# INEQUALITIES OF CORRELATION TYPE FOR SYMMETRIC STABLE RANDOM VECTORS

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Abstract. We point out a certain class of functions $f$ and $g$ for which random variables $f(X_1,\dots,X_m)$ and $g(X_{m+1},\dots,X_k)$ are non-negatively correlated for any symmetric jointly stable random variables $X_i$. We also show another result that is related to the correlation problem for Gaussian measures of symmetric convex sets.

## 1. Introduction

For $0<q\le 2$, let $Y$ be a symmetric $q$-stable random vector in $\mathbb{R}^n$ with characteristic function

$$\phi(\theta)=\exp(-\|\sum_{i=1}^n \theta_i s_i\|^q), \quad \theta\in\mathbb{R}^n, \tag{1}$$

where $s_1,\dots,s_n\in L_q([0,1])$, and the norm is taken from the space $L_q([0,1])$.

For any $k\in\mathbb{N}$, and any choice of vectors $\xi_1,\dots,\xi_k\in\mathbb{R}^n$, the inner products $X_1=(Y,\xi_1),\dots,X_k=(Y,\xi_k)$ are symmetric $q$-stable random variables. The random variables $X_1,\dots,X_k$ are jointly $q$-stable with zero mean, and we say that they are $\mathbb{R}^n$-generated in case we need to emphasize the dimension of the vector $Y$.

In this article, we show that, for any $m<k$, and any even continuous positive definite functions $f$ and $g$ on $\mathbb{R}^m$ and $\mathbb{R}^{k-m}$ respectively, the random variables $f(X_1,\dots X_m)$ and $g(X_{m+1},\dots X_k)$ are non-negatively correlated, i.e.

$$\mathbb{E}\big(f(X_1,\dots,X_m)\ g(X_{m+1},\dots,X_k)\big)\ge \mathbb{E}f(X_1,\dots X_m)\ \mathbb{E}g(X_{m+1},\dots,X_k), \tag{2}$$

where $\mathbb{E}$ stands for the expectation.

Inequality (2) reminds one of some results related to the concept of associated random variables. Recall that random variables $X_1,\dots,X_k$ are said to be associated if, for any choice of non-decreasing (in each variable) functions $f$ and $g$ on $\mathbb{R}^k$, the random variables $f(X_1,\dots,X_k)$ and $g(X_1,\dots,X_k)$ are non-negatively correlated whenever the expectations exist. Pitt (1982) proved that jointly Gaussian

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random variables are associated if and only if the correlation between each pair is non-negative. Lee, Rachev and Samorodnitsky (1990) generalized this result to the case of jointly $q$-stable random variables by giving a necessary and sufficient condition in terms of the spectral measure. Inequality (2) points out a special class of functions $f$ and $g$ for which the correlation between $f(X)$ and $g(X)$ is non-negative independently of relations between the jointly $q$-stable random variables $X_i$. For other results related to association of random variables, see Joag-dev, Perlman and Pitt (1983), and Suquet (1994).

Another celebrated result of Pitt (1977) shows that, for any jointly Gaussian $\mathbb{R}^2$-generated random variables $X_1, \ldots, X_k$, inequality (2) holds if $f$ and $g$ are the indicator functions of cubes in $\mathbb{R}^m$ and $\mathbb{R}^{k-m}$, namely, for each $t > 0$,

$$P(\max_{1\le i\le k} |X_i| < t) \ge P(\max_{1\le i\le m} |X_i| < t)\; P(\max_{m+1\le i\le k} |X_i| < t). \tag{3}$$

In other words, the quantity in the left-hand side is minimal (subject to the given marginal distributions) if for each choice of $i, j$ with $1 \le i \le m$ and $m+1 \le j \le k$ the random variables $X_i$ and $X_j$ are independent, that is to say, $b_{ij} = \mathrm{Cov}(X_i, X_j) = 0$. An equivalent formulation of the same fact is that, for any symmetric convex sets $F$ and $G$ in $\mathbb{R}^2$, $\mu(F \cap G) \ge \mu(F)\mu(G)$, where $\mu$ is a symmetric Gaussian measure in $\mathbb{R}^2$. The question of whether the same is true for symmetric convex sets in $\mathbb{R}^n$ ( and, correspondingly, for $\mathbb{R}^n$-generated Gaussians) remains open (see Schlumprecht, Schechtman and Zinn (1994) for a historical survey and partial results).

In Section 3, we consider the quantity in the left-hand side of (3) as a function of the $m(k-m)$ variables $b_{i,j}$, and prove that, for every dimension $n$, this function has a local minimum at the origin. Note that, to solve the problem completely, one has to prove that the function has global minimum at the origin.

## 2. A correlation inequality for positive definite functions of stable variables

In order to prove inequality (2) we need the following simple fact.

**Lemma 1.** *Let $0 < q \le 2$, and $\xi, \eta$ be any vectors from the space $L_q([0,1])$. Then*

$$\exp(-\|\xi+\eta\|^q) + \exp(-\|\xi-\eta\|^q) \ge 2\exp(-\|\xi\|^q - \|\eta\|^q).$$

*Proof.* A result of W. Orlicz (1933) (see also Clarkson (1936)) states that, for every $0 < q \le 2$ and $\xi, \eta \in L_q$,

$$\|\xi+\eta\|^q + \|\xi-\eta\|^q \le 2(\|\xi\|^q + \|\eta\|^q).$$

Now use the inequality relating the arithmetic and geometric means to obtain

$$\exp(-\|\xi+\eta\|^q) + \exp(-\|\xi-\eta\|^q) \ge$$
$$2\exp(-\|\xi+\eta\|^q/2 - \|\xi-\eta\|^q/2) \ge 2\exp(-\|\xi\|^q - \|\eta\|^q). \quad \square$$

**Theorem 1.** *Let $0 < q \le 2$ and $X_1, \dots, X_k$ be jointly $q$-stable random variables. Then for any $m < k$ and any even continuous positive definite functions $f, g$ on $\mathbb{R}^m$ and $\mathbb{R}^{k-m}$ respectively, the random variables $f(X_1, \dots, X_m)$ and $g(X_{m+1}, \dots, X_k)$ are non-negatively correlated.*

*Proof.* By Bochner's theorem, $f$ and $g$ are the characteristic functions of finite measures $\mu$ and $\nu$ on $\mathbb{R}^m$ and $\mathbb{R}^{k-m}$ respectively. The measures $\mu$ and $\nu$ are symmetric because the functions $f$ and $g$ are even.

Let $Y$ be the $q$-stable random vector in $\mathbb{R}^n$ generating $X_1, \dots, X_k$, and let $\xi_1, \dots, \xi_k \in \mathbb{R}^n$ be the vectors for which $X_1 = (Y, \xi_1), \dots, X_k = (Y, \xi_k)$. Denote by $\gamma$ the distribution of the vector $Y$, so $\gamma$ is a probability $q$-stable measure in $\mathbb{R}^n$ with the characteristic function given by (1).

Using Fubini's Theorem, we see that

$$\mathbb{E}\big(f(X_1, \dots, X_m)\ g(X_{m+1}, \dots, X_k)\big)$$

$$= \int_{\mathbb{R}^n} f\big((x, \xi_1), \dots, (x, \xi_m)\big)\ g\big((x, \xi_{m+1}), \dots, (x, \xi_k)\big)\ d\gamma(x)$$

$$= \int_{\mathbb{R}^n} \Big( \int_{\mathbb{R}^m} \exp(-i(u_1(x, \xi_1) + \cdots + u_m(x, \xi_m)))\ d\mu(u_1, \dots, u_m) \times$$

$$\int_{\mathbb{R}^{k-m}} \exp(-i(u_{m+1}(x, \xi_{m+1}) + \cdots + u_k(x, \xi_k)))\ d\nu(u_{m+1}, \dots, u_k) \Big)\ d\gamma(x)$$

(4)

$$= \int_{\mathbb{R}^m} \int_{\mathbb{R}^{k-m}} \Big( \int_{\mathbb{R}^n} \exp\big( -i(x, \sum_{j=1}^{k} u_j \xi_j)\big)\ d\gamma(x) \Big)\ d\mu(u_1, \dots, u_m)\ d\nu(u_{m+1}, \dots, u_k).$$

Let $\alpha = \sum_{j=1}^{m} u_j \xi_j, \beta = \sum_{j=m+1}^{k} u_j \xi_j \in \mathbb{R}^n$. Considering the coordinates of the vectors $\alpha$ and $\beta$ as linear functions of the coordinates of $u_1, \dots, u_m$ and $u_{m+1}, \dots, u_k$, respectively, and using (1) we see that the quantity in (4) is equal to

(5)

$$I_1 = \int_{\mathbb{R}^m} \int_{\mathbb{R}^{k-m}} \exp(-\| \sum_{j=1}^{n} \alpha_j s_j + \sum_{j=1}^{n} \beta_j s_j \|^q)\ d\mu(u_1, \dots, u_m)\ d\nu(u_{m+1}, \dots, u_k),$$

where the norm is taken from the space $L_q([0,1])$. Denote by $I_2$ the expression in (5) with minus instead of plus under the norm. Since the measure $\nu$ is symmetric, $I_1 = I_2$. By Lemma 1,

$$(I_1 + I_2)/2 \ge \int_{\mathbb{R}^m} \int_{\mathbb{R}^{k-m}} \exp(-\| \sum_{j=1}^{n} \alpha_j s_j \|^q) \times$$

$$\exp(-\| \sum_{j=1}^{n} \beta_j s_j \|^q)\ d\mu(u_1, \dots, u_m)\ d\nu(u_{m+1}, \dots, u_k)$$

$$= \int_{\mathbb{R}^m} \exp(-\|\sum_{j=1}^n \alpha_j s_j\|^q)\ d\mu(u_1,\dots,u_m)\ \times$$

$$\int_{\mathbb{R}^{k-m}} \exp(-\|\sum_{j=1}^n \beta_j s_j\|^q)\ d\nu(u_{m+1},\dots,u_k).$$

Repeating all the calculations in the reverse order we show that the latter quantity is equal to $\mathbb{E}f(X_1,\dots X_m)\ \mathbb{E}g(X_{m+1},\dots,X_k)$ which finishes the proof. $\square$

**Examples.** (i) Let $f(x_1,\dots,x_m) = (1-|x_1|)_+\cdots(1-|x_m|)_+$, and $g(x_{m+1},\dots,x_k) = (1-|x_{m+1}|)_+\cdots(1-|x_k|)_+$, where the function $(1-|t|)_+$ is equal to $1-|t|$ if $t\in[-1,1]$, and is equal to zero otherwise. It is well known that the function $(1-|t|)_+$ is positive definite, and hence $f$ and $g$ are positive definite. Thus, by Theorem 1, for every $m<k$ and every jointly stable random variables $X_1,\dots,X_k$,

$$\mathbb{E}\big((1-|X_1|)_+\cdot\ldots(1-|X_k|)_+\big) \geq$$

$$\mathbb{E}\big((1-|X_1|)_+\cdot\ldots(1-|X_m|)_+\big)\ \mathbb{E}\big((1-|X_{m+1}|)_+\cdot\ldots(1-|X_k|)_+\big).$$

The latter inequality can be generalized by taking any functions $f$ and $g$ of the form $f(x_1,\dots,x_m) = f_1(x_1)\dots f_m(x_m)$, $g(x_{m+1},\dots,x_k) = f_{m+1}(x_{m+1})\dots f_k(x_k)$, where $f_1,\dots,f_k$ are even functions on $\mathbb{R}$ which are convex and decreasing on $[0,\infty)$. Such functions $f_i$ are positive definite by a well-known result of Polya.

(ii) Let $q_1,\dots,q_k\in(0,2]$, $f(x_1,\dots,x_m) = \exp(-|x_1|^{q_1}-\cdots-|x_m|^{q_m})$, and $g(x_{m+1},\dots,x_k) = \exp(-|x_{m+1}|^{q_{m+1}}-\cdots-|x_k|^{q_k})$. Since for any $q\in(0,2]$ the function $\exp(-|t|^q)$ is positive definite, it follows that $f$ and $g$ are positive definite. Therefore, for every $m<k$

$$\mathbb{E}\big(\exp(-|X_1|^{q_1}-\cdots-|X_k|^{q_k})\big) \geq$$

$$\mathbb{E}\big(\exp(-|X_1|^{q_1}-\cdots-|X_m|^{q_m})\big)\mathbb{E}\big(\exp(-|X_{m+1}|^{q_{m+1}}-\cdots-|X_k|^{q_k})\big).$$

**Remarks.** (i) In the case of jointly Gaussian random variables the result of Theorem 1 can be extended to some classes of continuous functions $f$ and $g$ with power growth at infinity and such that their Fourier transforms (in the sense of distributions) are non-negative locally integrable functions with power growth at infinity. To do that, consider the convolutions of the functions $f$ and $g$ with Gaussian densities $e_n$ approaching the $\delta$-function as $n\to\infty$, and slightly modify the proof of Theorem 1.

(ii) Y. Hu has recently proved that, for any even convex functions $f$ and $g$ on $\mathbb{R}^n$ and jointly Gaussian random variables $X_1,\dots,X_n$, the random variables $f(X_1,\dots,X_n)$ and $g(X_1,\dots,X_n)$ are non-negatively correlated (private communication from T. Schlumprecht; compare the result of Hu with our Example 1).

## 3. On the local minimum in the correlation for Gaussian measures of symmetric convex sets

Let $\nu$ be the standard symmetric Gaussian measure on $\mathbb{R}^n$. Is it true that

$$\nu(F \cap G) \geq \nu(F)\nu(G) \tag{6}$$

for all symmetric convex sets $F$ and $G$ in $\mathbb{R}^n$ ? In 1977, L. Pitt proved that the answer is positive in the case $n = 2$. However, the question of whether the answer is positive for every dimension $n$ is still open.

It can be seen that it suffices to consider the sets $F = \{x \in \mathbb{R}^n : |(x, \xi_1)| \leq 1, \dots, |(x, \xi_k)| \leq 1\}$ and $G = \{x \in \mathbb{R}^n : |(x, \xi_{k+1})| \leq 1, \dots, |(x, \xi_{2k})| \leq 1\}$, where $k$ is an integer, and $\xi_1, \dots, \xi_k, \xi_{k+1}, \dots, \xi_{2k} \in \mathbb{R}^n$. For these sets $F$ and $G$, inequality (6) can be written in the form

$$P(\max_{1 \leq i \leq 2k} |X_i| < 1) \geq P(\max_{1 \leq i \leq k} |X_i| < 1)\ P(\max_{k+1 \leq i \leq 2k} |X_i| < 1), \tag{7}$$

where $X_1, \dots, X_{2k}$ are the jointly Gaussian random variables generated by the vectors $\xi_1, \dots, \xi_k, \xi_{k+1}, \dots, \xi_{2k} \in \mathbb{R}^n$ and a standard Gaussian random vector $Y$ in $\mathbb{R}^n$, so that $X_i = (Y, \xi_i)$ for each $i$.

It is easy to see that, to prove inequality (6), it suffices to consider the case where the vectors $\xi_i,\ i = 1, \dots, 2k$ are linearly independent. For example, if $n < 2k$ and the system of vectors $\xi_i$ has rank $n$, we can transfer everything to the space $\mathbb{R}^{2k}$, and consider the vectors $\eta_i = \xi_i + \epsilon e_i \in \mathbb{R}^{2k},\ i = 1, \dots, 2k$ where, for each $i$, either $e_i = 0$ or $\|e_i\| = 1$ and $e_i$ is orthogonal to each of the vectors $\xi_j,\ j = 1, \dots, 2k$ and $e_j,\ j \neq i,$ so that the vectors $\eta_i$ are linearly independent in $\mathbb{R}^{2k}$. Then inequality (7) for the random variables generated by the vectors $\eta_i$ would imply inequality (7) for the random variables generated by $\xi_i$'s by taking the limit as $\epsilon \to 0$ and applying the Lebesgue dominated convergence theorem.

Assume that the vectors $\xi_i \in \mathbb{R}^{2k},\ i = 1, \dots, 2k$ are linearly independent. Then the joint distribution $\mu$ of random variables $X_1, \dots, X_{2k}$ is a non-singular Gaussian measure in $\mathbb{R}^{2k}$, and the left-hand side of (7) is equal to

$$P(\max_{1 \leq i \leq 2k} |X_i| < 1) = \mu([-1, 1]^{2k}).$$

We fix the scalar products $(\xi_i, \xi_j)$ for all choices of $i, j$ with either $1 \leq i, j \leq k$ or $k+1 \leq i, j \leq 2k$, and consider the quantity $\mu([-1, 1]^{2k})$ as a function of $k^2$ variables $b_{i,j} = \mathrm{Cov}(X_i, X_j),\ i = 1, \dots, k,\ j = k+1, \dots, 2k$. To prove Pitt's inequality, one has to show that this function has a global minimum at zero. Being unable to do that we show instead that the function has a local minimum at zero. This fact is a simple consequence of Theorem 2 below.

In the proof of Theorem 2 we use one result about log-concave functions. A non-negative function $f$ on $\mathbb{R}^k$ is called log-concave if, for every choice of $x, y \in \mathbb{R}^k$, and $0 \leq t \leq 1$,

$$f(tx + (1 - t)y) \geq f(x)^t f(y)^{1-t}.$$

This means that the function $\log(f)$ is concave. Prekopa (1973) and Leindler (1972) have proved that if $f$ is a log-concave function on $\mathbb{R}^k$ and $0 < m < k$, then the function

$$g(x_1, \dots, x_m) = \int_{\mathbb{R}^{k-m}} f(x_1, \dots, x_m, z_1, \dots, z_{k-m})\, dz$$

is also log-concave.

**Theorem 2.** *Let $F$ and $G$ be symmetric convex sets in $\mathbb{R}^k$, and $\mu_B$ be a non-singular probability Gaussian measure in $\mathbb{R}^{2k}$ with the covariance matrix $\mathcal{A} = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$. Fix the $k \times k$ matrices $A$ and $C$, and consider $B = (b_{i,j})_{i,j=1}^k$ as a variable from the space $\mathbb{R}^{k^2}$. Then the function $B \mapsto \mu_B(F \times G)$ has a local minimum at the point $B = 0$.*

*Proof.* Without loss of generality, we may suppose that $F$ and $G$ have compact closure. Let $\chi_F,\ \chi_G$ be the indicator functions of the sets $F$ and $G$. Taking Fourier transforms, we obtain

$$\mu_B(F \times G) = \int_{\mathbb{R}^k} \int_{\mathbb{R}^k} \chi_F(x)\chi_G(y)\, d\mu_B(x, y)$$

$$= \int_{\mathbb{R}^k} \int_{\mathbb{R}^k} \hat{\chi_F}(x)\hat{\chi_G}(y) \exp(-\tfrac{1}{2}(x^T Ax + y^T Cy + 2x^T By))\, dx\, dy.$$

Taking the second partial derivative by $b_{i,j}$ and $b_{m,n}$, we get

$$H_{i,j,m,n} = \frac{\partial^2}{\partial b_{i,j} \partial b_{m,n}} \mu_B(F \times G)$$

$$= \int_{\mathbb{R}^k} \int_{\mathbb{R}^k} \hat{\chi_F}(x)\hat{\chi_G}(y)(x_i x_m y_j y_n) \exp(-\tfrac{1}{2}(x^T Ax + y^T Cy + 2x^T By))\, dx\, dy$$

$$= \frac{1}{(2\pi)^k |\mathcal{A}|^{1/2}} \int_F \int_G \frac{\partial^4}{\partial x_i \partial x_m \partial y_j \partial y_n} \exp(-\tfrac{1}{2}(x, y)^T \mathcal{A}^{-1}(x, y))\, dy\, dx.$$

The fact that $|\mathcal{A}| \neq 0$, and the validity of using Parseval's Equality in the latter equations, follow from the non-singularity of the measure $\mu_B$.

Since the sets $F$ and $G$ are symmetric, the partial derivative of the function $B \mapsto \mu_B(F \times G)$ by each $b_{i,j}$ is equal to zero at the point $B = 0$. In order to show that there is a local minimum at $B = 0$, we need to know that $H$ is positive definite when $B = 0$. Furthermore, by a change of variables, we see that it is sufficient to consider the special case when $A = C = I$. Hence, we need to show the positive definiteness of

$$H_{i,j,m,n} = \frac{1}{(2\pi)^{2k}} L_{i,m} K_{j,n}$$

where

$$L_{i,m} = \int_F (x_i x_m - \delta_{i,m}) \exp(-\tfrac{1}{2} x^T x)\, dx$$

and

$$K_{j,n} = \int_G (y_j y_n - \delta_{j,n}) \exp(-\tfrac{1}{2} y^T y)\, dy.$$

Since $H = L \otimes K$, it is sufficient to show that $L$ and $K$ are negative definite, and clearly it is enough just to prove it for $L$.

Thus we desire to show that

$$\sum_{i,m} L_{i,m} \alpha_i \alpha_m = \int_F ((\sum_i \alpha_i x_i)^2 - \|\alpha\|_2^2) \exp(-\tfrac{1}{2} x^T x)\, dx < 0$$

for all $\alpha \neq 0$. But by a change of variables, it is sufficient to show

$$\int_F (x_1^2 - 1) \exp(-\tfrac{1}{2} x^T x)\, dx < 0$$

for every convex symmetric set $F$ with compact closure.

To show this, we see this as

$$\int_{-\infty}^{\infty} (x_1^2 - 1) \exp(-\tfrac{1}{2} x_1^2) \phi(x_1)\, dx_1,$$

where

$$\phi(x_1) = \int_{\mathbb{R}^{k-1}} \chi_F(x_1, \dots, x_k) \exp(-\tfrac{1}{2}(x_2^2 + \cdots + x_k^2))\, dx_2 \dots dx_k.$$

Since $\chi_F(x) \exp(-\frac{1}{2}(x_2^2 + \cdots + x_k^2))$ is log-concave in $\mathbb{R}^k$, the result of Prekopa and Leindler mentioned before the formulation of Theorem 2 implies that $\phi$ is also log-concave. Since $\phi$ is also symmetric, it follows that $\phi(x_1) = \phi_1(|x|)$, where $\phi_1$ is a decreasing function. Furthermore, since $F$ has compact closure, $\phi_1$ is non-constant. Hence in order to show that

$$\int_{-\infty}^{\infty} (x_1^2 - 1) \exp(-\tfrac{1}{2} x_1^2) \phi(x_1)\, dx_1 < 0,$$

it is sufficient to show that for all $0 < a < \infty$

$$\theta(a) = \int_{-a}^{a} (x_1^2 - 1) \exp(-\tfrac{1}{2} x_1^2)\, dx_1 < 0.$$

The function under the latter integral has antiderivative $-x_1 \exp(-\frac{1}{2} x_1^2)$, so the result follows. □

Finally, we present one more argument showing that inequality (6) would be proved if one showed that the function from Theorem 2 had global minimum at zero.

Let $A = C = I$. Since the sets $F$ and $G$ are convex, their topological boundaries have zero Lebesgue measure. Let $\nu$ be standard Gaussian measure on $\mathbb{R}^k$. Then $\mu_0(F \times G) = \nu(F)\nu(G)$, whereas $\lim_{\lambda\to 1} \mu_{\lambda I}(F \times G) = \nu(F \cap G)$. To see this last assertion, note that

$$\mu_{\lambda I}(F \times G) = \frac{1}{((2\pi(1-\lambda^2))^k} \int_F \int_G \exp(-\frac{1}{2(1-\lambda^2)}(x^T x - 2\lambda x^T y + y^T y))\, dy\, dx$$

which, making the substitution $x = u + v$, $y = u - v$

$$= \frac{1}{(\pi(1-\lambda^2))^k} \int_{\mathbb{R}^k} \int_{(F-u)\cap(u-G)} \exp(-\frac{u^2}{1+\lambda} - \frac{v^2}{1-\lambda})\, dv\, du.$$

Now, if $u$ is not in the boundary of $F$ or the boundary of $G$, then it is easily seen that

$$\lim_{\lambda\to 1} \frac{1}{(\sqrt{\pi}(1-\lambda))^k} \int_{(F-u)\cap(u-G)} \exp(-\frac{v^2}{1-\lambda})\, dv = \chi_{F\cap G}(u).$$

Hence the last assertion follows by Lebesgue's law of dominated convergence.

It is clear now that, if the function $\mu_B$ has global minimum at zero then $\mu_{\lambda I}(F \times G) \geq \mu_0(F \times G)$, and, hence, $\nu(F \cap G) \geq \nu(F)\nu(G)$. However, the question of whether the function from Theorem 2 has global minimum at zero remains open.

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## References

1. J.A. Clarkson (1936), *Univormly convex spaces*, Trans. A.M.S. **40**, 396–414.
2. K. Joag-dev, M. D. Perlman and L. Pitt (1983), *Asociation of normal random variables and Slepian's inequality*, Ann. Probab. **11**, 451–455.
3. M.-L. T. Lee, S. Rachev and G. Samorodnitsky (1990), *Association of stable random variables*, Ann. Probab. **18**, 1759–1764.
4. L. Leindler (1972), *On a certain converse of Hölder's inequality II*, Acta. Sci. Math. Szeged **33**, 217–223.
5. W. Orlicz (1933), *Uber unbedingte Konvergenz in Funktionen Raumen I and II*, Studia Math. **4**, 33–37 and 41–47.
6. L.Pitt (1977), *A correlation inequality for gaussian measures of symmetric convex sets*, Ann. Probab. **5**, 470–474.
7. L. Pitt (1982), *Positively correlated normal variables are associated*, Ann. Probab. **10**, 496–499.
8. A. Prekopa (1973), *On logarithmic concave measures and functions*, Acta Sci. Math. (Szeged) **34**, 335–343.
9. T. Schlumprecht, G. Schechtman and J. Zinn (1994), *On the Gaussian measure of the intersection of symmetric convex sets, preprint.*
10. C. Suquet (1994), *Introduction a l'association*, Pub. IRMA, Lille **34 (no. XIII)**, 3–19.

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