MODEL FOR THE THERMOREGULATION

OF SEA LIONS

A Thesis

Presented to

the Faculty of the Graduate School

University of Missouri

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Chemical Engineering

----

by

V. Natarajan

August 1974

ACKNOWLEDGEMENTS

The author is indebted to Dr. Richard Luecke for his guidance and patience during the course of this work.

In addition, thanks is given to Dr. Frank South and his staff for their valuable advice.

During the preparation of the manuscript, the author was fortunate to have the assistance of his wife. Without her understanding and encouragement this work would not have been possible.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Mathematical modeling in biological systems is a relatively new and challenging field for chemical engineers. Application of heat and mass transfer and reaction kinetics allows development of these models for biological systems in accordance with the basic physical laws of nature.

Most physiological control systems are very complex and are difficult to describe on an intuitive approach. Prediction or interpretation of experimental results may involve separation of many interacting factors. Although thermoregulation has this characteristic in common with most physiological control systems, it has the distinction that parts of the overall system obey relatively simple physical laws which can be combined in quantitative form.

Marine mammals face particularly severe thermal challenges from their environment. An animal diving in the ocean not only encounters the latitudinal and seasonal temperature changes in oceanic waters, but also goes through a vertical stratification of temperatures on a very short term basis. While on land they encounter great seasonal and diurnal changes in temperature. So the study of the thermoregulatory system during both surface activity and diving could be of considerable importance to the understanding of thermoregulation of mammals in general.

Development of mathematical models enables us to interpret experimental data, design experiments and predict the outcome in situations where it would be difficult to make experimental measurements.

In the present study, a distributed parameter model is developed for the thermal behavior of the California sea lion (Zalophus californianus), while it is swimming, diving or hauled out on land. Many of the physiological parameters, such as local metabolic heat generation rates, local blood flow rates etc., are not available. Approximations for many of these system characteristics are taken from analogous situations in other mammals. The major source of these approximations is human models which have been used to study the thermal behavior of the miners in the hot humid environments (Wyndham and Atkins, 1968) of South African mines and for the development of space suits (Chambers et al., 1973).

A model for the thermal behavior of humans in the unsteady state was developed by Wissler (1964). The human body was divided into fifteen cylindrical sections. In each section, the large arteries and veins were represented by arterial and venous pools which were distributed radially throughout the section.

A study of temperature regulation in humans was done by Stolwijk and Hardy (1966). This model was largely a lumped parameter model. The body was represented by just

three cylindrical sections: the head, the trunk and the extremities. Each cylinder was divided into three homogeneous concentric volumes. All three skin layers exchange heat with the environment by conduction, convection, radiation and evaporation. The muscle layer exchanges heat by conduction with the skin and the core. The core likewise exchanges heat by conduction with the muscle.

It was assumed in this model that the central blood flows so fast through the arteries and veins that it loses very little heat directly to the external environment. Much greater amounts of heat are exchanged with various compartments of the core, muscle and skin by convective heat transfer in the small vessels. For this reason, the blood reservoir was separated from the other compartments.

The preceding model was refined later by Stolwijk (1972). The body was divided into six, instead of three segments: the head, the trunk, and left and the right arms and the left and the right legs. The head was considered as spherical in shape while the other body sections were taken as cylindrical. Each section had four layers: the core, the muscle, the fat and the skin. In addition, a central blood compartment linked the six sections together via the appropriate blood flows. There was a total of twenty-five compartments.

Initial conditions were obtained by simulating exposure to a 31°C environment with a 10 cm/sec air movement without any thermoregulation. The values computed were in reasonable

agreement with their experimental counterparts.

Recently efforts have been made in developing mathematical models for animals other than human beings.

A mathematical model, describing the dynamic generation and transfer of heat in marmots was used to analyze thermoregulatory behavior during. hibernation (Luecke et al., 1970). In this model, a spherical shape was selected to approximate the shape of the flattened ellipsoid of the hibernating marmot as it tucks its head under its tail. The sphere was divided into three concentric layers, the simultaneous partial differential equations, describing the heat transfer were solved numerically. By a small adjustment in the overall heat transfer coefficient to the environment the cooling curve measured as the animal entered into hibernation was closely reproduced.

This model was later used to correlate heart rate, EMG activity and metabolic heat generation by the marmot during hibernation (Luecke and South, 1971).

Most of the general biological information on sea lions has appeared in several review articles (Anderson, 1969; Ridgeway, 1972; Pierce, 1970; and Irving, et al., 1935).

In the present study, the model takes the form of seventeen non-linear, coupled, partial differential equations. Since they cannot be solved analytically finite difference techniques are employed for a numerical solution. The numerical results are compared with those from experiments and some of the estimated parameters and constants in the

control system and the model is adjusted so that computed

results agree closely with the experimental data.

Since only the experimental thermoregulatory data

for the animal in air are available at this time, the model

has been validated only for this case.

CHAPTER II

MATHEMATICAL MODEL OF HEAT EXCHANGE

A. Passive System:

    1. General Discussion:

        For computationally practical models it is necessary
to mathematically lump the body into a relatively small
number of sections. In this model, the animal is represented
by four main sections: the head, the trunk, the fore-flippers
and the hind-flippers.

        The sagittal view of a California sea lion was given
by Ridgeway (1972) which is reproduced here as Figure 1. The
head and the trunk of the animal are shaped somewhat like a
flat ellipsoid.

        The general anatomy of the left posterior limb and
the left anterior limb of the animal were given by Howell
(1929) which are produced here as Figures 2 and 3. The flip-
pers extend so that the flattened parts divide naturally into
triangles, connected by short cylindrical limbs to the body.

        An ellipsoid model for the head, and the trunk and
the flat triangular models for the flippers were not used
because the mathematical description would be too complicated.
A cylindrical model for each section was selected to simplify
the development and solution of the mathematical description.

        Each cylindrical section is divided into four concentric
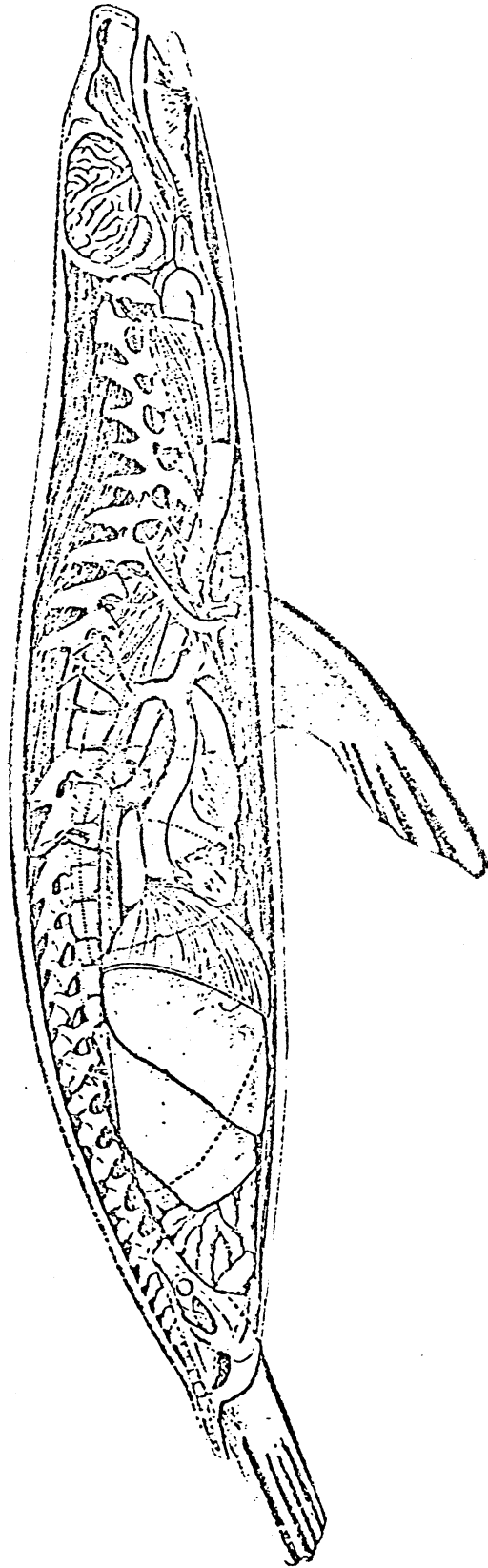
Figure 1. Sagittal View of California Sea Lion.
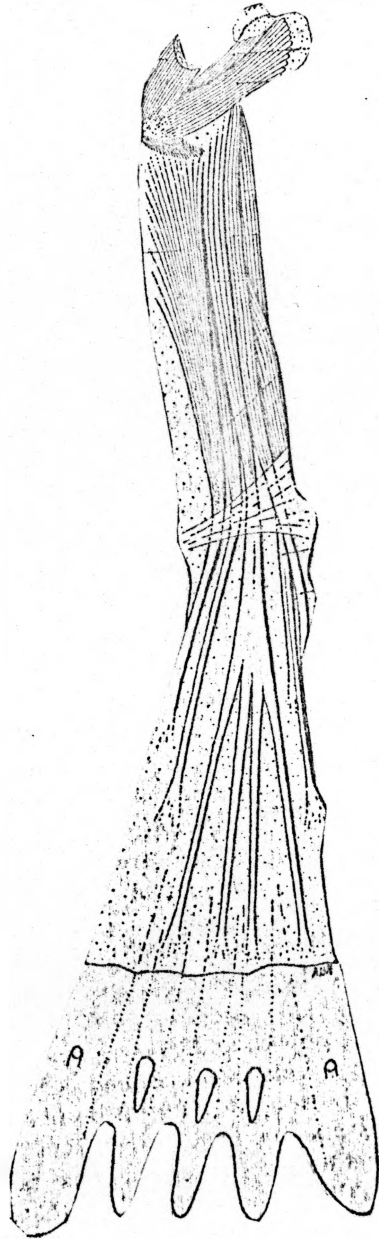
Figure 2. Left Posterior Limb of California Sea Lion.
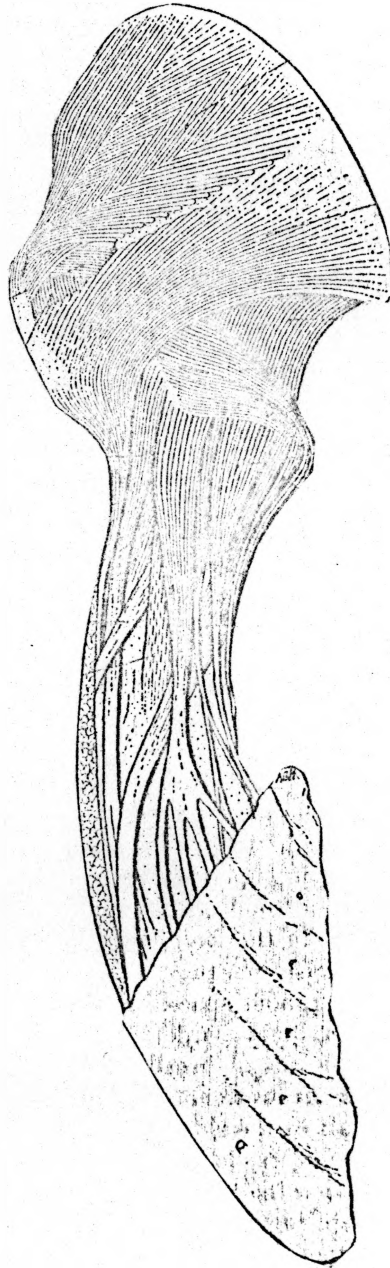
Figure 3. Left Anterior Limb of California Sea Lion.
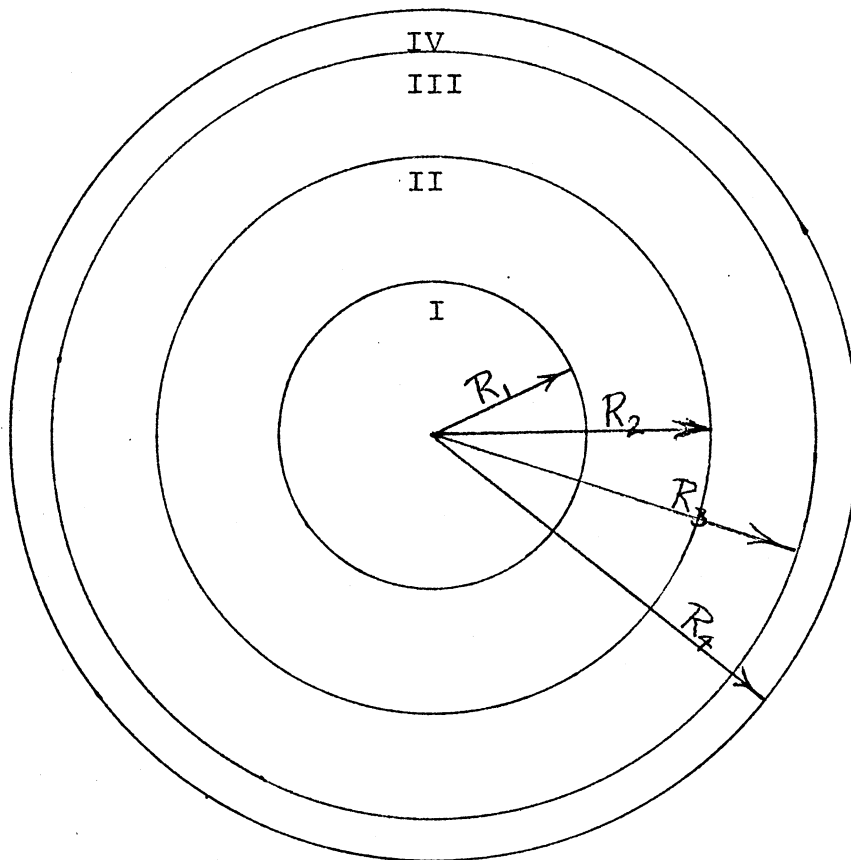
Figure 4. Cross sectional view of a section.
Layer I, central core; Layer II,
muscle; Layer III, blubber; Layer
IV, skin; R_1, R_2, R_3, R_4-radii of
core, muscle, blubber and skin
respectively.

contains visceral organs, layer II is the muscle, the layer III is the blubber, and layer IV is the skin.

Physical measurements were taken from two sea lions used as experimental subjects. This data is shown in table 1.

Relationships between surface area of the animal (TSA) in cm^2 and the weight of the animal in kg., were computed using the equation given by Irving et al., 1935:

$$TSA = 800 \ TW^{2/3} \qquad - 1$$

The calculated surface area is 7979.2 cm^2 compared with the measured values given in table 1.

The weight fractions (FW) of the different sections are computed on the basis of the measurements taken on the radii and lengths of the sections.

Since the physiological data on the weight distributions within different sections of the animal are not available, values are assumed or adopted for the most part from the human model (Stolwijk, 1972). Sea lions have more blubber than fat in humans. It is assumed that 20% of the animal weight is blubber, instead of 15% of the human body and adjustments are made.

The skin in each section is assumed to take up the outer 2mm. so the mass of the skin on each cylinder is found with only a slight error, by multiplying this 2mm thickness by the surface area of each section. The masses and physical dimensions of the various compartments are given in Table 2.

The central blood compartment (CB) has no clearly defined anatomical counterpart. The areas of heat transfer

Date: 3/26/74

TABLE 1

PHYSICAL MEASUREMENTS OF SEA LION

NAME	Joy		Noel	
SEX	Female		Female	
WEIGHT	31.5 kg.		31.6 kg.	
LENGTH:				
Trunk	94.5 cm		95.5 cm	
Head	22.0 cm		22.0 cm	
Fore-flippers	30.0 cm		30.0 cm	
Hind-flippers	30.0 cm		30.0 cm	
CIRCUMFERENCE:				
Trunk (ave.)	50.0 cm		49.0 cm	
Head (ave.)	20.0 cm		20.0 cm	
AVERAGE WIDTH:				
Fore-flippers	15.0 cm		15.5 cm	
Hind-flippers	8.0 cm		8.0 cm	
SURFACE AREA:				
Trunk	4725.0 cm^2	59.62%	4679.5 cm^2	58.94%
Head	440.0 cm^2	5.55%	440.0 cm^2	5.54%
Fore-flippers	1800.0 cm^2	22.72%	1860.0 cm^2	23.43%
Hind-flippers	960.0 cm^2	12.11%	960.0 cm^2	12.09%
TOTAL	7925.0 cm^2	100.00%	7939.5 cm^2	100.00%

TABLE 2

ANATOMICAL DIMENSIONS OF THE ANIMAL OF BODY WEIGHT 31.413 kg.

		Weight in kg.	Radius–Thickness in cm.	Number of Differential Layers	Thickness of differential Layers in cm.
D		2.033	5.560	80	
	Core	1.479	4.695	25	0.1878
	Muscle	0.264	0.368	25	0.0147
	Blubber	0.209	0.297	25	0.0118
	Skin	0.081	0.200	5	0.0400
NK		24.430	8.742	80	
	Core	7.900	5.293	25	0.2117
	Muscle	9.580	2.339	25	0.0936
	Blubber	5.320	1.110	25	0.0444
	Skin	1.630	0.200	5	0.0400
E-FLIPPERS		3.300	5.827	80	
	Core	1.05	3.424	25	0.1370
	Muscle	1.32	1.566	25	0.0626
	Blubber	0.62	0.637	25	0.0255
	Skin	0.31	0.200	5	0.0400
D-FLIPPERS		1.650	4.179	80	
	Core	0.525	2.421	25	0.0969
	Muscle	0.660	1.108	25	0.0443
	Blubber	0.310	0.450	25	0.0180
	Skin	0.155	0.200	5	0.0200
OTAL		31.413 31.413			

for blood in capillaries are much larger than that in the heart, arteries and veins. It is assumed, therefore, that the blood exchanges heat with the compartments by convection only in the capillary beds. For this reason, the heart, arteries and veins forming the blood pool are considered as a separate compartment.

Estimation of the size and location of the central blood compartment becomes very difficult since little anatomical data is available. A reasonable assumption would be that all blood in the heart and in vessels with a diameter greater than 1 cm belongs to the "central pool". Further difficulty was experienced in finding information for the total weight and distribution of the central pool in the sea lion. For the human model, Stolwijk and Hardy (1966), assumes that the central pool has a volume of roughly 1,350 ml: 300 ml for the heart, 375 ml for the venae cavae and 100 ml each in carotid arteries and jugular veins, bronchial arteries and veins. In this model, the central pool is assumed to have the volume, ratioed on the basis of weight of the human and the sea lion models. It is also assumed that it has the same distribution as the human model.

In order to create this pool, appropriate masses have to be subtracted from the core compartments of the trunk, head, fore-flippers and hind-flippers.

2. Heat Transfer Inside The Animal:

The heat transfer inside this model is assumed to

convection, via the blood flow and that part which occurs through direct tissue conduction.

The driving force for the convective heat transfer is the difference in temperature between the tissue and the blood. Because of the high surface area in tissue capillaries, complete heat transfer between the blood and the tissue is assumed. The blood leaves the tissue at the tissue temperature.

Conductive heat transfer occurs within each layer and is assumed to be occuring only in the radial direction. The surfaces on the ends of each section is assumed to be perfectly insulated and having no heat exchange. Axial temperature gradients in each section are neglected and all properties of each layer are assumed to be homogeneous with linear temperature profiles.

3. Heat Transfer To Environment:

The sea lion has a layer of short hair which does not seem to contribute much insulating value. The short hair on the body is completely wetted and lies so flat that its thickness is probably less than 1 mm. In this model, the effect of the hair on heat transfer has been neglected.

A representation of the entire model showing all of the channels of heat transfer is shown in Figure 5.

In general, heat transfer with the environment can occur by conduction, convection, radiation and evaporation. The rate of heat transfer for each mode with the environment depends on the physical properties of the surrounding medium. The type of heat transfer mechanism which is going to
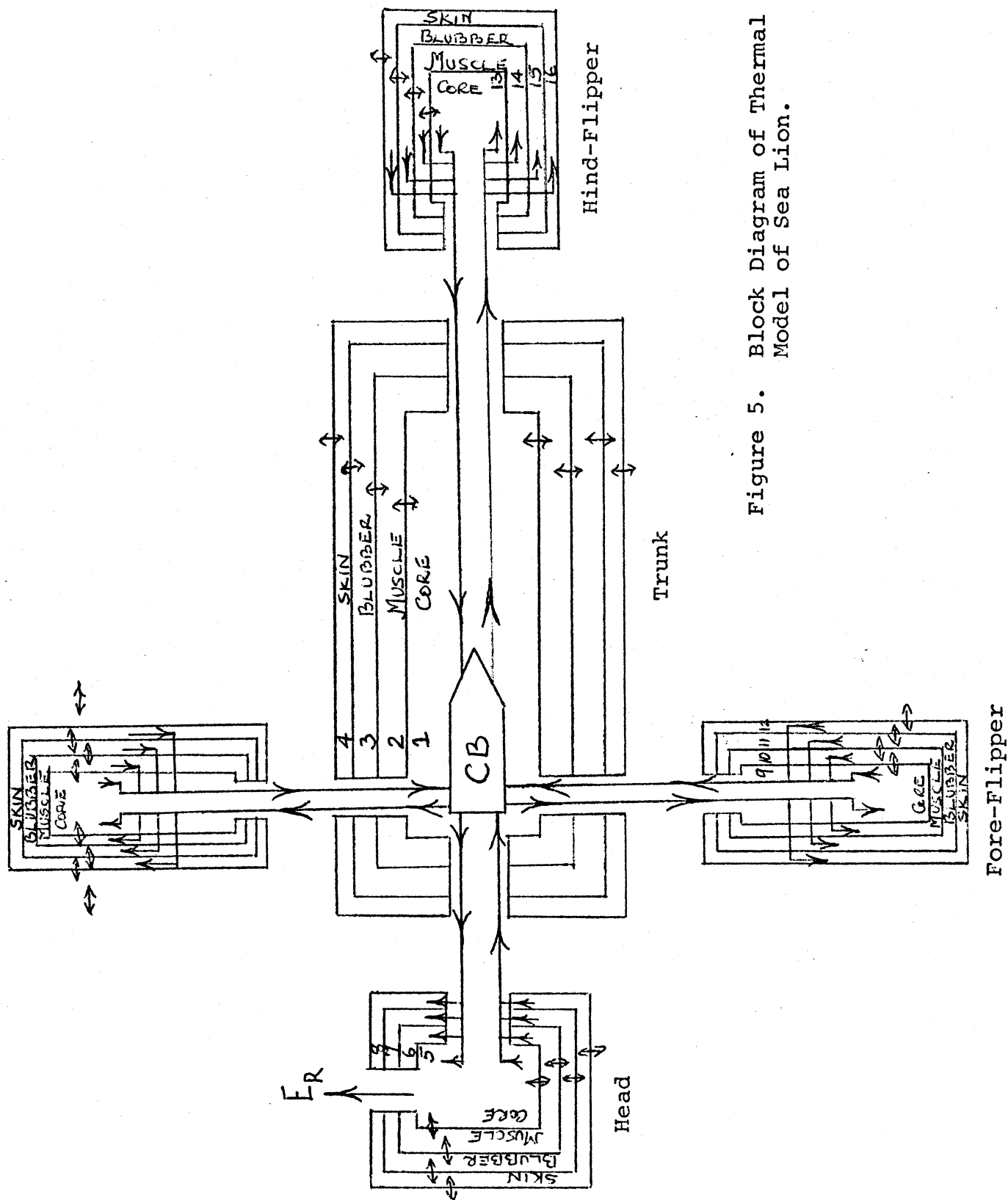
Figure 5. Block Diagram of Thermal Model of Sea Lion.

contribute to the heat transfer to the environment varies with sections of the body and the surrounding medium.

When the sea lion is on land, it will lose heat to environment by conduction, convection, radiation and evaporative cooling via respiration and insensible moisture losses.

When the animal is swimming in water, since most of its body is under water, it loses very little heat to the environment by radiation and skin evaporation. But it still exchanges heat with the environment by respiration.

When it is diving essentially all of the heat loss to the environment is by convection.

a.) Evaporative Cooling:

The actual sites and mechanisms of evaporative cooling involve active sweat secretion from the flippers (Matsura and Whittow, 1972) as well as from excessive salivation which may soak the fur of the entire ventrum of the neck. Since urination was observed, (South et al., 1973), to have occurred rather frequently during exposure to the higher temperatures, wetting of the circumanal area may also be a source of evaporative cooling. The method of evaluating the heat lost by evaporative cooling will be discussed in detail later.

b.) Radiation and Convection:

The radiation and convection heat transfer coefficients for a particular set of conditions can be estimated using empirical correlations.

The coefficient for radiant heat transfer (H_R) is

most easily determined. Heat lost by radiation per unit area
is given by Wissler (1961) as,

$$Q_{RAD} = 4.4846 \times 10^{-12} \ (T_s^4 - T_a^4) \ \frac{Kcal}{cm^2 hr} \qquad - 2$$

where,

T_s = temperature of the skin in $^\circ K$.

T_a = ambient temperature in $^\circ K$.

Defining the heat transfer coefficient for radiation (H_R) to
be $Q_{RAD}/(T_s - T_a)$, one obtains:

$$H_R = 4.4846 \times 10^{-12} \ (T_s^3 + T_s^2 T_a + T_s T_a^2 + T_a^3) \ \frac{Kcals}{cm^2 hr \ ^\circ C} \qquad - 3$$

Over the range of temperatures from $10^\circ C$ to $40^\circ C$ the
value for H_R increases from 0.000522 to 0.000599 Kcals/cm^{2}hr$^\circ$C.
Thus the heat transfer coefficient for radiation is nearly
constant and can be readily determined.

The convective heat transfer coefficient (H_c) was
taken to be that for forced convection across a flat plate as
given by Knudsen and Katz (1958).

$$H_c = 0.324 \ \frac{K}{L} \ \left(\frac{LU}{\gamma}\right)^{1/2} Pr^{1/3} \qquad - 4$$

where,

K = thermal conductivity of the fluid. (Kcals/cm $^\circ$c hr)

γ = kinematic viscosity of the fluid. (cm^2/hr)

Pr = Prandtl number of the fluid.

U = velocity of the fluid. (cm/hr)

L = Length of the animal. (cm) ·

In this model, the effective area while the animal is swimming and diving is taken to be the same as the total surface area.

The effective area for radiative and convective heat transfers when the animal is on land can not be easily determined because of the complex geometry of the sea lion. The area of heat transfer varies markedly as the posture of the animal changes. As the body temperature rises, the animal changes its posture to expose the maximum possible surface area. It is assumed that 90% of the total surface area of this model represents maximal exposure. Posture change is assumed to occur when the ambient air temperature is above $20^{\circ}C$. At ambient air temperature of $20^{\circ}C$ the animal is assumed to expose about 80% of its body. As the ambient air temperature falls, the animal folds its fore-flippers inside the hind-flippers to conserve heat. An exposure ratio of 65% is assumed for the lower temperatures.

c.) Respiration:

The expired air is assumed to be saturated with moisture at the core temperature. Thus the total rate of heat loss through the respiratory tract depends only on the respiration rate, the temperature of the core and on the temperature and humidity of the ambient air.

The respiration rate in turn is dependent on the total heat production in the body. Every kilocalorie of

heat liberated requires approximately 0.207 liters of oxygen. The inspired air has 20.9% volume of oxygen and the expired air has 15.9% volume of oxygen (South 1974). The respiratory volume (RV) in liter/hr can be calculated from the following equation:

$$RV = 4.145 \times TQ \qquad\qquad - 5$$

where,

TQ = Total heat production (Kcals/hr).

Some of the heat lost through the respiratory tract occurs in the core of the heat and some in the trunk but there is no available data on the actual distribution. It is assumed that 10% of the heat loss through the respiratory tract came from the head core and 90% from the trunk core. Further it is assumed that the expired air is saturated with water vapor at a mean temperature T_m:

$$T_m = 0.1 \, T_{HC} + 0.9 \, T_{TC} \qquad\qquad - 6$$

where the head core temperature (T_{HC}) and trunk core temperature (T_{TC}) are measured in $^{\circ}C$.

The rate of heat loss through the respiratory tract (Q_R) can be calculated from the following equation (Wissler 1961), knowing the ambient partial pressure of water (pc) in mm of Hg and the relative humidity of air (H).

$$Q_R = 15.12 \times 10^{-14} \; RV \; \{T_m - (T_A - 0.37 \, (1-H) \, pc)\}$$
$$\text{Kcals/hr} \qquad - 7$$

4. Basal Blood Flow:

In addition to its other physiological functions, blood performs a vital role as a heat exchange medium. For this reason, reliable data on the amount of blood flow and its distribution to different parts of the body would be very useful. However, unfortunately very little information for these values is available.

For a standard man of weight 70.94 kg., Stolwijk and Hardy (1966) report a basal blood flow of 5.407 liters/min. In the model for the sea lion, it is assumed that the total basal blood flow is the same as that of a human of the same weight.

Blood flow distribution is also modeled from that of human beings. Average values of the blood flow distribution in humans, given by Guyton (1966), are shown in Table 3. In this model, it is assumed that of the total blood flow 73.5% goes to core, 20% to the muscle, 3.5% to the blubber and 3% to the skin (South 1974).

As the function of the blood and that of the muscle, blubber and skin in different sections are the same, it is assumed that the basal blood flow to the blubber and skin are distributed to different sections according to their respective weights. The distribution of basal blood flow to the core of different sections is assumed to be the same as that in humans. The blood flow to the bones is distributed among different sections using the following assumptions:

TABLE 3

BLOOD FLOW TO DIFFERENT ORGANS UNDER BASAL
CONDITIONS FOR HUMANS

	Percent	ml/min
Brain	14	700
Heart	3	150
Bronchial	3	150
Kidneys	22	1100
Liver	27	1350
Portal	(21)	(1050)
Arterial	(6)	(300)
Muscle (inactive state)	15	750
Bone	5	250
Skin (cool weather)	6	300
Thyroid Gland	1	50
Adrenal Glands	0.5	25
Other Tissues	3.5	175
Total	100.0	5000.00

i) The fore-flippers of the animal are the same as the arms of the humans.

ii) The hind-flippers are just 1/3 of the fore-flippers.

iii) The trunk of the animal includes the trunk and legs of the human.

iv) The head of the animal is the same as that of the human.

So it is found to be that of the total blood flow to the core, 19.87% goes to the head, 78.36% goes to the trunk, 1,33% goes to the fore-flippers and 4.4% goes to the hind-flippers.

5. Metabolism:

Basal metabolism and exercise metabolism contribute to the total metabolic heat generation. These are discussed separately below.

a.) Basal Metabolism:

Kleiber (1947) developed the following equation, which relates the basal metabolism (QB) to the weight of the mammals "from mouse to elephant".

$$\text{Kilogram calories per day} = 70 \times \text{weight}^{3/4} \qquad - 8$$

But Irving and Hart (1957) reported that when the basal metabolic rates of harbor seals were plotted against their weights, the basal metabolism found in three separate studies was about the double the rate for the mammals of their size, as defined by Kleiber's formula. They also reported that the

basal metabolism of the two harbor seals was about 1.5-2 times
the "standard" values of their size. The experimental data
on basal metabolism for the sea lion reported by South et al.,
(1973) was about 2.06 Kcal/kg/hr. To match this, a correction
1.8 was made to the Kleibers formula so that it is applicable
for this model.

There are no reported data available for sea lions for
the distribution of basal metabolism to the various organs,
muscles, etc. In this model, the distributions of weights
and basal blood flow were assumed to be the same as that
of humans where applicable. Likewise it is assumed that
the distribution of basal metabolism is equivalent to that
of the human model (Stolwijk and Hardy, 1966). The detailed
distribution of basal metabolism along with the distribution
of basal flow is given in Table 4.

b.) Exercise Metabolism

During exercise, heat is generated in the
muscle layers of the trunk and fore-flippers. When the
animal is swimming or diving, it must develop enough work to
overcome the resistance due to acceleration and fluid
friction.

The amount of resistance offered by the fluid around
the animal's body can be evaluated if the pattern of flow of
the fluid around the animal was known. Lang (1966b) reports
that whales maintain a laminar flow over a high proportion of
the body while in water.

If the animal is swimming below the surface in deep

TABLE 4

DISTRIBUTION OF BASAL BLOOD FLOW AND METABOLISM

Compartment	Basal Metabolism in Kcal/hr	Basal Blood Flow in lit/min
Core	53.020	1.763
Muscle	12.049	0.481
Blubber	3.838	0.084
Skin	0.897	0.073
Total	69.804	2.401
Head Core	11.191	0.350
Muscle	0.268	0.011
Blubber	0.128	0.003
Skin	0.070	0.004
Head Total	11.657	0.368
Trunk Core	40.677	1.382
Muscle	9.884	0.389
Blubber	3.150	0.069
Skin	0.597	0.043
Trunk Total	54.308	1.883
Fore-Flippers Core	0.775	0.023
Muscle	1.276	0.054
Blubber	0.377	0.008
Skin	0.155	0.017
Fore-Flippers Total	2.583	0.102
Hind-Flippers Core	0.377	0.008
Muscle	0.621	0.027
Blubber	0.183	0.004
Skin	0.175	0.009
Hind Flippers Total	1.256	0.048
Animal Total	69.804	2.401

water (when the submersion depth of the body axis is equal to three times the body of revolution), the flow around the animal is laminar, (Anderson 1969). If the animal swims closer to the surface, it creates waves on the surface of the water. The consequent change in flow pattern increases the resistance. This does not quite create a turbulent motion of the fluid around the animal. It is said to be in the critical region, which is a transition between laminar to turbulent flow.

The drag coefficients were taken from experimental tests reported by Hoerner (1957) on rigid, smooth, streamlined bodies. The empirical equations were developed as a function of the Reynolds number (Re) to fit these experimental results. They are given (Rose 1956) as follows:

$$C_f = \frac{1.33}{Re^{1/2}} \qquad\qquad - 9$$

critical region,

$$C_f = \frac{0.074}{Re^{1/5}} \qquad\qquad - 10$$

Turbulent region,

$$\frac{1}{\sqrt{C_f}} = 4.15 \log Re\ C_f \qquad\qquad - 11$$

where C_f is the coefficient of frictional drag,

R_e is the Reynolds number which determines the flow pattern of the fluids.

$$R_e = \frac{X_L V}{\gamma} \qquad\qquad - 12$$

where, V is the speed at which the animal is swimming.

X_L is its length

and γ is the kinematic viscosity of seawater.

The power required to overcome the frictional drag (P_F) can be calculated using Lang's equation (1966)

$$P_F = \oint \frac{C_f \, \rho \, TSA \, v^3}{2\eta} \qquad - 13$$

where η is the efficiency of conversion of muscle energy to the power to overcome the drag.

Since no reported data is available it is assumed that the muscles of the animal convert the energy to work at 50% efficiency and the flippers convert this work to the power required to overcome the drag at 25% efficiency.

There is also resistance offered by the inertia due to acceleration. Although this is not taken into consideration in this model, provision has been made in the program for inclusion in future work.

b.) The Controlling System:

The feed back control parameters represent a part of the model that is particularly difficult to evaluate. The objective of this section is to formulate a controller concept which allows good simulation of observed phenomena. At the same time, it should be as flexible as possible so that the formulation can be modified without making extensive changes in the other parts of the model.

The controller is divided into two parts: sensor mechanisms and effector action. The sensor mechanisms are

not accessible to observation and study, but it is generally accepted that the temperature sensitive structures in the hypothalamus play an important role in thermoregulation (Nakayama et al., 1969).

Similarly, there is general agreement that thermal receptors in the skin play an important role (Hardy and Stolwijk, 1966). In addition there are indications that temperature sensitive structures exist in the voluntary muscle (Robinson et al., 1965).

The effector actions are well known in a qualitative sense: variations in skin blood flow and muscle blood flow and sweating with consequent evaporative loss.

The regulator which will be discussed below is by no means to be taken as final. It is anticipated that this hypothetical regulator will be useful in suggesting additional experimental work to challenge its validity.

The signals received from the sensors vary linearly with the local temperature (Stolwijk and Hardy, 1966). These sensors are assumed to be in the head core, in an even distribution over the skin and the muscles. Each of these sensor systems is taken to have zero output at a local temperature corresponding to a set point for the head core, $(\hat{T}_{IIC})$, mean skin $(\hat{T}_s)$ and muscle $(\hat{T}_m)$ temperatures. The head core temperature determines whether the major effector response is that for increasing heat loss by sweating and vasodilation or for increasing heat storage via vasoconstriction.

1) Evaporative Cooling:

Evaporative cooling depends on the amount
of surface area wetted by perspiration, urination and
salivation. Since the sweat glands are found only in the
flippers, it is easy to evaluate the surface area wetted by
perspiration. But the determination of surface area in the
trunk and the head where the evaporative cooling occurs due
to salivation and urination poses a great difficulty. Since
the quantitative information is unavailable it is assumed
that the animal wets about 10% of its surface area of the
trunk by urination and salivation. Therefore, the total
evaporative cooling is distributed among different sections
according to the surface area available for the evaporative
cooling.

The total evaporative cooling control (Ev) can be
written as,

$$Ev = Ev_1 \; (\hat{T}_{HC} - T_{HC}) \; (\hat{T}_S - T_S) \quad Kcals/hr \quad - 14$$

where,

Ev_1 = Gain for evaporative cooling

T_{HC} = Average temperature of the head core (^{O}C)

T_S = Weighted average temperature of the skin (^{O}C)

It is assumed that the positive deviation facilitates heat
loss and negative deviation facilitates heat storage, however
if the product is negative it is rejected. Since the evap-
orative heating is not possible, the restriction $Ev \geq o$ is
imposed.

The set temperatures for the head core ($\hat{T}_{HC}$) and the

skin ($\hat{T}_s$) were taken as those values found with no control with 20^oC ambient air. The gain Ev_1 was taken as 10.0.

2) Vasometer Action:

Blood performs two important functions. In addition to the function of serving as a transport medium for oxygen and nutrients, it also transports heat to the surface to prevent overheating of any sections. Control of blood flow to different sections enables the animal to control its overall body temperature.

The metabolic heat production in the core of each section is assumed to remain always at the basal level. This is not strictly true for the trunk where the heat produced by the heart, especially, is variable. This effect on heat production, however, is small. The heat production in the brain is quite constant. In the core of the flippers the heat production is low under all circumstances. So the oxygen and nutrient demand by the core of all the sections remains relatively constant. Thus the blood flow to the core of all the sections is taken to remain constant at the basal level.

The total heat production in the muscles can change over a wide range due to exercise. This variation in heat production requires different amounts of oxygen which, in turn, requires variation in blood flow to muscles.

The variation in blood flow to muscles can be evaluated from the oxygen demand. Every kilocalorie of energy produced

requires 0.207 liters of oxygen and every liter of blood
flow makes 0.20 liters of oxygen potentially available for the
muscle tissue (Guyton, 1966). Thus at least one liter of
blood is circulated in the muscles for every Kcal of energy
liberated. The minimal blood flow to the muscle (BFM) is
evaluated as

$$BFM = BF(2) + Exer \times 1000 \qquad cm^3/hr \qquad - 15$$

The blubber plays an important role as a heat insulator
It has a low thermal conductivity compared to other tissues.
The blood flow to the blubber varies the effective thermal
conductivity of the blubber. One of the principle ways the
animal preserves heat from colder environments is by reducing
its blood flow to the blubber from the basal level (BF(3)).
The blood flow to the blubber BFB is evaluated as

$$BFB - BF(3) + CoB \; (\hat{T}_{HC} - T_{HC}) \qquad - 16$$

The gain for blood flow to the blubber CoB is to the
value of 1000.0 to fit the experimental data. Since it is
not possible to have a negative blood flow to the blubber, a
restriction BFB$\geq$o is introduced.

Skin blood flow is highly dependant on the thermo-
regulatory controller. The basal blood flow to skin BF (4)
at thermal neutrality can be diminished by vasoconstriction
or increased by vasodilatation.

The animal is able to withstand colder environment,
but is not able to withstand the hotter environment. This
is an indication that the animal is able to decrease the

blood flow to the skin more efficiently to preserve heat than increase the blood flow to the skin to lose more heat. So in the model, vasodilatation and vasoconstriction are treated separately.

The quantitative relationships for the blood flow to skin (BFS) can be expressed separately as

$$BFS = BF(4) + VASOD (T_{HC} - \hat{T}_{HC}) \quad cm^3/hr \quad - 17$$

$$BFS = BF(4) + VASOC (T_{HC} - \hat{T}_{HC}) \quad cm^3/hr \quad - 18$$

with the restriction BFS>o.

The correction factors for the vasodilatation (VASOD) and vasoconstriction (VASOC) are adjusted to experimental data.

3) Blood Flow During Diving:

When the animal is diving, bradycardia - a reduction in **heart rate** - is developed. The bradycardia is not an isolated vascular activity but is associated with decreased cardic output and a generalized peripheral arterial vasoconstriction for maintenance of blood pressure. There is reduced blood flow in viscera, skeletal muscle and skin but cerebral and coronary flow is apparently less affected. (Anderson, 1969).

Elsner (1969) measured cardiac output by directly recording the blood flow in the pulmonary artery of zalophus californianus. Slowing of heart rate was clearly visible

while the stroke volume remained unchanged during the immersion. The heart rate fell from about 140 in air to 30-40 in water.

The above clearly demonstrates that the blood circulation during diving is reduced. A problem arises in evaluating the amount of reduction of blood flow to different parts of the body.

Flow practically stops in the muscles during diving (Scholander, 1940). In the mesenterial blood vessels flow is much reduced and it is often arrested in the flippers (Irving et al., 1941). The kidneys are effectively isolated from blood circulation during prolonged submersion (Murdough et al., 1961) or reduced. (Stone et al., 1973.)

In this model, it is assumed that during the diving the eyes, ears, cranial nerves and the base of the brain are receiving blood flow at the basal level. The organs in the trunk core such as liver etc., receive blood flow at half the basal blood flow. This amounts to about 37.5% of the total blood flow at steady state.

CHAPTER III

A. DEVELOPMENT OF EQUATIONS DESCRIBING THE MODEL:

The block diagram in Figure 6 summarizes heat flow
relationships between various compartments of the model.
These relationships can be expressed in the form of seventeen
simultaneous partial differential equations. For clarity
these equations will be presented in open loop form, that is,
describing the system without control. When the relationships
for the effector feedback actions are included, the loop
is effectively closed.

The heat transfer equations describing the heat
production and heat transfers in the four cylindrical sections
are developed below:

1. Head

    a. Core

    The head core exchanges heat with the head
muscles through conduction and with the blood compartment
via convection. Heat input via generation by metabolism and
heat output through respiratory tract also occurs in the
head core. The combined mass and energy balance lead to the
equation:

$$\rho_1 C_{p_1} \left(\frac{\partial T_1}{\partial t}(t,r) \right) = \frac{1}{r} \frac{\partial}{\partial r}\left(r K_1 \frac{\partial T_1}{\partial r} \right) + \frac{Q_1}{V_1} - \frac{E_{V_{R_1}}}{V_1} + \frac{BF_1}{V_1} \lambda_\psi (T_{BL} - T_1) \qquad -19$$

$$0 \doteq R_1 \leq r \leq R_2$$
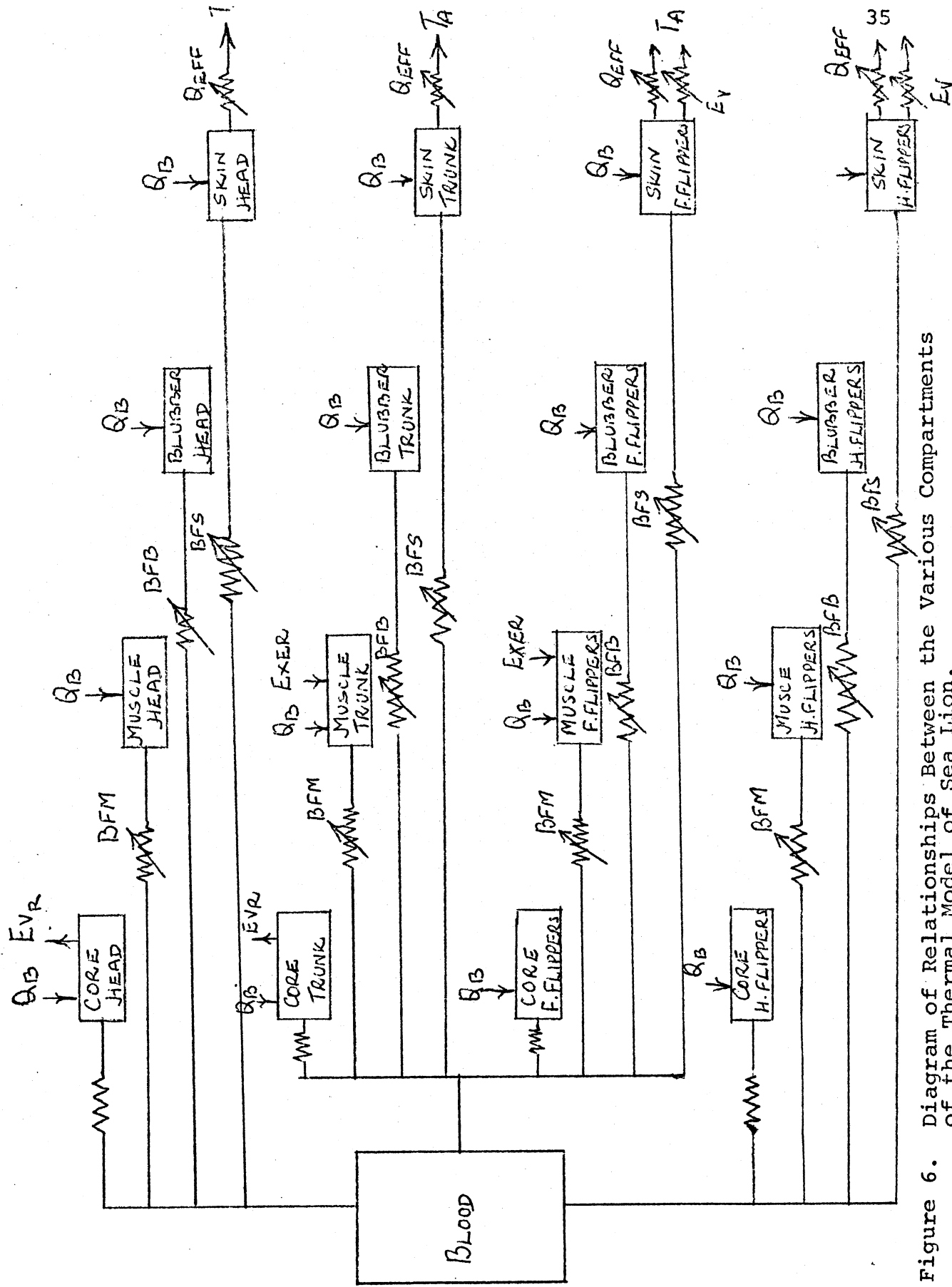
Figure 6. Diagram of Relationships Between the Various Compartments of the Thermal Model of Sea Lion.

where,

ρ_i = Density of ith layer (kg/cm^3);

Cp_i = Specific heat of ith layer (Kcal/kg-$^\circ$C)

T_i = Temperature of the ith layer ($^\circ$C)

t = time (hr)

K_i = Thermal conductivity of the ith layer

  (Kcal · cm/cm^2·hr·$^\circ$C)

γ = Radius (cm)

Q_i = Basal metabolism of the ith layer (Kcal/hr)

V_i = Volume of the ith layer (cm^3)

Evr_i = Respiratory water loss of ith layer (Kcals/hr)

BF_i = Blood flow to ith layer (cm^3/hr)

λ = Dimensionless function accounting for the effect

  of counter current heat exchange between veins

  and arteries (assumed to be unity).

Ψ = Product of density and specific heat of blood

  (Kcal/cm^3 $^\circ$C)

T_{BL} = Temperature of the central blood compartment

  ($^\circ$C)

b. Muscle

The muscle has a conductive heat path to the core and the blubber, a convective heat flow path to the central blood compartment and heat input from the metabolism of the muscle. The equation describing this layer is

$$\rho_2 \, C_{p_2} \left[\frac{\partial T_2}{\partial t} (t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left(r \, K_2 \frac{\partial T_2}{\partial r} \right) + \frac{Q_2}{V_2} + \frac{BF_2}{V_2} \lambda \Psi \left(T_{BL} - T_2 \right) \qquad - 20$$

$$R_2 \leq r \leq R_3$$

c. Blubber

There is a conductive path to the muscle and the skin, a convective path to the central blood compartment and the heat input from the metabolism of the blubber. The total expression for the blubber is:

$$\rho_3 C_{P_3} \left[\left(\frac{\partial T_3}{\partial t}\right)(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r}\left(r k_3 \frac{\partial T_3}{\partial r}\right) + \frac{Q_3}{V_3} + \frac{BF_3}{V_3} \lambda \psi \left(T_{BL} - T_3\right)$$

$$R_3 \leq r \leq R_4 \qquad - 21$$

d. Skin

Heat is exchanged between the skin and the environment through convection, conduction and radiation. There is a conductive path to the muscle, a convective path to the central blood compartment and a heat input from the basal metabolism of the skin. The total expression is:

$$\rho_4 C_{P_4} \left[\frac{\partial T_4}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r}\left(r k_4 \frac{\partial T_4}{\partial r}\right) + \frac{Q_4}{V_4} + \frac{BF_4}{V_4} \lambda \psi \left(T_{BL} - T_4\right)$$

$$R_4 \leq r \leq R_5 \qquad - 22$$

Similar heat transfer equations can be developed for the core, the muscle, the blubber and the skin of the trunk, the fore-flippers and the hind-flippers. They are summarized below:

2. Trunk

a. Core

$$\rho_5 C_{P_5} \left[\frac{\partial T_5}{\partial r}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r}\left[r k_5 \frac{\partial T_5}{\partial r}\right] + \frac{Q_5}{V_5} - \frac{E V_{R_5}}{V_5}$$

$$- 23$$

$$+ \frac{BF_5}{V_-} \lambda \psi \left(T_{BL} - T_5\right) \qquad 0 = R_b \leq r \leq R_7$$

b. Muscle

$$\rho_6 \, C_{P_6} \left[\frac{\partial T_6}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left(r \, K_6 \, \frac{\partial T_6}{\partial r} \right) + \frac{Q_6}{V_6} + \frac{EXER_6}{V_6}$$

$$+ \frac{BF_6}{V_6} \, \lambda \, \psi \, (T_{BL} - T_6) \qquad R_7 \leq r \leq R_8 \qquad - 24$$

where,

$$Exer_i \rightarrow \text{Exercise Metabolism assigned to the ith}$$

layer (Kcals/hr)

c. Trunk Blubber

$$\rho_7 \, C_{P_7} \left[\frac{\partial T_7}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r \, K_7 \, \frac{\partial T}{\partial r} \right] + \frac{Q_7}{V_7}$$

$$+ \frac{BF_7}{V_7} \, \lambda \, \psi \, (T_{BL} - T_7) \qquad - 25$$
$$R_8 \leq r \leq R_9$$

d. Trunk Skin

$$\rho_8 \, C_{P_8} \left[\frac{\partial T_8}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r \, K_8 \, \frac{\partial T_8}{\partial r} \right] + \frac{Q_8}{V_8}$$

$$+ \frac{BF_8}{V_8} \, \lambda \, \psi \, (T_{BL} - T_8) \qquad R_9 \leq r \leq R_{10} \qquad - 26$$

3. Fore-Flippers

a. Core

$$\rho_9 \, C_{P_9} \left[\frac{\partial T_9}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r \, K_9 \, \frac{\partial T_9}{\partial r} \right] + \frac{Q_9}{V_9}$$

$$+ \frac{BF_9}{V_9} \, \lambda \, \psi \, (T_{BL} - T_9) \qquad - 27$$
$$0 = R_{11} \leq r \leq R_{12}$$

b. Muscle

$$\rho_{10} \, C_{P_{10}} \left[\frac{\partial T_{10}}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r \, K_{10} \, \frac{\partial T_{10}}{\partial r} \right] + \frac{Q_{10}}{V_{10}} + \frac{EXER_{10}}{V_{10}}$$

$$+ \frac{BF_{10}}{V_{10}} \, \lambda \, \psi \, (T_{BL} - T_{10}) \qquad R_{12} \leq r \leq R_{13} \qquad - 28$$

c. Blubber

$$\rho_{11} \, C_{P_{11}} \left[\frac{\partial T_{11}}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r \, K_{11} \, \frac{\partial T_{11}}{\partial r} \right] + \frac{Q_{11}}{V_{11}} - \qquad - 29$$

$$+ \frac{BF_{11}}{V_{11}} \, \lambda \, \psi \, (T_{BL} - T_{11})$$
$$R_{13} \leq r \leq R_{14}$$

d. Skin

$$\rho_{12} C_{P_{12}} \left[\frac{\partial T_{12}}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r K_{12} \frac{\partial T_{12}}{\partial r} \right] + \frac{Q_{12}}{V_{12}} - \frac{E_{V_{12}}}{V_{12}}$$

$$+ \frac{BF_{12}}{V_{12}} \lambda \psi \left(T_{BL} - T_{12} \right) \qquad R_{14} \leq r \leq R_{15} \qquad - 30$$

where,

$E_{vi} \rightarrow$ Heat lost from the ith layer by evaporative cooling (Kcals/hr)

4. Hind-Flippers

a. Core

$$\rho_{13} C_{P_{13}} \left[\frac{\partial T_{13}}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left(r K_{13} \frac{\partial T_{13}}{\partial r} \right) + \frac{Q_{13}}{V_{13}}$$

$$+ \frac{BF_{13}}{V_{13}} \lambda \psi \left(T_{BL} - T_{13} \right) \qquad 0 = R_{16} \leq r \leq R_{17} \quad 31$$

b. Muscle

$$\rho_{14} C_{P_{14}} \left[\frac{\partial T_{14}}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r K_{14} \frac{\partial T_{14}}{\partial r} \right] + \frac{Q_{14}}{V_{14}}$$

$$+ \frac{BF_{14}}{V_{14}} \lambda \psi \left(T_{BL} - T_{14} \right) \qquad R_{17} \leq r \leq R_{18} \quad - 32$$

c. Blubber

$$\rho_{15} C_{P_{15}} \left[\frac{\partial T_{15}}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r K_{15} \frac{\partial T_{15}}{\partial r} \right] + \frac{Q_{15}}{V_{15}}$$

$$+ \frac{BF_{15}}{V_{15}} \lambda \psi \left(T_{BL} - T_{15} \right) \qquad R_{18} \leq r \leq R_{19} \quad - 33$$

d. Skin

$$\rho_{16} C_{P_{16}} \left[\frac{\partial T_{16}}{\partial t}(t,r) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left(r K_{16} \frac{\partial T_{16}}{\partial r} \right) + \frac{Q_{16}}{V_{16}}$$

$$+ \frac{BF_{16}}{V_{16}} \lambda \psi \left(T_{BL} - T_{16} \right) \qquad R_{19} \leq r \leq R_{20} \quad - 34$$

5. Central Blood Compartment

Central blood compartment exchanges heat by

convection with the sixteen other compartments. The mass and energy balance for the central blood compartment is given below:

$$V_{BL} \; \psi \left[\frac{\partial T_{BL}}{\partial t} (t) \right] = \sum_{i=1}^{16} BF_i \; \lambda \; \psi \left(\overline{T}_i - T_{BL} \right)$$

- 35

where,

$V_{BL} \rightarrow$ Volume of central blood compartment (cm^3)

$T_i \rightarrow$ Average temperature of the ith layer $(^\circ C)$

$$= \frac{\int_{R_{i-1}}^{R_i} T_i(t,r) \; 2\pi r \, dr \; L_i}{\pi \left(R_i^2 - R_{(i-1)}^2 \right) L_i}$$

$L_i \rightarrow$ Length of the ith layer (cm)

B. BOUNDARY CONDITIONS:

In order to solve the seventeen partial differential equations developed in the proceeding section, constraining conditions must be specified. Some of these take the form of initial conditions which specify all of the temperatures at the instant the transient begins:

$$T_i \; (o, \; r) = To_i (r) \qquad\qquad - 36$$

$$T_{BL} \; (o) \;\; = T_{BLo} \qquad\qquad - 37$$

Also needed are the boundary conditions which relate the animal to its environment. In general, they are based on the fact that the local rate of conduction of heat to the

surface through the skin is equal to the rate of heat transfer from the surface to the environment.

$$-\left[K_i \frac{\partial T_i}{\partial r}\right]_{r=R_i} = H_{eff}\left[T_i(t, R_i) - T_A\right] \qquad - 38$$

Finally since each element possesses axial symmetry

$$\left[\frac{\partial T_i}{\partial r}\right]_{r=0} = 0 \qquad - 39$$

C. KNOWN PARAMETERS:

The values of many parameters in the partial differential equations such as thermal conductivity of the layers, the density and the specific heat of the layers and the blood for the sea lions are not available in the literature. It is necessary to assume these values from analogous situations in other mammals. Because of basic similarities of mammals these assumptions should introduce very minor error. The sources and the values of these parameters are given below:

1. Thermal Conductivity

The specific conductivity of the core, the muscle and the skin are taken from the human model (Stolwijk and Hardy, 1966) to be 0.0036 Kcal $\cdot$ cm/cm$^2\cdot$hr$\cdot^\circ$C.

Scholandar et al., (1950) measured the thermal conductivity of the dead blubber for seals as 1.6 cal/cm$^2\cdot$hr$\cdot^\circ$C

Hart and Irving (1959) made measurements of heat flow through local skin areas as harbor seals in different gradients induced by varying water temperatures. It indicated the conductivity of living blubber to be 2.5 cal/cm$^2 \cdot$hr$\cdot^o$C. This difference can be ascribed to the heat transport by the vascularization which provides the variable component in the insulation.

Bryden (1964) has investigated the insulating capacity of the subcutaneous fat of Southern Elephant Seal and found the specific conductivity to be .00017 $\pm$ .0004 cal/sec-cm-^{o}C. In this model, specific conductivity given for Southern Elephant Seal's subcutaneous fat is assumed.

2. Density

Density of human tissue is .001 kg/cm^3 (Stolwijk and Hardy, 1966), therefore it is assumed that the densities of the core, muscle, skin and the blood are to be that of the human tissue.

Reid and Mackay (1968) report the density of dolphins' blubber to be ,98 g/cm^3. There is not much difference between the densities of the human tissue and the dolphin's blubber. The density of the sea lion's blubber is taken to be that of the dolphin's blubber.

3. Specific heat

The specific heat of the human tissue and blood were reported by Stolwijk and Hardy (1966) to be 0.83 and 0.92 Kcal/kg-^{o}C, respectively. These values are assumed in this model, as there are no data available for the sea lions.

D. SOLUTIONS TO THE PARTIAL DIFFERENTIAL EQUATIONS:

The partial-differential equations are solved using the finite-difference technique, since this method allows variation of physical properties with position.

Basically, the procedure consists of subdividing each of the circular elements into a number of annular shells and assigning a single characteristic temperature to the material in each of the shells. A marching procedure is employed in which the initial temperatures are used to compute the temperatures after a short interval of time Δt. These new temperatures are then used to compute the temperatures at time $2\Delta t$ and so on as long as necessary.

Normally, the temperatures in a given region are all specified at t=o by the initial conditions. The problem is to compute the temperatures in the next row (t=Δt) as shown in Figure 7.

There are formulations explicit or implicit to express these partial differential equations in finite-difference form. The major drawback in the explicit method is that in order to insure stability only very small increments in the time variable are permissible. This frequently leads to excessive computation time.

An implicit-recurrence formula is one in which two or more unknown values of the function at t_{i+1} are expressed in terms of previous values. Such formulations require solutions of sets of simultaneous equations.
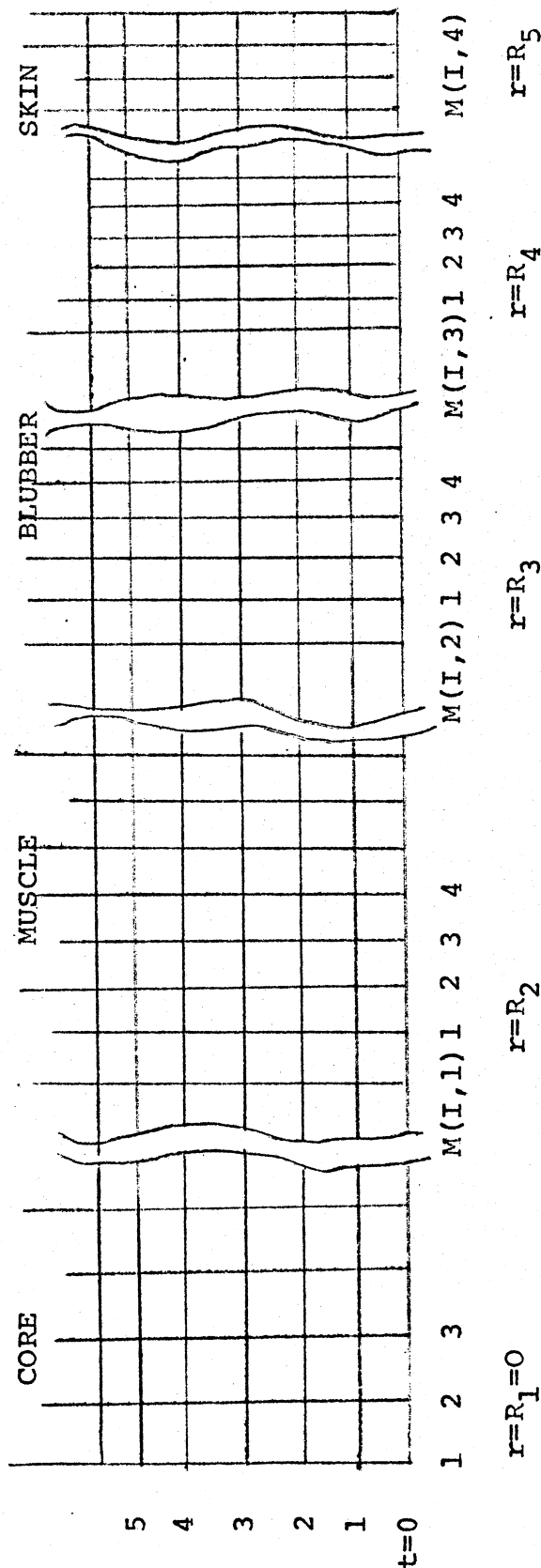
Figure 7. Diagram Depicting the Time Increments and the Special Steps Used.

The Crank-Nicholson formulation scheme, which is an implicit-recurrence formula with the weighing factor of 1/2 is used here. This method is theoretically stable for all net sizes.

Equation 19 which represents the mass energy balance for the head core will be used to illustrate the development of the finite-difference formulation.

The second order partial derivatives are reduced to first order by a method suggested by Wissler (1964). The partial differential equations are integrated once with respect to "R" assuming the temperature in each cell to be homogeneous.

The difference equation used to approximate the first differential layer in the head core is obtained by integrating each term in the equation (19) over a cylindrical region ranging from $r=R_1=0$ to $r=R_1+\frac{\Delta r_1}{2}$, in which Δr_1 is the space increment in the Head Core. Then the equation 19 becomes:

$$\int_{R_1=0}^{R_1+\frac{\Delta r_1}{2}} \rho_1\, C_{P_1}\left[\frac{\partial T(t,r)}{\partial r}\right] r\, dr = \int_{R_1=0}^{R_1+\frac{\Delta r_1}{2}} \frac{\partial}{\partial r}\left(r\, K_1\, \frac{\partial T_1}{\partial r}\right) dr$$

$$+ \int_{R_1=0}^{R_1+\frac{\Delta r_1}{2}} \left[\frac{Q_{1,1}}{V_{1,1}} - \frac{E_{V_{R_{1,1}}}}{V_{1,1}} + \frac{\beta F_{1,1}}{V_{1,1}}\left(T_{BL}-T_{1,1}\right)\right] r\, dr \qquad - 40$$

Applying the boundary condition - 39, equation - 40 becomes:

$$\frac{\rho_1\, C_{P_1}}{2}\left[\left(R_1+\frac{\Delta r_1}{2}\right)^2 - R_1^2\right]\frac{\partial T_{1\,i,1}}{\partial t} \approx \left[K_1\left(R_1+\frac{\Delta r_1}{2}\right)\frac{\partial T_{1\,i}\left(3/2\right)}{\partial r}\right]$$

$$+ \frac{Q_{11}}{2V_{11}} + \frac{\beta F_{1\,i}}{2V_{11}}\lambda\, \psi\left(T_{BL}-T_{1\,i,1}\right) - \frac{E_{V_{R_{1\,i,1}}}}{2V_{11}}\right]\left[\left(R_1+\frac{\Delta r_1}{2}\right)^2 - R_1^2\right] \qquad - 41$$

For the rest of the differential layers in the head core (except the last one), the following equation is obtained, letting T_{1ij} be the temperature at the jth radial point, rj, after (i-1)th time step, t=t_i and integrating each term in the equation (19) over a cylindrical region ranging from r=R_1+rj-$\frac{\Delta r_1}{2}$ to r=R_1+rj+$\frac{\Delta r_1}{2}$,

$$\frac{\rho_1 C_{P1}}{2} \left[\left(R_1 + rj + \frac{\Delta r_1}{2}\right)^2 - \left(R_1 + rj - \frac{\Delta r_1}{2}\right)^2 \right] \frac{\partial T_{1ij}}{\partial t} \cong$$

$$\left[k_1 \left(R_1 + rj + \frac{\Delta r_1}{2}\right) \frac{\partial T_{1i(j+1/2)}}{\partial r} - k_1 \left(R_1 + rj - \frac{\Delta r_1}{2}\right) \frac{\partial T_{1i(j-1/2)}}{\partial r} \right.$$

$$\left. + \frac{Q_{1ij}}{2r_{ij}} + \frac{\beta F_{1ij}}{2r_{ij}} \lambda \psi \left(T_{BL} - T_{1ij}\right) - \frac{E V_{R 1ij}}{2 V_{ij}} \right] \times$$

$$\left[\left(R_1 + rj + \frac{\Delta r_1}{2}\right)^2 - \left(R_1 + rj - \frac{\Delta r_1}{2}\right)^2 \right]$$

- 42

For the last (outer) differential layer in the head core and the first differential layer in the head muscle, the following difference equation is obtained, by integrating over the range of r=R_2-$\frac{\Delta r_1}{2}$ to r=R_2+$\frac{\Delta r_1}{2}$ and letting $T_{1i,(M(1,1)+}$ be the temperature at the boundary between the core and the muscle of the head.

$$\rho_{AV_1} C_{P_{AV_1}} \left[\left(R_2 + \frac{\Delta r_1}{2}\right)^2 - \left(R_2 - \frac{\Delta r_1}{2}\right)^2 \right] \frac{\partial T_{1i(M(1,1)+1)}}{\partial t} \cong \left[K_2 \left(R_2 + \frac{\Delta r_2}{2}\right) \frac{\partial T_{2i}}{\partial r} \right.$$

$$(M(1,1)+3/2) - K_1 \left(R_2 - \frac{\Delta r_1}{2}\right) \frac{\partial T_{1i}}{\partial r} (M(1,1)+1/2) \right] + \left[\frac{Q_{1i(M(1,1)+1)}}{2 V_{1i(M(1,1)+1)}} + \frac{\beta F_{1i(M(1,1)+1)}}{2 V_{1i(M(1,1)+1)}} \times \psi \right]$$

$$\left(T_{BL} - T_{1i(M(1,1)+1)}\right) + \frac{E V_{R1i(M(1,1)+1)}}{2 V_{1i(M(1,1)+1)}} + \frac{Q_{2i(M(1,1)+1)}}{2 V_{2i(M(1,1)+1)}} + \frac{\beta F_{2i(M(1,1)+1)}}{2 V_{2i(M(1,1)+1)}} \lambda \psi \left(T_{BL}\right.$$

where,

- 43

As the basal metabolism (Q), blood flow (BF), heat loss through the respiratory tract (Evr) and the exercise metabolism (Exer) are known for the layers, they are distributed among the differential layers on weight basis.

For the muscle, blubber and the skin (except the last layer of the skin), the same procedure is used.

For the outer layer of the skin, the following difference equation is obtained, by integrating each term in the equation, over a cylindrical region ranging from $r+R_5-\frac{\Delta r_4}{2}$ to $r=R_5$ and using the outer boundary condition equation -24.

$$\frac{P_4 \, CP_4}{2} \left[R5^2 - \left(R5 - \frac{\Delta r_4}{2} \right)^2 \right] \frac{\delta T_{1\,i,\,L_1}}{\delta t} \cong$$

$$\left[-R5 \, H_{EFF} \left[T_{1\,i,\,L_1} - T_A \right] - K_4 \, \frac{\delta T_{1\,i}}{\delta r} \left(L_1 - \frac{1}{2} \right) \right.$$

$$+ \left[\frac{Q_4\,i,\,L}{2\,V_4} + \frac{BF_4\,i,\,L}{2\,V_4} \, \lambda \, \psi \, (T_{BL} - T_1\,i,\,L_1) - \frac{E}{2} \right.$$

$$\left[R5^2 - \left(R5 - \frac{\Delta r_4}{2} \right)^2 \right]$$

-46

It is assumed that the heat loss by evaporation is lost from the last layer of the skin.

The partial derivatives appearing in the proceeding equations are approximated as follows:

$$\frac{\partial T_{1i,j}}{\partial t} = \frac{T_{1(i+1)j} - T_{1i,j}}{\Delta t} \qquad -47$$

$$\frac{\partial T_{1i(j+\frac{1}{2})}}{\partial r} = \frac{T_{1i(j+1)} - T_{1i,j}}{\Delta r} \qquad -48$$

$$\frac{\partial T_{1i(j-\frac{1}{2})}}{\partial r} = \frac{T_{1i,j} - T_{1i,(j-1)}}{\Delta r} \qquad -49$$

Substituting the proceeding approximations into the equations and using the arithmetic mean of the values of the right-hand side at time t_i and t_{i+1} to approximate the values of the right-hand side of time $(t_i+t_{i+1})/2$ (the Crank-Nicholson scheme), a set of equations of the following form is obtained:

$$A_{1j} T_{1(i+1)(j-1)} + B_{1j} T_{1(i+1)j} + C_{1j} T_{1(i+1)(j+1)}$$

$$= D_{1j} \qquad 1 \leq j \leq L_1 \qquad -50$$

where, A_{1j}, B_{1j} and C_{1j} are constants determined by the physical properties and the mesh size, and D_{1j} is determined by the temperature at time t_i.

The same procedure is used and the following equations are obtained for the trunk, fore-flippers and hind-flippers respectively.

$$A_{2j} T_{2(i+1)(j-1)} + B_{2j} T_{2(i+1)j} + C_{2j} T_{2(i+1)(j+1)} \qquad -51$$

$$= D_{2j} \qquad 1 \leq j \leq L_2$$

$$A_{3j} T_{3(i+1)(j-1)} + B_{3j} T_{3(i+1)j} + C_{3j}(i+1)(j+1)$$

$$= D_{3j} \qquad 1 \leq j \leq L_3 \qquad - 52$$

$$A_{4j} T_{4(i+1)(j-1)} + B_{4j} T_{4(i+1)j} + C_{4j} T_{4(i+1)(j+1)}$$

$$= D_{4j} \qquad 1 \leq j \leq L_4 \qquad - 53$$

The detailed expressions for the constants are given in Appendix I.

The equations (50), (51), (52) and (53) to be solved can be expressed in the tridiagonal matrix form as follows:

$$
\begin{bmatrix}
B_1\,T_1 + C_1\,T_2 \\
A_2\,T_1 + B_2\,T_2 + C_2 T_3 \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \cdot \\
A_L T_{(L-1)} + B_L T_{(L)}
\end{bmatrix}
=
\begin{bmatrix}
D_1 \\
D_2 \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
D_L
\end{bmatrix}
$$

Where the subscripts for the sections and the time steps are suppressed. For the solution of the above equations concerning T_i's the following recursion formula, which is a Gaussian elimination procedure, is employed.

$$T_L = r_L \qquad - 54$$

$$T_i = r_i - \left(c_i\, T_{i+1} / \beta_i \right) \qquad i = 2, \cdots L-1 \qquad - 55$$

where β_i's and r_i's are determined from the recursion formulae such as

$$\beta_1 = B_1 \qquad\qquad - 56$$

$$r_1 = D_1 / \beta_1 \qquad\qquad - 57$$

$$T_1 = r_1 - \frac{(A_1 + B_1)\, T_1}{B_1} \qquad\qquad - 58$$

$$\beta_i = B_i - \frac{A_i\, C_{i-1}}{\beta_{i-1}} \qquad i = 2, \cdots L \qquad - 59$$

$$r_i = D_i - \frac{A_i\, r_{i-1}}{\beta_i} \qquad i = 2, \cdots L \qquad - 60$$

The development of the recursion formula used is given in Appendix II.

E.   CALCULATIONS FOR THE BLOOD TEMPERATURE:

Blood temperature is an important criteria in keeping the body temperature at any level, since the blood plays an important role as a heat exchange medium.

Equation 35 can be rewritten as

$$\frac{d\,T_{BL}}{dt} = \sum_{i=1}^{16} \frac{B\,F_i\, T_i}{V_{BL}} - \sum_{i=1}^{16} \frac{B\,F_i\, T_{BL}}{V_{BL}} \qquad - 61$$

$$\text{Letting } A = \sum_{i=1}^{16} \frac{B\,F_i}{V_{BL}}$$

$$\text{and } B = \sum_{i=1}^{16} \frac{B\,F_i\, T_i}{V_{BL}}$$

Equation 61 becomes:

$$\frac{dT_{BL}}{dt} + AT = B$$

<div align="right">- 62</div>

By solving equation 62, the blood temperature $T_{BL(i+1)}$ at time $t = t_{(i+1)}$ can be calculated, knowing the blood temperature T_{BLi} at time $t = t_i$. It is given as follows:

$$T_{BL(i+1)} = \frac{B}{A} + \left[T_{BLi} - \frac{B}{A} \right] e^{-A(t_{i+1} - t_i)}$$

<div align="right">- 63</div>

It is assumed that the blood leaving the layers is at the average temperatures of the respective layers. The average temperature of the layers are calculated on weight basis.

Under most blood flow conditions the turnover ratio is very high and the blood pool temperature remains constant.

CHAPTER IV

RESULTS AND DISCUSSION

The heat lost by the animal to the environment by conduction, convection and radiation can be calculated by two methods. One way is on the basis of temperature gradient between the last differential layer of the skin and the environment. Another is on the basis of the change in total internal energy of all the differential layers in the body of the animal.

The heat lost to the environment was computed by these two methods, with the animal at steady state in $20^\circ C$ ambient air. It was found that the values given by the first method was about 10% higher than that by the second method. Although the second method is a more tedious way of calculating the heat loss, it yields a more accurate value. The reason for this discrepency can be attributed to the accumulation of round-off errors in numerical solutions at the last (outer) differential layer of the skin.

Air:

The measured body temperature of the animal as a function of time, when the animal was exposed to the air temperatures of 15°, 20° and $25^\circ C$ (South et al., 1973) is shown in Figure 8. Figure 9 shows the computed average temperature of the trunk core as a function of time, when the
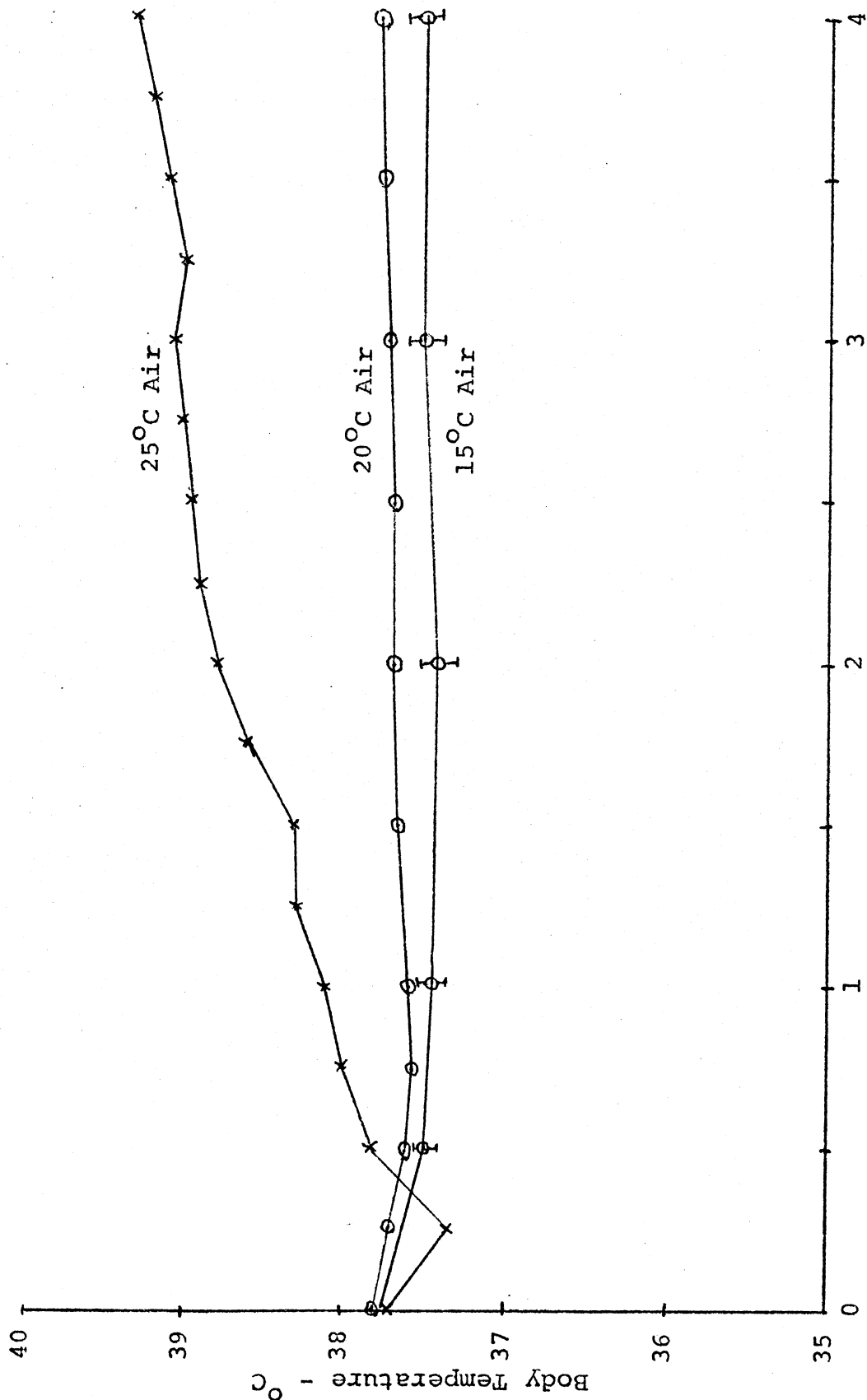
Figure 8. Measured Body Temperature Time in calorimeter - Hours of the Animal in Air Function of Time.
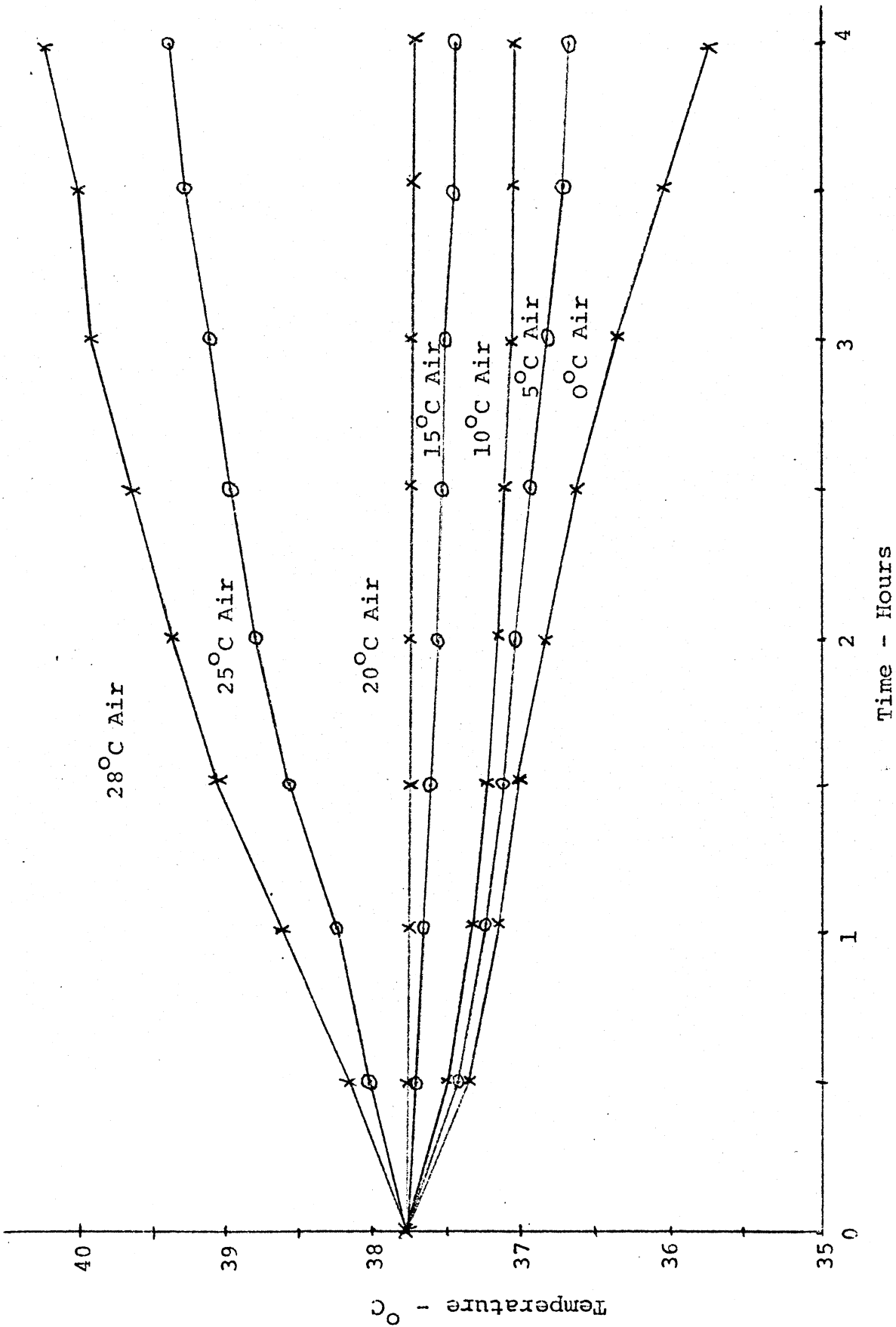
Figure 9. Computed Average Temperature of the Trunk Core of the Animal in Air as a Function of Time.

animal is hauled out at 0°, 5°, 10°, 15°, 20°, 25^0 and $28^\circ C$ air temperatures. These computed values closely reproduce the measured values. The body temperature of the animal is relatively constant at 5°, 10°, 15° and 20° air temperatures. But at $0^\circ C$ air temperature, the body temperature drops by $1.5^\circ C$ over a period of 4 hours. During this time, the animal might move around or use other routes of thermogenesis to keep its body temperature from falling.

The animal exhibits discomfort in air temperatures above $25^\circ C$. The reason for this can be attributed to the high insulation offered by the blubber and the animal's inability to circulate more blood to the skin, as well as its limited ability to lose heat by evaporative cooling. Under these circumstances the animal might spend most of the time in water.

The measured (South et al., 1973) and the computed heat loss by radiation, convection, respiration and skin evaporation as a function of air temperatures are shown in Figure 10 and 11 respectively. It can be seen that the measured values are quite well reproduced by the computations. These values indicate that the radiant loss was the most important route (as high as 50 or 55%) for the heat loss to the environment.

The temperature of the last (outer) differential layer of the skin at 4 hours and at various ambient air temperatures are given in Table 5. Hart and Irving (1959) reported the skin temperature of harbor seals, (mean of two points posterior
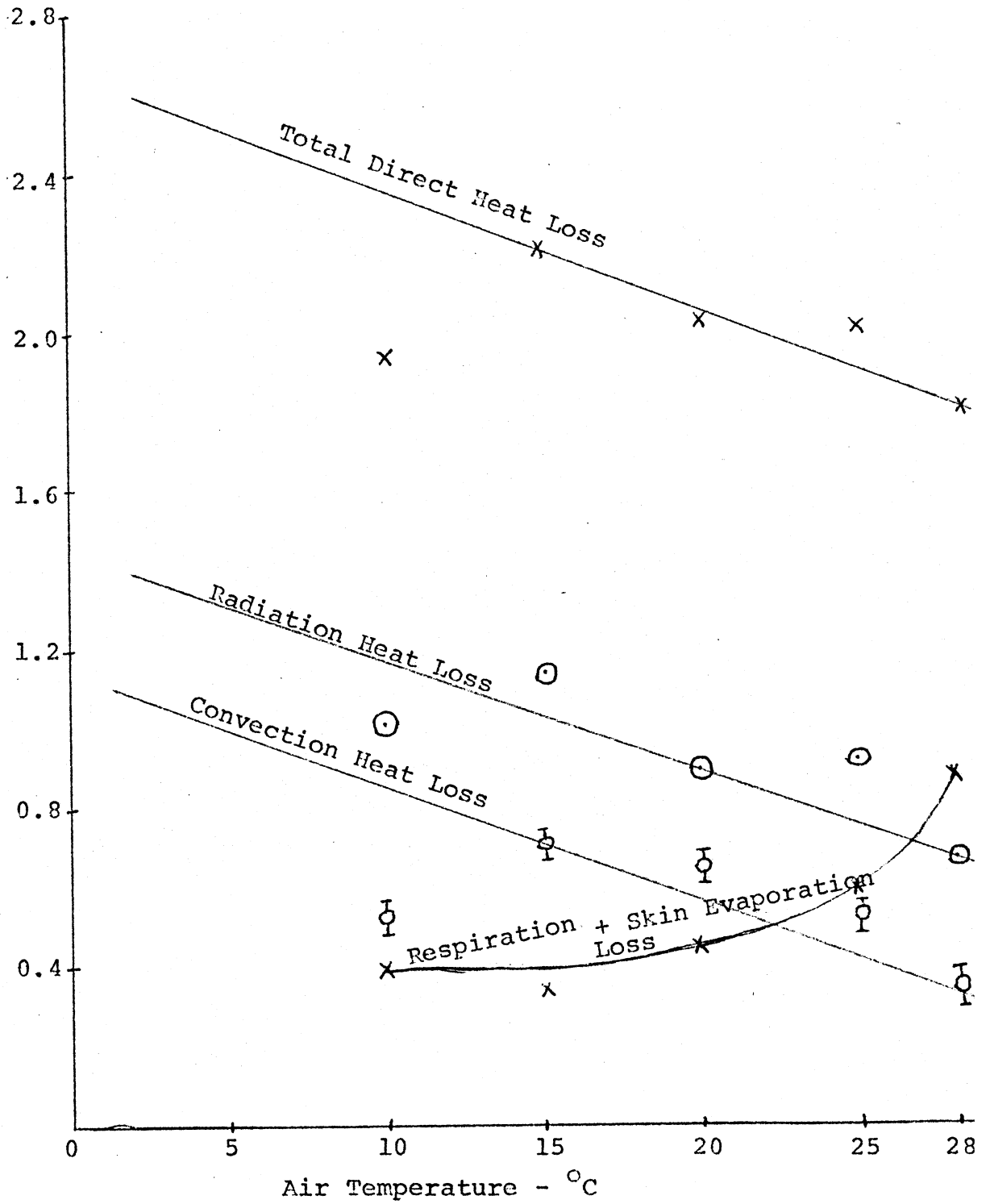
Figure 10. Measured Heat Loss as a Function
of Air Temperature.
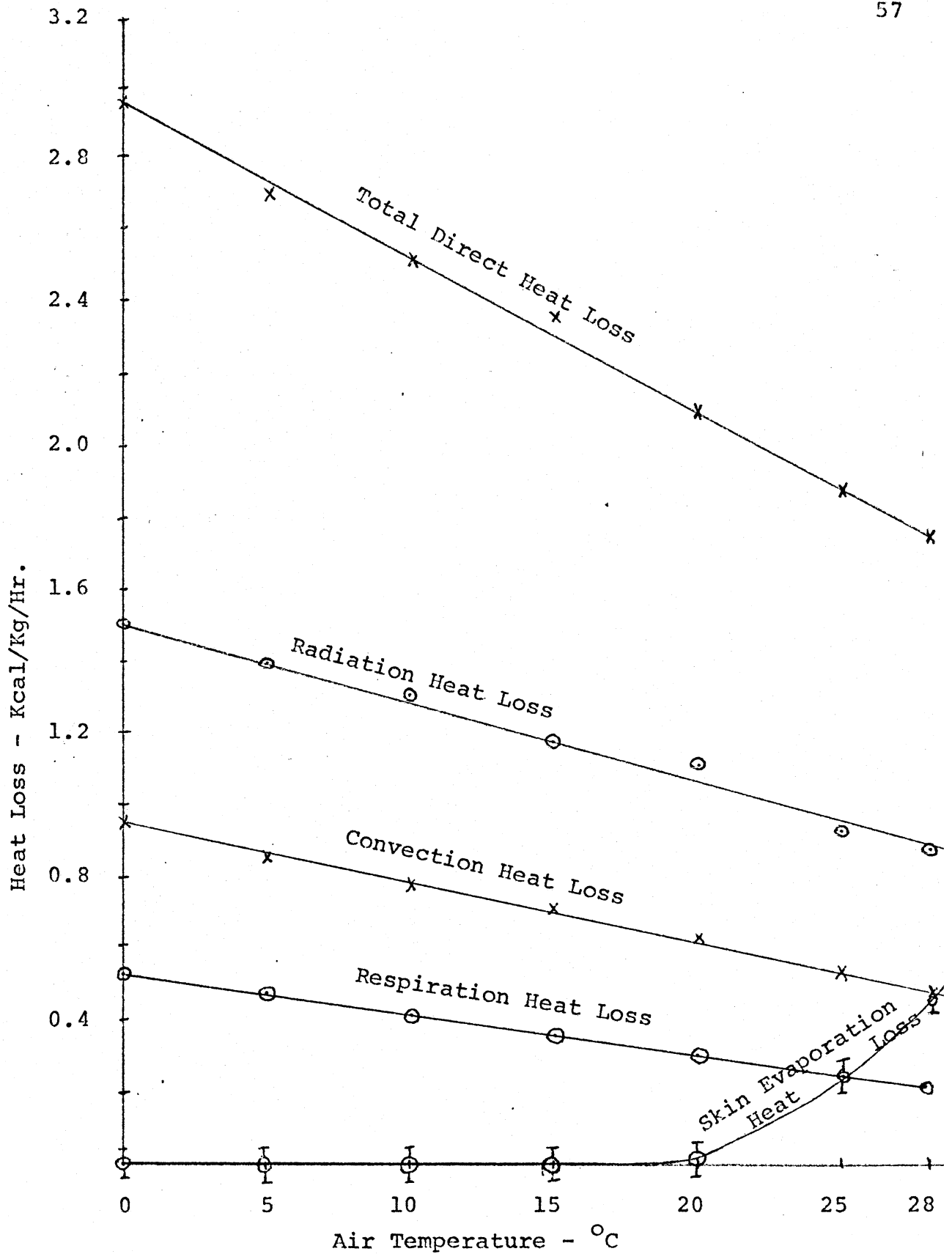
Figure 11. Computed Heat Loss as a Function of Air Temperature

58

TABLE 5

COMPUTED TEMPERATURE OF THE LAST (OUTER) DIFFERENTIAL LAYER OF THE
SKIN AT 4.0 HOURS AND AT VARIOUS AIR TEMPERATURES WHEN
THE ANIMAL IS HAULED OUT ON LAND

Air Temperature in °C	Head Skin Temp. in °C	Trunk Skin Temp. in °C	Fore-Flippers Skin Temp. in °C	Hind-Flippers Skin Temp. in °C
0	25.73	17.50	20.59	21.81
5	27.62	20.80	25.16	24.16
10	29.42	25.25	26.92	27.23
15	31.97	30.68	32.11	31.67
20	32.10	30.90	32.15	31.75
25	34.75	33.61	32.57	32.71
28	36.24	35.11	32.00	32.62

to axilla and differing less than 0.5°C) at an air temperature of 20°C was 30°C. The simulated value is 30.9°C.

Swimming:

The computed average trunk core temperature of the animal swimming at 6 and 7 miles/hr as a function of temperature at 0°, 5°, 10°, 15° and 20° water temperatures are shown in Figures 12 and 13.

In this model, the same control equations are used for the animal on land and swimming. Since there are no reported data available for the animal swimming, it is not possible to differentiate between the control equations for the animal on land and the animal swimming in water.

It can be seen from the figures 12 and 13, that the animal is able to maintain its body temperature in 0°C water by swimming 1 mile/hr faster.

The computed heat loss is given as a function water temperature in Figure 14. It clearly indicates that the convection heat loss (amounts to as high as 80%) is the most important single route for the heat loss.

Tables 6 and 7 show the temperatures of the last differential layer of the skin at various water temperatures when the animal is swimming at 6 and 7 miles/hour respectively.

Diving:

Figure 15 shows the computed average temperature of the trunk core of the animal as a function of time when the animal is diving at 9 miles/hour. A point of interest is
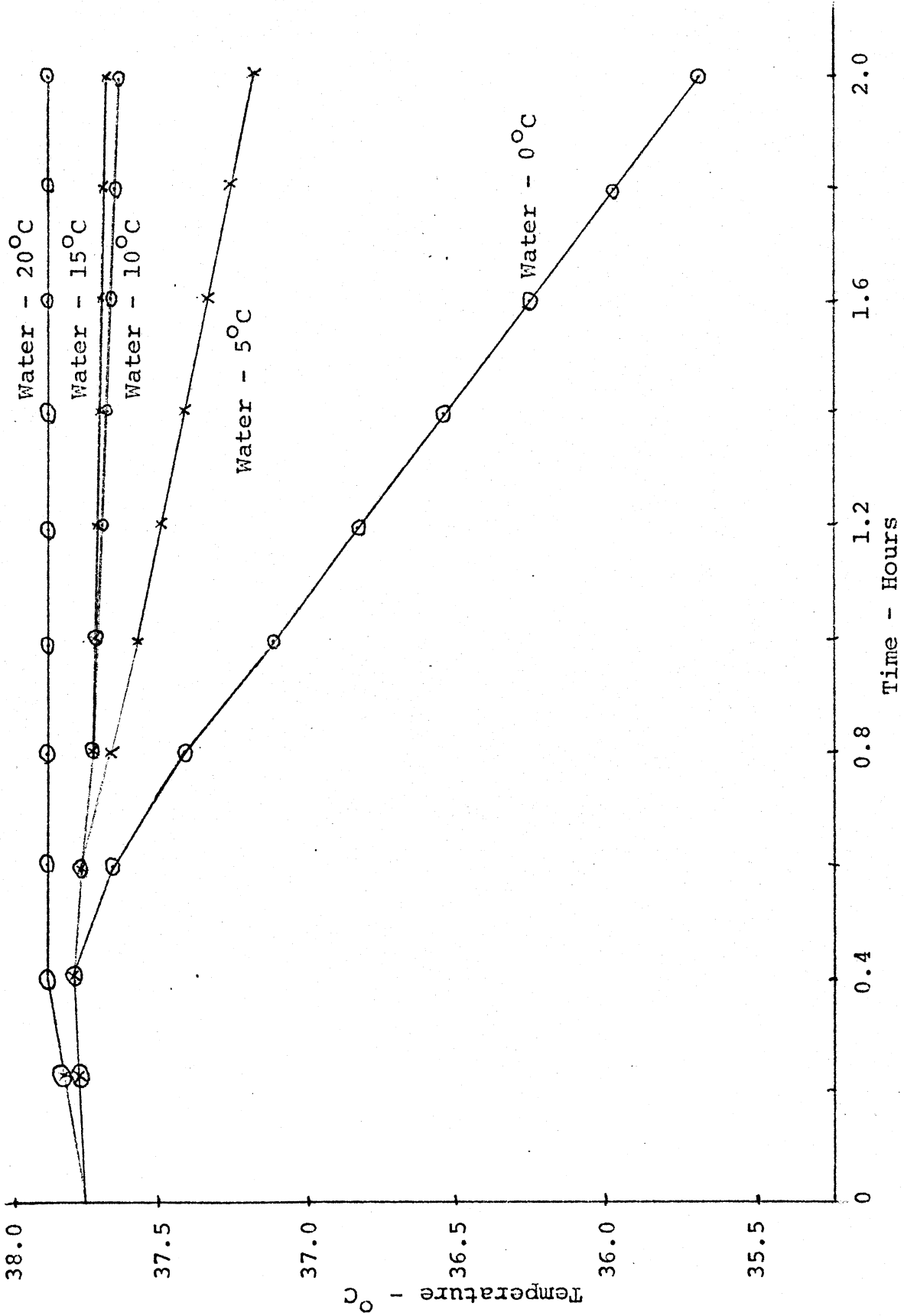
Figure 12. Computed Average Trunk Core Temperature of the Animal Swimming at 6 miles/hour as a Function of Time.
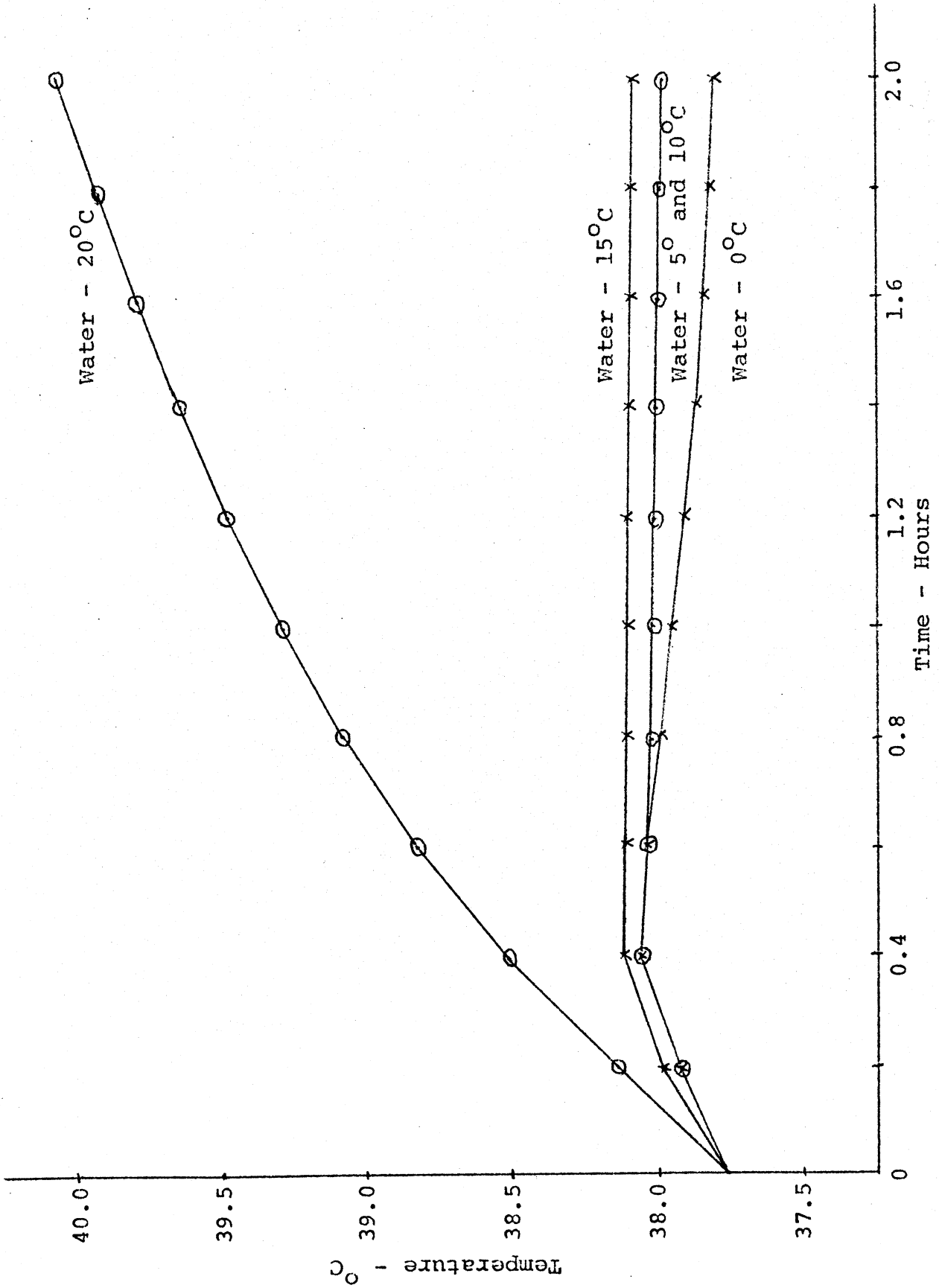
Figure 13. Computed Average Trunk Core Temperature When the Animal is Swimming
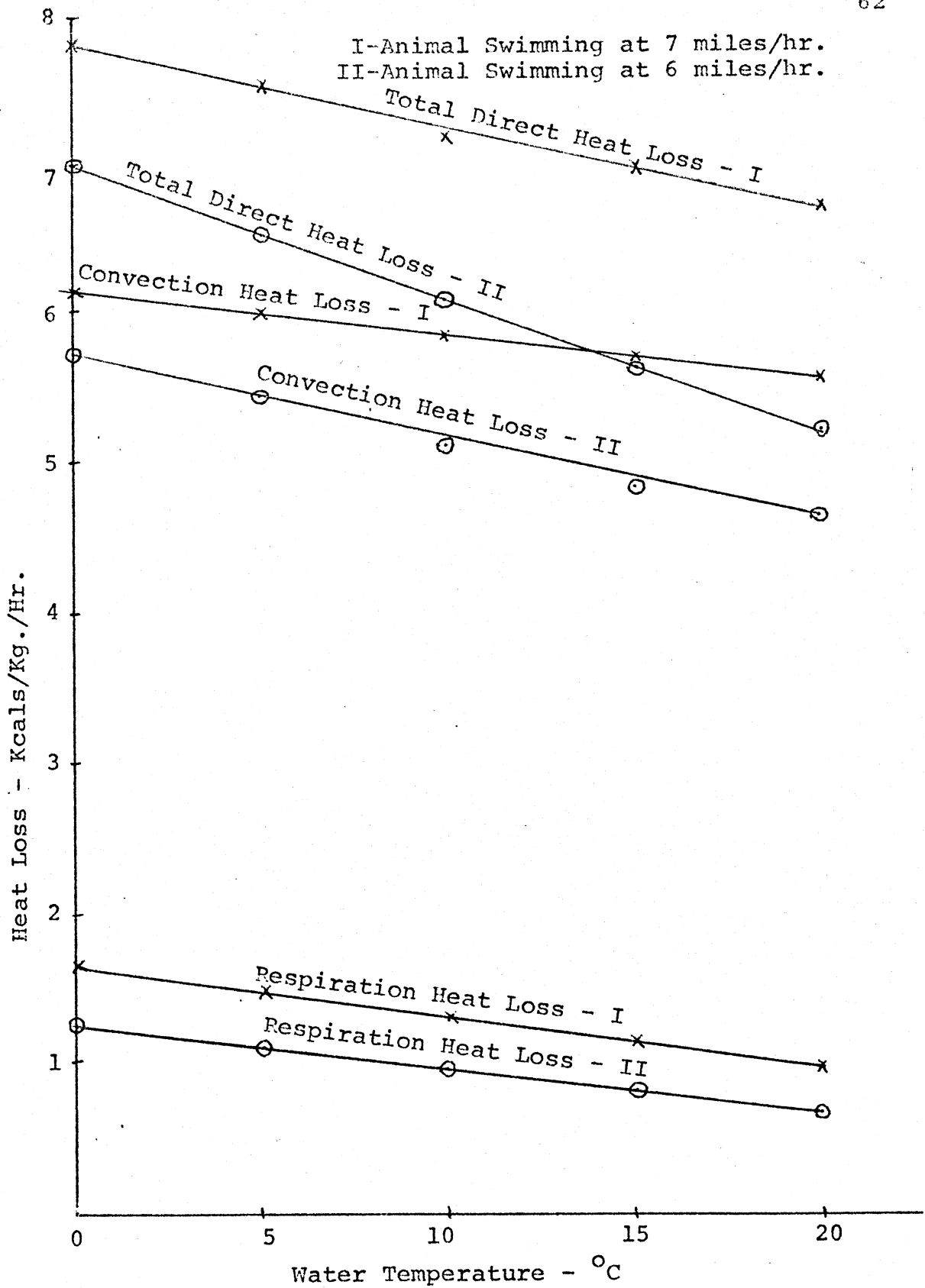
Figure 14. Computed Heat Loss When the Animal is Swimming, as a Function of Water Temperature.

TABLE 6

COMPUTED TEMPERATURE OF THE LAST (OUTER) DIFFERENTIAL LAYER OF THE
SKIN IN 2.0 HOURS AND AT VARIOUS WATER TEMPERATURES WHEN THE
ANIMAL IS SWIMMING AT 6 MILES/HOUR

Water Temperature in $^{\circ}C$	Head Skin Temp. in $^{\circ}C$	Trunk Skin Temp. in $^{\circ}C$	Fore-Flippers Skin Temp. in $^{\circ}C$	Hind-Flippers Skin Temp. in $^{\circ}C$
0	4.76	1.87	2.88	3.45
5	9.26	6.68	7.60	8.10
10	13.77	11.75	12.44	12.80
15	18.29	16.90	17.66	17.75
20	22.83	22.04	22.86	22.70

TABLE 7

COMPUTED TEMPERATURE OF THE LAST (OUTER) DIFFERENTIAL LAYER OF THE
SKIN AT 2.0 HOURS AND AT VARIOUS WATER TEMPERATURES WHEN
THE ANIMAL IS SWIMMING AT 7 MILES/HOUR

Water Temperature in ^{o}C	Head Skin Temp. in ^{o}C	Trunk Skin Temp. in ^{o}C	Fore-Flippers Skin Temp. in ^{o}C	Hind-Flippers Skin Temp. in ^{o}C
0	4.84	1.88	2.91	3.52
5	9.36	7.05	7.90	8.32
10	13.90	12.22	13.15	13.28
15	18.46	17.38	18.37	18.24
20	23.1	22.27	23.22	23.02

that the body temperature of the animal increases about 1.2°C over a period of 20 minutes. This type of response has not been reported in any literature. However, it is quite possible as the animal is liberating about 99 Kcals/hour of heat in the muscles, in addition to the 70 Kcals/hour of basal metabolism. Furthermore the blood, which plays such an important role as a heat transfer medium, is completely shut off during diving from all the sections except the head core and half of the trunk core. Blood flow to the blubber was permitted for thermoregulatory purposes.

It is interesting to note from Table 8 that the body temperature of the animal increases at a steady rate at all water temperatures down to 0°C.

Computed temperatures of the differential layers in the trunk of the animal after 16.2 minutes of diving at 9 miles/hour are shown in Figure 15. The reason for the rise in body temperature of the animal while diving becomes apparent. In the muscles, there is heat liberation of about 99 Kcals/hr due to exercise metabolism. Since there is no blood flow to the muscles this heat cannot be transferred by convection. There is only the conductive heat transfer occuring. Because of the temperature gradient between the muscle and the core, heat is conducted to the core, and hence the rise in core temperature.

The insulation offered by the blubber against heat loss to the environment by the animal can be clearly understood from the temperature gradient of about 20°C through
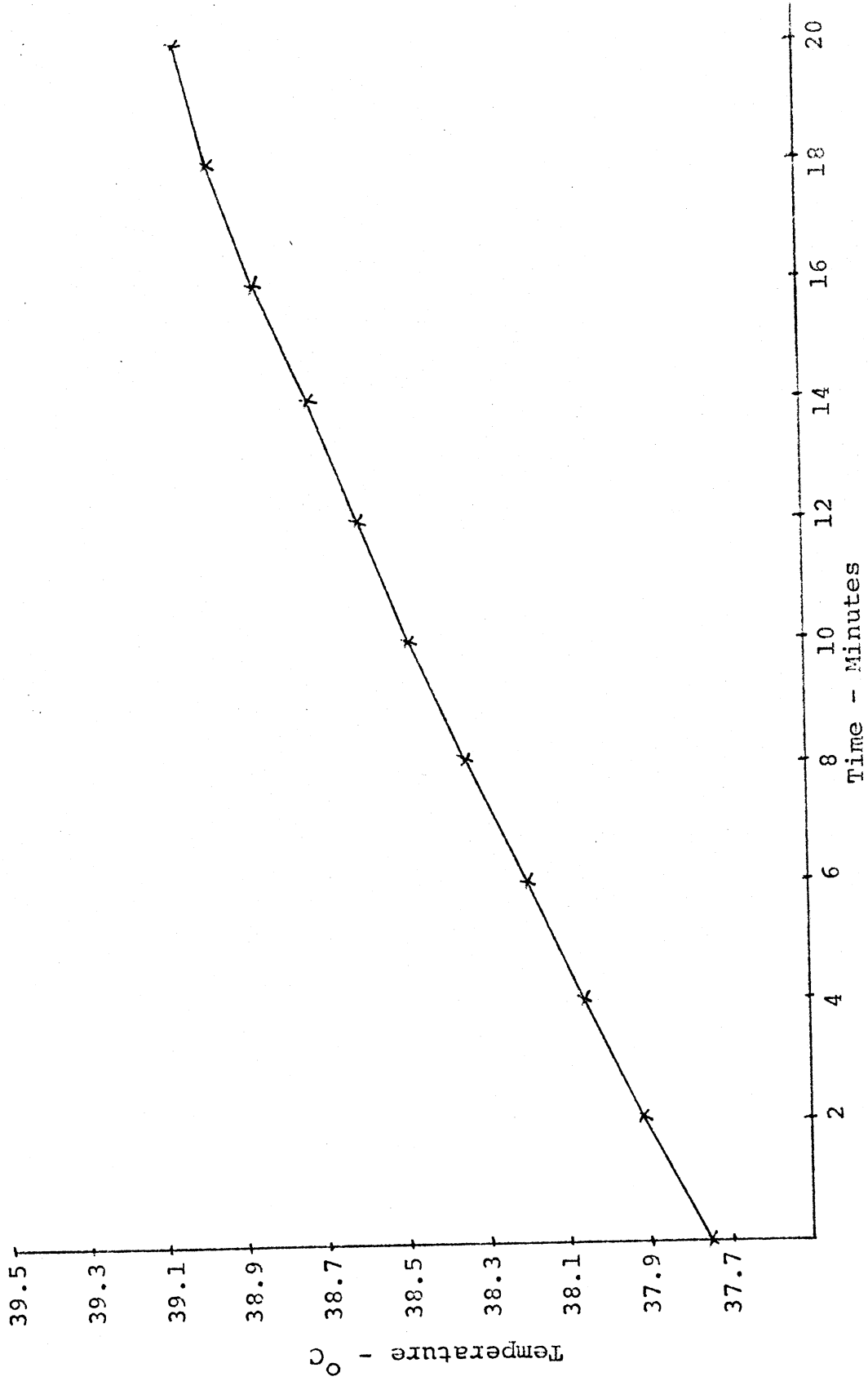
Figure 15. Computed Average Temperature of the Trunk Core of the Animal When It is Diving at 9 Miles/Hr. in Water of Temperatures 0°, 5°, 10°, 15° and 20°C as a Function of Time.

TABLE 8

COMPUTED AVERAGE TEMPERATURE OF THE TRUNK CORE,
WHILE THE ANIMAL IS DIVING AT 9 MILES/HR.

Time in Minutes	Water Temperature in $^{\circ}$C				
	0°	5°	10°	15°	20°
0	37.75	37.75	37.75	37.75	37.75
2	37.90	37.90	37.90	37.90	37.90
4	38.05	38.05	38.06	38.06	38.06
6	38.19	38.20	38.20	38.21	38.21
8	38.33	38.33	38.34	38.35	38.36
10	38.46	38.46	38.47	38.48	38.50
12	38.59	38.59	38.60	38.62	38.63
14	38.72	38.72	38.725	38.74	38.76
16	38.84	38.84	38.84	38.86	38.89
18	38.96	38.95	38.96	38.98	39.01
20	39.08	39.07	39.07	39.07	39.13

the blubber, shown in Figure 16.

The only important route of heat transfer, which is convective heat transfer, when the animal is diving is shown as a function of water temperature in Figure 16.

The temperatures of the last (outer) differential layer of the skin at various water temperatures when the animal is diving, are given in table 9. Hart and Irving (1959) reported the skin temperature of harbor seals, immersed in $0^{\circ}C$ water was about $2.5^{\circ}C$ after 15 minutes. The simulated value is $2.31^{\circ}C$.
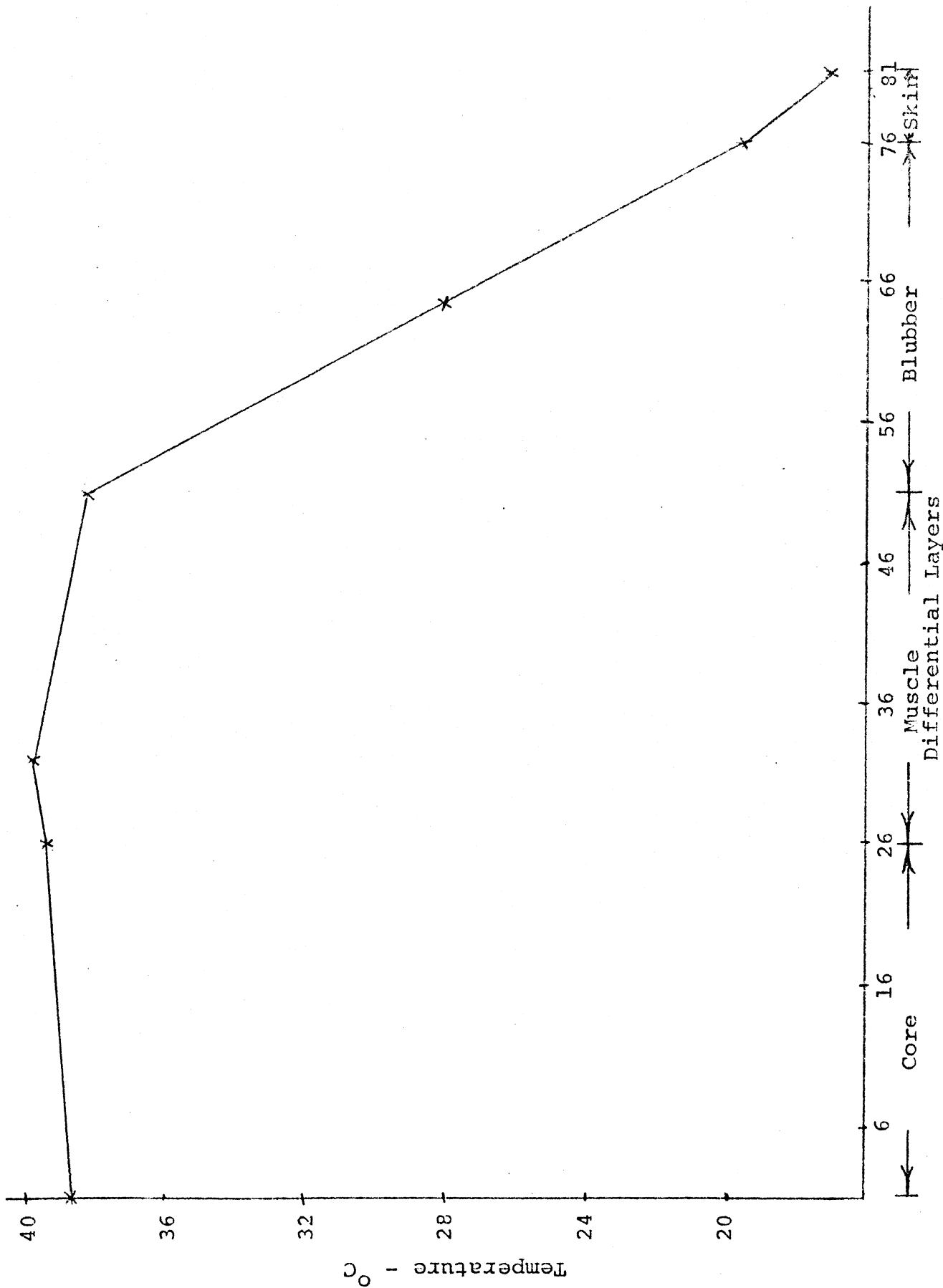
Figure 16. Computed Temperatures at 16.2 Minutes as a Function of Differential Layers.
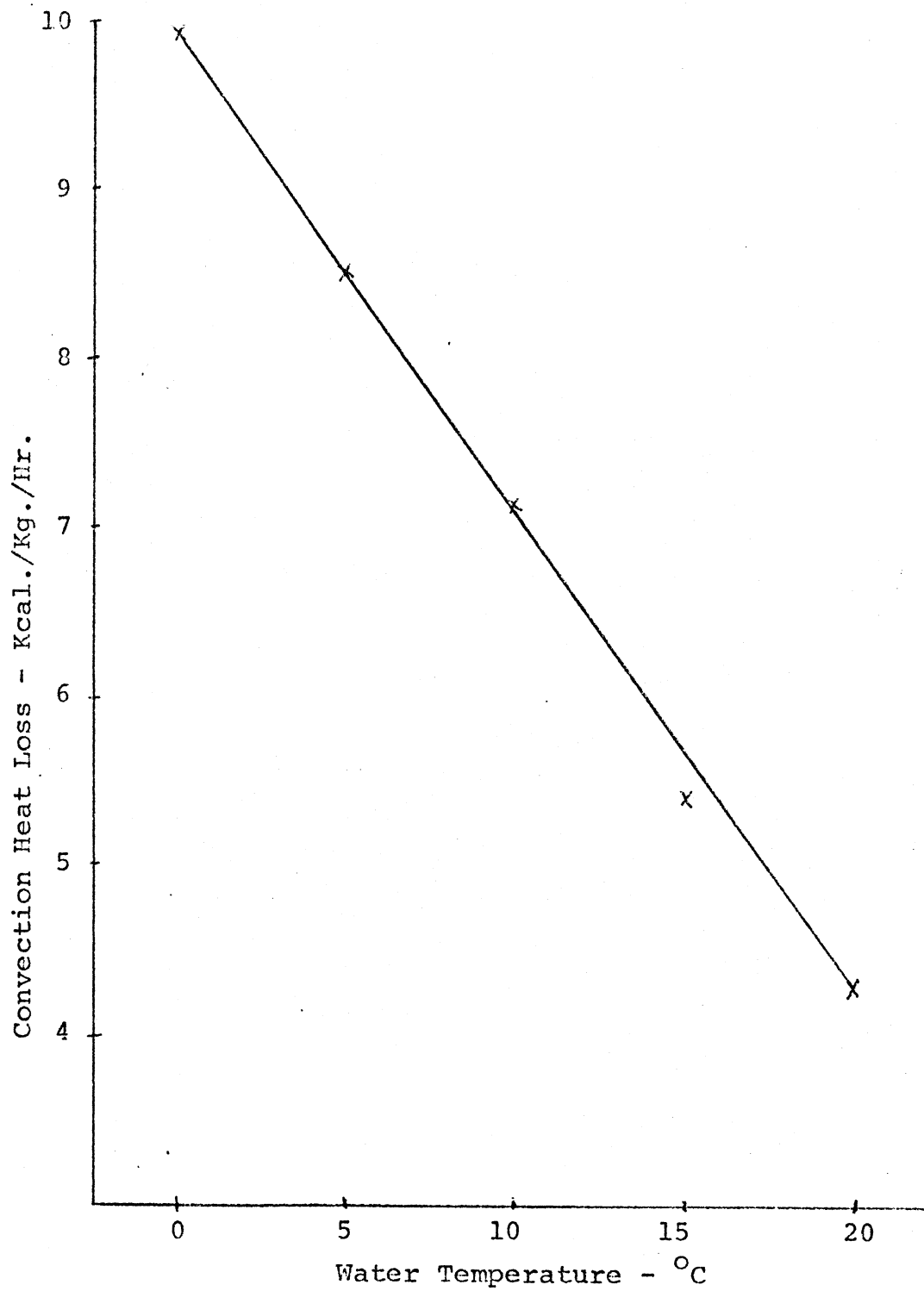
Figure 17.   Computed Convection Heat Loss as a Function of
Water Temperature, When the Animal is Diving
at 9 Miles/Hr.

TABLE 9

COMPUTED TEMPERATURE OF THE LAST (OUTER) DIFFERENTIAL LAYER OF THE
SKIN AT 16.2 MINUTES AND AT VARIOUS WATER TEMPERATURES
WHEN THE ANIMAL IS DIVING AT 9 MILES/HR.

Water Temperature in °C	Head Skin Temp. in °C	Trunk Skin Temp. in °C	Fore-Flippers Skin Temp. in °C	Hind-Flippers Skin Temp. in °C
0	3.65	2.31	2.52	2.59
5	8.21	7.03	7.23	7.27
10	12.76	11.74	11.93	11.94
15	17.30	16.45	16.63	16.61
20	21.84	21.14	21.31	21.27

CHAPTER V

SUMMARY

A mathematical model was developed describing the heat exchange taking place in a sea lion during swimming or diving or while hauled out on land. The model consisted of four cylinders simulating the head, the trunk, the fore-flippers and the hind-flippers. Each cylinder was divided into four concentric layers - core, muscle, blubber and skin. Both conductive and convective heat exchange were taking place in the model - conductive heat exchange radially through the layers and convective heat exchange via the blood flow.

Much of the data needed for quantification of the model was unavailable so that many assumptions and estimates were made, whenever possible, from analogous situations in other mammals.

The partial differential equations describing the heat transfer were solved numerically using a set of computer programs on the IBM 360/70 computer (see Appendix III).

The computed temperatures from the model for the sea lion hauled out on land was compared with experimental temperatures. They were in good agreement.

An increase in body temperature when the animal was diving was noted. It will be interesting to see whether

this can be proved experimentally.

With minor changes in the model, such as changing the parameters, this model can be used for understanding the thermoregulation of any marine mammal.

TABLE OF SYMBOLS

Symbol	Meaning	Dimension	Equation
A, B, C, D	Constants	None	50
BF	Blood flow	cm^3/hr	15
C_f	Frictional drag coefficient	None	9
CoB	Gain for the blood flow to blubber	$cm^3/hr\text{-}^\circ C$	16
C_p	Specific heat	$Kcals/kg\text{-}^\circ C$	19
D	Diameter	cm	4
Ev	Heat lost by evaporative cooling	$Kcal/hr$	14
Ev_1	Gain for evaporative cooling	$Kcal/hr\text{-}^\circ C$	14
Ev_R	Respiratory water loss	$Kcals/hr$	19
$Exer$	Exercise metabolism	$Kcals/hr$	15
H	Relative humidity	None	7
H_C	Convective heat transfer coefficient	$Kcals/cm^2\text{-}hr\text{-}^\circ C$	4
H_{EFF}	Effective heat transfer coefficient	$Kcals/cm^2\text{-}hr\text{-}^\circ C$	19
H_R	Radiative heat transfer coefficient	$Kcals/cm^2\text{-}hr\text{-}^\circ C$	3
K	Thermal conductivity	$Kcals/cm\text{-}hr\text{-}^\circ C$	4
P_F	Power required to overcome the frictional drag	$Kcals/h4$	13
Pr	Prandtl number	None	4
Pc	Ambient partial pressure of water	mm of Hg	7
Q	Basal metabolism	$Kcals/hr$	19
Q_R	Respiration heat loss	$Kcals/hr$	7

74

Symbol	Meaning	Dimension	Equation
Q_{RAD}	Radiative heat loss	Kcals/hr	2
R, r	Radius	cm	19
Re	Renold's number	None	9
RV	Respiratory volume	liter/hr	5
T_i	Temperature of the i-th layer	^oC	19
T_A	Ambient temperature	^oC	2
T_{BL}	Blood temperature	^oC	19
T_{HC}	Head Core temperature	^oC	6
$\hat{T}_{HC}$	Set head core temperature	^oC	14
T_m	Mean temperature of expired air	^oC	6
TQ	Total heat production	^oC	5
T_S	Weighted average skin temperature	^oC	2
$\hat{T}_S$	Set skin temperature	^oC	14
TSA	Total surface area	cm^2	1
T_{TC}	Trunk core temperature	^oC	6
TW	Total weight	kg	1
t	Time	hr	19
V	Volume	cm^3	19
v	Velocity	cm/hr	12
VASOC	Gain for vasoconstriction	$cm^3/hr-^oC$	17
VASOD	Gain for vasodilatation	$cm^3/hr-^oC$	18
X_L	Total length	cm	12
γ	Kinematic viscosity	cm^2/hr	4
ρ	Density	kg/cm^3	19

Symbol	Meaning	Dimension	Equation
λ	Quantity accounting for counter current heat exchange between veins and arteries	None	19
Ψ	Product of density and specific heat of blood	Kcals/cm^3-$^\circ$C	19

BIBLIOGRAPHY

BIBLIOGRAPHY

Anderson, H. T., ed. (1969) The Biology of Marine Mammals. Academic Press, New York.

Bryden, M. M. (1964) Insulating capacity of subcutaneous fat of the southern elephant seal. Nature. London, 203: 1299-1300.

Chambers, A. B., J. R. Blackaby and J. B. Miles (1973). A study of the thermoregulatory characteristics of a liquid-cooled garment with automatic temperature control based on sweat rate: Experimental Investigation and Biothermal Man-Model Development. NASA TN D-7311.

Elsner, R. (1969) Cardiovascular adjustments to diving. In: The Biology of Marine Mammals, edited by H. T. Anderser Academic Press, New York, pp. 117-143.

Guyton, A. C. (1966) Text-Book of Medical Physiology. Third edition, W. B. Saunders Company, Philadelphia and London.

Hart, J. S., and L. Irving. (1959) The energetics of harbor seals in air and in water with special consideration of seasonal changes. Can. J. Zool., 37: 447-457.

Hoerner, S. F. (1957) Fluid Dynamic Drag. Hoerner, New York, pp. 6-16.

Howell, A. B. (1929) Contributions to the comparative anatomy of the eared and earless seals (Genera Zalophus and Phoca). Proc. U. S. Nat. Mus. No. 2736, Vol. 73, art. 15:1-142.

Irving, L., D. M. Solandt, D. Y. Solandt and K. C. Fisher. (1935) The respiratory metabolism of the seal and its adjustment to diving. J. Cell. Comp. Physiol. 7(1): 137-151.

Irving, L., P. F. Scholander and S. W. Grinnel. (1941) Significance of the heart rate to the diving ability of the seals. J. Cell. Comp. Physiol. 18:283-297.

Irving, L. and J. S. Hart. (1957) The metabolism and insulation of seals as bare-skinned mammals in cold water. Can. J. Zool., 35: 497-511.

Kleiber, M. (1947) Body size and metabolic rate. Physiol. Revs. 27: 511-541.

Lang, T. G. (1966) Hydro dynamic analysis of dolphin fin profiles. Nature 209 (5028) 1110-1111.

Luecke, R. H., E. W. Gray and F. E. South. (1970) Simulation of passive behavior of a cooling biological system. Pflügers Archiv., 327: 37-52.

Luecke, R. H., and F. E. South. (1971) A possible model for thermoregulation during deep hibernation. In: Hibernation - Hypothermia: Perspectives and Challenges Elsevier Publishing Company, Amsterdam.

Nakayama, T. H., J. S. Eisenman, and J. D. Hardy. (1961) Single unit activity of anterior hypothalamus during local heating. Science 134, 560-561.

Matsuura, D. T. and G. C. Whittow. (1972) Partition of evaporative heat loss in the California Sea lion. Fed. Proc., 31: 826.

Murdough, H. V. Jr., B. Schmidt-Nielsen, J. W. Wood and W. L. Mitchell. (1961) Cessation of renal function during diving in the trained seal (Phoca Vitulina). J. Cell. Comp. Physiol. 58:261.

Pierce, R. W. (1970) Design and Operation of Metabolic Chamber for Marine Mammals. Ph.D. Dissertation, University of California, Berkeley.

Reid, M. H. and R. S. MacKay. (1968) Application of a new technique for system identification to Dolphin temperature regulation and rabbit intra-cular pressure control. Med. & Biol. Engng. Vol. 6, pp. 269-290.

Ridgeway, S. H., ed. (1972) Mammals of the sea, Biology and Medicine. C. C. Thomas, Springfield, Illinois.

Robinson, S., F. R. Meyer, J. L. Newton, C. H. Ts'ao and L. O. Holgersen. (1965) Relations between sweating, cutaneous blood flow, and body temperature in work. J. Appl. Physiol. 20, 575.

Scholander, P. F. (1940) Experimental investigations of the respiratory function in diving mammals and birds. Hvalradskr, 22: 1-131.

Scholander, P. F., V. Walters, R. Hock and L. Irving. (1950) Body insulation of some arctic and tropical mammals and birds. Biol. Bulletin. 99, 225-236.

Seagrave, R. C. (1971) Biomedical Applications of Heat and Mass Transfer. The Iowa State University Press, Ames, Iowa.

South, F. E., R. H. Luecke, M. D. Shanklin and M. L. Zatzman. (1973) Direct partional calorimetry of the California sea lion. Proc. 10th Ann. Conf. on Biol. Sonar and Marine Mammals. (In press)

South, F. E., (1974) (Personal Communications)

Stolwijk, J. A. J., and J. D. Hardy. (1966) Temperature regulation in man - A theoretical study. Pflügers Archiv., 291: 129-162.

Stolwijk, J. A. J., (1972) Mathematical Model of Thermoregulation. In: Physiological and Behavioral Temperature Regulation. Charles C. Thomas, Springfield, Illinois.

Wissler, E. H. (1961) Steady-state temperature distribution in man. Jour. Appl. Physiol., 16, 734-740.

Wissler, E. H. (1964) A mathematical model of the human thermal system. Bull. of Math. Biophys. 26: 147.

Wyndham, C. H. and A. R. Atkins. (1968) A physiological scheme and mathematical model of temperature regulation in man. Pflügers Archiv. 303: 14-30.

Stone, H. L., K. Gray, R. Stabe and J. M. Chandler. (1973) Renal blood flow in a diving trained sea lion. Nature. Vol. 242, No. 5399, pp. 530-531.

APPENDIX I

The finite-difference approximation for the four partial differential equations for the head are obtained as follows:

In the following equations, the subscript for section has been suppressed.

A. Core

$$A_1 = 0$$

$$B_1 = 1 + 2AA_1 \qquad \text{where} \qquad AA_1 = \frac{K_1 \Delta t}{\rho_1 C_{p_1} (\Delta r_1)^2}$$

$$C_1 = -2AA_1$$

$$D_1 = (1 - 2AA_1) T_{i1} + 2AA_1 T_{i2} + \left[Q_{11} + B F_{1i1} \lambda \psi (T_{BL} - T_{i1}) - E_{VR_{i1}} \right] \times \left[\frac{\Delta t}{C_{p_1} W_{11}} \right]$$

$$A_j = -AA_1 \left(R_1 + \Delta r_1 (j - 1.5) \right) \div \left[2 (R_1 + \Delta r_1 (j-1)) \right] \qquad j = 2 \cdots M_1$$

$$B_j = 1 + AA_1$$

$$C_j = -AA_1 (R_1 + \Delta r_1 (j - 0.5)) \div \left[2(R_1 + \Delta r_1 (j-1)) \right] \qquad j = 2 \cdots M$$

$$D_j = -A_j T_{i(j-1)} + (1 - AA_1) T_{ij} - C_j T_{i(j+1)} + \left[Q_{1j} + B F_{1ij} \lambda \psi (T_{BL} - T_i) - E_{VR_{ij}} \right] \left[\frac{\Delta t}{C_{p_1} W_{1j}} \right] j = 2 \cdots$$

B. Muscle

$$A_{M_1 + 1} = -BG_1 \times BH_1$$

$$B_{M_1 + 1} = 1 + BH_1 (BE_1 + BG_1)$$

$$C_{M_1 + 1} = -BE_1 \times BH_1$$

$$D_{M_1 + 1} = -A_{M_1 + 1} T_{i, M_1} + (2 - B_{M_1 + 1}) T_{i, (M_1 + 1)} - C_{M_1 + 1} T_{i, (M_1}$$

$$+ \left[Q_{1(M_1+1)} + BF_{1i(M_1+1)} \lambda \psi \left(T_{BL} - T_{i,(M_1+1)} \right) + Q_{2,(M_1+1)} \right.$$

$$\left. + BF_{2i(M_1+1)} \lambda \psi \left(T_{BL} - T_{i(M_1+1)} \right) + EXER_{(M_1+1)} \right] \left[\frac{\Delta t}{C_{PAV_1} W_{AV_1}} \right]$$

where,

$$BG_1 = K_1 \left(R_2 - \frac{\Delta r_1}{2} \right) \div \Delta r_1$$

$$BH_1 = \Delta t \div \left\{ \rho_{AV_1} C_{PAV_1} \left[\left(R_2 + \frac{\Delta r_2}{2} \right)^2 - \left(R_2 - \frac{\Delta r_1}{2} \right)^2 \right] \right\}$$

$$BE_1 = K_2 \left(R_2 + \Delta r_{2/2} \right) \div \Delta r_2$$

$$AA_2 = K_2 \Delta t^2 / \rho_2 C_{P_2} (\Delta r_2)^2$$

$$A_j = -AA_2 \left(R_2 + \Delta r_2 (j - 1.5) \right) \div 2 \left(R_2 + \Delta r_2 (j-1) \right)$$
$$j = (M_1 + 2) \cdots M$$

$$B_j = 1 + AA_2$$

$$C_j = -AA_2 \left(R_2 + \Delta r_2 (j - 0.5) \right) \div 2 \left(R_2 + \Delta r_2 (j-1) \right) \quad j = (M_1 + 2) \cdots$$

$$D_j = -A_j T_{i(j-1)} + (1 - AA_2) T_{ij} - C_j T_{i(j+1)}$$

$$+ \left[Q_{2j} + B F_{2ij} \lambda \psi \left(T_{BL} - T_{ij} \right) + EXER_j \right] \left[\frac{\Delta t}{C_{P_2} W_{2j}} \right]$$

$$j = (M_1 + 2) \cdots M_2$$

C. **Blubber**

$$A_{M_2+1} = -BG_2 \quad BH_2$$

$$B_{M_2+1} = 1 + BH_2 \left(BE_2 + BG_2 \right)$$

$$C_{M_2+1} = -BE_2 \, BH_2$$

$$D_{M_2+1} = -A_{M_2+1} \, T_{i\,M_2} + (2 - B_{M_2+1}) \, T_{i\,(M_2+1)}$$

$$- C_{M_2+1} \, T_{i\,(M_2+2)} + \Big[Q_{2\,(M_2+1)} + \Big(B \, F_{2\,i\,(M_2+1)}$$

$$+ B \, F_{3\,i\,(M_2+1)} \Big) \, \lambda \, \psi \Big(T_{BL} - T_{i\,(M_2+1)} \Big) \, Q_{3\,(M_2+1}$$

$$+ EXER_{(M_2+1)} \Big] \left[\dfrac{\Delta t}{C_{P_{AV_2}} \, W_{AV_2}} \right]$$

where,

$$BG_2 = K_2 \left[R_3 - \Delta r_2 / 2 \right] \div \Delta r_2$$

$$BH_2 = \Delta t \Big/ P_{AV_2} \, C_{P_{AV_2}} \left[\left(R_3 + \dfrac{\Delta r_3}{2} \right)^2 - \left(R_3 - \dfrac{\Delta r_2}{2} \right)^2 \right]$$

$$BE_2 = K_3 \left(R_3 + \dfrac{\Delta r_3}{2} \right) \div \Delta r_3$$

$$AA_3 = K_3 \, \Delta t \Big/ P_3 \, C_{P_3} \, (\Delta r_3)^2$$

$$A_j = -AA_3 \Big(R_3 + \Delta r_3 (j - 1.5) \Big) \div 2 \Big(R_3 + \Delta r_3 (j-1) \Big) \quad J = (M_2 + 2) \cdots$$

$$B_j = 1 + AA_3$$

$$C_j = -AA_3 \left(R_3 + \Delta r_3 (j - 0.5) \right) \div 2 \left(R_3 + \Delta r_3 (j-1) \right) \qquad j = (M_2 + 2) \cdots$$

$$D_j = -A_j \, T_{i(j-1)} + (1 - AA_3) \, T_{ij} - C_j \, T_{i(j+1)}$$

$$+ \left[Q_{3j} + BF_{3ij} \, \lambda \, \psi \, (T_{BL} - T_{ij}) \right] \left[\frac{\Delta t}{C_{P3} \, W_{3j}} \right]$$

$$j = (M_2 + 2) \cdots\cdots M_3$$

D. Skin

$$A_{M_3+1} = -BG_3 \; BH_3$$

$$B_{M_3+1} = 1 + BH_3 \, (BE_3 + BG_3)$$

$$C_{M_3+1} = -BE_3 \; BH_3$$

$$D_{M_3+1} = -A_{M_3+1} \, T_{i \, M_3} + (2 - B_{M_3+1}) \, T_{i \, (M_3+1)}$$

$$- C_{M_3+1} \, T_{i \, (M_3+2)} + \left[Q_{3 \, (M_3+1)} + Q_{4 \, (M_3+1)} \right.$$

$$+ \left(BF_{3 \, i \, (M_3+1)} + BF_{4 \, i \, (M_3+1)} \right) \left(T_{BL} - T_{i \, (M_3+1)} \right) \Big] \; x$$

$$\left[\frac{\Delta t}{C_{P_{AV_3}} \, W_{AV_3}} \right]$$

where,

$$BG_3 = K_3 \left(R_4 - \Delta r_{3/2} \right) \div \Delta r_3$$

$$BH_3 = \frac{\Delta t}{\rho_{AV_3} \, C_{P_{AV_3}}} \left[\left(R_4 + \Delta r_{4/2} \right)^2 - \left(R_4 - \Delta r_{3/2} \right)^2 \right]$$

$$BE_3 = K_4 \left(R_4 + \Delta r_{4/2} \right) \div \Delta r_4$$

$$AA_4 = \frac{K_4 \, \Delta t}{\rho_4 \, C_{P_4} \, (\Delta r_4)^2}$$

$$A_j = -AA_4 \left(R_4 + \Delta r_4 \, (j - 1.5) \right) \div 2 \left(R_4 + \Delta r_4 (j-1) \right)$$
$$j = (M_3 + 2) \, \cdots$$

$$B_j = 1 + AA_4$$

$$C_j = -AA_4 \left(R_4 + \Delta r_4 \, (j - 0.5) \right) \div 2 \left(R_4 + \Delta r_4 \, (j-1) \right)$$
$$j = (M_3 + 2) \, \text{-----} \quad N$$

$$D_j = -A_j \, T_{i(j-1)} + (1 - AA_4) \, T_{ij} - C_j \, T_{i(j+1)}$$
$$+ \left[Q_{4j} + BF_{4ij} \, \lambda \, \psi \, (T_{BL} - T_{ij}) \right] \left[\frac{\Delta t}{C_{P_4} \, W_{4j}} \right]$$
$$j = (M_3 + 2) \, \text{-----} \quad M_4$$

$$A_{M_4 + 1} = -AA_4 \left(R_5 - \Delta r_{4/2}\right) \div \left(R_5 - \Delta r_{4/4}\right)$$

$$B_{M_4 + 1} = 1 + \frac{R_s \, H_{eff} \, DELT}{\rho_4 \, c_{P_4} \, \Delta r_4 \left(R_5 - \frac{\Delta r_4}{4}\right)} - A_{M_4 + 1}$$

$$C_{M_4 + 1} = 0$$

$$D_{M_4 + 1} = \left(2 - B_{M_4 + 1}\right) T_{i \, (M_4 + 1)} - A_{M_4 + 1} \, T_{i \, M_4}$$

$$+ \frac{2 \, R_s \, H_{eff} \, DELT \, T_A}{\rho_4 \, c_{P_4} \, \Delta r_4 \left(R_5 - \Delta r_{4/4}\right)} + \left[\frac{\Delta t}{c_{P_4} \, W_{4 \, (M_4 + 1)}}\right] X$$

$$\left[Q_{4 \, (M_4 + 1)} + BF_{4 \, i \, (M_4 + i)} \, \lambda \, \psi \, T_{BL} - T_{i \, (M_4 + 1)} - E_v\right]$$

APPENDIX II

A recursion formula, which is known as a Graussian procedure, is developed as follows:

Consider the following tridiagonal matrix equation:

$$\begin{bmatrix} B_1X_1+C_1X_2 \\ A_2X_1+B_2X_2+C_2X_3 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ A_iX_{i-1}+B_iX_i+C_iX_{i+1} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ B_NX_{N-1}+C_NX_N \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_i \\ \cdot \\ \cdot \\ Y_N \end{bmatrix}$$

$$(A-1)$$

Let

$$X_i = \Gamma_i - \frac{C_i X_{i+1}}{\beta_i} \qquad (A-2)$$

in which γ_i and β_i are to be determined. Substituting the equation (A-2) into i_th row in both sides of the matrix equation (A-1),

$$X_i = \frac{Y_i - A_i \Gamma_{i-1}}{B_i - \dfrac{A_i C_{i-1}}{\beta_{i-1}}} - \frac{C_i X_{i+1}}{B_i - \dfrac{A_i C_{i-1}}{\beta_{i-1}}} \quad \cdots \cdots \quad (A-3)$$

From equation (A-3)

$$\beta_i = B_i - \frac{A_i C_{i-1}}{\beta_{i-1}} \qquad (A-4)$$

$$\gamma_i = \frac{Y_i - A_i \, \gamma_{i-1}}{\beta_i} \tag{A-5}$$

From the first equation in the matrix equation (A-1)

$$X_1 = \left(Y_1/\beta_1\right) - \left(A_1 + C_1\right) X_2 / B_1$$

whence,

$$\beta_1 = B_1 \quad \text{and} \quad \gamma_1 = Y_1/\beta_1 \tag{A-6}$$

Finally, substitution of the recursion solution into the Nth equation of the matrix equation)A-2), gives

$$X_N = \frac{Y_N - A_N X_{N-1}}{B_N} = \frac{Y_N - A_N\left(\gamma_{N-1} - \frac{C_{N-1}}{\beta_{N-1}} X_N\right)}{B_N} \tag{A-7}$$

whence

$$X_N = \frac{Y_N - A_N \gamma_{N-1}}{B_N - \dfrac{A_N \, C_{N-1}}{\beta_{N-1}}} = \gamma_N \tag{A-8}$$

Therefore, we have the following relationships:

$$X_N = \gamma_N$$

$$X_i = \gamma_i - \frac{C_i \, X_{i+1}}{\beta_i} \qquad i = 1, 2, \cdots N-1$$

where β's and γ's are determined from the recursion formula

$$\beta_1 = B_1$$

$$r_1 = Y_1 / \beta_1$$

$$X_1 = \frac{r_1 - (A_1 + c_1) X_2}{\beta_1}$$

$$\beta_i = B_i - \frac{A_i \, c_{i-1}}{\beta_{i-1}} \qquad i = 2, \cdots N$$

$$r_i = \frac{Y_i - A_i \, r_{i-1}}{\beta_i} \qquad i = 2, \cdots N$$

APPENDIX III

A computer program was written to solve the seventeen partial differential equations describing the model. The temperatures of the different parts of the body are printed as a function of time.

The equations are set up for the general case so that any sea lion weight may be used. This is done by assuming that the blood flow, metabolism etc., are functions of the total weight of the animal.

Fortran IV language for the IBM 360 system is used in the program. The flow diagram for the main program can be found in Figure III-1.

```
                    ┌─────────┐
                    │  START  │
                    └─────────┘
                         │
                         ▼
                ┌──────────────────┐
                │ Read, I, Time    │
                │      Delt        │
                └──────────────────┘
                         │
                         ▼
                ┌──────────────────────┐
                │ Calculate Physiological │
                │      Dimensions      │
                └──────────────────────┘
                         │
                         ▼
                ┌────────────────────────────┐
                │ Calculate Initial Temp. Distri- │
                │   bution and Average Temp.  │
                └────────────────────────────┘
                         │
                         ▼
                ┌────────────────────────────┐
                │ Calculate Basal Blood Flow, │
                │ Metabolism, & Exercise Metabolism │
                │        Distributions        │
                └────────────────────────────┘
                         │
                         ▼
                       ◇ I>4 ◇
        ┌─────────┐  No ◇     ◇
        │ I=I+1   │◄────◇   ◇
        └─────────┘      Yes
                         │
                         ▼
                ┌────────────────────────┐
                │ Adjust Blood Flow With │
                │   Feed Back Control    │
                └────────────────────────┘
                         │
                         ▼
                ┌────────────────────────────┐
                │ Calculate Heat Transfer Coeffi- │
                │  cient & Evaporation Losses │
                │     and Blood Temperature   │
                └────────────────────────────┘
                         │
                         ▼
                     ┌───────┐
                     │  I=1  │
                     └───────┘
                         │
                         ▼
                ┌────────────────────────────┐
                │ Calculate New Temp. Distri- │
                │  bution & Ave. Temperatures │
                └────────────────────────────┘
                         │
                         ▼
                       ◇ I>4 ◇
        ┌─────────┐  No ◇     ◇
        │ I=I+1   │◄────◇   ◇
        └─────────┘      Yes
                         │
                       Time>5.0
                         No
                         Yes
                         │
        ┌─────────────────┐
        │ Time=Time+Delt  │
        └─────────────────┘
                         ▼
                     ┌────────┐
                     │  STOP  │
                     └────────┘
```
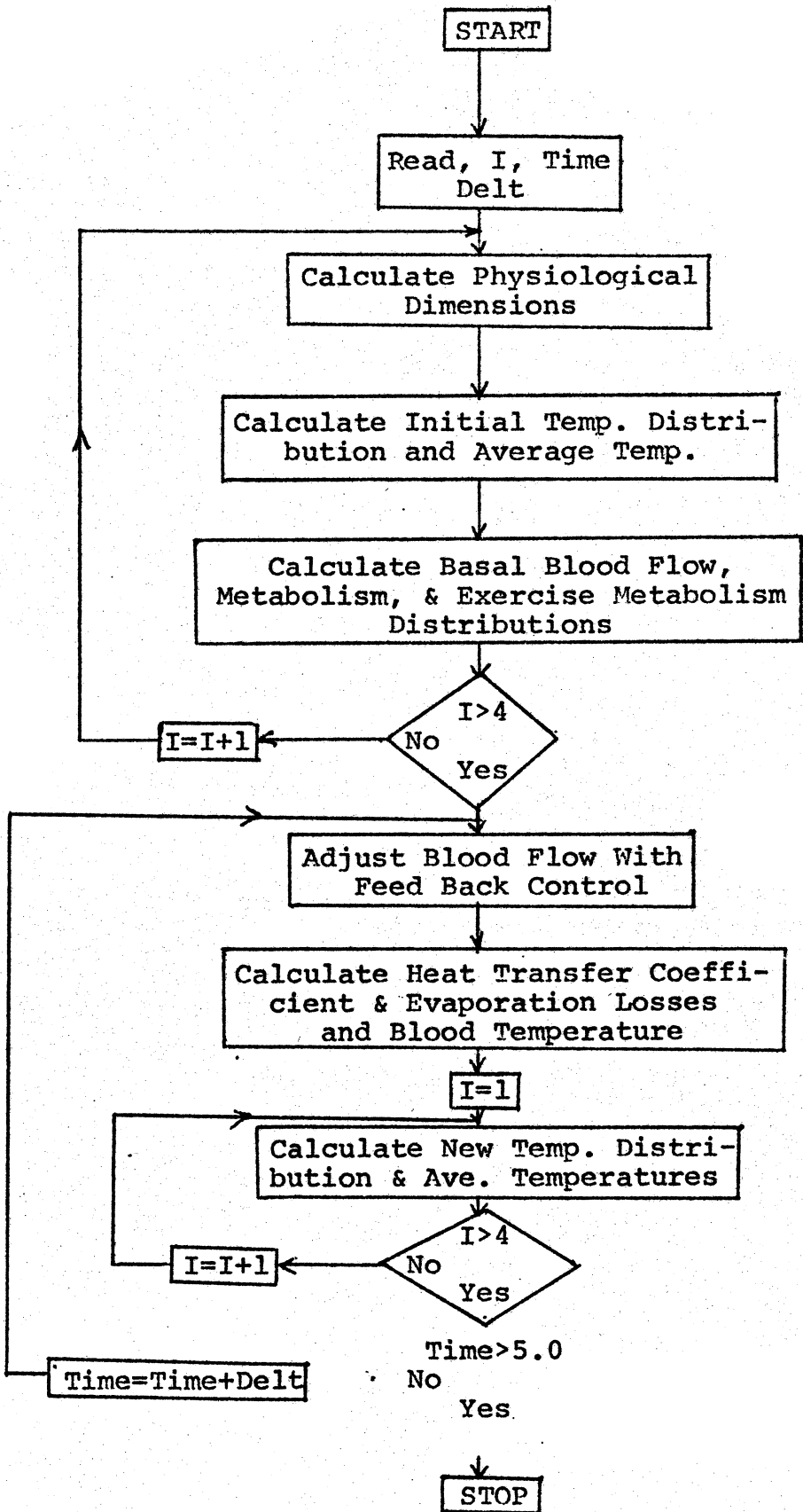
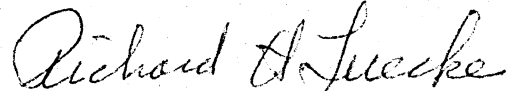Figure III-1.   Flow Chart for Program

The undersigned, appointed by the Dean of the Graduate Faculty, have examined a thesis entitled
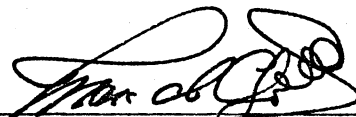
MODEL FOR THE THERMOREGULATION OF SEA LIONS

presented by   V. NATARAJAN
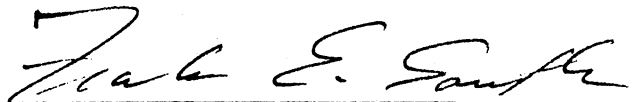
a candidate for the degree of   MASTER OF SCIENCE

and hereby certify that in their opinion it is worthy of acceptance.

_____
DR. R.H. LUECKE

_____
DR. L.E. MARC de CHAZAL

_____
DR. FRANK SOUTH

University Libraries
University of Missouri

Digitization Information Page

Local identifier   Natarajan1974

Source information

    Format   Book
    Content type   Text
    Source ID   Gift copy from department; not added to MU
collection.
    Notes   Includes handritten documents in appendix

Capture information

    Date captured   July 2023
    Scanner manufacturer   Fujitsu
    Scanner model   fi-7460
    Scanning system software ScandAll Pro v. 2.1.5 Premium
    Optical resolution   600 dpi
    Color settings   8 bit grayscale
    File types   tiff
    Notes

Derivatives - Access copy

    Compression   Tiff: LZW compression
    Editing software   Adobe Photoshop
    Resolution   600 dpi
    Color   grayscale
    File types   pdf created from tiffs
    Notes   Images cropped, straightened, brightened