moreover, the percent improvement in cost and time achieved by the JOCR-U model over SHCR-U is considerably greater for long-range drones. However, the purchasing cost of such drones are likely to be more expensive than short-range drones. Therefore, decision-makers should consider a trade-off between the additional purchasing cost of long-range drones and their benefits.

Table 4.5: Impact of changing each of the four factors on min-cost/min-time and performance of proposed solution approaches

Factor	Level	Minimum total cost					Minimum total completion time				
		$\mathcal{C}^{R}_{JOCR}$	$\mathcal{C}^{U}_{SHCR}$	$\mathcal{C}^{U}_{JOCR}$	$\mathcal{C}^{gap}_{SHCR-JOCR}$	$\mathcal{C}^{gap}_{R-U}$	$\mathcal{T}^{R}_{JOCR}$	$\mathcal{T}^{U}_{SHCR}$	$\mathcal{T}^{U}_{JOCR}$	$\mathcal{T}^{gap}_{SHCR-JOCR}$	$\mathcal{T}^{gap}_{R-U}$
N	20	139.3	130.6	117.0	-10.8%	-16.8%	6.3	6.2	5.5	-11.4%	-12.5%
	50	192.6	197.7	172.7	-13.2%	-10.9%	9.0	9.9	8.3	-17.5%	-9.1%
ξ	0%	149.8	142.2	119.8	-15.3%	-20.8%	7.0	7.6	6.0	-20.8%	-15.3%
	20%	182.0	186.1	169.8	-8.8%	-6.9%	8.3	8.5	7.8	-8.0%	-6.3%
V^D	15	165.9	164.2	144.8	-12.0%	-13.8%	9.0	9.3	8.2	-11.2%	-8.9%
	35	165.9	164.2	144.8	-12.0%	-13.8%	6.3	6.8	5.6	-17.6%	-12.7%
F_l	7.5	173.4	167.9	151.9	-9.7%	-13.2%	8.0	8.1	7.1	-11.9%	-11.7%
	12.5	158.4	160.5	137.8	-14.3%	-14.5%	7.3	8.0	6.7	-16.9%	-9.9%

4.5 Conclusion

In this chapter, the policy of allowing truck-drone stops to be anywhere in the delivery area (i.e., customer or non-customer locations) is considered. Joint optimization models are proposed for the clustering of customer locations and routing of truck and multiple drones. The most two common objectives in the literature are adopted in this chapter − minimizing total cost and delivery completion time. A comparison is conducted with the best known approach in the literature (i.e., the SHCR heuristic) handling the same problem. The results show that our proposed optimization models can significantly outperform the SHCR heuristic by an average cost reduction of 6%-22% and time savings up to 32%.

A sensitivity analysis is performed for the three approaches by varying some key parameters. The results show that operational cost and delivery time do not change proportionally with N, but increase at a lower rate. Furthermore, investing in drones with higher payload capacity (i.e., to eliminate truck-only locations) can save significant cost and delivery time. Also, it is recommended to employ drones with higher speed to achieve faster delivery completion time. Lastly, results of instances with long and short drone flight ranges are provided to illustrate the possible tradeoff between purchasing cost of long-range drones and the expected benefits.

Chapter 5

Routing and scheduling of truck and drones for package delivery with flexible launch and recovery sites

While recent literature has extensively studied truck-drone routing, most of them restrict the drone launch and recovery operations (LARO) to customer locations [10, 47, 48, 52, 55]. However, the truck can stop at other feasible locations in the service region to deploy or recover the drones. As a matter of fact, real-life testing by UPS shows the drone being launched from the roof of a delivery truck at non-customer locations [36]. Moreover, scheduling of truck operator tasks at a given stop, which possibly includes customer service and launch/recovery of multiple drones, is usually neglected [32, 53, 75]. Nevertheless, it is crucial to consider the sequencing dynamics to ensure safe and realistic operations [76].

An illustrative example is provided in Fig. 5.1 to highlight both the potential of allowing non-customer nodes (also referred to as flexible locations or sites) for drone LARO and the importance of scheduling at truck stops. The network in this example consists of a depot, 8 customer locations to be served by a fleet of truck and four drones, and 8 non-customer locations. Note that the flexible LARO sites can be established in advance by considering the feasible areas within the delivery region. It can be observed from the optimal plan that the delivery completion time is substantially lower when flexible stop locations are permitted. In this particular example, the distance traveled by truck is shorter and drone fleet utilization is higher if trucks are allowed to launch and recover drones at flexible locations. For instance, in the case of restricted LARO, node 4 is served by a truck instead of a drone since existing locations along the truck route do not meet the flight range restriction. As a result, the truck route is significantly longer when compared to the case of using flexible sites. Furthermore, while the truck carries four drones in both settings, only three of them were utilized when the LARO are restricted. Hence, the drones play a secondary role in aiding the truck when restricting LARO to customer locations. In contrast, when flexible stop locations are considered, the truck assists the drones for faster delivery completion.

Motivated by the potential and feasibility of flexible locations in practice, this chapter aims to minimize the completion time of last-mile delivery operations by employing a mixed fleet (truck and multiple drones) and accounting for flexible truck stops for drone LARO. Specifically, this study contributes to the research stream of collaborative truck-drone system for package delivery in the following ways.

- A new variant of drone-enabled logistics for last-mile package delivery called the collaborative truck-drone routing and scheduling problem with flexible launch and recovery locations (CTDRSP-FL) is introduced.
- To the best of our knowledge, this is the first work to consider flexible truck stops with the following three logistical decisions: (i) *assignment*: whether a drone or truck must serve a specific location, (ii) *routing*: in which order should the truck visit the assigned locations (truck routing), and at which truck stop should a drone be launched and recovered to serve a customer (drone routing or sortie), (iii) *scheduling*: launch and retrieval times of multiple drones at a given truck stop. Unlike the prior works, the truck-drone fleet is not required to visit all the nodes because certain flexible stop locations may never be visited (see unselected flexible locations in Fig. 5.1). Besides, incorporating the scheduling of drones at each truck stop further complicates the problem.

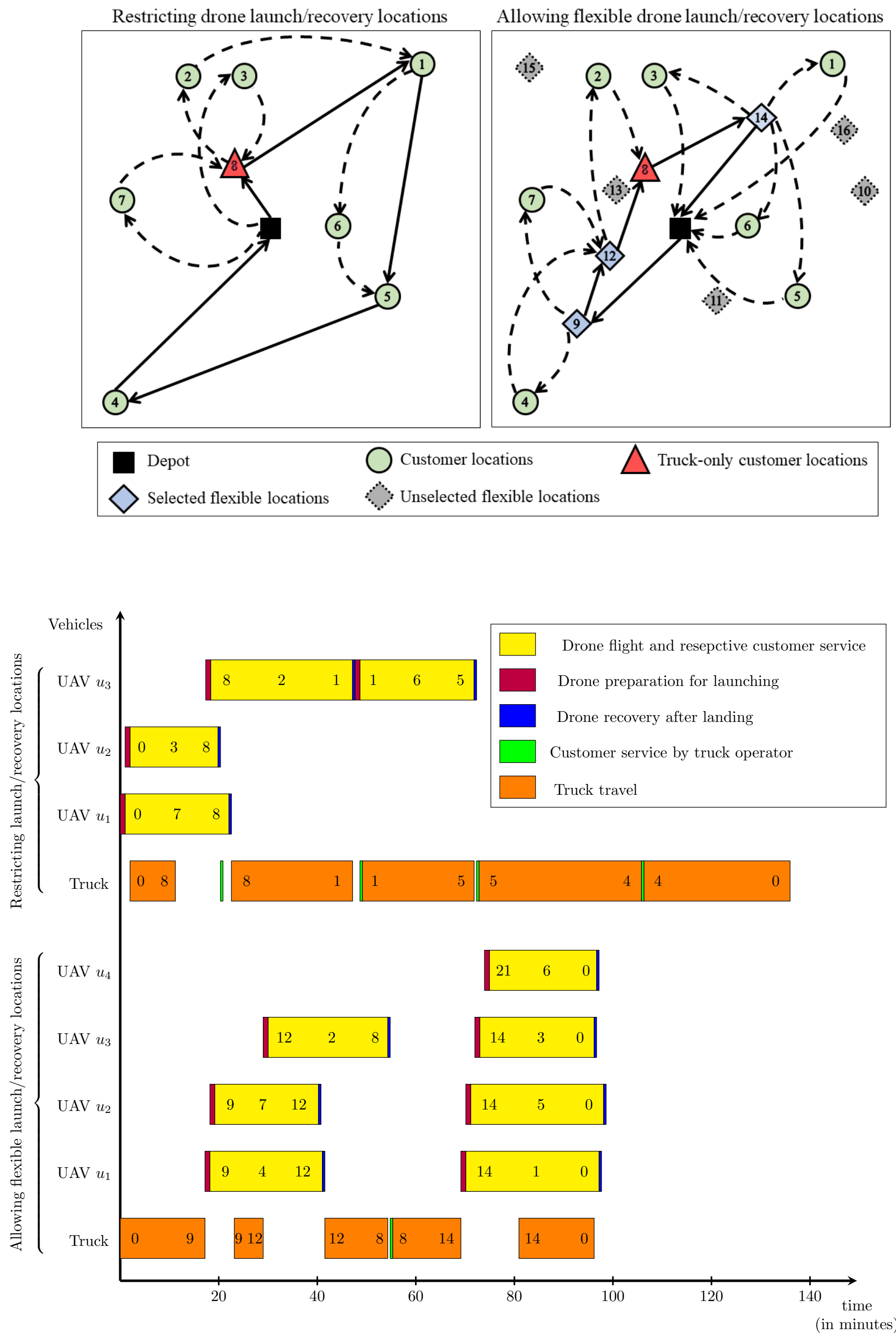

Figure 5.1: Illustrative example of using flexible locations for drone launching and recovery. The solid arcs are for truck route and the dash arcs are for drone sorties. Numbers in the Gantt chart are for the locations on the truck/drone routes.

- A fleet of heterogeneous drones characterized by their endurance and speed is considered. As drone technology is expected to evolve rapidly, companies are bound to have UAVs of various capabilities (e.g., quadcopter vs. fixed-wing drones), and it becomes critical to account for these attributes during route planning.
- A new mixed integer linear programming (MILP) model is developed to handle the flexible locations for drone LARO and simultaneously optimize the three aforementioned logistical decisions.
- An optimization-enabled two-phase search (OTS) algorithm is developed to solve large size instances in a reasonable time. Specifically, simulated annealing (SA) algorithm is employed in the first phase to explore specific areas of the solution space, and a variable neighborhood search (VNS) algorithm expands the search in the second phase by leveraging the solution obtained by the SA algorithm. Besides, a modular approach is adopted as it provides greater flexibility to handle evolving regulations and drone characteristics, while also enabling scalable deployment.
- Several new test instances are introduced and extensive analysis is conducted to evaluate the proposed models. The delivery completion time associated with the proposed CTDRSP-FL model is benchmarked against the conventional approach of restricting the LARO to be at customer locations.

5.1 Problem description

This research considers the last-mile delivery of packages to a set of customer locations $\mathcal{N} = \{i_1, i_2, i_3, \dots i_N\}$ distributed in a 2D Euclidean space using a truck and a set of heterogeneous drones $\mathcal{U} = \{u_1, u_2, u_3, \dots u_U\}$. While all locations are typically reachable by truck, drones can only serve a subset of locations $\mathcal{N}^U \subseteq \mathcal{N}$ due to technical and operational restrictions (e.g., limited payload capacity), and therefore the remaining customers $\mathcal{N}^T \subseteq \mathcal{N}$ (where $\mathcal{N}^T = \mathcal{N} \backslash \mathcal{N}^U$) must be served by truck. In contrast to most of the last-mile delivery literature, our problem expands the delivery network to include an additional set of F feasible truck stop locations (non-customer locations and reachable by truck) besides the depot and customer locations, thereby providing flexibility for drone LARO. The resultant set of feasible locations can therefore be denoted by $\mathcal{F} = \{i_0\} \cup \mathcal{N} \cup \{i_{N+1}, i_{N+2}, i_{N+3}, \dots i_{N+F}\} \cup \{i_{N+F+1}\}$, where i_0 and i_{N+F+1} represent the starting and ending depot, respectively, which can be distinct or same location. Among the $N + F$ feasible locations, up to $\widehat{S}$ of customer and non-customer locations can be visited by the truck. Therefore, a set of potential (or selected) truck stops $\mathcal{S} = \{s_0, s_1, s_2, s_3, \dots s_{\widehat{S}}, s_{\widehat{S}+1}\}$ is considered, which includes the $\widehat{S}$ locations in addition to two stops s_0 and $s_{\widehat{S}+1}$ corresponding to the depot indices i_0 and i_{N+F+1} in set $\mathcal{F}$, respectively.

The truck carrying packages and a fleet of drones must start from the depot, visit a subset of feasible locations, and return to the depot. The truck travel time from location i to $j \in \mathcal{F}$ is denoted by R^T_{ij}. All customer locations that coincide with the truck route are assumed to be served by the truck operator, requiring service time V^T_i for customer $i \in \mathcal{N}$. Besides, the truck serves as a mobile drone platform for battery swap (or recharging) and LARO. Each UAV $u \in \mathcal{U}$ is allowed to carry one package per trip, where each UAV sortie consists of launching from a truck stop $i \in \mathcal{F}$, visiting a customer location $j \in \mathcal{N}^U$ to deliver the package, and returning to the next truck stop $k \in \mathcal{F}$ requiring flying time R^U_{ijku} and service time V^U_j. All customers must be served either by a drone $u \in \mathcal{U}$ or truck before the fleet returns to the depot. Without loss of generality, the total time of a drone $u \in \mathcal{U}$ between its launching and landing must be within its endurance E_{ijku} for the task from location i to customer j then to location $k \in \mathcal{F}$. Before a drone sortie, the truck operator spends a duration H_i at truck stop (or depot) $i \in \mathcal{F}$ to retrieve the respective order and prepare the drone for launching. Likewise, after completing an order delivery by drone, it requires a recovery time G_i when landing at truck stop $i \in \mathcal{F}$. Thus, overall delivery completion time is the total time between starting the first vehicle preparation to depart from the depot and completing the last vehicle recovery at the depot. Given the aforementioned problem characteristics, the objective of this research is to minimize the delivery completion time.

There are three categories of decisions to be made for solving the CTDRSP-FL. First, a vehicle (truck or drone) should be assigned to visit/serve each customer. In addition, truck stops, either customer location or flexible site, must be selected, and then truck route must be established. Furthermore, launching locations of drones (which must be the depot or truck stops) to serve their assigned customers are determined, while the landing of drones are assumed to be at the subsequent truck stop [53, 77]. Second, the departure and arrival times of all vehicles at each truck stop and depot must be determined. Third, since the truck operator is assumed to perform only one task at a time, sequencing (or scheduling) of tasks at truck stops (i.e., drone launching and landing, and customer service) should be ascertained. An MILP is developed to jointly optimize the three decisions and solve the CTDRSP-FL.

5.2 Mixed integer linear programming model

The following notation are used to formulate the optimization model.

Indices and Sets

$i, j, k \in \mathcal{N}$	set of customer locations, $\mathcal{N} = \{i_1, i_2, i_3, \ldots i_N\}$
$i, j, k \in \mathcal{N}^U$	set of customer locations that can be served by drone or truck, $\mathcal{N}^U \subseteq \mathcal{N}$
$i, j, k \in \mathcal{N}^T$	set of customer locations that must be served by truck, $\mathcal{N}^T = \mathcal{N} \backslash \mathcal{N}^U$
$i, j, k \in \mathcal{F}$	set of potential truck stops, $\mathcal{F} = \{i_0\} \cup \mathcal{N} \cup \{i_{N+1}, i_{N+2}, i_{N+3}, \ldots i_{N+F}\} \cup \{i_{N+F+1}\}$
$s \in \mathcal{S}$	set of selected truck stops, $\mathcal{S} = \{s_0, s_1, s_2, s_3, \ldots s_{\widehat{S}}, s_{\widehat{S}+1}\}$

$u \in \mathcal{U}$	set of UAVs (or drones), $\mathcal{U} = \{u_1, u_2, u_3, \ldots u_U\}$

Parameters

N	number of customer locations
F	number of feasible truck stops at non-customer locations
$\widehat{S}$	maximum number of stops allowed for the truck
U	number of drones
R^T_{ij}	travel time of truck from feasible truck stop i to $j \in \mathcal{F}$
R^U_{ijku}	travel time of drone $u \in \mathcal{U}$ from feasible truck stop $i \in \mathcal{F}$ to visit delivery location $j \in \mathcal{N}^U$ and then returning to feasible truck stop $k \in \mathcal{F}$
V^T_i	service time of truck at delivery location $i \in \mathcal{N}$
V^U_i	service time of drone at delivery location $i \in \mathcal{N}^U$
E_{ijku}	endurance (in time unit) of drone $u \in \mathcal{U}$ to fly from location $i \in \mathcal{F}$ for delivering a package at location $j \in \mathcal{N}^U$ and flying back to location $k \in \mathcal{F}$
H_i	launch preparation time of drone at truck stop $i \in \mathcal{F}$
G_i	recovery time of drone at truck stop $i \in \mathcal{F}$

Decision variables

d^T_s	departure time of the truck from stop $s \in \mathcal{S}$ heading to $s + 1 \in \mathcal{S}$

a_s^T	arrival time of the truck to stop $s \in \mathcal{S}$ coming from $s-1 \in \mathcal{S}$
d_i^U	departure (or launching) time of a drone to visit location $i \in \mathcal{N}$
a_i^U	arrival (or landing) time of a drone after visiting location $i \in \mathcal{N}$
b_i^U	service begin time at customer location $i \in \mathcal{N}$ if it is selected to be a truck stop
e_i^U	service end time at customer location $i \in \mathcal{N}$ if it is selected to be a truck stop
w_{ijs}	1 if customer location $i \in \mathcal{N}$ is served (by launching or recovering the respective drone, or by delivery via truck operator) before another location $j \in \mathcal{N}$ at truck stop $s \in \mathcal{S}$, 0 otherwise
x_{ius}	1 if a delivery location $i \in \mathcal{N}$ is served by UAV $u \in \mathcal{U}$ dispatched from truck stop $s \in \mathcal{S}$, 0 otherwise
y_{is}	1 if location $i \in \mathcal{F}$ is assigned as a truck stop $s \in \mathcal{S}$, 0 otherwise
$\widehat{t}$	delivery completion time of all vehicles

The objective function (5.1) minimizes the completion time of delivery operations, which is determined by constraints (5.2) and (5.3) as the maximum of truck arrival to the depot and the latest recovery of drone after landing (i.e., $\widehat{t} = \max\limits_{\forall i \in \mathcal{N}}(a_s^T, a_i^U + \sum_{u \in \mathcal{U}} G_j x_{ius})$, $j = \{j_{N+F+1}\}, s = \{s_{\widehat{S}}, s_{\widehat{S}+1}\})$. The binary and continuous variables are specified by constraints (5.4) and (5.5), respectively.

$$Minimize \quad \widehat{t} \tag{5.1}$$

$S.t.$

$$\widehat{t} \geq a_s^T \qquad \forall s \in \{s_{\widehat{S}+1}\} \subseteq \mathcal{S} \tag{5.2}$$

$$\widehat{t} \geq a_i^U + \sum_{u \in \mathcal{U}} G_j x_{ius} \qquad \forall i \in \mathcal{N}, j \in \{j_{N+F+1}\} \subseteq \mathcal{N}, s \in \{s_{\widehat{S}}\} \subseteq \mathcal{S} \tag{5.3}$$

$$w_{ijs}, x_{ius}, y_{is} \in \{0,1\} \qquad \forall i, j \in \mathcal{F}, u \in \mathcal{U}, s \in \mathcal{S} \tag{5.4}$$

$$d_s^T, a_s^T, d_i^U, a_i^U, \widehat{t} \in \mathbb{R}_+ \qquad \forall i \in \mathcal{N}, s \in \mathcal{S} \tag{5.5}$$

5.2.1 Assignments and routing of hybrid vehicle fleet

Constraints (5.6)-(5.13) determine the assignments of truck stops (that can be customer locations or flexible drone launch/recovery locations), assignments of drones to customers (who were not selected to be visited by truck), and the routing of vehicles (both drones and truck).

$$y_{is} = 1 \qquad \forall i \in \{i_0\} \subseteq \mathcal{F}, s \in \{s_0\} \subseteq \mathcal{S} \tag{5.6}$$

$$y_{is} = 1 \qquad \forall i \in \{i_{N+F+1}\} \subseteq \mathcal{F}, s \in \{s_{\widehat{S}+1}\} \subseteq \mathcal{S} \tag{5.7}$$

$$\sum_{i \in \mathcal{F}} y_{is} = 1 \qquad \forall s \in \mathcal{S} \tag{5.8}$$

$$\sum_{s \in \mathcal{S}} y_{is} \leq 1 \qquad \forall i \in \mathcal{F} \backslash \{i_{N+F+1}\} \tag{5.9}$$

$$y_{is} \leq y_{i,s+1} \qquad \forall i \in \{i_{N+F+1}\} \subseteq \mathcal{F}, s \in \mathcal{S} \backslash \{s_{\widehat{S}+1}\} \tag{5.10}$$

$$\sum_{s \in \mathcal{S}} y_{is} = 1 \qquad \forall i \in \mathcal{N}^T \tag{5.11}$$

$$\sum_{u \in \mathcal{U}} \sum_{s \in \mathcal{S} \backslash \{s_{\widehat{S}+1}\}} x_{ius} + \sum_{s \in \mathcal{S}} y_{is} = 1 \qquad \forall i \in \mathcal{N}^U \tag{5.12}$$

$$\sum_{i \in \mathcal{N}} x_{ius} \leq 1 \qquad \forall u \in \mathcal{U}, s \in \mathcal{S} \tag{5.13}$$

The starting and ending depots are represented using two different indices (or stops), $\{s_0\}$ and $\{s_{\widehat{S}+1}\}$, to account for their spatial distribution. Such a modeling approach provides the flexibility to represent the depot(s) as a single physical location or two distinct nodes in the $2D$ Euclidean delivery region. Constraints (5.6) and (5.7) ensure that the truck starts and ends its route at the depot. Constraint (5.8) guarantees that each selected truck stop $s \in \mathcal{S}$ is assigned to exactly one potential location $i \in \mathcal{F}$, while constraint (5.9) ensures that each potential stop can be visited at most once by a truck. Furthermore, constraint (5.10) limits the assignments to the depot index $\{j_{N+F+1}\}$ to a consecutive subset of truck stop indices ending with $\widehat{S}+1$. Each customer location that must be visited only by truck, $i \in \mathcal{N}^T$, is enforced using constraint (5.11). Constraint (5.12) guarantees that each customer location is either assigned to be served by a truck or drone, while constraint (5.13) allows each drone $u \in \mathcal{U}$ to be dispatched at most once from each truck stop.

5.2.2 Timing decision I: departure and arrival times of truck

Constraints (5.14)-(5.18) determine the optimal departure and arrival times of truck at its stops.

$$a^T_{s+1} \geq d^T_s + R^T_{ij} - M(2 - y_{is} - y_{j,s+1}) \qquad \forall i, j \in \mathcal{F}, s \in \mathcal{S}\backslash\{s_{\widehat{S}+1}\} \tag{5.14}$$

$$d^T_s \geq a^T_s \qquad \forall s \in \mathcal{S} \tag{5.15}$$

$$d^T_s \geq a^U_i + \sum_{j \in \mathcal{F}} G_j y_{js} - M(1 - \sum_{u \in \mathcal{U}} x_{iu,s-1}) \qquad \forall i \in \mathcal{N}, s \in \mathcal{S}\backslash\{s_0\} \tag{5.16}$$

$$d^T_s \geq d^U_i - M(1 - \sum_{u \in \mathcal{U}} x_{ius}) \qquad \forall i \in \mathcal{N}, s \in \mathcal{S} \tag{5.17}$$

$$d^T_s \geq e_i - M(1 - y_{is}) \qquad \forall i \in \mathcal{N}, s \in \mathcal{S}\backslash\{s_0\} \tag{5.18}$$

The arrival time of truck at stop $s+1 \in \mathcal{S}$ is the sum of truck departure time from stop $s \in \mathcal{S}$ and the travel time between these stops R^T_{ij}, as given by constraint (5.14). It should be noted that this constraint is active only if locations i and $j \in \mathcal{F}$ are selected as stops s and $s+1 \in \mathcal{S}$, respectively. Moreover, constraints (5.14) and (5.15) also facilitate subtour elimination for the truck route. The truck departure time from stop $s \in \mathcal{S}$ is the maximum of the following four events, which are captured by constraints (5.15)-(5.18), respectively: (i) arrival time of truck to stop $s \in \mathcal{S}$, (ii) recovery completion time of UAVs launched from the preceding stop, (iii) departure time of all UAVs from stop $s \in \mathcal{S}$, and (iv) service completion time at truck stop $s \in \mathcal{S}$, if it is a customer location.

5.2.3 Timing decision II: launching and landing times of drones

Constraints (5.19)-(5.23) determine the optimal time of launching and recovering drones at each truck stop.

$$d_i^U \geq a_s^T + \sum_{j \in \mathcal{F}} H_j y_{js} - M(1 - \sum_{u \in \mathcal{U}} x_{ius}) \qquad \forall i \in \mathcal{N}, s \in \mathcal{S} \tag{5.19}$$

$$d_j^U \geq a_i^U + \sum_{k \in \mathcal{F}} (G_k + H_k) y_{ks} - M(2 - x_{ius} - x_{ju,s+1})$$
$$\forall i, j \in \mathcal{N}, u \in \mathcal{U}, s \in \mathcal{S} \backslash \{s_{\widehat{S}+1}\} \tag{5.20}$$

$$a_j^U \geq d_j^U + \sum_{u \in \mathcal{U}} R_{ijku}^U x_{jus} + V_j^U - M(3 - \sum_{u \in \mathcal{U}} x_{jus} - y_{is} - y_{k,s+1})$$
$$\forall i, k \in \mathcal{F}, j \in \mathcal{N}, s \in \mathcal{S} \backslash \{s_{\widehat{S}+1}\} \tag{5.21}$$

$$a_i^U \geq a_{s+1}^T - M(1 - \sum_{u \in \mathcal{U}} x_{ius}) \qquad \forall i \in \mathcal{N}, s \in \mathcal{S} \backslash \{s_{\widehat{S}+1}\} \tag{5.22}$$

$$a_j^U - d_j^U \leq \sum_{u \in \mathcal{U}} E_{ijku} x_{jus} + M(3 - \sum_{u \in \mathcal{U}} x_{jus} - y_{is} - y_{k,s+1})$$

$$\forall i, k \in \mathcal{F}, j \in \mathcal{N}, s \in \mathcal{S} \backslash \{s_{\widehat{S}+1}\} \qquad (5.23)$$

Constraint (5.19) ensures the dispatch time of each drone to be after the truck arrival to its stop plus the respective launch preparation time. Likewise, constraint (5.20) guarantees the departure of each drone $u \in \mathcal{U}$ from a truck stop to be after the sum of the following three times: drone arrival time from the preceding location, drone recovery time, and preparation time for the next delivery. Constraint (5.21) stipulates that the arrival of each drone must be after the flight duration and service time at the assigned customer location. In addition, constraint (5.22) ensures that the drone landing is allowed only after the truck reaches its stop. However, for each UAV sortie (from truck stop $i \in \mathcal{F}$ to deliver a package to a customer location $j \in \mathcal{N}$ then returning to rendezvous at stop $k \in \mathcal{F}$), the total flight and service duration must be within its endurance, as stated by constraint (5.23).

5.2.4 Timing decision III: customer service at truck stops

Each customer $i \in \mathcal{N}$ can be served by the truck operator or a drone. When a customer location is assigned to be a truck stop, the delivery is assumed to be performed by the truck operator. The following constraints control the beginning and end of such delivery services.

$$b_i \geq a_s^T - M(1 - y_{is}) \qquad \forall i \in \mathcal{N}, s \in \mathcal{S} \tag{5.24}$$

$$e_i \geq b_i + V_i^T - M(1 - \sum_{s \in \mathcal{S}} y_{is}) \qquad \forall i \in \mathcal{N} \tag{5.25}$$

$$b_i \leq M \sum_{s \in \mathcal{S}} y_{is} \qquad \forall i \in \mathcal{N} \tag{5.26}$$

$$e_i \leq M \sum_{s \in \mathcal{S}} y_{is} \qquad \forall i \in \mathcal{N} \tag{5.27}$$

If a customer location is assigned to be a truck stop, then the service start and end time by the truck operator are specified by constraints (5.24) and (5.25), respectively. Specifically, constraint (5.24) ensures that the service can begin only after the truck arrival to the respective stop $s \in \mathcal{S}$, while constraint (5.25) determines its end time depending on the service duration V_i^T. On the other hand, if a customer $i \in \mathcal{N}$ is not assigned to be served by a truck (i.e., $\sum_{s \in \mathcal{S}} y_{is} = 0$), then constraints (5.26) and (5.27) forces the service begin and end time to be zero.

5.2.5 Scheduling of tasks at truck stops

At each truck stop, it is crucial to coordinate the drone launch and recovery operation to avoid multiple drones departing/arriving at the same time. In addition, the scheduling of tasks at each stop must account for the truck operator availability.

An operator can perform only one of the following tasks at a given time: (i) prepare a drone for launching, (ii) recover a drone, or (iii) deliver the respective package to the customer at truck stop. Constraints (5.28)-(5.37) control the optimization of scheduling these tasks.

$$w_{ijs} + w_{jis} \geq \sum_{u\in\mathcal{U}} x_{ius} + \sum_{u\in\mathcal{U}} x_{jus} + \sum_{u\in\mathcal{U}} x_{iu,s-1} + \sum_{u\in\mathcal{U}} x_{ju,s-1} + y_{is} + y_{js} - 1$$
$$\forall i,j \in \mathcal{N}, s \in \mathcal{S}\backslash\{s_0\}, i > j \qquad (5.28)$$

$$w_{ijs} + w_{jis} \geq \sum_{u\in\mathcal{U}} x_{ius} + \sum_{u\in\mathcal{U}} x_{jus} - 1$$
$$\forall i,j \in \mathcal{N}, s \in \{s_0\} \subseteq \mathcal{S}, i > j \qquad (5.29)$$

$$a_i^U \geq a_j^U + \sum_{k\in\mathcal{F}} G_k y_{ks} - M(3 - w_{jis} - \sum_{u\in\mathcal{U}} x_{iu,s-1} - \sum_{u\in\mathcal{U}} x_{ju,s-1})$$
$$\forall i,j \in \mathcal{N}, s \in \mathcal{S}\backslash\{s_0\} \qquad (5.30)$$

$$d_i^U \geq d_j^U + \sum_{k\in\mathcal{F}} H_k y_{ks} - M(3 - w_{jis} - \sum_{u\in\mathcal{U}} x_{ius} - \sum_{u\in\mathcal{U}} x_{jus})$$
$$\forall i,j \in \mathcal{N}, s \in \mathcal{S} \qquad (5.31)$$

$$a_j^U \geq d_i^U - M(2 + w_{jis} - \sum_{u\in\mathcal{U}} x_{ju,s-1} - \sum_{u\in\mathcal{U}} x_{ius})$$
$$\forall i,j \in \mathcal{N}, s \in \mathcal{S}\backslash\{s_0\} \qquad (5.32)$$

$$d_i^U \geq a_j^U + \sum_{k\in\mathcal{F}}(G_k + H_k)y_{ks} - M(3 - w_{jis} - \sum_{u\in\mathcal{U}} x_{ju,s-1} - \sum_{u\in\mathcal{U}} x_{ius})$$

$$\forall i,j \in \mathcal{N}, s \in \mathcal{S}\backslash\{s_0\} \qquad (5.33)$$

$$a_i^U \geq e_j - M(3 - w_{jis} - \sum_{u\in\mathcal{U}} x_{iu,s-1} - y_{js}) \qquad \forall i,j \in \mathcal{N}, s \in \mathcal{S}\backslash\{s_0\} \qquad (5.34)$$

$$b_i \geq a_j^U + \sum_{k\in\mathcal{F}} G_k y_{ks} - M(3 - w_{jis} - \sum_{u\in\mathcal{U}} x_{ju,s-1} - y_{is}) \quad \forall i,j \in \mathcal{N}, s \in \mathcal{S}\backslash\{s_0\} \qquad (5.35)$$

$$b_i \geq d_j^U - M(3 - w_{jis} - y_{is} - \sum_{u\in\mathcal{U}} x_{jus}) \qquad \forall i,j \in \mathcal{N}, s \in \mathcal{S} \qquad (5.36)$$

$$d_i^U \geq e_j + \sum_{k\in\mathcal{F}} H_k y_{ks} - M(3 - w_{jis} - \sum_{u\in\mathcal{U}} x_{ius} - y_{js}) \qquad \forall i,j \in \mathcal{N}, s \in \mathcal{S} \qquad (5.37)$$

The sequencing of launching and landing operations of drones as well as delivery service tasks of customers at truck stops is controlled using constraints (5.28) and (5.29) for each pair of customer locations i and $j \in \mathcal{N}$ at truck stop $s \in \mathcal{S}$. In particular, constraint (5.28) stipulates sequencing the pertaining tasks of locations i and $j \in \mathcal{N}$ only when they are served (by drone launching/retrieval or delivery by the truck operator) at the same truck stop, while constraint (5.29) similarly handles the case of launching drones from the depot. If multiple drones are recovered at a truck stop $s \in \mathcal{S}$, constraint (5.30) ensures collision avoidance by determining the arrival times of drones such that a drone can arrive only after the preceding drone has been recovered. Similarly, if multiple drones are launched from a truck stop $s \in \mathcal{S}$, constraint (5.31) specifies the launching times of drones while considering a preparation duration between each consecutive sorties. For each pair of drones, with one recovered and the other launched at the same truck stop $s \in \mathcal{S}$, constraints (5.32) and (5.33) determine their landing and launching times, respectively, while considering a recovery and preparation duration if the landing is before the launching task. When a customer location $i \in \mathcal{N}$ is assigned to a truck stop $s \in \mathcal{S}$, constraints (5.34)-(5.37) control the start and end times of delivery by the truck operator while accounting for the recovery and launching tasks of drones at the same truck stop.

5.3 Optimization-enabled two-phase search algorithm

The strong NP-hardness of the addressed problem is clear based on less complex cases in the literature [11, 48, 53]. Therefore, it is impractical to solve large size problems using the proposed MILP model. Hence, in this section, we develop an optimization-enabled two-phase search (OTS) algorithm that decomposes the problem into three modules to handle the different decisions associated with the CTDRSP-FL. The goal of Module I is to establish the truck route, which implicitly determines the assignment of customers to be served either by a drone or truck (i.e., assignment and truck routing decisions). Module II assigns a specific drone $u \in \mathcal{U}$ to each customer who must be served by a drone (based on Module I), and establishes the drone sorties, while Module III aims to optimize the scheduling decision at each truck stop.

Fig. 5.2 provides an overview of the proposed algorithm and highlights the interaction between the three modules. We develop two metaheuristics, simulated annealing (SA) and variable neighborhood search (VNS), which are solved sequentially to explore and exploit the search space associated with Module I decisions. The SA determines a feasible truck route by considering only the customer locations in the delivery network, thereby restricting the search to a limited region of the CTDRSP-FL solution space. Subsequently, the VNS leverages the solution obtained by the SA and widens the search to flexible truck stop locations. Hence, the proposed two-phase sequential search approach initially focuses on a subset of the feasible solutions and then considers the entire search space to improve the solution quality. The motivation for such a two-phase sequential search approach is to obtain a good solution in a reasonable time by progressively focusing on different parts of the solution space. Furthermore, an additional benefit of our modeling approach is that the second phase can be easily integrated into existing models proposed in the literature to relax their restrictions on LARO to be at customer locations.

Given the candidate solution of Module I at each iteration of the search process, Module II identifies which drone in the set $\mathcal{U}$ should be deployed to serve each customer (who were selected in Module I to be served by drones) and subsequently determines the routing of drones. Next, Module III optimizes the scheduling of truck operator tasks at each stop using a decomposed optimization model. Thus, the proposed approach collaboratively employs the three modules as the neighborhood search of the metaheuristics advances. The following subsections provide a detailed description of the three modules.

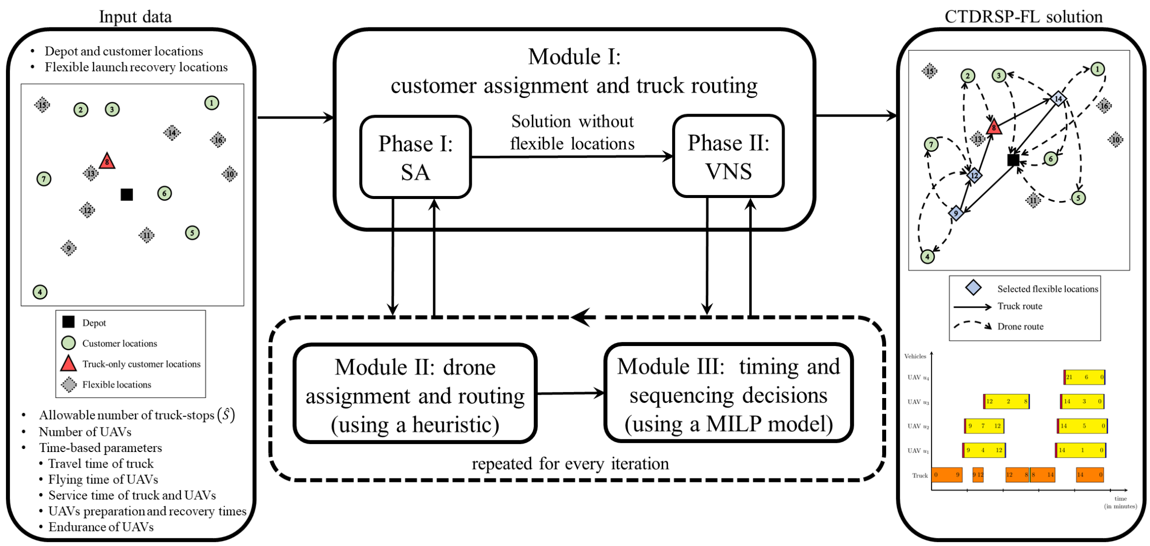

Figure 5.2: Overview of the developed OTS algorithm

5.3.1 Module I: customer assignment and truck routing

This module jointly determines the assignment of customers to each of the two vehicle types (truck or drone) and the truck route. An illustrative solution representation of Module I is exhibited in Fig. 5.3. It depicts the customer locations assigned to the truck, flexible truck stops selected for drone LARO (e.g., nodes i_{N+3}, i_{N+7}), and sequence in which the stops are visited. The remaining customers (e.g., nodes i_2, i_3, and i_4) that are not present in Fig. 5.3 are assigned to be served by drones. It is important to note that the solution representation throughout the search process must be constructed by taking into account several problem characteristics to ensure feasibility. First, a drone or truck must be assigned to serve each customer $i \in \mathcal{N}^U$, while the truck is assigned by default to serve all customers in set $\mathcal{N}^T$. Let $\mathcal{N}^{U-}$ and $\mathcal{N}^{U+}$ denote the subset of customers who are selected to be served by truck and drones, respectively, where $\mathcal{N}^{U-} \cup \mathcal{N}^{U+} = \mathcal{N}^U$. Then, the truck route must cover the set of customers $\mathcal{N}^{TR-} = \mathcal{N}^T \cup \mathcal{N}^{U-}$. In addition, the truck route must start and end its route at the depot (represented by indices $\{i_0\}$ and $\{i_{N+F+1}\}$) and may also stop at non-customer (or flexible) locations belonging to set $\mathcal{F}$. If $\mathcal{N}^{TR+}$ denote the depot and selected flexible truck stops, then, without loss of generality, the cardinality of the set $\mathcal{N}^{TR} = \mathcal{N}^{TR-} \cup \mathcal{N}^{TR+}$ must not exceed the maximum allowable number of truck stops $\widehat{S}$, as indicated in Fig. 5.3.

s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	...	...	$s_{\widehat{S}}$	$s_{\widehat{S}+1}$
i_0	i_5	i_{12}	i_8	i_{N+7}	i_6	i_1	i_{N+3}	i_{14}	...	...	i_{N+9}	i_{N+F+1}

Figure 5.3: Solution representation of truck assignment and routing in Module I

To determine the Module I decisions, we develop a two-phase search strategy using SA and VNS (see Fig. 5.2). SA is a well-known probabilistic search technique developed by [78] with a mechanism to escape from local optimal solutions by probabilistically accepting low quality solutions. It has been successfully used to solve various routing problems [10, 49, 65, 79, 80]. Also, VNS is a popular local search technique developed by [81] to improve an incumbent solution by systematically visiting multiple distant neighborhoods in the solution space [75, 82, 83]. Therefore, a two-phase approach that employs SA to obtain a good feasible solution for a defined solution space, and then leverages VNS to improve it by considering flexible LARO sites is promising for the CTDRSP-FL, especially to achieve a good trade-off between solution quality and runtime.

To generate the neighborhood solutions at each iteration of the search process, we consider three neighborhood operators for both SA and VNS. Algorithm 2 shows the procedure for sequentially generating the three neighborhood solutions. The first search operator (B_1) considers a random swap of two truck stops (lines 4-9). Given the current truck route in the search algorithm (B_0) and truck travel time ($R_{ij}^T \quad \forall i, j \in \mathcal{N}$), two random indices ($i^{\text{stop}}$ and i^{location}) are generated (line 4), one for position of the truck route (i.e., an index from set $\mathcal{S}$) and another for a potential physical location (i.e., an index from set $\mathcal{N}$) to be included in the resultant truck route. If the chosen truck stop (i.e., i^{stop}) is a customer location that must be visited by truck (i.e., $B_0[i^{\text{stop}}] \in \mathcal{N}^T$), then the swapping operation is confined to be within the existing truck route (lines $5-7$), otherwise the originally chosen location (i.e., i^{location}) is considered as shown on line 9. The second operator (B_2) rearranges B_1 using the TSP nearest neighbor algorithm [84] (lines $11-13$), while the third neighborhood search operator (B_3) reverses the truck route of B_2 (lines 14 and 15). Finally, the three neighborhood solutions B_1, B_2, and B_3 are returned.

Algorithm 2 Neighborhood search operators

Input: B_0 and $R^T_{ij} \quad \forall i, j \in \mathcal{N}$
Output: truck routes of the three neighbors
$B_1 \leftarrow B_0$
generate two random indices, $i^{\text{stop}} \sim U(1, \widehat{S})$ and $i^{\text{location}} \sim U(1, N)$
if $B_1[i^{\text{stop}}] \in \mathcal{N}^T$ **then**
    generate a random index, $j^{\text{stop}} \sim U(1, \widehat{S})$
    swap $B_1[i^{\text{stop}}]$ with $B_1[j^{\text{stop}}]$
else
    swap $B_1[i^{\text{stop}}]$ with i^{location}
end if
$B_2[s_0] \leftarrow i_0$
for $s \in \mathcal{S}$ **do**
    set $j^* = \text{argmin}(R^T_{B_2[s],j} | j \in B_1 \wedge j \notin B_2)$
    $B_2[s+1] \leftarrow j^*$
end for
for $s \in \mathcal{S}$ **do**
    $B_3[s] \leftarrow B_2[\widehat{S} - s]$
end for
return B_1, B_2, B_3

5.3.1.1 Constructing routing solution without flexible locations

The pseudocode of the SA is shown in Algorithm 3. An initial feasible routing solution is given as an input to initiate the neighborhood search, in addition to SA parameters, namely initial and final temperature (T^0 and T^{min}), cooling rate (K), number of iterations at each temperature level (iter^{max}) and maximum runtime (τ^{max}_{SA}). Upon initializing the current values of the SA algorithm (lines $3-5$), the loop on lines $6-20$ explores the solution space over different temperature levels until one of the following two termination criteria is met: (i) current temperature level (T^{current}) falls below T^{min} or (ii) current runtime (τ) exceeds τ^{max}_{SA}. For each level, a predefined number of search iterations is conducted by loop $7-18$. In each iteration, three neighbors to the current best solution are generated using Algorithm 2 (line 8). Next, the algorithms of Modules II and III (see Fig. 5.2) are executed to obtain fully defined solutions of the three neighbors (line 9), and the best among them is identified (line 10). If the delivery completion of the best neighborhood solution ($\widehat{t'}$) is better than the incumbent solution's objective value ($\widehat{t^*}$), then the incumbent solution is updated with the best neighborhood solution (along with the associated objective value), as given by lines 11 and 12. Besides, if the best neighbor outperforms the current best solution, the former replaces the latter (lines 13 and 14). Otherwise, for the purpose of enhancing the exploration of SA algorithm, the selected neighbor may be considered as the current best solution with a probability based on the metropolis acceptance criterion (lines $15-18$) [85]. Based on the truck route obtained by the SA algorithm (B^*), the customers are categorized into $\mathcal{N}^{U-}$ and $\mathcal{N}^{U+}$ (line 21). Finally, both customer assignments and truck route are returned. The obtained truck route is then given as an input to the VNS in the second phase of Module I (as shown in Fig. 5.2) for improvement by allowing truck stops to be at flexible locations.

5.3.1.2 Improving routing solution with flexible locations

The VNS procedure adopted in this research is given in Algorithm 4. In addition to using Module I outputs as parameters (B^*, $\widehat{t^*}$), the VNS procedure also requires the following inputs: number of iterations for exploring near and distant neighbors ($\text{iter}_1^{\max}$ and $\text{iter}_2^{\max}$) and runtime threshold ($\tau_{VNS}^{\max}$). The VNS is comprised of two

Algorithm 3 Module I $-$ phase I (SA)

Input: T^0, $T^{\min}$, K, iter$^{\max}$, and $\tau_{SA}^{\max}$
Output: customer assignments and truck route
initialize a solution B and compute its objective value $\widehat{t}(B)$
set the current best solution ($B^* \leftarrow B$) and objective value ($\widehat{t}^* \leftarrow \widehat{t}(B)$)
$T^{\text{current}} \leftarrow T^0$
while $T^{\text{current}} \geq T^{\min}$ **and** $\tau < \tau_{SA}^{\max}$ **do**
  for iter := 1 **to** iter$^{\max}$ **do**
    generate three neighbors (B_1, B_2, B_3) to current solution B (using Algorithm 2)
    run Modules II and III (Algorithms 5 and 6) sequentially for B_1, B_2 and B_3
    set $B' = B_{\operatorname{argmin}(B_1,B_2,B_3)}$ and $\widehat{t}' = \min(\widehat{t}(B_1), \widehat{t}(B_2), \widehat{t}(B_3))$
    if $\widehat{t}' \leq \widehat{t}^*$ **then**
      $B^* \leftarrow B'$ and $\widehat{t}^* \leftarrow \widehat{t}'$
    end if
    if $\widehat{t}' \leq \widehat{t}(B)$ **then**
      $B \leftarrow B'$ and $\widehat{t}(B) \leftarrow \widehat{t}'$
    else
      generate rand_num $\sim U(0,1)$
      if rand_num $\leq e^{(\frac{\widehat{t}'-\widehat{t}(B)}{\widehat{t}'} \times 100)/T^{\text{current}}}$ **then**
        $B \leftarrow B'$ and $\widehat{t}(B) \leftarrow \widehat{t}'$
      end if
    end if
  end for
  $T^{\text{current}} \leftarrow K \times T^{\text{current}}$
  update elapsed time (τ)
end while
$\mathcal{N}^{U-} = \{i \in \mathcal{N}^U | i \in \mathcal{B}^*\}$ and $\mathcal{N}^{U+} = \{i \in \mathcal{N}^U | i \notin \mathcal{B}^*\}$
return $\mathcal{N}^{U-}$, $\mathcal{N}^{U+}$, B^* and $\widehat{t}^*$

search levels (loops on lines $4-11$ and $6-11$) that are executed for iter$_1^{\max}$ and iter$_2^{\max}$ iterations, respectively. The first level generates a single neighbor to the current best solution (line 5), while the second level widens the search by using that neighbor in generating multiple distant neighbors from the incumbent solution (lines 7-9). The random swap operator (in Algorithm 2) is used in the first level after allowing truck stops to be at flexible locations. Specifically, on line 4 of Algorithm 2, the randomly generated potential location is modified to be $i^{\text{location}} \sim U(1, N+F)$. In the second level (i.e., loop $6-11$ of Algorithm 4), six neighbors are generated (line 8). Three of them are obtained by Algorithm 2, while the other three are created by adapting Algorithm 2 to only allow non-customer locations to be included (i.e., $i^{\text{location}} \sim U(N+1, N+F)$ on line 4 of the algorithm). Subsequently, the fully defined solutions of the six neighbors are obtained by running Algorithms 5 and 6 sequentially (i.e., Modules II and III as shown in Fig. 5.2), and the best among them is identified (lines 8 and 9). If the best neighbor outperforms the incumbent solution, the former replaces the latter (lines 10 and 11). The algorithm is terminated if the current runtime (τ) goes beyond $\tau_{VNS}^{\max}$. Finally, the best found customer assignment ($\mathcal{N}^{U-}$ and $\mathcal{N}^{U+}$) and improved truck route obtained by allowing stops at flexible locations (B^* and $\widehat{t}^*$) are returned (line 13).

Algorithm 4 Module I − phase II (VNS)

1: **Input:** B^*, $\widehat{t}^*$, $\text{iter}_1^{\max}$, $\text{iter}_2^{\max}$, $\tau_{VNS}^{\max}$
2: **Output:** customer assignments and truck route
3: **while** $\tau < \tau_{VNS}^{\max}$ **do**
4:     **for** iter1 := 1 **to** $\text{iter}_1^{\max}$ **do**
5:         generate a neighbor (B_1^{l1}) around the current best solution (using Algorithm 2)
6:         **for** iter2 := 1 **to** $\text{iter}_2^{\max}$ **do**
7:             generate neighbors (B_1^{l2}, B_2^{l2}, …, B_6^{l2}) to B_1^{l1}
8:             run Modules II and III (Algorithms 5 and 6) sequentially for B_1^{l2}, B_2^{l2}, …, B_6^{l2}
9:             set $B = B_{\text{argmin}(B_1^{l2}, B_2^{l2}, \ldots, B_6^{l2})}$ and $\widehat{t}^B = \min(\widehat{t}^{B_1^{l2}}, \widehat{t}^{B_2^{l2}}, \ldots, \widehat{t}^{B_6^{l2}})$
10:             **if** $\widehat{t}^B \leq \widehat{t}^*$ **then**
11:                 $B^* \leftarrow B$, $\widehat{t}^* \leftarrow \widehat{t}^B$
12:             **end if**
13:         **end for**
14:     **end for**
15: **end while**
16: $\mathcal{N}^{U-} = \{i \in \mathcal{N}^U | i \in \mathcal{B}^*\}$ and $\mathcal{N}^{U+} = \{i \in \mathcal{N}^U | i \notin \mathcal{B}^*\}$
17: **return** $\mathcal{N}^{U-}$, $\mathcal{N}^{U+}$, B^* and $\widehat{t}^*$

5.3.2 Module II: drone assignment and routing

For each tested truck route in Module I, Module II is used to identify the drone-customer assignment and drone sorties (see line 9 in Algorithm 3 and line 8 in Algorithm 4). A rule-based heuristic is developed for Module II such that the truck waiting and drone hovering are minimized. In particular, a priority is given to drone sorties that do not require truck waiting and have minimal drone hovering, and then the sorties are assigned in ascending order of truck waiting duration. A pseudocode of the developed heuristic is provided in Algorithm 5. First, a list of all feasible drone sorties is established, where each sortie is identified by a tuple (s, j, u) (line 3). Subsequently, a preprocessing step is conducted to calculate the actual truck travel time (A^T_s) and drone travel times (A^U_{sju}) from each truck stop based on the partial solution constructed in Module I (lines 4 and 5). The heuristic rule of minimizing truck waiting and drone hovering is developed in the loops of lines $6-13$ and $14-21$. First, the drone sorties that can be performed without truck waiting are considered (lines $6-13$). If a customer can be served by only one feasible drone sortie, then that sortie is assigned by default (lines 7 and 8). Otherwise, the assignments are selected such that the time delay between the truck and drone arrival at each truck stop is minimized (lines 11 and 12). In both cases, any assigned sortie and its mutually exclusive sorties are removed from the list $\mathcal{L}$ (lines 9 and 13). Specifically, once a customer location is assigned to a drone, it should not be served again by a drone launched from a different truck stop. Furthermore, the assigned drone and launching truck stop cannot be selected again to serve a different customer. Similar to the loop on lines $6-13$, assignments of drone sorties are applied until all customers are served (lines $14-21$). Finally, the assignments of drone sorties and their routing are returned (line 22).

Algorithm 5 Module II − assignment of drones to customer locations and their routing

1: **Input:** B^* and time-based parameters
2: **Output:** assignments and routing of drones
3: initialize a list of all feasible sorties $\mathcal{L} = (s, j, u) \quad \forall s \in \mathcal{S}\setminus\{s_{\widehat{S}+1}\}, j \in \mathcal{N}^U, u \in \mathcal{U}$
4: determine actual truck travel time (A_s^T) from each stop $s \in \mathcal{S}\setminus\{s_{\widehat{S}+1}\}$ to the next stop
5: calculate travel time of each done (A_{sju}^U) if dispatched $s \in \mathcal{S}\setminus\{s_{\widehat{S}+1}\}$ to serve $j \in \mathcal{N}^U$ and return to the next truck stop
6: **while** a drone sortie $(s, j, u) \in \mathcal{L}$ can be performed without truck waiting **do**
7:     **if** location $j \in \mathcal{N}^U$ can be served only by a single feasible sortie in $\mathcal{L}$ **then**
8:         $x^*_{jus} \leftarrow 1$
9:         remove the corresponding sortie from the list $\mathcal{L}$
10:     **else**
11:         $(s^*, j^*, u^*) \leftarrow \text{argmin}_{s\in\mathcal{S}\setminus\{s_{\widehat{S}+1}\}, j\in\mathcal{N}^U, u\in\mathcal{U}}(A_s^T - A_{sju}^U|_{A_s^T \geq A_{sju}^U})|_{(s,j,u)\in\mathcal{L}}$
12:         $x^*_{j^*u^*s^*} \leftarrow 1$
13:         remove sorties $(s, u, j^*) \quad \forall s \in \mathcal{S}\setminus\{s_{\widehat{S}+1}\}, u \in \mathcal{U}$ and $(s^*, u^*, j) \quad \forall j \in \mathcal{N}^U$ from the list $\mathcal{L}$
14:     **end if**
15: **end while**
16: **while** at least one customer $j \in \mathcal{N}^U$ is not served **do**
17:     **if** location $j \in \mathcal{N}^U$ can be served only by a single feasible sortie in $\mathcal{L}$ **then**
18:         $x^*_{jus} \leftarrow 1$
19:         remove the corresponding sortie from the list $\mathcal{L}$
20:     **else**
21:         $(s^*, j^*, u^*) \leftarrow \text{argmin}_{s\in\mathcal{S}\setminus\{s_{\widehat{S}+1}\}, j\in\mathcal{N}^U, u\in\mathcal{U}}(A_{sju}^U - A_s^T|_{A_{sju}^U \geq A_s^T})|_{(s,j,u)\in\mathcal{L}}$
22:         $x^*_{j^*u^*s^*} \leftarrow 1$
23:         remove sorties $(s, u, j^*) \quad \forall s \in \mathcal{S}\setminus\{s_{\widehat{S}+1}\}, u \in \mathcal{U}$ and $(s^*, u^*, j) \quad \forall j \in \mathcal{N}^U$ from the list $\mathcal{L}$
24:     **end if**
25: **end while**
26: **return** $x^*_{jus} \quad \forall j \in \mathcal{N}, u \in \mathcal{U}, s \in \mathcal{S}$

5.3.3 Module III: scheduling decisions

The mathematical programming model with objective function (6.1) and constraints (6.2)-(5.37) is decomposed in Module III to independently optimize the scheduling decisions at each truck stop. The tasks to be optimized at each truck stop $s \in \mathcal{S}$ are launching and landing of drones. Besides, if the truck stop is at a customer location, then the truck operator's service start and end time for that location must also be sequenced. To model the scheduling decisions at each truck stop as an optimization problem, the following three mutually exclusive subsets of locations are identified for each truck stop $s \in \mathcal{S}$: (i) $\mathcal{N}_s^+$ is the subset of customer locations that are visited by drones to be recovered at truck stop $s \in \mathcal{S}$, (ii) $\mathcal{N}_s^0$ is the subset containing the customer location that is assigned to be the truck stop $s \in \mathcal{S}$, otherwise $\mathcal{N}_s^0 = \phi$, and (iii) $\mathcal{N}_s^-$ is the subset of customer locations to be visited by drones launched from truck stop $s \in \mathcal{S}$.

Algorithm 6 provides the pseudocode of Module III. The outputs of Modules I and II are used as inputs and the decomposed model is then solved sequentially for each truck stop $s \in \mathcal{S}$. In the decomposed model, the binary variables y_{is} and x_{ius} are fixed (lines 8 and 9) based on the truck routing and drone-customer assignments in Modules I and II, respectively, as illustrated in Fig. 5.2. The arrival times of both truck and drones are fixed based on the previous run of the decomposed model (lines 10 and 11). Furthermore, the sequencing binary variable w_{ijs} is limited to customer locations in sets $\mathcal{N}_s^+$, $\mathcal{N}_s^0$, and $\mathcal{N}_s^-$ and to the truck stop $s \in \mathcal{S}$ for each run of the decomposed model. Therefore, the maximum number of binary variables in each model run does not exceed $(2 \times U + 1)^2$. For each run, the arrival times of truck and drones are stored (lines 12 and 13) to be used in the subsequent model run. Finally, a fully defined solution to the delivery problem is obtained at the end of Module III. An illustrative solution representation of Module III at a truck stop is shown in Fig. 5.4, where $\mathcal{N}_s^+ = \{i_2, i_{11}\}$, $\mathcal{N}_s^0 = \{i_6\}$, and $\mathcal{N}_s^- = \{i_3, i_9, i_{13}\}$. Given the assignment of drones (both recoveries and launches) to a truck stop (i.e., customer location i_6 in the illustrative example), the tasks are sequenced as following: launch drone u_2 $\rightarrow$ recover drone u_4 $\rightarrow$ recover drone u_3 $\rightarrow$ serve customer i_6 $\rightarrow$ launch drone u_4 $\rightarrow$ launch drone u_1.

Algorithm 6 Module III − scheduling of delivery tasks

1: **Input:** B^* and $x^*_{jus} \quad \forall j \in \mathcal{N}, u \in \mathcal{U}, s \in \mathcal{S}$
2: **Output:** fully defined solution for the delivery problem
3: define binary parameter $y^*_{is} \quad \forall i \in \mathcal{F}, s \in \mathcal{S} \quad$ based on B^*
4: **for** each truck stop $s' \in \mathcal{S}$ **do**
5:     minimize objective function (6.1)
6:     *S.t.*
7:         constraints (6.2) − (5.34) by updating set $\mathcal{S}$ to $\mathcal{S} = \{s'\}$
8:         $x_{jus} = x^*_{jus} \quad \forall j \in \mathcal{N}, u \in \mathcal{U}, s \in \{s'\} \subseteq \mathcal{S}$
9:         $y_{is} = y^*_{is} \quad \forall i \in \mathcal{F}, s \in \{s'\} \subseteq \mathcal{S}$
10:         $a^T_s = a^{T*}_s \quad \forall s \in \{s'\} \subseteq \mathcal{S}$
11:         $a^U_i = a^{U*}_i \quad \forall i \in \mathcal{N}^+_{s'}$
12:     $a^{T*}_{s'+1} \leftarrow a^T_{s'+1}$
13:     $a^{U*}_i \leftarrow a^U_i \quad \forall i \in \mathcal{N}^-_{s'}$
14: **end for**
15: **return** fully defined solution

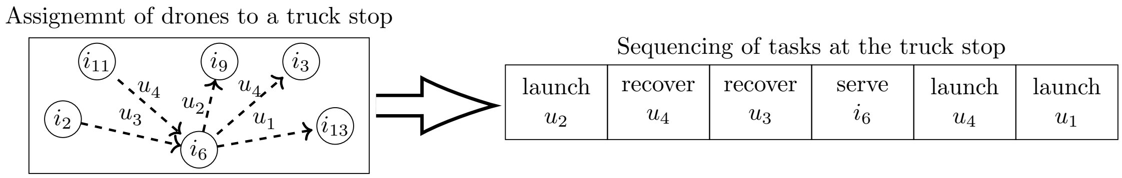

Figure 5.4: An illustrative solution representation of sequencing decisions in Module III

5.4 Computational experiments

This section includes extensive computational experiments to evaluate the impact of adopting flexible drone launch and recovery sites. The MILP model has been developed on GAMS 30.1 and solved using CPLEX 12.9. The proposed OTS algorithm is implemented in Python 3.6. All experiments were performed on a computer with an AMD Ryzen $7-2700X$ @ 3.7 GHz processor and 16 GB RAM. First, the global optimal solutions are obtained for small size problems using the proposed MILP model. We also use these instances to benchmark the performance of the OTS algorithm. Next, we demonstrate the capability of the proposed solution approach to solve relatively large-sized problems. Finally, a comprehensive analysis is conducted on large instances to assess the impact of three critical parameters, namely, number of flexible locations, UAV fleet size, and flight range. In order to quantify the impact of considering flexible locations, we benchmark its performance against the problem that assumes truck stops to be at customer locations only (i.e., $F = 0$), which is referred to as the collaborative truck-drone routing and scheduling problem with restricted drone launch and recovery locations (CTDRSP-RL). Note that the proposed OTS algorithm is adapted to solve large instances of CTDRSP-RL by restricting the search of VNS to customer-only locations.

5.4.1 Experimental setup of problem instances and OTS parameters

Test instances are created by generating randomly distributed customer and flexible stop locations on a delivery region. We consider three network configurations with 8, 25 and 50 customers distributed within areas of 15×15, 25×25, and 35×35 miles^2, respectively. Besides, we consider 10% of customers to be served only by a truck, i.e., $|\mathcal{N}^T| = \lceil 0.1 \times N \rceil$. The number of flexible locations (F) is set as $2N$ in our initial analysis. For each problem configuration (delivery locations, flexible stops, truck-only customers), we randomly generate and solve 10 instances. For each instance, the fleet of delivery vehicles is assumed to comprise a truck carrying four drones. The maximum number of allowable truck stops ($\widehat{S}$) is fixed at $\lfloor N/2 \rfloor$. Similar to prior works, the velocity of both truck and drones is set at 25 mph [33, 34, 55], and the drone flight range is considered to be 12 miles [47]. The travel time of vehicles and drone endurance are computed based on the coordinates of customer and flexible locations. Specifically, the truck travel time is determined by considering the rectilinear distance between nodes, while the drone travel time is calculated based on Euclidean distance. The motivation for such an approach is to emulate the truck path on a road network, and the drone route in the low-altitude airspace [34]. Customer service times by truck operator and drone are set at 0.5 and 1 minute, respectively [52]. Furthermore, for all numerical experiments, we compare the performance of CTDRSP-FL with CTDRSP-RL in terms of the (i) average percentage reduction in delivery completion time (referred to as saving percent), (ii) improvement in drone utilization (η^{IMP}), which is computed as the percentage improvement in the number of customers that are served by drones instead of truck, and (iii) percent of truck stops that are selected at flexible sites (S^F).

Since the first phase of the OTS (i.e., the SA algorithm) is used to provide a good initial solution (i.e., customer assignments and truck route without flexible stops) for Phase 2, its runtime is limited to 5 minutes. For the second phase of the OTS (i.e., the VNS algorithm), the runtime threshold is set to 60 minutes since it is commonly adopted by prior research works on truck-drone routing [48, 52]. Further, the numerical results in Section 5.4.3 illustrate the convergence patterns for different problem sizes and support the chosen runtime threshold. The other key parameters of SA and VNS algorithms are tuned by considering a representative set of instances using CALIBRA, a well-established fine-tuning procedure employing a Taguchi's fractional factorial experimental design combined with a local search [86]. The ranges explored for T^0, K, and iter^{max} (SA-related parameters) are [100, 1000], [0.8, 0.99], and [10, 30], respectively. The best performance is obtained for values: $T^0 = 663$, $K = 0.94$, and iter$^{max} = 23$. Similarly, the values explored for the number of iterations of the VNS algorithm (iter$_1^{\max}$ and iter$_2^{\max}$) is also between [10, 30], and the selected values are 13 and 15 iterations, respectively.

5.4.2 Results for Small Test Instances

The results of test instances with 8 customers for both the MILP model and OTS algorithm are presented in Table 5.1. The proposed models are solved for the new variant (CTDRSP-FL) and the benchmark problem (CTDRSP-RL). When employing the MILP model, all the instances converged to proven optimal solutions. It is evident that the consideration of flexible sites for drone LARO can yield substantial savings in the delivery completion time as opposed to restricting them to customer locations. On average, the package delivery was 14.4% faster for the newly introduced variant. Besides, the CTDRSP-FL achieved lower delivery completion for all the 10 instances with improvements ranging between 7.0% - 27.5%. Also, the gap in Table 5.1 denotes the percentage deviation between the solution obtained by OTS algorithm and global optimal. For both CTDRSP-RL and CTDRSP-FL, the proposed OTS algorithm is able to achieve the global optimal for a majority of the cases, and near-optimal solution in other instances. The average gap is less than 1% for both CTDRSP-RL and CTDRSP-FL. With regard to the computational performance, the average runtime to obtain the optimal solutions using the MILP model was 6,490 seconds (min: 930, max: 14,028). For the OTS algorithm, all instances in the first phase converged to the minimum temperature within 300 seconds (min: 62, average: 171, max: 288), while the second phase could obtain the results in Table 5.1 (which are optimal for most instances) in less than 60 seconds (min: 4, average: 17, max: 52).

Table 5.1: Delivery completion time ($\hat{t}$) of ten instances with $N = 8$ using the MILP model and OTS algorithm

	CTDRSP-RL			CTDRSP-FL			
Instance	MILP	OTS	Gap %	MILP	OTS	Gap %	Saving %
1	109.7	109.7	0.0%	100.3	100.3	0.0%	8.6%
2	105.7	105.7	0.0%	82.9	82.9	0.0%	21.6%
3	86.8	89.6	3.3%	69.9	71.4	2.1%	19.5%
4	129.4	129.4	0.0%	117.1	117.1	0.0%	9.5%
5	103.7	103.7	0.0%	88.3	88.3	0.0%	14.8%
6	136.0	136.0	0.0%	98.6	98.7	0.1%	27.5%
7	113.1	113.7	0.5%	101.4	101.4	0.0%	10.4%
8	100.0	100.5	0.5%	86.7	88.0	1.4%	13.3%
9	117.9	117.9	0.0%	103.6	104.5	0.8%	12.1%
10	93.6	93.6	0.0%	87.1	88.0	1.1%	7.0%
Average	**109.6**	**110.0**	**0.4%**	**93.6**	**94.0**	**0.6%**	**14.4%**

Fig. 5.5 shows the characteristics of the optimal solutions for $N = 8$. In Fig. 5.5 (a), the percentage improvement in drone utilization when considering flexible truck stops is shown as opposed to restricting them to customer locations. The drone utilization is calculated as the ratio of customers served by drones to the total number of droneable locations. For all cases under consideration, drone utilization is better for the proposed CTDRSP-FL. This may be because of the fact that an additional customer must be visited by a drone every time a flexible site is used as a truck stop instead of a customer location, thereby engaging the drones in more sorties. Moreover, a flexible stop may allow parallelization of drone delivery such that the fleet synchronization at a truck stop is efficient (i.e., minimum waiting time), which, in turn, enables faster delivery completion. Fig. 5.5 (b) illustrates that 77% of

truck stops are at flexible locations. Another interesting observation in the optimal solution of CTDRSP-FL is that the flexible stops that were neither close nor far away from the depot were mostly chosen, while stops in the periphery were rarely selected. Specifically, we observed that the majority of the chosen flexible stops are within the area of 25% to 75% of the distance from the depot to the delivery area borders. This suggests that it is beneficial to shorten the truck path by employing drones to faraway locations.

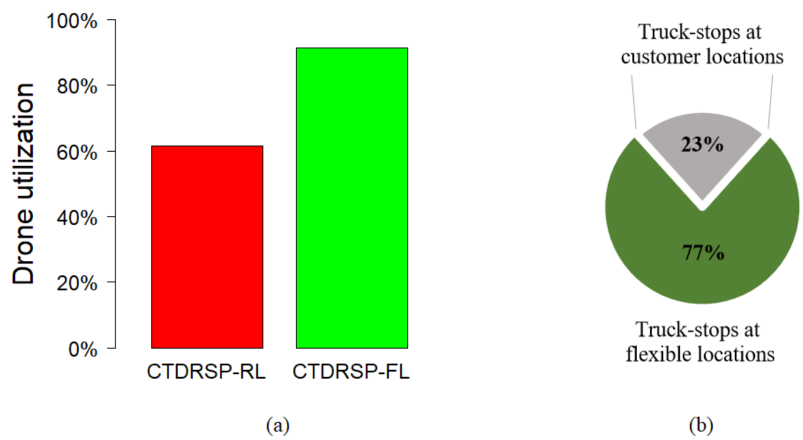

Figure 5.5: Characteristics of optimal solutions for $N = 8$: (a) drone utilization of CTDRSP-RL and CTDRSP-FL; and (b) percentage of truck stops at customer and flexible locations for CTDRSP-FL

5.4.3 Performance of OTS algorithm on larger instances

The proposed OTS algorithm is employed to obtain solutions for larger problem instances ($N = 25$ and 50) of both CTDRSP-RL and CTDRSP-FL, and the corresponding results are presented in Table 5.2. The average savings in delivery completion time for the two problem sizes is 12.5% and 11.3%, respectively. In contrast to the results of $N = 8$, the average savings decreased marginally as problem size increases. In particular, the average improvement achieved by CTDRSP-FL decreases by 1.9% (from 14.4% to 12.5%) when $N = 25$ instead of $N = 8$ and by only 1.2% (from 12.5% to 11.3%) when an additional 25 customer locations are considered. Nevertheless, the standard deviation of the improvement achieved tends to decrease as the problem size increases from 8 to 50 customers. In other words, the percentage savings in delivery time is consistent and close to average as problem size increases, especially for 50 customers. Besides, consistent with our initial findings, the drone utilization is higher for all the instances if flexible drone LARO are allowed. Therefore, even for large size problems, using flexible locations for drone LARO can achieve substantial savings in delivery completion time and better drone utilization.

For all instances with $N = 25$ and 50, the SA phase terminated only upon reaching the runtime threshold of 300 seconds. Subsequently, the VNS phase was executed for one hour, which resulted in the convergence patterns in Fig. 5.6. It can be observed that the convergence pattern for the different problem sizes is similar, but the convergence speed becomes slower with the increase in the problem size. For example, while the best solutions for $N = 8$ (which are optimal for most instances according to Table 5.1) could be achieved in less than 60 seconds, runtimes of $1,600$ and $2,500$ seconds were required to obtain an average gap of 0.5% to the best-known solutions for the instances with $N = 25$ and 50, respectively. Thus, Fig. 5.6 can provide operators with insights on the tradeoff between solution quality and corresponding

Table 5.2: Delivery completion time of ten instances with $N = 25$ and $N = 50$ using the OTS algorithm

		Delivery completion time ($\widehat{t}$)				
N	Instance	CTDRSP-RL	CTDRSP-FL	Saving %	η^{IMP}	S^F
25	1	320.0	262.4	18.0%	30.8%	44.4%
	2	322.6	276.0	14.4%	38.5%	55.6%
	3	296.4	262.4	11.5%	46.2%	66.7%
	4	290.2	263.6	9.2%	30.8%	44.4%
	5	301.4	271.3	10.0%	21.4%	33.3%
	6	265.2	237.5	10.4%	12.6%	18.2%
	7	264.8	234.3	11.5%	23.1%	33.3%
	8	308.6	269.2	12.8%	30.8%	44.4%
	9	292.4	253.5	13.3%	38.5%	55.6%
	10	300.0	259.9	13.4%	30.8%	44.4%
	Average	**296.2**	**259.0**	**12.5%**	**30.4%**	**44.0%**
50	1	568.4	498.3	12.3%	40.0%	50.0%
	2	496.7	444.4	10.5%	28.0%	30.0%
	3	509.1	449.4	11.7%	28.0%	35.0%
	4	550.8	480.0	12.9%	24.0%	30.0%
	5	596.4	539.6	9.5%	40.0%	50.0%
	6	557.6	498.1	10.7%	28.0%	35.0%
	7	519.2	456.8	12.0%	19.2%	30.0%
	8	540.4	485.9	10.1%	28.0%	30.0%
	9	515.9	458.7	11.1%	7.7%	15.0%
	10	534.5	466.8	12.7%	25.9%	45.0%
	Average	**538.9**	**477.8**	**11.3%**	**26.9%**	**35.0%**

required computational time.

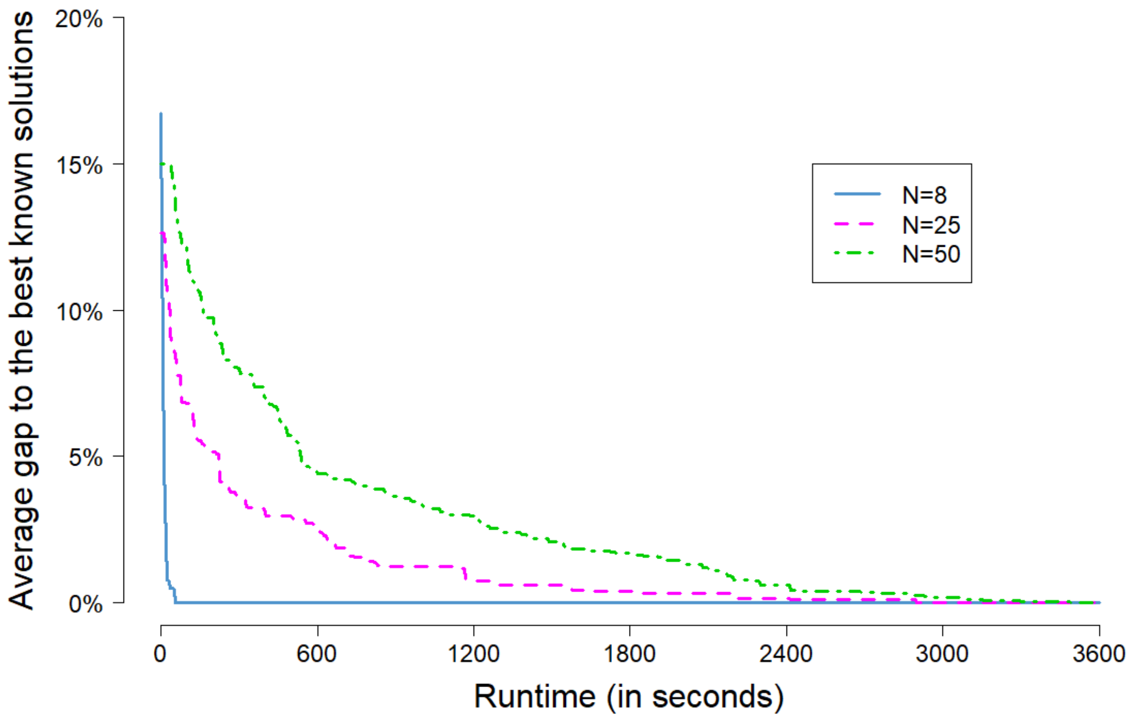

Figure 5.6: Convergence of the second phase in the OTS algorithm for different problem sizes

5.4.4 Impact of critical parameters on delivery completion time

The delivery completion time may be affected by the problem parameters. In this section, we assess the sensitivity of the CTDRSP-FL to three critical parameters by considering three levels for each of them, as shown in Table 5.3. First, the influence of decreasing the number of flexible locations (that is set at $2N$ in the baseline experiments) by 50% and 75% is evaluated. Second, the impact of UAV fleet size per truck is assessed. Finally, longer flight ranges than those used in the baseline experiments are considered. We vary the factor levels one at a time to assess its impact, while fixing the other parameters to the baseline setting. Moreover, similar to the previous analysis, we run 10 replications for each problem configuration. While the three levels of each factor are evaluated, values of the other two factors are fixed based on the baseline setting in Section 5.4.1. Thus, a total of 180 test instances (3 factors $\times$ 3 levels $\times$ 10 replications $\times$ 2 variants) are generated and analyzed.

Table 5.3: Levels of key factors affecting the impact of using flexible locations

	Levels		
Factor	Low	Medium	High
Number of flexible locations	$0.5N$	N	$2N$
Number of UAVs	2	4	6
Flight range (in miles)	12	15	20

Table 5.4 provides the impact of varying the number of potential flexible locations (F) on the delivery completion time, drone utilization, and assignment of truck stops to flexible sites. The same tested instances in the previous section are considered. For both 25 and 50 customer instances, the reduction in delivery completion time increases as the number of potential sites for flexible drone LARO is increased. This could be because the feasible solution space is larger when the number of potential flexible locations is varied from $0.5N$ to $2N$, thereby increasing the likelihood of establishing a flexible stop that facilitates better synchronization of truck and drones to serve the nearby customers. For this same reason, the number of flexible stops that are selected as truck stops tends to increase with F, as shown in Table 5.4. Likewise, the percentage improvement in drone utilization increases when F is increased. Since

drones have limited flight range, restricting their LARO to customer-only locations may limit the number of feasible drone sorties. However, increasing the number of flexible stops improves the likelihood of deploying a drone, while shortening the truck route and reducing its waiting time. With regards to the characteristics of the best solution obtained from the OTS algorithm, we observed that when a flexible location is closer to a customer (e.g., within one mile), it is less likely to be selected as a truck stop. Moreover, if it is selected, the resulting reduction in delivery time is usually marginal. Also, we observed that a shorter truck route can yield substantial savings in delivery completion time. Therefore, flexible locations around the depot are more likely to be selected. In particular, flexible locations should neither be very near to the depot nor the periphery of the delivery area. Furthermore, irrespective of the allowable number of flexible sites (F), analysis of the different instances shows that, in certain cases, a solution with fewer flexible locations serving as truck stops can achieve a better reduction in delivery time than a plan with more such locations. Thus, not only the number of flexible truck stops but also their relative positions to the depot and customer locations are crucial to efficiently reduce delivery time in the CTDRSP-FL.

Table 5.5 shows the impact of varying the drone fleet size. It is clear that operating more drones per truck can reduce the delivery completion time. The delivery time obtained for CTDRSP-FL is better than CTDRSP-RL even if only two drones are employed. However, it is also observed that there is no monotonic relation between

Table 5.4: Average results for using different levels of number of flexible locations

N	F	Delivery completion time			η^{IMP}	S^F
		CTDRSP-RL	CTDRSP-FL	Saving %		
25	$0.5N$	296.2	277.2	6.4%	10.7%	16.7%
	N	296.2	268.7	9.3%	21.4%	32.2%
	$2N$	296.2	259.0	12.5%	30.4%	44.0%
50	$0.5N$	538.9	503.3	6.6%	7.8%	12.0%
	N	538.9	498.1	7.6%	15.3%	22.5%
	$2N$	538.9	477.8	11.3%	26.9%	35.0%

the number of drones and the corresponding savings in delivery completion time by using flexible locations, especially for test instances with $N = 25$. Also, an important finding is that using flexible truck stops can achieve faster deliveries with fewer drones as opposed to restricting LARO to customer locations (CTDRSP-RL) and employing more drones. For example, in the instances of $N = 25$, the average delivery completion time for CTDRSP-FL with two drones is lower than CTDRSP-RL with six drones, as shown in Table 5.5. Likewise, for instances with 50 customers, the delivery completion time with four drones and flexible LARO is earlier than adopting six UAVs but restricting truck stops to customer locations. Furthermore, the average improvement in drone utilization with $U = 4$ and 6 is similar for both problem sizes. Thus, using flexible locations for drone LARO can aid the operators not only in reducing delivery time but also in better utilizing their available fleet of drones. It is observed that the proportion of stops that were flexible tends to be higher for $N = 25$ when compared to instances with 50 customers, as shown in Table 5.5. As the number of customer locations increases within a delivery region, the likelihood of choosing some of them for drone LARO also increases, which, in turn, could lead to the selection of fewer flexible stops. Also, S^F increases with the drone fleet size for instances with 25 customers, but such a trend was not observed for $N = 50$.

Table 5.5: Average results for using different levels of number of drones

		Delivery completion time				
N	U	CTDRSP-RL	CTDRSP-FL	Saving %	η^{IMP}	S^F
25	2	300.1	277.6	7.5%	23.9%	34.6%
	4	296.2	259.0	12.5%	30.4%	44.0%
	6	287.9	257.9	10.4%	31.4%	48.3%
50	2	543.2	526.6	3.1%	8.4%	9.5%
	4	538.9	477.8	11.3%	26.9%	35.0%
	6	516.7	451.8	12.6%	24.4%	33.0%

The reduction in delivery completion time of the two variants, η^{IMP}, and S^F achieved by employing drones with different flying ranges are presented in Table 5.6. Utilizing longer range drones results in faster delivery completion for both CTDRSP-RL and CTDRSP-FL. A drone with a longer range could be capable of reaching more customer locations (due to larger coverage) from a given truck stop. In other words, sorties that were not feasible with a short-range drone might now be possible. Thus, such a capability allows greater parallelization of delivery tasks and could be a reason for the observed pattern in Table 5.6. Nevertheless, allowing flexible truck stops always yielded superior delivery time as opposed to restricting them to customer-only locations. Also, the utilization of drones is not drastically affected as the range increases. This may be because the range primarily impacts the launch and recovery locations, but not the number of customers served by the drone. Most importantly, allowing flexible stops can enable faster delivery with short-range UAVs when compared to CTDRSP-RL employing long-range drones. For example, in Table 5.6, the average delivery time for $N = 50$ and flight range of 12 miles is 477.8 minutes if flexible locations are used, which is lower than operating drones with longer flight ranges but without allowing flexible locations (i.e., CTDRSP-RL). Also, the S^F is not substantially affected by the range of drones.

Table 5.6: Average results for using different levels of flight range

		Delivery completion time				
N	Flight range	CTDRSP-RL	CTDRSP-FL	Saving %	η^{IMP}	S^F
25	12	296.2	259.0	12.5%	30.4%	44.0%
	15	261.4	240.8	7.9%	31.5%	46.7%
	20	250.1	236.5	5.4%	32.3%	47.8%
50	12	538.9	477.8	11.3%	26.9%	35.0%
	15	502.3	461.8	8.1%	29.2%	37.0%
	20	486.0	454.7	6.4%	24.0%	30.5%

5.5 Conclusion

This chapter addresses the problem of last-mile delivery using a fleet of heterogeneous drones working in tandem with a single truck. In contrast to most of the previous works, this study does not restrict drone launch and recovery operations to be at customer locations. Specifically, we propose a new variant of the truck-drone tandem called the collaborative truck-drone routing and scheduling problem with flexible launch and recovery locations (CTDRSP-FL). While prior research used drones to aid the trucks in package delivery, the proposed variant seeks to better exploit the drones for deliveries while embracing the assistance from truck stopping at flexible sites (feasible non-customer locations). Furthermore, the addressed problem variant considers the scheduling of truck operator tasks at a given stop, which possibly include customer service and launch/recovery of multiple drones.

A new MILP model is formulated to optimally solve the CTDRSP-FL with the objective of minimizing delivery completion time. The proposed optimization model considers several practical aspects, such as the necessity of serving some customers by trucks and scheduling of tasks at truck stops (e.g., launching and recovery of drones and customer service by the truck operator). Furthermore, an optimization-enabled two-phase search (OTS) algorithm is developed to efficiently solve large instances. While the first phase shares many characteristics with existing studies (e.g., truck-drone routing through a network of customer nodes), the second phase is developed to serve as an improvement module that leverages the presence of flexible drone launch and recovery sites. Numerical analysis shows that substantial savings in delivery completion time ($> 25\%$ in some instances) can be achieved by using non-customer locations for drone launch and recovery operations. Besides, the utilization of drones was also substantially higher for the proposed variant as opposed to the traditional truck-drone tandem. Our findings also led to several practical insights on selecting the potential flexible stops such as avoiding locations with high demand density and selecting areas that are likely to shorten the truck route.

Chapter 6

Aerial and ground drone assisted delivery routing with autonomous repositioning and docking stations

Hybrid systems of a truck working in tandem with unmanned vehicles (aerial and ground drones) are proven to substantially reduce delivery time [8, 11, 52]. However, a potential source of inefficiency is that the truck waits to recover the unmanned vehicles, especially that ground drones may spend relatively long time until the customers picks up their packages. This time consuming task can be eliminated by utilizing a network of stations recovering the unmanned vehicles after performing their delivery operations and docking them. Therefore, the truck is allowed to move directly after dispatching the unmanned vehicles, thereby yielding faster last-mile deliveries for the next customers. Moreover, the truck can return faster to the depot and deployed for another batch of orders. On the other hand, rather than assuming that the unmanned vehicles accompanying the truck throughout its route, they can gather at a truck stop (not necessarily the depot) when required and then separate permanently once the truck dispatch the unmanned vehicle for delivery. Therefore, the idle time of drones can be minimized as they are required only for a portion of the truck route travel time. The investment cost for establishing the supporting stations is expected to be relatively low as the existing public properties can be utilized (possibly after minor modifications) for docking. For example, there is a patent for docking drones on lamp posts [87]. In addition, parking of robots can be similar to that of scooters and bikes [88].

A schematic illustration of the role of supporting stations is provided in Fig. 6.1. It shows a truck route starting from the depot with the orders to be delivered. Next, the truck stops to pick up a drone coming from a nearby station to its route. The operator loads the drone to truck and continues its route until another stop near to the customer. Then, the truck operator retrieves the drone and respective order and launches the drone for the last-mile delivery. The truck continues its route towards the depot, while the drone conducts the delivery to customer and then travel for docking at the closest available station. Whereas the schematic illustration in Fig. 6.1 shows a typical operation in this study, there are many other possible variants as follows: (i) the truck can load drones at the depot, (ii) the drones can be collected at and launched from the depot, (iii) the truck can stop at a station or a non-customer location to pick up and launch drones, (iv) the truck can stop at a customer location to deliver the order by the operator and launch a drone to another customer, and (v) the truck can stop at multiple locations along its route after a drone collection before launching the drone.

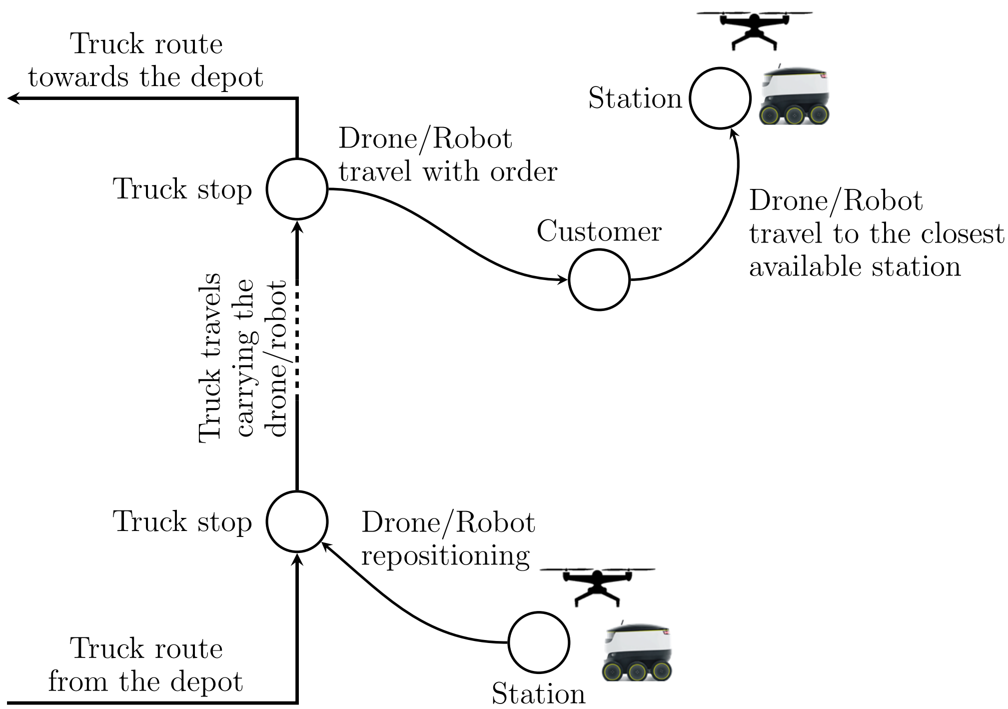

Figure 6.1: Schematic illustration of the role of supporting stations and autonomous repositioning

Utilization of supporting stations for last-mile delivery by a combination of truck and unmanned vehicles is studied in the literature in different ways [8, 14, 55, 89, 90]. Karak and Abdelghany considered an equipped truck for carrying drones stops only by stations in the delivery area for attaching packages to drones before launching, while all customers are served only by drones [8]. Also, the truck is assumed to wait at each station on its route until collecting all respective drones, and all drones must return to the depot by the end of the delivery plan. A similar approach is developed by Ostermeier et al. [14] but with using delivery robots instead of drones, and allowing the truck to stop at drop-off locations in addition to the stations, while the customers are only visited by robots. However, they avoided the truck waiting to recover the robots after delivery tasks, where they return to the closest stations to the respective customers for recharging and parking until a next truck delivery tour. A two-tier delivery system is developed by Bakach et al. [89] comprising a truck distributing the goods from a depot to stationary hubs, then robots perform the last-mile deliveries from the hubs to respective customers. Wang and Sheu [55] assumed docking hubs with an unlimited supply of drones, and that the landing operations can be conducted only at these hubs or the depot. Hong et al. [7] developed an approach to design a network of recharging stations for drones to extend the service coverage. However, all drones are assumed to dispatch directly from and return to the depot, not a truck, thereby losing the advantages of the economies of scale. Similarly, Shavarani et al. [90] jointly optimize the locations of warehouses and recharging stations in the delivery area with the minimum total cost. This chapter contributes to the literature as following:

- A hybrid system with three types of vehicles (truck, drones, and robots) is considered along with a network of supporting stations and autonomous repositioning capabilities of unmanned vehicles.

- The truck is allowed to stop by three different types of locations - customers (to directly deliver the packages), launching/drop-off sites, and stations.
- Eligibility of vehicles to perform the last-mile deliveries is respected. For example, some customers can be served by truck or drone, truck or robot, or truck only.
- A MILP model is formulated to optimally solve the delivery problem of using a hybrid truck-drone-robot system with supporting stations and repositioning capabilities.
- Routing of vehicles and scheduling of delivery operations are obtained with a detailed plan about which drones/robots to use and their collection and launching/drop-off locations.
- A scenario of using the existing fleet of traditional trucks (not equipped for carrying drones nor robots) is studied to assess the impact of investing in replacing the traditional trucks with hybrid systems.
- The impact of operating repositioning-enabled drones is evaluated in terms of estimated reduction in delivery time to exhibit the outcome of investing in advanced models of drones and autonomous recharging (or battery replacement) facilities at stations.

6.1 Problem description

This chapter addresses the problem of last-mile package deliveries from a depot to a set of customers using a truck working in tandem with multiple heterogeneous aerial and ground drones. In this chapter, the aerial drones are referred to as unmanned aerial vehicles (UAVs) and the ground drones are denoted as autonomous delivery robots (ADRs). The delivery network is comprised of a set of depot and low-cost stations (for UAV/ADR docking and recharging) $\mathcal{S} = \{s_0, s_1, s_2, s_3, \ldots s_S\}$, a set of drop-off sites $\mathcal{F} = \{f_1, f_2, f_3, \ldots f_F\}$ for temporary truck parking to launch UAVs and dispatch ADRs, and a set of customers $\mathcal{N} = \{i_1, i_2, i_3, \ldots i_N\}$ to be served by either truck, UAV, or ADR.

The three sets $\mathcal{S}$, $\mathcal{F}$, and $\mathcal{N}$ are divided into subsets as following. First, the set of depot and stations $\mathcal{S}$ are categorized into three subsets based on the eligibility of docking unmanned vehicle into $\mathcal{S}^D \subseteq \mathcal{S}$ for stations designed only for UAV docking, $\mathcal{S}^R \subseteq \mathcal{S}$ for ADR docking, and $\mathcal{S}\backslash\mathcal{S}^D\backslash\mathcal{S}^R$ for docking both vehicle types. Second, the drop-off sites can be used for UAV launching only ($\mathcal{F}^D \subseteq \mathcal{F}$), ADR dispatching only ($\mathcal{F}^R \subseteq \mathcal{F}$), or for dropping both vehicles ($\mathcal{F}\backslash\mathcal{F}^D\backslash\mathcal{F}^R$). Third, the set of customers $\mathcal{N}$ is categorized into locations can be served by UAV or truck ($\mathcal{N}^D \subseteq \mathcal{N}$), ADR or truck ($\mathcal{N}^R \subseteq \mathcal{N}$), truck only ($\mathcal{N}^T \subseteq \mathcal{N}$), or any of the three vehicle types ($\mathcal{N}\backslash\mathcal{N}^D\backslash\mathcal{N}^R\backslash\mathcal{N}^T$). Furthermore, with respect to launching UAVs and dispatching ADRs, customer locations can be used for UAVs only ($\mathcal{N}^{WD} \subseteq \mathcal{N}$), ADRs only

($\mathcal{N}^{WR} \subseteq \mathcal{N}$), both launching UAVs and dispatching ADRs ($\mathcal{N}^{WDR} \subseteq \mathcal{N}$), or cannot be used for launching UAVs nor dispatching ADRs ($\mathcal{N} \backslash \mathcal{N}^{WD} \backslash \mathcal{N}^{WR} \backslash \mathcal{N}^{WDR}$).

The truck is allowed to stop by any location in the aforementioned three sets, resulting in a set of potential truck stops $\mathcal{P} = \mathcal{S} \cup \mathcal{F} \cup \mathcal{N}$. Among those locations, up to K truck stops can be selected, where $\mathcal{K} = \{k_0, k_1, k_2, k_3, \dots k_K, k_{K+1}\}$ represents the actual set of truck stops and the indices $\{k_0, k_{K+1}\}$ refer to starting and ending the truck route at the depot. The fleet of delivery vehicles consists of a truck, a set of heterogeneous UAVs $\mathcal{D} = \{d_1, d_2, d_3, \dots d_D\}$, and a set of heterogeneous ADRs $\mathcal{R} = \{r_1, r_2, r_3, \dots r_R\}$.

At the beginning of the delivery planning horizon, the truck is assumed to be at the depot while the positions of the fleet of UAVs and ADRs are known (i.e., each autonomous vehicle is at a specific station in set $\mathcal{S}$). Each UAV $d \in \mathcal{D}$ and ADR $r \in \mathcal{R}$ takes a travel time of E^D_{ds} and E^R_{rs}, respectively, to reach a station $s \in \mathcal{S}$ from the respective initial position. Such relocation can be applied only if the travel time of UAV $d \in \mathcal{D}$ and ADR $r \in \mathcal{R}$ is within the allowed ranges Q^D_d and Q^R_r, respectively. Furthermore, a relocation is considered when a UAV or ADR is assigned to collaborate in the delivery plan, where the autonomous vehicle and truck are coordinated to rendezvous at a station $s \in \mathcal{S}$. Then, the truck driver can either attach a package to the autonomous vehicle and dispatch it to perform a delivery task, or pick the vehicle up and load it on the truck to be dispatched afterwards from another stop on the truck route. However, the truck can concurrently accommodate up to C^D UAVs and C^R ADRs at any part of its route. The travel time of truck from a potential truck stop p to $p' \in \mathcal{P}$ is denoted as $T^T_{pp'}$, while the flying time of UAV $d \in \mathcal{D}$ and travel time of ADR $r \in \mathcal{R}$ from truck stop $p \in \mathcal{P}$ to serve customer $i \in \mathcal{N}$ and return to the closest docking station are denoted as T^D_{dpi} and T^R_{rpi}, respectively. A delivery package is attached to UAV $d \in \mathcal{D}$ or ADR $r \in \mathcal{R}$ only if the total travel is within the ranges thereof, G^D_d and G^R_r, respectively. A preparation time H^D_d and H^R_r for order and delivery vehicle retrieval is required for each UAV $d \in \mathcal{D}$ and ADR $r \in \mathcal{R}$, respectively. Furthermore, a service time of truck, UAV, and ADR (V^T_i, V^D_i, and V^R_i) are considered at each customer location $i \in \mathcal{N}$ depending on the assigned delivery vehicle. All customers must be served by either of the three vehicle types. At the end of the delivery plan, the truck must return to the depot. The objective in the addressed problem is to minimize the delivery completion time, represented by the latest arrival of vehicles (truck, UAVs, or ADRs) to their designated destinations.

The following decisions are optimized to obtain a delivery plan for the addressed problem. First, number of UAVs and ADRs to be loaded onto the truck before traveling from the depot are determined. Second, the truck route is specified, which must start and end at the depot and include the customers in set $\mathcal{N}^T$. In addition, the truck stops can be at other customer sites, docking stations, and drop-off sites. At each truck stop $k \in \mathcal{K}$, the driver can perform one or more of three tasks - (i) loading autonomous vehicle(s) onto the truck (which may incur idle time in waiting the vehicle(s) arrival) to be employed later on the truck route, (ii) unloading autonomous vehicle(s) and prepare it for a delivery operation, and (iii) launching/dispatching autonomous vehicle(s) not loaded on the truck (i.e., rendezvous with the truck at its stop). Third, a subset of UAVs and ADRs is selected to work in tandem with truck in performing delivery operations. Fourth, assignments of selected UAVs and ADRs to loading locations onto truck (if required) and to both launching/dispatching locations and customers are specified. Finally, departure and arrival times of all vehicles are scheduled. A MILP model is developed in this chapter to optimize these decisions and achieve the minimum delivery completion time.

6.2 Mixed integer linear programming model

The following notation are used to formulate the optimization model.

Indices and Sets

$s \in \mathcal{S}$	set of depot and low-cost stations for UAV/ADR docking and recharging, $\mathcal{S} = \{s_0, s_1, s_2, s_3, \dots s_S\}$
$f \in \mathcal{F}$	set of drop-off sites for launching UAVs and dispatching ADRs to perform last-mile deliveries, $\mathcal{F} = \{f_1, f_2, f_3, \dots f_F\}$
$i, j \in \mathcal{N}$	set of customer locations, $\mathcal{N} = \{i_1, i_2, i_3, \dots i_N\}$

$p, p' \in \mathcal{P}$	set of potential truck stops (i.e, low-cost stations, drop-off sites, and customers), $\mathcal{P} = \mathcal{S} \cup \mathcal{F} \cup \mathcal{N}$
$k, k' \in \mathcal{K}$	set of actual truck stops, $\mathcal{K} = \{k_0, k_1, k_2, k_3, \dots k_K, k_{K+1}\}$
$d \in \mathcal{D}$	set of UAVs, $\mathcal{D} = \{d_1, d_2, d_3, \dots d_D\}$
$r \in \mathcal{R}$	set of ADRs, $\mathcal{R} = \{r_1, r_2, r_3, \dots r_R\}$

Parameters

S	number of low-cost stations
F	number of drop-off sites
N	number of customer locations
K	maximum allowable number of truck stops
D	number of UAVs
R	number of ADRs
C^D	number of UAVs that can be docked on the truck roof
C^R	number of ADRs that the truck can carry
E^D_{ds}	duration required for UAV $d \in \mathcal{D}$ to fly from its location at the start of the planning horizon to station $s \in \mathcal{S}$
E^R_{rs}	duration required for ADR $r \in \mathcal{R}$ to travel from its location at the start of the planning horizon to station $s \in \mathcal{S}$
$T^T_{pp'}$	travel time of truck from potential truck stop p to $p' \in \mathcal{P}$

T^D_{dpi} travel time of UAV $d \in \mathcal{D}$ from potential truck stop $p \in \mathcal{P}$ to customer $i \in \mathcal{N}$ and then to the closest station

T^R_{rpi} travel time of ADR $r \in \mathcal{R}$ from potential truck stop $p \in \mathcal{P}$ to customer $i \in \mathcal{N}$ and then to the closest station

Q^D_d flight range of UAV $d \in \mathcal{D}$ for relocation

Q^R_r travel range of ADR $r \in \mathcal{R}$ for relocation

G^D_{id} flight range of UAV $d \in \mathcal{D}$ serving customer $i \in \mathcal{N}$

G^R_{ir} travel range of ADR $r \in \mathcal{R}$ serving customer $i \in \mathcal{N}$

H^D_d preparation time of UAV $d \in \mathcal{D}$

H^R_r preparation time of ADR $r \in \mathcal{R}$

V^T_i service time of truck at delivery location $i \in \mathcal{N}$

V^D_i service time of UAV at delivery location $i \in \mathcal{N}$

V^R_i service time of ADR at delivery location $i \in \mathcal{N}$

Decision variables

a^T_k arrival time of truck to truck stop $k \in \mathcal{K}$

a^D_d arrival time of UAV $d \in \mathcal{D}$ to the closest station after completing its delivery task

a^R_r arrival time of ADR $r \in \mathcal{R}$ to the closest station after completing its delivery task

d^T_k departure time of truck from truck stop $k \in \mathcal{K}$

u^D_{dk} 1 if UAV $d \in \mathcal{D}$ is carried by the truck before traveling to stop $k \in \mathcal{K}$, 0 otherwise

u^R_{rk} 1 if ADR $r \in \mathcal{R}$ is carried by the truck before traveling to stop $k \in \mathcal{K}$, 0 otherwise

q^D_{dk} 1 if a UAV $d \in \mathcal{D}$ is collected by truck at stop $k \in \mathcal{K}$, 0 otherwise

q^R_{rk} 1 if a ADR $r \in \mathcal{R}$ is collected by truck at stop $k \in \mathcal{K}$, 0 otherwise

w^D_{dk} 1 if a UAV $d \in \mathcal{D}$ is launched from truck stop $k \in \mathcal{K}$, 0 otherwise

w^R_{rk} 1 if a ADR $r \in \mathcal{R}$ is dispatched from truck stop $k \in \mathcal{K}$, 0 otherwise

x_{pk} 1 if station, drop-off site, or customer location $p \in \mathcal{P}$ is assigned to be truck stop $k \in \mathcal{K}$, 0 otherwise

y^D_{id} 1 if a delivery location $i \in \mathcal{N}$ is served by UAV $d \in \mathcal{D}$, 0 otherwise

y^R_{ir} 1 if a delivery location $i \in \mathcal{N}$ is served by ADR $r \in \mathcal{R}$, 0 otherwise

$\widehat{t}$ delivery completion time of all vehicles

The following specifies the developed MILP model for obtaining the optimal last-mile delivery plan using the hybrid truck-drone-robot system. The objective function (6.1) minimizes the delivery completion time, which is the maximum of arrival times of delivery vehicles to their respective destinations as computed by constraints (6.2), (6.3), and (6.4), for the truck, UAVs, and ADRs, respectively. The binary and continuous decision variables are defined by constraints (6.5) and (6.6), respectively.

$$Minimize \quad \widehat{t} \tag{6.1}$$

S.t.

$$\widehat{t} \geq a_k^T \qquad \forall k \in \mathcal{K} \ni k = \{k_{K+1}\} \tag{6.2}$$

$$\widehat{t} \geq a_d^D \qquad \forall d \in \mathcal{D} \tag{6.3}$$

$$\widehat{t} \geq a_r^R \qquad \forall r \in \mathcal{R} \tag{6.4}$$

$$u_{dk}^D, u_{rk}^R, q_{dk}^D, q_{rk}^R, w_{dk}^D, w_{rk}^R, x_{pk}, y_{id}^D, y_{ir}^R \in \{0,1\} \quad \forall i \in \mathcal{N}, d \in \mathcal{D}, r \in \mathcal{R}, p \in \mathcal{P}, k \in \mathcal{K} \tag{6.5}$$

$$a_k^T, a_d^D, a_r^R, d_k^T, \widehat{t} \in \mathbb{R}_+ \qquad \forall k \in \mathcal{K}, d \in \mathcal{D}, r \in \mathcal{R} \tag{6.6}$$

6.2.1 Routing of delivery vehicles

Routing of truck, UAVs, and ADRs is controlled by constraints (6.7)-(6.14) guaranteeing feasible routes of vehicles as well as inclusion of all customers in the delivery plan.

$$x_{pk} = 1 \qquad \forall p \in \mathcal{P} \ni p = \{s_0\}, k \in \mathcal{K} \ni k = \{k_0, k_{K+1}\} \tag{6.7}$$

$$\sum_{p \in \mathcal{P}} x_{pk} = 1 \qquad \forall k \in \mathcal{K} \tag{6.8}$$

$$\sum_{k \in \mathcal{K}} x_{pk} \leq 1 \qquad \forall p \in \mathcal{P} \backslash \{s_0\} \tag{6.9}$$

$$x_{pk} \leq x_{p,k+1} \qquad \forall p \in \mathcal{P} \ni p = \{s_0\}, k \in \mathcal{K} \backslash \{k_0, k_{K+1}\} \tag{6.10}$$

$$\sum_{k \in \mathcal{K}} x_{ik} + \sum_{d \in \mathcal{D}} y_{id}^D + \sum_{r \in \mathcal{R}} y_{ir}^R = 1 \quad \forall i \in \mathcal{N} \tag{6.11}$$

$$\sum_{k \in \mathcal{K}} x_{ik} + \sum_{d \in \mathcal{D}} y_{id}^D = 1 \qquad \forall i \in \mathcal{N}^D \tag{6.12}$$

$$\sum_{k \in \mathcal{K}} x_{ik} + \sum_{r \in \mathcal{R}} y_{ir}^R = 1 \qquad \forall i \in \mathcal{N}^R \tag{6.13}$$

$$\sum_{k \in \mathcal{K}} x_{ik} = 1 \qquad \forall i \in \mathcal{N}^T \tag{6.14}$$

Constraint (6.7) ensures that the truck starts and ends its route at the depot ($s_0 \in \mathcal{P}$). Along the truck route, each stop $k \in \mathcal{K}$ must be assigned to a potential truck stop $p \in \mathcal{P}$ as specified by constraint (6.8), while constraint (6.9) guarantees that each potential stop $p \in \mathcal{P} \backslash \{s_0\}$ is allowed only once to serve as a truck stop $k \in \mathcal{K}$. However, in constraint (6.9), multiple truck stops in set $\mathcal{K}$ can be assigned to the depot $\{s_0\} \in \mathcal{P}$ allowing the total number of truck stops to be less than K. Those redundant stops are assigned by constraint (6.10) to be at the end of the truck route. Therefore, constraint (6.10) implicitly stipulates that the depot cannot be visited by truck until the end of its route. For each customer $i \in \mathcal{N}$, constraint (6.11) ensures exactly one delivery service by the truck, an UAV, or an ADR. Based on the feasibility of serving customers by different vehicle types, constraints (6.12) -(6.14) confine the assignments in the delivery plan to the eligible vehicles for the customers in sets $\mathcal{N}^D$, $\mathcal{N}^R$, and $\mathcal{N}^T$, respectively.

6.2.2 Collection of UAVs and ADRs

Along the truck route, collection of UAVs and ADRs from eligible stations is controlled by constraints (6.15)-(6.22).

$$\sum_{k\in\mathcal{K}} q^D_{dk} \leq 1 \qquad \forall d \in \mathcal{D} \tag{6.15}$$

$$\sum_{k\in\mathcal{K}} q^R_{rk} \leq 1 \qquad \forall r \in \mathcal{R} \tag{6.16}$$

$$\sum_{k\in\mathcal{K}} q^D_{dk} = \sum_{i\in\mathcal{N}} y^D_{id} \qquad \forall d \in \mathcal{D} \tag{6.17}$$

$$\sum_{k\in\mathcal{K}} q^R_{rk} = \sum_{i\in\mathcal{N}} y^R_{ir} \qquad \forall r \in \mathcal{R} \tag{6.18}$$

$$q^D_{dk} \leq 1 - x_{pk} \qquad \forall d \in \mathcal{D}, k \in \mathcal{K}, p \in \mathcal{F} \cup \mathcal{N} \tag{6.19}$$

$$q^R_{rk} \leq 1 - x_{pk} \qquad \forall r \in \mathcal{R}, k \in \mathcal{K}, p \in \mathcal{F} \cup \mathcal{N} \tag{6.20}$$

$$\sum_{r\in\mathcal{R}} q^R_{rk} \leq R(1 - x_{sk}) \qquad \forall k \in \mathcal{K}, s \in \mathcal{S}^D \tag{6.21}$$

$$\sum_{d\in\mathcal{D}} q^D_{dk} \leq D(1 - x_{sk}) \qquad \forall k \in \mathcal{K}, s \in \mathcal{S}^R \tag{6.22}$$

Each UAV $d \in \mathcal{D}$ or robot $r \in \mathcal{R}$ is allowed by constraints (6.15) and (6.16), respectively, to be collected at most once by the truck. However, constraints (6.17) and (6.18) stipulate the UAV or ADR collection, respectively, only if it is assigned to serve one of the customer locations. Moreover, collection of UAVs and ADRs is restricted by constraints (6.19) and (6.20), respectively, to be performed only at stations, not customer nor drop-off locations. Furthermore, depending on the station eligibility to accommodate delivery vehicles, constraints (6.21) and (6.22) determine if to allow vehicle collection by truck at each station $s \in \mathcal{S}$ or not. The rationale of constraints (6.19)-(6.22) is to avoid waiting of autonomous vehicles (i.e., UAV hovering or ADR parking) at unequipped locations.

6.2.3 Launching UAVs and dispatching ADRs

The last-mile delivery tasks by UAVs and ADRs are steered by constraints (6.23)-(6.32) to optimize the assignments of these autonomous vehicles to both truck stops as dispatching locations and customers for package delivery.

$$\sum_{k \in \mathcal{K}} w_{dk}^{D} \leq 1 \qquad \forall d \in \mathcal{D} \tag{6.23}$$

$$\sum_{k \in \mathcal{K}} w_{rk}^{R} \leq 1 \qquad \forall r \in \mathcal{R} \tag{6.24}$$

$$\sum_{k \in \mathcal{K}} w_{dk}^{D} = \sum_{i \in \mathcal{N}} y_{id}^{D} \qquad \forall d \in \mathcal{D} \tag{6.25}$$

$$\sum_{k\in\mathcal{K}} w_{rk}^{R} = \sum_{i\in\mathcal{N}} y_{ir}^{R} \qquad \forall r \in \mathcal{R} \tag{6.26}$$

$$\sum_{r\in\mathcal{R}} w_{rk}^{R} \leq R(1 - x_{sk}) \qquad \forall k \in \mathcal{K}, s \in \mathcal{S}^{D} \tag{6.27}$$

$$\sum_{d\in\mathcal{D}} w_{dk}^{D} \leq D(1 - x_{sk}) \qquad \forall k \in \mathcal{K}, s \in \mathcal{S}^{R} \tag{6.28}$$

$$\sum_{r\in\mathcal{R}} w_{rk}^{R} \leq R(1 - x_{fk}) \qquad \forall k \in \mathcal{K}, f \in \mathcal{F}^{D} \tag{6.29}$$

$$\sum_{d\in\mathcal{D}} w_{dk}^{D} \leq D(1 - x_{fk}) \qquad \forall k \in \mathcal{K}, f \in \mathcal{F}^{R} \tag{6.30}$$

$$\sum_{r\in\mathcal{R}} w_{rk}^{R} \leq R(1 - x_{ik}) \qquad \forall k \in \mathcal{K}, i \in \mathcal{N}\backslash\mathcal{N}^{WR}\backslash\mathcal{N}^{WDR} \tag{6.31}$$

$$\sum_{d\in\mathcal{D}} w_{dk}^{D} \leq D(1 - x_{ik}) \qquad \forall k \in \mathcal{K}, i \in \mathcal{N}\backslash\mathcal{N}^{WD}\backslash\mathcal{N}^{WDR} \tag{6.32}$$

Constraints (6.23) and (6.24) ensure, respectively, that each UAV $d \in \mathcal{D}$ and ADR $r \in \mathcal{R}$ can be assigned to travel with a delivery package from at most one truck stop $k \in \mathcal{K}$. In addition, dispatching of an autonomous vehicles to perform a delivery task is stipulated by constraints (6.25) and (6.26) when it is assigned to serve one of the customers. Furthermore, from constraints (6.17), (6.18), (6.25), and (6.26), an autonomous vehicle cannot be used for package delivery unless it has been collected by the truck driver at a stop $k \in \mathcal{K}$. Feasibility of UAVs and ADRs dispatching locations is guaranteed by constraints (6.27)-(6.32). In particular, since the stations can be designed to accommodate only one type of autonomous vehicles, constraints (6.27) and (6.28) accordingly guide the assignments of vehicles to stations. Similarly, constraints (6.29) and (6.30) control whether UAVs and ADRs, respectively, can be assigned to deliver a package from each drop-off location $f \in \mathcal{F}$, while constraints (6.31) and (6.32) determine if a customer location $i \in \mathcal{N}$ can be used for launching UAVs/dispatching ADRs.

6.2.4 Relation between the truck and unmanned vehicles

Dynamics of autonomous vehicles collection, transportation from a truck stop to a next, and dispatching for delivery tasks are handled by constraints (6.33)-(6.38).

$$u^D_{dk} = 0 \qquad \forall k \in \mathcal{K} \ni k = \{k_0\}, d \in \mathcal{D} \tag{6.33}$$

$$u^R_{rk} = 0 \qquad \forall k \in \mathcal{K} \ni k = \{k_0\}, r \in \mathcal{R} \tag{6.34}$$

$$u^D_{dk} + q^D_{dk} = u^D_{d,k+1} + w^D_{dk} \qquad \forall d \in \mathcal{D}, k \in \mathcal{K} \backslash \{k_{K+1}\} \tag{6.35}$$

$$u^R_{rk} + q^R_{rk} = u^R_{r,k+1} + w^R_{rk} \qquad \forall r \in \mathcal{R}, k \in \mathcal{K} \backslash \{k_{K+1}\} \tag{6.36}$$

$$\sum_{d \in \mathcal{D}} u^D_{dk} \leq C^D \qquad \forall k \in \mathcal{K} \tag{6.37}$$

$$\sum_{r \in \mathcal{R}} u^R_{rk} \leq C^R \qquad \forall k \in \mathcal{K} \tag{6.38}$$

Constraints (6.33) and (6.34) state that the truck does not carry any UAVs nor ADRs before commencing the delivery plan. Therefore, all involved autonomous vehicles are tracked starting with a possibility of collecting some of them at the depot before the truck travel. Flow balance of the UAVs and ADRs at each truck stop is governed by constraints (6.35) and (6.36), respectively. Particularly, the number of UAVs (or ADRs) carried by truck before arriving the stop $k \in \mathcal{K}$ and those collected after arrival (i.e., $u^D_{dk} + q^D_{dk}$) must be equal to the number of dispatched vehicles from the truck stop to perform delivery tasks and the onboard vehicles after leaving the truck stop (i.e., $u^D_{d,k+1} + w^D_{dk}$). Since the truck has a limited space for UAVs (C^D) and ADRs (C^R), constraints (6.37) and (6.38) limit their capacity on the truck.

6.2.5 Scheduling of delivery vehicles

While the previous constraints aim at achieving feasible delivery routing plan, scheduling the departure and arrival times of vehicles at truck stops and stations is specified by constraints (6.39)-(6.44).

$$d^T_k \geq a^T_k + \sum_{d \in \mathcal{D}} H^D_d w^D_{dk} + \sum_{r \in \mathcal{R}} H^R_r w^R_{rk} + \sum_{i=p \in \mathcal{N}} V^T_i x_{pk} \quad \forall k \in \mathcal{K} \tag{6.39}$$

$$d^T_k \geq E^D_{ds} q^D_{dk} - M(1 - x_{sk}) \quad \forall k \in \mathcal{K} \backslash \{k_{K+1}\}, d \in \mathcal{D}, s \in \mathcal{S} \tag{6.40}$$

$$d^T_k \geq E^R_{rs} q^R_{rk} - M(1 - x_{sk}) \quad \forall k \in \mathcal{K} \backslash \{k_{K+1}\}, r \in \mathcal{R}, s \in \mathcal{S} \tag{6.41}$$

$$a^T_k \geq d^T_{k-1} + T^T_{pp'} - M(2 - x_{p,k-1} - x_{p'k}) \quad \forall k \in \mathcal{K} \backslash \{k_0\}, p, p' \in \mathcal{P} \tag{6.42}$$

$$a_d^D \geq d_k^T + T_{dpi}^D + V_i^D - M(3 - x_{pk} - y_{id}^D - w_{dk}^D) \qquad \forall d \in \mathcal{D}, p \in \mathcal{P}, k \in \mathcal{K}, i \in \mathcal{N} \tag{6.43}$$

$$a_r^R \geq d_k^T + T_{rpi}^R + V_i^R - M(3 - x_{pk} - y_{ir}^R - w_{rk}^R) \qquad \forall r \in \mathcal{R}, p \in \mathcal{P}, k \in \mathcal{K}, i \in \mathcal{N} \tag{6.44}$$

Constraint (6.39) determines the truck departure time from each stop $k \in \mathcal{K}$ by stipulating preparation times for the assigned UAVs and ADRs. Furthermore, when the truck collects UAV $d \in \mathcal{D}$ or ADR $r \in \mathcal{R}$ at stop $k \in \mathcal{K}$, constraints (6.40) and (6.41) ensure that its departure time must be later than the arrival of the respective autonomous vehicle to that truck stop. Arrival time of the truck to a stop $k \in \mathcal{K}$ is determined by constraint (6.42). Similarly, constraints (6.43) and (6.44) compute the arrival times of UAVs and ADRs to their destination stations based on departure and travel times thereof, in addition to service time at customer location.

6.2.6 Travel ranges of UAVs and ADRs

A feasible delivery plan must guarantee the limited travel ranges of UAVs and ADRs, which is addressed by constraints (6.45)-(6.48).

$$E_{ds}^D q_{dk}^D - M(1 - x_{sk}) \leq Q_d^D \qquad \forall d \in \mathcal{D}, k \in \mathcal{K}, s \in \mathcal{S} \tag{6.45}$$

$$E_{rs}^R q_{rk}^R - M(1 - x_{sk}) \leq Q_r^R \qquad \forall r \in \mathcal{R}, k \in \mathcal{K}, s \in \mathcal{S} \tag{6.46}$$

$$a^D_d - d^T_k - M(2 - w^D_{dk} - y^D_{id}) \leq G^D_{id} \qquad \forall d \in \mathcal{D}, k \in \mathcal{K}, i \in \mathcal{N} \tag{6.47}$$

$$a^R_r - d^T_k - M(2 - w^R_{rk} - y^R_{ir}) \leq G^R_{ir} \qquad \forall r \in \mathcal{R}, k \in \mathcal{K}, i \in \mathcal{N} \tag{6.48}$$

Constraints (6.45) and (6.46) ensure that the relocation time of each UAV $d \in \mathcal{D}$ and ADR $r \in \mathcal{R}$ is within the respective travel ranges Q^D_d and Q^R_r, respectively. Similarly, when delivery tasks are assigned, flying ranges of UAVs and travel ranges of ADRs are respected by constraints (6.47) and (6.48).

6.2.7 Valid inequalities and knowledge-based constraints

The MILP model of objective function (6.1) and constraints (6.2)-(6.48) takes relatively long computational time to obtain optimal solutions even for small size problems, as observed from initial experiments. Therefore, valid inequalities and knowledge-based constraints are developed in this section to enhance the proposed model performance, thereby yielding optimal solutions of reasonable size instances to efficiently benchmark solution approaches for larger size problems. However, both sets of constraints only provide cuts of the solution space, and therefore do not exclude any of the optimal solutions. Initially, the following lemmas and propositions are presented to clarify the concepts of the proposed valid inequalities and knowledge-based constraints.

Lemma 1. For three locations indexed by k, $k+1$, and $k+2 \in \mathcal{K}$, the travel time $T^T_{k,k+2}$ is a lower bound to the overall time of traveling from k to the intermediate stop $k+1$ and then to $k+2$.

Proof. Based on the triangular law of inequality, $T^T_{k,k+2} \leq T^T_{k,k+1} + T^T_{k+1,k+2}$. $\square$

Proposition 1. For any two different truck stops k and $k' \in \mathcal{K}$, $a^T_k > (d^T_{k'} + T^T_{k',k})|_{k'<k}$.

Proof. Consider two consecutive truck stops $k-1$ and $k \in \mathcal{K}$. Based on constraint (6.42), truck arrival at stop k equals the departure time from $k-1$ plus its travel time from $k-1$ to k, $a^T_k = d^T_{k-1} + T^T_{k-1,k}$. Likewise, $a^T_{k-1} = d^T_{k-2} + T^T_{k-2,k-1}$. Since $d^T_{k-1} > a^T_{k-1}$, by definition, then $a^T_k > d^T_{k-2} + T^T_{k-2,k-1} + T^T_{k-1,k}$. From Lemma 1, $T^T_{k-2,k} < T^T_{k-2,k-1} + T^T_{k-1,k}$. Thus, $a^T_k > d^T_{k-2} + T^T_{k-2,k}$. By induction, $a^T_k > d^T_{k'} + T^T_{k'k}, \forall k, k' \in \mathcal{K}, k > k'$. $\square$

Proposition 2. Arrival time of a vehicle (e.g., UAV or ADR) carried by a truck from the depot $\{s_0\} \in \mathcal{P}$ to a customer $i \in \mathcal{N}$ is larger than the minimum of independent travel time spent by either that vehicle or the truck.

Proof. Assume that both the autonomous vehicle and truck can independently travel from $\{s_0\} \in \mathcal{P}$ to $i \in \mathcal{N}$ on the same geometric route in times T^A and T^T, respectively. When the truck carries the vehicle from the depot $\{s_0\} \in \mathcal{P}$ and dispatch it to perform the last-mile delivery to customer $i \in \mathcal{N}$, the resultant travel time is $\widehat{T} = (1-p) \times T^A + p \times T^T$, where $p \sim [0,1]$ is the proportion of the route traveled by the truck carrying the autonomous vehicle. Therefore, $\widehat{T} \geq (1-p) \times T^A$ and $\widehat{T} \geq p \times T^T$ irrespective of the value of p. Moreover, $\widehat{T} \geq min(T^A, T^T)$. Thus, arrival to $i \in \mathcal{N}$ after a travel time $\widehat{T}$ is larger than the minimum of T^A and T^T. $\square$

Lemma 2. The truck can be assigned to stop $k \in \mathcal{K}$ and an autonomous vehicle can be dispatched from k as long as the autonomous vehicle travel time does not visit a customer $i \in \mathcal{N}$ causing violation of its allowable travel range.

Proof. For a potential truck stop $p \in \mathcal{P}$, if $x_{pk} = 1$ for a stop $k \in \mathcal{K}$, a travel time of an unmanned vehicle longer than the allowable range contradicts with the delivery problem feasibility. $\square$

Lemma 3. The truck can be assigned to stop $k \in \mathcal{K}$ and an autonomous vehicle can be dispatched to serve a customer $i \in \mathcal{N}$ as long as the vehicle is not assigned to be dispatched from stop k in case that it violates its allowable travel range.

Proof. The same feasibility conditions in Lemma 2 are applied. $\square$

Lemma 4. An autonomous vehicle can be dispatched from a stop $k \in \mathcal{K}$ to serve a customer $i \in \mathcal{N}$ as long as k does not coincide with a geographical location $p \in \mathcal{P}$

causing violation of the autonomous vehicle's allowable travel range.

Proof. The same feasibility conditions in Lemma 2 are applied. □

Proposition 3. If vehicle travel time from truck stop $k \in \mathcal{K}$ coinciding with a potential location $p \in \mathcal{P}$ to visit customer $i \in \mathcal{N}$ and return to the closest station is larger than the allowable travel range, the following three assignments cannot be conducted simultaneously: coinciding location $p \in \mathcal{P}$ with stop $k \in \mathcal{K}$, dispatching the vehicle from stop $k \in \mathcal{K}$, and employing the vehicle to serve customer $i \in \mathcal{N}$.

Proof. Lemmas 2, 3, and 4 together prove the preposition. □

Proposition 4. If autonomous vehicle relocation time to station $s \in \mathcal{S}$ is longer than its travel range, the station cannot be assigned as a truck stop collecting that autonomous vehicle.

Proof. Feasibility of the delivery plan includes two conditions. First, the truck can be assigned to stop at station $s \in \mathcal{S}$ unless picking an autonomous vehicle up from that station requires a relocation violating the vehicle travel range. Second, pick up of an autonomous vehicle at a station $s \in \mathcal{S}$ can be performed unless it requires a relocation to reach s violating its travel range. □

Based on these lemmas and propositions, the valid inequalities (6.49)-(6.51) are used to apply lower bounds to the three determinants of delivery completion time, a_k^T, a_d^D, and a_r^R (c.f., objective function (6.1) and constraints (6.2)-(6.4)), while the knowledge-based constraints (6.52)-(6.55) capitalizes on the travel ranges information of UAVs and ADRs to guide the branching procedure of the MILP solver.

$$a_k^T \geq d_{k'}^T + T_{p'p}^T - M(2 - x_{pk} - x_{p'k'}) \qquad \forall p, p' \in \mathcal{P}, k, k' \in \mathcal{K}, k > k' \tag{6.49}$$

$$a_d^D \geq min(T_{pi}^T + \min_{s \in \mathcal{S}}(T_{is}^T), T_{dpi}^D) + V_i^D - M(1 - y_{id}^D) \quad \forall d \in \mathcal{D}, p \in \mathcal{P} \ni p = \{s_0\}, i \in \mathcal{N} \tag{6.50}$$

$$a_r^R \geq min(T_{pi}^T + \min_{s \in \mathcal{S}}(T_{is}^T), T_{rpi}^R) + V_i^R - M(1 - y_{ir}^R) \quad \forall r \in \mathcal{R}, p \in \mathcal{P} \ni p = \{s_0\}, i \in \mathcal{N} \tag{6.51}$$

$$x_{pk} + w_{dk}^D + y_{id}^D \leq 2 \qquad \forall p \in \mathcal{P}, i \in \mathcal{N}, k \in \mathcal{K}, d \in \mathcal{D} | T_{dpi}^D + V_i^D > G_{id}^D \tag{6.52}$$

$$x_{pk} + w_{rk}^R + y_{ir}^R \leq 2 \qquad \forall p \in \mathcal{P}, i \in \mathcal{N}, k \in \mathcal{K}, r \in \mathcal{R} | T_{rpi}^R + V_i^R > G_{ir}^R \tag{6.53}$$

$$x_{sk} + q_{dk}^D \leq 1 \qquad \forall s \in \mathcal{S}, k \in \mathcal{K}, d \in \mathcal{D} | E_{ds}^D > Q_d^D \tag{6.54}$$

$$x_{sk} + q_{rk}^R \leq 1 \qquad \forall s \in \mathcal{S}, k \in \mathcal{K}, r \in \mathcal{R} | E_{rs}^R > Q_r^R \tag{6.55}$$

6.3 Computational experiments

In this section, we conduct numerical experiments to demonstrate the effectiveness of last-mile delivery by hybrid truck-drone systems with a network of supporting stations. The tested instances are benchmarked with the traditional delivery by truck only. Furthermore, the impact of UAV/ADR relocation to rendezvous with truck is assessed. Finally, a tradeoff between operating hybrid versus traditional trucks is studied. The developed MILP model has been coded on GAMS 30.1 and solved by CPLEX 12.10. All computational experiments were conducted on a PC with an AMD Ryzen 7-2700X processor and 16 GB RAM.

6.3.1 Setup of test instances

The test instances are created by randomly generating locations of network nodes within an area of 25×25 miles^2 around a depot at the center of delivery area. The number of locations for stations, drop-off sites, and customers is listed in Table 6.1 for each set of allowable vehicles. When establishing the stations, a minimum of 5 miles between them is considered, while all drop-off sites are far by at least 3 miles from stations and other sites. Based on this setting, ten different instances are generated. Customer locations that can be served by a specific vehicle type (e.g., UAV) may be assigned as a dispatching site for the respective vehicle type. The maximum allowable number of truck stops (K) is set at $50\% \times |N|$. The system is assumed to contain a total of 15 UAVs and 15 ADRs and their locations are uniformly distributed within a range of [0,3] per station. The truck can simultaneously carry up to 4 UAVs on its roof and 4 ADRs along with the customer packages. The truck and UAV speed is set at 25 mph, while ADRs can travel 10 mph [12]. Travel times of vehicles are computed based on the distances between coordinates of stations and the aforementioned velocities. The travel ranges of UAVs and ADRs are set at 10 and 16 miles, respectively [91]. Before each UAV launching or ADR dispatching, a one minute preparation time is required to retrieve the vehicle and respective order, while the truck, UAVs, and ADRs spend a service time of 0.5, 1.0, and 3.0 minutes, respectively, at customer locations.

Table 6.1: Number of locations of stations, drop-off sites, and customers for each set of allowable vehicles

	UAVs and truck	ADRs and truck	Truck only	All vehicles	Total
Stations	2	2	-	6	10
Drop-off sites	2	2	-	6	10
Customers	2	2	1	5	10

6.3.2 Effectiveness of using UAVs and ADRs with docking stations

In order to evaluate the effectiveness of using UAVs and ADRs with supporting stations, a comparison is conducted with delivery operations by truck only. The traveling salesman problem is solved to obtain the optimal solutions for the ten instances, while the proposed MILP model is solved with a 6-hr runtime limit resulting in the optimal solutions for six out of the ten instances. Table 6.2 exhibits the delivery completion time of both cases and the saving percent of using hybrid system with supporting stations as compared to truck only delivery operations. The saving in delivery completion time ranges from 23.7% to 41.4%, which demonstrates the effectiveness of using the hybrid delivery systems while utilizing supporting stations.

Table 6.2: Delivery completion time by solving the TSP model and the developed MILP model for ten instances

Instance	Truck only	Hybrid system with supporting stations	Saving %
1	216.6	157.6	27.3%
2	164.1	101.9	37.9%
3	220.2	129.1	41.4%
4	243.0	163.6	32.7%
5	230.8	155.1	32.8%
6	180.4	116.3	35.5%
7	215.0	154.8	28.0%
8	252.0	148.8	40.9%
9	203.4	155.2	23.7%
10	221.6	133.7	39.7%
Average	214.7	141.6	34.0%

6.3.3 Impact of UAVs and ADRs repositioning

Relocation of unmanned vehicles may require recharging or autonomous battery replacement facilities at stations and advanced models of UAVs and ADRs. Therefore, the impact of establishing facilities and investing in advanced vehicles to allow the relocation is studied in this section. Table 6.3 shows the delivery completion time for two cases, without and with relocation. While a marginal saving of less than 3.3% is achieved when the relocation is considered for seven instances, a substantial saving in delivery time is obtained for the remaining three instances, resulting in an average saving of 9.8%. Thus, delivery operators may consider the tradeoff between the estimated saving in delivery time and the corresponding required investment in facilities and vehicles.

Table 6.3: Delivery completion time when unmanned vehicles can be operated without and with repositioning

Instance	Without repositioning	With repositioning	Saving %
1	185.2	157.6	14.9%
2	140.2	101.9	27.3%
3	129.4	129.1	0.2%
4	243.0	163.6	32.7%
5	155.1	155.1	0.0%
6	116.3	116.3	0.0%
7	154.8	154.8	0.0%
8	153.9	148.8	3.3%
9	155.7	155.2	0.3%
10	137.3	133.7	2.6%
Average	157.1	141.6	9.8%

6.3.4 Tradeoff between traditional and hybrid trucks

Replacing the existing fleet of trucks or even retrofitting them to accommodate UAVs and ADRs may require a considerable investment. Therefore, delivery operators may consider a tradeoff between the required investment amount and the corresponding estimated yield versus maintaining their traditional fleet of trucks. In this section, we quantify the average saving in delivery time by using traditional trucks with unmanned vehicles and supporting stations as compared to both using stand-alone trucks and hybrid trucks for delivery operations. The optimal solutions for the former can be achieved by the proposed MILP model by fixing variables u_{dk}^{D} and u_{rk}^{R} to zero. Figure 6.2 shows that using traditional trucks with UAVs and ADRs outperforms the truck-only delivery for all the tested instances. Nevertheless, the difference is marginal in some cases (e.g., instance 4, 7, and 9) as the truck travels longer to rendezvous the unmanned vehicles at nearby locations to the target customers. Furthermore, additional saving can be achieved by using hybrid trucks. However, using hybrid verses traditional trucks may result in either small or no saving in delivery time (e.g., instances 3, 5, and 8). On average, using UAVs and ADRs with the traditional trucks saves 22.3% of delivery time, while investing in hybrid trucks can yield an additional saving of 11.7%.

6.4 Conclusion

This chapter studies the impact of establishing a network of supporting stations on reducing delivery time. A MILP model is developed for the addressed delivery problem. Furthermore, valid inequalities and knowledge-based constraints are formulated. The obtained delivery completion time by the system under consideration is benchmarked with the conventional delivery operations by truck only. We found that up to 41.4% of delivery time can be saved when using hybrid truck-drone-robot systems with supporting stations. Next, two cases are separately studied to exhibit the tradeoff between the required investment in vehicles and facilities versus the cor-

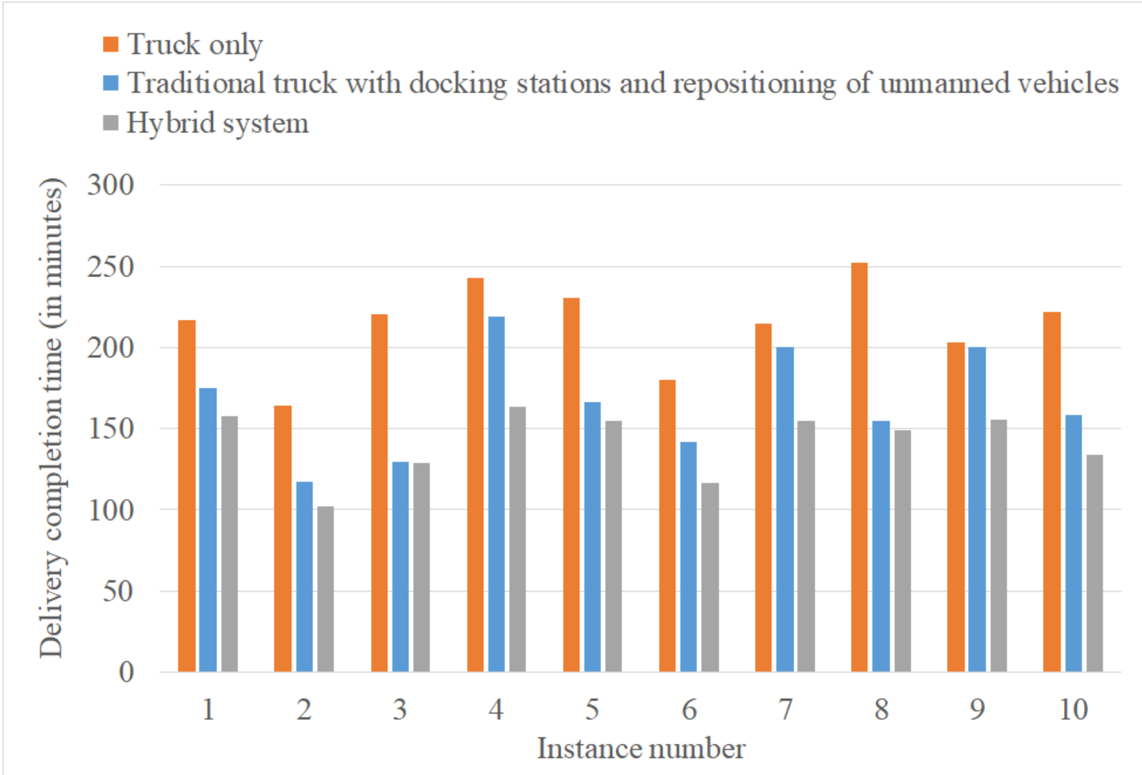

Figure 6.2: Comparison between delivery completion time by using three settings of traditional and hybrid trucks

responding estimated reduction in delivery time. In particular, we found that the relocation of UAVs and ADRs can contribute with an additional 9.8% saving in delivery time, while using hybrid instead of traditional trucks can yield an additional saving of 11.7%. The proposed model in this chapter demonstrated the benefit of a hybrid delivery system and also yielded some preliminary insights.

Chapter 7

Conclusions and future research directions

This dissertation considers the problem of last-mile delivery using multiple UAVs and a single truck. Four variants are addressed in this dissertation. We adopt the approach of grouping customer locations into non-overlapping clusters and routing the truck via each cluster's focal point to facilitate simultaneous UAV deliveries in that cluster. Unlike the common sequential approaches in the literature, this work presents a new integrated method for clustering and routing decisions. We propose mathematical programming models to solve the problem for two different policies - (i) restricting truck stops to a customer-only location (JOCR-R), (ii) allowing truck stops to be anywhere in the delivery region (JOCR-U). Besides, we formulate our models to consider the two common objectives, namely, minimizing total costs and minimizing delivery completion time. Thus, we provide the flexibility to treat the problem as a single objective to optimize one of the performance measures or handle it as a multi-objective to achieve the best compromise solutions. Furthermore, we introduce a knowledge-based constraint and machine learning-based heuristic to accelerate the JOCR-U models.

An extensive numerical analysis is conducted for the proposed models and benchmarked with a recently introduced sequential heuristic approach in the literature. Solving the test instances independently for the two objectives revealed that the proposed joint optimization approach outperforms the sequential heuristic method for all the cases. Besides, allowing the focal points to be anywhere in the delivery region instead of restricting it to a customer location provides substantial savings with respect to cost and delivery completion time. In addition, a sensitivity analysis of key model parameters is also conducted to establish their independent effects on the performance measures. Also, the ϵ-constraint method achieves the best compromise between the two objectives. Finally, numerous insights drawn from our analysis could aid the practitioners and researchers in the effective routing of one truck and multiple UAVs.

Furthermore, we consider a common assumption in the existing literature of truck-drone tandems, which predominantly restricts the UAV launch and recovery operations (LARO) to customer locations. Such a constrained setting may not be able to fully exploit the capability of UAVs. Moreover, this assumption may not accurately reflect the actual delivery operations. In this research, we address these gaps and introduce a new variant of truck-drone tandem that allows the truck to stop at non-customer locations (referred to as flexible sites) for UAV LARO. The proposed variant also accounts for three key decisions - (i) assignment of each customer location to a vehicle, (ii) routing of truck and UAVs, and (iii) scheduling UAV LARO and truck operator activities at each stop, which are always not simultaneously considered in the literature. A mixed integer linear programming model is formulated to jointly optimize the three decisions with the objective of minimizing the delivery completion time. To handle large problem instances, we develop an optimization-enabled two-phase search algorithm by hybridizing simulated annealing and variable neighborhood search. Numerical analysis demonstrates substantial improvement in delivery efficiency of using flexible sites for LARO as opposed to the existing approach of restricting truck stop locations. Finally, several insights on UAV utilization and flexible site selection are provided based on our findings.

Establishing of docking stations and the repositioning feature of unmanned vehicles are also considered. The proposed delivery system is benchmarked with the conventional delivery operations by truck only. In addition, the impact of investing in autonomous recharging (or battery replacement) and repositioning-enabled vehicles on reducing the delivery time is evaluated. Moreover, as replacing the existing fleet of trucks with hybrid systems may incur large investment, the tradeoff between them in terms of the corresponding estimated reduction in delivery time is considered. Since the proposed MILP model for using docking stations and repositioning of unmanned vehicles can only obtain optimal solutions for small size problems, efficient solution approaches should be developed in future research to allow using such delivery systems at scale. In addition, the following directions should be considered in the future research: (i) stochastic operations, especially due to road congestion and weather conditions, (ii) time windows for delivery of packages, (iii) service coverage of delivery vehicles, (iv) rectilinear travel of drones in a city with tall buildings, and (v) drones capable of delivering multiple packages in each sortie.

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Vita

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