Mathematical Problems from Cryobiology

James Dale Benson

Professor Carmen Chicone and Professor John Critser, Dissertation Supervisors

ABSTRACT

Cryobiology is the study of life and death at low temperatures and provides a fascinating setting for applied mathematics. The interdisciplinary nature of cryobiology mirrors the diversity of applications ranging from animal agriculture to laboratory cell and species preservation to critical human clinical applications for the preservation of life and for the killing of cells during cryosurgery. The work comprising this thesis develops approaches for optimization of cryobiological protocols, and defines a new model for common cryobiological procedures. The first step is to advance an understanding of the optimal control of a classical ODE system describing the mass transport that occurs during cryopreservation. This investigation leads to the description of exact solutions to this 70-year-old nonlinear system, a global stability result for the generalized system with n-solutes, controllability and existence of optimal controls in the n-solute case, and a complete synthesis of optimal controls in the 2-solute case. After defining optimal controls, the question arises whether the predicted continuous optimal control of the extracellular environment affects the hypotheses of the ODE model, namely, perfect stirring inside and outside of the cell/tissue-media boundary. We constructed a new model coupling the ODE mass transport at the cell/tissue boundary of changing radius with convection-diffusion and potential flow models and a numerical integration scheme to explore the effects of advection on the cell-media interface.