ESSAYS ON MONEY AND BANKING: A SEARCH-THEORETIC APPROACH

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In Partial Fulfillment

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Doctor of Philosophy

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by

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The undersigned, appointed by the Dean of the Graduate School, have examined the dissertation entitled

ESSAYS ON MONEY AND BANKING:
A SEARCH-THEORETIC APPROACH

presented by Heon Lee,

a candidate for the degree of Doctor of Philosophy,

and hereby certify that, in their opinion, it is worthy of acceptance.

Professor Chao Gu

Professor Joseph Haslag

Professor Aaron Hedlund

Professor Xuemin (Sterling) Yan

To My Father

ACKNOWLEDGEMENTS

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(PO I I H IO 1:3)

I sincerely appreciate my advisor, Dr. Chao Gu. Without her invaluable advice and overwhelming support, I could not have completed this dissertation. She has been the best academic advisor I could have ever imagined.

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ESSAYS ON MONEY AND BANKING:
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ABSTRACT

My dissertation studies the role of banking in the monetary transmission.

In the first chapter, I develop a monetary-search model where the money multiplier is endogenously determined. I show that when the central bank pays interest on reserves, the money multiplier and the quantity of the reserve can depend on the nominal interest rate and the interest on reserves. The calibrated model can explain the evolution of the money multiplier and the excess reserve-deposit ratio in the pre- and post-2008 periods. The quantitative analysis suggests that the dramatic changes in the money multiplier after 2008 were driven by the introduction of interest on reserves with a low nominal interest rate.

In the second chapter, I develop a dynamic general equilibrium model to study the (in)stability of the fractional reserve banking system. The model shows that the fractional reserve banking system can endanger stability in that equilibrium is more prone to exhibit endogenous cyclic, chaotic, and stochastic dynamics under lower reserve requirements, although it can increase welfare in the steady-state. Introducing endogenous unsecured credit to the baseline model does not change the main results. This chapter also provides empirical evidence that is consistent with the prediction of the model.

Chapter 1

Money Creation and Banking: Theory and Evidence

> [A] model of the banking system in which currency, reserves, and deposits play distinct roles ... seems essential if one wants to consider policies like reserves requirements, interest on deposits, and other measures that affect different components of the money stock differently. Lucas (2000)

1.1 Introduction

Since the Great Recession, many economists have been trying to better understand the substantial changes in the conduct of monetary policy. Most of the literature focuses on the independence of the quantity of reserves from interest rate management in an ample-reserves regime (e.g., Keister, Martin, and McAndrews, 2008; Bech and Klee, 2011; Curdia and Woodford, 2011; Kashyap and Stein, 2012; Cochrane, 2014; Ennis, 2018).[1] The rationale is that the interest rate paid on reserves provides a floor for the short-term interest rate that the central bank seeks to control. Then, as the central bank's target reaches the interest rate on reserves, paying interest on reserves "divorces" money from interest rate management, and the central bank can determine the amount of reserves independently of the interest rate. This implies market rates are equal to or higher than the interest rate on reserves.

However, as Figure 1.1 shows, since the Federal Reserve introduced the target interest rate range in December 2008, the interest on reserves has been higher than the federal funds rate and has been equal to the upper limit of the target range. Also, the short-term interest rate is closely correlated to the amount of reserves rather than independent of them. The left panel of Figure 1.2 plots the excess reserves ratio against the spread between the federal funds rate and the interest on reserves. It shows that the excess reserves ratio and the interest rate spread move in opposite directions. It suggests that, contrary to the previous work, the quantity of reserves is

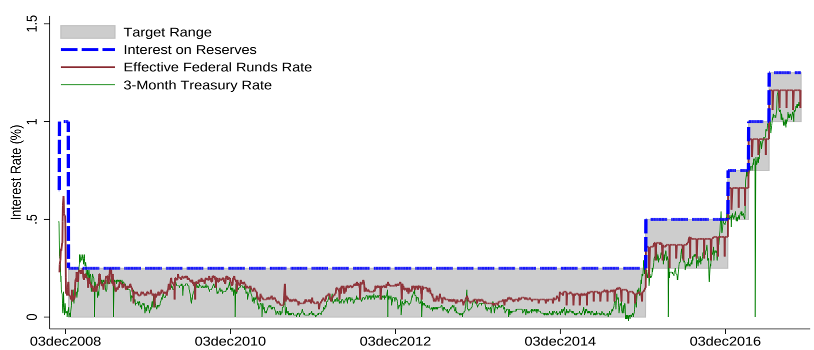

Figure 1.1: US Short-term Interest Rates and the Target Range by the Federal Reserve

[1] Many works on the central bank' large-scale asset purchases also could be cited here. Since the central bank purchases securities from financial intermediaries by paying with reserves, the independence of the quantity of reserve from interest rate management implies the independence of the amount of asset purchases from interest rate management and vice versa.

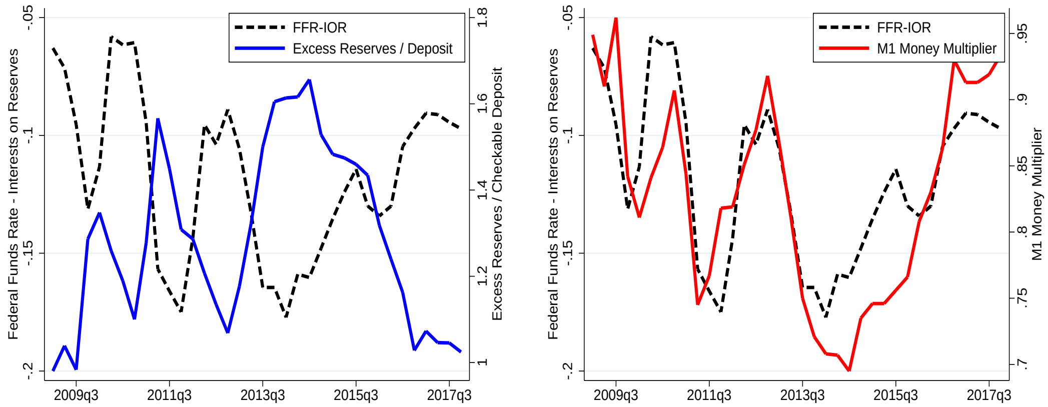

Figure 1.2: US Excess Reserves and M1 Multiplier in the Post-2008 Period

not independent of the central bank's interest rate management even in the post-2008 period with ample reserves. In addition, the right panel of Figure 1.2 shows that the money multiplier moves together with the interest rate spread, which is just opposite to the excess reserves ratio. The time-series variation in the excess reserve ratio is directly reflected in the M1 money multiplier. This implies that the monetary policy in the post-2008 period is closely related to the bank's money creation activity,[2] whereas many of monetary models did not pay attention to the money multiplier during the last decades. These discrepancies call for a model that better captures the transmission of monetary policy through the banking system.

This paper revisits the issue of money creation to understand the role of banking in conducting monetary policy during the pre- as well as post-2008 period. To guide the modeling, Section 1.2 presents a series of empirical facts on money creation and money demand. In contrast to the textbook explanation of the money multiplier, there is no negative relationship between the required reserve ratio and the M1 money multiplier

[2] As mentioned above, the central bank buys assets by paying with reserves. In the ample-reserves regime, the central bank's purchases are directly reflected in the central bank's liability with the same amount of excess reserves.

during the period of zero excess reserves. I identify two structural breaks in the money creation process since 1960: one associated with consumer credit, and another associated with interest on reserves. The first structural break in 1992 coincides with a structural break of the deposit component of money demand. A stable long-run money demand relationship, however, is recovered if one accounts for the impact of unsecured credit. The second structural break in 2008 coincides with the Fed's introduction of the interest on reserves. Banks have been holding excess reserves since then.

These findings suggest that if one wants to study monetary transmission, a desirable monetary model should have the following properties. First, the model should feature the distinct role of the interest on reserves and the nominal interest rate. Second, the model should be able to answer why banks are holding excess reserves now, whereas they did not before 2008. Last, the model needs to capture the interaction between money and credit.

This paper builds a model based on Lagos and Wright (2005) and Berentsen, Camera, and Waller (2007) of money and banking to understand the monetary transmission under different regimes (e.g., ample-reserves regime, scarce-reserves regime) and identify the factors that changed the regimes from one to another. The bank's demand for reserves and inside money creation are endogenously determined.[3] The model includes the explicit structure of monetary exchange and the role of financial intermediation. Agents can trade by using cash, claims on deposits (e.g., check or debit card), banknotes, and unsecured credit (e.g., credit card).[4] The bank creates

[3]See Sargent and Wallace (1982), Freeman (1987), Freeman and Huffman (1991), Haslag and Young (1998), and Freeman and Kydland (2000) for the previous works on inside money creation. Recent works based on a search-theoretic framework include Gu, Mattesini, Monnet, and Wright (2013a) and Andolfatto, Berentsen, and Martin (2020).

[4]By modeling unsecured credit with an exogenous credit limit, this paper follows Gu, Mattesini,

banknotes by making loans, and its lending is constrained by the reserve requirement. The equilibrium falls into one of the following three cases: a scarce-reserves equilibrium, an ample-reserves equilibrium, and a no-banking equilibrium.

In the scarce-reserves equilibrium, the nominal interest rate is sufficiently high. The bank's lending limit binds. If the central bank lowers the nominal interest rate, the reserve balance increases. The bank creates loans proportional to reserves, which implies the reserve requirement affects the money multiplier. If the central bank pays interest on reserves and sets the nominal interest rate at some moderate level, the bank holds excess reserves. We call this an ample-reserves equilibrium. In the ample-reserves equilibrium, the bank's lending limit does not bind. The reserve requirement does not change the money multiplier. Instead, the money multiplier is determined by the nominal interest rate and the interest on reserves. Lowering the nominal interest rate increases reserves, but the banks do not create banknotes proportionally, which lowers the money multiplier. A higher interest on reserves decreases the money multiplier because the bank has more incentive to hold reserves and less incentive to create banknotes. The interest on reserves and the nominal interest rate play distinct roles and they jointly determine the quantity of reserves. These are new findings compared with the literature. In both cases, better credit conditions reduce the real balance of inside money and reserves, but not cash, which results in a lower money multiplier. When the nominal interest rate is low enough, there is sufficient outside money to facilitate trade so buyers do not need to deposit their balances in the bank. The bank can not create inside money because it does not hold any reserves. This is called a no-banking equilibrium.

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and Wright (2016). For other approaches to introducing credit to the monetary economy, see Sanches and Williamson (2010), Lotz and Zhang (2016) and Williamson (2016).

Next I quantify the model to determine the impact of the monetary policy and the introduction of consumer credit on reserves and the money multiplier. The model is parameterized to match pre-2008 US data. Quantitatively, the calibrated model can account for the behavior of money creation before and after 2008. The model-generated series can mimic the historical behavior of the M1 money multiplier, the excess reserves to deposit ratio, and the currency deposit ratio. The welfare analysis shows that lowering the reserve requirement or paying interest on reserves can reduce the welfare cost of inflation. Also, the quantitative analysis identifies the source of changes in the money multiplier and means of payment. The counterfactual analysis shows that the pre-2008 trend of a decreasing money multiplier is driven by an increase in unsecured credit, whereas the post-2008 trend of a decreasing money multiplier is not attributed to the increase in unsecured credit. From the model and data, I provide evidence that suggests the dramatic changes in the money multiplier after 2008 are mainly driven by the Federal Reserve's monetary policy: the introduction of the interest on reserves with low nominal interest rate.

This paper is organized as follows. Section 1.2 provides motivating evidence. Section 1.3 constructs the search-theoretic monetary model of money creation. Section 1.4 calibrates the model to quantify the theory. Section 1.5 concludes.

1.2 Motivating Evidence

Whereas the money multiplier decreased drastically since 2008, the decrease in the money multiplier itself is not is a recent phenomenon. The top-left panel of Figure 1.3 shows that the US M1 money multiplier has been decreasing since 1992. However,

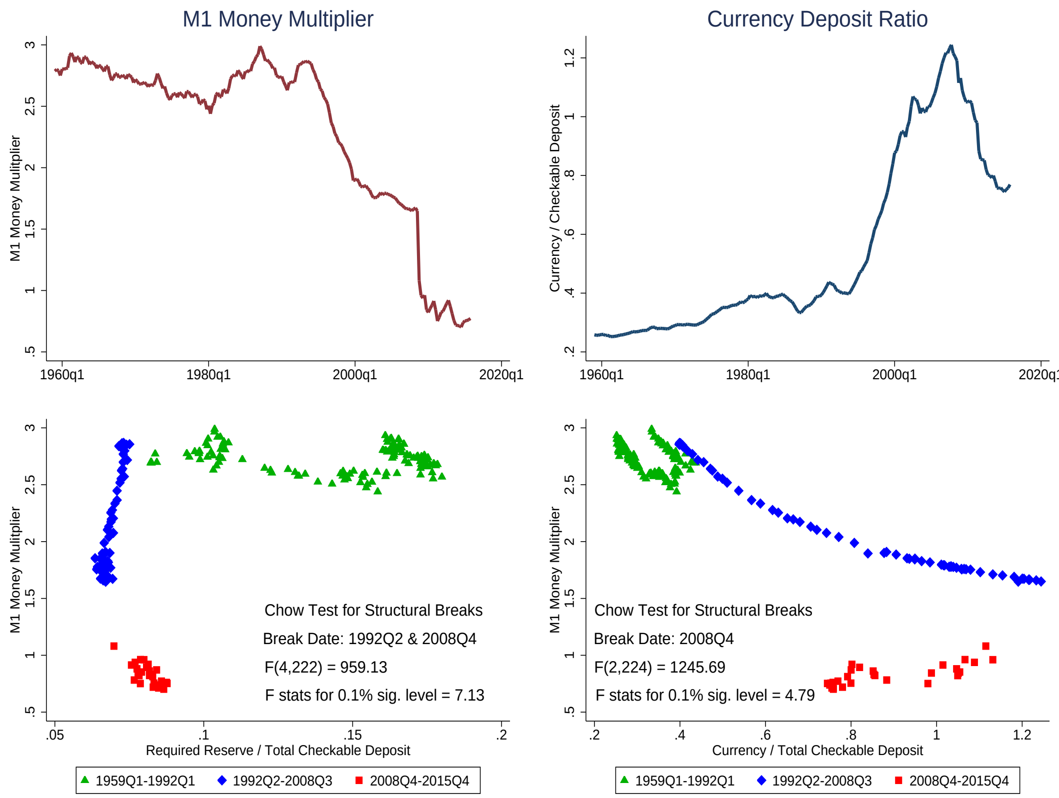

Figure 1.3: Money Multiplier, Currency/Deposit Ratio and Required Reserve Ratio

Chow tests for structural breaks are implemented. The bottom-left panel reports a test statistic with the null hypothesis of no structural breaks in 1992Q2 and 2008Q4 and the bottom-right panel reports a test statistic with the null hypothesis of no structural break in 2008Q4. Sample periods are 1959Q1-2015Q4. Appendix A.2.1 contains details of the Chow tests.

the declining trends before and after 2008 are different. As the top-right panel of Figure 1.3 shows, the decline during 1992-2007 is accompanied by a huge increase in the ratio of currency to deposit, whereas the decline after 2008 is accompanied by a huge drop in the ratio of currency to deposit. The bottom-left panel of Figure 1.3 reports two structural breaks in the relationship between M1 multiplier and the required reserve ratio: 1992Q3 and 2008Q4. It also shows that there is no negative relationship between M1 money multiplier and the required reserve ratio from 1992Q2 to 2008Q3; the excess reserve ratio had been zero until 2008, which suggests that the

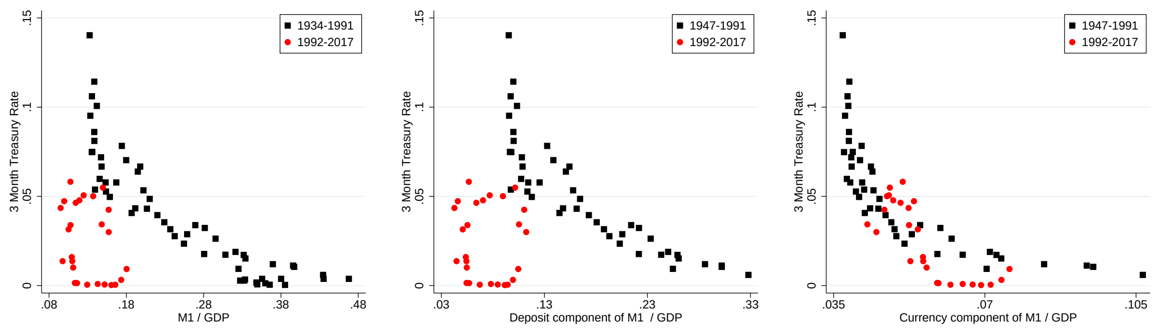

Figure 1.4: US Money Demand for M1 and Its Components

required reserve ratio does not drive the money multiplier.[5,6]

To offer a better picture of what had happened at the structural break of 1992, Figure 1.4 plots the ratio of M1 to GDP and the ratio of M1's components to GDP against the 3 month Treasury Bill rate. There is a breakdown in M1 in 1992 that coincides with the structural break in Figure 1.3. As pointed out by Lucas and Nicolini (2015), this is due to the breakdown of the deposit component, not the currency component. My hypothesis is that the increased availability of consumer credit crowded out deposit but not cash, which implies that once one accounts for the substitution effect of the newly available consumer credit, there should still be a negative relationship between the real money balance and the interest rate.[7]

[5]The required reserve ratio presented in Figure 1.3 is computed by (Required Reserves)/(Total Checkable Deposits). The legal reserve requirement for net transaction accounts was 10% from April 2, 1992, to March 25, 2020, but some banks are imposed upon by lower requirements or exempt depending on the size of their liabilities. These criteria changed 27 times from the 1st quarter of 1992 to the last quarter of 2019. From March 2020, all the required reserve ratios have become zero. See Feinman (1993) and https://www.federalreserve.gov/monetarypolicy/reservereq.htm for more details on the historical evolution of the reserve requirement policy of the United States.

[6]Appendix A.2.2 contains more detail on the Chow tests reported in Figure 1.3.

[7]One may think this is due to the relaxation of bank deposit regulation in the 1980s and 1990s that stimulated financial innovations such as money market deposit accounts (MMDAs) in the 1980s or retail sweep accounts in the 1990s (e.g., VanHoose and Humphrey, 2001, Teles and Zhou, 2005, Lucas and Nicolini, 2015, Berentsen, Huber, and Marchesiani, 2015). However, Appendix A.2.3 shows there is a breakdown in M2 in 1992 as well and MMDAs and retail sweeps are part of M2.

Table 1.1: Cointegration Regressions and Tests

Dependent Variable:	$ln(m_t)$		$ln(d_t)$	
	OLS	CCR	OLS	CCR
	(1)	(2)	(3)	(4)
r_t	0:016	0:027	0:049	0:053
	(0.004)	(0.004)	(0.009)	(0.009)
$ln(uc_t)$		0:279		0:574
		(0.033)		(0.040)
$adj R^2$	0.109	0.970	0.229	0.962
N	112	112	112	112
Johansen $r = 0$	15.004	41.744	14.934	49.174
5% Critial Value for $r = 0$	15.41	29.68	15.41	29.68
1% Critial Value for $r = 0$	20.04	35.65	20.04	35.65
Johansen $r = 1$	0.027	12.163	0.26	14.319
5% Critial Value for $r = 1$	3.76	15.41	3.76	15.41
1% Critial Value for $r = 1$	6.65	20.04	6.65	20.04

Notes: Columns (1) and (3) report OLS estimates and columns (2) and (4) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Johansen cointegration test results are reported in columns (1)-(4). Appendix A.2.2 contains unit root tests for each series. The data are quarterly from 1980Q1 to 2007Q4.

Following Cagan (1956), and Ireland (2009), I relate the natural logarithm of m, the ratio of money balances to income, to the short-term nominal interest rate, denoted by r. I also regress r on the natural logarithm of d, the ratio of deposit balances to income.

$$ln(m_t) = {}_0 + {}_1 r_t + {}_t; \quad ln(d_t) = {}_0 + {}_1 r_t + {}_t$$

In addition to the above specifications, to capture the impact of the improved availability of consumer credit that can substitute the deposit, I add a logarithm of uc, the ratio of unsecured credit to income as another regressor as follows.[8]

$$ln(m_t) = {}_0 + {}_1 r_t + {}_2 ln(uc_t) + {}_t; \quad ln(d_t) = {}_0 + {}_1 r_t + {}_2 ln(uc_t) + {}_t$$

I focus on the post-1980 period, until the arrival of the Great Recession. In Table

[8] Following to Krueger and Perri (2006), I use revolving consumer credit.

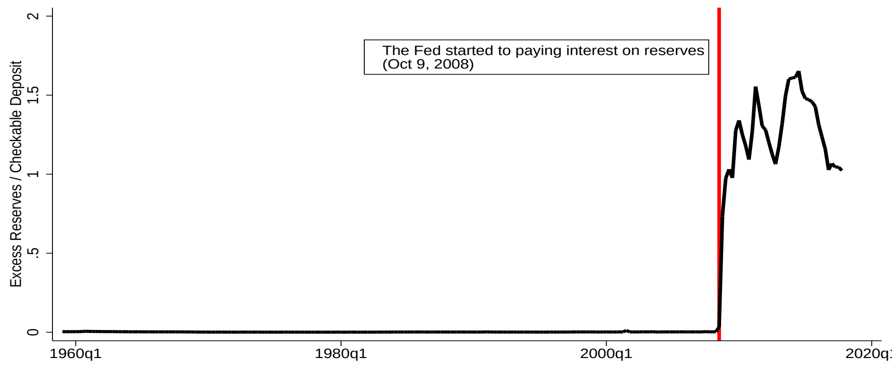

Figure 1.5: Excess Reserves Ratio

1.1, columns (1) and (3) report the estimates without unsecured credit, and columns (2) and (4) report the estimates with unsecured credit. The Johansen tests in columns (1) and (3) fail to reject the null hypothesis of no cointegration, which confirms the apparent breakdowns from Figure 1.4, and ordinary least squares (OLS) estimates from columns (1) and (3) both report positive coefficients on r_t that contradict the conventional notion of money demand: the stable downward-sloping relationship between real balances and interest rates. In columns (2) and (4), however, the Johansen tests reject their null of no cointegration at a 99 percent confidence level, suggesting there exists a stable relationship between real money balances, interest rates, and real balances of unsecured credit. To estimate the cointegration relationship, I implement the canonical cointegrating regression, proposed by Park (1992), in columns (2) and (4). The estimated coefficients on r_t and $ln(uc_t)$ both are negative and significantly different from zero. Thus, using the cointegrating regressions and tests, I document the evidence that once one accounts for the substitution effect of consumer credit, there still exists a stable negative relationship between real money balances and the interest rates. This substitution effect is a potential explanation for the decline of the money multiplier during 1992Q2-2008Q3.

The second structural break at 2008Q3, which can be detected from the bottom-right panel of Figure 1.3 as well as the bottom-left panel, coincides with the Fed's introduction of the interest on reserves. As Nakamura (2018) points out, the amount of reserves had skyrocketed before interest rates hit zero and its dramatic increase was simultaneous with the Fed's introduction of the interest on reserves.

To interpret the observed patterns from data, in the following section I develop a model that can incorporate the evolution of the money creation process.

1.3 Model

The model constructed here extends the standard monetary search model (Lagos and Wright, 2005) by introducing fractional reserve banking and unsecured credit. Time is discrete and two markets convene sequentially in each time period: (1) a frictionless centralized market (CM, hereafter), where agents work, consume, and adjust their balances, following after (2) a decentralized market (DM, hereafter), where buyers and sellers meet and trade bilaterally. The DM trade features imperfect record-keeping and limited commitment. Due to these two frictions, some means of payment are needed in DM trades. Below I describe the economic agents in this economy and the different types of DM meetings.

Buyers and Sellers The economy consists of a unit mass of buyers and a unit mass of sellers who discount their utility each period by . The preferences of buyers and sellers for each period are

$$U^b = U(X) \quad H + u(q) \quad \text{and} \quad U^s = U(X) \quad H \quad c(q);$$

where X is the CM consumption, H is the CM disutility from production, and q is the DM consumption. As standard, assume U', u', $c' > 0$, U'', $u'' < 0$, c'' 0, and $u(0) = c(0) = 0$. Consumption goods are perishable. One unit of H produces one unit of X in the CM. The efficient consumption in the CM and DM is denoted by X and q , respectively, which solve $U'(X\ ) = 1$ and $u'(q\ ) = c'(q\ )$, respectively.

The Bank There are measure n of active banks that is endogenously determined by the free entry condition in the equilibrium. In the CM, the banks accept deposit and decide how much to deposit at the central bank as reserves and how many banknotes to issue. Managing a deposit payment facility incurs a cost. The cost is represented by a cost function $(\tilde{d})$, where $\tilde{d}$ is the amount of deposit in real terms, $'; \ '' > 0$, and $(0) = \ '(0) = 0$. The reserve earns a nominal interest rate of i_r. The bank extends loans by issuing banknotes. The loans are paid back with interest rate i_ℓ. Enforcing repayment is costly. The cost function is described by $(\tilde{\ })$, where $\tilde{\ }$ is the loan in real terms, $'; \ '' > 0$, and $(0) = \ '(0) = 0$. The bank's lending is constrained by the reserve requirement, i.e., the bank cannot lend more than $(1 \quad)\tilde{r}/$, where   is the reserve requirement and $\tilde{r}$ is the real balance of reserves.

Types of DM meetings There are three types of DM meetings. In DM1, there is no record-keeping device, and the seller can only recognize cash. In DM2, the seller can recognize cash, the claims on bank accounts, and private banknotes. So she accepts cash, deposit receipts and banknotes. In DM3, in addition to the means of payment accepted in DM2, the buyer can trade using unsecured credit with credit limit   as the trading is monitored imperfectly.[9] The probability of joining a type j

[9] The acceptance of different means of payment can be endogenized as in Lester, Postlewaite, and

meeting is $_j$. The agents get to know which type of meeting they will be going to in the preceding CM.

The Central Bank The central bank controls the base money supply M in the CM. Let denote the base money growth rate. Then the changes in the real balance of base money can be written as

$$M = M^+ \quad M;$$

where x^+ is the value of (any variable) x in the next period. The base money is held in two ways: (1) C as currency in circulation, i.e., outside money held by agents; (2) R as reserves held by a representative bank. Thus,

$$M = C + R:$$

The central bank can control the base money supply in two ways. First, it can conduct a lump-sum transfer or collect a lump-sum tax in the CM. Second, it can increase the money supply by paying interest on reserves, i_r. Let T represents a lump-sum transfer (or tax if it is negative). The central bank's constraint is

$$M = \quad (M^+ \quad M) = T + i_r \ R;$$

where is the price of money in terms of the CM consumption good.

Wright (2012) or Lotz and Zhang (2016) but here we assume the types of meetings are exogenously given. Lester, Postlewaite, and Wright (2012) endogenize the meeting types by allowing sellers' costly *ex ante* choice to acquire the technology for recognizing certain type of assets. Similarly, Lotz and Zhang (2016) study the environment with costly record-keeping technology where sellers must invest in a record-keeping technology to accept credit.

1.3.1 The CM Problem

Buyers' Decisions At the beginning of the CM, each buyer's subsequent DM meeting type is realized. Therefore, the buyers' CM problem depends on their DM meeting type. Let $W_j^B(m; d; b; \ell; \ )$ denote the CM value function where j is the type of the following DM meeting, m is the cash holding, d is the deposit balance, b is the private banknote holding, ℓ is the loan borrowed from the bank during the last CM period, and $\ $ is the unsecured debt owed to the seller from the previous DM. All the state variables are in unit of the current CM consumption good. Let $\ $ denote the lump-sum transfer (or tax if it is negative) to the buyer in the CM. Now, consider the value of the CM. For an agent who is going to a type j DM meeting, the CM problem is

$$W_j^B(m; d; b; \ell; \ ) = \max_{X; H; \hat{m}_j; \hat{d}_j; \hat{b}_j; \hat{\ }_j} U(X) \quad H + \quad V_j^B(\hat{m}_j; \hat{d}_j; \hat{b}_j; \hat{\ }_j)$$

$$\text{s.t. } (1 + \quad)\hat{m}_j + (1 + \quad)\hat{d}_j + X = m + (1 + i_d)d + b \quad (1 + i_\ell)\ell \quad\quad + H + $$

$$\hat{b}_j = \hat{\ }_j;$$

where $\hat{m}_j$, $\hat{d}_j$, $\hat{b}_j$ and $\hat{\ }_j$ are the cash holding, deposit balance, private banknote balance, and debt balance, respectively, carried to the next DM. The first-order conditions (FOCs) are $U'(X) = 1$ and

$$(1 + \quad) + \quad \partial V_j^B(\hat{m}_j; \hat{d}_j; \hat{b}_j; \hat{\ }_j)/\partial \hat{m}_j \quad 0; = \text{ if } \hat{m}_j > 0 \tag{1.1}$$

$$(1 + \quad) + \quad \partial V_j^B(\hat{m}_j; \hat{d}_j; \hat{b}_j; \hat{\ }_j)/\partial \hat{d}_j \quad 0; = \text{ if } \hat{d}_j > 0 \tag{1.2}$$

$$\partial V_j^B(\hat{m}_j; \hat{d}_j; \hat{b}_j; \hat{\ }_j)/\partial \hat{b}_j + \quad \partial V_j^B(\hat{m}_j; \hat{d}_j; \hat{b}_j; \hat{\ }_j)/\partial \hat{\ }_j \quad 0; = \text{ if } \hat{\ }_j > 0. \tag{1.3}$$

The first term on the left-hand side (LHS) of equation (1.1) is the marginal cost of acquiring cash. The second term is the discounted marginal value of carrying cash to the following DM. Therefore, the choice of $\hat{m}_j > 0$ equates the marginal cost and the marginal return on cash. A similar interpretation applies to equation (1.2) for the decision on deposit. For equation (1.3), the first term on the LHS captures the discounted marginal value of carrying privately issued banknotes from the CM to the following DM, and the second term captures the discounted marginal cost of getting a bank loan. The envelope conditions for $W_j^B(m, d, b, \ell, \tau)$ are

$$\frac{\partial W_j^B}{\partial m} = 1, \quad \frac{\partial W_j^B}{\partial d} = 1 + i_d, \quad \frac{\partial W_j^B}{\partial b} = 1, \quad \frac{\partial W_j^B}{\partial \ell} = -(1 + i_\ell), \quad \frac{\partial W_j^B}{\partial \tau} = -1$$

for all $j = 1, 2, 3$, which implies $W_j^B(m, d, b, \ell, \tau)$ is linear. This linearity allows us to write

$$W_j^B(m, d, b, \ell, \tau) = m + (1 + i_d)d + b - (1 + i_\ell)\ell - \tau + W_j^B(0, 0, 0, 0, 0).$$

Let $W^B(m, d, b, \ell, \tau)$ be the buyers' expected value function before the CM at period t opens, i.e., before their subsequent DM meeting type is realized. Then one can write the buyer's expected value function in the CM as $W^B(m, d, b, \ell, \tau) = \sum_j \alpha_j W_j^B(m, d, b, \ell, \tau)$.

Sellers' Decisions A seller enters the CM with cash, m, deposits, d, private banknotes, b, and unsecured credit τ that a buyer owes to the seller from the previous DM. The seller does not borrow from the bank as long as $i_\ell > 0$. Let $W_j^S(m, d, b, 0, \tau)$ be the sellers' value function in the CM at period t. It can be written as follows:

$$W_j^S(m, d, b, 0, \tau) = \max_{X, H, \hat{m}_j, \hat{d}_j, \hat{b}_j, \hat{\tau}_j} U(X) - H + \beta V_j^S(\hat{m}_j, \hat{d}_j, \hat{b}_j, \hat{\tau}_j)$$

$$\text{s.t. } (1 + \pi)\hat{m}_j + (1 + \pi)\hat{d}_j + X = m + (1 + i_d)d + b + \tau + H + T$$

$$\hat{b}_j = \hat{\tau}_j$$

As we will see below, the DM terms of trade does not depend on the seller's portfolio, there is no incentive for the sellers to carry any liquidity to the next DM as the cost of holding liquidity is positive. The envelope conditions are

$$\frac{\partial W_j^S}{\partial m} = 1; \quad \frac{\partial W_j^S}{\partial d} = 1 + i_d; \quad \frac{\partial W_j^S}{\partial b} = 1; \quad \frac{\partial W_j^B}{\partial} = 1$$

for all $j \in \{1, 2, 3\}$, which implies $W_j^S(m, d, b, 0, \ )$ is linear. By linearity, the CM value function can be written as

$$W_j^S(m, d, b, 0, \ ) = m + (1 + i_d) + b + \quad + W_j^S(0, 0, 0, 0, 0).$$

1.3.2 The DM Problem

In the DM, the buyer and seller trade bilaterally. Let q_j and p_j be the DM consumption and payment in a type-j DM meeting. The bilateral trade is characterized by (p_j, q_j). This trade is subject to $p_j \quad z_j$ where z_j is the total liquidity of the buyer in a type-j meeting. The liquidity position for each type of buyer is

$$z_1 = m_1 \tag{1.4}$$

$$z_2 = m_2 + d_2(1 + i_d) + b_2 \tag{1.5}$$

$$z_3 = m_3 + d_3(1 + i_d) + b_3 + \quad . \tag{1.6}$$

The DM terms of trade is determined by Kalai (1977) proportional bargaining. Kalai bargaining solves the following problem:

$$\max u(q) \quad p \quad s.t \quad u(q) \quad p = \quad [u(q) \quad c(q)]$$

where $\ \in [0, 1]$ denotes the buyers' bargaining power. The payment, p, is a function of DM consumption, q. This can be expressed as $p = v(q) = (1 \quad)u(q) + \quad c(q)$.

Define *liquidity premium*, (q), as follows:

$$(q) = \frac{u'(q)}{v'(q)} \quad 1 = \frac{[u'(q) \quad c'(q)]}{(1 \quad)u'(q) + \ c'(q)}$$

where $(q) > 0$ for $q < q$ and $(q\ ) = 0$ with $'(q) < 0$ for $q \in [0; q\ )$. When $z_j \quad p$, the buyer has sufficient liquidity to purchase efficient DM output q . In this case, the payment to the seller is $p \ = v(q\ )$.

By the linearity of W_j^B, we can write a DM value function for a buyer in a type-j meeting as follows:

$$V_j^B(m_j; d_j; b_j; \ell_j) = u(q_j) \quad p_j + W^B(m_j; d_j; b_j; \ell_j; 0) \tag{1.7}$$

where $p_j \quad z_j$. The third term on the right-hand side (RHS) is the continuation value when there is no trade. The rest of the RHS is the surplus from the DM trade. DM payments are constrained by $p_j \quad z_j$. With $v(q_j) = p_j$ and $z_j \quad p$, differentiating V_j^B and substituting its derivatives into the FOCs from the CM problem yields

$$(1 + \ ) + \quad 1 + \ (q_1) \ = 0 \tag{1.8}$$

$$(1 + \ ) + (1 + i_d) \qquad 0; = \text{ if } d_1 > 0 \tag{1.9}$$

$$i_\ell \qquad 0; = \text{ if } \ell_1 > 0 \tag{1.10}$$

$$(1 + \ ) + \quad 1 + \ (q_i) \qquad 0; = \text{ if } m_j > 0 \text{ for } j = 2; 3 \tag{1.11}$$

$$(1 + \ ) + (1 + i_d) \quad 1 + \ (q_i) \qquad 0; = \text{ if } d_j > 0 \text{ for } j = 2; 3 \tag{1.12}$$

$$i_\ell + \ (q_i) \qquad 0; = \text{ if } l_j > 0 \text{ for } j = 2; 3; \tag{1.13}$$

where $q_j = \min\{q\ ; v^{-1}(z_j)\}$ and $(q\ ) = 0$.

Similarly, the sellers' DM value function is

$$V_j^S(m_j; d_j; b_j; `_j) = p_j \quad c(q_j) + W_j^S(m_j; d_j; b_j; `_j; 0):$$

1.3.3 The Bank's Problem

A bank maximizes its profit subject to the lending constraint.

$$\max_{r; d; \tilde{}} \quad i_r r \quad i_d d \quad (d) + i_` \tilde{} \quad (\tilde{}) \tag{1.14}$$

$$s.t. \quad \tilde{} \quad ` \quad \frac{1}{\quad} r \tag{1.15}$$

$$r \quad d \tag{1.16}$$

Let $_L$ denote the Lagrange multiplier for the lending constraint. It is straightforward to show $r = d$. The FOCs for the bank's problem can be written as

$$0 = i_r \quad i_d \quad {}^0(r) + \quad _L \frac{1}{\quad} \tag{1.17}$$

$$0 = i_` \quad {}^0(\tilde{}) \quad _L: \tag{1.18}$$

The bank's *ex post* profit equals to the entry cost, k

$$(i_r \quad i_d) r + i_` \tilde{} \quad (r) \quad (\tilde{}) = k: \tag{1.19}$$

Suppose there are active banks. Consider two cases. In the first case, the bank's lending constraint is binding, i.e., $_L > 0$. In the second case, the bank's lending constraint is loose, i.e., $_L = 0$. We call the first case a "scarce-reserves case," and the second an "ample-reserves case."

The Scarce-Reserves Case The bank does not have enough reserves. It needs to acquire reserves to make more loans, which implies a binding constraint. With $_L > 0$, the bank's FOCs (1.17) and (1.18) give

$$0 = i_r \quad i_d \quad {}^0(\bar{r}) + \left[i_\ell \quad {}^0(\tilde{\ell}) \right] \frac{1}{\quad} : \tag{1.20}$$

The Ample-Reserves Case The bank has sufficient reserves. Its lending constraint does not bind. Then the two FOCs for the bank's problem are

$$0 = i_r \quad i_d \quad {}^0(\bar{r}) \tag{1.21}$$

$$0 = i_\ell \quad {}^0(\tilde{\ell}): \tag{1.22}$$

The bank's unconstrained optimal lending, ℓ, satisfies

$$i_\ell = {}^0(\ell)$$

and increases as i_ℓ rises.

1.3.4 Stationary Equilibrium

I focus on a symmetric stationary monetary equilibrium in which the same type of agents make the same decisions and the real balances are constant over time. Given that $= {}^+ = M^+/M = C^+/C = 1 +$, the net inflation rate, , is equal to the currency growth rate, , in the stationary monetary equilibrium. By the Fisher equation, $1 + i = (1 + \quad)/$. The market clearing conditions are

$$_2 \ell_2 + \ _3 \ell_3 = n\tilde{\ell} = \ell \tag{1.23}$$

$$_2 d_2 + \ _3 d_3 = n\bar{r} = r = \quad R \tag{1.24}$$

$$_1m_1 + \quad _2m_2 + \quad _3m_3 = m = \quad C; \tag{1.25}$$

where $M = C + R$. Note that $i \quad 0$, i.e., $\quad 1$ is necessary for the existence of equilibrium.[10] Define the stationary equilibrium as follows:

Definition 1 (Stationary Equilibrium). *Given monetary policy, i, i_r, and and credit limit , a stationary monetary equilibrium consists of real balances $(m_j; d_j; \ell_j)_{j=1}^3$; allocation $(q_1; q_2; q_3; X)$; the measure of banks n, and prices $(i_\ell; i_d)$, such that*

(i) $(i_d; i_\ell; q_1; q_2; q_3)$ *solves* (1:8)-(1:13) *and* (1:17)-(1:19) *with* $q_j = \min\{q \ ; v \ ^1(z_j)\}$*, where* $z_1 = m_1$, $z_2 = m_2 + (1 + i_d)d_2 + \ell_2$*, and* $z_3 = m_3 + (1 + i_d)d_3 + \ell_3 + \ .$

(ii) *The bank lending* $\tilde{\ell} = \min(\ell; \ell \ )$*, where* $\ell = (1 \quad)r= \$ *and* ℓ *solves* $i_\ell = \ ^0(\ell \ )$

(iii) *Asset markets clear* (1:23)-(1:25).

Given Definition 1, there are three types of equilibria, which are defined as follows:

Definition 2. *In a no-banking equilibrium,* $\ell = r = n = 0$*. In an ample-reserves equilibrium,* $\ell > \ell \quad 0$*. In a scarce-reserves equilibrium,* $\ell \quad \ell > 0$*.*

In the no-banking equilibrium, the deposit interest rate is zero, $i_d = 0$ and there is no active bank. Because the return to holding deposits is dominated or equal to the return to holding currency, agents do not have any incentive to deposit their balances. With zero reserve, the lending limit is zero. Therefore, in this equilibrium, agents only use cash for DM trading. All agents hold the same balance of cash and consume the same amount of consumption goods.

$$i = L(m_j) \text{ for } j = 1; 2 \tag{1.26}$$

[10] Whereas the lower bound of the nominal interest rate is zero in this setting, one can relax this by introducing liquid assets or threats of theft. See Rocheteau, Wright, and Xiao (2018), Lee (2016) and Williamson (2019) for details.

and $m_3 = \max\{0; L^{-1}(i) \quad \}$. In this equilibrium, it is straightforward to see that DM consumption is efficient when $i = 0$, i.e. the Friedman rule applies. In the ample-reserves equilibrium, each bank holds sufficient reserves to lend ℓ , i.e., $\ell \ \ < \bar{\ell} = (1 \quad)r/\ $, where r represents the equilibrium reserve balance of each bank. Thus, the unconstrained optimal lending is less than the lending limit. Given i and i_r, $(r; \tilde{\ell})$ solves

$$\left\{1 + \ ^{\prime}(\tilde{\ell})\right\}[1 + i_r \quad ^{\prime}(r)] = 1 + i; \quad ^{\prime}(r)r \quad (r) + \ ^{\prime}(\tilde{\ell})\tilde{\ell} \quad (\tilde{\ell}) = k.$$

In the scarce-reserves equilibrium, however, the lending limit is lower than the bank's unconstrained optimal lending. Therefore, the lending constraint (1.15) binds, i.e., $\ell \ \ > \tilde{\ell} = \bar{\ell} = (1 \quad)r/\ $. Given i_ℓ and i_d, the equilibrium real balance of reserves satisfies

$$r = \begin{cases} \dfrac{(\ _2 + \ _3)}{1 + i_d} L^{-1}(i_\ell) \quad \dfrac{\ _3}{1 + i_d} & \text{if } \hat{\ } > \\ \dfrac{\ _2}{1 + i_d} L^{-1}(i_\ell) & \text{if } \hat{\ } \end{cases} \tag{1.27}$$

where $\hat{\ } = L^{-1}(i_\ell)$.

For each type of equilibrium, the following results are proved in Appendix A.1.

Proposition 1. *(i) In the ample-reserves and scarce-reserves equilibrium, $\partial i_d/\partial i > 0$, $\partial i_\ell/\partial i > 0$, $\partial i_d/\partial i_r > 0$, and $\partial i_\ell/\partial i_r < 0$. (ii) In the no-banking equilibrium, $\partial i_d/\partial i = \partial i_d/\partial i_r = \partial i_\ell/\partial i = \partial i_\ell/\partial i_r = 0$.*

Proposition 1 tells us that as long as the measure of banking is positive, the monetary policy rates pass through the deposit rate and lending rate. The deposit rate is strictly increasing in the nominal interest rate and the interests on reserves. The lending rate is strictly increasing in the nominal interest rate but is strictly

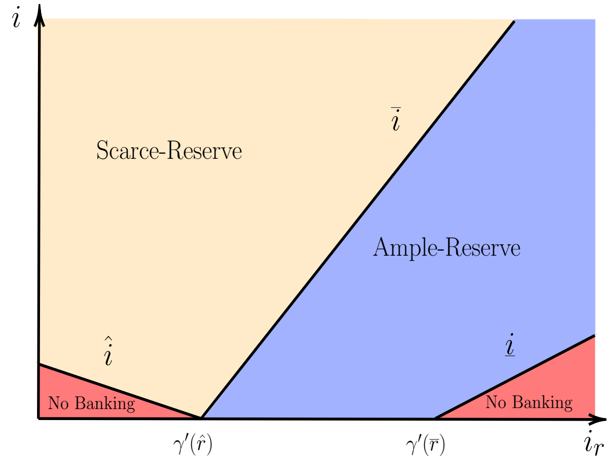

Figure 1.6: Monetary Equilibrium Regions in $(i; i_r)$ Space

decreasing in interest on reserves. I also establish conditions for the determination of equilibrium types.

Proposition 2. *Given* $(i_r; \ )$*: (i)* $\exists!$ *ample-reserves equilibrium if and only if* $i_r > \gamma'(\hat{r})$ *and* $i \in (\underline{i}; \bar{i})$*; (ii)* $\exists!$ *scarce-reserves equilibrium if and only if either* $i > \hat{\imath}$ *and* $i_r < \gamma'(\hat{r})$ *or* $i > \bar{i}$ *and* $i_r > \gamma'(\hat{r})$*; (iii)* $\exists!$ *no banking equilibrium if and only if either* $i \in [0; \hat{\imath})$ *and* $i_r < \gamma'(\hat{r})$*, or* $i \in [0; \underline{i})$ *and* $i_r > \gamma'(\underline{r})$*; and the thresholds satisfy*

$$\hat{\imath} = \frac{}{1 - }[\gamma'(\hat{r}) - i_r] + \gamma'\left(\frac{1}{}\hat{r}\right); \quad \bar{i} = [1 + i_r - \gamma'(\hat{r})]\left[1 + \gamma'\left(\frac{1}{}\hat{r}\right)\right] - 1$$

and $\underline{i} = i_r - \gamma'(\underline{r})$*, where* $\hat{r}$ *solves* $\gamma'(\hat{r})\hat{r} - \gamma(\hat{r}) + \gamma'\left(\frac{1}{}\hat{r}\right)\frac{1}{}\hat{r} - \gamma\left(\frac{1}{}\hat{r}\right) = k$ *and* $\underline{r}$ *solves* $\gamma'(\underline{r})\underline{r} - \gamma(\underline{r}) = k$*.*

Banks hold excess reserves when the central bank pays sufficiently high interest on reserves with the nominal interest rate at some moderate level. To see the intuition consider the case in which the bank holds reserves. The reserve requirement and the reserve balances determine the lending limit. Due to monotone pass-through from the nominal interest rate to the bank's lending rate, the bank's unconstrained optimal lending ℓ is increasing in the nominal interest rate. There exists a threshold of the nominal interest rate below which the lending limit is lower than the bank's unconstrained lending (scarce-reserves), and above which the lending limit is higher than the bank's unconstrained lending (ample-reserves). In other words, there is a critical value i that satisfies $\ell = \ell = (1 \quad)r = \quad$.

However, when the equilibrium deposit rate is zero, agents have no incentive to deposit their balance in the bank, implying the no-banking equilibrium. As shown in Proposition 1, the deposit rate is monotone in the nominal interest rate and the deposit rate could be zero given some nominal interest rate. There exists a threshold $\hat{\imath}$, below which the deposit rate is zero and above which the deposit rate is positive. The bank's constraint plays a crucial role in determining the equilibrium type. Lowering the nominal interest rate or increasing interest on reserves loosens the bank's lending constraint. These lead to the following results:

Corollary 1. *The thresholds i and $\underline{i}$ are increasing in i_r and $\hat{\imath}$ is decreasing in i_r.*

As Figure 1.6 illustrates, the equilibrium type is determined by $(i; i_r)$. The bank holds excess reserves when the interest on reserves i_r is high enough and the nominal interest rate is not too high.

In the ample-reserves equilibrium, $q_2 = q$ if $i_r \quad i = \; {}'(\underline{r})$. Therefore, the DM2 meeting consumption can be efficient even though the economy is not under the Friedman rule. This result can be formally summarized in the following proposition.

Proposition 3. *In the ample-reserves equilibrium with $i > 0$, DM2 consumption is efficient if $i_r = i + \; {}'(\underline{r})$.*

The intuition behind the efficient DM2 consumption is straightforward. In many monetary models, a high inflation or interest rate increases the opportunity cost of

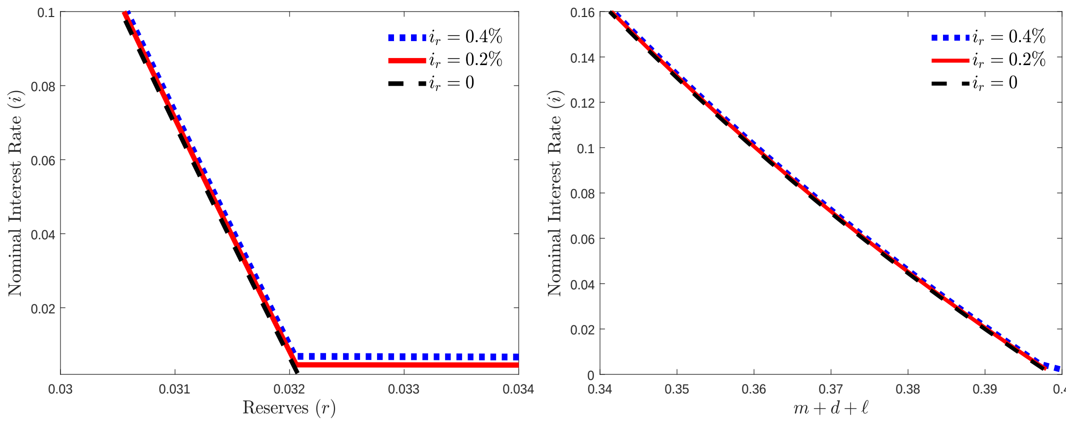

Figure 1.7: Demand for Reserves and the Monetary Aggregate

holding money. In the environment where money is valued as a medium of exchange, having less liquidity in the economy because of an opportunity cost of holding money is inefficient. However, the interest on reserves provides a proportional return. If this return is properly distributed across agents, it eliminates the inefficiency that arises from the opportunity cost of holding money, which results in efficient DM2 consumption.

In the scarce-reserves equilibrium, it is easy to show that the equilibrium reserve balance is decreasing in i

$$\frac{\partial r}{\partial i} = \begin{cases} \frac{-(\gamma_2 + \gamma_3) L'^{-1}(i_\ell)}{1 + i_d} \frac{\partial i_\ell}{\partial i} - \frac{[(\gamma_2 + \gamma_3) L'^{-1}(i_\ell) - \gamma_3]}{(1 + i_d)} \frac{\partial i_d}{\partial i} < 0 & \text{if } \wedge > \\ \frac{-\gamma_2 L'^{-1}(i_\ell)}{1 + i_d} \frac{\partial i_\ell}{\partial i} - \frac{\gamma_2 L^{-1}(i_\ell)}{(1 + i_d)^2} \frac{\partial i_d}{\partial i} < 0 & \text{if } \wedge \end{cases}$$

because $\partial i_\ell / \partial i < 0$ and $\partial i_d / \partial i > 0$, as shown in Proposition 1.

Whereas Proposition 1 shows that how the deposit rate can change as the central bank sets the nominal interest rate, the pass-through of nominal interest rate to deposit rate depends on other policy variables and the equilibrium type.

Proposition 4. *Higher reserve requirement weakens the pass-through from the nom-*

inal interest rate to the deposit rate in the scarce-reserve equilibrium, i.e.,

$$\frac{\partial^2 i_d}{\partial \quad \partial i} < 0 \quad \text{if } \ell < \ell \, ;$$

Higher interest on the reserve raises the pass-through from the nominal interest rate to the deposit rate in the ample-reserves equilibrium, i.e.,

$$\frac{\partial^2 i_d}{\partial i_r \partial i} > 0 \quad \text{if } \ell \quad \ell \, ;$$

Regardless of the equilibrium type, there exists a downward-sloping demand curve for total liquidity. To illustrate this, I define the monetary aggregate as a sum of cash holdings, reserves, and banknotes in the economy. The right (left) panel of Figure 1.7 shows that there exist stable downward-sloping demand curves for monetary aggregates (reserves) with different interest on reserves. An increase in i_r shifts the money demand to the right, both in the scarce-reserves equilibrium and in the ample-reserves equilibrium. In the scarce-reserves case, higher i_r raises $\tilde{r}$ and $\tilde{}$ with a looser lending constraint and shifts the money demand to the right. However, in the ample-reserves equilibrium, higher i_r raises reserves $\tilde{r}$ but decreases $\tilde{}$. Last, a rise in i_r increases i, which allows the monetary authority to induce the ample-reserves equilibrium with higher nominal interest rates. As the central bank pays interest on reserves, the economy can shift to the ample-reserves equilibrium, which has a flatter demand curve for reserves. This is consistent with the observation from Nakamura (2018) that the quantity of reserves skyrocketed before interest rates hit zero, but its dramatic increase was simultaneous with the Fed's introduction of the interest on reserves.

The Role of Access to Unsecured Credit One also can check the effect of changes in the access to unsecured credit, $_3$, or changes in the credit limit, . An

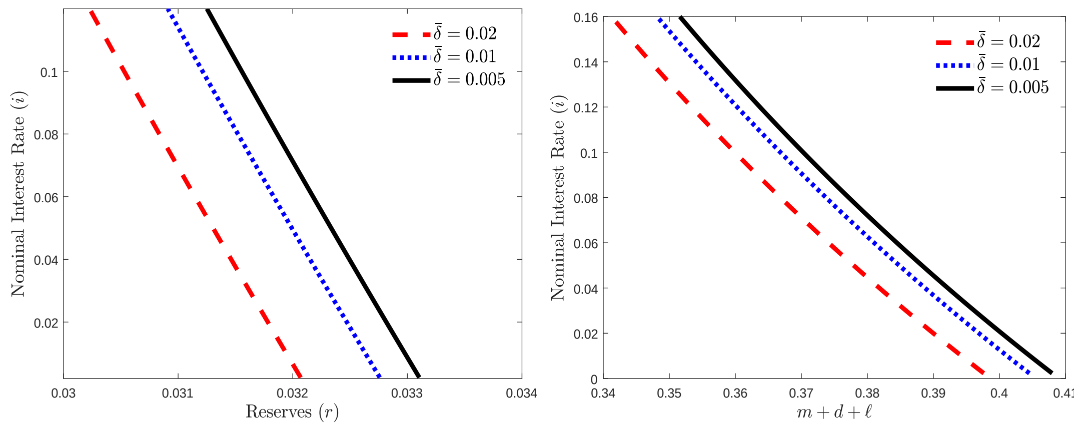

Figure 1.8: Demand for Reserves and the Monetary Aggregate with Different Credit Limits

increase in $_3$ implies that more buyers can use unsecured credit in the DM. Some DM2 buyers become DM3 buyers and they hold less deposit than they used to hold. With higher , although the measure of DM3 buyers stays same, DM3 buyers can use more unsecured credit for the DM trade. As long as there are positive amount of reserve balances, $r > 0$, an increase in $_3$ or lowers r. Appendix A.1 verifies the following:

Proposition 5. *Let $\hat{} >$. In scarce-reserve and ample-reserves equilibria, better credit condition and more credit access decrease the real balance of reserves, i.e.,*

$$\frac{@r}{@} < 0; \quad \frac{@r}{@ _3} < 0:$$

This result is consistent with the finding in Section 1.2. As more unsecured credit becomes available, real balances of inside money decrease. By summing up the results, we now establish the results on the money multiplier. Define money multiplier $(m + r + `)=(m + r)$, then we have following results:

Proposition 6. *In ample-reserves equilibrium, for small m, we have*

$$\frac{\partial}{\partial i} > 0, \quad \frac{\partial}{\partial i_r} < 0.$$

Let $\hat{} >$. *In the ample-reserves and scarce-reserves equilibria, a better credit condition lowers the money multiplier as long as* $m > 0$ *and* < 1*, i.e.,*

$$\frac{\partial}{\partial} < 0 \quad \text{if } m > 0 \ \& \ < 1.$$

Thus, the model can successfully address the mechanism illustrated in Section 1.2. We can interpret the decline in the money multiplier in the pre-2008 economy as a result of improved availability of consumer credit under the scarce-reserve equilibrium. For the post-2008 period, after the Fed started paying interest on reserves the economy moved to the ample-reserves equilibrium. The model suggests that the changes in the money multiplier and the excess reserve ratio are the results of the Fed's management of two interest rates, the nominal interest rate and the interest on reserves.

1.4 Quantitative Analysis

To evaluate the theory quantitatively, I calibrate the model to match several targets using pre-2008 data. Using calibrated parameters, I compare the model predictions with the data of the pre- and post-2008 periods. Given the parameters, the stationary equilibrium is characterized by $(i, i_r, \ , \ )$. The required reserves ratio is computed by dividing the required reserves by total checkable deposits. Whereas the first three series are easy to obtain, it is hard to get the unsecured credit limit, , from either macro or micro data. Since the use of unsecured credit corresponds to the credit limit in the model, the unsecured credit limit is computed using the unsecured credit to

Table 1.2: Model Parametrization

Parameter	Value	Target/source	Data	Model
		External Parameters		
DM3 matching prob, $_3$	0.69	SCF 1970-2007		
		Internal Parameters		
Bargaining power,	0.454	avg. retail markup	1.384	1.384
Enforcement cost level, E	0.001	avg. UC/DM	0.387	0.370
Deposit operating cost level, A	0.0017	avg. R/Y	0.014	0.017
Entry cost, k	0.0011	avg. $/Y$	0.0016	0.0011
DM1 matching prob, $_1$	0.187	avg. C/D	0.529	0.523
DM utility level, B	0.825	avg. C/Y	0.044	0.044
DM utility curvature, b	0.398	semi-elasticity of C/Y to i	3.713	3.712

Note: C, R, DM, D, UC, and Y denote currency in circulation, reserves, DM transactions, deposit, unsecured credit, and nominal output, respectively. denotes the net income of banks.

output ratio, instead of using explicit credit limit data. In the model, the unsecured credit to output ratio is given by $_3/(B + \sum_j^3 \, _j z_j)$, so we can compute using the model with the given policies $(i, i_r, \ )$ and other parameters. Following Krueger and Perri (2006), the revolving consumer credit is used as the unsecured credit. For this exercise, I generate simulated data by using 4 series: (i) nominal interest rates[11]; (ii) the interest on reserves; (iii) the required reserve ratio; and (iv) the unsecured credit to GDP ratio.

1.4.1 Calibration

The utility functions for the DM and the CM are $u(q) = Bq^{1-b}/(1-b)$ and $U(X) = \log(X)$ implying $X^* = 1$ (a normalization). The cost function for the DM is $c(q) = q$. The enforcement cost for lending is assumed to be quadratic, $(\tilde{\ }) = E\tilde{\ }^2$, and the

[11] I use 3-month Treasury-bill rates as standard. Whereas I use 3-month Treasury-bill rates instead of the federal funds rates because the model does not include interbank lending, Section 1.4.5 and Appendix A.3.1 check robustness by using different measures of monetary policy. The sensitivity analysis suggests that the main results are not overly sensitive to the choice for the measure of monetary policy.

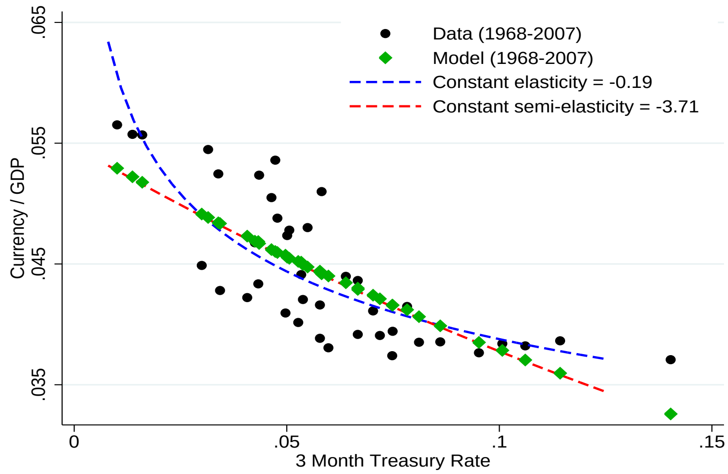

Figure 1.9: Money Demand for Currency

management cost for deposit facility takes the form, $(\tilde{d}) = A\tilde{d}^{a}$.[12] The fraction of buyers who can use unsecured credit is set to $_3 = 0{:}69$.[13] The remaining 7 parameters $(;A;B;b;k;E;\ _1)$ are set to match the following eight targets: (i) the average retail market markup; (ii) the average credit share of the DM transactions, $_3 = DM$; (iii) the average currency to deposit ratio, $C=D$; (iv) the average reserves to output ratio, $R=Y$; (v) the average currency to output ratio, $C=Y$; (vi) the semi-elasticity of $C=Y$ to i, where i denotes the nominal interest rate; and (vii) the average net income of banks to output ratio;[14] (viii) the average net income of banks to deposit ratio. The targets are computed on the basis of 1968-2007 data, except for the markup, which uses the average from 1993 to 2007, and the net income of banks, which uses the average from 1984 to 2007.

[12] To rule out the no-banking equilibrium from the exercise, I set the curvature parameter of ( ), a, to be 1.2 so that ( ) will be su□ciently less convex than ( ). Appendix A.3.2 includes a sensitivity analysis for the curvature parameter of ( ).

[13] The Survey of Consumer Finances provides triennial series for the percentage of U.S. households holding at least one credit card from 1970 to 2007. The average percentage during 1970-2007 is 69%.

[14] I use the net attributable income of FDIC-insured commercial banks and savings institutions as the net income of banks.

The bargaining power is set to match the DM markup to the retail markup.[15] Set $(B;b)$ to match the currency to output ratio and the semi-elasticity of $C=Y$ with respect to i. The costs of operating deposit services and issuing loans from the bank, captured by A and E, are set to match the reserves to output ratio, the unsecured credit to DM transaction ratio and the banking industry profit to deposit ratio. The entry cost k is set to match banking industry profit to output ratio. Lastly, I set $_1$ to match the currency-deposit ratio. The calibrated parameters and the targets are summarized in Table 1.2, and the calibrated money demand of currency is shown in Figure 1.9.

1.4.2 Results and the Model Fit

Figure 1.10 compares the model and data for the sample period, 1968 to 2007. The top-left panel of Figure 1.10 shows the M1 money multiplier from 1968 to 2007. The model generated decreases in the M1 multiplier during 1987-2007, while the peak was in 1987 in the data and 1984 in the model. During this period, the excess reserves to deposit ratio had been almost zero both in the data and in the model, suggesting the US economy had been in the scarce-reserves equilibrium. In the model, the declining trend of the M1 multiplier in the pre-2008 period is driven by the increase in unsecured credit, which crowds out the inside money (private banknotes and reserves) but not currency. This induces increases in the currency to deposit ratio, as shown in the bottom-left panel of Figure 1.10. The bottom-right panel of Figure 1.10 compares how unsecured credit crowds out reserves in the model and the data.

The next step is to evaluate model projections by comparing them with the data

[15] In the Annual Retail Trade Survey, the average ratio of gross margins to sales from 1993-2007 is 0.2776, implying the average markup is 1.3844.

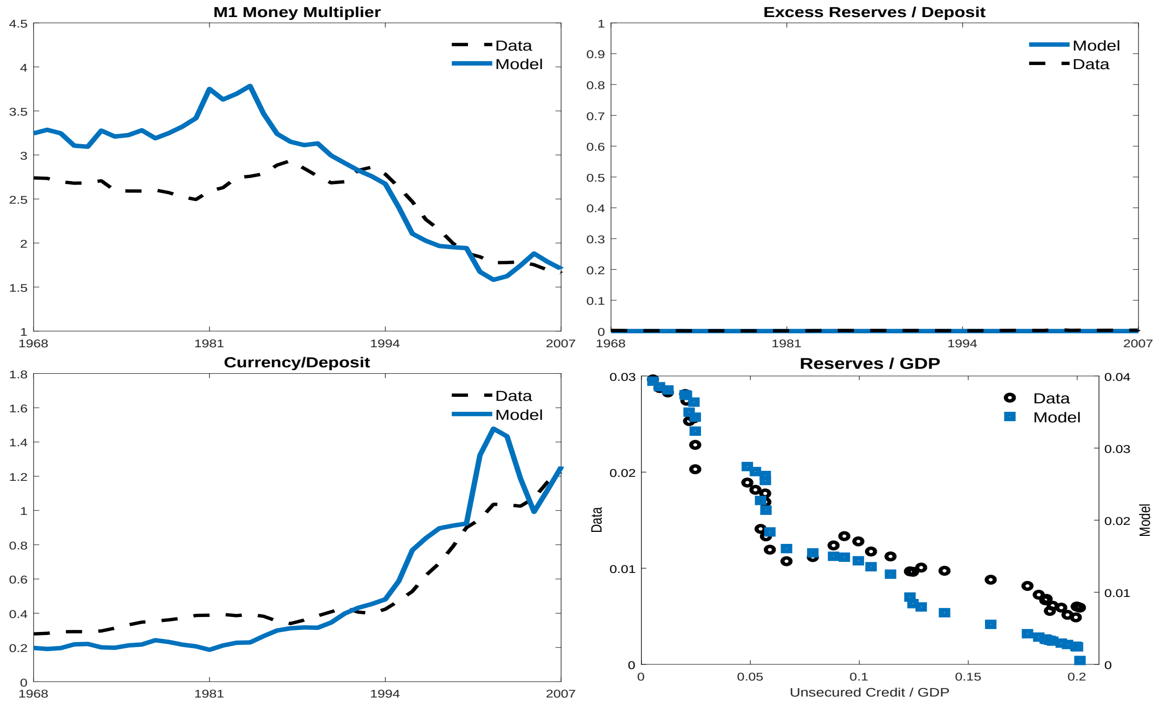

Figure 1.10: In-sample Fit: 1968-2007

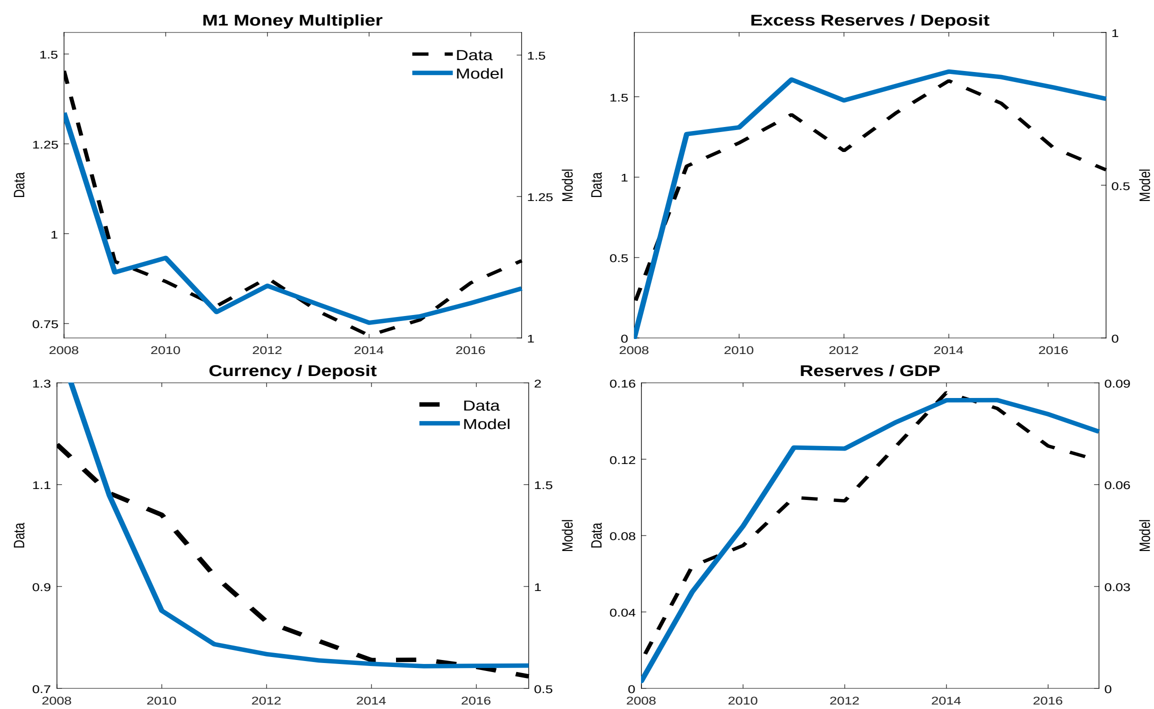

Figure 1.11: Out-of-sample Fit: 2008-2017

after 2007. Overall, the model can match the patterns in the data. Timeplots of Figure 1.11 compares the model projections for the M1 money multiplier, the excess reserves to deposit ratio, the currency to deposit ratio, and the reserves to output ratio with data from 2008 to 2017. The model-implied series shows similar patterns to the actual data series. The model can generate the change in the equilibrium type, from scarce-reserves to ample-reserves, and a similar pattern of excess reserves to deposit ratio. This change in the equilibrium type is represented by a huge drop in the money multiplier in the top-left panel and a huge increase in the excess reserves to deposit ratio in the top-right panel.

Regression estimates shown in Table 1.3 illustrate the main mechanism of the model. Columns (1) and (2) show the regression coefficient estimates using the following equation for 1968-2007.

$$\text{Reserves}_t/\text{GDP}_t = \beta_0 + \beta_1 \text{UnsecuredCredit}_t/\text{GDP}_t + \beta_2 \text{Tbill3}_t + \varepsilon_t$$

Since all three series have a unit root and are cointegrated, both in the data and in the model-generated series, the coefficients are estimated using the canonical cointegrating regression.[16] The estimated negative coefficient on the 3-month T-bill rate suggests a downward sloping demand for reserves with respect to the interest rate; but other coefficients on unsecured credit suggest that this demand for reserves can shift as the credit condition changes, as shown in Figure 1.8. This is consistent with Proposition 5, and the model-implied regression produces similar results.

Columns (3) and (4) regress the M1 multiplier on the 3 month T-bill rate and the interest on reserves, and Columns (5) and (6) regress the excess reserves ratio on the same variables. Because the number of observations in the data is too small, Columns

[16] Unit root and cointegration test results are reported in Appendix A.2.2.

Table 1.3: Model-implied Regression Coefficients, Model vs. Data

Dependent Variable:	Reserves/GDP (1968-2007)		M1 Money Multiplier (2009-2017)		Excess Reserve/Deposit (2009-2017)	
	Data (1)	Model (2)	Data (3)	Model (4)	Data (5)	Model (6)
Unsecured Credit/GDP	0:125 (0:010)	0:200				
3 Month T-bill Rate	0:001 (0:000)	0:001	0:686 (0:184)	0:864	1:561 (0:498)	1:697
Interest on Reserves			0:648 (0:210)	0:848	1:461 (0:567)	1:688
adj R^2	0:830	0:814	0:656	0:918	0:577	0:997

Notes: Columns (1)-(2) report the canonical cointegrating regression (CCR) estimates. First stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. Columns (3)-(6) report OLS estimates. For (3) and (5) Newey-West standard errors with lag 1 are reported in parentheses. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Intercepts are included but not reported.

(3) and (5) use data from the 1st quarter of 2009 to the 4th quarter of 2017. Regressions using the data and the model-implied series provide similar results. Based on the regression results in (3)-(6), for a given interest on reserves, raising the 3-month T-bill rates increases the M1 multiplier while it decreases excess reserves. At the same time, for a given 3-month treasury rate, lowering the interest on reserves decreases the M1 multiplier while it increases excess reserves. In the model, when the bank faces higher interests on reserves, it holds more reserves and does not lend as much as before. This is because interest on reserves yields profits to the bank with low cost, but lending is associated with the enforcement cost. This increases the excess reserve ratio and lowers the money multiplier. The model also provides the composition of the monetary base over time. Figure 1.12 compares the composition of the monetary base from the data and the model. The model successfully generates the changes for each component of the monetary base - currency, required reserves, and excess reserves - both before and after 2008.

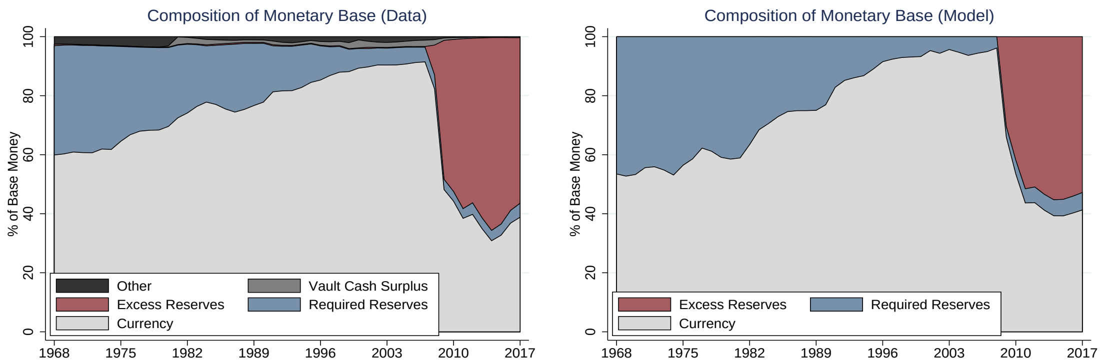

Figure 1.12: Composition of Monetary Base: Data vs. Model

A Digression on Model Fit For the post-2007 period, although the model projections can match the patterns in the data well, they do not fit very well in levels. This discrepancy is from the fact that the theoretical lower bound for the money multiplier is 1 in the model. In reality, however, the U.S. economy has experienced M1 multipliers lower than 1.[17] There are two potential explanations for this.

One possible reason is that monetary policy can be conducted in different ways than the lump-sum transfer in the model. In the model, all the base money is distributed to agents through the lump-sum transfer, and they keep some in their bank accounts. Reserves are in the bank deposits in this setup, and this implies the money multiplier can not be lower than 1. In contrast to most of the monetary models that assume money is injected as a lump-sum transfer across the agents (buyers and sellers, in this model), much money injection is made to the banking system directly in the real economy. For example, in the quantitative easing program, the Fed purchased large amounts of financial assets from financial intermediaries and gave them

[17] The M1 multiplier of the U.S. was lower than 1 from December 2008 (0.975) until June 2018 (0.991).

the same amount in reserves. These reserves are directly injected into the banking system and this is different from lump-sum transfers. In this case, reserves can only be held by banks, not by the public, and banks do not lend out reserves. One may need to consider a more explicit mechanism for monetary policy implementation.[18]

Another possible reason is that reserves could be kept in saving accounts or time deposits, which is in M2 but not in M1. Even though one assumes that the monetary base is distributed through a lump-sum transfer, it does not have to be kept in a checkable account. In this case, there is no discrepancy between the data and the theoretical lower bound for the money multiplier because the M2 money multiplier has never been lower than 1.[19] From a balance sheet point of view, reserves are recorded as a cash asset on the commercial bank's balance sheet because reserves are held as an account for the commercial banks at the Federal Reserve Bank, but deposits are liabilities. In this case, reserves cannot exceed total liabilities (total deposits), but they can exceed checkable deposits.

[18] Previous works on the explicit model of the interbank market with monetary policy implementation include Armenter and Lester (2017), Afonso, Armenter, and Lester (2019), Bianchi and Bigio (2014), and Chiu, Eisenschmidt, and Monnet (2020). Those models explicitly describe search frictions and the market structure of the interbank market for reserves, whereas this paper assumes a centralized market for reserves. Noting that the Fed controls the effective federal funds rates, which are interbank rates, introducing the interbank market can allow more realistic monetary transmission.

[19] The lowest M2 money multiplier during 1959-2019 was 2.812 at August 2014.

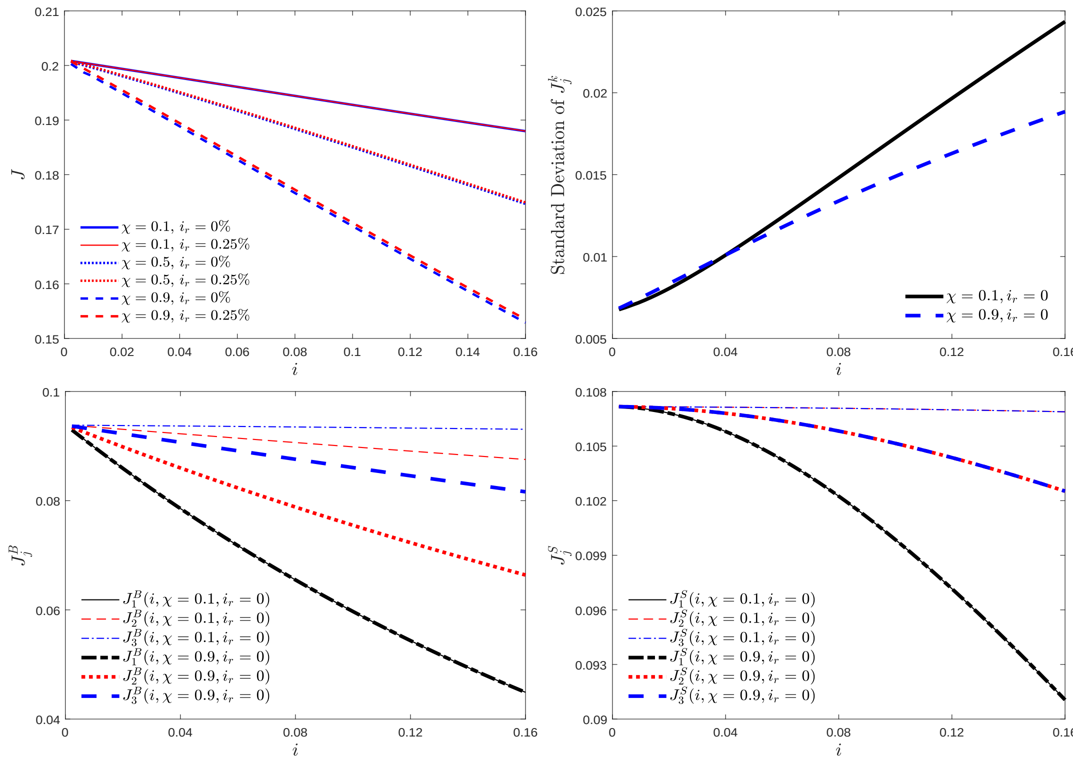

Figure 1.13: Welfare

1.4.3 Welfare

So far, I have shown that different tools of monetary policy have distinct roles in monetary transmission. This section focuses on the impacts of these different tools of monetary policy in terms of welfare. I measure the welfare of the seller in type j meeting using her DM trade surplus.

$$J_j^S(i; \ ; i_r) = (1 \quad)[u(q_j) \quad c(q_j)];$$

and the welfare of the buyer who trades in the j type DM meeting is a DM trade surplus with the cost for acquiring the cash and reserves.

$$J_j^B(i; \ ; i_r) = \quad im_j(i; \ ; i_r) \quad (i \quad i_d)r_j(i; \ ; i_r) + (1 \quad)[u(q_j) \quad c(q_j)]$$

I define the total welfare as a weighted sum of each agent's welfare.

$$J(i; \quad ; i_r) = \sum_{j=1}^{3} {}_j [J_j^B(i; \quad ; i_r) + J_j^S(i; \quad ; i_r)]$$

The top-left panel of Figure 1.13 illustrates the effects of monetary policy, i, on total welfare J ranging from 0% to 16% and how its impact can change depending on different reserve requirements and different interest rates on reserves. Each curve denotes the welfare under the different reserve requirements and interest rates on reserves. The welfare is monotonically decreasing in i, and each curve can be shifted up by paying interest on reserves or lowering the reserve requirement. These results are well expected. The higher opportunity cost of holding money lowers welfare. Paying interest on reserves, however, is welfare improving because it compensates the opportunity cost of holding money. Lowering reserve requirement also improves welfare since it provides more liquidity to the economy. The difference becomes smaller as the economy faces lower i, and the Friedman rule gives the optimal level of welfare.

In addition to total welfare, one can examine the distributional effect of monetary policy. The top-right panel of Figure 1.13 plots the standard deviation of each agent's welfare, J_j^k under different policies. Clearly, the effects of the monetary policy are different across the agents. The bottom-left (bottom-right) panels of Figure 1.13 plots the buyer's (seller's) welfare in different DM meetings depending on different policies. Among buyers, DM1 buyer's welfare is lower than others. Whereas DM2 and DM3 buyers consume the same amount in the DM, given that the access to unsecured credit allows the DM3 buyer carry less money than the DM2 buyer, the DM3 buyer's welfare is higher than the DM2 buyer's. For the sellers, because the consumption

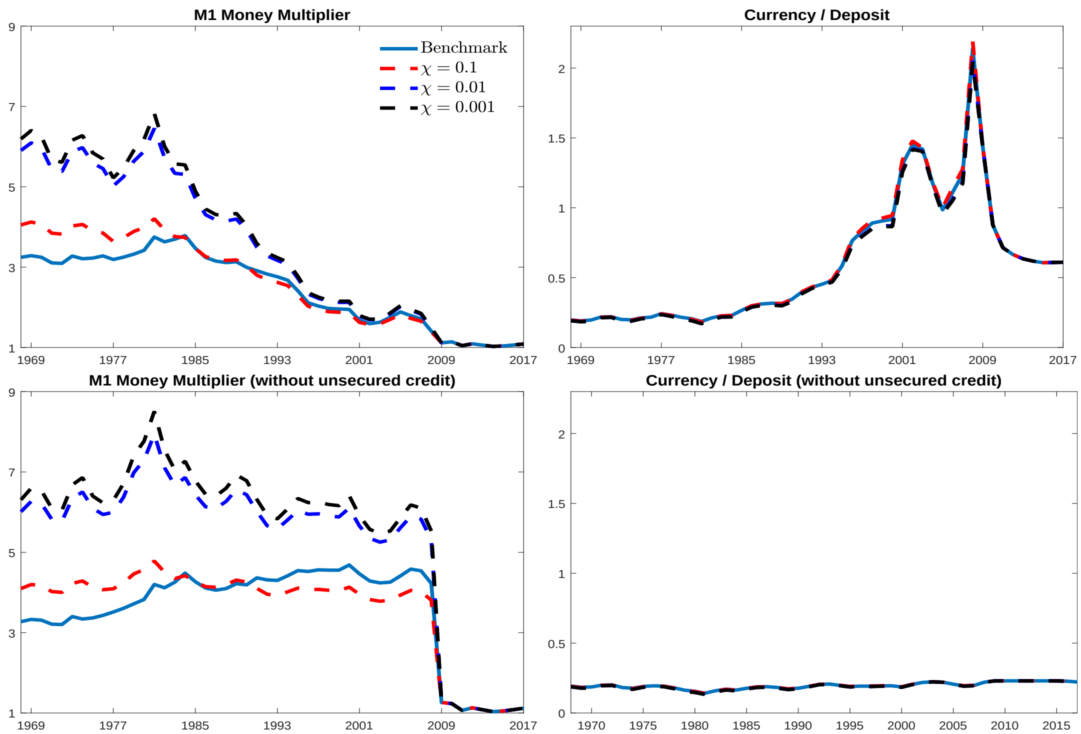

Figure 1.14: Counterfactual Analysis

in DM2 and DM3 are equal, they enjoy the same welfare. As in the buyers' case, DM1 seller's welfare is the lowest among all the sellers. Lowering reserve requirement increases the welfare of buyers and sellers in DM2 and DM3. However, DM1 agents' welfare is independent of reserves requirement, access to credit, and the interest on reserves but only depends on the nominal interest rate.

1.4.4 Counterfactual Analysis

In this section, I use calibrated parameters to assess how the money multiplier and currency deposit ratio would be changed by setting different reserve requirements. I also demonstrate that it is important to distinguish the effect of the reserve requirement from the effect of credit.

The top panel of Figure 1.14 shows the counterfactual under different reserve requirements while keeping $(i; i_r; \;)$ the same as in the benchmark case. With a lower reserve requirement, the money multiplier increases. However, we see a trend of decreasing multipliers regardless of reserve requirement. As illustrated in Table 1.3, this gradual decrease in the money multiplier since the late 1980s is driven by an increase in unsecured credit in the model. The currency deposit ratio increases with a higher reserve requirement and all the cases show the similar trend.

The bottom panel of Figure 1.14 shows the counterfactual under a different reserve requirement with $\; = 0$, and $(i; i_r)$ set to the data. In the bottom-right panel, we see almost no changes in the currency deposit ratio. Since the money demand for currency and inside money is stable, if unsecured credit did not crowd out inside money, there would not be a substantial increase in the currency deposit ratio, as the U.S. economy witnessed. In the case of the money multiplier, whereas it shows a stationary pattern before 2009, it drops drastically once the Federal Reserve started paying interest on reserves and lowered the nominal interest rate. This suggests that the gradual decline of the money multiplier from the late 1980s to 2007 can be attributed to an increase in unsecured credit, whereas the dramatic decline in the money multiplier since 2008 can be explained by the monetary policies of the Federal Reserve.

1.4.5 Robustness

This section briefly summaries a few results from Appendix A.3. Appendix A.3.1 examines the sensitivity of the results using different measures of the monetary policy target: the federal funds rate, and the commercial paper rate. Using different measures does not change the main results. In Appendix A.3.2, to check whether the curvature parameter 1.2 for the deposit operating cost is sensitive, I change the benchmark parameter from 1.2 to 1.15 or 1.25. Changing these parameters does not have a significant impact on the results.

1.5 Concluding Remarks

This paper develops a monetary-search model with fractional banking and unsecured credit and studies the money creation process. In the fractional reserve system, the money is created when banks make loans. The bank's lending, however, can be constrained by the reserve requirement and the reserves.

Banks hold excess reserves when the central bank pays sufficiently high interest on reserves with the nominal interest rate at some moderate level. In this case, the money multiplier and the quantity of the reserve depend on the nominal interest rate and the interest on reserves rather than the reserve requirement. In contrast to the previous works, these two interest rates play distinct roles. Whether the banks hold excess reserves or not, there exists a downward-sloping demand curve for reserves, and the Friedman rule is optimal. Paying interest on reserves with low nominal interest rates can move the economy from the scarce reserves regime to the ample reserves regime, which is consistent with what we have seen in the US economy. The quantitative analysis can generate simulated data that resemble the actual data. This paper provides evidence from the model and the data that suggests that the dramatic changes in the money multiplier after 2008 are mainly driven by the introduction of the interest on reserves with the low nominal interest rate.

This work can be extended in various ways. Although I focus on the centralized market for the reserves with homogeneous banks, in reality, the market for reserves is a decentralized interbank market and banks have different portfolios. Therefore, one can further investigate how much the market structure and heterogeneity matter for the transmission of monetary policy (e.g., Afonso and Lagos, 2015; Armenter and Lester, 2017; Afonso, Armenter, and Lester, 2019). Second, it would be worthwhile to study how inside creation via loan extension is related to investment and firms' dynamics (e.g., Ennis, 2018; Bianchi and Bigio, 2014; Altermatt, 2019). This will allow us to understand the investment channels of monetary policy more explicitly. Moreover, I assume that bank assets are composed of loans and reserves. But commercial banks' assets are mainly composed of securities, loans, and reserves. Extending the model to incorporate banks' portfolio choices and analyzing the role of investment, financial regulation, and monetary policy can open up other research avenues. (e.g., Rocheteau, Wright, and Zhang, 2018).

Chapter 2

On the Instability of Fractional Reserve Banking

> Motivated partly by a desire to avoid such [excessive] price-level fluctuations and possible Wicksellian price-level indeterminacy, quantity theorists have advocated legal restrictions on private intermediation. ... Thus, for example, Friedman (1959, p. 21) ... has advocated 100 percent reserves against bank liabilities called demand deposit. Sargent and Wallace (1982)

2.1 Introduction

There have been claims that fractional reserve banking is an important cause of boom-bust cycles, based on the notion that banks create excess credit under fractional reserve banking. (e.g., Fisher, 1935; Von Mises, 1953; Minsky, 1957; Minsky, 1970). For instance, Fisher (1935) views fractional reserve banking as one of several important factors in explaining economic fluctuations. Others believe that this is a primary cause of boom-bust cycles. According to Von Mises (1953), the overexpansion of bank credit as a result of fractional reserve banking is the root cause of business cycles. Minsky (1970) claims that economic booms and structural characteristics of the financial system, such as fractional reserve banking, can result in an economic collapse even when fundamentals remain unchanged.

This idea leads to policy debates on fractional reserve banking. Earlier examples include Peel's Banking Act of 1844 and the Chicago plan of banking reform with a 100% reserve requirement proposed by Irving Fisher, Paul Douglas, and others in 1939. Later, Friedman (1959) supported this banking reform, whereas Becker (1956) took the opposite position of supporting free banking with 0% reserve requirement.[1] Recently in 2018, Switzerland had a referendum of 100% reserve banking, which was rejected by 75.72% of the voters. The referendum aimed at making money safe from crisis by constructing full-reserve banking.[2] Whereas the debate on whether a fractional reserve banking system is inherently unstable has been an important policy discussion since a long time ago, the debate has never stopped.

This paper examines the instability of fractional banking by answering the following questions: (i) Can fractional reserve banking be inherently volatile even if we shut down the stochastic component of the economy? (ii) If so, under what condition can fractional reserve banking generate endogenous cycles without the presence of exogenous shocks and changes in fundamentals? To assess the claim that fractional reserve banking causes business cycles, this paper constructs a model of money and

[1] Sargent (2011) provides a novel review of the historical debates between narrow banking and free banking as tensions between stability versus efficiency.

[2] The official title of the referendum was *the Swiss federal popular initiative "for crisis-safe money: money creation by the National Bank only! (Sovereign Money Initiative)"* and also titled as "*debt-free money*."

banking that captures the role of fractional reserve banking.

In the model, each agent faces an idiosyncratic liquidity shock. Banks accept deposits and extend loans to provide risk-sharing among the depositors whereas the bank's lending is constrained by the reserve requirement. The real balance of money is determined by two factors: storage value and liquidity premium. The storage value is increasing in the future value of money. However, the liquidity premium, the marginal value of its liquidity function, is decreasing if the money becomes more abundant. When the liquidity premium dominates the storage value, the economy can exhibit endogenous fluctuations. Fractional reserve banking amplifies the liquidity premium because it allows the bank to create inside money through lending. Due to this amplified liquidity premium, the fractional reserve banking system is more prone to endogenous cycles.

In the baseline model, lowering the reserve requirement increases welfare in the steady state. However, lowering the reserve requirements can induce two-period cycles as well as three-period cycles, which implies the existence of periodic cycles of all order and chaotic dynamics. This also implies it can induce sunspot cycles. This result holds in the extended model with unsecured credit. The model also can deliver a self-fulfilling bubble burst. It is worth noting that the full reserve requirement does not necessarily exclude the possibility of endogenous cycles. However, the economy will be more susceptible to cycles with lower reserve requirement.[3]

This paper departs from previous works in two ways. First, in contrast to the previous works on banking instability, which mostly focus on bank runs following the seminal model by Diamond and Dybvig (1983), this paper focuses on the volatility

[3] Gu, Monnet, Nosal, and Wright (2019) show that introducing banks to the economy could induce instability in various settings which is in line with this result.

of real balances of inside money. It is another important focal point of banking instability because recurring boom-bust cycles associated with banking are probably be more prevalent than bank runs. Second, the approach here differs from a traditional approach to economic fluctuations with financial frictions. To understand economic fluctuations, there are two major points of view. The first one is that economic fluctuations are driven by exogenous shocks disturbing the dynamic system, and the effects of exogenous shocks shrink over time as the system goes back to its balanced path or steady-state. The second one is that they instead reflect an endogenous mechanism that produces boom-bust cycles. While there has been a lot of work on the role of financial friction in the business cycles including Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), and Gertler and Karadi (2011), most of them focused on the first approach, in which all economic fluctuations are caused by exogenous shocks and the financial sectors only serve as an amplifier. This paper, however, takes the second approach and focuses on whether the endogenous cycles arise in the absence of the stochastic components of the economy.

To evaluate the main prediction from the theory that fractional reserve banking induces excess volatility, I test the relationship between the required reserves ratio and the volatilty in real balance using cointegrating regression. A significant negative relationship between the two variables are found, and the results are robust to different measures of inflation and different frequency of time series. Both theoretical and empirical evidence indicate a link between the reserve requirement and the (in)stability.

Related Literature This paper builds on Berentsen, Camera, and Waller (2007), who introduce financial intermediaries with enforcement technology to Lagos and

Wright (2005) framework. The approach to introduce unsecured credit to the monetary economy is related to Lotz and Zhang (2016) and Gu, Mattesini, and Wright (2016) which are based on the earlier work by Kehoe and Levine (1993).

This paper is related to the large literature on fractional reserve banking. Freeman and Huffman (1991) and Freeman and Kydland (2000) develop general equilibrium models that explicitly capture the role of fractional reserve banking. Using those models, they explain the observed relationships between key macroeconomic variables over business cycles. Chari and Phelan (2014) study the condition under which fractional reserve banking can be socially useful by preventing bank runs in the cash-in-advance framework. More recently, Andolfatto, Berentsen, and Martin (2020) integrates Diamond (1997) into Lagos and Wright (2005) to provide a model in which fractional reserve banking emerges endogenously and a central bank can prevent bank panic as a lender of last resort. Whereas many previous work on instability focuses on bank runs, this paper studies a different type of instability in the sense that fractional reserve banking induces endogenous monetary cycles.

This paper is also related to the large literature on endogenous fluctuations, chaotic dynamics, and indeterminacy that have been surveyed by Brock (1988), Baumol and Benhabib (1989), Boldrin and Woodford (1990), Scheinkman and Woodford (1994) and Benhabib and Farmer (1999). For a model of bilateral trade, Gu, Mattesini, Monnet, and Wright (2013b) show that credit markets can be susceptible to endogenous fluctuations due to limited commitment. Gu et al. (2019) show that introducing financial intermediaries to an economy can engender instability in four distinct setups that capture various functions of banking. The model in this paper is closely related to Gu et al. (2019), whereas the model here is extended to incorporate fractional reserve banking.

The rest of the paper is organized as follows. Section 2.2 constructs the baseline search-theoretic monetary model. Section 2.3 provides main results. Section 2.4 introduces unsecured credit. Section 2.5 discusses the empirical evaluation of the model. Section 2.6 concludes.

2.2 Model

The model is based on Lagos and Wright (2005) with a financial intermediary as in Berentsen, Camera, and Waller (2007). Time is discrete and infinite. In each period, three markets convene sequentially. First, a centralized financial market (FM), followed by a decentralized goods market (DM), and finally a centralized goods market (CM). The FM and CM are frictionless. The DM is subject to search frictions, anonymity, and limited commitment. Therefore, a medium of exchange is needed to execute trades.

There is a continuum of agents who produce and consume perishable goods. At the beginning of the FM, a preference shock is realized: With probability , an agent will be a buyer in the following DM and with probability $1 \quad$, she will be a seller. The buyers and the sellers randomly meet and trade bilaterally in the DM. Agents discount their utility each period by . Within-period utility is represented by

$$U = U(X) - H + u(q) - c(q);$$

where X is the CM consumption, H is the CM disutility from production, and q is the DM consumption. As standard $U', u', c' > 0$, $U'', u'' < 0$, $c'' \geq 0$, and $u(0) = c(0) = 0$. The CM consumption good X is produced one-for-one with H, implying the real wage is 1. The efficient consumption in CM and DM is X and q that solve $U'(X) = 1$

and $u'(q^*) = c'(q^*)$, respectively.

There is a representative bank who accepts deposits and lends loans in the FM. In the FM, the agent can borrow money from the bank for a promise to repay money in the subsequent CM at nominal lending rate i_l. The agent can also deposit money to the bank and receive money in the subsequent CM at nominal deposit rate i_d. The banking market is perfectly competitive. The bank can enforce the repayment of loans at no cost. Last, there is a central bank that controls the money supply M_t. Let γ be the growth rate of the money stock. Changes in money supply are accomplished by lump-sum transfer if $\gamma > 0$ and by lump-sum tax if $\gamma < 0$.

2.2.1 Agent's Problem

Let W_t, G_t, and V_t denote the agent's value function in the CM, FM, and DM, respectively, in period t. There are two payment instruments for the DM transaction: fiat money (outside money) and loans from the bank (inside money). I will allow the agents to use unsecured credit as a means of payment in the next section. An agent entering the CM with nominal balance m_t, deposit d_t, and loan ℓ_t, solves the following problem:

$$
\begin{aligned}
W_t(m_t, d_t, \ell_t) &= \max_{X_t, H_t, \hat{m}_{t+1}} U(X_t) - H_t + \beta G_{t+1}(\hat{m}_{t+1}) \\
\text{s.t.} \quad \phi_t \hat{m}_{t+1} + X_t &= H_t + T_t + \phi_t m_t + (1 + i_{d,t}) \phi_t d_t - (1 + i_{l,t}) \phi_t \ell_t,
\end{aligned}
\tag{2.1}
$$

where T_t is the lump-sum transfer (or tax if it is negative), $i_{d,t}$ is the deposit interest rate, $i_{l,t}$ is the loan interest rate, ϕ_t is the price of money in terms of the CM goods, and $\hat{m}_{t+1}$ is the money balance carried to the FM where banks take deposits and makes loans. The first-order conditions (FOCs) result in $X_t = X$ and

$$_t = G'_{t+1}(\hat{m}_{t+1}); \quad (2.2)$$

where $G'_{t+1}(\hat{m}_{t+1})$ is the marginal value of an additional unit of money taken into the FM of period $t+1$. The envelope conditions are

$$\frac{\partial W_t}{\partial m_t} = {}_t; \qquad \frac{\partial W_t}{\partial d_t} = {}_t(1 + i_{d;t}); \qquad \frac{\partial W_t}{\partial \ell_t} = {}_t(1 + i_{l;t});$$

implying W_t is linear in m_t, d_t, and ℓ_t.

The value function of an agent at the beginning of FM is

$$G_t(m) = G_{b;t}(m) + (1 \quad)G_{s;t}(m); \quad (2.3)$$

where $G_{j \in \{b;s\};t}$ is the value function of type j agent in the FM. Agents choose their deposit balance d_j and loan ℓ_j based on the realization of their types in the following DM. The value function $G_{j;t}$ can be written as

$$G_{j;t}(m) = \max_{d_{j;t};\ell_{j;t}} V_{j;t}(m \quad d_{j;t} + \ell_{j;t}; d_{j;t}; \ell_{j;t}) \quad \text{s.t} \quad d_{j;t} \quad m; \quad (2.4)$$

where $V_{j;t}$ is the value function of type j agent in the DM. The FOCs are

$$\frac{\partial V_{j;t}}{\partial \ell_{j;t}} \quad 0 \quad (2.5)$$

$$\frac{\partial V_{j;t}}{\partial d_{j;t}} \quad _d \quad 0 \quad (2.6)$$

where $_d$ is the Lagrange multiplier for $d_{j;t} \quad m$.

The terms of trade in the DM are determined by an abstract mechanism that is studied in Gu and Wright (2016). The buyer must pay $p = v(q)$ to the seller to get q where $v(q)$ is some payment function satisfying $v'(q) > 0$ and $v(0) = 0$. As shown in Gu and Wright (2016), if the trading protocol satisfies four common axioms, then the terms of trade can be written in the following form.

$$p = \begin{cases} z & \text{if } z < p^* \\ p^* & \text{if } z \geq p^* \end{cases} \qquad q = \begin{cases} v^{-1}(z) & \text{if } z < p^* \\ q^* & \text{if } z \geq p^* \end{cases}, \tag{2.7}$$

where p^* is the payment required to get efficient consumption q^*, and z is the total liquidity, $\phi(m - d + \ell)$, held by the buyer. Many standard mechanisms, such as Kalai and generalized Nash bargaining, are consistent with this specification.

With probability α, a buyer meets a seller in the DM while a seller meets a buyer with probability α_s. Since the CM value function is linear, the DM value function for the buyer can be written as

$$V_{b,t}(m_t - d_{b,t} + \ell_{b,t}, d_{b,t}, \ell_{b,t}) = \alpha[u(q_t) - p_t] + W(m_t - d_{b,t} + \ell_{b,t}, d_{b,t}, \ell_{b,t}), \tag{2.8}$$

where $p_t \equiv \phi_t(m_t - d_{b,t} + \ell_{b,t})$. Assuming interior solution, differentiating $V_{b,t}$ yields

$$\frac{\partial V_{b,t}}{\partial m} = \phi_t[\alpha\lambda(q_t) + 1], \qquad \frac{\partial V_{b,t}}{\partial d} = \phi_t[-\alpha\lambda(q_t) + i_{d,t}], \qquad \frac{\partial V_{b,t}}{\partial \ell} = \phi_t[\alpha\lambda(q_t) - i_{l,t}],$$

where $\lambda(q) = u'(q)/v'(q) - 1$ if $p^* > z$ and $\lambda(q) = 0$ if $z \geq p^*$. Combining the buyer's FOCs in the FM and the derivatives of V_b yields

$$i_{d,t} - \alpha\lambda(q_t) - \mu_d \leq 0, \text{ “}=\text{” } 0 \text{ iff } d_{b,t} > 0 \tag{2.9}$$

$$-i_{l,t} + \alpha\lambda(q) \leq 0, \text{ “}=\text{” } 0 \text{ iff } \ell_{b,t} > 0. \tag{2.10}$$

A seller's DM value function is

$$V_{s,t}(m_t - d_{s,t} + \ell_{s,t}, d_{s,t}, \ell_{s,t}) = \alpha_s[p_t - c(q_t)] + W_t(m_t - d_{s,t} + \ell_{s,t}, d_{s,t}, \ell_{s,t}). \tag{2.11}$$

Differentiating $V_{s,t}$ yields

$$\frac{\partial V_{s,t}}{\partial m_t} = \quad _t, \qquad \frac{\partial V_{s,t}}{\partial d_t} = \quad _t i_{d,t}, \qquad \frac{\partial V_{s,t}}{\partial \ell_t} = \quad _t i_{l,t}.$$

Similar to the buyer's case, combining the seller's FOCs in the FM and the first-order derivatives of $V_{s,t}$ yields

$$_t i_{d,t} \quad _d \quad 0, \setminus = \text{"}0 \text{ iff } d_{s,t} > 0 \tag{2.12}$$

$$_t i_{l,t} \quad 0, \setminus = \text{"}0 \text{ iff } \ell_{s,t} > 0. \tag{2.13}$$

One can show that buyers do not deposit and sellers always deposit whereas buyers always borrow loans but sellers do not. This is because the buyer needs liquidity to trade for q in the DM but the seller does not. Formally, for $m > 0$, we have $\partial V_{b,t}/\partial d_{b,t} < \partial V_{s,t}/\partial d_{s,t} = 0$ and $\partial V_{s,t}/\partial \ell_{s,t} < \partial V_{b,t}/\partial \ell_{b,t} = 0$ because

$$0 = \overbrace{i_{d,t} \quad _d / \quad _t}^{\partial V_{s,t}/\partial d_{s,t}} > \overbrace{i_{d,t} \quad _d / \quad _t \qquad (q_t)}^{\partial V_{b,t}/\partial d_{b,t}} \tag{2.14}$$

$$0 = \underbrace{\quad _t i_{l,t} + \quad _t \quad (q_t)}_{\partial V_{b,t}/\partial \ell_{b,t}} > \underbrace{\quad _t i_{l,t}}_{\partial V_{s,t}/\partial \ell_{s,t}} \tag{2.15}$$

implying $i_{l,t} = \quad (q_t)$, $d_{s,t} = m$, $d_{b,t} = 0$, $\ell_{s,t} = 0$, and $\ell_b > 0$ as long as $\quad (q_t) > 0$.

Using the above results, we can rewrite the value functions in the FM as follows:

$$G_{b,t}(m_t) = \quad [u(q_t) \quad p_t] + W(m_t + \ell_{b,t}, 0, \ell_{b,t}) \tag{2.16}$$

$$G_{s,t}(m_t) = \quad _s[p_t \quad c(q_t)] + W(m_t \quad d_{s,t}, d_{s,t}, 0) \tag{2.17}$$

where $q_t = v^{\ 1}(p_t)$ and $p_t = \min\{p^{\ }, \quad (m_t + \ell_{b,t})\}$. Take derivative of $G_{j,t}(m_t)$ with respect to m_t to get

$$G'_{b,t}(m_t) = \quad _t + \quad _t \quad (q_t) \tag{2.18}$$

$$G'_{s;t}(m_t) = \quad _t + \quad _t i_{d;t}. \tag{2.19}$$

Since $G'_t(m_t) = \quad G'_{b;t}(m_t) + (1 \quad)G'_{s;t}(m_t)$, we have the following:

$$G'_t(m_t) = \quad _t \; [1 + \quad (q_t)] + \quad _t(1 \quad)(1 + i_{d;t}). \tag{2.20}$$

Combine (2.2) and (2.20) to get the Euler equation

$$_t = \begin{cases} \quad _{t+1} \; [\quad f1 + \quad (q_{t+1})g + (1 \quad)(1 + i_{d;t+1})] & \text{if } z_{t+1} < p \\ \quad _{t+1} & \text{if } z_{t+1} \quad p \; ; \end{cases} \tag{2.21}$$

where $q_{t+1} = v \; ^1(z_{t+1})$ and $z_{t+1} = \quad _{t+1}(m_{t+1} + \ell_{b;t+1})$

2.2.2 Bank's Problem

A representative bank accepts deposits d and makes loans ℓ. The depositors are paid at the nominal interest rate i_d by the bank, and the borrowers need to repay their borrowing with a nominal interest rate i_l. The central bank sets reserve requirement . The representative bank solves the following profit maximization problem.

$$\max_{d;\ell} \; (i_l \ell \quad i_d d) \quad s.t. \quad \ell \quad d \tag{2.22}$$

The FOCs for the bank's problem are

$$0 = i_l \quad _L \tag{2.23}$$

$$0 = \quad i_d + \quad _L = \; ; \tag{2.24}$$

where $_L$ is the Lagrange multiplier with respect to the bank's lending constraint. For $_L > 0$, we have

$$i_l = \quad i_d \tag{2.25}$$

while $_L = 0$ implies $i_d = i_l = 0$. Given the bank's problem and the agent's problem, we can define an equilibrium as follows:

Definition 3. *Given (;), an equilibrium consists of sequences of prices* $\{ _t; i_{l,t}; i_{d,t}\}_{t=0}^{\infty}$, *real balances* $\{m_t; \ell_{b,t}; \ell_{s,t}; d_{b,t}; d_{s,t}\}_{t=0}^{\infty}$, *and allocations* $\{q_t; X_t; \ell_t\}_{t=0}^{\infty}$ *satisfying the following:*

- *Agents solve CM, FM and DM problems: (2.1) and (2.4)*
- *The terms of trade in the DM satisfy (2.7), (2.8) and (2.11)*
- *A representative bank solves its profit maximization problem: (2.22)*
- *Markets clear in every period:*

  (i) Deposit Market: $d_{b,t} + (1 \quad)d_{s,t} = d_t$

  (ii) Loan Market: $\ell_{b,t} + (1 \quad)\ell_{s,t} = \ell_t$

  (iii) Money Market: $m_t = M_t$

The next step is to characterize the equilibrium. With binding bank's lending constraint, the equilibrium lending satisfies $\ell_t = (1 \quad)m_t/ \quad$ and $\ell_{b,t} = (1 \quad)m_t/(\quad)$. Combine equations (2.10), (2.21) and (2.25), and use equilibrium condition $m_{t+1} = M_{t+1}$ to get

$$_t = \begin{cases} _{t+1} \dfrac{1 \quad + \quad}{\quad} \quad v^{-1}(z_{t+1}) + 1 & \text{if } z_{t+1} < p \\ _{t+1} & \text{if } z_{t+1} \quad p \, ; \end{cases} \tag{2.26}$$

where $z_{t+1} = \ _{t+1}M_{t+1}(1 \quad + \quad)/ \quad$. Then multiplying both sides of (2.26) by $M_t(1 \quad + \quad)/ \quad$ allows us to reduce the equilibrium condition to one difference equation of real balances z:

$$z_t = f(z_{t+1}) \quad \frac{z_{t+1}}{1+i} \; \frac{1 \quad +}{} \; L(z_{t+1}) + 1 \; ; \tag{2.27}$$

where $(1+i) \quad = \quad$ and $L(z) \qquad v \; ^1(z)$ is the liquidity premium.[4]

2.3 Results

This section establishes key results on the instability of banking. Consider a stationary equilibrium, which is a fixed point that satisfies $z = f(z)$. There always exists an non-monetary equilibrium with $z = 0$. Given $i \; 2 \; [0; \;)$ and $\; 2 \; (0; 1]$, where $\; = \; (1 \quad + \quad)L(0)= \;$,[5] an unique stationary monetary equilibrium exists and satisfies

$$i = (1 \quad + \quad) \; L(z_s):$$

Since $L'(z) < 0$ (see Gu and Wright, 2016), the following result holds:

Proposition 7. *In the stationary equilibrium, lowering i or lowering $\;$ increases q.*

The dynamics of monetary equilibrium is characterized by equation (2.27). We can derive the condition that the economy exhibits a two-period cycle that satisfy $z_1 < z_s \quad p \; < z_2$.

Proposition 8 (Two-period Monetary Cycle)**.** *There exists a two-period cycle with $z_1 < z_s \quad p \; < z_2$ if $\; 2 \; (0; \; _m)$, where*

$$_m \qquad \frac{(1 \quad) \; L \; \frac{p}{1+i}}{(1+i)^2 \quad 1 \qquad L \; \frac{p}{1+i}}:$$

---

[4] In the stationary equilibrium, $i = \; = \quad 1$ is the nominal interest rate.

[5] Nash and Kalai bargaining provides simple examples for $\;$. Under the Inada condition $u'(0) = 1$, with Kalai, $\; = \; (1 \quad + \quad)= \; (1 \quad)$; whereas with Nash bargaining, $\; = 1$.

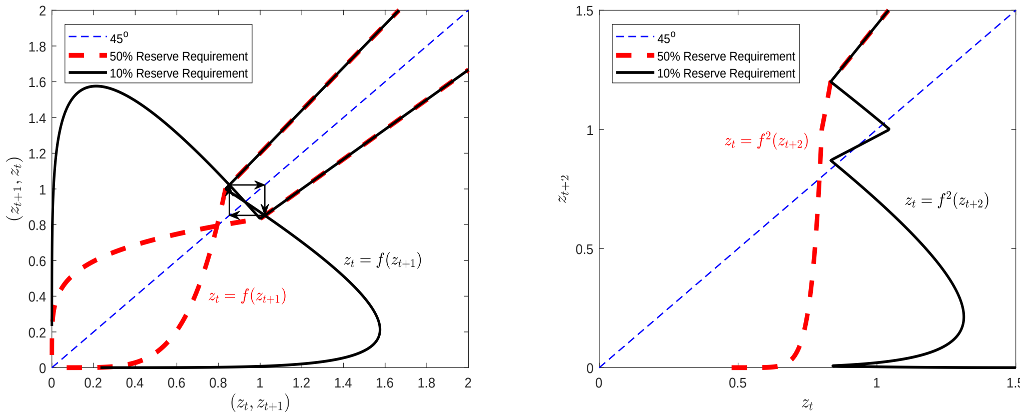

Figure 2.1: A Two-period Cycle under Fractional Reserve Banking

Proof. See Appendix B.1. ■

Proposition 8 shows that lowering the reserve requirement can induce a two-period cycle under the general trading mechanism. However, in general, a two period cycle with $z_1 < z_s < z_2$, could be either $z_2 > p$ or $z_2 < p$. Following the standard textbook method (see Azariadis, 1993), we can show that if $f'(z_s) <$ 1, there exists a two-period cycle with $z_1 < z_s < z_2$. Consider a special case where $qu''(q)/u'(q) =$, $c(q) = q$ and the buyer makes take-it-or-leave-it (TIOLI) offer. The following proposition says that there exists a two-period cycle if is low.

Proposition 9. *Assume* $qu''(q)/u'(q) =$ *and* $c(q) = q$*. If* $2\ (0;\ _m)$*, where*

$$_m \quad \frac{(1 \quad)}{(1 \quad) + (2 \quad)(1+i)}; \qquad (2.28)$$

then $f'(z_s) <$ 1*.*

Proof. See Appendix B.1. ■

Whereas (2.28) is written in terms of , this condition can be written in terms of

i, as follows:

$$0 < i < \frac{[\;(1\quad)\quad(1\quad)]}{(2\quad)} \tag{2.29}$$

The role of i on cycles depends on . By (2.29), if < 2, lowering either or i can induce a cycle. If $2=(\quad) > \quad > 2$, $_m$ is negative when $i > \frac{\quad 2}{2}$ and positive when $i < \frac{\quad 2}{2}$. In this case, setting i higher than $\frac{\quad 2}{2}$ eliminates cyclic equilibria. If $2=(\quad)$, $_m$ is negative for all i, implying the cycle does not exist. When $= 2$, $_m$ is constant, implying that the i has no effect on the cycle in this case.

To interpret the results, recall $f(z_{t+1})$ from equation (2.27). The first term, $z_{t+1}=(1+i)$ on the right-hand side, reflects the store of value, which is monotonically increasing in z_{t+1}. The second term $(1\quad+\quad)\ L(z_{t+1})=\quad+1$, reflecting the liquidity premium, is decreasing in z_{t+1}. Because $f^0(z_{t+1})$ depends on both terms, $f(z_{t+1})$ is nonmonotone in general. If the liquidity premium dominates the storage value, we can have $f^0(\ ) < \quad 1$, which is a standard condition for the existence of cyclic equilibria. Lowering the reserve requirement amplifies the liquidity premium because it allows the bank to create more liquidity through lending. This amplification of liquidity generates endogenous cycles.

In addition to the condition for two-period cycles, the next result provides the condition for three-period cycles under the general trading mechanism. The existence of three period-cycles implies cycles of all orders as well as chaotic dynamics (see Sharkovskii, 1964 and Li and Yorke, 1975).

Proposition 10 (Three-period Monetary Cycle and Chaos). *A three-period cycle with $z_1 < z_2 < p\quad < z_3$ does not exist. There exists a three-period cycle with*

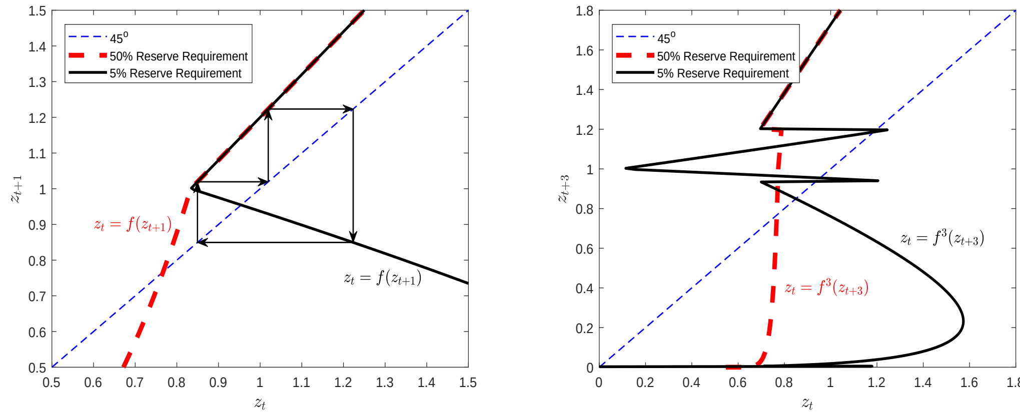

Figure 2.2: A Three-period Cycle under Fractional Reserve Banking

$z_1 < p < z_2 < z_3$ if $\in (0; \hat{}_m)$, where

$$\hat{}_m \quad \frac{(1 \quad)\ L\ \frac{p}{1+i}}{(1+i)^3 \quad 1 \quad L\ \frac{p}{1+i}}:$$

Proof. See Appendix B.1. ■

The following corollary is a direct result from proposition 10.

Corollary 2 (**Binding Liquidity Constraint**). *In any n-period cycle, the liquidity constraint binds, $z_t < p$, at least one periodic point over the cycle.*

Proof. See Appendix B.1. ■

The model can also generate sunspot cycles. Consider a Markov sunspot variable $S_t \in \{1; 2\}$. This sunspot variable is not related to fundamentals but may affect equilibrium. Let $\Pr(S_{t+1} = 1 | S_t = 1) = {}_1$ and $\Pr(S_{t+1} = 2 | S_t = 2) = {}_2$. The sunspot is realized in the FM. Let W_t^S be the CM value function in state S in period

t, then

$$W_t^S(m_t; d_t; `_t) = \max_{X_t; H_t; \hat{m}_{t+1}} U(X_t) \quad H_t + \quad {}_s G_{t+1}^S(\hat{m}_{t+1}) + (1 \quad {}_s) G_{t+1}^{\;S}(\hat{m}_{t+1})$$

$$\text{s.t.} \quad {}_t^S \hat{m}_{t+1} + X_t = H_t + T_t + \; {}_t^S m_t + (1 + i_{d;t}) \; {}_t^S d_t \quad (1 + i_{l;t}) \; {}_t^{S} `_t:$$

The FOC can be written as

$$_t^S + \quad {}_s G_{t+1}^{0S}(\hat{m}_{t+1}) + \quad (1 \quad {}_s) G_{t+1}^{0\;S}(\hat{m}_{t+1}) = 0: \tag{2.30}$$

Solving the FM problem results in

$$G_{t+1}^{0S}(m_{t+1}^S) = \; {}_{t+1}^S \; \frac{1 \quad +}{} \quad L(z_{t+1}^S) + 1 \; : \tag{2.31}$$

We substitute (2.31) into (2.30) and use the money market clearing condition $m_{t+1} = M_{t+1}$ to get the Euler equation.

$$_t^S = \quad {}_s \; {}_{t+1}^S \; \frac{1 \quad +}{} \quad L(z_{t+1}^S) + 1 \quad + \quad (1 \quad {}_s) \; {}_{t+1}^{\;S} \; \frac{1 \quad +}{} \quad L(z_{t+1}^{\;S}) + 1 \; :$$

where $z_{t+1}^S = \; {}_{t+1}^S M_{t+1}(1 \quad + \quad)=$. Then multiply both sides of the Euler equation by $M_t(1 \quad + \quad)=$ to reduce the equilibrium condition into one difference equation of real balances z_{t+1}^S:

$$\begin{aligned} z_t^S &= \frac{{}_s z_{t+1}^S}{1 + i} \; \frac{1 \quad +}{} \quad L(z_{t+1}^S) + 1 \quad + \frac{(1 \quad {}_s) z_{t+1}^{\;S}}{1 + i} \; \frac{1 \quad +}{} \quad L(z_{t+1}^{\;S}) + 1 \\ &= {}_s f(z_{t+1}^S) + (1 \quad {}_s) f(z_{t+1}^{\;S}): \end{aligned} \tag{2.32}$$

We define a sunspot equilibrium as follows:

Definition 4 (**Proper Sunspot Equilibrium**). *A proper sunspot equilibrium consists of the sequences of real balances $f z_t^S g_{t=0;S=1;2}^{1}$ and probabilities $(\; {}_1; \; {}_2)$, solving (2.32) for all t.*

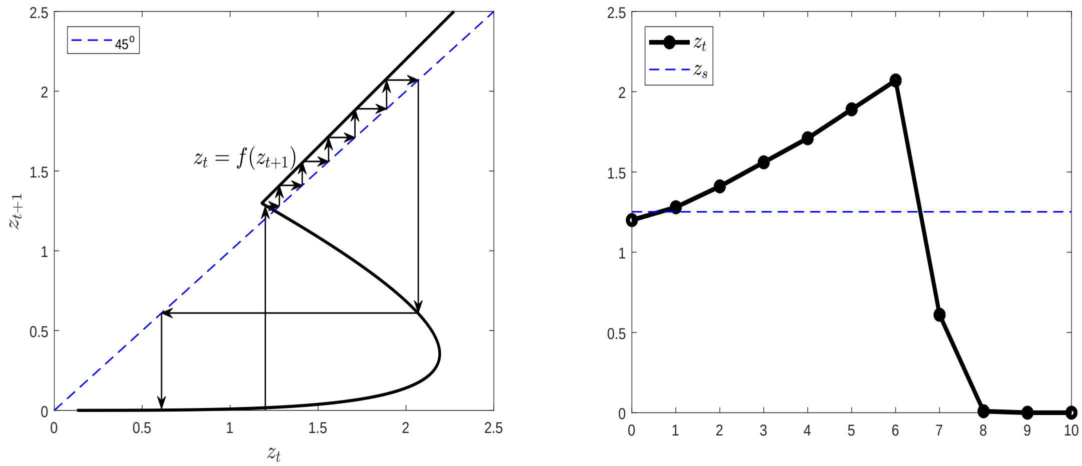

Figure 2.3: Self-Fulfilling Bubble and Burst Equilibria

Consider stationary sunspot equilibria with $z^1 < z^2$ that only depend on the state, not the time. The liquidity constraint is binding in state $S = 1$. By the standard approach (see again Azariadis, 1993 for the textbook treatment), the condition for two-period cycles is also sufficient and necessary for two-state sunspot equilibrium. If $f'(z_s) < -1$, there exists $(\pi_1, \pi_2) \in (0, 1)^2$, $\pi_1 + \pi_2 < 1$, such that the economy has a proper sunspot equilibrium in the neighborhood of z_s.

In addition to the deterministic and stochastic cycles, the model also features the equilibria where real balance increases above the steady-state until certain time, T, and crashes to zero. Consider a sequence of real balance $\{z_t\}_{t=0}^{\infty}$ with $z_T \equiv \max\{z_t\}_{t=0}^{\infty} > z_s$ (bubble) that crashes to 0 (burst) as $t \to \infty$, where $T \geq 1$ and $z_T > z_0$. We refer to this equilibrium as a self-fulfilling bubble and burst equilibria:

Definition 5 (Self-Fulfilling Bubble and Burst Equilibria). *For initial real balance $z_0 > 0$, a self-fulfilling bubble and burst is a sequence of $\{z_t\}_{t=0}^{\infty}$ satisfying (2.27) and $0 < z_s < z_T$, $\lim_{t \to \infty} z_t = 0$, and $z_T = \max\{z_t\}_{t=0}^{\infty}$ with $T \geq 1$.*

The next step is to check under which condition this type of equilibria can occur.

When $z_s > z$, where z solves $f'(z) = 0$, there exist multiple equilibria. Then, if $f(z) \quad q$, the self-fulfilling bubble and burst equilibria exist. Proposition 11 uses some specific functions and a buyer's TIOLI offer and shows lowering the reserve requirement can induce this type of equilibria.

Proposition 11 (Existence of Self-Fulfilling Bubble and Burst Equilibria). *Assume $qu''(q)/u'(q) = \quad$ and $c(q) = q$. There exist self-fulfilling bubble and burst equilibria, if*

$$0 < \quad < \min \left\{ \frac{(1 \quad) \quad (1+i)}{(1 \quad)^2 q \quad + (1+i)[(1 \quad)(3+i \quad) \quad]}; \frac{(1 \quad)}{1+i \quad (i+ \quad)} \right\}$$

Proof. See Appendix B.1. ■

2.4 Money and Unsecured Credit

Consider an alternative payment instrument in the DM - unsecured credit. The buyer can pay for DM goods using unsecured credit that will be redeemed to the seller in the following CM and she can borrow up to her debt limit, b_t. For simplicity, I assume that the buyer makes a TIOLI offer to the seller in the DM, which means the buyer maximizes her surplus subject to the seller's participation constraint. The DM cost function is $c(q) = q$. Suppose the buyer has issued b_t units of unsecured debt in the previous DM. The CM value function is

$$\begin{aligned} W_t(m_t; d_t; `_t; \quad b_t) &= \max_{X_t; H_t; \hat{m}_{t+1}} U(X_t) \quad H_t + \quad G_{t+1}(\hat{m}_{t+1}) \\ \text{s.t.} \quad {}_t\hat{m}_{t+1} + X_t &= H_t + T_t + \quad {}_t m_t + (1+i_{d;t}) \quad {}_t d_t \quad (1+i_{l;t}) \quad {}_t `_t \quad b_t; \end{aligned} \tag{2.33}$$

which is the same as before except that the agent needs to pay or collect the debt. The agent's FM problem is identical to the previous section. Then, $1 \quad$ fraction of agents will deposit $\hat{m}_{t+1}$, and fraction of agents will borrow loan from the bank. The DM value function is

$$V_t^b(m_t + \ell_t; 0; \ell_t) = [u(q_t) \quad q_t] + W_t(m_t + \ell_t; 0; \ell_t; 0);$$

where $q_t = \min\{q ; b_t + \quad_t(m_t + \ell_t)\}$. Given b_t, solving equilibrium yields

$$z_t = \begin{cases} \frac{z_{t+1}}{1+i} \left[1 \quad + \frac{}{} \quad u'(z_{t+1} + b_{t+1}) \quad 1 \; + 1 \right] & \text{if } z_{t+1} + b_{t+1} < q \\ \frac{z_{t+1}}{1+i} & \text{if } z_{t+1} + b_{t+1} \quad q \; ; \end{cases} \tag{2.34}$$

where $z_{t+1} = (1 \quad + \quad) \;_{t+1}M_{t+1}/(\quad)$.

Next, I am going to endogenize the debt limit. The buyer cannot commit to pay back the debt. If the buyer reneges she is captured with probability . The punishment for a defaulter is permanent exclusion from the DM trade but she can still produce for herself in the CM. The value of autarky is $\underline{W}(0; 0; 0; 0) = [U(X \;) \quad X \; + T]/(1 \quad)$. The incentive condition for voluntary repayment is

$$\underbrace{\quad b_t + W_t(m_t; d_t; \ell_t; 0)}_{\text{value of honoring debts}} \quad \underbrace{(1 \quad)W_t(m_t; d_t; \ell_t; 0) + \quad \underline{W}(m_t; d_t; \ell_t; 0)}_{\text{value of not honoring debts}} :$$

One can write the debt limit b_t as $b_t \quad b_t \quad W_t(0; 0; 0; 0) \quad \underline{W}(0; 0; 0; 0)$. Recall the CM value function. Using the solution of FM, we can rewrite the buyer's CM value function as

$$\begin{aligned} W_t(0; 0; 0; 0) = {} & U(X \;) \quad X \; + T_t + \quad W_{t+1}(0; 0; 0; 0) \\ & + \max_{\hat{m}_{t+1}} \{ \quad _t\hat{m}_{t+1} + \quad [u(q_{t+1}) \quad q_{t+1}] + \quad _{t+1}\hat{m}_{t+1} \}; \end{aligned}$$

where $q_{t+1} = \min\{q ; z_{t+1} + b_{t+1}\}$. Substituting $W_t(0; 0; 0; 0) = b_t/ \quad + \underline{W}(0; 0; 0; 0)$

yields

$$\frac{b_t}{} = \quad {}_t M_{t+1} + \quad [u(z_{t+1} + b_{t+1}) \quad z_{t+1} \quad b_{t+1}] + \frac{b_{t+1}}{} + \quad {}_{t+1} M_{t+1};$$

where M_{t+1} and z_{t+1} solve (2.34). Rearranging terms yields

$$b_t = \begin{cases} b_{t+1} + \dfrac{[\quad z_t + \quad z_{t+1}]}{1 \quad +} + \quad S(z_{t+1} + b_{t+1}) & \text{if } z_{t+1} + b_{t+1} < q \\ b_{t+1} + \dfrac{[\quad z_t + \quad z_{t+1}]}{1 \quad +} + \quad S(q \;) & \text{if } z_{t+1} + b_{t+1} \quad q \;; \end{cases} \tag{2.35}$$

where $S(z_{t+1} + b_{t+1}) \quad [u(z_{t+1} + b_{t+1}) \quad z_{t+1} \quad b_{t+1}]$ is the buyer's trade surplus. The equilibrium can be collapsed into a dynamic system satisfying (2.34)-(2.35).

In the stationary equilibrium, (2.34) becomes

$$\frac{i}{(1 \quad + \quad)} + u^{0}(q) \quad 0; = \text{ if } z > 0 \tag{2.36}$$

and (2.35) becomes

$$(1 \quad)b = \begin{cases} \dfrac{[\quad]z}{1 \quad +} + \quad [u(z + b) \quad z \quad b] & \text{if } z + b < q \\ \dfrac{[\quad]z}{1 \quad +} + \quad [u(q \;) \quad q \;] & \text{if } z + b \quad q \;; \end{cases} \tag{2.37}$$

where $q = \min f z + b; q \; g$. The stationary equilibrium solves the above two equations, and it falls into one of the three cases: the pure money equilibrium, the pure credit equilibrium, and the money-credit equilibrium. First, if no one can capture the buyer after she reneges, $= 0$, the unsecured credit is not feasible, $b = 0$. In this case, the equilibrium will be the pure money equilibrium. Second, when b solving (2.37) satisfies $u^{0}(b) < i \; =[\;(1 \quad + \quad)]$ then money is not valued, $z = 0$. We have the pure credit equilibrium in this case. Third, if solutions of (2.36)-(2.37), $(z; b)$ are strictly positive then we have the money-credit equilibrium.

The debt limit at the stationary equilibrium, b, is a fixed point satisfying $b = (b)$ where

$$(b) = \begin{cases} \frac{}{r}[u(\tilde{q}) \quad \tilde{q}] \quad \frac{i}{r(1 \quad + \quad)}(\tilde{q} \quad b) & \text{if } \tilde{q} > b \quad 0 \\ \frac{}{r}[u(b) \quad b] & \text{if } q \quad > b \quad \tilde{q} \\ \frac{}{r}[u(q \ ) \quad q \ ] & \text{if } b \quad q \end{cases} \tag{2.38}$$

where $\tilde{q}$ solves $u^{0}(\tilde{q}) = 1 + i \ =[\ (1 \quad + \quad)]$ and $r \quad 1= \quad 1$. The DM consumption q_S is determined by $q_S = \min\{q \ ; \max\{\tilde{q}; b\}\}$. Money and credit coexist if and only if $0 < b < \tilde{q}$, which holds when $0 < \quad < \min\{1; \tilde{\ }\}$, where

$$\tilde{\ } \quad \frac{r(1 \quad + \quad)}{[u(\tilde{q})=\tilde{q} \quad 1](1 \quad + \quad) \quad 2i}:$$

The DM consumption is decreasing in i in the stationary monetary equilibrium.

Consider the dynamics of equilibria where money and credit coexist. I claim the main results from Section 2.3 - lowering the reserve requirement can induce endogenous cycles - still hold even after unsecured credit is introduced. For compact notation, let $\quad \max\{i; r\}$ and $w_j \quad z_j + b_j$. The following proposition establishes the conditions for two-period cycles, three-period cycles, and chaotic dynamics.

Proposition 12 (**Monetary Cycles with Unsecured Credit**). *There exists a two-period cycle of money and credit with* $w_1 < q \ < w_2$ *if* $\ 2 (0; \ _c)$, *where*

$$_c \quad \frac{(1 \quad) \quad u^{0}\left(\frac{q}{1+}\right) \quad 1}{(1 + i)^2 \quad 1 \qquad u^{0}\left(\frac{q}{1+}\right) \quad 1}:$$

There exists a three-period cycle of money and credit with $w_1 < q \ < w_2 < w_3$, *if*

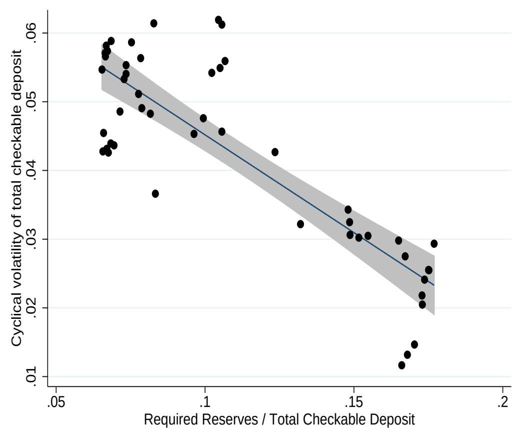

Figure 2.4: Scatter Plot for Inside Money Volatility and Required Reserve Ratio

2 $(0; \hat{}_c)$, *where*

$$\hat{}_c \qquad \frac{(1 \qquad) \;\; u^0 \; \frac{q}{1+} \qquad 1}{(1+i)^3 \quad 1 \qquad\quad u^0 \; \frac{q}{1+} \qquad 1}:$$

Proof. See Appendix B.1. ■

2.5 Empirical Evaluation: Inside Money Volatility

In the previous sections, the theoretical results show that lowering the required reserve ratio can induce instability. To evaluate the model prediction, I examine whether the required reserve ratio is associated with the cyclical volatility of the real balance of the inside money.

Following Jaimovich and Siu (2009) and Carvalho and Gabaix (2013), I measure the cyclical volatility in quarter t as the standard deviation of a filtered log real total checkable deposit during a 41-quarter (10-year) window centered around quarter t. Total checkable deposits are from the H.6 Money Stock Measures published by the Federal Reserve Board and converted to real value using the Consumer Price Index (CPI). Seasonally adjusted series are used to smooth the seasonal fluctuation. I adopt the Hodrick-Prescott (HP) filter with a 1600 smoothing parameter as standard. To construct an annual series, quarterly observations are averaged for each year. The sample period is from 1960:I to 2018:IV so that there are annual series from 1965 to 2013. To check whether the results are sensitive to different measures of the price level, I also use the core CPI, the Personal Consumption Expenditures (PCE), and the core PCE to transform the total checkable deposit into real value.

The legal reserve requirement for the demand deposits has been 10% since April 2, 1992. However, the Federal Reserve imposes different reserve requirements depending on the size of a bank's liability. These criteria have changed over time. For example, during 1992:Q1-2019:Q4, this changed 27 times. To consider these changes, I divide the required reserves by total checkable deposits to compute the required reserve ratio.

Figure 2.4 presents a scatter plot of the cyclical volatility of the real inside money balance and the required reserve ratio. Column (1) of Table 2.1 reports its regression estimates with Newey-West standard errors. The plot and estimates show a negative relationship between the cyclical volatility of the real inside money balance and the required reserve ratio with statistically significant regression coefficients. However, this result can be driven by a spurious regression. Table 2.2 provides unit root test results for the federal funds rate, the required reserve ratio, and the cyclical volatility of inside money. Both augmented Dickey-Fuller tests and Phillips-Perron tests fail to reject the null hypotheses of unit roots for these series, whereas they reject the null hypotheses of unit roots at their first differences. In addition to that, the Johansen cointegration test in Column (1), suggests that there is no cointegration relationship between two variables. So it is hard to rule out that Column (1)'s results are driven by a spurious regression.

To overcome this issue, I adopt the cointegrating regression with an additional

Table 2.1: Effect of Required Reserve Ratio

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent variable: $^{Roll}_{t}$	OLS (1)	CCR (2)	OLS (3)	CCR (4)	OLS (5)	CCR (6)	OLS (7)	CCR (8)
	0:283	0:245	0:267	0:221	0:306	0:227	0:307	0:220
	(0:027)	(0:002)	(0:027)	(0:003)	(0:029)	(0:004)	(0:027)	(0:005)
ffr		0:109		0:125		0:187		0:207
		(0:002)		(0:003)		(0:004)		(0:004)
Constant	0:074	0:074	0:070	0:071	0:074	0:075	0:073	0:073
	(0:003)	(0:000)	(0:004)	(0:000)	(0:004)	(0:000)	(0:004)	(0:000)
Obs.	49	49	49	49	49	49	49	49
$adj R^2$	0.700	0.621	0.728	0.648	0.740	0.650	0.764	0.665
$_{trace}(r = 0)$	9.807	35:688	9.120	35:145	9.109	35:367	8.593	35:028
5% CV	15:41	29:68	15:41	29:68	15:41	29:68	15:41	29:68
$_{trace}(r = 1)$	3:324	10.682	2:839	10.065	2:723	9.894	2:417	9.345
5% CV	3:76	15:41	3:76	15:41	3:76	15:41	3:76	15:41

Note: For (1), (3), (5) and (7), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2), (4), (6), and (8), first-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West fixed lag, 4 $(T=100)^{2=9}$; denotes the required reserve ratio, ffr denotes federal funds rates and $^{Roll}_{t}$ denotes the cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Table 2.2: Unit Root Tests

		Phillips-Perron test		ADF test
		$Z(\ )$	$Z(t)$	$Z(t)$ w/ lag 1
ffr		6:766	1:704	2:362
		1:492	1:173	1:341
$^{Roll}_{t}$	(CPI)	4:708	2:191	2:090
$^{Roll}_{t}$	(Core CPI)	4:681	2:189	1:978
$^{Roll}_{t}$	(PCE)	4:329	2:038	2:047
$^{Roll}_{t}$	(Core PCE)	4:076	1:954	1:930
ffr		28:373	5:061	6:357
		31:818	4:802	3:693
$^{Roll}_{t}$	(CPI)	24:905	3:416	2:942
$^{Roll}_{t}$	(Core CPI)	24:758	3:509	2:942
$^{Roll}_{t}$	(PCE)	23:691	3:330	2:842
$^{Roll}_{t}$	(Core PCE)	22:826	3:296	2:768

Note: ffr denotes federal funds rates, denotes required reserve ratio, and $^{Roll}_{t}$ denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

variable, the federal funds rate. Column (2) of Table 2.1 provides Johansen cointegration test results for the federal funds rate, the required reserves, and the cyclical volatility of inside money. The trace test suggests a cointegration relationship among these three variables, which is consistent with the theoretical result: The instability depends on the reserve requirement and the interest rate. With the cointegration relationship, we may not have to worry about a spurious relationship. Column (2) of Table 2.1 reports the estimates for the cointegrating relationship. Because of the potential bias from long-run variance, I estimate a canonical cointegrating regression (CCR). The estimates are statistically significant with a sizeable level and consistent with the prediction from the model.

To check the sensitivity of the results, I redo all the analyses using the core CPI, the Personal Consumption Expenditures (PCE), and the core PCE to transform the total checkable deposit into real value. Columns (3), (5), and (7) of Table 2.1 regress required reserve ratio on the inside money volatility and report its Newey-West standard errors. They also report the trace test statistics of Johansen cointegration test between these two variables. The results are consistent with the benchmark case in Column (1). Columns (4), (6), and (8) of Table 2.1 report CCR estimates regressing the required reserve ratio and federal funds rate on the inside money volatility and the trace test statistics of Johansen cointegration test between these three variables. All the results are consistent with the benchmark case in Column (2).

Appendix B.2 includes more sensitivity analyses: (1) Using quarterly series instead of annual series; (2) Using time series before 2008. All the results are not sensitive with respect to different frequencies and time periods.

2.6 Conclusion

The goal of this paper is to examine the (in)stability of fractional reserve banking. To that end, this paper builds a simple monetary model of fractional reserve banking that can capture the conditions for (in)stability under different specifications. Lowering the reserve requirement increases the welfare at the steady state. However, it can induce instability. The baseline model and its extension establish the conditions for endogenous cycles and chaotic dynamics. The model also features stochastic cycles and self-fulfilling boom and burst under explicit conditions. The model shows that fractional reserve banking can endanger stability in the sense that equilibrium is more prone to exhibit cyclic, chaotic, and stochastic dynamics under lower reserve requirements. This is due to the amplified liquidity premium. This result holds in the extended model with unsecured credit.

This paper also provides some empirical evidence that is consistent with the prediction of the model. I test the association between the required reserves ratio and the real inside money volatility using cointegrating regression. I find a significant negative relationship between the two variables. Both theoretical and empirical evidence find a link between the reserve requirement policy and (in)stability.

Appendix A

Appendix for Chapter 1

A.1 Proofs

Proof of Proposition 1. First, consider the ample-reserves equilibrium. Each bank's the real balance of reserves, $\tilde{r}$, and lending, $\tilde{\ell}$, are determined by following two equations

$$0 = \underbrace{\frac{1+i}{1+i_r \quad {}^{0}(\tilde{r})} \quad 1 \quad {}^{0}(\tilde{\ell})}_{P} \tag{A.1}$$

$$0 = \underbrace{{}^{0}(\tilde{r})\tilde{r} \quad (\tilde{r}) + {}^{0}(\tilde{\ell})\tilde{\ell} \quad (\tilde{\ell}) \quad k}_{E} \tag{A.2}$$

where $\frac{1+i}{1+i_r \quad {}^{0}(\tilde{r})}$ 1. By the implicit function theorem, we have

$$P_r d\tilde{r} + P_\ell d\tilde{\ell} + P_i di + P_{i_r} di_r = 0 \tag{A.3}$$

$$E_r d\tilde{r} + E_\ell d\tilde{\ell} + E_i di + E_{i_r} di_r = 0 \tag{A.4}$$

where

$$P_r = (1+i)\,\phi''(\tilde{r})/\{1+i_r-\phi'(\tilde{r})\}^2 \qquad E_r = -\phi''(\tilde{r})\tilde{r}$$

$$P_\ell = -\phi''(\tilde{\ell}) \qquad E_\ell = -\phi''(\tilde{\ell})\tilde{\ell}$$

$$P_i = 1/\{1+i_r-\phi'(\tilde{r})\} \qquad E_i = 0$$

$$P_{i_r} = -(1+i)/\{1+i_r-\phi'(\tilde{r})\}^2 \qquad E_{i_r} = 0.$$

Applying Cramer's Rule to (A.3)-(A.4) yields the following comparative statics results:

$$\frac{\partial \tilde{\ell}}{\partial i} = \frac{P_i E_r}{P_r E_\ell - P_\ell E_r} > 0, \qquad \frac{\partial \tilde{r}}{\partial i} = \frac{P_i E_\ell}{P_r E_\ell - P_\ell E_r} < 0$$

$$\frac{\partial \tilde{\ell}}{\partial i_r} = \frac{P_{i_r} E_r}{P_r E_\ell - P_\ell E_r} < 0, \qquad \frac{\partial \tilde{r}}{\partial i_r} = \frac{P_{i_r} E_\ell}{P_r E_\ell - P_\ell E_r} > 0.$$

From the above results, we can get immediate results $\partial i_d/\partial i > 0$ and

$$\frac{\partial i_d}{\partial i_r} = 1 - \frac{1+i}{1+i+[1+i_r-\phi'(\tilde{r})]} > 0 \tag{A.5}$$

since $i_d = i_r - \phi'(\tilde{r}) > 0$.

Given $\partial \tilde{\ell}/\partial i_r < 0$, $\partial \tilde{\ell}/\partial i > 0$ and $i_\ell = \phi'(\tilde{\ell})$, we have $\partial i_\ell/\partial i_r < 0$ and $\partial i_\ell/\partial i > 0$, which implies that, in DM2, $L^{-1}(i_\ell)$ is increasing in i_r and decreasing in i.

Now consider the no-banking equilibrium. The no-banking equilibrium satisfies

$$i = L(m_j) \text{ for } j = 1, 2; \quad m_3 = \max\{0, L^{-1}(i)\}$$

with $r = 0$, $i_d = 0$ and $n = 0$. In the no-banking case, it is straight forward to show

$$\frac{\partial r}{\partial i} = 0, \quad \frac{\partial i_d}{\partial i} = 0, \quad \frac{\partial r}{\partial i_r} = 0, \quad \frac{\partial i_d}{\partial i_r} = 0.$$

In the scarce-reserve equilibrium, $\tilde{r}$ is determined by

$$(i_r - i_d)\tilde{r} + i_\ell\left(\frac{1}{\ } - \tilde{r}\right) - \phi(\tilde{r}) - \left(\frac{1}{\ } - \tilde{r}\right) = k$$

$$\left[i_r \quad i_d \quad {}^0(\tilde{r}) + \quad i_\ell \quad {}^0 \frac{1}{\quad} \tilde{r} \quad \frac{1}{\quad} \right] = 0;$$

where $i_\ell = \frac{1+i}{1+i_d} \quad 1$. This implies that $\tilde{r}$ solves

$$ {}^0(\tilde{r})\tilde{r} \quad (\tilde{r}) + \quad {}^0 \frac{1}{\quad} \tilde{r} \quad \frac{1}{\quad} \tilde{r} \quad \frac{1}{\quad} \tilde{r} = k \qquad \text{(A.6)}$$

which is independent of i and i_r. The right-hand side of (A.6) is a positive constant, $k > 0$. Because the left-hand side of (A.6) is equal to 0 when $\tilde{r} = 0$ and strictly increasing in $\tilde{r}$, equation (A.6) uniquely pins down $\tilde{r}$. Let's define this $\tilde{r}$ as $\hat{r}$. The equilibrium deposit rate can be expressed as

$$i_d = i_r \quad {}^0(\hat{r}) + \quad \frac{1+i}{1+i_d} \quad 1 \quad {}^0 \frac{1}{\quad} \hat{r} \quad \frac{1}{\quad} > 0. \qquad \text{(A.7)}$$

Because $\hat{r}$ is independent of i and i_r, the implicit function theorem yields $\frac{\partial i_d}{\partial i_r} = 1 + \frac{(1 \quad)(1+i)}{(1+i_d)^2} > 0$ and

$$\frac{\partial i_d}{\partial i} = \frac{(1+i_d)}{1} + \frac{1+i}{1+i_d}^{\ 1} > 0. \qquad \text{(A.8)}$$

Using above results, one can show that

$$\frac{\partial i_\ell}{\partial i} = \frac{(1+i_d)}{(1+i)(1 \quad) + \quad (1+i_d)^2} > 0; \quad \frac{\partial i_\ell}{\partial i_r} = \frac{1+i}{(1+i_d)^2}\frac{\partial i_d}{\partial i_r} < 0.$$

■

Proof of Proposition 2. First, consider the ample-reserves equilibrium. Each bank's real balance of reserves, $\tilde{r}_s$, and lending, $\tilde{\ell}$, are determined by equations (A.1)-(A.2). By applying implicit function theorem to (A.3),

$$\frac{\partial \tilde{r}}{\partial \tilde{\ell}} = \frac{\partial P}{\partial P_r} = \frac{{}^{00}(\tilde{\ell})[1 + i_r \quad {}^0(\tilde{r})]^2}{(1+i) \quad {}^{00}(\tilde{r})} > 0.$$

By applying implicit function theorem to (A.4),

$$\frac{@\tilde{r}}{@\tilde{\ }} = \frac{@E}{@E_{\tilde{r}}} = \frac{{}^{00}(\tilde{r})\tilde{r}}{{}^{00}(\tilde{\ })\tilde{\ }} < 0:$$

By (A.1), $\tilde{r} = \max f\, 0;\ {}^{0\ 1}(i_r\quad i)g$ when $\tilde{\ } = 0$. De ne this $\tilde{r}$ as $\tilde{r}_1$. By equation (A.2), $\tilde{r} > 0$ when $\tilde{\ } = 0$, where $\tilde{r}_2$ solves ${}^{0}(\tilde{r})\tilde{r}\quad (\tilde{r}_2) = k$. De ne this $\tilde{r}$ as $\tilde{r}_2$. Therefore equations (A.1)-(A.2) have a single crossing point in $\mathbb{R}^2_+$ space as long as $\tilde{r}_1 < \tilde{r}_2$. This condition ($\tilde{r}_1 < \tilde{r}_2$) is satis ed under the ample-reserves equilibrium, which will be shown at the end of this proof.

Given this and $i_{`} = (1 + \ i)=(1 + i_d) \quad 1$, n solves

$$(\ _2 + \ \ _3)L\ ^1(i_{`}) \quad _3 = n(\tilde{r} + \tilde{\ }) \tag{A.9}$$

when $\hat{\ } > \quad$, whereas n solves

$$_2L\ ^1(i_{`}) = n(\tilde{r} + \tilde{\ })$$

when $\hat{\ } \quad$. Because n is uniquely determined by above equations, solving for the ample-reserves equilibrium provides unique solution.

Now consider the scarce-reserve equilibrium. Recall that (A.6) uniquely pins down $\tilde{r}$. Given $\tilde{r}$, i_d is uniquely determined by (A.7). Given $i_{`} = \frac{1+i}{1+i_d} \quad 1$, n solves

$$(\ _2 + \ \ _3)L\ ^1(i_{`}) \quad _3 = n\tilde{r}\ \frac{1 + \ i\ _d}{} \tag{A.10}$$

when $\hat{\ } > \quad$, whereas n solves

$$_2L\ ^1(i_{`}) = n\tilde{r}\ \frac{1 + \ i\ _d}{}$$

when $\hat{\ } \quad$. Because n is uniquely determined by above equations, it is straightforward to show that there is an unique solution.

In case of the no banking equilibrium, the equilibrium satisfies $i = L(m_j)$ for $j = 1; 2$ and $m_3 = \max\{0; L^{-1}(i) \quad \}$, which yield an unique solution.

The next step is characterizing the equilibrium type for given policies. When bank's unconstrained optimal lending $\ell = \;{}'^{-1}(i_\ell)$ is bigger than ℓ, the equilibrium is a scarce reserve equilibrium, whereas when $\ell \quad = \;{}'^{-1}(i_\ell)$, the equilibrium is an ample-reserves equilibrium. There exists a threshold that satisfies $\ell = \ell$ and

$$i = [1 + i_r \quad {}'(\hat{r})] \; 1 + {}' \frac{1}{\quad} \hat{r} \quad 1:$$

Because $\partial = \partial i < 0$ and $\ell = r(1 \quad) = $, it is straightforward to show $\partial = \partial i < 0$ and $\partial \ell = \partial i > 0$. Therefore, i is a threshold above which the equilibrium is a scarce reserve equilibrium and below which the equilibrium is an ample-reserves equilibrium.

The no-banking equilibrium arises when the equilibrium deposit rate i_d is zero and lowering i decreases the deposit rate. The threshold between the no-banking case and the scarce-reserve case $\hat{i}$ satisfies $i_d = i_r \quad {}'(\tilde{r}) + \left[\frac{1+\hat{i}}{1+i_d} \quad 1 \quad {}' \frac{1}{\quad} \tilde{r} \quad \frac{1}{\quad}\right] = 0$, implying

$$\hat{i} = \frac{\quad}{1}[\;{}'(\hat{r}) \quad i_r] + {}' \frac{1}{\quad} \hat{r} \; :$$

When $i < i_d$, there is no equilibrium in the market for reserves since demand for reserves is infinite. In this this case, there is no equilibrium with banks. The threshold that satisfies $i = i_d$ is $\underline{i} = i_r \quad {}'(\underline{r})$ where $\underline{r}$ solves ${}'(\underline{r})\underline{r} \quad (\underline{r}) = k$. Since the ample-reserve equilibrium always satisfies $i > \underline{i}$, the condition for the existence of a unique ample-reserve equilibrium, $r_1 < r_2$, is satisfied.

Therefore, we can conclude that given $(i_r; \quad)$: (i) $\exists!$ ample-reserves equilibrium if and only if $i_r > \;{}'(\hat{r})$ and $i \in (\underline{i}; i)$; (ii) $\exists!$ scarce-reserves equilibrium if and only if either $i > \hat{i}$ and $i_r < \;{}'(\hat{r})$ or $i > i$ and $i_r > \;{}'(\hat{r})$; (iii) $\exists!$ no banking equilibrium if and only if either $i \; 2 \; [0;\hat{\imath})$ and $i_r < \; {}^0(\hat{r})$, or $i \; 2 \; [0;\underline{i})$ and $i_r > \; {}^0(\underline{r})$. ■

Proof of Corollary 1. Because $\underline{r}$ solves ${}^0(\underline{r})\underline{r} \quad (\underline{r}) = k$, $\underline{r}$ is independent of i_r. Therefore, given $\underline{i} = i_r \quad {}^0(\underline{r})$, $@\underline{i}=@i_r = 1 > 0$. Similarly, because $\hat{r}$ solves ${}^0(\hat{r})\hat{r}$ $(\hat{r}) + \; {}^0 \frac{1}{\quad}\hat{r} \; \frac{1}{\quad}\hat{r} \quad \frac{1}{\quad}\hat{r} = k$, $\hat{r}$ is independent of i_r. Therefore, given

$$\hat{\imath} = \frac{}{1}[\; {}^0(\hat{r}) \quad i_r] + \; {}^0 \frac{1}{\quad}\hat{r} \; ; \quad i = [1 + i_r \quad {}^0(\hat{r})] \;\; 1 + \; {}^0 \frac{1}{\quad}\hat{r} \qquad 1;$$

we have $@\hat{\imath}=@i_r = \frac{}{1} < 0$ and $@=@i_r = \left[1 + \; {}^0 \frac{1}{\quad}\hat{r}\right] > 0$.

■

Proof of Proposition 4. First, consider the scarce-reserve equilibrium. Recall equations (A.6) and (A.7)

$$0 = \underbrace{\; {}^0(\tilde{r})\tilde{r} \quad (\tilde{r}) + \; {}^0 \frac{1}{\quad}\tilde{r} \; \frac{1}{\quad}\tilde{r} \quad \frac{1}{\quad}\tilde{r} \quad k}_{}$$

$$0 = \underbrace{i_d + i_r \quad {}^0(\tilde{r}) + \frac{1+i}{1+i_d} \;\; 1 \quad {}^0 \frac{1}{\quad}\tilde{r} \quad \frac{1}{\quad}}_{} :$$

Applying the implicit function theorem yields

$$\begin{bmatrix} \; {}_{i_d} & \; {}_r \\ \; {}_{i_d} & \; {}_r \end{bmatrix} \begin{bmatrix} di_d=d \\ d\tilde{r}=d \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

where

$${}_{i_d} = 0; \qquad {}_{i_d} = \quad 1 \quad \frac{(1+i)(1 \quad)}{(1+i_d)^2}$$

$${}_r = \; {}^{00}(\;)\tilde{r} + \; {}^{00}(\;) \; \frac{1}{\quad}{}^2 \tilde{r}; \qquad {}_r = \; {}^{00}(\;) \quad \frac{1}{\quad}{}^2 \; {}^{00}(\;)$$

$$= \frac{1}{3}\tilde{r}^2 \; {}^{00}(\;); \qquad = \frac{1}{3}\tilde{r} \; {}^{00}(\;) \quad \frac{1}{2}\left[\frac{1+i}{1+i_d} \quad 1 \quad {}^0(\;)\right] :$$

By Cramer's rule,

$$\frac{@i_d}{@} = \frac{r \quad r}{i_d \quad r \quad i_d \quad r} = \frac{\frac{1}{2}\left[\frac{1+i}{1+i_d}\quad 1 \quad {}^0\ \frac{1}{}\hat{r}\right]}{1+\frac{(1+i)(1\quad)}{(1+i_d)^2}} < 0:$$

Recall (A.8) $\frac{@i_d}{@i} = \left[\frac{(1+i_d)}{1} + \frac{1+i}{1+i_d}\right]^{1}$, by taking a derivative with respect to

$$\frac{@i_d}{@\ @i} = \frac{(1+i_d)}{(1\quad)^2[1+i+\frac{}{1}(1+i_d)]^2} + \frac{1+i}{[1+i+\frac{}{1}(1+i_d)^2]^2}\frac{@i_d}{@} < 0$$

because $\frac{@i_d}{@} < 0$.

Now consider the ample-reserve equilibrium. Recall equation (A.5) from the ample reserve case,

$$\frac{@i_d}{@i} = 1 \quad \frac{1+i}{1+i+[1+i_r \quad {}^0(\hat{r})]}:$$

Taking derivative with respect to i gives the following

$$\frac{@^2 i_d}{@i@i} = \frac{[1+i_r \quad {}^0(\ )] \quad (1+i)\ {}^{00}(\ )\frac{@}{@i}}{\{2+i+i_r \quad {}^0(\ )\}^2}$$

$$= \frac{[1+i_r \quad {}^0(\ )]^3}{\{2+i+i_r \quad {}^0(\ )\}^2\{`+[1+i_r \quad {}^0(\ )]^2\}} > 0$$

using the results from the proof of Proposition 1. ■

Proof of Proposition 5. Assume $\hat{} >$. For the scarce reserve equilibrium, recall (1.27)

$$r = \frac{(\ _2 + \ _3)}{1+i_d}L^{\ 1}(i_`) \quad \frac{3}{1+i_d}:$$

Given $\hat{\ } >$, it is straightforward to show that

$$\frac{@r}{@} = \frac{_3}{1+i_d} < 0; \qquad \frac{@r}{@_3} = \frac{}{1+i_d} < 0:$$

For the ample reserve equilibrium, recall (A.9)

$$(\ _2 + \ _3)L\ ^1(i_`) \qquad _3 = n(\tilde{r} + \tilde{\ }):$$

Because $\tilde{r}$, $\tilde{\ }$, i_d, and $i_`$ are independent of , we have

$$\frac{@n}{@} = \frac{_3}{\tilde{r} + \tilde{\ }} < 0; \qquad \frac{@n}{@_3} = \frac{}{\tilde{r} + \tilde{\ }} < 0$$

implying $\frac{@r}{@} < 0$ and $\frac{@r}{@_3} < 0$ where $r = n\tilde{r}$. ■

Proof of Proposition 6. De ne money multiplier $(m + r + `)=(m + r)$. In the scarce reserves equilibrium, $= \frac{m+r=}{m+r}$. In this case we have

$$\frac{@}{@} = \frac{m(1 \quad)}{(m+r)^2}\frac{@r}{@} < 0$$

as long as $m > 0$ and $\ 2\ (0;1)$. In the ample-reserves equilibrium, we have

$$\frac{@}{@} = \frac{m\tilde{\ }}{(m+r)^2}\frac{@n}{@} < 0$$

as long as $m > 0$. Now consider the e ect of i and i_r under the ample-reserves equilibrium. By taking a derivative with respect to i, we have

$$\frac{@}{@i} = \frac{1}{(m+r)^2}\left[\underbrace{\frac{@\tilde{\ }}{@i}}\ n(m+r) \qquad \underbrace{\frac{@m_`}{@i}} + \underbrace{\frac{@n_{\tilde{\ }}}{@i}}_{?} m\right];$$

which is positive when m is small enough. By taking a derivative with respect to i_r,

$$\frac{\partial}{\partial i_r} = \frac{1}{(m+r)^2}\left[\underbrace{\frac{\partial \tilde{}}{\partial i_r}} n(m+r) + \underbrace{\frac{\partial n}{\partial i_r}}_{?} \tilde{} m\right]$$

which is negative when m is small enough. Therefore, for small m, we have

$$\frac{\partial}{\partial i} > 0; \quad \frac{\partial}{\partial i_r} < 0$$

in the ample-reserves equilibrium. ■

A.2 Additional Results

A.2.1 Chow Test

Figure 1.3 includes the Chow test for structural breaks. The test result reported in the bottom-left panel of Figure 1.3 is implemented by estimating following regression.

$$
\begin{aligned}
\text{Money multiplier}_t = \; & {}_0 + {}_1(\text{RequiredReserves/Deposit}_t) \\
& + 1_{t \; 1992Q2}[\,{}_0 + {}_1(\text{RequiredReserves/Deposit}_t)] \\
& + 1_{t \; 2008Q4}[\,{}_0 + {}_1(\text{RequiredReserves/Deposit}_t)] + {}_t
\end{aligned}
$$

Table A.1a reports F-statistics which are obtained by testing ${}_0 = {}_1 = {}_0 = {}_1 = 0$.

The Chow test in the bottom-right panel of Figure 1.3 is implemented by estimating following regression.

$$
\begin{aligned}
\text{Money multiplier}_t = \; & {}_0 + {}_1(\text{Currency/Deposit})_t \\
& + 1_{t \; 2008Q4}[\,{}_0 + {}_1(\text{Currency/Deposit})_t] + {}_t
\end{aligned}
$$

Table A.1b reports F-statistics is obtained by testing ${}_0 = {}_1 = 0$. The regression estimates and the Chow test results are summarized at the below table.

Table A.1: Chow Test for Structural Breaks

(a) Require Reserve Ratio

Dependent Variable: Money Multiplier	
RR	0:601
	(0:365)
$RR \quad 1_{t \;\; 1992Q2}$	132:279
	(0:031)
$RR \quad 1_{t \;\; 2008Q4}$	147:943
	(8:574)
$1_{t \;\; 1992Q2}$	9:091
	(0:557)
$1_{t \;\; 2008Q4}$	0:074
	(0:611)
Constant	2:813
	(0:053)
Obs.	228
R^2	0.963
DF for numerator	4
DF for denominator	222
F Statistic for Chow test	1711.32
F Statistic for 1% sig. level	3.40
F Statistic for 0.1% sig. level	4.79

(b) Currency Deposit Ratio

Dependent Variable: Money Multiplier	
CD	1:301
	(0:027)
$CD \quad 1_{t \;\; 2008Q4}$	52:018
	(4:995)
$1_{t \;\; 2008Q4}$	3:061
	(0:409)
Constant	3:159
	(0:015)
Obs.	228
R^2	0.974
DF for numerator	2
DF for denominator	224
F Statistic for Chow test	1245.69
F Statistic for 1% sig. level	4.70
F Statistic for 0.1% sig. level	7.13

Notes: Newy-West standard errors are in parentheses. ***, **, and * denote signi cance at the 1, 5, and 10 percent levels, respectively. Degree of freedom is denoted by DF.

A.2.2 Unit Root and Cointegration Test

Columns (2) and (4) in Table 1.1 includes the canonical cointegrating regression estimates and the cointegration tests. This section reports unit root tests for the series used in Columns (2) and (4). For all the four variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their rst di erence rejects the null hypothesis of non-stationarity at 1% signi cance level. All series are demeaned before implementing the unit root test following to Elliott and Müller (2006) and Harvey, Leybourne, and Taylor (2009), because the magnitude of the initial value can be problematic. Let ***, **, and * denote signi cance at the 1, 5, and 10 percent levels, respectively. The data are quarterly from 1980Q1 to 2007Q4.

Table A.2: Unit Root Test

	Phillips-Perron test	
	Z( )	Z(t)
ln(m)	0:567	0297
ln(d)	1:275	1054
ln(uc)	1:114	1:710
r	7:721	2:471
ln(m)	46:623	5:335
ln(d)	42:267	5:060
ln(uc)	41:998	5:107
r	94:183	9:263

Columns (1) and (2) in Table 1.3 includes the canonical cointegrating regression estimates since all three series R=Y, UC=Y, and 3-month treasury rate - have unit roots and cointegrated both for the data and the model-implied series. This section also reports unit root tests, cointegration tests and sensitivity checks using federal funds rates. For all the ve variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their rst di erence rejects the null hypothesis of non-stationarity at 1% signi cance level. The Johansen tests reject their null of no cointegration at 99 percent con dence level, suggesting there exists a stable relationship between real reserves balances, interest rates, and real balances of unsecured credit both in the data and the model. This result is robust with respect to di erent measures of interest rate, the federal funds rate. The data are yearly from 1968 to 2007.

Table A.3: Unit Root Test and Additional CCR Estimates

(a) Unit Root Test

	Phillips-Perron test	
	Z( )	Z(t)
Tbill3	7:683	1:967
ffr	8:683	2:121
UC=Y	0:315	0:450
R=Y(Data)	1:735	2:240
R=Y(Model)	1:094	1:861
Tbill3	24:363	4:514
ffr	25:127	4:747
UC=Y	24:204	4:202
R=Y(Data)	26:473	4:329
R=Y(Model)	33:542	5:176

(b) Canonical Cointegrating Regression

Dependent Variable:	Reserves/GDP (1968-2007)
UC=Y	0:122
	(0:004)
ffr	0:064
	(0:009)
Constant	3:058
	(0:095)
Obs.	40
R^2	0:854
adj R^2	0:846
Long run S.E.	0:141

Notes: All series are demeaned before implementing the unit root test because the magnitude of the initial value can be problematic, as pointed out by Elliott and Müller (2006) and Harvey, Leybourne, and Taylor (2009). ***, **, and * denote signi cance at the 1, 5, and 10 percent levels, respectively.

A.2.3 Unit Root and Cointegration Test for M2

To check the breakdown of the stable relationship between M2 and interest rate, Figure A.1 plots the ratio of M2 to GDP for the US and the ratio of M2's components to GDP against the 3 month Treasury Bill rate. There is also a breakdown in M2 in

Table A.4: Johansen Test for Cointegration

(a) UC/Y, Tbill3 and R/Y (Data)

Max rank	$_{trace}(r)$	5% CV	1% CV
0	39.5289	29.68	35.65
1	6.3521	15.41	20.04
2	1.7359	3.76	6.65
Max rank	$_{max}(r; r+1)$	5% CV	1% CV
0	33.1768	20.97	25.52
1	4.6162	14.07	18.63
2	1.7359	3.76	6.65

(b) UC/Y, Tbill3 and R/Y (Model)

Max rank	$_{trace}(r)$	5% CV	1% CV
0	46.8658	29.68	35.65
1	10.2012	15.41	20.04
2	3.2950	3.76	6.65
Max rank	$_{max}(r; r+1)$	5% CV	1% CV
0	36.6646	20.97	25.52
1	6.9063	14.07	18.63
2	3.2950	3.76	6.65

(c) UC/Y, r and R/Y (Data)

Max rank	$_{trace}(r)$	5% CV	1% CV
0	42.2554	29.68	35.65
1	6.1539	15.41	20.04
2	1.7615	3.76	6.65
Max rank	$_{max}(r; r+1)$	5% CV	1% CV
0	36.1015	20.97	25.52
1	4.3924	14.07	18.63
2	1.7615	3.76	6.65

(d) UC/Y, r and R/Y (Model)

Max rank	$_{trace}(r)$	5% CV	1% CV
0	46.4585	29.68	35.65
1	10.1184	15.41	20.04
2	3.1302	3.76	6.65
Max rank	$_{max}(r; r+1)$	5% CV	1% CV
0	36.3401	20.97	25.52
1	6.9882	14.07	18.63
2	3.1302	3.76	6.65

Figure A.1: Money Demand for M2 and Its Components

1992 that coincides with the structural break in Figure 1.3 and Figure 1.4.

Table A.5: Cointegration Regressions and Tests (M2)

Dependent Variable:	$\ln(m_t)$		$\ln(d_t)$	
	OLS	CCR	OLS	CCR
	(1)	(2)	(3)	(4)
r_t	0:009	0:019	0:013	0:020
	(0.002)	(0.002)	(0.002)	(0.003)
$\ln(uc_t)$		0:182		0:225
		(0.024)		(0.027)
$adjR^2$	0.133	0.306	0.201	0.288
N	112	112	112	112
Johansen $r = 0$	18.582	40.396	19.210	39.421
5% Critial Value for $r = 0$	15.41	29.68	15.41	29.68
1% Critial Value for $r = 0$	20.04	35.65	20.04	35.65
Johansen $r = 1$	2.762	13.177	2.713	13.364
5% Critial Value for $r = 1$	3.76	15.41	3.76	15.41
1% Critial Value for $r = 1$	6.65	20.04	6.65	20.04

Notes: Columns (1) and (3) report OLS estimates and columns (2) and (4) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote signi cance at the 1, 5, and 10 percent levels, respectively. Johansen cointegration test results are reported in column (1)-(4). The data are quarterly from 1980Q1 to 2007Q4.

To see whether the unsecured credit can account for this breakdown in M2, I repeated the analysis of Table 1.1 using M2 instead of M1. Table A.5 reports the results. Again, I focus on the post-1980 period, until the arrival of the Great Recession. In columns (2) and (4) the Johansen tests reject their null of no cointegration at 99 percent confidence level, suggesting there exists a stable relationship between M2 real money balances, interest rates, and real balances of unsecured credit. The canonical cointegrating regression estimates in columns (2) and (4) show that the estimated coefficients on r_t and $\ln(uc_t)$ both are negative and significantly different from zero. Thus, using the cointegrating regressions and tests, I document the evidence that once we account for the substitution effect of consumer credit, there still exists a stable negative relationship between M2 real balances and the interest rates.

Table A.6 provides the unit root test results for M2 to output ratio and deposit component of the M2 to output ratio. For all the two variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first differences reject the null hypothesis of non-stationarity at 1% significance level. All series are demeaned before implementing the unit root test

Table A.6: Unit Root Test (M2)

	Phillips-Perron test	
	$Z( )$	$Z(t)$
$\ln(m)$	0:567	0:297
$\ln(d)$	1:275	1:054
$\ln(m)$	46:623	5:335
$\ln(d)$	42:267	5:060

Notes: All series are demeaned before implementing the unit root test because the magnitude of the initial value can be problematic, as pointed out by Elliott and Müller (2006) and Harvey, Leybourne, and Taylor (2009). ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

A.3 Quantitative Robustness

A.3.1 Robustness I: Di erent Measure of Monetary Policy

Di erent papers have used di erent series as monetary instruments. For example, Lucas (2000) and Lagos and Wright (2005) use commercial paper rate. Bethune, Choi, and Wright (2020) uses 3 month treasury bill rate. New Keynesian literature usually use federal funds rate (e.g, Christiano, Eichenbaum, and Evans, 2005 Smets and Wouters, 2007, Christiano, Eichenbaum, and Trabandt, 2016) or 3 month treasury bill rate (e.g., Ireland, 2011) as measure of monetary policy while those models don't t to the money demand. This section checks the robustness of the main quantitative results by using di erent measures of monetary policy: commercial paper rate, and federal funds rate.

Table A.7: Parametrizations with Di erent Measure of Monetary Policy

Interest	3 Month T-bill		CP		Federal Funds	
	Data	Model	Data	Model	Data	Model
Targets						
avg. retail markup	1.384	1.384	1.384	1.384	1.384	1.388
avg. C=Y	0.044	0.044	0.044	0.044	0.044	0.043
avg. R=Y	0.014	0.017	0.014	0.017	0.014	0.017
avg. C=D	0.529	0.520	0.529	0.520	0.529	0.512
avg. UC=DM	0.387	0.370	0.387	0.370	0.387	0.371
avg. =Y	0.0016	0.0011	0.0016	0.0011	0.0016	0.0011
semi-elasticity of C=Y	-3.716	-3.724	-3.713	-3.712	-3.020	-3.719

Note: C, R, DM , UC, Y denote currency in circulation, reserves, DM transactions, unsecured credit and nominal GDP, respectively.

Table A.7 compares how model moments change depending on di erent measures of monetary policy. The table shows that the results are not very sensitive because the model matches the target moments well, even when di erent measures of monetary policy are used. Figure A.2 compares the model ts using di erent measures of

Figure A.2: Model Fit with Di erent Measure of Monetary Policy

monetary policy. The model-generated series also show similar patterns for the M1 money multiplier, the excess reserve ratio, and the currency deposit ratio.

A.3.2 Robustness II: Other Specifications

This section summarizes alternative parameterization results. For robustness, I examine how results are sensitive with respect to different parameter for curvature parameter for deposit operating cost while keeping other parameters are same. For Model 1, I set $(\tilde{d}) = A\tilde{d}^{1.15}$ while I set $(\tilde{d}) = A\tilde{d}^{1.25}$ in the Model 2.

Table A.8 compares how model moments change depending on different parameterization. The table shows that the results are not very sensitive depending on different parameterizations. Figure A.3 compares the model fits under different parameterizations. The model-generated series also show similar patterns for the M1 money multiplier, the excess reserve ratio, and the currency deposit ratio.

Table A.8: Alternative Parametrizations

	Data	Baseline	Model 1	Model 2
External Parameters				
a		1.2	1.15	1.25
$_3$		0.69	0.69	0.69
Calibration targets				
avg. retail markup	1.384	1.384	1.384	1.384
avg. C=Y	0.044	0.044	0.044	0.044
avg. R=Y	0.014	0.017	0.017	0.017
semi-elasticity of C=Y	-3.713	-3.712	-3.712	-3.712
avg. C=D	0.529	0.520	0.520	0.520
avg. UC=DM	0.387	0.370	0.370	0.370
avg. =Y	0.0016	0.0011	0.0011	0.0011

Note: C, R, DM , UC, Y denote currency in circulation, reserves, DM transactions, unsecured credit and nominal GDP, respectively.

Figure A.3: Model Fit with Di erent Speci cations

A.4 Data Sources and Variable De nitions

- Federal funds rates: E ective Federal Funds Rate" (FRED series FEDFUNDS).
- Interest on reserves: Interest Rate on Excess Reserves" (FRED series IOER) and Interest Rate on Required Reserves" (FRED series IORR).
- 3-month treasury rate: 3-Month Treasury Bill: Secondary Market Rate" (FRED series TB3MS).
- Deposit / Total checkable deposits: Total Checkable Deposits" (FRED series TCDSL).
- Excess reserves: Excess Reserves of Depository Institutions" (FRED series EXCRESNS) and (FRED series EXCSRESNS).
- Excess reserve ratio: $\frac{\text{Excess reserves}}{\text{Total checkable deposits}}$.
- Required reserves: Required Reserves of Depository Institutions" (FRED series REQRESNS).
- Required reserves ratio: $\frac{\text{Required reserves}}{\text{Total checkable deposits}}$.
- Reserves: Total Reserves of Depository Institutions" (FRED series TOTRESNS).
- M1: M1 Money Stock" (FRED series M1SL).
- M2: M2 Money Stock" (FRED series M2SL).
- Monetary base: Monetary Base; Total" (FRED series BOGMBASE).
- M1 money multiplier: $\frac{\text{M1}}{\text{Monetary Base}}$.

- Currency: Currency Component of M1" (FRED series CURRSL).
- Deposit Component of M1: M1 Currency.
- Deposit Component of M2: M2 Currency.
- Unsecured credit: Revolving Consumer Credit Owned and Securitized" (FRED series REVOLSL) transformed from monthly to quarterly by summing monthly data.
- GDP: Gross Domestic Product" (FRED series GDP), quarterly and Gross Domestic Product" (FRED series GDPA), annual.
- Commercial paper rate: Nominal interest rate" from Ireland (2009), updated to 2017 using 3-Month AA Non nancial Commercial Paper Rate" (FRED series CPN3M).

The quantitative analysis uses the annual average of above series.

Appendix B

Appendix for Chapter 2

B.1 Proofs

Proof of Proposition 8. Let there exists a two-period cycle satisfying $z_1 < z_s < p < z_2$. Since $z_2 > p$, we have $z_2 = (1 + i)z_1$. Using (2.27) with $z_1 < p$ gives

$$= \frac{(1 \quad) L (z_1)}{(1+i)^2 \quad 1 \quad L \ (z_1)} \tag{B.1}$$

This two-period cycle should satisfy $z_1 < z_s < p$ and $z_2 = (1 + i)z_1 > p$. First one can be easily shown using

$$0 = L(p) < L(z_s) = \frac{i}{(1 \quad + \quad)} < \frac{(1+i)^2 \quad 1}{(1 \quad + \quad)} = L(z_1)$$

since we have $L^0( ) < 0$. Because $dz_1 = d < 0$, the latter one, $z_1 > p = (1 + i)$, is held when

$$0 < \quad < \frac{(1 \quad) L \frac{p}{1+i}}{(1+i)^2 \quad 1 \quad L \frac{p}{1+i}}:$$

■

Proof of the Existence of a Two-period Monetary Cycle where $f'(z) < 1$.
Let $f^2(z) = f \; f(z)$. With given the unique steady state, $f(z) > z$ for $z < z_s$ and $f(z) < z$ for $z > z_s$. Because $f(z)$ is linear increasing function for $z > p$, there exist a $\bar{z} > p$ s.t $f(\bar{z}) > p$. Since $\bar{z} > p$ and $f(\bar{z}) < \bar{z}$, $\bar{z}$ satis es $f^2(\bar{z}) < f(\bar{z}) < \bar{z}$. We can write slope of $f^2(z)$ as follows.

$$\frac{@f^2(z)}{@z} = f'[f(z)]f'(z) = f'(z)f'(z) = [f'(z)]^2$$

which implies $@f^2(z)=@z > 1$ when $f(z) < 1$. And it is easy to show $@f^2(0)=@z > 0$. With given $i > 0$ and > 0, there exist a $(z_1; z_2)$, satisfying $0 < z_1 < z_s < z_2$ which are x points for $f^2(z)$ ■

Proof of Proposition 9. When DM trade is based on take-it-or-leave-it o er from buyer to seller with $c(q) = q$ and $qu''(q)=u' =$, f' can be written as

$$f'(q) = \frac{1}{1+i} \frac{1 \quad +}{} [u''(q)q + u'(q) \quad 1] + 1 \quad < \quad 1$$

Using $u''(q)q = \quad u'(q)$ gives

$$\frac{1 \quad +}{} [u'(q)(1 \quad) \quad 1] + 1 < \quad (1+i)$$

where $u'(q) = 1 + \frac{i}{(1 \quad + \quad)}$. Substituting $u'(q)$ and rearranging terms give

$$0 < \quad < \frac{(1 \quad)}{(1 \quad) + (2 \quad)(1+i)}$$

■

Proof of Proposition 10. I divide three period cycles into two cases.
Case 1: Let there exists a three-period cycle satisfying $z_1 < z_s < p < z_2 < z_3$. Since $z_2; z_3 > p$, we have $z_2 = (1 + i)z_1$, $z_3 = (1 + i)z_2 = (1 + i)^2 z_1$. Using (2.27) with

$z_1 < p$ gives

$$= \frac{(1 \quad) L \ (z_1)}{(1+i)^3 \quad 1 \quad L \ (z_1)} \tag{B.2}$$

This three-period cycle should satisfy $z_1 < z_s < p$ and $z_2 = (1 + i) z_1 > p$. First one can be easily shown using

$$0 = L(p) < L(z_s) = \frac{i}{(1 \quad + \quad)} < \frac{(1+i)^3 \quad 1}{(1 \quad + \quad)} = L(z_1)$$

since we have $L^0( ) < 0$. Because $dz_1 = d < 0$, the latter one, $z_1 > p = (1 + i)$, is held when

$$0 < \quad < \frac{(1 \quad) L \ \frac{p}{1+i}}{(1+i)^3 \quad 1 \quad L \ \frac{p}{1+i}}:$$

Case 2: Let there exists a three-period cycle satisfying $z_1 < z_2 < p \quad z_3$. Since $z_3 > p$, we have $z_3 = z_2(1 + i)$ and $(z_2; z_1)$ solves (B.3)-(B.4).

$$z_1 = f(z_2) = \frac{1 \quad +}{} L \ (z_2) + 1 \quad \frac{z_2}{1+i} \tag{B.3}$$

$$z_2 \quad \tilde{f}(z_1) = \frac{1 \quad +}{} L \ (z_1) + 1 \quad \frac{z_1}{(1+i)^2}: \tag{B.4}$$

These functions satisfies $f(x) > x$ for $x < z_s$, $f(x) < x$ for $x > z_s$, $\tilde{f}(x) > x$ for

Figure B.1: Intersection of $\tilde{f}(z)$ and $f(z)$

$x < \bar{z}$ and $\tilde{f}(x) < x$ for $x > \bar{z}$ where $\bar{z}$ solves $\bar{z} = \tilde{f}(\bar{z})$. One can easily show $\bar{z} < z_s$. Therefore any intersection between $z_1 = f(z_2)$ and $z_2 = \tilde{f}(z_1)$ satisfies $z_1 > z_2$ which contradicts to our initial conjecture $z_1 < z_2$. This implies there is no three-period cycles satisfying $z_1 < z_2 < p \quad z_3$. Therefore we can conclude that a three-period cycle exist when

$$0 < \quad < \frac{(1 \quad) L \quad \frac{p}{1+i}}{(1+i)^3 \quad 1 \quad L \quad \frac{p}{1+i}}.$$

The existence of a three-cycle implies the existence of cycles of all orders and chaotic dynamics by the Sarkovskii theorem (Sharkovskii, 1964) and the Li-Yorke theorem (Li and Yorke, 1975). ■

Proof of Corollary 2: Proposition 10 shows that at least one periodic point satisfies $z_t < z_s < p$ in 3- period cycles. Two period cycles satisfies $z_1 < z_s < z_2$ also implies at least one periodic point satisfies $z_t < z_s < p$ in 2-period cycles since $z_1 < z_s < p$. This result holds for any n-periodic cycles. Let $z_1 < z_2 < \ldots z_n$ be the periodic points of an n-cycle. Suppose $z_j > z_s$ for all $j = 1, 2, \ldots n$. By the definition of a n-period cycle, $z_1 = f(z_n) < z_n$ since $f(z) < z$ for $z > z_s$.

$$z_n = f(z_{n-1}) < z_{n-1} = f(z_{n-2}) < z_{n-2} \ldots < z_1.$$

which shows the contradiction implying at least one periodic point satisfies $z_t < z_s < p$. ■

Proof of Proposition 11. Consider $z_t = f(z_{t+1})$. If $z_s > z$ where z solves $f'(z) = 0$. In this case, there exist multiple equilibria. If $q \quad f(z)$, then there exist equilibria $\{z_t\}_{t=0}^{\infty}$ with $z_T \quad \max\{z_t\}_{t=0}^{\infty} > q$ (bubble) which crashes to 0 (burst) as $t \to \infty$, where $T \quad 1$ and $z_T > z_0$. Then there exist equilibria with bubble-burst as a self-fulfilling crisis. Conditions for this case are shown as below. Similar to Proposition 9, consider take-it-leave-it o er with $qu^{00} = u^0 =$ and $c(q) = q$. Then we have following di erence equation:

$$z_t = f(z_{t+1}) \begin{cases} \frac{z_{t+1}}{1+i} \quad \frac{1 \quad +}{} \quad [u^0(z_{t+1}) \quad 1] + 1 & \text{if } z_{t+1} < q \\ \frac{z_{t+1}}{1+i} & \text{if } z_{t+1} \quad q \end{cases} \qquad \text{(B.5)}$$

Step 1: [Multiplicity i.e., $z_s > z$ where z solves $f^0(z) = 0$] Consider the following condition.

$$f^0(z) = \frac{1}{1+i} \quad \frac{(1 \quad + \quad)}{}[u^0(z)(1 \quad) \quad 1] + 1 \quad = 0$$

Since $z_s > z$! $u^0(z_s) < u^0(z)$, we have

$$u^0(z_s) = 1 + \frac{i}{(1 \quad + \quad)} < \frac{1}{1} \quad 1 \quad \frac{}{(1 \quad + \quad)} = u^0(z):$$

This can be reduced as

$$< \frac{(1 \quad)}{1+i \quad (i + \quad)}$$

Step 2: [Show $q \quad f(z)$] It is straightforward to show that $q < f(z)$ holds when

$$< \frac{(1 \quad) \quad (1+i)}{(1 \quad)^2 q + (1+ i)[(1 \quad)(3 + i \quad) \quad]}$$

Therefore, when

$$0 < \quad < \min \quad \frac{(1 \quad) \quad (1+i)}{(1 \quad)^2 q + (1+ i)[(1 \quad)(3 + i \quad) \quad]}; \frac{(1 \quad)}{1+i \quad (i + \quad)}$$

there exist $f z_t g_{t=0}^1$ satisfying $z_T \quad \max f z_t g_{t=0}^1 > q$ and $\lim_{t!1} z_t = 0$, where $T \quad 1$ and $z_T > z_0 > q = (1+i)$. ■

Proof of Proposition 12. A two period cycle result is presented and three-period case will follow. Let there exists a two-period cycle satisfying $w_1 < q < w_2$ where

$w_j = z_j + b_j$. Since $w_2 > q$, we have $z_2 = (1 + i)z_1$ and $b_2 = (1 + r)b_1$ where q_1, b_1, and z_1 solve

$$u^0(q_1) = 1 + \frac{(1+i)^2 \quad 1}{(1 \quad + \quad)}$$

$$b_1 = [(1 + r)^2 \quad 1]^{1} \frac{i}{1 \quad +} \; 1 \quad \frac{(1+i)^2}{} z_1 + \quad [u(q_1) \quad q_1]$$

and $z_1 = q_1 \quad b_1$. This two-period cycle should satisfy $q_1 < q$ and $w_2 = (1 + i)z_1 + (1 + r)b_1 > q$. For given $i > 0$ and > 0, rst one can be easily shown using

$$1 = u^0(q\ ) < u^0(q_s) = 1 + \frac{i}{(1 \quad + \quad)} < 1 + \frac{(1+i)^2 \quad 1}{(1 \quad + \quad)} = u^0(q_1)$$

since we have $u^{00}(\ ) < 0$. Now we also can check the latter using the below conditions

$$(1 + r)q_1 > (1 + i)z_1 + (1 + r)b_1 = w_2 > q \; > q_1 = z_1 + b_1 \quad \text{if } r > i$$

$$(1 + i)q_1 > (1 + i)z_1 + (1 + r)b_1 = w_2 > q \; > q_1 = z_1 + b_1 \quad \text{if } i > r:$$

The su cient conditions to have $w_2 > q$ is $q_1 > q = (1 + r)$ for $r > i$ and $q_1 > q = (1 + i)$ for $i > r$. Since we have $dq_1 = d < 0$, there exist a three period cycle $q_1 = w_1 < q_s < q < w_2 < w_3$ when

$$0 < \; < \frac{(1 \quad)\;[u^0 \frac{q}{1+} \quad 1]}{(1+i)^2 \quad 1 \quad [u^0 \frac{q}{1+} \quad 1]}$$

where $= \max f\, i; r\, g$. Now, let there exists a three-period cycle satisfying $q_1 = w_1 < q_s < q < w_2 < w_3$ where $w_j = z_j + b_j$. Since w_3, $w_2 > q$, we have $z_2 = (1 + i)z_1$, $z_3 = (1 + i)^2 z_1$, $b_2 = (1 + r)b_1$ and $b_3 = (1 + r)^2 b_1$ where q_1, b_1, and z_1 solve

$$u^0(q_1) = 1 + \frac{(1+i)^3 \quad 1}{(1 \quad + \quad)}$$

$$b_1 = [(1 + r)^3 \quad 1]^{1} \frac{i}{1 \quad +} \; 1 \quad \frac{(1+i)^2}{} z_1 + \quad [u(q_1) \quad q_1]$$

and $z_1 = q_1 \; b_1$. This three-period cycle should satisfy $q_1 < q_s < q$ and $w_2 = (1+i)z_1 + (1+r)b_1 > q$. For given $i > 0$ and > 0, rst one can be easily shown using

$$1 = u^0(q) < u^0(q_s) = 1 + \frac{i}{(1 \quad + \quad)} < 1 + \frac{(1+i)^3 \quad 1}{(1 \quad + \quad)} = u^0(q_1)$$

since we have $u^{00}(\;) < 0$. Now we also can check the latter using below conditions

$$(1+r)q_1 > (1+i)z_1 + (1+r)b_1 = w_2 > q \; > q_1 = z_1 + b_1 \quad \text{if } r > i$$

$$(1+i)q_1 > (1+i)z_1 + (1+r)b_1 = w_2 > q \; > q_1 = z_1 + b_1 \quad \text{if } i > r:$$

The su cient conditions to have $w_2 > q$ is $q_1 > q \; =(1+r)$ for $r > i$ and $q_1 > q \; =(1+i)$ for $i > r$. Since we have $dq_1 = d \; < 0$, there exist a three period cycle $q_1 = w_1 < q_s < q \; < w_2 < w_3$ when

$$0 < \; < \frac{(1 \quad) \; [u^0 \; \frac{q}{1+} \quad 1]}{(1+i)^3 \quad 1 \qquad [u^0 \; \frac{q}{1+} \quad 1]}$$

where $= \max f\, i; r\, g$. Again, the existence of a three-cycle implies the existence of cycles of all orders and chaotic dynamics by the Sarkovskii theorem and the Li-Yorke theorem. ■

B.2 Empirical Appendix

This section provides robustness checks for empirical results. To check the sensitivity of the results, Table B.1 and B.2 repeat all the empirical analysis, reported in Table 2.1 and 2.2, using quarterly series instead of annual data. This section also provides robustness checks using time-series before 2008. Table B.3 and B.4 repeat the analysis using time-series before 2008. All the results are similar to the benchmark analysis shown in Table 2.1 and 2.2.

Table B.1: E ect of Required Reserve Ratio: Robustness Check (Quarterly)

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent variable: $^{Roll}_{t}$	OLS (1)	CCR (2)	OLS (3)	CCR (4)	OLS (5)	CCR (6)	OLS (7)	CCR (8)
	0:282	0:452	0:266	0:400	0:305	0:485	0:306	0:476
	(0:016)	(0:001)	(0:014)	(0:003)	(0:015)	(0:000)	(0:014)	(0:006)
ffr		0:050		0:058		0:015		0:047
		(0:000)		(0:002)		(0:000)		(0:005)
Constant	0:074	0:085	0:070	0:079	0:074	0:089	0:073	0:086
	(0:002)	(0:000)	(0:002)	(0:000)	(0:002)	(0:000)	(0:002)	(0:001)
Obs.	196	196	196	196	196	196	196	196
adjR^2	0.696	0.240	0.725	0.263	0.737	0.222	0.761	0.268
$_{trace}$ (r = 0)	9.496	31:950	11.045	33:808	10.930	34:481	12.103	35:951
5% CV	15:41	29:68	15:41	29:68	15:41	29:68	15:41	29:68
$_{trace}$ (r = 1)	1:677	11.162	1:959	12.266	1:938	12.094	1:887	12.485
5% CV	3:76	15:41	3:76	15:41	3:76	15:41	3:76	15:41

Note: For (1), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2) and (3), rst-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West xed lag, $4(T/100)^{2/9}$; ffr denotes federal funds rates and $^{Roll}_{t}$ denotes cyclical volatility of real inside money balances. ***, **, and * denote signi cance at the 1, 5, and 10 percent levels, respectively.

Table B.2: Unit Root Tests: Robustness Check (Quarterly)

		Phillips-Perron test		ADF test
		Z( )	Z(t)	Z(t) w/ lag 1
ffr		8:611	1:956	2:183
		1:335	1:145	1:199
$^{Roll}_{t}$	(CPI)	4:320	2:062	1:554
$^{Roll}_{t}$	(Core CPI)	4:388	2:201	1:924
$^{Roll}_{t}$	(PCE)	3:822	1:946	1:868
$^{Roll}_{t}$	(Core PCE)	3:565	1:928	2:023
ffr		139:701	10:792	10:288
		163:796	12:272	9:909
$^{Roll}_{t}$	(CPI)	23:132	2:604	3:576
$^{Roll}_{t}$	(Core CPI)	30:423	3:544	4:894
$^{Roll}_{t}$	(PCE)	24:507	2:874	4:362
$^{Roll}_{t}$	(Core PCE)	28:054	3:373	5:138

Note: ffr denotes federal funds rates, denotes required reserve ratio, and $^{Roll}_{t}$ denotes cyclical volatility of real inside money balances. ***, **, and * denote signi cance at the 1, 5, and 10 percent levels, respectively.

Table B.3: E ect of Required Reserve Ratio: Robustness Check (pre-2008)

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent	OLS	CCR	OLS	CCR	OLS	CCR	OLS	CCR
variable: $^{Roll}_{t}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	0:266	0:297	0:266	0:268	0:307	0:288	0:305	0:277
	(0:030)	(0:001)	(0:030)	(0:001)	(0:032)	(0:002)	(0:029)	(0:002)
ffr		0:107		0:124		0:189		0:210
		(0:001)		(0:001)		(0:002)		(0:002)
Constant	0:070	0:080	0:070	0:076	0:074	0:082	0:072	0:080
	(0:004)	(0:000)	(0:004)	(0:000)	(0:004)	(0:000)	(0:004)	(0:002)
Obs.	43	43	43	43	43	43	43	43
adjR^2	0.727	0.659	0.727	0.710	0.739	0.708	0.759	0.734
$_{trace}$ (r = 0)	8.373	32:228	7.438	31:299	7.661	31:867	6.897	31:250
5% CV	15:41	29:68	15:41	29:68	15:41	29:68	15:41	29:68
$_{trace}$ (r = 1)	1:504	9.554	1:125	8.428	1:146	8.603	0:938	7.693
5% CV	3:76	15:41	3:76	15:41	3:76	15:41	3:76	15:41

Note: For (1), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2) and (3), rst-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West xed lag, 4 $(T=100)^{2=9}$; ffr denotes federal funds rates and $^{Roll}_{t}$ denotes cyclical volatility of real inside money balances. ***, **, and * denote signi cance at the 1, 5, and 10 percent levels, respectively.

Table B.4: Unit Root Tests: Robustness Check (pre-2008)

		Phillips-Perron test		ADF test
		Z( )	Z(t)	Z(t) w/ lag 1
ffr		9:476	2:258	2:868
		0:768	0:660	0:877
$_t^{Roll}$	(CPI)	2:966	1:738	1:770
$_t^{Roll}$	(Core CPI)	2:860	1:641	1:495
$_t^{Roll}$	(PCE)	2:662	1:515	1:627
$_t^{Roll}$	(Core PCE)	2:412	1:371	1:400
ffr		25:378	4:773	5:833
		28:208	4:594	3:658
$_t^{Roll}$	(CPI)	25:627	4:281	3:813
$_t^{Roll}$	(Core CPI)	25:836	4:329	3:764
$_t^{Roll}$	(PCE)	24:420	4:101	3:594
$_t^{Roll}$	(Core PCE)	23:848	4:034	3:464

Note: ffr denotes federal funds rates, denotes required reserve ratio, and $_t^{Roll}$ denotes cyclical volatility of real inside money balances. ***, **, and * denote signi cance at the 1, 5, and 10 percent levels, respectively.

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