ESSAYS IN FINANCIAL MACROECONOMICS

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by

MOHAMMAD NABI ARJMANDI

Dr. Chao Gu

Dr. Joseph H. Haslag

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The undersigned, appointed by the Dean of the Graduate School, have examined the dissertation entitled

ESSAYS IN FINANCIAL
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presented by Mohammad Nabi Arjmandi,

a candidate for the degree of Doctor of Philosophy,

and hereby certify that, in their opinion, it is worthy of acceptance.

Dr. Chao Gu

Dr. Joseph H. Haslag

Dr. Shawn Ni

Dr. Kuntara Pukthuanthong

DEDICATION

I dedicate this dissertation to my cherished family. Their unwavering love and support have fueled my academic journey, and I'm grateful for their sacrifices and belief in my pursuit of knowledge.

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I would like to express my sincere gratitude to my family, advisors, and professors especially, Dr. Haslag, Dr. Gu, and Dr. Dybvig, for their unwavering support and guidance throughout my dissertation. Their expertise, encouragement, and belief in my abilities have been instrumental in my success. I am also thankful to my friends, colleagues, and research participants for their valuable contributions. Their support has made this journey meaningful and rewarding.

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Mohammad Nabi Arjmandi

Dr. Chao Gu, Dissertation Supervisor

Dr. Joseph H. Haslag, Dissertation Supervisor

ABSTRACT

This dissertation delves into critical aspects of financial markets and intermediaries, shedding light on optimal portfolio rebalancing strategies in the presence of cash distributions and transaction costs, as well as the simultaneous determination of deposit and loan contracts within financial intermediaries.

The first chapter concentrates on optimizing portfolio rebalancing by considering cash distributions and proportional transaction costs. It introduces sweep accounts as pivotal components, exploring their roles and demonstrating how transaction costs influence investment choices. This chapter also investigates the impact of asset cash distributions and investor investment horizons on asset demand, revealing shifts in liquidity premiums.

In the second chapter, the research presents a comprehensive framework where deposit and loan contracts coexist in equilibrium, offering a deeper understanding of financial intermediaries' dual roles. With a focus on managing repayment risk while having access to a safe yet lower-yielding asset, this chapter examines how varying collateral fractions influence loan terms and deposit contracts. The findings provide valuable insights into the correlation between borrowers' commitment and loan terms.

Through its multifaceted exploration of portfolio management, transaction costs, deposit, and loan contracts, this dissertation enhances our comprehension of financial intermediation dynamics and contributes to the broader knowledge of financial markets.

Chapter 1

Optimal Portfolio Rebalancing with Sweep Under Transaction Cost

Absrtract

This paper investigates the optimal portfolio rebalancing strategy for assets with cash distributions and proportional transaction costs. A sweep account is an account that is used as the default destination for coupon and dividend proceeds as they arrive. In this study, we incorporate this account and investigate the optimal strategy for the sweep account manager. Our results indicate that the "no-transaction" region is split into two sub-regions, where the cash proceeds are either invested entirely in the riskless asset or in the risky asset, depending on the transaction costs. Additionally, we analyze the impact of the assets' cash distributions and the investors' investment horizon on the demand for the assets. Our findings suggest that changes in the cash distribution of assets, depending on the relative magnitude of transaction costs for risky and riskless assets, can have a varying impact on asset demand. In particular, our results indicate that when the transaction cost for the risk-

less asset is low, an increase in the cash distributions from the risky asset and an increase in the investor's investment horizon have a positive impact on the liquidity premium of the risky asset.

1.1 Introduction

A sweep account is a type of cash management account that automatically transfers excess cash balances into higher-yielding investment vehicles, such as money market funds or equity investment accounts. The function of a sweep account is to help investors manage the cash received from their investments more effectively and gain higher returns on their cash earnings without having to actively manage their cash balances. It's also worth noting that sweep accounts have become increasingly popular in recent years due to the low interest rate environment and the need for investors to find ways to earn a higher return on their cash. As a result, many banks and financial institutions now offer sweep accounts as a way to help their customers manage their cash more effectively.

This study examines the effect of assets' cash distributions on the optimal portfolio rebalancing strategy. Specifically, this paper studies the problem of optimal portfolio allocation of assets with cash distributions under the assumption of a constant relative risk aversion (CRRA) investor facing proportional transaction costs when purchasing the asset. In this economy, the CRRA investor maximizes their final wealth. We assume that the investor's investment horizon is finite and stochastic. Following the literature, the economy is modeled with two assets - a risky equity and a riskless bond. The investor's portfolio, in turn, consists of two accounts: a bond account that receives coupon payments and an equity account that receives dividend payments.

However, in order to incorporate the impact of assets' cash distribution on the investors' portfolio rebalancing strategy, we add a third account called the sweep account. All cash distributions originating from the bond and equity accounts are automatically deposited into the sweep account, which is then managed to optimize the allocation of cash balances back into the bond and equity accounts. In the absence of liquidity shocks, optimal cash allocation would entail the reinvestment of cash proceeds into either the bond or equity account. This study is designed to provide an analysis of the impact of cash distributions and proportional transaction costs on the optimal portfolio allocation, and how they influence the investment behavior of the investor. Notably, the study's results reveal a binary optimal sweep decision, where the sweep account's entire balance is allocated to either bond or equity purchases.

The results of this analysis indicate that an increase in the dividend payout ratio of the equity asset leads to a rise in demand for equity and a decrease in demand for bonds, assuming that the transaction cost associated with the bond is comparatively lower than those associated with equity. However, beyond a critical ratio of transaction cost of bond to equity, the impact of a higher dividend payout ratio reverses, and the demand for equity starts to wane. Moreover, this paper delves into the impact of a higher coupon rate of bonds on the investment decision of fund managers. We show that when the coupon rate on bonds rises, as the transaction cost of the bond correspondingly increases, the sweep policy does not change regardless of whether coupon rates are high or low. However, this increase in the coupon rate leads to a decrease in the demand for both equity and bonds.

Another critical aspect of the proposed financial model lies in the consideration of the time horizon of the investors. Although these investors are technically maximizing their utility over an indefinite span of time, the assumption that the end of their investment horizon is randomly arriving presents us with an opportunity to gain a deeper understanding of the impact of the expected time horizon on their investment behavior. In this paper, we demonstrate that the relative size of the transaction cost of bonds and equity influences investors' tendency to change their investment behavior as they approach their investment horizon. Specifically, as the investment horizon approaches, in an environment where the transaction cost of bond is comparatively lower than the transaction cost of equity, investors decrease their rate of equity purchases and increase the pace of bond purchases. This result is consistent with the findings of Hopenhayn and Werner (1996), who demonstrated that as investors' investment horizons shorten and they desire to consume earlier, their demand for a more liquid asset with a lower expected payoff would increase.

On the other hand, when the transaction cost of risk-free assets becomes relatively larger compared to the transaction cost of equity, the behavior of investors changes in response to the shortening of their expected time to their investment horizon. In this scenario, investors tend to increase their allocation towards the risky asset and decrease their demand for the risk-free asset, reflecting the impact of cash distribution associated with these assets on the demand for these assets. Furthermore, the study demonstrates that the degree of risk aversion among investors has an adverse effect on the demand for the equity and a positive effect on the demand for the bond.

Finally, this paper builds upon previous studies on the liquidity premium of equities and examines how it is affected by the equity dividend payout and the investment horizon of investors. Constantinides (1986), defines liquidity premium as the extra return that an investor earns as compensation for the lack of liquidity associated with a particular asset. In other words, investors demand a higher rate of return for assets that are less liquid or difficult to sell quickly, compared to assets that are highly liquid and easily tradeable.

Drawing from this definition, we demonstrate that higher dividends would have a positive impact on the demand for equities when the transaction cost for bond is low. This increased demand would, in turn, result in higher transaction costs associated with acquiring the asset which leads to an increase in the asset's liquidity premium to compensate investors for their elevated expenditure on transaction costs.

In addition to exploring the relationship between cash payments and the liquidity premium of an asset, this study also examines the impact of investors' investment horizons on the liquidity premium for an asset with transaction costs. It is shown that as investors' investment horizons shorten, their demand for the risky asset would decrease and their demand for the riskless asset would increase, depending on the gap in the two transaction costs. This decrease in demand for the risky asset would then reduce the investor's expenditure on transaction costs associated with acquiring this asset. As the demand for the risky asset decreases and the expenditure on transaction costs associated with acquiring the asset decline, the asset would lose some of its liquidity premia.

Constantinides (1986) showed that transaction costs can increase the liquidity premium of an asset. He asserted that investors in equilibrium should receive a higher mean return for an asset with transaction costs, as compensation for the increased expenditure associated with acquiring the asset. Constantinides further claimed that the liquidity premium is typically lower in magnitude than the transaction cost.

However, recent studies have challenged this view. Papers such as Lynch and Tan (2011) have demonstrated that the liquidity premium can be of the same magnitude as the transaction cost, in certain cases. Lynch and Tan showed that when predictable returns and wealth shocks to labor income are introduced, transaction costs can result in liquidity premia that are on the same order of magnitude as the transaction cost spread. Here, we demonstrate that incorporating two additional factors, namely asset cash payouts and investors' average investment horizon, into the analysis can help explain some of the underestimations of the liquidity premium observed in Constantinides' research.

The findings of this research suggest that the coupon rate of bonds and the dividend payout ratio of equities play a crucial role in determining the optimal portfolio allocation of assets. Furthermore, the results of this study can provide valuable insights for investment decision-making regarding the benefits of sweep accounts. Additionally, it can contribute to a deeper understanding of the impact of cash distributions and transaction costs on the demand for assets within the economy.

1.2 Related Literature

The academic literature on optimal portfolio allocation in the presence of transaction costs has been widely explored. Merton (1971) first addressed the issue of optimal portfolio allocation when there are no transaction costs involved. He showed that there exists an optimal ratio of bonds to equity in a portfolio that an investor should maintain by continuously rebalancing the portfolio.

However, when transaction costs are considered, the problem becomes more complex. The problem of optimal portfolio rebalancing when investors are facing transaction costs has been studied under different specifications. Constantinides (1979) and Constantinides (1986) analyzed the problem of optimal portfolio choice under the condition of maximizing infinite lifetime consumption and proportional transaction costs. Taksar et al. (1988), Davis and Norman (1990), and Dumas and Luciano (1991) also tackled the issue using the stochastic singular control problem and showed that proportional transaction costs create three distinct regions in the allocation space.

The first region, referred to as the no-transaction (NT) region, is a convex cone in the portfolio allocation space where investors do not rebalance their portfolio. The second region is known as the Buy (B) region where the investor rebalances the portfolio by buying the risky asset and selling the riskless asset. The boundary between the Buy region and the

No-Transaction region is called the Buy-Boundary. The third region is known as the Sell (S) region, where the investor rebalances the portfolio by selling the risky asset and buying the riskless asset. The boundary between the Sell region and the No-Transaction region is called the Sell-Boundary.

Liu and Wu (2001)further extended the literature and explored the impact of transaction costs on an optimal consumption and investment decision, assuming that security returns have bounded uncertainty. Some other papers studied the impact of fixed transaction cost on optimal portfolio choice. Among them, Liu (2004) studied the optimal portfolio allocation problem with fixed transaction costs and multiple risky assets for investors with constant absolute risk aversion (CARA) utility, while Dybvig (2020) analyzed the problem for mean-variance utility maximizer investors. They found that with fixed transaction costs, investors optimally maintain their portfolio allocation between two constant levels and rebalance their positions as soon as their portfolio allocation reaches either boundary to reach an optimal target.

Other research has focused on the optimal consumption and portfolio strategy when taking into account labor income. Bodie et al. (1992) added to this body of research by examining the influence of the labor-leisure choice on portfolio and consumption decisions throughout an individual's life cycle. Furthermore, Dybvig and Liu (2010) investigated the optimal consumption and portfolio problem in the context of voluntary or mandatory retirement and the presence or absence of a non-negative wealth constraint, which restricts borrowing against future wages.

Several academic studies have explored the topic of optimal portfolio selection with transaction costs and a finite investment time horizon. Gennotte and Jung (1994) and Boyle and Lin (1997) are among the seminal works in this field, and they have demonstrated that as the time horizon of investors increases, the no-transaction boundaries grow in a monotonic fashion, ultimately converging to the infinite horizon case in the limit.

The literature in this field has primarily focused on either self-financing portfolios or assets that do not have any cash distribution. To the best of my knowledge, the only study that has briefly mentioned the case of dividend-paying assets is Dumas and Luciano (1991), who state that "the case of a dividend-paying asset would be an interesting case and the extension would require an additional state variable," but they do not provide any in-depth examination. The importance of considering cash payments from assets in investment portfolios was highlighted by Blume (1980), who conducted a survey revealing a strong preference among individual investors for dividend-paying stocks.

This trend of dividend-paying stocks being more attractive than non-dividend-paying stocks in the investment world is at odds with the findings of Modigliani and Miller (1958) and Modigliani and Miller (1963), who showed that dividends are irrelevant under certain stringent assumptions. To address this apparent contradiction, researchers have attempted to explain why investors prefer dividend-paying stocks. Dybvig and Zender (1991) and Ofer and Thakor (1987) proposed that information asymmetry models could help explain the signals firms send to investors by paying dividends. Frankfurter and Lane (1992) suggested that behavioral biases could also play a role in investors' preference for dividends.

In Section 1.3.1, we present a solution to the allocation problem when the transaction cost for bonds is zero. The solution is based on a free boundary ordinary differential equation, which is similar to the solution found in Davis and Norman (1990). In this scenario, the boundary for the sweep falls on the boundary for buying equity, and the no-transaction region is always the region where cash is transferred from the sweep account to the bond account. The main difference between the solution in this paper and the solution in Davis and Norman is the consideration of non-zero dividend payout ratios for equity.

In Section 1.3.2, we extend the analysis to the case where the transaction costs for both bonds and equity are non-zero. This paper shows that in this scenario, the sweep boundary is strictly inside the no-transaction region. This means that the no-transaction region splits into two sub-regions, and based on the transaction costs for bonds and equity, the investor can transfer cash from the sweep account to either the bond or equity account. By incorporating the impact of dividends in the model, we analyzes the effect of dividends on the demand for the risky asset and it's liquidity premia. And in section 1.4 we conclude.

1.3 The Continuous Time Model

In this economy, we assume there is a continuum of investors who live for a finite period, but their investment horizon arrival is not deterministic. The investors can trade two assets, one riskless (bond) and another risky (equity).

The shares of the assets are infinitely divisible. Investors take the price of the assets as given and they can only long the assets with zero capital gain tax at the time of sale. Let the equity price follows geometric Brownian motion with mean μ, standard deviation σ, and a constant dividend yield of q. The bond pays a constant rate of return of r with a coupon rate of c. We assume that $\mu - r > 0$.

To formalize the investor's portfolio holdings, we denote H_t^b as the dollar value of the investor's bond holdings at time t, and H_t^s as the dollar value of their equity holdings. Assume the investor incurs proportional transaction costs of λ^b only when purchasing bond and λ^s only when purchasing equity. Following the assumption that the sale of assets is free of transaction costs, the investor's wealth at time t is given by $W_t = H_t^b + H_t^s$. Suppose that $H_0^b = x$ is the initial endowment of bond, and $H_0^s = y$ is the initial endowment of equity. Thus, the initial wealth of the investor is $W_0 = x + y$. As a result, the law of motion for the investor's holdings, in the absence of any transactions at time t, is as follows:

$$dH_t^b = H_t^b(r-c)dt + cH_t^b dt \tag{1.1}$$

$$dH_t^s = (\mu - q)H_t^s dt + qH_t^s dt + \sigma H_t^s dz_t \tag{1.2}$$

Equation 1.1 reflects the investor's bond holding law of motion at time t, and Equation 1.2 shows the investor's equity holding law of motion at time t. In Equation 1.2, z_t follows standard Brownian motion, which represents the stochastic component of the risky asset. In the absence of any transaction costs, changes to the coupon rate or dividend yield will not affect the path of the accounts since these earnings can be reinvested in the same account without incurring any costs.

Suppose the investor has a finite time horizon that is exponentially distributed with a parameter value of η. Thus, the probability of the investor meeting their horizon at time $\tau \in dt$ is $\eta e^{-\eta t}$. Consequently, the expected arrival time of the investment horizon is $\frac{1}{\eta}$. This assumption allows us to convert the current finite horizon problem to an infinite time horizon problem, which will later help us to find the stationary solution to the problem.

Assumption 1.1. *The investor has a finite horizon τ which arrives randomly and it follows an exponential distribution with a parameter value of η. The investor also has a constant relative risk aversion (CRRA) preference over their final wealth, $\frac{W_\tau^{1-\gamma}}{1-\gamma}$, where W_τ is the investor's wealth at time τ, and $0 < \gamma \neq 1$.*

Following this assumption, the investors' objective is to maximize their expected utility over their final wealth. Thus, based on the investor's horizon assumption, the expected value of the investor's utility over their final wealth can be transformed into an expected value over an infinite horizon.

$$E^*\Big[\frac{W_\tau^{1-\gamma}}{1-\gamma}\Big] = E\Big[\int_0^\infty \eta e^{-\eta t}\frac{W_t^{1-\gamma}}{1-\gamma}dt\Big] \tag{1.3}$$

Expectation E^* has measure on everything and the expectation E has measure on everything other than the horizon arrival. This transformation of the objective function allows us to study the stationary solution for the optimal portfolio rebalancing strategy. If we maximize Equation 1.3 in the absence of transaction costs subject to constraints of Equations 1.1 and 1.2, the solution would be similar to Merton (1971), where the investor's optimal portfolio allocation is a constant ratio of bonds to equity equal to $\frac{\gamma\sigma^2}{\mu-r} - 1$, and the investor would continuously trade to maintain this optimal ratio of bonds to equity.

Lemma 1.1. *Let $\eta > (1-\gamma)(r + \frac{(\mu-r)^2}{2\gamma\sigma^2})$, x be the value of the bond account, and y be the value of the equity account of the investor's portfolio. If the investor's objective is to maximize Equation* 1.3 *subject to Equations* 1.1*, and* 1.2*, the investor's optimal portfolio rebalancing policy is $\frac{x}{y} = \frac{\gamma\sigma^2}{\mu-r} - 1$, and the optimal value function is*
$v(x,y) = \eta\Big(\eta - (1-\gamma)(r + \frac{(\mu-r)^2}{2\gamma\sigma^2})\Big)^{-1}\frac{(x+y)^{1-\gamma}}{1-\gamma}$

Proof. Merton (1971) □

In this lemma, the assumption of $\eta > (1-\gamma)(r + \frac{(\mu-r)^2}{2\gamma\sigma^2})$ ensures the concavity of the optimal value function and it is necessary for the existence of an optimal solution. In the next section, we extend Merton's problem by introducing a positive proportional transaction cost for purchasing equity. We show that this extension would transform the problem into a similar one as discussed in Davis and Norman (1990). However, unlike their study, this paper considers the existence of equity dividends that are transferred to the bond account in the equilibrium.

1.3.1 Optimal Portfolio When Only Equity is Subject to Transaction Cost

To incorporate the transaction cost of purchasing the equity, we define non-decreasing, adapted, and right-continuous processes U_t and L_t, where U_t represents the cumulative dollar value of bond purchase and L_t represents the cumulative dollar value of equity purchase from time 0 until time t.

To optimize the expected utility of the final wealth, investors must determine the optimal bond, equity, and cash sweep values. Additionally, we define the Sweep account as a repository in which the cash proceeds from bonds and equity accumulate. The fraction χ_t denotes the proportion of the sweep account at time t, which the investor directs towards the equity account, while $1 - \chi_t$ denotes the proportion directed towards the bond account.

Problem 1.1. *Consider an investor facing a proportional transaction cost of $\lambda^s > 0$ when purchasing equity and a zero transaction cost for bonds. Then, the investor seeks to choose χ_t, U_t, L_t for $t \in [0, \infty]$ in order to maximize the following problem:*

$$v(x, y) = \max_{L_t, U_t, \chi_t} \mathbb{E} \int_0^\infty \left[\eta e^{-\eta t} \frac{W_t^{1-\gamma}}{1-\gamma} \right] dt \tag{1.4}$$

subject to:

$$dH_t^b = \Big[(r - c)H_t^b + (1 - \chi_t)\Big(cH_t^b + qH_t^s \Big) \Big] dt - (1 + \lambda^s) dL_t + dU_t \tag{1.5}$$

$$dH_t^s = \Big[(\mu - q)H_t^s + \frac{1}{1 + \lambda^s} \chi_t \Big(cH_t^b + qH_t^s \Big) \Big] dt + \sigma H_t^s dz_t + dL_t - dU_t \tag{1.6}$$

$$\forall t \quad H_t^b, H_t^s \geq 0, \qquad H_0^b = x, \qquad H_0^s = y$$

In Problem 1.1, the investor seeks to maximize their expected utility of their final wealth over a random horizon that arrives at a rate of η by choosing the optimal equity strategy L_t, bond strategy U_t, and sweep strategy χ_t. The first constraint, as stated in Equation 1.5, is the law of motion for the bond account. Bonds held in this account earn a deterministic interest rate of r and receive a coupon rate of c. To purchase dU_t units of bonds, the investor must sell dU_t units of equity, as the sale of equity incurs no transaction costs. The second constraint, as stated in Equation 1.6, is the law of motion for the equity account.

Equity is a risky asset that follows GBM with mean return μ and standard deviation σ. The investor receives dividend payments from the equity at a rate of q at time t. To add dL_t units of equity to the equity account, the investor must sell $(1 + \lambda^s)dL_t$ units of bonds to cover the transaction cost of purchasing the equity. The third condition reflects the assumption that short selling of any asset is not allowed, and that investors can only hold positive balances of each asset. It also ensures that the investment strategy set is compact.

At each time, the investor receives cash distributions from the bond and equity assets held in their portfolio, which are automatically deposited into the sweep account as they arrive. As a result, the cash balance in the sweep account at time t is given by the sum of the cash distributions received from the bond account, which are represented by cH_t^b, and the cash distributions from the equity account, which are represented by qH_t^s. Given the sweep account balance, the investor reallocates a fraction χ_t of this account into the equity account and a fraction $1 - \chi_t$ into the bond account.

Proposition 1.1 addresses the solution to Problem 1.1, which pertains to dividend-paying assets and positive transaction costs for the equity asset, while the bond has zero transaction costs (i.e., $\lambda^b = 0$). When the transaction cost of one asset is zero, investors treat that account as a sweep account and deposit the cash proceeds from all accounts into the account with zero transaction cost, which in this case is the bond account. As a result, the boundaries of the sweep account and the zero-transaction cost account overlap in the portfolio allocation space.

The usual assumptions in the relevant literature, such as in Davis and Norman (1990), are that assets do not pay dividends, or if they do, the dividend is assumed to be costlessly reinvested in the same asset. The solution to Problem 1.1 is similar to that of Davis and Norman, except for the presence of a sweep account to manage the dividend from the risky asset. However, this difference leads to some variations in the no-transaction region.

Lemma 1.2. *Assume that a solution to Problem 1.1 exists for all initial values x and y > 0 , and that the value function is twice continuously differentiable with respect to x and y. Then the value function is concave and has the following homothetic property:* $v(x, y) = y^{1-\gamma}\psi(\frac{x}{y})$*, and* $\psi(\frac{x}{y})$ *is a* C^2 *function.*

Proof. Davis and Norman (1990), Muzere (2001) □

Proposition 1.1. *In Problem 1.1, the optimal sweep decision of the investor is to always transfer cash into the bond account. Having the optimal rebalancing conditions, setting* $y = 1$*, there exists a unique sell boundary* x_0*, and a unique buy boundary* x_T*.*

(i) *There exists a boundary, called the sell boundary at* x_0*, such that if* $x \leq x_0$*, the investor's optimal strategy is to sell equity and buy bonds until the portfolio reaches the sell boundary. The region in the allocation space to the left of the sell boundary is called the sell region. The investor's value function in the sell region is given by,* $\psi(x) = \frac{1}{1-\gamma}A(x+1)^{1-\gamma}$*, for a constant value of* A*.*

(ii) *There exists a boundary, called the buy boundary at* x_T*, such that if* $x \geq x_T$*, the investor's optimal strategy is to buy equity and sell bonds until the portfolio reaches the buy boundary. The region in the allocation space to the right of the buy boundary* *is called the buy region. The investor's value function in the buy region is given by,* $\psi(x) = \frac{1}{1-\gamma}B(x+1+\lambda^s)^{1-\gamma}$, *for a constant value of* B.

(iii) *The region in between the sell boundary,* x_0, *and the buy boundary,* x_T, *is called the no transaction region, where the investor would not trade. The value function in this region is solved through the following free boundary differential equation.*

$$\beta_3 x^2 \psi''(x) + (\beta_2 x + q)\psi'(x) + \beta_1 \psi(x) + \frac{\eta}{1-\gamma}(x+1)^{1-\gamma} = 0 \tag{1.7}$$

Where, $\beta_1 = (\mu - q - \frac{1}{2}\sigma^2\gamma)(1-\gamma) - \eta, \quad \beta_2 = \sigma^2\gamma + r - \mu + q, \quad \beta_3 = \frac{1}{2}\sigma^2$

Such that at the boundaries, x_0, *and* x_T, *the following conditions hold:*

$$\frac{\psi'(x_T)}{\psi(x_T)} = \frac{1-\gamma}{x_T + 1 + \lambda^s} \qquad \frac{\psi'(x_0)}{\psi(x_0)} = \frac{1-\gamma}{x_0 + 1} \tag{1.8}$$

$$\frac{\psi''(x_T)}{\psi'(x_T)} = \frac{-\gamma}{x_T + 1 + \lambda^s} \qquad \frac{\psi''(x_0)}{\psi'(x_0)} = \frac{-\gamma}{x_0 + 1}$$

Proof. *See Appendix* □

Figure 1.1 displays the results of Proposition 1.1, which illustrates the asset allocation space, where the horizontal axis represents the value of the bond account, and the vertical axis represents the value of the equity account. The middle cone-shaped area denotes the no-transaction (NT) region where investors refrain from trading until their portfolio value reaches the boundaries of the cone. The boundaries are straight lines passing through the origin and $(x_0, 1)$ and $(x_T, 1)$, respectively, owing to the homothetic property of the value function and the boundary conditions.

The Merton line, located inside the shaded region, shows the portfolio where investors

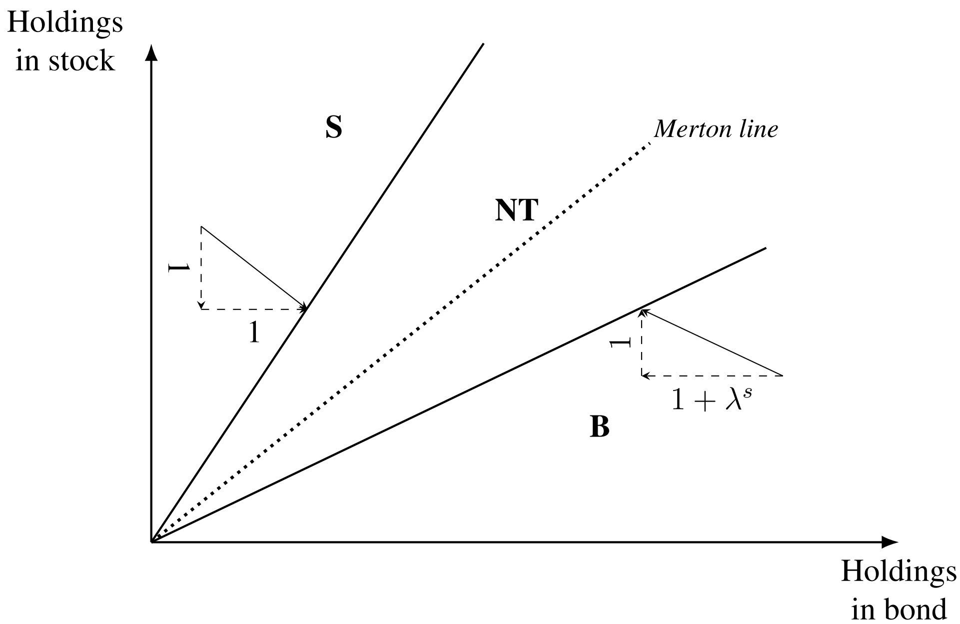

Figure 1.1: Space of Bond and Stock with regions of trading and no-trading $\lambda^b = 0$, $\lambda^s > 0$.

continuously rebalance their portfolios to remain on the line, given zero transaction costs. However, the presence of transaction costs creates this wedge in the asset allocation space. Within the wedge, the cost of rebalancing the portfolio exceeds the benefits, prompting investors to let the portfolio deviate from the Merton line until it reaches either boundary of the NT region. At the boundary, the investor is indifferent, and hence, only aims to maintain the portfolio at the boundaries once it drifts out of the NT region.

The region on the left side of the no-transaction region, marked by *S*, is known as the sell region where investors sell equity and buy bonds to maintain the sell boundary of the no-transaction region. The value function in this region along the trading direction (a line with a slope of -1) is constant. On the right side of the no-transaction region, the buy region, labeled as *B*, represents the area where investors purchase equity and sell bonds to stay on the buy boundary. In this region, the value function is constant along the trading direction, which is a line with a slope of $-\frac{1}{1+\lambda^s}$.

Proposition 1.1 also demonstrates that the sweep boundary coincides with the buy boundary. In the no-transaction region, the investor uses all the cash proceeds from bonds and equities to buy bonds. In the region to the right of the buy boundary, the investor would use the cash balance in the sweep account to buy equity. As the investor would never let the portfolio drift out of the NT region, the sweep action always moves cash toward buying bonds.

Proposition 1.2. *Lets assume that* $\eta > (1-\gamma)(r + \frac{(\mu-r)^2}{2\gamma\sigma^2})$. *The exact solution to the ordinary differential equation in Proposition 1.1 is given by,* $\psi(x) = C_1\Psi_1(x) + C_2\Psi_2(x) + \psi_p(x)$, where,

$$\Psi_i(x) = x^{-k_i}\Phi(a_i, b_i; \frac{q}{\beta_3}x^{-1})$$

$$\psi_p(x) = \Psi_2(x)\int_0^x \Psi_1(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)} - \Psi_1(x)\int_0^x \Psi_2(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)}$$

$$W(x) = \Psi_1(x)\Psi_2'(x) - \Psi_2(x)\Psi_1'(x)$$

$$k_i = \frac{(\beta_2-\beta_3) \pm \sqrt{(\beta_2-\beta_3)^2 - 4\beta_3\beta_1}}{2\beta_3}, \qquad i = 1, 2$$

$$a_i = k_i, \qquad b_i = -\frac{\beta_2}{\beta_3} + 2k_i + 2$$

$$\Phi(a, b; y) = 1 + \Sigma_{j=1}^{\infty}\frac{(a)_j}{(b)_j}\frac{y^j}{j!} \quad \text{where,} \quad (a)_j = a(a+1)...(a+j-1), \quad (a)_0 = 1.$$

Proof. *See Appendix* □

Proposition 1.2 indicates that the exact solution to the ordinary differential equation that governs the optimal value function in the no-transaction region (as introduced in Proposition 1.1) takes the form of Kummer's confluent hypergeometric function. This Proposition implies that the solution to Problem 1.1 is unique boundaries denoted by x_0 and x_T, such that it is optimal not to trade when the portfolio value lies between these boundaries.

The importance of this extension in our model is that it treats cash distribution optimally, enabling us to examine the impact of the dividend payout ratio on the asset demand in the economy or variations in the no-transaction region.

Studying the sensitivity of the no-transaction region is crucial because it determines the optimal trading behavior of investors. For example, if the transaction region widens, investors will trade less frequently to maintain the optimal level of portfolio selection. This, in turn, reduces the demand for assets, which is relevant to the liquidity premium demanded by investors in the economy. In the next subsection, we present the sensitivity of the no-transaction region and the equity's liquidity premium with respect to the dividend payout ratio and investment horizon.

Comparative Statics

In this part, we show how different parameters such as the dividend payout ratio of equity, and the investor's horizon could affect the no-transaction region boundaries.

Figure 1.2 displays the no-transaction region boundaries' shape concerning the transaction cost size for the following parameters: $\mu = 0.12$, $\sigma = 0.4$, $\gamma = 0.45$, $\eta = 0.05$, $r = 0.065$, $q = 0.01$. These parameters were selected to satisfy, $0 < \frac{\gamma\sigma^2}{\mu - r} - 1 < 1$.

We adhere to this condition throughout the numerical exercises in the paper to maintain the value of holdings in bonds and equities strictly positive in the absence of transaction costs. Davis and Norman (1990) refer to this condition as the "hedging" condition, where assets are divided between stocks and bonds to reduce volatility. Dumas (1991) demonstrated the non-dividend paying equity version of this outcome. The result indicates that the no-transaction region widens as the cost of trading equity increases.

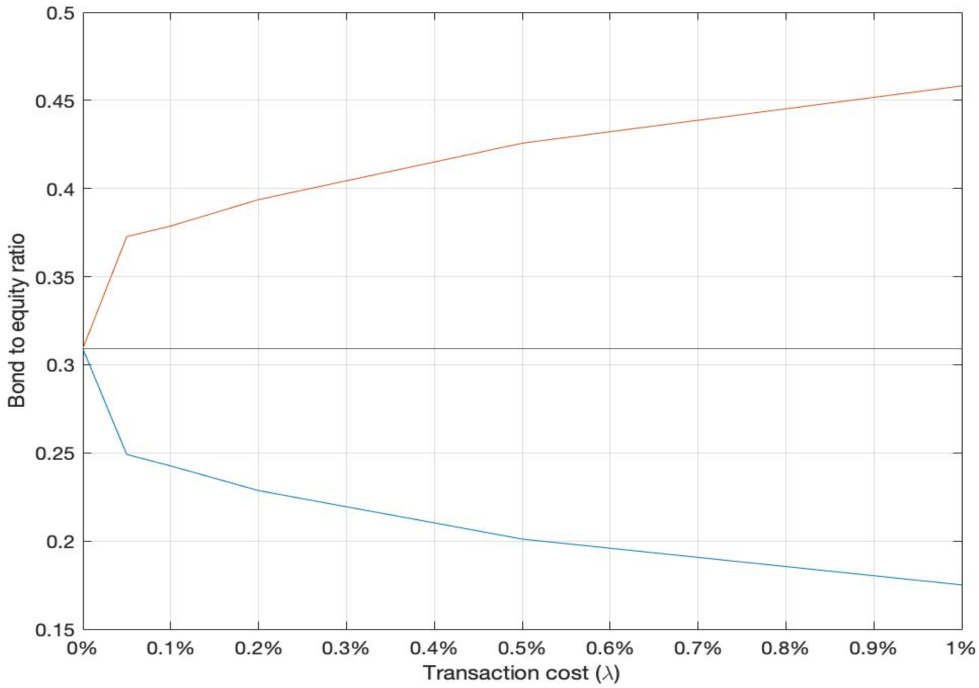

Figure 1.2: The effect of transaction cost on trading boundaries, $\lambda^b = 0$, $\lambda^s > 0$.

In this graph, the increasing line represents the sell boundary's position in relation to different equity transaction cost values, while the decreasing line represents the buy boundary. The Merton line, which is the optimal bond-to-equity ratio when the transaction cost is zero, is the straight line in the middle.

The graph demonstrates that as the cost of purchasing equity rises, investors decrease their demand for both equity and bonds. The reduction in equity demand is apparent since investors need a higher return on equity to purchase it when the cost of trading equity rises. Therefore, they wait to buy equity at lower prices to earn a greater return. The decrease in bond demand is less evident. However, as the cost of purchasing equity rises, investors are less inclined to sell their current equity balance because it would be costly for them to repurchase it later if doing so would be optimal. Since investors must sell equity to buy bonds, their demand for bonds also decreases.

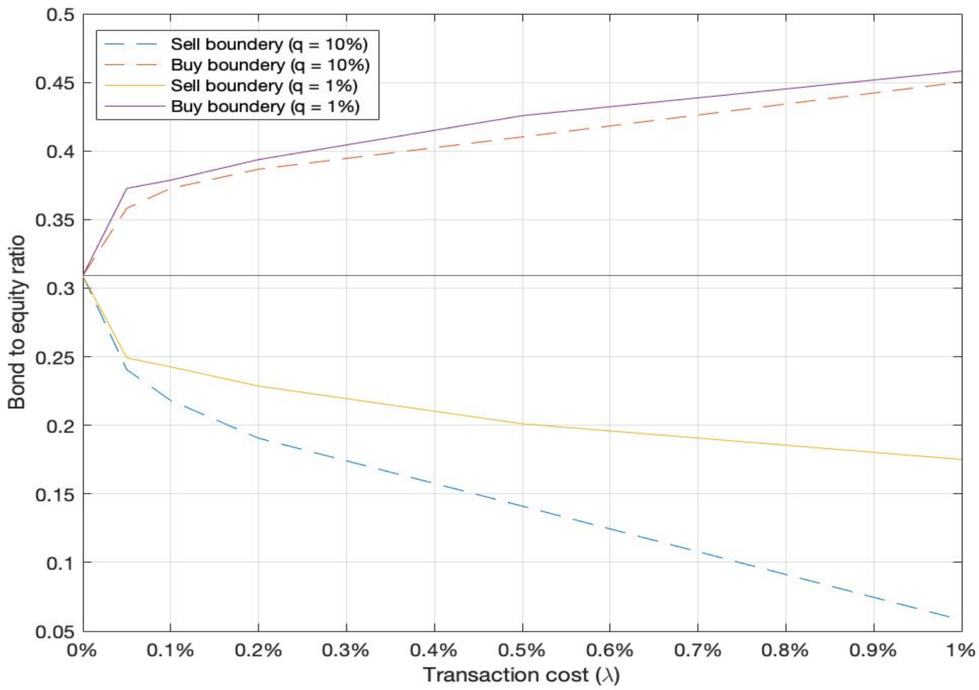

Figure 1.3: The effect of transaction cost and dividend yield on trading boundaries, $\lambda^b = 0$, $\lambda^s > 0$.

Figure 1.3 depicts the variation in the no-transaction (NT) region with respect to different equity transaction cost values for two distinct dividend payout ratios. It compares the no-transaction region when the dividend payout ratio increases from 0.01 to 0.1. In this graph, the purple line represents the buy boundary when the dividend payout ratio is 0.1, the red line is the buy boundary when the dividend payout ratio is 0.01, the yellow line corresponds to the sell boundary for a dividend yield of 0.1, and the blue line corresponds to the sell boundary for a dividend payout ratio of 0.01. The graph demonstrates that increasing the equity's dividend payout ratio results in a downward shift in both the sell and buy boundaries.

Proposition 1.1 showed that in a scenario where the transaction cost on the bond is zero, the best strategy is to convert all cash received from dividends into bonds in each period. When the equity payout ratio increases, more cash flows from the equity account into the bond account through the equity's dividend payout channel. In other words, as the dividend payout ratio rises, equity gets converted to bonds faster. This means that the investor would want to buy the stock sooner, at any transaction cost value, so the buy boundary moves down.

On the other hand, if the dividend yield goes up, the sell boundary should become more relaxed. Because the equity payout ratio is converting equity to bonds faster, investors have less reason to sell bonds and buy equity. So the sell boundary also moves down.

Another important aspect of the model is its consideration of the transformed expected utility that assumes investors seek to maximize their utility over an infinite period of time. However, the model also accounts for the fact that investors have a randomly arriving horizon. This means that investors may have a certain expectation about when they will need to use their investments.

Specifically, the model defines the probability of the horizon's arrival as η. Since the investment horizon is exponentially distributed, an increase in η leads to a decrease in the expected time until the horizon's arrival, which is represented as $\frac{1}{\eta}$. When the investor expects their horizon to arrive sooner, they treat bonds and equity differently due to the fact that trading equity incurs costs while trading bonds does not. As the investor approaches their investment horizon, they convert bonds to equity at a slower pace and equity to bonds at a faster pace. This anticipated behavior stems from the investor's desire to spend less on rebalancing their portfolio when they are closer to maturity.

It's noteworthy to mention that the benefit for investing in the equity is forward-looking, and the time to maturity is directly related to the investor's marginal benefit from purchasing the equity. As the investor's investment horizon approaches, the marginal cost of trading equity remains fixed at any given equity price, but the marginal benefit of purchasing the equity decreases. Therefore, the model suggests that the expected time until the investor's horizon arrival affects their trading behavior, which has important implications for portfolio management.

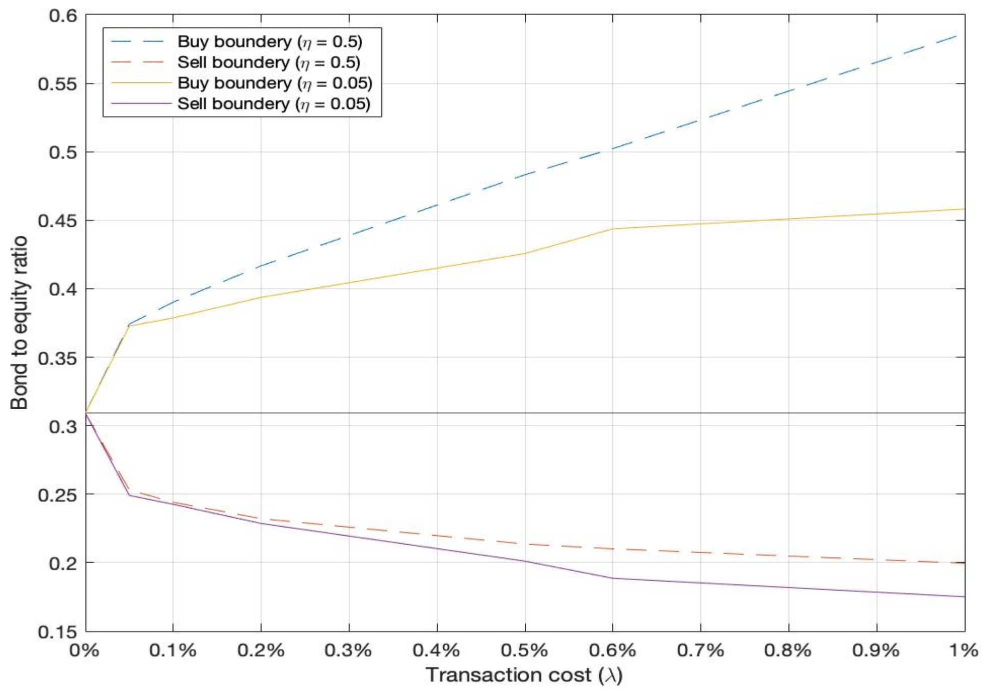

Figure 1.4: The effect of transaction cost and investor's horizon on trading boundaries, $\lambda^b = 0$, $\lambda^s > 0$.

In Figure 1.4, the sensitivity of the no-transaction boundaries is depicted with respect to the investors' horizon. The graph highlights that as the η value increases from 0.05 to 0.5, or in other words, as the expected time to maturity or investment horizon decreases, the buy boundary of the no-trading region increases. This implies that the investor's demand for bond increases as they approach their investment horizon, which is in line with their goal of preserving their capital and reducing risk as they get closer to needing their investments.

Conversely, the sell boundary of the no-transaction region also increases as η increases. This means that the investor's demand for equity decreases as they approach their investment horizon. Therefore, as the investor approaches their investment horizon, they would prefer to hold a more conservative portfolio with a higher allocation of bond, which has zero transaction costs, and a lower allocation of equity, which has positive transaction costs.

Finally, the paper examines the impact of transaction costs on the liquidity premium, following the work of Constantinides (1986). Constantinides suggests that transaction costs have a second-order effect on the liquidity premium, which is defined as the excess in the mean return of an asset subject to transaction costs compared to an asset that is exempt from transaction costs.

To measure the liquidity premium, Constantinides considers the case of two assets with perfectly correlated rates of return and equal standard deviations of their rates of return. In this scenario, the expected rate of return of the asset with transaction costs must exceed that of the exempted asset in equilibrium. The liquidity premium, denoted by $\delta(\lambda)$, represents the additional return that investors require to be compensated for the transaction costs, λ, and is given by the difference in the mean returns of the two assets.

Constantinides defines the liquidity premium, $\delta(\lambda)$, as the excess in the mean return of the asset with transaction cost compared with the asset without transaction cost which makes the investor indifferent between holding either of the assets at the optimal portfolio allocation under no transaction cost, $x^* = \frac{\gamma\sigma^2}{\mu - r} - 1$. In other words, in equilibrium the liquidity premium, $\delta(\lambda)$ that the investors require to be compensated for the transaction cost λ, must satisfy the following equation.

$$\psi(x^*) = \eta\Big(\eta - (1-\gamma)(r + \frac{(\mu - \delta(\lambda) - r)^2}{2\gamma\sigma^2})\Big)^{-1}\frac{(x^*+1)^{1-\gamma}}{1-\gamma} \tag{1.9}$$

The left-hand side of Equation 1.9 represents the expected utility of the investor from holding portfolio x^* under transaction cost, which we derived explicitly from Proposition 1.2. On the other hand, the right-hand side of the equation represents the expected utility under no transaction cost, which is derived in Lemma 1.1. This equation provides a measure of liquidity premium that estimates the excess return required to maintain the optimal portfolio inside the no-transaction region.

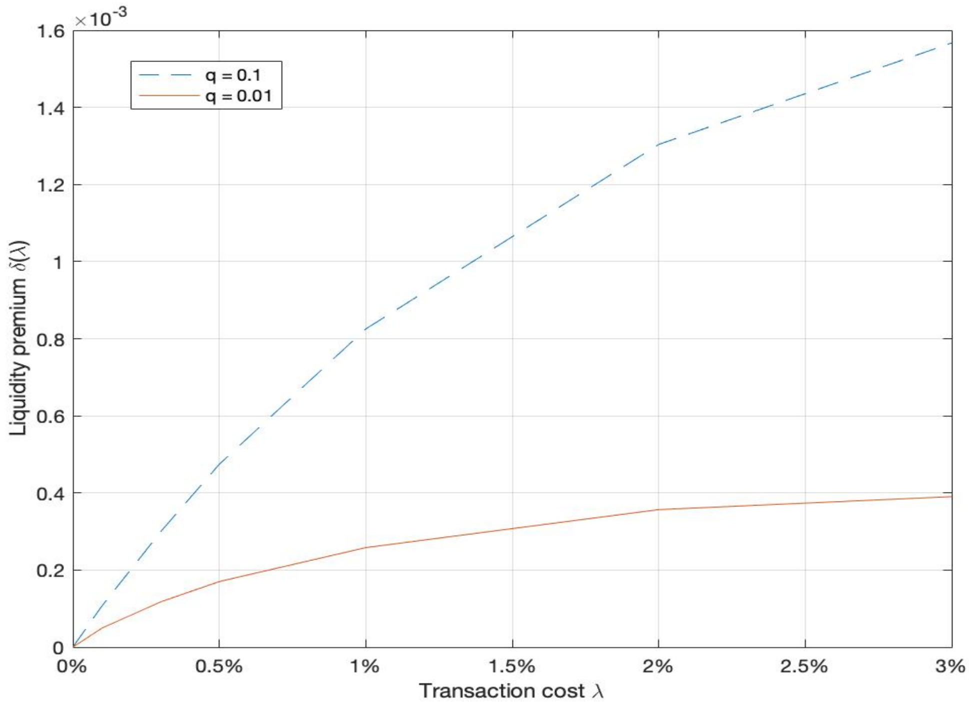

Figure 1.5: The effect of transaction cost and dividend yield on the stock's liquidity premium, $\lambda^b = 0$, $\lambda^s > 0$.

Figure 1.5 illustrates the liquidity premium, $\delta(\lambda)$, under two distinct dividend payout policies. The dashed line pertains to $q = 0.1$, whereas the solid line corresponds to $q = 0.01$. Subsequent to the rise in the transaction cost, the liquidity premium almost doubles.

As Constantinides (1986) argued, the liquidity premium is typically an order of magnitude below the transaction cost. Nevertheless, current literature suggests that the liquidity premium may be of the same order of magnitude as the transaction cost spread.

This study aims to establish that dividend payout from the asset is one of the contributing factors that could escalate the liquidity premium. The rationale behind the increase in premium lies in the fact that higher dividend payout leads to a surge in the demand for the asset, which, in turn, lowers the buy boundary. Consequently, the investor ends up procuring the asset more frequently, leading to higher transaction costs. Hence, the investor has to be compensated for the amplified cost of buying the asset.

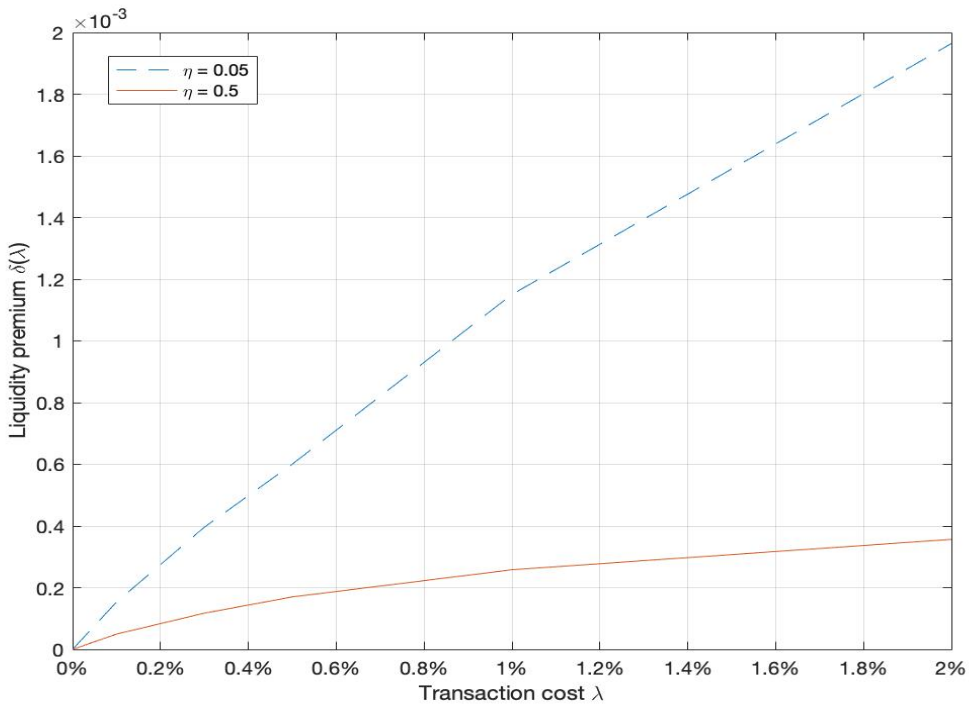

Figure 1.6: The effect of transaction cost and investor's horizon on the stock's liquidity premium, $\lambda^b = 0$, $\lambda^s > 0$.

In Figure 1.6, we observe a positive association between the liquidity premium and the investors' horizon. Specifically, as the parameter η increases from 0.05 to 0.5, the expected time to maturity or investment horizon of the investors decreases. Consequently, the liquidity premium decreases as the investor's expected horizon shorten. The reason for this positive correlation is rooted in the behavior of the investor as they approach their investment horizon. Investor tends to reduce their demand for the asset and trade equity less frequently. As a result of the decreased transaction cost, the investor would be required to be compensated less for trading the asset.

We can conclude that in an environment where the transaction cost of bonds is zero, high investment horizon and high dividend payout ratio are two potential contributing factors to the higher liquidity premium observed in empirical findings as compared to Constantinides' findings. The interplay between these factors leads to a shift in the demand for the asset with transaction costs, ultimately influencing the liquidity premium.

However, these relationships may change when transaction cost of bond is positive. In the next section, we will explore a comprehensive model that accounts for positive transaction costs for both riskless and risky assets.

1.3.2 Optimal Portfolio When Bond and Equity are Subjected to Transaction Cost

In this section, we delve into the full problem that includes positive values of proportional transaction costs for purchasing bonds and equity. As in the previous case, the transaction cost is proportional to the size of the trade and is only incurred when the investor purchases the asset. However, this time, the investor does not have a trivial sweep decision like in the case of zero transaction cost of bond, where the investor always converts cash to the bond. The presence of transaction cost of bond alters the investor's sweep decision process and influences their choice of asset.

In this problem, the investor at time t chooses how much to buy/sell bond and equity and also chooses how to reallocate the sweep account cash balance in the bond or the equity account. As before, let U_t be the cumulative dollar value of bond purchase at time t, L_t be the cumulative dollar value of equity purchase at time t, and χ_t be the fraction of the sweep account at time t that the investor transfers from the sweep account into the equity account.

Problem 1.2. *Consider an investor facing a proportional transaction cost of $\lambda^s > 0$ when purchasing equity, and $\lambda^b > 0$ when purchasing bond. Then, the investor seeks to choose χ_t, U_t, L_t for $t \in [0, \infty]$ in order to maximize the following problem:*

$$v(x,y) = \max_{L_t, U_t, \chi_t} E \int_0^\infty \Big[\eta e^{-\eta t} \frac{W_t^{1-\gamma}}{1-\gamma}\Big] dt \tag{1.10}$$

Subject to:

$$dH_t^b = \Big[(r-c)H_t^b + \frac{1}{1+\lambda^b}(1-\chi_t)\Big(cH_t^b + qH_t^s\Big)\Big]dt - (1+\lambda^s)dL_t + dU_t$$

$$dH_t^s = \Big[(\mu-q)H_t^s + \frac{1}{1+\lambda^s}\chi_t\Big(cH_t^b + qH_t^s\Big)\Big]dt + \sigma H_t^s dz_t + dL_t - (1+\lambda^b)dU_t$$

$$\forall t \quad H_t^b, H_t^s \geq 0, \qquad x = H_0^b, \qquad y = H_0^s$$

Similar to Problem 1.1, the first constraint is the law of motion for the bond account. To purchase dU_t units of bonds, the investor must sell $(1+\lambda^b)dU_t$ units of equity to cover the transaction cost of purchasing the bond. The second constraint is the law of motion for the equity account. To add dL_t units of equity to the equity account, the investor must sell $(1+\lambda^s)dL_t$ units of bonds to cover the transaction cost of purchasing the equity.

In Problem 1.2 the cash proceeds from the equity account are given by qH_t^s, while the cash proceeds from the bond account are given by cH_t^b. As a result, the cash balance that is held in the sweep account at time t is given by $cH_t^b + qH_t^s$. The investor must then decide how to allocate this cash balance between the equity account and the bond account.

Specifically, the investor can allocate a fraction $0 \leq \chi_t \leq 1$ of the sweep account balance to the equity account, incurring a transaction cost of λ^s in the process. The remaining balance can then be allocated to the bond account, incurring a transaction cost of λ^b proportional to the size of the transaction.

Proposition 1.3. *Let λ^s, $\lambda^b > 0$, and assume there exists a solution to the Problem 1.2. Under this specification,*

(i) *There exists a boundary, called the sell boundary at x_0, such that if $x \leq x_0$, the investor's optimal strategy is to sell equity and buy bonds until the portfolio reaches the sell boundary. The region in the allocation space to the left of the sell boundary is called the sell region. The investor's value function in the sell region is given by, $\psi(x) = \frac{1}{1-\gamma}A(x + \frac{1}{1+\lambda^b})^{1-\gamma}$, for a constant value of A.*

(ii) *There exists a boundary, called the buy boundary at x_T, such that if $x \geq x_T$, the investor's optimal strategy is to buy equity and sell bonds until the portfolio reaches the buy boundary. The region in the allocation space to the right of the buy boundary is called the buy region. The investor's value function in the buy region is given by, $\psi(x) = \frac{1}{1-\gamma}B(x + 1 + \lambda^s)^{1-\gamma}$, for a constant value of B.*

(iii) *The region in between the sell boundary, x_0, and the buy boundary, x_T, is called the no transaction region, where the investor would not trade. In this region there exists a sweep boundary, x_e, which $x_0 < x_e < x_T$. In the region where $x_0 \leq x \leq x_e$ (NT_0) the investor's optimal strategy is to sweep the cash into the bond account. The value function in this region is solved through the following free boundary differential* *equation,*

$$\beta_3 x^2 \psi_2''(x) + (\beta_2 x + \frac{q}{1+\lambda^b})\psi_2'(x) + \beta_1 \psi_2(x) + \frac{\eta}{1-\gamma}(x+1)^{1-\gamma} = 0$$

Where, $\beta_1 = (\mu - q - \frac{1}{2}\sigma^2\gamma)(1-\gamma) - \eta, \quad \beta_2 = \sigma^2\gamma + r - \frac{\lambda^b}{1+\lambda^b}c - \mu + q, \quad \beta_3 = \frac{1}{2}\sigma^2$

(iv) *In the region where* $x_e \leq x \leq x_T$, (NT_1)*, the investor's optimal strategy is to sweep the cash into the equity account. The value function in this region follows,*

$$\beta_3 x^2 \psi_1''(x) + (\beta_2 x - \frac{1}{1+\lambda^s}cx^2)\psi_1'(x) + \Big(\beta_1 + \frac{1-\gamma}{1+\lambda^s}cx\Big)\psi_1(x) + \frac{\eta}{1-\gamma}(x+1)^{1-\gamma} = 0$$

$$\beta_1 = \Big(-\frac{1}{2}\sigma^2\gamma + \mu - \frac{\lambda^s}{1+\lambda^s}q\Big)(1-\gamma) - \eta, \qquad \beta_2 = \sigma^2\gamma + r - c - \mu + \frac{\lambda^s}{1+\lambda^s}q, \qquad \beta_3 = \frac{1}{2}\sigma^2$$

(v) *The following conditions hold at the boundaries:*

$$\frac{\psi_1'(x_T)}{\psi_1(x_T)} = \frac{1-\gamma}{x_T + 1 + \lambda^s} \qquad \frac{\psi_2'(x_0)}{\psi_2(x_0)} = \frac{(1-\gamma)(1+\lambda^b)}{x_0(1+\lambda^b)+1}$$

$$\frac{\psi_1''(x_T)}{\psi_1'(x_T)} = \frac{-\gamma}{x_T + 1 + \lambda^s} \qquad \frac{\psi_2''(x_0)}{\psi_2'(x_0)} = \frac{-\gamma(1+\lambda^b)}{x_0(1+\lambda^b)+1}$$

$$\psi_1(x_e) = \psi_2(x_e) \qquad \psi_1'(x_e) = \psi_2'(x_e) \qquad \psi_1''(x_e) = \psi_2''(x_e)$$

Proof. *See Appendix* □

Proposition 1.3 demonstrates that the optimal solution to Problem 1.2 is a unique buying boundary of the equity, a unique selling boundary of the equity, and a unique boundary for the sweep account. Similar to the previous case, the no-transaction region is a convex cone, where the investor would not rebalance the portfolio in that area. However, the sweep decision boundary for the investor, in this case, lies strictly inside the no-transaction region.

As shown in Figure 1.7, there are two sub-regions in the no-transaction region. One of the sub-regions is denoted by NT_0, and the other line is denoted by NT_1. In NT_0, the investor fully reinvests the cash balance in the sweep account into the bond account, and in NT_1, the investor fully reinvests the cash balance in the sweep account into the equity account.

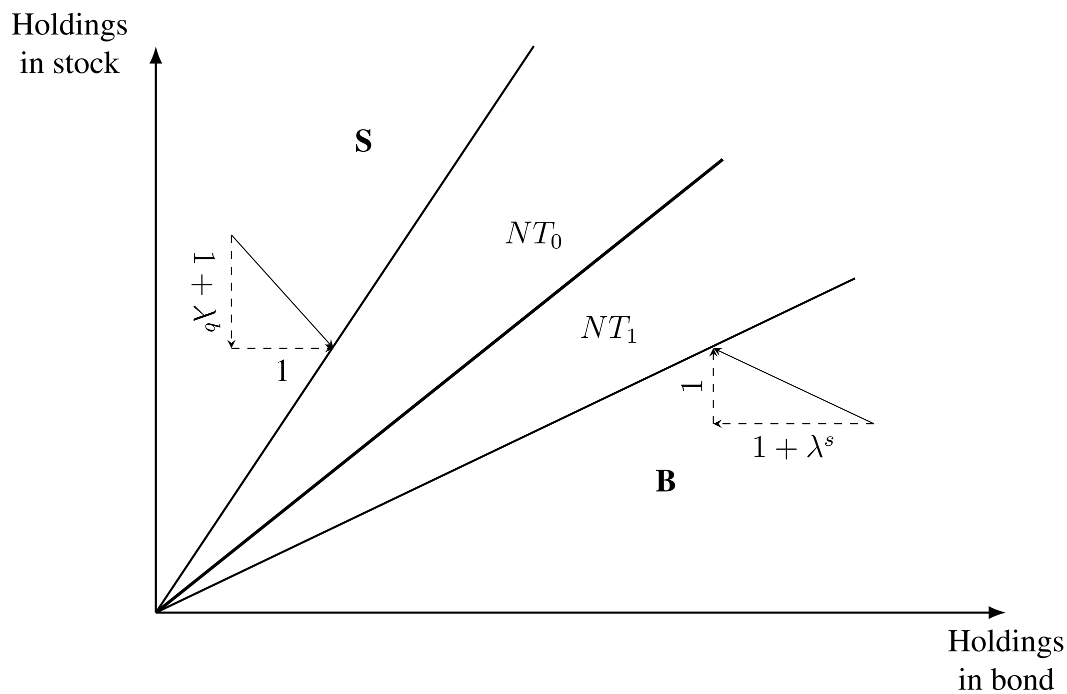

Figure 1.7: Space of Bond and Stock with regions of trading, no-trading and sweep, $\lambda^b > 0$, $\lambda^s > 0$.

The sweep boundary indicates that, in the area to the left of this boundary, the investor would only purchase bonds with the cash balance in the sweep account, and in the region to the right of the sweep boundary, the investor would reallocate the cash balance in the sweep account into the equity account.

Proposition 1.4. *Lets assume that $\eta > (1-\gamma)(r + \frac{(\mu-r)^2}{2\gamma\sigma^2})$. The exact solution to* *the ordinary differential equation in Proposition 1.3 for the NT_1 region is as followed,* $\psi_1(x) = C_{11}\Psi_{11} + C_{12}\Psi_{12}(x) + \psi_{1p}(x)$ where,

$$\Psi_{1i}(x) = x^{k_i}\Phi(a_i, b_i; \frac{c}{\beta_3(1+\lambda^s)}x)$$

$$\psi_{1p}(x) = \Psi_{12}(x)\int_0^x \Psi_{11}(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)} - \Psi_{11}(x)\int_0^x \Psi_{12}(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)}$$

$$W(x) = \Psi_{11}(x)\Psi'_{12}(x) - \Psi_{12}(x)\Psi'_{11}(x)$$

$$k_i = \frac{(\beta_3-\beta_2)+\sqrt{(\beta_3-\beta_2)^2-4\beta_3\beta_1}}{2\beta_3} \qquad i = 1,2$$

$$a_i = k_i - 1 + \gamma, \qquad b_i = \frac{\beta_2}{\beta_3} + 2k_i$$

$$\Phi(a,b;y) = 1 + \Sigma_{j=1}^{\infty}\frac{(a)_j}{(b)_j}\frac{y^j}{j!} \quad \text{where,} \quad (a)_j = a(a+1)...(a+j-1), \quad (a)_0 = 1.$$

And the solution to the ordinary differential equation in Proposition 1.3 for the NT_0 region is as followed, $\psi_2(x) = C_{21}\Psi_{21}(x) + C_{22}\Psi_{22}(x) + \psi_{2p}(x)$, where,

$$\Psi_{2i}(x) = x^{-k_i}\Phi(a_i, b_i; \frac{q}{(1+\lambda^b)\beta_3}x^{-1})$$

$$\psi_{2p}(x) = \Psi_{22}(x)\int_0^x \Psi_{21}(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)} - \Psi_{21}(x)\int_0^x \Psi_{22}(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)}$$

$$W(x) = \Psi_{21}(x)\Psi'_{22}(x) - \Psi_{22}(x)\Psi'_{21}(x)$$

$$a_i = k_i, \qquad b_i = -\frac{\beta_2}{\beta_3} + 2k_i + 2$$

Proof. See Appendix □

Proposition 1.4 demonstrates that the exact solution to the ODE introduced in Proposition 1.3 takes the form of Kummer's Confluent Hypergeometric function. This proposition indicates that the solution to the problem would be a unique set of boundaries denoted by x_0, x_T, and x_e, such that it is optimal not to trade when the portfolio value lies in the region between x_0 and x_T. Additionally, it shows that it is optimal to reinvest the cash balance from the sweep account into the bond account when the portfolio is in the region between x_0 and x_e, and it is optimal to reinvest the cash balance from the sweep account into the equity account when the portfolio is in the region between x_e and x_T.

Comparative Statics

In this section, we will be examining the results derived from Proposition 1.3. The first step is to replicate the changes in the no-transaction region as the transaction costs of bonds and equity are varied.

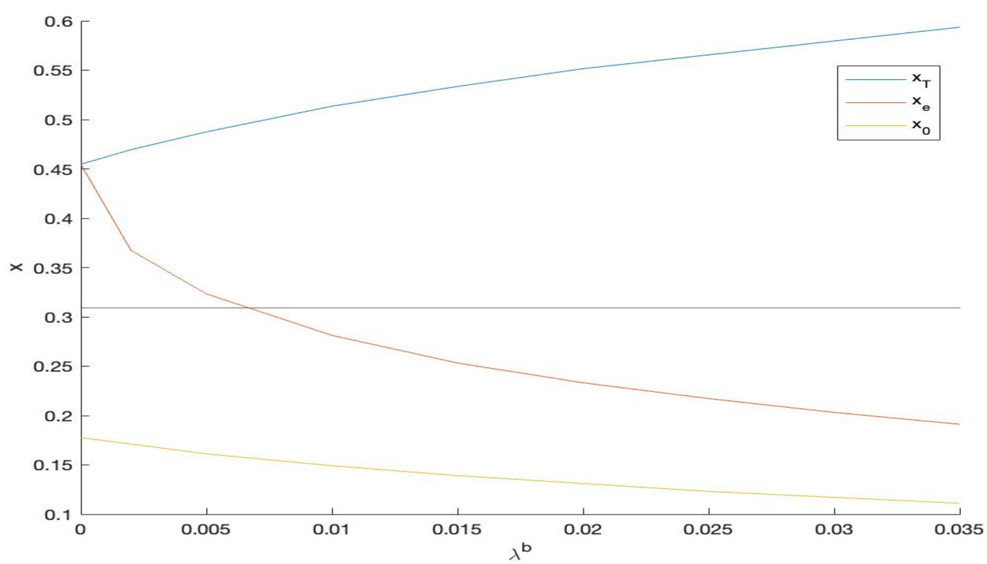

Figure 1.8: The effect of transaction cost of bond on trading and sweep boundaries, $\lambda^b > 0$, $\lambda^s > 0$.

Figure 1.8 depicts the changes in the boundaries with respect to variations in the transaction cost of bond while fixing transaction cost equity. In this section, the parameters are $\mu = 0.12, \sigma = 0.4, \gamma = 0.45, r = 0.065, q = 0.01, \eta = 0.05, c = 0.02, \lambda^s = 0.01$ unless it is specified otherwise.

The buy boundary is increasing, which indicates that the investor's demand for equity is decreasing. This is because the investor is less inclined to sell their bond to purchase equity since it would be more expensive to buy back their bond. The sell boundary is decreasing as well, as the cost of bond purchases is increasing. That is because the investor would want the price of equity to increase enough to cover the higher cost of bond purchase. The straight line in the center is the Merton line.

The sweep boundary overlaps with the buy boundary when the transaction cost of bonds is zero, as we learned in the first section. However, as the transaction cost of bonds increases, the sweep boundary declines and creates two sub-regions within the NT region. The first sub-region, labeled NT_1, lies between the buy boundary and the sweep boundary, where the investor invests the entire cash balance of the sweep account in the equity account. The second sub-region, labeled NT_0, lies between the sell boundary and the sweep boundary, where the investor invests the entire cash balance of the sweep account in the bond account. The area of NT_1 increases as the transaction cost for bonds increases, while the area of NT_0 decreases.

Figure 1.9 displays the changes in the boundaries as the transaction cost of bonds is held constant at $\lambda^b = 0.01$, while the transaction cost of equity varies. Notably, the sweep boundary starts at the sell boundary when the transaction cost of equity is zero and subsequently rises as the cost of purchasing equity escalates. This trend is attributed to investors being more likely to allocate the cash balances of the sweep account to the bond account when the cost of purchasing equity becomes high. Knowing the boundary variations with

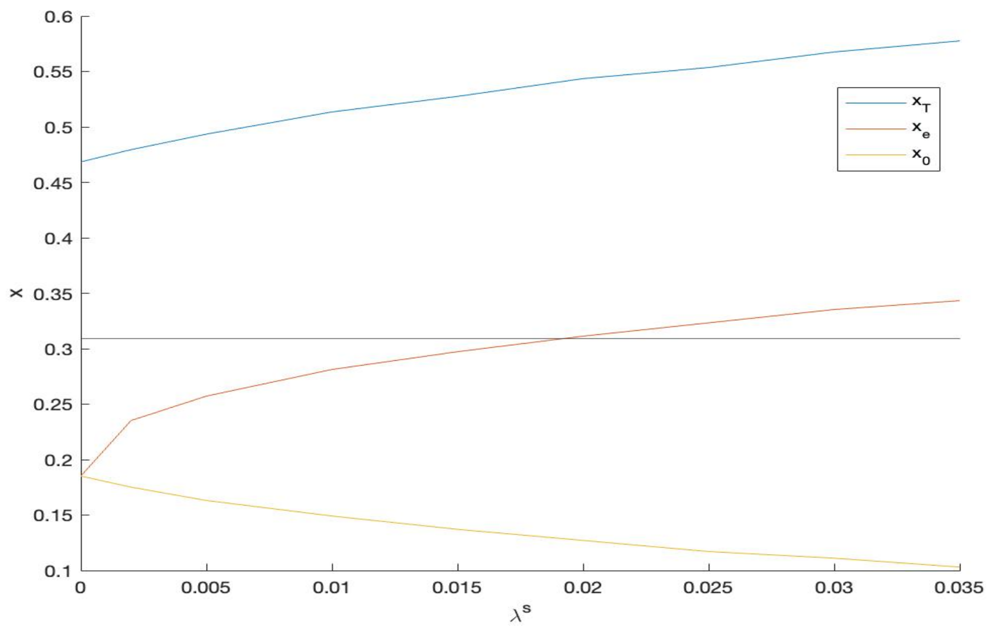

Figure 1.9: The effect of transaction cost of stock on trading and sweep boundaries, $\lambda^b > 0$, $\lambda^s > 0$.

respect to changes in the transaction costs of assets, we can compare these graphs under different asset characteristics.

Figure 1.10 depicts the boundaries under two different dividend payout policies as the transaction cost of bonds increases. The solid lines indicate the buy, sell, and sweep boundaries for a dividend payout policy of 0.01, while the dashed lines represent the corresponding boundaries for a dividend payout policy of 0.1.

From this graph, we can observe that at lower levels of bond transaction costs, the demand for equity increases as the equity pays more dividends. A similar case was previously demonstrated the section when the transaction cost of bonds was zero. However, as the transaction cost of bonds increases, the buy boundary for the higher dividend payout case intersects with the buy boundary under the low dividend payout case at some level of

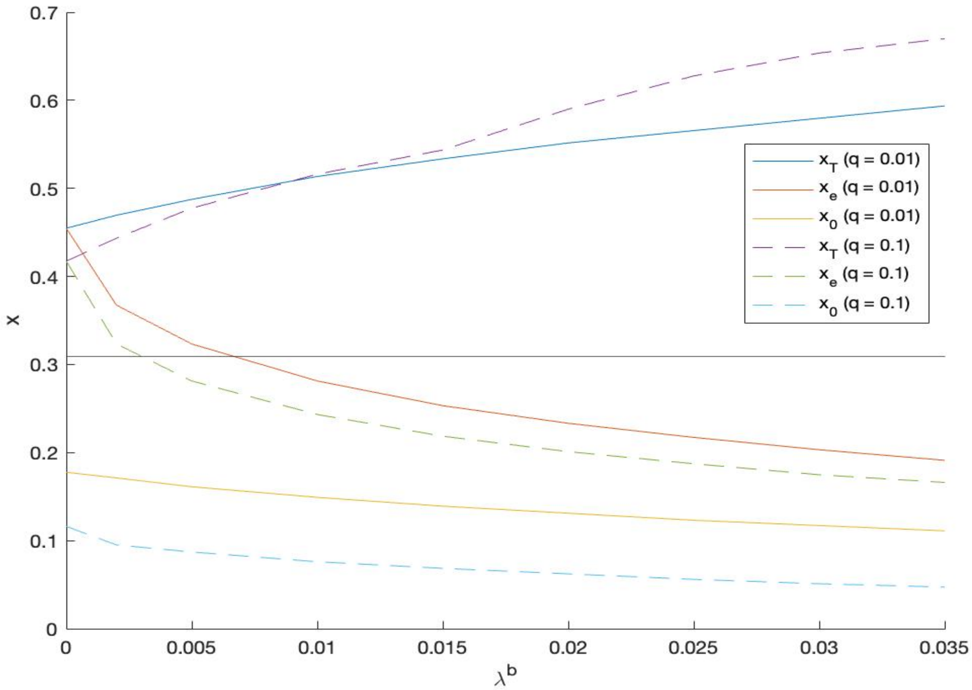

Figure 1.10: The effect of transaction cost of bond and dividend yield on trading and sweep boundaries, $\lambda^b > 0$, $\lambda^s > 0$.

the bond transaction cost. This implies that if the bond transaction cost is relatively high compared to the equity transaction cost, the demand for equity would decrease after an increase in the equity payout.

At lower levels of bond transaction cost, investors sweep cash into bonds at low cost and prefer to reinvest the cash in the equity account as the equity balance is declining due to the high dividend level. This results in higher demand for equity. Once the transaction cost of bonds surpasses a certain threshold, investors reduce their demand for both equity and bonds as the dividend payout increases. At higher levels of bond transaction cost, investors face high costs for sweeping cash into the bond account, which increases the likelihood of transferring cash distribution into the equity account. In this case, the demand for equity would decrease for two reasons.

First, dividends are more likely to be reinvested in the equity account, resulting in a lower need for purchasing more equity. Second, since the bond is more expensive, investors hesitate to sell bonds and buy equity in return because they will have to pay high transaction costs on that trade when they want to sell that equity in the future to buy bonds. To sum up, the effect of the dividend payout ratio on demand for equity is ambiguous and depends on the relative magnitude of the transaction cost of equity and bonds.

Figure 1.11 shows how a coupon bond affects asset demand under two scenarios: a low coupon rate of 0.02 and a high rate of 0.06, with $\lambda^b = 0.01$. When equity transaction costs are very low, investors demand more bonds to replace depleted bond balances due to high coupon rates. But as equity transaction costs rise, investors demand fewer bonds due to decreased likelihood of transferring cash to equity.

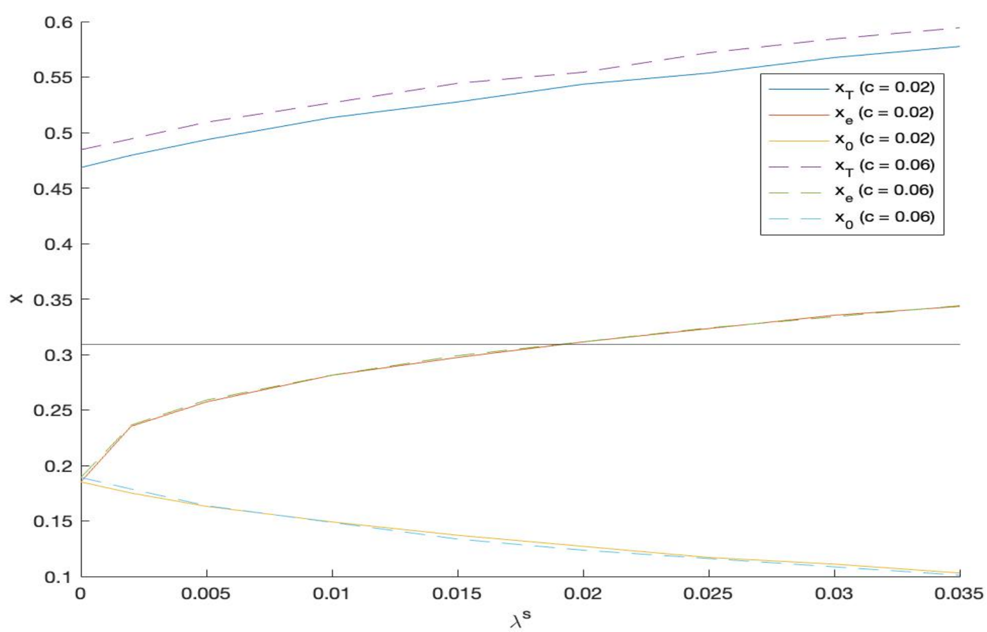

Figure 1.11: The effect of transaction cost of stock and coupon rate on trading and sweep boundaries, $\lambda^b > 0$, $\lambda^s > 0$.

Figure 1.12 shows the impact of investor horizon on asset demand. This figure shows the position of boundaries under a high investment horizon, $\eta = 0.05$ and a low investment horizon, $\eta = 0.5$, when transaction cost for equity, λ^s, is 0.01.

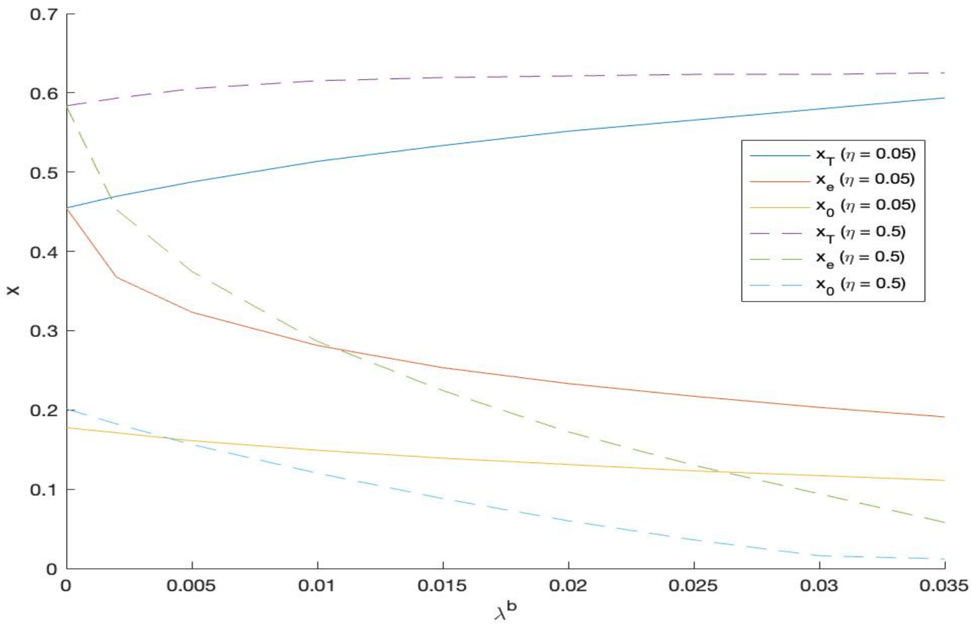

Figure 1.12: The effect of transaction cost of bond and investor's horizon on trading and sweep boundaries, $\lambda^b > 0$, $\lambda^s > 0$.

This figure shows that when the transaction cost of bond is low relative to equity investors demand the riskless asset more and demand the risky asset. This is consistent with the literature. However, the controversial effect of investment horizon happens when the transaction cost of equity gets elevated. Demand for bonds decreases and the demand for equity actually increases under very high transaction costs for bonds.

1.4 Conclusion

This paper explores the optimal allocation of cash precedes received from a portfolio of dividend-paying equity and coupon bonds in the presence of proportional transaction cost. Merton (1971) showed that the optimal allocation of non-dividend paying bonds and equities in the absence of transaction cost is a constant allocation in which investors continuously trade assets to maintain the optimal allocation.

Davis and Norman (1990) showed that when investors face proportional transaction cost and non-dividend-paying assets, the investors would not rebalance their portfolio in a convex cone region in the allocation space around the Merton line. In this paper, we argue that when investors are investing in dividend-paying stocks the optimal decision of reinvesting the cash proceeds is determined by a boundary which is called *Sweep* boundary. This paper illustrates that the No Transaction region when the assets pay higher dividends shifts downward such that the buy boundary and the sell boundary decrease.

Additionally, this paper displays the impact of the investment horizon and illustrates that when the investors' investment horizon becomes shorter, they delay the portfolio rebalancing by lowering their demand for the illiquid asset and they increase the pace at which they purchase the more liquid asset. Furthermore, we show the impact of asset cash payout and investors' horizon on the liquidity premium. We argue that when the investor's investment horizon shortens they would decrease their demand to decrease their expenditure on transaction costs. The investors' lower expenditure on transaction costs translates to lower liquidity premia in equilibrium.

We also show that when assets increase their cash distributions in terms of dividends or coupons, investors would increase their demand for these assets and the increased demand for the asset means that the investors' expenses on transaction cost would increases therefore the liquidity premia would rise as a result.

This paper extends the literature on optimal portfolios with transaction cost by including cash payments from the assets. This extension sheds light on the impact of dividends and coupons on the portfolio optimization decision and subsequently demand and liquidity premia for the assets with transaction cost.

Chapter 2

Optimal Intermediary Contracts

Abstract

Financial intermediaries simultaneously engage in two key relationships: they accept deposits and extend loans. In this paper, we develop a theoretical framework in which deposit contracts and loan contracts are determined simultaneously in equilibrium. Loan terms consist of the interest rate, the loan amount, and the collateral value required by banks when borrowers cannot commit to repayment. Across various trading protocols, we investigate how changes in borrower commitment affect the terms of the intermediary contract. The primary contribution of our model is that loan interest rates can decrease (resulting in declining spreads), and loan sizes can increase when borrower commitment declines. We also provide evidence of bank loan terms responding in accordance with the model's predictions following the enactment of the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA). Our findings reveal that BAPCPA, which heightens the commitment level among borrowers, leads to an increase in interest rates and a reduction in loan sizes for a specific subset of borrowers within the U.S. economy.

2.1 Introduction

Financial intermediaries simultaneously engage in two core relationships: accepting deposits and extending loans. These two relationships are intricately linked, as they directly impact the bank's balance sheet.[1]

Reserchers have traditionally concentrated on either deposit contracts or loan contracts. On the deposit side, a substantial body of work has evolved since Diamond and Dybvig's seminal 1983 study, which introduced the concept of deposit contracts partially insuring against consumption risk.[2] On the loan side, researchers have examined optimal loan contracts with varying information frictions. [3]

In this paper, we explore the interplay between deposit contracts, addressing idiosyncratic liquidity needs, and loan contracts, subject to repayment risk. While the existing banking literature primarily models the relationship between a bank's assets and liabilities, we contribute to the field by introducing a depository institution that offers welfare-improving risk-sharing opportunities to depositors. Our approach delves into the dynamics of optimal contracts offered by this bank when borrowers cannot commit to loan repayment, and depositors seek risk-sharing arrangements. In essence, we investigate how changes in loan contract terms impact the risk-sharing options available to depositors.

Deposit contracts in our framework are constructed following the Diamond-Dybvig model, incorporating idiosyncratic liquidity shocks. The bank has the capability to allocate

[1] We will discuss in greater detail below. The literature goes back to Bernanke, Bernanke et al. (1998) and their study with costly state verification. Financial frictions take on a variety of forms, including withdrawal shocks. In general, frictions illuminate the role that maturity transformation has on bank's balance sheets.

[2] See, for example, papers by Bryant (1980), Jacklin (1987), Bhattacharya and Fulghieri (1987), Hellwig (1994), Diamond (1997), Holmström and Tirole (1998), Von Thadden (1999), Allen and Gale (2003),Franklin and Douglas (2004), Caballero and Krishnamurthy (2004), Farhi et al. (2009).

[3] See, for example work by Hester (1979), Williamson (1987), Besanko and Thakor (1987), Bernanke et al. (1998), Berger and Udell (1990)Berger and Udell (1995), and Dowd (1992), Klapper (1999), and John et al. (2003)for a summary of the literature on collateral in loan contracts.

deposits across different technologies that vary in maturity, return, and risk. This includes both short-term and long-term safe technologies, as well as loans where the borrower's returns exceed those of the safe technologies. It's worth noting that borrowers lack the ability to commit to repayment, necessitating self-enforcing loan contracts. The key components of the loan contract consist of the interest rate, loan amount, and collateral requirements.

Changes in a borrower's commitment capacity have a ripple effect, influencing the terms of the loan contract, and consequently, the deposit contract. Furthermore, there exists an extensive margin that involves a delicate balance between the trade-off of repayment risk and the returns generated from safe investments. Consequently, when borrower's commitment deteriorate, the risk of repayment can become substantial, prompting the bank to diversify its asset portfolio.

Our primary experiment aims to investigate how changes in a borrower's commitment level impact the interplay between deposit contracts and loan contracts. In this paper, commitment is assessed through the borrower's ability to divert investments away from loan repayment for their own benefit, as outlined in works such as Biais et al. (2007) or DeMarzo and Fishman (2007). It serves as a summary metric encompassing a range of factors. For example, this parameter can signifies the impact of strict bankruptcy laws in deterring moral hazards through regulatory rules. Alternatively, we can view it as a measure of how much collateral is required. In this context, commitment is linked to the extent to which a borrower's collateral can be pledged, as seen in Kiyotaki and Moore (1997). A borrower's capacity to pledge a larger fraction implies greater commitment level compared to a borrower pledging a smaller fraction.[4] Here, we concentrate on the connection between

[4]In models in which an existing security is pledged as collateral in say, a repurchase agreement, then the association between pledgeability and haircuts are yet another link. Because we focus on direct lending, we do not really have a model that speaks to haircuts. By modifying the model to consider a secondary market for the loan, then the issue of the equilibrium haircut would be natural.

the borrower's commitment level and the terms offered by creditors to the borrower.

We examine the optimal contracts within competitive and over-the-counter (OTC) like loan market structures, including bilateral bargaining and competitive search. In each market structure, the equilibrium loan contract falls into one of three regions, contingent on the borrower's commitment level. In economies featuring highly committed borrowers, the repayment constraint is not binding, resulting in unconstrained efficiency. As commitment levels decline, the repayment constraint becomes binding, prompting banks to adjust loan terms, such as collateral requirements, loan size, and loan rates, while still directing all long-term resources towards loans. Finally, there exists a critical threshold of commitment level below which banks allocate some of their resources to the lower-return, direct investment. These critical values that delineate the three regions are consistent across various market structures.

A surprising finding emerges as the terms of the loan contract do not consistently change with variations in commitment level across the three regions. In other words, the comparative statics results vary with shifts in borrower's commitment level. Additionally, this relationship is influenced by changes in market structure and parameter values. With multiple components of the loan contract affecting borrower repayment at different levels, these interactions become intricate. This complexity becomes particularly apparent in scenarios where the bank and borrower engage in Nash bargaining, illustrating that both loan rates and collateral requirements can exhibit non-monotonic behavior in response to changes in borrower's commitment capacity.

The non-monotonicity presents three significant implications. Firstly, our results reveal that the relationship between collateral and commitment is state-dependent. When borrowers pledge a smaller fraction of their loans, the need for greater collateral diminishes because other loan terms can effectively incentivize borrowers to repay. Secondly, our findings challenge conventional wisdom, particularly concerning whether rising spreads and decreasing loan quantities uniformly signal deteriorating commitment intensity or credit quality. Lastly, the connection between risk-sharing and commitment remains ambiguous. Under Nash bargaining, risk-sharing demonstrates an inverse relationship with commitment in certain regions, leading to increased payments for impatient consumers and decreased returns for patient consumers.

Additionaly, considering competitive search allows us to explore several critical issues. First, we examine the impact of entry costs on financial inclusion. A lower entry cost borne by borrowers leads to improved financial inclusion, but it results in less favorable loan terms for borrowers. Second, we analyze the consequences of an unforeseen reduction in the returns on borrowers' projects for financial stability. In regions where the repayment constraint becomes binding, borrowers default due to the shock, potentially triggering a bank run if the bank lacks alternative revenue sources. However, the occurrence of a bank run may be mitigated if the bank has invested sufficiently in safe technologies. The key takeaway is that the likelihood of a bank run decreases as the bank diversifies its portfolio, particularly when borrowers are less reliable.

Regarding the impact of commitment on deposit contracts, we observe the following result. Suppose borrower's commitment intensity deteriorates. In a competitive loan market, as loan demand tightens, the loan rate decreases, subsequently reducing both long-term and short-term deposit rates. In an OTC market using Nash bargaining, the bank internalizes the effect of diminished long-term resources, influencing rates, loan size, and collateral size. Consequently, the long-term deposit rate declines. However, the short-term deposit rate can rise, partially compensating depositors in terms of their ex-ante expected utility. This might give the impression of improved risk-sharing in bilaterally negotiated contracts. Nonetheless, the reduced spread between long- and short-term rates doesn't necessarily indicate welfare improvements; in reality, the commitment level worsens.

Furthermore, we also present empirical evidence of the impact of borrowers' commitment levels on loan terms in the United States. Specifically, we demonstrate that an improvement in commitment levels has led to certain creditors raising interest rates and reducing credit amounts for some debtors.

Our analysis capitalizes on a significant external shock to borrowers' commitment capacity in the United States, specifically the enactment of the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA). This legislation, passed by the House on April 14, 2005, and signed into law by the U.S. President on April 20, 2005, introduced stricter bankruptcy regulations. It rendered cases filed on or after October 17, 2005, subject to more stringent rules, making it notably more challenging for both consumers and businesses to file for Chapter 7 bankruptcy. Instead, it encouraged them to opt for Chapter 13, a process that became more time-consuming and financially burdensome for those seeking bankruptcy protection.

Under Chapter 7 bankruptcy, most of the debtor's debts are discharged, while Chapter 13 mandates that the debtor repays a larger portion of the debt to the creditor through a restructured payment plan. This change in bankruptcy law had significant implications for borrowers' commitment levels, subsequently affecting the terms of their loan agreements. As our model illustrates, this change in bankruptcy law which lead to an enhancement of the borrower's commitment, effectively shifted bargaining power from debtors to creditors.

Our findings reveal that the BAPCPA legislation, aimed at reforming bankruptcy laws, led to an increase in interest rates for some U.S. borrowers who were seeking loans with maturities exceeding 55 months. On average, the interest rates on their loans surged by more than 52 basis points, resulting in higher borrowing costs for these borrowers. Furthermore, the legislation's impact extended to the size of loans granted, which saw a noteworthy reduction. On average, loan sizes decreased by a substantial \$216 million. We also show that the increase in interest rates is more significant among the subset of the borrowers with lower S&P Quality Ranking. As described in our model, the option to abscond becomes a bargaining tool for borrowers as they try to secure better loan terms from creditors. This bargaining power becomes even more critical when borrowers are looking for loans with longer maturities or have weaker credit profiles, as the risk of default rises with extended loan duration, Leland and Toft (1996). As a result, borrowers who choose longer-maturity loans are particularly affected by the consequences of this new legislation.

In practical terms, this effect translates into higher interest rates being charged to these borrowers, and this, in turn, results in an increased cost of loans. As a consequence, the demand for loans among these borrowers has declined, ultimately leading to a reduction in the equilibrium loan sizes for this group.

Finally, in this paper, we utilize our model to investigate the impact of restricting intermediaries to storage and lending activities. This particular inquiry pertains to the ongoing debate on universal banking, particularly in relation to the Glass-Steagall Act.

Our findings in this simplified model indicate that regulations akin to Glass-Steagall would primarily benefit borrowers but potentially come at the cost of depositors. If banks were constrained in their investment options, they would likely extend more credit to borrowers at lower interest rates. This, in turn, would increase the loan amounts on the banks' balance sheets but may also expose banks to shocks in the return on borrowers' investments. These findings may contrast with the main argument of Glass-Steagall supporters, who believe that limiting banks' direct investment activities would enhance financial stability.

2.2 Related literature

Our main contribution lies in the explicit link between deposit contracts and loan contracts. On the theoretical front, our paper lies at the intersection of the literature on deposit market and the works on optimal loan contracts. Antinolfi and Prasad (2008) study the deposit contract in a Diamond-Dybvig economy with depsitors-formed banks of limited commitment. Individual depositors are constrained in the selling of long-term assets because they may renege on the delivery of asset. Banks can allocate resources more effectively than individuals by pooling collateral capacity. The debt-incentive constraint does not bind for the bank, but would bind for individuals.

There is large literature that examines how a bank's deposit and asset structure are affected by various risks. Van den Heuvel (2008), for example, creates a need for a bank by specifying a deposit-in-the utility function. Given these deposits, deposit insurance creates moral hazard problem by encouraging banks to choose risky investment. The capital requirement is the means by which regulators force the bank to have "skin in the game" to mitigate the moral hazard problem. On the other hand, capital requirement limits the bank's capacity to accept deposits. Therefore, the welfare costs of capital requirement is the focus of the paper.

More recently, Piazzesi and Schneider (2021) specify deposits-in-advance to explain why banks are useful. These banks are subject to uncertain payment flows. In particular, each bank faces an idiosyncratic deposit-flow shock; effectively, one can consider their setup as a net negative payment shock realized in one bank is offset by net positive payment shocks to all other banks. This setup accounts for why an interbank market is useful; that is, a bank can obtain the necessary reserves when suffering large enough negative payment shocks. Hence, reserves act as a form of collateral backing bank deposits.

Bianchi and Bigio (2022) focus on monetary policy transmitted through banks also subject to withdrawal shocks. Macroeconomic outcomes depend on how monetary policy is implemented through two independent tools: interest rates on reserves and the central bank's balance sheet.[5] Our point is that this literature specifies bank deposits as satisfying a basic notion of maturity transformation and risk. Our goal is to incorporate risk sharing into the analysis.

On the empirical side, our results bear on how theory overlaps with applications in two ways. First, the literature has provided a basis for interpreting movements in interest rates and loan quantities as indicators of changing commitment levels. For example, Boot and Thakor (1991) develop a model predicting that the interest-rate spread between risky and risk-free debt is monotonically declining in the value of collateral.

To illustrate its empirical implementation, note that the Federal Reserve Bank of Chicago uses a multi-factor model to compute the National Financial Conditions Index (NCFI).[6] From the list of 105 indicators, we count 30 different interest rate spreads used to construct the NCFI. There are also 28 variables that measure quantities or ratios. The NFCI tends to put negative weights on yield spreads while the index tends to put positive weights on quantities. Our results suggest that the relationship between commitment and these indicators–collateral value, loan rates and loan size–are non-monotone, varying with the market structure. Our results raise questions that there is an unconditional statistical relationship between rates and commitment.[7]

Second, the literature commonly uses collateral intensity as an indicator of credit con-

[5]Bigio and Bianchi structure is an extension on the interbank market studied in Afonso and Lagos (2015).

[6]See Brave and Kelley (2017) for a complete description.

[7]There is an extensive literature on the role of collateral in the optimal loan contracts. For example, are secured loans are riskier? See, for example, Hester (1979), Berger and Udell (1990), Berger and Udell (1995) and Klapper (1999) provide empirical support for the hypothesis that less creditworthy borrowers tend to be required to post collateral. John et al. (2003) provide additional empirical evidence supporting a negative relationship between collateral and loan rates.

dition. For example, Benmelech and Bergman (2009) use collateral redeployability as a proxy for the intensity of collateral pledged, and present evidence that there is a negative impact of collateral values on loan rates and size. Our model economy reports changes in collateral and loan rates as both endogenous responses to change in commitment levels. More generally, our model economy links risk sharing in the deposit market with limited commitment in the loan market, which allows us to gain additional insight into the nature of the correlations between the two markets.

There is a literature that has studied how pledgeability affects loan contracts following Kiyotaki and Moore (1997). Capital production can be nonmonotone in pledgeability due to two opposing effects: more capital reduces its marginal product making it less attractive. Whereas the additional production allows for more borrowing. In related monetary models, pledged capital is used as liquidity (see Lagos and Rocheteau (2008), and Venkateswaran and Wright (2013), among others). Gu et al. (2022) shows as pledgeability increases, capital production first increases owing to the high marginal utility of liquidity. As pledgeability increases further, capital production falls as liquidity becomes less scarce, and eventually stays constant as the marginal utility flattens. Our paper also features multiple roles of capital but allows for more general preferences. We show that the nonmonotone patterns are more intricate and they depend on the preference of the borrowers and other parameters.

The rest of the paper is organized as follows: Section 2.3 demonstrates the data and presents empirical evidence. In Section 2.4, we introduce the model environment. Section 2.5 focuses on solving the equilibrium in the competitive loan market. Section 2.6 explores Nash bargaining. Section 2.7 discusses competitive search. In Section 2.8, we study the implications of the Glass-Steagall Act and universal banking on intermediary contract terms. Finally, Section 2.9 provides the conclusion.

2.3 Data and Empirical Evidence

In this section, we provide an overview of our dataset and present our empirical findings. Loan data is obtained from the LPC's Dealscan repository. This dataset encompasses privately issued loans, originating from both traditional banks, and spans the period from 2004 to 2006. According to Carey and Hrycay (1999), the Dealscan database encapsulates a significant portion, estimated at 50% to 75%, of the total value of commercial loans within the United States during the early 1990s. Additionally, we acquire credit ratings for borrowing firms from the Compustat-CapitalIQ dataset and integrate these two datasets using the methodology developed by Chava and Roberts (2008).

Our primary focus centers on loans issued to non-government U.S. borrowers within the 60-day window before and after the signing of the Bankruptcy Abuse Prevention and Consumer Protection Act by the US president which occurred on April 20, 2005.

It's important to highlight that there was a grace period of about six months between the law's enactment and its enforcement. This time gap gives rise to a scenario in which loans issued after the law's enactment but before its enforcement could belong to either the 'treated' or 'not treated' category. Let's take the case of a loan issued on April 21, 2005. If the borrower of this loan filed for bankruptcy prior to October 17, 2005, they would remain unaffected by the law's changes. However, if the legal process occurred after the enforcement date, they would become subject to the new bankruptcy regulations. Therefore, loans that were originated in this period could be impacted by the law to a lesser degree than the loans that are originated after the law's implementation date.

To address this concern, we limit our analysis to loans with a maturity of 55 months or more. In doing so, we assume that the likelihood of bankruptcy within six months of the grace period between the law's passage date and its implementation date is minimal for borrowers who are granted long-term loans. In essence, we are supposing that lenders issuing these longer-term loans after the law was signed would consider the new bankruptcy laws, even though the law was not enforced at the time of origination.

Moreover, we exclusively concentrate on loans linked to the London Inter-Bank Offered Rate (LIBOR). LIBOR is the most prevalent reference rate in our dataset, and we make this selection to shield our analysis from the fluctuations of other reference rates that may influence loan terms. In Figure 2.1, we observe the relative stability of the LIBOR rate throughout the data period, in contrast to the prime rate, another widely employed reference rate, which displays greater variability. Between 2004 and 2006, the U.S. Federal Reserve initiated a series of interest rate hikes in response to inflation and rising energy costs. This difference between the LIBOR and the prime rate can be attributed to the Federal Reserve's policy during that period.

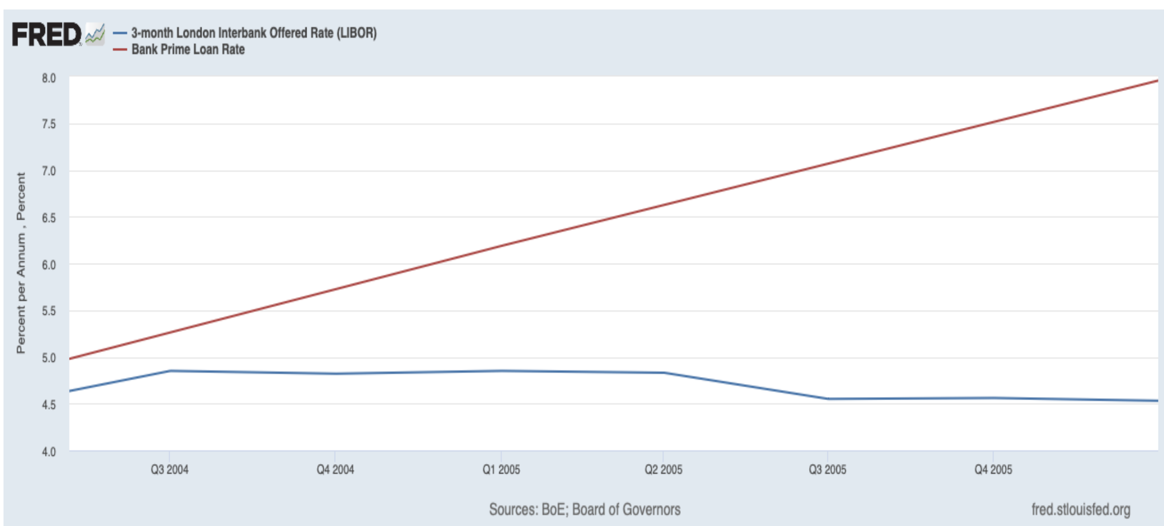

Figure 2.1: 3-month LIBOR and bank prime rate between 2004 and 2006.

2.3.1 Results

Utilizing Regression Discontinuity Design in Time (RDDiT), we run the following regressions to evaluate the impact of the new bankruptcy law on interest rates and loan size for our borrowers of interest. In our analysis, we employ two distinct models to evaluate the policy's impact. In the first model, we assess the policy's effects through the following regression:

$$y_{it} = \beta_0 + \beta_1 * \text{time\_dummy}_t + \beta_3 * \text{threshold}_t + \beta_4 * \text{time\_dummy}_t * \text{threshold}_t + \epsilon_{it}$$

Which measures the average instantaneous change in the variable y_{it} after the law's enactment across all loans in that period. In the second model, we run the following regression:

$$y_{it} = \beta_0 + \beta_1 * \text{time\_dummy}_t + \beta_3 * \text{threshold}_t + \beta_4 * \text{time\_dummy}_t * \text{threshold}_t + \beta_5 \text{spcsrc.B}_{i,t} + \beta_6 \text{spcsrc.C}_{i,t} + \beta_7 \text{spcsrc.B}_{i,t} * \text{threshold}_t + \beta_8 \text{spcsrc.C}_{i,t} * \text{threshold}_t + \epsilon_{it}$$

This model assesses the same variable while accounting for heterogeneity across borrowers with different qualities. First, our analysis focuses on the policy's influence on loan interest rates. In this context , we define "$y_{i,t}$ = Margin_bps" as the metric that quantifies the additional spread above LIBOR for a given loan at a specific time, measured in basis points.

To incorporate the temporal aspect, we employ the "time_dummy" variable. It starts at zero on the reference date of '2005-04-20' and increases by one unit for each subsequent day, decreasing by one unit for each preceding day. For example, the "time_dummy" for '2005-04-01' registers as -19. Simultaneously, the "threshold" functions as a binary variable, assuming a value of 1 if the observation's date falls after the reference date and 0 if it occurs before.

In our second regression model, we introduce the variable "spcsrc," representing the S&P Quality Ranking for the borrower. This ranking assesses a company's historical financial performance, taking into account earnings, dividends, and its position concerning the firm's current fiscal year-end. The rankings range from "A+," signifying the highest rank, to "C," representing the lowest rank. To address potential correlations in loan interest rate residuals within the borrower group, we implement error clustering in both models, enhancing the robustness of our analysis.

Table 2.1: The impact of BAPCPA on loan interest rate.

Model	Model 1	Model 2
Intercept	224.46 (22.43)	300.20 (68.66)
Time Dummy	-0.99 (0.77)	0.995 (1.42)
Threshold	52.45* (31.77)	-116.44 (73.11)
Time Dummy $\times$ Threshold	-0.33 (1.00)	-1.20 (1.28)
spcsrc.B $\times$ Threshold	-	119.43** (57.11)
spcsrc.C $\times$ Threshold	-	145.86*** (51.56)
No Observation	4321	1793

*The left value is the coefficient, the right value in parentheses is the standard error, and standard errors are clustered by borrower. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.*

In Table 2.1, Model 1, we pay particular attention to β_3, the coefficient of *Threshold*. This coefficient reveals an average increase of 52 basis points in the interest rate following the policy's enactment, applying universally across all credit rating categories.

In Model 2, β_3 shows the policy's impact on interest rates for loans obtained by borrowers with an S&P Quality Ranking of "A" (highest ranking) at the time of origination. Table 2.1 illustrates that interest rates for borrowers with the highest ranking have decreased by

116 basis points. Moreover, "$\beta_3 + \beta_7$" characterizes the policy's effect on loans acquired by borrowers with an an S&P Quality Ranking of "B" (below-average), and "$\beta_3 + \beta_8$" shows the policy's influence on loans for borrowers who held a ranking of "C" (lowest ranking) at the time of loan origination. Table 2.1 demonstrates that the interest rate for borrowers with below-average ranking has increased by 3 basis points, and the interest rate for borrowers with the lowest ranking has increased by approximately 30 basis points.

These findings underscore the policy's significantly stronger impact on borrowers with lower rankings. Lax bankruptcy protection laws often offer advantages to borrowers in the lower ranking spectrum or those who demand loans with longer maturities. Our results reveal that these borrowers are disproportionately affected by the stricter regulations, resulting in a reduction in their bargaining power within this context.

Additionally, we have included a figure to emphasize the persistent effect of the policy on interest rates. This figure illustrates the 10-day average interest rates both before and after the policy implementation. We have categorized the data into short-term loans, which are loans with a maturity of 12 months or less, and long-term loans, which have maturities of 55 months or more. It's worth noting that, in our dataset, the maximum maturity is 240 months.

In Figure 2.2, we observe that the policy has effectively stabilized the decreasing interest rate trend for loans with maturities greater than 55 months. However, the trend for shorter-maturity loans remains largely unaffected. This visual description clearly highlights the policy's varying impact on loans with different maturities.

In our model, we assume that borrowers all have similar preferences for loan maturity. Therefore, offering a detailed analytical explanation for this phenomenon is beyond the current scope of our model, and we defer it for future research.[8]

[8] One possible reason for this diverse behavior could be related to the concept of a 'preferred habitat,'

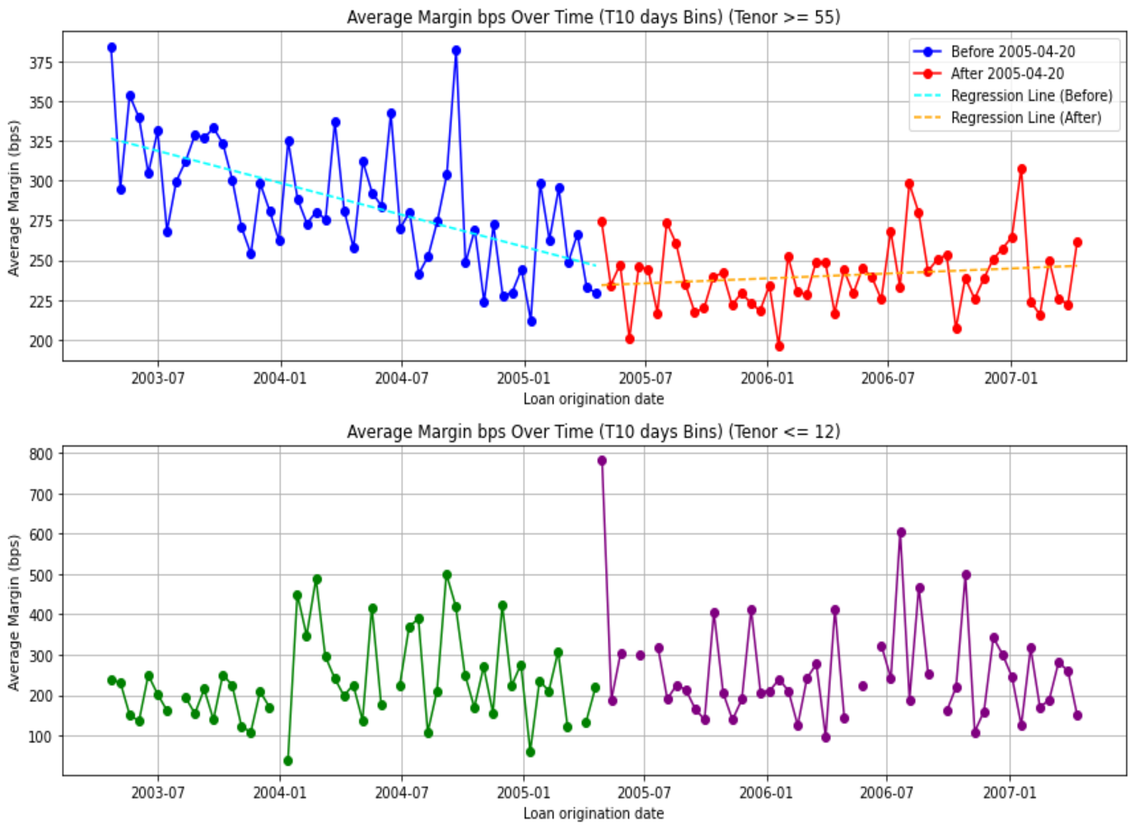

Figure 2.2: Difference in the persistent impact of the BAPCPA on the loan interest rate based on loan maturity.

Next, we examine the size of the loan that is extended to our set of borrowers. We define "y_{it} = Tranche_Amount" which is the size of the loan offer at the origination date measured in millions of dollars.

In Table 2.2, Model 1 indicates an average decrease of $216 million in loan sizes following the enactment of BAPCPA, spanning all credit rating categories. Model 2 highlights that the loan size for borrowers with the highest "A" ranking has increased by $537 million.

as discussed by Culbertson (1957) and popularized by Modigliani and Sutch (1966), which is also widely recognized in practical applications. According to this perspective, there are groups of investors who have preferences for specific loan maturities. The interest rate for a particular maturity is influenced by demand and supply shocks specific to that maturity.

Table 2.2: The impact of BAPCPA on loan size.

Model	Model 1	Model 2
Intercept	396.30 (127.54)	-241.65 (390.49)
Time Dummy	-0.73 (5.61)	-7.46 (8.41)
Threshold	-216.91^{*} (131.64)	537.68 (399.40)
Time Dummy $\times$ Threshold	4.11 (5.74)	-1.20 (1.28)
spcsrc.B^{-} $\times$ Threshold	-	-335.73^{**} (188.23)
spcsrc.C $\times$ Threshold	-	-573.46^{***} (240.94)
No Observation	4321	1793

*The left value is the coefficient, the right value in parentheses is the standard error, and standard errors are clustered by borrower. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.*

Furthermore, the loan size for borrowers with a lower "B-" ranking has increased by around $200 million, while the loan size for borrowers with the lowest "C" ranking has decreased by approximately $36 million.

As demonstrated in Table 2.1, the policy's effect is notably seen in increased borrowing costs for those with lower rankings. The elevated cost of credit leads to reduced demand from these borrowers, consequently causing a decrease in the equilibrium loan sizes for this specific group, as depicted in Table 2.2.

2.3.2 Policy Discussion

The BAPCPA law had a primary goal of preventing the misuse of bankruptcy regulations. It's important to note that although the banking lobby pushed for this law, its proponents argued that it would benefit consumers and small businesses. They believed the benefits of this law would pass to the borrowers by reducing credit costs and potentially increasing credit availability. For instance, when President Bush signed the law, he explained that it would lead to "credit being extended to more people at better rates"[9] (Bush, 2005).

However, this paper reveals that this argument is not entirely accurate, and the law doesn't uniformly benefit all types of debtors. Our model suggests that specific borrowers, influenced by a combination of commitment, creditworthiness, and other factors, may face higher credit costs and reduced access to credit due to the stricter bankruptcy regulations.

As we will explain in the model, borrowers effectively utilize their ability to abscond as leverage to gain advantages over creditors. For example, our model demonstrates that borrowers with high commitment capacity are fully exploited by the borrower. It's important to note that in this paper, our measure of commitment aligns more closely with the legal bankruptcy powers granted to borrowers, rather than being solely about credit. This is because we assume that, in the model, borrowers cannot manipulate their commitment level to maximize their own utility. In our model, commitment is considered an exogenous characteristic bestowed upon the borrower.

In summary, while this paper refrains from taking a specific stance on the welfare implications of the BAPCPA law, we assert that the law did not achieve the intended results for all consumers and businesses. Instead, we illustrate that a specific subset of debtors ends up bearing the burden of the stricter bankruptcy regulations.

2.4 The Model

The model environment is based on Diamond-Dybvig. There are three time periods indexed by $t = 0, 1, 2$. Two types of agents live for three periods; there is a measure one of depositors and a measure n of borrowers. At date $t = 0$, all depositors are alike and all

[9] Press Release, George W. Bush, President Signs Bankruptcy Abuse Prevention, Consumer Protection Act (April 20, 2005).

borrowers are alike.

Each depositor is endowed with one unit of capital in $t = 0$ and nothing in $t = 1, 2$. Depositors have the access to a long-term investment technology that turns 1 unit of capital at $t = 0$ into $\underline{R} > 1$ units of consumption good at $t = 2$ or 1 unit of consumption good at $t = 1$. There is also a storage technology transforming capital into the consumption good at a one-for-one rate in either period. At date $t = 1$, each depositor receives an idiosyncratic preference shock. With probability λ, the depositor is impatient, meaning he derives utility exclusively from consuming in $t = 1$. With probability $1 - \lambda$, the depositor is patient, meaning he values consumption in $t = 2$.

Let $u(x_1)$ and $u(x_2)$ denote the utility function of the impatient and patient depositors, respectively, where x_t is the consumption in period t, $u' > 0 > u''$ and $u(0) = 0$. The coefficient of relative risk aversion (CRRA), $-xu''(x)/u'(x)$, is greater than 1 for $x \geq 1$. Whether a depositor is patient or impatient is his private information.[10] By the law of large numbers, λ is also the fraction of depositors in the population who are impatient.

Borrowers can access a technology that turns one unit of capital at $t = 0$ into $\bar{R} > \underline{R}$ units of consumption good at $t = 2$ or one unit of consumption good at $t = 1$. They are not endowed with capital. However, they can produce capital by incurring a utility cost at $t = 0$. Let $c(k)$ be the cost function of producing k units of capital, where $c', c'' > 0$ and $c'(0) = c(0) = 0$. Borrowers consume x_B units of consumption good at $t = 2$ with utility function $v(x_B)$, where $v' > 0 > v''$ and $v(0) = 0$.

Following the literature, depositors have the incentive to form a coalition that acts like a bank by providing themselves with a deposit contract to insure against the consumption shock. Because $\bar{R} > \underline{R}$, the bank would lend all their capital to the borrowers. However,

[10]The private information feature of the model is not important for our analysis since we do not focus on bank runs. It only matters in Section 2.7.2.

the borrowers cannot commit to repayment. When a borrower receives a loan, he can liquidate the investment at $t = 1$, abscond with χ fraction of it, transform capital into the consumption good at a one-for-one rate, and store it to consume at $t = 2$. For simplicity, assume when the borrower absconds with the investment, the remaining $1 - \chi$ fraction of it is lost, which captures the loss from diverting investment. To reduce the severity of the commitment problem, the borrower can pledge his own capital as collateral. So when the borrower absconds with the investment, he loses a fraction of the collateral. For simplicity, let this fraction be $1 - \chi$ as well.

In this environment, banks and borrowers can work together to economize on the investment and exploit the higher-return technology. We show that the gains from trade are determined and are divided, depending on the market structure. To develop intuition, we first study a competitive loan market where the banks and borrowers take the loan rate as given and choose the loan size. We then consider bilateral trade where the terms of trade are determined according to Nash bargaining. Lastly, we consider competitive search so the trade surplus is divided according to the market tightness.

To set up the benchmark for comparison, we first calculate the autarky payoff of the banks and the borrowers. Let borrowers solve:

$$\hat{W}_B = \max_k \left[-c(k) + v\left(\bar{R}k\right) \right]$$

The first-order condition is $c'(k) = \bar{R}v'\left(\bar{R}k\right)$. Let the solution for capital be represented by $\hat{k}$. Borrowers consume $\hat{x}_B = \bar{R}\hat{k}$.

As a coalition of depositors, the bank seeks to maximize the expected welfare of its depositors. Let the bank solve

$$\hat{W}_D = \max_{x_1,x_2}[\lambda u(x_1) + (1-\lambda)u(x_2)] \tag{2.1}$$

$$\text{st } (1-\lambda x_1)\underline{R} = (1-\lambda)x_2 \tag{2.2}$$

$$x_2 \geq x_1 \tag{2.3}$$

where (2.2) is the resource constraint and (2.3) is the incentive constraint so that patient depositors withdraw at $t = 2$. The bank liquidates λx_1 from its investment at $t = 1$ to pay the impatient depositors and leaves the rest until $t = 2$ with return $\underline{R}$ to pay the patient ones. As standard, the solution to (2.1), denoted by $(\hat{x}_1, \hat{x}_2)$ satisfies the first-order condition $u'(x_1) = \underline{R}u'(x_2)$ and (2.2). At $(\hat{x}_1, \hat{x}_2)$, (2.3) does not bind.

2.5 Competitive loan market

In a competitive loan market, a borrower takes the market rate as given subject to pledging enough capital as collateral to satisfy the repayment constraint. By not reneging, the borrower consumes

$$x_B = \big(\bar{R} - r\big)\ell + \bar{R}k, \tag{2.4}$$

where r is the loan rate and ℓ is the loan size. The consumption comes from two sources: he borrows ℓ from the bank, invests it, gets return $\bar{R}$ and pays r; in addition, his own capital yields $\bar{R}$. Conversely, by reneging, the borrower gets χ fraction of the *total* investment. Formally, the repayment constraint is represented as

$$x_B \geq \chi(\ell + k). \tag{2.5}$$

A borrower chooses loan quantity and collateral solving the following problem.[11]

$$\max_{k,\ell}[-c(k)+v(x_B)]$$

$$\text{st (2.4) and (2.5)}$$

The first-order conditions are

$$-\big(\chi+r-\bar{R}\big)c'(k)+r\chi v'(x_B) = 0 \tag{2.6}$$

$$\eta-\frac{\big(\bar{R}-r\big)v'(x_B)}{\chi+r-\bar{R}} = 0 \tag{2.7}$$

where η is the Lagrangian multiplier associated with (2.5). The shadow value of the repayment constraint sets up two cases. First, if $r=\bar{R}$, then $k=\hat{k}$ and $\ell\leq\big(\bar{R}-\chi\big)\hat{k}/\chi$, where the last inequality describes the demand for loan given $r=\bar{R}$. Second, with $\bar{R}-\chi\leq r<\bar{R}$, the demand for loan solves (2.6) with binding (2.5). That is,

$$c'\left(\frac{\chi+r-\bar{R}}{\bar{R}-\chi}\ell\right)=\frac{r\chi}{\chi+r-\bar{R}}v'\left(\frac{r\chi}{\bar{R}-\chi}\ell\right) \tag{2.8}$$

With (2.5) binding, the total investment versus capital (or leverage) is $r/\big(\chi+r-\bar{R}\big)$. The RHS of (2.8) says for a marginal unit of k, the borrower's consumption increases by $r\chi/\big(\chi+r-\bar{R}\big)$ as he is paid χ fraction of total investment when (2.5) binds. So the RHS is the marginal benefit of producing capital, which equals the marginal cost (the LHS) in equilibrium.

The bank chooses loan quantity and the deposit contract (x_1,x_2), taking the loan rate

[11] This setup can also be interpreted as follows. Given the market loan rates for borrowers with different k, the borrowers choose k and ℓ. The bank chooses markets to extend loans. As they always lend in the market with the highest r, the equilibrium interest rate is unique.

as given. The bank keeps λx_1 in its own technology to pay the impatient depositors. It can invest an additional amount of a in its own long-term technology, lending the rest. The bank solves

$$\max_{x_1,x_2,a} [\lambda u(x_1) + (1-\lambda)u(x_2)]$$
$$\text{st } (1-\lambda)x_2 = (1-\lambda x_1 - a)r + a\underline{R} \tag{2.9}$$

and (2.3). Equation (2.9) is the feasibility constraint for the bank. The amount of loan extended to the borrowers is $1-\lambda x_1 - a$. With loan repayment at date $t=2$, the bank is paid the interest rate r. The safe-haven investment, a, matures with return $\underline{R}$. From its revenues, the bank pays the patient depositors at $t=2$.

The first-order condition with respect to x_1 is

$$u'(x_1) - ru'(x_2) = 0 \tag{2.10}$$

By (2.9), $a=0$ if $r > \underline{R}$, $0 < a < 1-\lambda x_1$ if $r = \underline{R}$, and $a = 1-\lambda x_1$ if $r < \underline{R}$. With the bank lending $1-\lambda x_1 - a$, (2.10) also describes the loan supply as a function of r.

The loan-market clearing condition is

$$1-\lambda x_1 - a = n\ell \tag{2.11}$$

which pins down equilibrium r. The equilibrium loan contract is in one of the three distinct regions with respect to fraction of loan pledged, depending on whether (2.5) binds and whether $a>0$ or $a=0$. We refer to the High commitment region for χ such that (2.5) does not bind. As χ increases, the repayment constraint binds and $a=0$, which we refer to as the Intermediate commitment region. Lastly the Low commitment region is characterized by values of χ such that (2.5) binds and $a > 0$. The High region corresponds to the most creditworthy borrowers (lowest χ), followed by the Intermediate region and ultimately by the Low region as χ continues to increase. A more detailed discussion follows.

High commitment region: With $\eta = 0$, we solve (2.9)-(2.11) to get $a = 0$, $r = \bar{R}$, $k = \hat{k}$, $x_1 = x_1^*$ and $x_2 = x_2^*$, where (x_1^*, x_2^*) satisfies $u'(x_1) = \bar{R}u'(x_2)$ and $(1 - \lambda x_1)\bar{R} = (1 - \lambda)x_2$. Total loan size is $1 - \lambda x_1^*$. Each borrower gets $\ell^* = (1 - \lambda x_1^*)/n$. The High region applies for $\chi \leq \chi_1^C$, where $\chi_1^C \equiv \hat{k}\bar{R}/\left(\ell^* + \hat{k}\right)$.[12]

In the High region, the bank takes full advantage of the borrower's technology. The marginal rate of substitution between x_1 and x_2 is equal to the marginal rate of technological transformation implied by the borrower's technology. The borrower's capital production and consumption are the same as in autarky.

Intermediate commitment region: With $\eta > 0$ and $a = 0$, the Intermediate region holds for $\chi \in (\chi_1^C, \chi_2^C]$. (We derive χ_2^C in the Appendix proof of Proposition 2.1.) In this region, $\underline{R} < r < \bar{R}$.

Note that the bank's loan supply does not depend on χ. In the Intermediate region, the borrower's demand for loan is constrained by (2.5). For a constant loan rate, r, an increase in χ results in borrowers demand fewer loans. In this setting, borrowers could produce more capital to borrow more, but an additional unit of loan requires more capital, with the marginal cost equal to c'. Leverage declines and v' is lower because of the additional loan and capital. From (2.8), we find that the demand for loan decreases with χ, which results in a lower loan rate. In equilibrium, the bank lends more to partially make up the loss in long-term revenue, and the borrowers take advantage of lower r to borrow more.

[12]Our notational convention is as follows: the superscript C denotes the "competitive" market structure, and the subscript "1" denotes the first cutoff.

Given a lower loan rate, the borrowers get a higher return on the loan. Whether they will produce more capital to pledge as collateral depends on the elasticity of intertemporal substitution (EIS). If EIS is elastic (i.e., $-v'/xv'' > 1$), a higher return encourages higher consumption growth rate, which requires more capital production. If it is inelastic (i.e., $-v'/xv'' < 1$), less capital will be produced as a weaker response to the change in the loan rate.

Borrower's consumption, x_B, is not necessarily monotone in χ for two reasons: (1) the production of collateral may go either way, and (2) the repayment constraint binds so the borrowers cannot borrow freely in response to the change in r. However, when evaluated at χ_1^C, the borrower's consumption and lifetime utility are strictly increasing as represented by $-c+v$. To understand this, assume first that the market rate r stays constant as χ increases from χ_1^C. Borrowers choose ℓ and k along the the envelope frontier of $-c+v$ as (2.5) just binds. There is no first-order effect. However, r is not constant; it is lower, which improves the loan terms and increases borrower's welfare.

Low commitment region: With $\eta > 0$ and $a > 0$, we see that $r = \underline{R}$. It follows immediately that $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$ as in autarky. The borrowers take full advantage of the loan market as they pay bank's reservation rate. This region requires $\chi > \chi_2^C$. As r cannot be lowered further, ℓ decreases with χ to satisfy (2.5). It follows that x_B is decreasing in χ and a is strictly increasing. Again, the change in k depends on $-v'/xv''$. For elastic intertemporal substitution, borrowers produce less capital as borrowing is more responsive to changes in the return, which produces larger response in x_B. For inelastic intertemporal substitution, borrowers produce more capital to partially offset the effect of tightened borrowing condition so consumption does not fall too much.

When χ increases and the economy moves from High to Intermediate and eventually to the Low region, the borrowing conditions can change in several dimensions in order to alleviate the commitment problem: the borrower can provide more collateral, the market loan rate can be lowered, and/or the banks can reduce lending by allocating more funds to the safe-haven project. We find that banks invest in the safe-haven asset as a last resort. This is because increasing k and lowering r at their interior optimum does not have a first-order impact on efficiency. However, raising a from 0 (at the corner solution) reduces total output, which does have a first-order impact.

The result is summarized in the following proposition.

Proposition 2.1 *In a perfectly competitive loan market, there exists χ_1^C and χ_2^C, with $\chi_1^C < \chi_2^C$ such that (1) with $\chi < \chi_1^C$, the equilibrium is in the high commitment region; (2)with $\chi_1^C \leq \chi < \chi_2^C$, the equilibrium is in the Intermediate commitment region; (3) with $\chi > \chi_2^C$, the equilibrium is in the Low commitment region.*

The top panel of Table 2.3 reports the comparative statics with respect to χ for each region. Here, $dk/d\chi \doteq v''x/v' + 1$ in the Intermediate commitment region, $sgn(dk/d\chi) = sgn[-(v''x/v' + 1)]$ in the Low commitment region, where sgn stands for the sign. In addition, $dx_B/d\chi|_{\chi_1^C} > 0$.

Table 2.3: Comparative Statics, Competitive Market

	$\frac{dx_1}{d\chi}$	$\frac{dx_2}{d\chi}$	$\frac{dx_B}{d\chi}$	$\frac{dk}{d\chi}$	$\frac{dr}{d\chi}$	$\frac{d\ell}{d\chi}$	$\frac{da}{d\chi}$
High	0	0	0	0	0	0	N.A.
Intermediate	-	-	?	?	-	+	N.A.
Low	0	0	-	?	0	-	+
	$\frac{dx_1}{dn}$	$\frac{dx_2}{dn}$	$\frac{dx_B}{dn}$	$\frac{dk}{dn}$	$\frac{dr}{dn}$	$\frac{d\ell}{dn}$	$\frac{da}{dn}$
High	0	0	0	0	0	-	N.A.
Intermediate	+	+	-	?	+	-	N.A.
Low	0	0	0	0	0	0	-

To illustrate the equilibrium outcomes for different values of the borrower's fraction of loan pledged, we turn to a numeric example. In all examples in the paper, unless mentioned, the following functions and parameters are used:

$$u(x) = \frac{(x+b)^{1-\gamma} - b^{1-\gamma}}{1-\gamma}, c(k) = Bk^{\alpha}, v(x) = A\frac{(x+b)^{1-\delta} - b^{1-\delta}}{1-\delta}$$

where $b = 0.001$, $\gamma = 2$, $B = 1$, $\alpha = 2$, $A = 0.2$ and $\delta = 0.5$ or 2. Other parameters are $\bar{R} = 1.5$, $\underline{R} = 1.2$, $\lambda = 0.5$, and $n = 1$. The results are plotted in Figure 2.3.[13]

We begin by discussing what happens in the intermediate commitment region. With $\delta = 0.5$, $-v'/xv'' > 1$ (left column), collateral is inversely related to fraction of the loan pledged. Note that the relationship changes sign when we move into the low commitment region. The borrower's consumption and lifetime utility (labeled by W_B) follow the sign-switching pattern observed in the analysis of collateral. Regarding the deposit contract, both patient and impatient depositors are hurt with the greater loss suffered by patient depositors. Their expected utility (labeled by W_D) decreases in the Intermediate region and stays at the autarky level in the Low region. In the Low region, we observe the floor on the loan rate and loan size decreasing in χ.

With $\delta = 2$, $-v'/xv'' < 1$ (right column). The pattern of k is reversed in this case: it decreases in χ in the Intermediate region and then increases in the Low region. There is no significant change in the pattern of other variables. The results indicate how the elasticity of intertemporal substitution matters for the quantity of collateral in the equilibrium loan contract.

The competitive market raises questions about the quality of credit indicators. For the case in which $\delta = 0.5$, the loan rate and collateral intensity (i.e., k/ℓ ratio) decline and

[13] The dash blue curve in the top panel is x_2, and the solid blue curve is x_1.

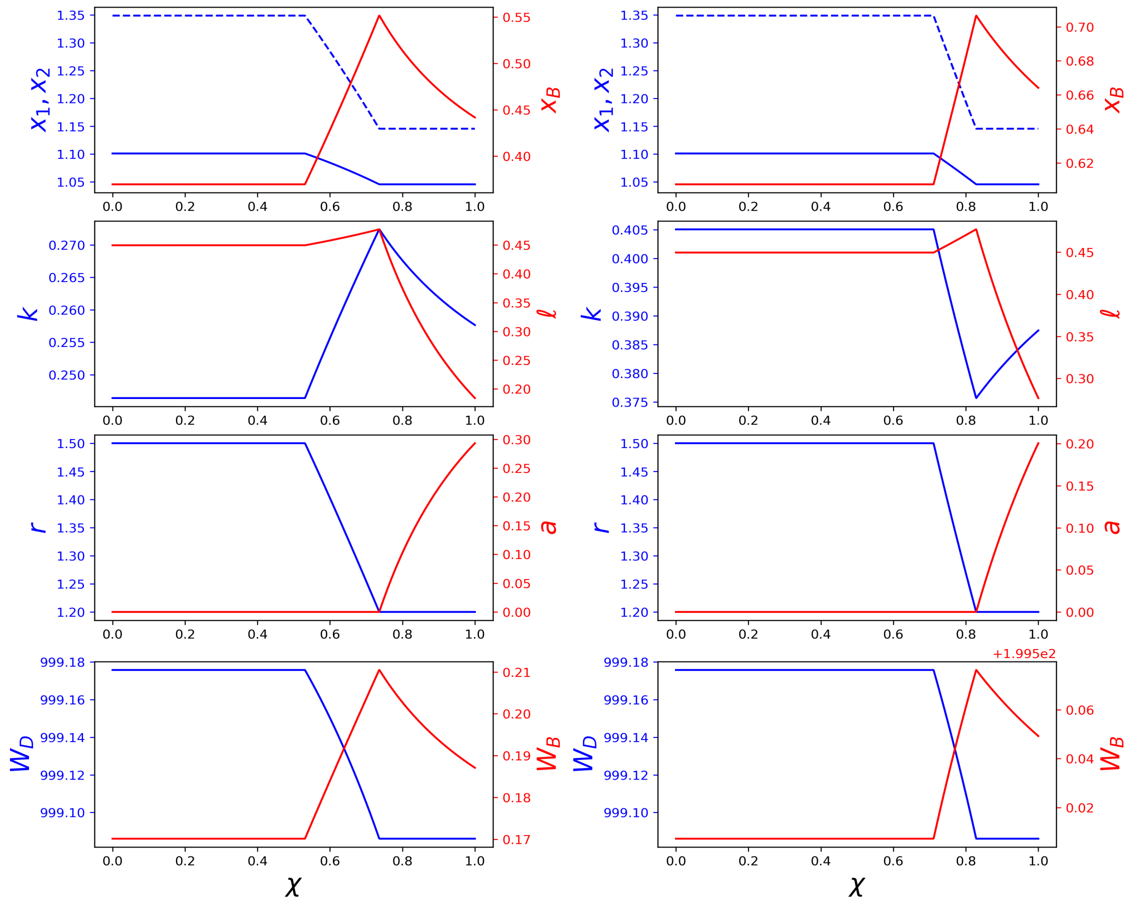

Figure 2.3: Experiments with Varying χ in Competitive Equilibrium: $\delta = 0.5$ left, $\delta = 2$ right

the loan size increases as the credit conditions deteriorates in the Intermediate commitment region. It is only when the economy enters the Low commitment region that the indicators begin to match with the interpretation of credit conditions behind summary indicators like the NCFI.

We also conduct the comparative statics with respect to market tightness. For exogenous changes in n, the results are reported in the bottom panel of Table 2.3. As n increases, the competition for loans becomes more fierce. It is straight-forward to show that χ_1^C

and χ_2^C increase with n. In the High region of $r = \bar{R}$, bank's supply of loans is fixed at $1 - \lambda x_1^*$. As there are more borrowers, each gets a smaller loan, which makes the repayment constraint looser and expands the High region. In the Intermediate region, as n increases, ℓ decreases, r increases, and banks receive higher return on the investment and pay more to both types of depositors. The borrowers get lower x_B and will pledge more k if $-v'/v''x > 1$. Our intuition is straightforward: as the demand for loans increases, the market works in favor of the bank. The more borrowers, the less likely that the market will be in the Low region where the borrowers take full advantage of the market. Consequently, the Low region shrinks.

The overall point of both parameter settings is that the loan terms adjust so that there is no default in equilibrium. Borrowers receive more attractive loan rates and possible larger loans in the Intermediate region. The model economy predicts that indicator values–such as loan rate, loan size and collateral values–need to be interpreted based on the state of credit conditions in the economy.

2.6 Nash bargaining

Next, we consider bilateral trade. Our goal is to consider a market structure that resembles the OTC market. Does the market structure affect the response to changes in fraction of loan pledged?

Suppose a bank and a borrower decide on the terms of trade according to Nash bargaining solution. Let θ be the bargaining power of the bank. The generalized Nash problem is

$$\max_{x_1, x_2, x_B, r, k, a} \left[\lambda u(x_1) + (1-\lambda) u(x_2) - \hat{W}_D\right]^{\theta} \left[-c(k) + v(x_B) - \hat{W}_B\right]^{1-\theta} \tag{2.12}$$

st (2.3), (2.9)

$$x_B = (1 - \lambda x_1 - a)\big(\bar{R} - r\big) + k\bar{R} \tag{2.13}$$

$$x_B \geq \chi(1 - \lambda x_1 - a + k) \tag{2.14}$$

Trade only occurs if both parties agree on the terms of trade. Autarky is the alternative. Once consumption, capital, loan size and rate are agreed upon, bank and borrower split the trade surplus according to their bargaining power.

We write the efficiency conditions as follows:

$$\begin{aligned}
u'(x_1)[c'(k) - \chi v'(x_B)] - \big(\bar{R} - \chi\big)u'(x_2)c'(k) &= 0 && (2.15)\\
a[-u'(x_1) + \underline{R}u'(x_2)] &= 0 && (2.16)\\
\theta u'(x_1)S_B - (1-\theta)c'(k)S_D &= 0 && (2.17)\\
\chi\eta - \frac{(1-\theta)S_D c'(k)}{u'(x_1)}\big[\bar{R}u'(x_2) - u'(x_1)\big] &= 0 && (2.18)
\end{aligned}$$

where $S_D \equiv \lambda u(x_1) + (1-\lambda)u(x_2) - \hat{W}_D$ is bank's trade surplus, $S_B \equiv -c(k) + v(x_B) - \hat{W}_B$ is borrower's surplus, and η is the Lagrangean multiplier associated with (2.14). As in the competitive equilibrium, there are three regions, depending on χ.

Because one's surplus is not necessarily positively related to the bargaining set under Nash (see Kalai (1977)), it is hard to derive unambiguous comparative statics for all variables in the three regions. In other words, when χ goes up, we cannot determine whether the terms of trade improve for the borrower or not.

We do find that a is positively related to χ in the Low commitment region. Thus, there exist a unique cutoff dividing the Intermediate and the Low regions. It also implies that investing in the safe-haven long-term project is the last tool that the bank uses to cope with deteriorating credit conditions. The intuition is the same as in Section 2.5: by starting to invest in the safe-haven long-term asset (a increasing from 0), there is a discrete drop in the total resources for consumption. However, changing k, r and/or ℓ in the interior solution moves the economy along the envelope frontier and does not have a first-order impact.

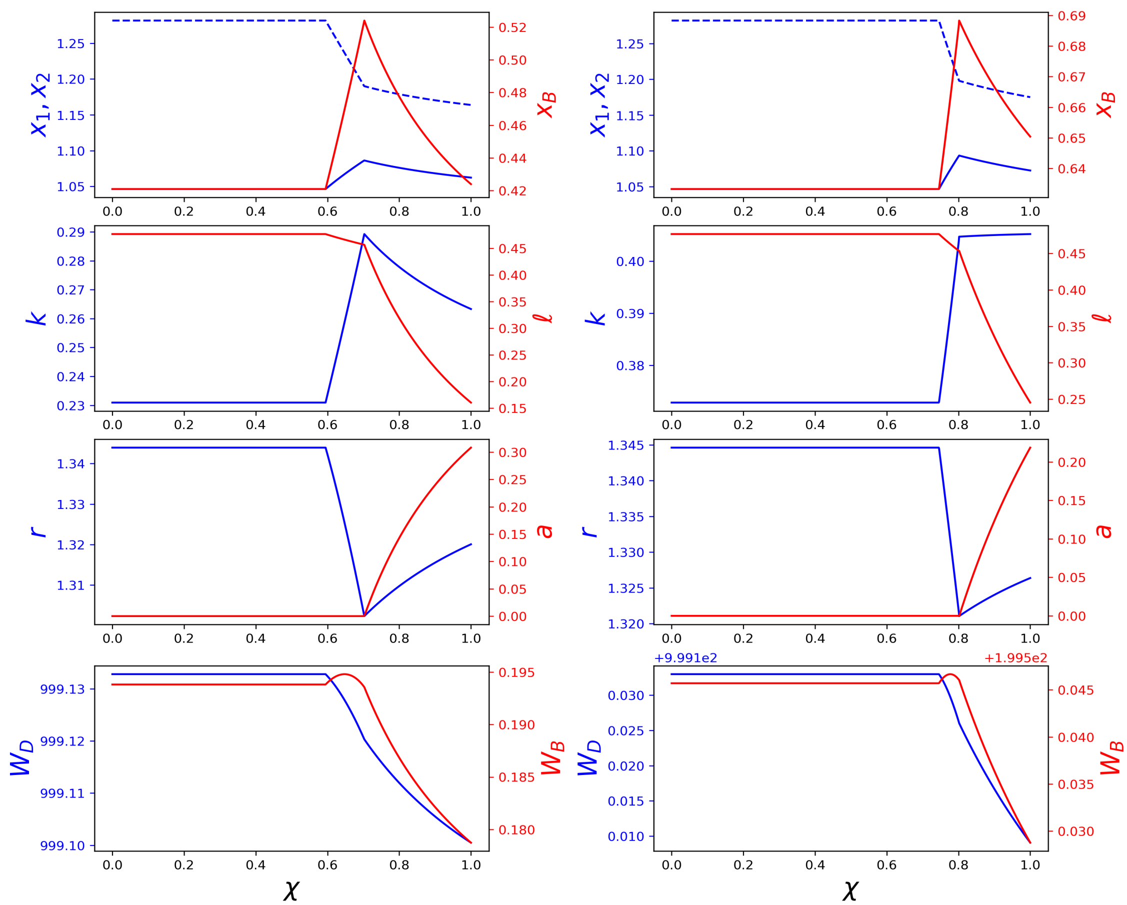

Figure 2.4: Experiments with Varying χ in Nash Bargaining: $\delta = 0.5$ left, $\delta = 2$ right

Figure 2.4 plots the numerical result with $\theta = 0.5$. There are some differences in the pattern of contract variables when compared with the optimal contracts in the competitive market: for increases in χ, (i) x_1 is monotonically increasing in the Intermediate commitment region; (ii) r is strictly increasing in the Low commitment region; (iii) ℓ is strictly decreasing in the Intermediate and Low commitment regions, (iv) W_B increases and then decreases in the Intermediate commitment region, and (v) in the example of $\delta = 2$, k strictly increases in the Intermediate and the Low commitment regions. Notice that when x_1 increases and x_2 decreases, risk sharing improves. The cost, however, is fewer resources are invested in the high-return borrower's project, and the bank offers more short-term consumption and lower rate on long-term deposit as a compromise.

Now that banks and borrowers are bargaining over the terms of the loan contract, we still see the non-monotonic relationship between changes in commitment level and changes in capital; capital is positively related to commitment level in the Intermediate region and negatively related to commitment level in the Low region. However, the elasticity of intertemporal substitution no longer affects the sign of the comparative statics in the Intermediate region.

2.7 Competitive search

Next, we consider a competitive search loan market. Suppose a bank can open a loan market characterized by deposit and loan terms $(x_1, x_2, x_B, r, k, \ell)$. Borrowers observe those posted terms and pay an entry cost, denoted ϕ, choosing to go to a specific market. In a market, banks and borrowers are matched according to the function $M(n_D, n_B)$, where n_D and n_B are the measures of the banks and borrowers, respectively. Assume M is strictly increasing, strictly concave, and homogeneous of degree 1 in both arguments. Let $\tau = n_B/n_D$ be the market tightness. The probability that a bank meets a borrower is $\sigma(\tau) \equiv M(1, \tau)$ and the probability that a borrower meets a bank is $\sigma(\tau)/\tau$.

Banks post terms of trade to solve the following problem:

$$\max_{x_1,x_2,x_B,r,k,a,\tau} \sigma(\tau)[\lambda u(x_1)+(1-\lambda)u(x_2)]+[1-\sigma(\tau)]\hat{W}_D \tag{2.19}$$

$$\text{st } (2.3), (2.9), (2.13), (2.14)$$

$$\frac{\sigma(\tau)}{\tau}\left\{-c(k)+v\left[(1-\lambda x_1-a)\left(\bar{R}-r\right)+k\bar{R}\right]\right\}+\left[1-\frac{\sigma(\tau)}{\tau}\right]\hat{W}_B=\phi+\hat{W}_B \tag{2.20}$$

The LHS of (2.20) is the expected utility for a borrower in the market. A borrower is willing to pay ϕ to enter the market as long as the expected trade surplus is larger than ϕ.

The first-order conditions are (2.15), (2.16) and

$$[1-\varepsilon(\tau)]u'(x_1)S_B-\varepsilon(\tau)c'(k)S_D = 0 \tag{2.21}$$

$$\eta-\frac{\sigma(\tau)u_2'(x_2)}{c'(k)-\chi v'(x_B)}\left[c'(k)-\bar{R}v'(x_B)\right] = 0 \tag{2.22}$$

where $\varepsilon(\tau)=\sigma'(\tau)\tau/\sigma(\tau)$ is the elasticity of the matching function and η is the Lagrangian multiplier associated with (2.14).

Compared with Nash bargaining, the contracts obtained in competitive search are qualitatively similar. With competitive search, bargaining power is endogenous, owing to the elasticity of the matching function. In general, he market tightness and trade surplus are pinned down jointly by (2.20) and (2.21).[14]

As with the other economies, we find that there are three regions for the equilibrium contracts varying with the borrower's fraction of loan pledged when $\varepsilon'<0$. In the competitive search economy, many comparative statics are ambiguous; however, $dk/d\chi<0$, $dr/d\chi<0$ and $da/d\chi>0$ in the Low commitment region, and $dr/d\chi<0$ at χ_1^S, where

[14] Note that when the matching elasticity is constant, the solution is the same as under Nash bargaining and the market tightness is solely pinned down by (2.20).

χ_1^S is the cutoff χ between High and Intermediate regions.

Though not included in this paper, the numerical analysis are qualitatively similar to those reported in the Nash bargaining economy.[15] One key difference is how the bank alters payments to impatient consumers relative to patient consumers in the Intermediate region. Specifically, payments to impatient consumers *increase* when the bank meets borrowers in bilateral matches and has some market power. Thus, the existence of market power matters for risk sharing.

2.7.1 Changes to entry cost

Suppose the entry cost is lower in the competitive search economy. More borrowers enter the market, and the market gets tighter, which lowers the matching probability of the borrowers. Holding everything else constant, a tighter market lowers the expected utility of entering. Whether the fall in the matching probability is sufficient to offset the fall in entry cost depends on the elasticity of the matching function. With the assumption that $\varepsilon' < 0$. We have the following proposition.

Proposition 2.2 *Suppose $\varepsilon' < 0$. The cutoffs, χ_1^S and χ_2^S, strictly increase in ϕ.*

If ϕ decreases, for example, the high commitment region characterized by $\chi \leq \chi_1^S$ shrinks while the Low commitment region expands. Although low entry fee expands entry, the contract is more likely to be in the Low commitment region. The implication is that some deposits are allocated to the safe project. This finding may sound counter-intuitive. But, as more borrowers enter the market, banks provide less favorable terms. With higher rates and more collateral the borrower's repayment constraint tightens.

[15]The working paper version of the paper has the plots. A notable difference is that the elasticity of intertemporal substitution does not change the patterns of the contract variables.

Table 2.4: Comparative Statics, Competitive Search, $\varepsilon' < 0$, Changes in Entry Cost

	$\frac{dx_1}{d\phi}$	$\frac{dx_2}{d\phi}$	$\frac{dx_B}{d\phi}$	$\frac{dk}{d\phi}$	$\frac{dr}{d\phi}$	$\frac{d\ell}{d\phi}$	$\frac{d\tau}{d\phi}$	$\frac{da}{d\phi}$
High	$-$	$-$	$+$	$-$	$-$	$+$	$-$	N.A.
Intermediate	?	?	?	$-$	$-$	?	$-$	N.A.
Low	$-$	$-$	$+$	$-$	$-$	$+$	$-$	$-$

Table 2.4 lists the comparative statics. As entry cost decreases, we see that the terms of loan contracts worsen in the Low commitment region; in other words, borrowers in the market are worse off, but depositors are better off. The result in the Intermediate region is ambiguous.

Figure 2.5 plots equilibrium outcomes with $A = 1.1$ and $\delta = 1.05$, using the same values for other parameters as previous experiments. In addition, let $M(n_d, n_b) = n_d n_b/(n_d + n_b)$. Set $\chi = 0.99$.[16] The borrowers do not enter if $\phi > 0.14$. The economy moves from the Low credit region to the Intermediate region at $\phi = 0.005$ and then moves to the High region at $\phi = 0.137$. As the entry cost increases, market tightness and loan size increase, loan rate, collateral, and depositor's consumption decline.

Consider how a change in entry cost affects deposit and loan contracts. The results of this experiment shed light on changes in the distribution of the gains and the mechanism operating on welfare.[17] In addition, we consider the impact of an unexpected change in the return to borrower's projects on bank stability.

[16]This extreme value of χ is used to show that the economy transits through all three regions when ϕ increases. For smaller value of χ, the economy transits through two regions or stays in one region.

[17]Of course, we can model entry decision in competitive market or bilateral trading. However, free entry in the competitive market will rule out the High region if borrowers enter or the Low region if banks enter. While the terms of trade do not change under bilateral negotiation as the entry cost is sunk when the two parties meet.

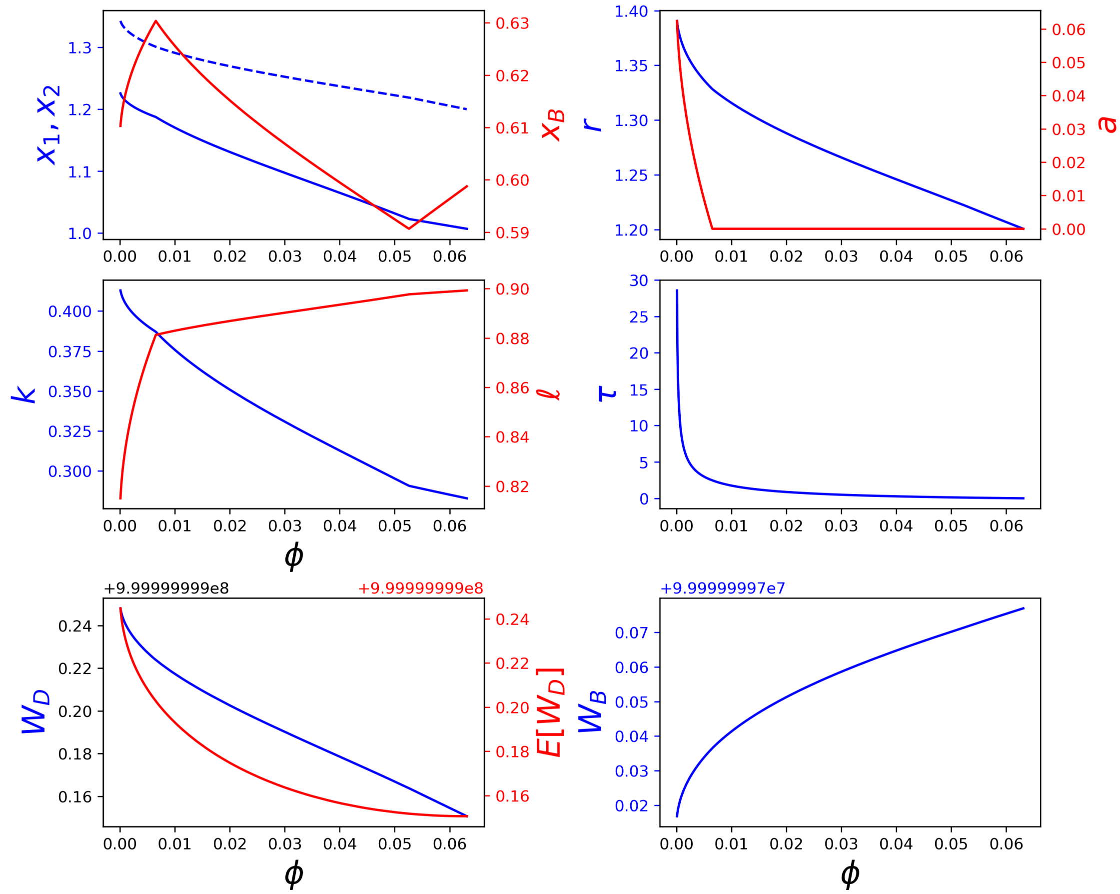

Figure 2.5: Experiments with Varying ϕ in Competitive Search

2.7.2 Discussion on financial stability

Throughout the paper, we do not discuss bank runs because a demand deposit with suspension clauses can eliminate panic runs in the environment with no aggregate uncertainty. In the following experiment, we consider an unexpected negative shock to the return on the borrower's project, providing some insight into the financial stability in our model economy.

To formalize the experiment, suppose there is an unanticipated decrease in the return of borrower's project. Let $\tilde{R}$ denote the new realized return, where $\tilde{R} < \bar{R}$. This information is revealed to the public at $t = 1$. The borrower honors the repayment if

$$\ell\left(\tilde{R} - r\right) + k\tilde{R} \geq \chi(\ell + k). \tag{2.23}$$

Otherwise, the borrower defaults. Because the shock is unexpected, default is not accommodated by the contract. Depositors learn the borrower's repayment decision, and then decide whether to withdraw at $t = 1$. Even with a suspension clause, the depositor's withdrawal decision depends on the resources available in $t = 2$. With fewer resources, patient depositors are tempted to withdraw at $t = 1$.

In the high commitment region, (2.14) does not bind. As long as the fall in $\bar{R}$ is not too big, the borrowers honor the debt. In the Intermediate and Low commitment regions, however, the repayment constraint binds. Consequently, a decrease in borrower's return violates (2.14) and triggers a default on the loan contract.

In the Intermediate commitment region, the bank opts to allocate all long-term assets to the borrowers' projects. With all-the-eggs-in-one basket, default triggers a bank run. However, bank runs may not occur in the Low region where $a > 0$. The bank can pay each patient consumer $a\underline{R}/(1-\lambda)$ from its own technology. If $a\underline{R}/(1-\lambda) \geq x_1$, patient depositor's best response is to not withdraw at $t = 1$ if all other patient ones wait. So the tipping point for run to occur is where $a\underline{R}/(1-\lambda) = x_1$. fraction of loan pledged affects the tipping point through two channels. First, the bank's direct investment, a is increasing in χ. Second, the payments to impatient depositors, x_1, is decreasing in χ. Through both channels, a bank runs is less likely to occurs with deteriorating credit conditions.

We extend the numerical analysis with $\lambda = 0.2$, $\gamma = 1.1$, $\delta = 0.5$ and $A = 0.05$. The cutoffs are $\chi_1^S = 0.29$ and $\chi_2^S = 0.44$. For $\chi < \chi_1^S$, lower χ can tolerate bigger negative shocks. In the Low commitment region, $a\underline{R} \geq (1-\lambda)x_1$ for $\chi \geq 0.88$, which means even if the borrowers renege, there is no bank run.

2.8 Glass-Steagall Act. And Universal Banking

There has always been an ongoing debate regarding the involvement of intermediaries, particularly commercial banks, in direct investment activities and whether they should be regulated. Supporters of these regulations argue that intermediary direct investments could potentially pose a threat to the stability of financial markets.

The Glass-Steagall Act of 1933 stands as a significant legislative attempt to address the division between intermediation and direct investment. This act, which restricted commercial banks from participating in securities underwriting, played a pivotal role in shaping the financial landscape. Kroszner and Rajan (1994) study conducted a comparative analysis of the performance of commercial banks, which were constrained from direct investment, and investment banks with the autonomy to invest directly. Their findings disclosed no substantial disparities in the quality of securities underwritten by these institutions.

Furthermore, in the context of the German economy, Gorton and Schmid (2000) unveiled the positive influence of universal banking on the performance of firms under bank holdings. Berlin et al. (1996) embarked on an exploration of the potential contributions of bank equity investments in addressing the challenges faced by financially distressed businesses. Additionally, Santos (1996) delved into the welfare implications arising from restrictions on bank investments in non-financial enterprises.

In our basic model in section 2.5, we delve into the dynamics of bank loan contract terms and deposit contract terms, both of which are affected by borrowers' commitment issues. Our model can serve as a valuable tool for comparing the implications of universal banking versus non-universal banking and their impact on borrowers and depositors.

In this section, we compare and contrast two different scenarios. The first scenario involves an intermediary with the ability to universally allocate assets, allowing for storage, risky direct investments, and lending to external borrowers. The second scenario considers an intermediary limited to storing deposits or lending them to borrowers but not allowed to make direct investments. It's essential to clarify that our model does not consider moral hazards in the banking system. This study focuses on how allocation differs when we either restrict or grant banks universal autonomy in utilizing their liabilities.

To adapt our model for this task, we are introducing modifications to our competitive equilibrium model. In this updated model, we incorporate an element of risk associated with the bank's direct investments. Specifically, we assume that the bank's direct investment can lead to two possible outcomes: a desired return, denoted as R_h (where $R_h > 1$), with a probability of ρ, and an undesired return, denoted as R_l (where $R_l < 1$), with a probability of $1 - \rho$. Regarding the return on the borrower's technology, denoted as $\bar{R}$, we maintain our initial assumption that $R_h < \bar{R}$.

In the case of universal banking, we assume that the bank has the ability to utilize its own direct investment, storage, and lending. In this scenario, the bank, which represents a coalition of borrowers, seeks to solve the following problem within the competitive market:

$$\max_{x_1,s,a} \lambda u(x_1) + (1-\lambda)\Big[u(x_{2h})\rho + u(x_{2l})(1-\rho)\Big]$$

st.

$$(1-\lambda)x_{2h} = (s - \lambda x_1) + (1-s-a)r + aR_h \tag{2.24}$$

$$(1-\lambda)x_{2l} = (s - \lambda x_1) + (1-s-a)r + aR_l \tag{2.25}$$

$$u(x_{2h})\rho + u(x_{2l})(1-\rho) \geq u(x_1) \tag{2.26}$$

In this problem, the bank aims to optimize the expected utility of depositors by determining the ideal deposit rate for impatient depositors (x_1), the optimal storage amount (s), and the optimal level of investment in the bank's direct technology (a). The bank can subsequently figures out the optimal deposit rate for patient depositors and the ideal loan amount for borrowers, using the optimal values of x_1, s, and a.

The first constraint, represented by equation 2.24, deals with how consumption is allocated between patient and impatient borrowers when the direct investment results in a the high outcome. The second constraint, equation 2.25, focuses on consumption allocation when the direct investment yields the low outcome. The third constraint, equation 2.26, plays a role in preventing patient investors from triggering a bank run when they discover their type in time 1. The deposit contract ensures that the expected return from withdrawing in time 2 exceeds that of withdrawing in time 1. This condition is essential for the stability of the system. On the borrower's side, they face a similar problem as described in Section 2.5. The market clearing condition, now considering storage, is given by $1 - s - a = n\ell$.

Solving the competitive problem reveals that the high commitment region remains the same as described in Section 2.5. In this region, the lender fully exploits the borrowers. This is because, in this region, the bank strategically chooses not to invest in direct investments, effectively keeping the riskiness of these investments separate from the bank's contracts.

In the Intermediate commitment region, the bank, as discussed in Section 2.5, begins to reduce the interest rate on the loans. The interest rate on loans decreases from $\bar{R}$ to $\max\{1, \rho R_h + (1-\rho)R_l\}$. The optimal intermediary contract terms offered by the bank are the same as those presented in Section 2.5, but the span of this region widens since $\max\{1, \rho R_h + (1-\rho)R_l\} < R_h$.

The Low commitment region becomes relevant when the expected return on the bank's direct investment surpasses the return on storage. We assume that $1 < \rho R_h + (1 - \rho) R_l$, indicating that the bank initiates investments in its direct investment in the low commitment region. Within this region, the bank reduces loan sizes and allocates deposits into its direct investment. This region further divides into two sub-regions.

In the first sub-region, the bank increases its investment in the direct investment and decreases its loan size. As the commitment level of the borrowers declines in this sub-region, the bank diverts more deposits into the direct investment until the interest rate on the loan reaches 1, which equals the storage return. In the second sub-region of the lower commitment level, the bank begins reallocating funds towards storage, ceasing further allocation to the direct investment.

In the High commitment region and intermediate commitment region, storage is binding, meaning that banks store just enough to fulfill the impatient withdrawals in time 1. Although the bank faces risks from both the borrower and its own direct investment, it still dominates storage in terms of expectations. In the low commitment region, banks initially hold enough in storage technology to pay off the impatient withdrawals in time 1, but in the second part of the low commitment region, banks increase their storage beyond the impatient's expected withdrawal amount. At this point, the bank's return on the loan and storage are equal, and the bank sets the loan size and collateral such that, at this rate, the borrower doesn't abscond.

In the Non-Universal Banking case, when banks are barred from direct investment they would solve the following problem:

$$\max_{x_1, s} \lambda u(x_1) + (1 - \lambda) u(x_2)$$

$$\text{st.} \qquad (1 - \lambda) x_2 = (s - \lambda x_1) + (1 - s) r$$

$$x_1 \leq x_2$$

And borrowers are solving the usual problem. All the agents in the economy would face the same allocation in the first region as they faced in the previous cases. However banks allocation changes at the intermediate commitment level.

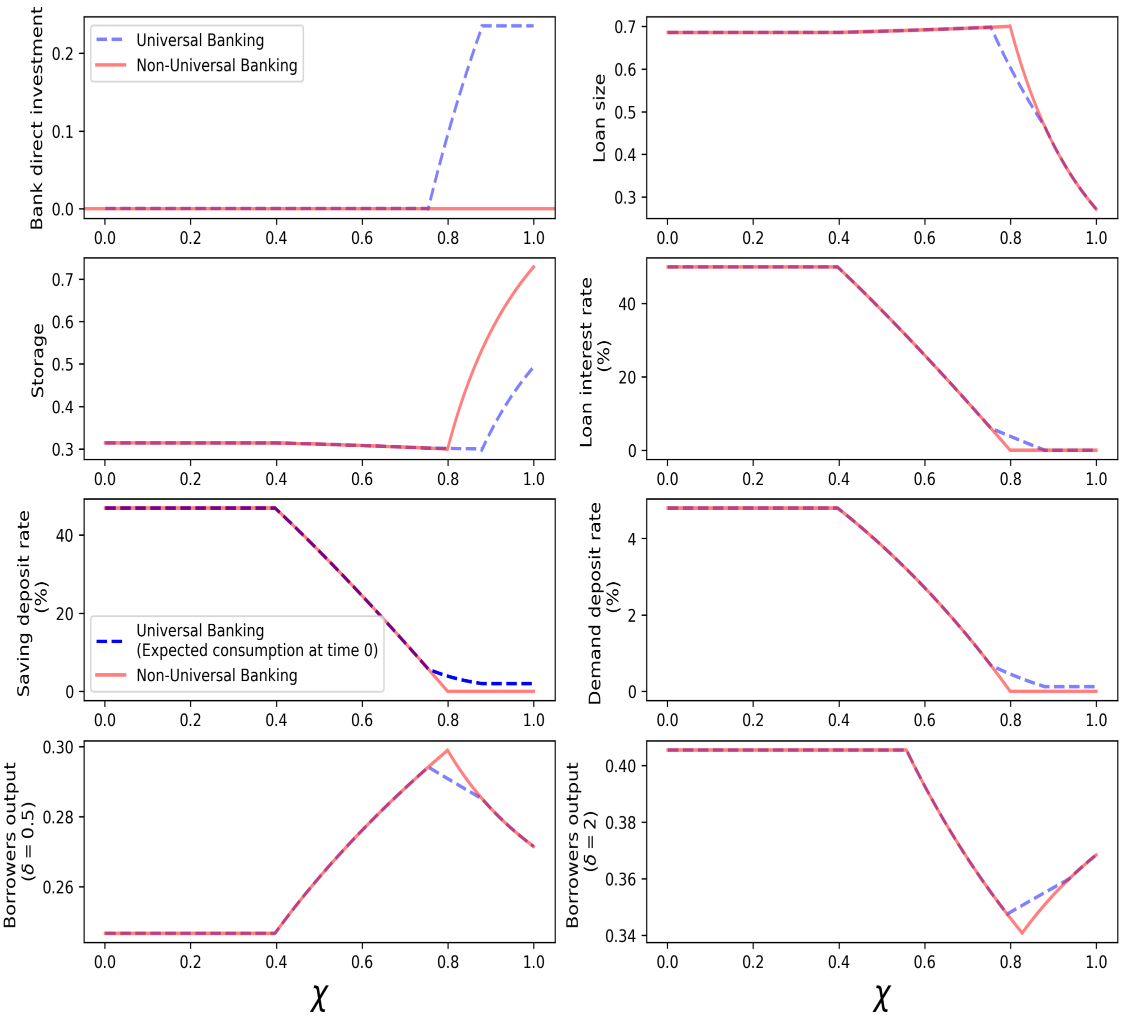

Figure 2.6: Experiments with Varying χ comparing universal banking and non-universal banking.

Figure 2.6 compares intermediary contract terms when banks are allowed to invest directly versus when they are prohibited from direct investment. In this figure, we can observe how the consumption levels of depositors and borrowers differ between universal banking and non-universal banking.

Universal banking primarily benefits depositors, while non-universal banking is more favorable to borrowers. When banks face constraints on their investments, their return on loans must account for their reduced investment capacity. As a result, banks may be willing to extend more loans at more favorable rates to borrowers in certain commitment scenarios.

This figure also illustrates that borrower output increases under non-universal banking when their relative risk aversion is less than one. However, if the relative risk aversion of borrowers exceeds one, universal banking leads to higher borrower output.

Contrary to the beliefs of supporters of regulations like Glass-Steagall, our findings suggest that restricting banks to storage and lending activities may not always improve financial system stability. Although these restrictions can increase bank storage, they also lead to expanded credit extension. Consequently, banks become more vulnerable to MIT shocks to the private sector's return, as explained in section 2.7.2, which can weaken their balance sheets during periods of financial stress.

To sum up, our model explains the implications of restricting banks from direct investment in an environment where banks are not subjected to moral hazard. This limitation is detrimental for depositors but beneficial for borrowers. We also demonstrate that, in this simplified environment, output levels can either increase or decrease depending on the risk preferences of borrowers when banks' direct investment is limited. Furthermore, we demonstrate that universal banking does not necessarily increase the riskiness of the financial system; it can reduce the risk associated with banks' balance sheets due to lower loan levels.

2.9 Summary and Conclusion

In this paper, we have developed a theoretical framework to derive equilibrium loan and deposit contracts in an economy where borrowers lack the ability to commit to loan repayment, while depositors seek risk sharing. Our results reveal that changes in commitment levels impact loan terms and risk sharing differently, depending on the market structure and the elasticity of intertemporal substitution.

One of the most intriguing findings pertains to the non-monotonic relationship between commitment levels, loan contract terms, and risk sharing in deposit contracts. This non-monotonicity has significant implications for interpreting financial indicators. Financial analysts often interpret movements in indicators, such as increasing loan rates or credit-constrained loan sizes, as signs of deteriorating borrower quality. However, in our model, loan rates decrease as borrower's commitment level worsen due to binding repayment constraints. Additionally, despite a decline in depositor's expected welfare with commitment levels, risk sharing is predicted to improve. The presence of a safe-return option introduces another dimension to the relationship between commitment levels and loan contract terms, shedding light on when and to what extent banks use outside options to diversify their portfolios.

We examine these relationships within over-the-counter (OTC) market setups and search and matching frameworks. We identify conditions in which aggregate shocks can lead to borrower defaults on bank loans, potentially causing bank insolvency and runs. Interestingly, the likelihood of bank runs is not monotonically related to borrower commitment levels, as extensive asset margins allow banks to diversify their portfolios and reduce the risk of runs, particularly under low-credit conditions.

To empirically illustrate this unexpected relationship between borrower commitment level and loan terms, we analyze the impact of the U.S. Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) on borrowers. The enactment of this law, which can be seen as improving the overall commitment levels of U.S. borrowers, led to increased loan rates and decreased loan sizes for specific borrower segments.

In the final part of our study, we adapt our foundational model to investigate the effects of Glass-Steagall and universal banking on intermediary contracts. Our findings indicate that limiting banks' direct investment opportunities can reduce returns for depositors while increasing returns for borrowers. Since banks are restricted from diversifying their investments among a broader range of opportunities, they become more inclined to extend larger amounts of credit at lower rates to borrowers with lower commitment levels. This, however, can make banks' balance sheets riskier by exposing them to shocks in borrowers' technology returns, contrasting the arguments of supporters of banking legislation such as Glass-Steagall.

The theoretical insights provided in this paper offer valuable guidance, particularly in understanding the interplay between limited commitment and bank portfolio diversification. On a theoretical front, there are opportunities for building models that explore how shocks operate in this interactive setup, potentially delving into the crisis amplification mechanism in banking within a dynamic context.

Appendix A

Appendix for Chapter 1

Proof of Proposition 1.1

Define the following Martingale,

$$M_t \equiv \frac{\eta}{1-\gamma}\int_0^t e^{-\eta s} w_s^{1-\gamma} ds + e^{-\eta t} v(x,y)$$

By the Ito formula,

$$M_t - M_0 = \frac{\eta}{1-\gamma}\int_0^t e^{-\eta s} w_s^{1-\gamma} ds + \int_0^t e^{-\eta s} dv(x,y) - \eta \int_0^t e^{-\eta s} v(x,y) dt$$

$$dM = e^{-\eta t}\Big\{\Big[\frac{\eta}{1-\gamma} w_t^{1-\gamma} - \eta v + \frac{1}{2}\sigma^2 y^2 v_{yy} + (\mu - q) y v_y + \Big(rx + qy\Big) v_x\Big] dt$$

$$+\Big(\frac{1}{1+\lambda^s} v_y - v_x\Big)\chi_t\Big(cx + qy\Big) dt + \Big(v_y - (1+\lambda^s) v_x\Big) dL_t + \Big(v_x - v_y\Big) dU_t + \sigma y v_y dz_t\Big\}$$

The HJB to be solved for v in 1.4 is:

$$\max_{l,u,\chi}\Big\{\frac{1}{2}\sigma^2 y^2 v_{yy} + \Big(rx + qy\Big) v_x + \Big(\mu - q\Big) y v_y + \frac{\eta}{1-\gamma} w^{1-\gamma} - \eta v$$

$$+\Big(\frac{1}{1+\lambda^s}v_y - v_x\Big)\chi\Big(cx+qy\Big) + \Big(v_y - (1+\lambda^s)v_x\Big)E[\frac{dL}{dt}] + \Big(v_x - v_y\Big)E[\frac{dU}{dt}]\Big\} = 0$$

Where $\quad v_x = \frac{\partial v}{\partial x}, \quad v_y = \frac{\partial v}{\partial y}, \quad v_{xx} = \frac{\partial^2 v}{\partial x^2}, \quad$ and $\quad v_{yy} = \frac{\partial^2 v}{\partial y^2}.$

To maximize the HJB, the choices of χ, L, and U are:

$$\chi\begin{cases} = 1 & \textit{if } \ v_y > (1+\lambda^s)v_x \\ \in [0,1] & \textit{if } \ v_y = (1+\lambda^s)v_x \\ = 0 & \textit{if } \ v_y < (1+\lambda^s)v_x \end{cases} \tag{A.1}$$

$$dL\begin{cases} > 0 & \textit{if } \ v_y \geq (1+\lambda^s)v_x \\ = 0 & \textit{if } \ v_y \leq (1+\lambda^s)v_x \end{cases} \tag{A.2}$$

$$dU\begin{cases} > 0 & \textit{if } \ v_x \geq v_y \\ = 0 & \textit{if } \ v_x \leq v_y \end{cases} \tag{A.3}$$

Thus, χ has a bang-bang solution. The No-Trade region, in this case, is exactly the same as Davis and Norman's NT, i.e., $v_x \leq v_y \leq (1+\lambda^s)v_x$. Thus in the NT region $\chi = 0$. In the NT region, $dL = dU = 0$ and $\chi = 0$. The partial differential equation that defines the value function in the NT region is as followed:

$$\frac{1}{2}\sigma^2 y^2 v_{yy} + \Big(rx + qy\Big)v_x + \Big(\mu - q\Big)yv_y + \frac{\eta}{1-\gamma}(x+y)^{1-\gamma} - \eta v = 0 \tag{A.4}$$

By the homothetic property of the utility function,weconjecture the value function that satisfies the above is, $\quad v(x,y) = y^{1-\gamma}\psi(\frac{x}{y})$.

As v is constant along lines of slope -1 in region S and along $-(1+\lambda^s)^{-1}$ in region B,

At $x \leq x_0$, there is constant A such that:

$$\psi(x) = \frac{1}{1-\gamma}A(x+1)^{1-\gamma} \tag{A.5}$$

And at $x \geq x_T$, there is a constant B such that:

$$\psi(x) = \frac{1}{1-\gamma} B(x+1+\lambda^s)^{1-\gamma} \tag{A.6}$$

So that we can write the differentials at A.4 in terms of ψ as:

$$v_y(x,y) = (1-\gamma)y^{-\gamma}\psi(\frac{x}{y}) - y^{-\gamma}\frac{x}{y}\psi'(\frac{x}{y}), \qquad v_x(x,y) = y^{-\gamma}\psi'(\frac{x}{y})$$

$$v_{yy}(x,y) = -\gamma y^{-\gamma-1}\Big((1-\gamma)\psi(\frac{x}{y}) - \frac{x}{y}\psi'(\frac{x}{y})\Big) - y^{-\gamma-1}\Big((1-\gamma)\frac{x}{y}\psi'(\frac{x}{y}) - \frac{x}{y}\psi'(\frac{x}{y}) - (\frac{x}{y})^2\psi''(\frac{x}{y})\Big)$$

$$v_{xx} = y^{-\gamma-1}\psi''(\frac{x}{y}) \qquad v_{xy} = -\gamma y^{-\gamma-1}\psi'(\frac{x}{y}) - y^{-\gamma-1}\frac{x}{y}\psi''(\frac{x}{y})$$

Setting $y = 1$, equation A.4 reduces to below for $x \in [x_0, x_T]$,

$$\beta_3 x^2\psi''(x) + (\beta_2 x + q)\psi'(x) + \beta_1\psi(x) + \frac{\eta}{1-\gamma}(x+1)^{1-\gamma} = 0 \tag{A.7}$$

$$\beta_1 = (\mu - q - \frac{1}{2}\sigma^2\gamma)(1-\gamma) - \eta, \qquad \beta_2 = \sigma^2\gamma + r - \mu + q, \qquad \beta_3 = \frac{1}{2}\sigma^2$$

Using A.2 at $x = x_T$, or by the value matching property, at this boundary, the value of the utility function in the direction of trade must be unchanged. The direction of trade at this boundary is selling $(1+\lambda^s)dL$ bonds and buying dL stocks.

$$v(x,y) = v(x-(1+\lambda^s)dL, y+dL)$$

Expanding above we get

$$v_y = (1+\lambda^s)v_x \quad \Rightarrow \quad (1-\gamma)\psi(x_T) = (x_T+1+\lambda^s)\psi'(x_T) \tag{A.8}$$

Similarly at $x = x_0$ direction of trade is to sell dU stocks and buy dU bonds. By A.3 or by the value-matching property at this boundary.

$$v(x, y) = v(x - dU, y + dU)$$

$$v_y = v_x \quad \Rightarrow \quad (1-\gamma)\psi(x_0) = (x_0 + 1)\psi'(x_0) \tag{A.9}$$

For optimality[1], at the boundaries, the derivative of the value function must stay unchanged in the direction of the trade. At $x = x_T$ the smooth pasting property dictates

$$v_y(x - (1+\lambda^s)dL, y + dL) = (1+\lambda^s)v_x(x - (1+\lambda^s)dL, y + dL)$$

Expanding above

$$(1+\lambda^s)v_{yx} - v_{yy} = (1+\lambda^s)^2 v_{xx} - (1+\lambda^s)v_{xy} \quad \Rightarrow \quad \frac{\psi''(x_T)}{\psi'(x_T)} = \frac{-\gamma}{x_T + 1 + \lambda^s} \tag{A.10}$$

And, based on the smooth pasting condition at x_0;

$$v_{yx} - v_{yy} = v_{xx} - vxy \quad \Rightarrow \quad \frac{\psi''(x_0)}{\psi'(x_0)} = \frac{-\gamma}{x_0 + 1} \tag{A.11}$$

Having the differential equation A.7, for $\psi(x)$ as in A.6, we can find B at $x = x_T$, and for $\psi(x)$ as in A.5, we can find A at $x = x_0$.

[1]Harrison and Taksar (1983) named this condition 'smooth pasting' or 'high contact' when the optimality requires the marginal utility before and after the regulation to be equal. Dumas (1991) names this condition 'super contact' when the optimality requires a higher degree of tangency which in this case the second derivative is required for optimality.

Proof of Proposition 1.2

To solve for the homogeneous solution of the above differential equation, let $\psi(x) = x^{-k}w(x^{-1})$ where k is the root for the following quadratic equation, $\beta_3 k^2 + (\beta_3 - \beta_2)k + \beta_1 = 0$ and w satisfied the following equation, (Polyanin and Zaitsev (2002)):

$$\beta_3 z w''(z) - [qz + \beta_2 - 2\beta_3(k+1)]w'(z) - kqw(z) = 0$$

$$k_i = \frac{(\beta_2 - \beta_3) \pm \sqrt{(\beta_2 - \beta_3)^2 - 4\beta_3\beta_1}}{2\beta_3}, \qquad i = 1, 2$$

Where k_1 is the positive root and k_2 is the negative root of the above equation. Note, this quadratic function has two roots, one is negative and one is positive. To solve for the solution of the transformed differential equation we use transformation $y = -\frac{q}{\beta_3}x^{-1}$, and for every k_i, $w(x^{-1}) = \Phi(a_i, b_i; y)$, where Φ is the Kummer's confluent hypergeometric function. Defining the parameters a_i, b_i for Kummer's confluent hypergeometric function:

$$a_i = k_i, \qquad b_i = -\frac{\beta_2}{\beta_3} + 2k_i + 2 \qquad \Phi(a, b; y) = 1 + \Sigma_{j=1}^{\infty} \frac{(a)_j}{(b)_j}\frac{y^j}{j!}$$

where, $\quad (a)_j = a(a+1)...(a+j-1), \quad (a)_0 = 1.$

If b is not a non-positive integer, then the general solution has the following form:

$$\psi(x) = C_1\Psi_1(x) + C_2\Psi_2(x) + \psi_p(x) \tag{A.12}$$

$$\Psi_i(x) = x^{-k_i}\Phi(a_i, b_i; \frac{q}{\beta_3}x^{-1})$$

$$\psi_p(x) = \Psi_2(x)\int_0^x \Psi_1(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)} - \Psi_1(x)\int_0^x \Psi_2(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)}$$

$$W(x) = \Psi_1(x)\Psi_2'(x) - \Psi_2(x)\Psi_1'(x)$$

Based on the optimal value function, A.12, and from the boundary conditions of A.8, A.9, A.10, and A.11 we can find C_1, C_2, x_0, and x_T,

Proof of Proposition 1.3

In this case the assumption is, $\lambda^b > 0 \qquad \lambda^s > 0.$

Define the following Martingale, $M_t \equiv \frac{\eta}{1-\gamma}\int_0^t e^{-\eta s} w_s^{1-\gamma} ds + e^{-\eta t} v(x,y)$

By the Ito formula,

$$M_t - M_0 = \frac{\eta}{1-\gamma}\int_0^t e^{-\eta s} w_s^{1-\gamma} ds + \int_0^t e^{-\eta s} dv(x,y) - \eta \int_0^t e^{-\eta s} v(x,y) dt$$

$$dM = e^{-\eta t}\Big\{\Big[\frac{\eta}{1-\gamma} w_t^{1-\gamma} - \eta v + \frac{1}{2}\sigma^2 y^2 v_{yy} + (\mu - q) y v_y + \Big((r - \frac{\lambda^b c}{1+\lambda^b})x + \frac{q}{1+\lambda^b} y\Big) v_x\Big] dt$$

$$+ \Big(\frac{1}{1+\lambda^s} v_y - \frac{1}{1+\lambda^b} v_x\Big)\chi_t\Big(cx + qy\Big)dt + \Big(v_y - (1+\lambda^s)v_x\Big)dL_t + \Big(v_x - (1+\lambda^b)v_y\Big)dU_t + \sigma y v_y dz_t\Big\}$$

The HJB becomes:

$$\max_{L,U,\chi}\Big\{\frac{1}{2}\sigma^2 y^2 v_{yy} + \Big((r - \frac{\lambda^b c}{1+\lambda^b})x + \frac{q}{1+\lambda^b} y\Big) v_x + (\mu - q) y v_y + \frac{\eta}{1-\gamma} w^{1-\gamma} - \eta v$$

$$+ \Big(\frac{1}{1+\lambda^s} v_y - \frac{1}{1+\lambda^b} v_x\Big)\chi\Big(cx + qy\Big) + \Big(v_y - (1+\lambda^s)v_x\Big)E[\frac{dL}{dt}] + \Big(v_x - (1+\lambda^b)v_y\Big)E[\frac{dU}{dt}]\Big\} = 0$$

Solving for the optimal χ :

$$\chi \begin{cases} = 0 & \text{if } \frac{1+\lambda^s}{1+\lambda^b} v_x > v_y \\ \in [0,1] & \text{if } \frac{1+\lambda^s}{1+\lambda^b} v_x = v_y \\ = 1 & \text{if } \frac{1+\lambda^s}{1+\lambda^b} v_x < v_y \end{cases} \qquad \text{(A.13)}$$

As $\frac{1}{1+\lambda^b} \leq \frac{1+\lambda^s}{1+\lambda^b} \leq 1+\lambda^s$, Thus the optimal choice of χ in A.13 splits the NT region into two sub-regions; NT_0 where $\chi = 0$, and NT_1 where $\chi = 1$.

Solving for the HJB at NT_1:

$$\frac{1}{2}\sigma^2 y^2 v_{1yy} + (\mu - \frac{\lambda^s}{1+\lambda^s}q)yv_{1y} + \frac{c}{1+\lambda^s}xv_{1y} + (r-c)xv_{1x} + \frac{\eta}{1-\gamma}(x+y)^{1-\gamma} - \eta v_1 = 0 \tag{A.14}$$

As before, based on the homothetic property of the value function, We conjecture that the value function has the following form: $v_1(x,y) = y^{1-\gamma}\psi_1(x/y)$.

Based on this transformation of the value function, equation A.14 reduces to below for $x \in [x_e, x_T]$:

$$\beta_3 x^2 \psi_1''(x) + (\beta_2 x - \frac{1}{1+\lambda^s}cx^2)\psi_1'(x) + (\beta_1 + \frac{1-\gamma}{1+\lambda^s}cx)\psi_1(x) + \frac{\eta}{1-\gamma}(x+1)^{1-\gamma} = 0 \tag{A.15}$$

$$\beta_1 = \Big(-\frac{1}{2}\sigma^2\gamma + \mu - \frac{\lambda^s}{1+\lambda^s}q\Big)(1-\gamma) - \eta \qquad \beta_2 = \sigma^2\gamma + r - c - \mu + \frac{\lambda^s}{1+\lambda^s}q, \qquad \beta_3 = \frac{1}{2}\sigma^2$$

The free boundaries in this region are:

$$\chi \begin{cases} = 0 & \text{if } \frac{1+\lambda^s}{1+\lambda^b}v_{1x} > v_{1y} \\ \in [0,1] & \text{if } \frac{1+\lambda^s}{1+\lambda^b}v_{1x} = v_{1y} \\ = 1 & \text{if } \frac{1+\lambda^s}{1+\lambda^b}v_{1x} < v_{1y} \end{cases} \tag{A.16}$$

$$dL \begin{cases} > 0 & \text{if } v_{1y} \geq (1+\lambda^s)v_{1x} \\ = 0 & \text{if } v_{1y} \leq (1+\lambda^s)v_{1x} \end{cases} \tag{A.17}$$

A.17 reflects value matching property at $x = x_T$. At this boundary, the trade occurs in the direction of $(1+\lambda^s)dL$ bond sale and purchase of dL stocks. By the value matching property, the value function should be unchanged when this trade occurs at this boundary.

$$v_1(x,y) = v_1(x-(1+\lambda^s)dL, y+dL)$$

$$v_{1y} = (1+\lambda^s)v_{1x} \quad \Rightarrow \quad (1-\gamma)\psi_1(x_T) = (x_T+1+\lambda^s)\psi_1'(x_T) \tag{A.18}$$

Equation A.16 reflects the value matching property at $x = x_e$, where the Sweep regulator must be indifferent between buying bonds or buying stock with the cash at the center.

$$v_1(x+\frac{1}{1+\lambda^b}(cx+qy)dt, y) = v_1(x, y+\frac{1}{1+\lambda^s}(cx+qy)dt)$$

Expanding above gives us the value-matching property at the Sweep boundary.

$$v_{1y} = \frac{1+\lambda^s}{1+\lambda^b}v_{1x} \quad \Rightarrow \quad (1-\gamma)\psi_1(x_e) = (x_e+\frac{1+\lambda^s}{1+\lambda^b})\psi_1'(x_e) \tag{A.19}$$

Based on the smooth pasting condition at $x = x_T$, the derivative of the value function at the boundary of $x = x_T$ must be fixed at the direction of trade by the regulator.

$$v_{1y}(x-(1+\lambda^s)dL, y+dL) = (1+\lambda^s)v_{1x}(x-(1+\lambda^s)dL, y+dL)$$

$$2(1+\lambda^s)v_{1yx} = v_{1yy}+(1+\lambda^s)^2v_{1xx} \quad \Rightarrow \quad \frac{\psi_1''(x_T)}{\psi_1'(x_T)} = \frac{-\gamma}{x_T+1+\lambda^s} \tag{A.20}$$

Based on the optimality of smooth pasting condition at the Sweep boundary, $x = x_e$, derivative of the value function must stay unchanged at the direction of the trade at this boundary.

$$v_{1y}(x+\frac{1}{1+\lambda^b}(cx+qy)dt, y) = \frac{1+\lambda^s}{1+\lambda^b}v_{1x}(x+\frac{1}{1+\lambda^b}(cx+qy)dt, y) \quad \Rightarrow \quad v_{1yx} = (\frac{1+\lambda^s}{1+\lambda^b})^2v_{1xx}$$

$$v_{1y}(x, y+\frac{1}{1+\lambda^s}(cx+qy)dt) = \frac{1+\lambda^s}{1+\lambda^b} v_{1x}(x, y+\frac{1}{1+\lambda^s}(cx+qy)dt) \quad \Rightarrow \quad v_{1yy} = (\frac{1+\lambda^s}{1+\lambda^b})^2 v_{1xy}$$

Thus

$$v_{1yy} = (\frac{1+\lambda^s}{1+\lambda^b})^2 v_{1xx} \quad \Rightarrow \quad \frac{\psi_1''(x_e)}{\psi_1'(x_e)} = \frac{-\gamma(1+\lambda^b)}{x_e(1+\lambda^b)+(1+\lambda^s)} \tag{A.21}$$

In the NT_0 region the HJB is as followed:

$$\frac{1}{2}\sigma^2 y^2 v_{2yy} + \Big((r - \frac{\lambda^b c}{1+\lambda^b})x + \frac{q}{1+\lambda^b} y\Big) v_{2x} + (\mu - q) y v_{2y} + \frac{\eta}{1-\gamma}(x+y)^{1-\gamma} - \eta v_2 = 0 \tag{A.22}$$

Based on the homothetic property of the value function, We conjecture that the value function has the following form, $\quad v_2(x, y) = y^\gamma \psi_2(x/y)$.

Equation A.22 reduces to below for $x \in [x_0, x_e]$,

$$\beta_3 x^2 \psi_2''(x) + (\beta_2 x + \frac{q}{1+\lambda^b})\psi_2'(x) + \beta_1 \psi_2(x) + \frac{\eta}{1-\gamma}(x+1)^{1-\gamma} = 0 \tag{A.23}$$

$$\beta_1 = (\mu - q - \frac{1}{2}\sigma^2\gamma)(1-\gamma) - \eta, \quad \beta_2 = \sigma^2\gamma + r - \frac{\lambda^b}{1+\lambda^b}c - \mu + q, \quad \beta_3 = \frac{1}{2}\sigma^2$$

The free boundaries in NT_0 are:

$$\chi \begin{cases} = 0 & \text{if } \frac{1+\lambda^s}{1+\lambda^b} v_{2x} \geq v_{2y} \\ = 1 & \text{if } \frac{1+\lambda^s}{1+\lambda^b} v_{2x} \leq v_{2y} \end{cases} \tag{A.24}$$

$$dU \begin{cases} > 0 & \text{if } \frac{1}{1+\lambda^b} v_{2x} \geq v_{2y} \\ = 0 & \text{if } \frac{1}{1+\lambda^b} v_{2x} \leq v_{2y} \end{cases} \tag{A.25}$$

At the Buy boundary,$x = x_0$, equation A.25 reflects the value matching property of v_2 in the direction of trade at this point.

$$v_2(x, y) = v_2(x + dU, y - (1+\lambda^b)dU)$$

$$v_{2x} = (1+\lambda^b)v_{2y} \quad \Rightarrow \quad (1-\gamma)(1+\lambda^b)\psi_2(x_0) = (x_0(1+\lambda^b)+1)\psi_2'(x_0) \qquad \text{(A.26)}$$

Similar to A.19, the value matching of v_2 at the sweep boundary, $x = x_e$, is reflected by A.24.

$$v_{2y} = \frac{1+\lambda^s}{1+\lambda^b} v_{2x} \quad \Rightarrow \quad (1-\gamma)\psi_2(x_e) = (x_e + \frac{1+\lambda^s}{1+\lambda^b})\psi_2'(x_e) \qquad \text{(A.27)}$$

Based on the smooth pasting condition at $x = x_0$, the derivative of the value function in the NT_1 region must be unchanged at the direction of trade at the buy boundary.

$$v_{2x}(x+dU, y-(1+\lambda^b)dU) = (1+\lambda^b)v_{2y}(x+dU, y-(1+\lambda^b)dU)$$

$$-v_{2xx} + (1+\lambda^b)v_{2xy} = -(1+\lambda^b)v_{2yx} + (1+\lambda^b)^2 v_{2yy} \quad \Rightarrow \quad \frac{\psi_2''(x_0)}{\psi_2'(x_0)} = -\frac{\gamma(1+\lambda^b)}{x_0(1+\lambda^b)+1} \qquad \text{(A.28)}$$

And similar to A.21, based on the smooth pasting condition at $x = x_e$.

$$v_{1yy} = (\frac{1+\lambda^s}{1+\lambda^b})^2 v_{1xx} \quad \Rightarrow \quad \frac{\psi_2''(x_e)}{\psi_2'(x_e)} = -\frac{\gamma(1+\lambda^b)}{x_e(1+\lambda^b)+(1+\lambda^s)} \qquad \text{(A.29)}$$

Combining A.16 and A.24 at $x = x_e$:

$$\psi_1'(x_e) = \psi_2'(x_e) \qquad \text{(A.30)}$$

Combining A.21 and A.29 at $x = x_e$:

$$\psi_1''(x_e) = \psi_2''(x_e) \qquad \text{(A.31)}$$

Also, at $x = x_e$:

$$\psi_1(x_e) = \psi_2(x_e) \qquad \text{(A.32)}$$

Conditions of A.30, A.31, A.32 show that at the sweep boundary on top of the value matching and the first derivative the second derivatives must be similar in the regions on both sides of the sweep boundary.

We can find A at $x = x_0$ for $\psi(x) = \frac{1}{1-\gamma}A(x(1+\lambda^b)+1)^{1-\gamma}$ in A.23.

We can find B at $x = x_T$ for $\psi(x) = \frac{1}{1-\gamma}B(x+1+\lambda^s)^{1-\gamma}$ in A.15.

Proof of Proposition 1.4

To solve for the homogeneous solution of the above differential equation, using transformations $\psi_1(x) = x^k w(x)$ where k is the root for the quadratic equation $\beta_3 k^2 + (\beta_2 - \beta_3)k + \beta_1 = 0$. This leads to the following equation:

$$\beta_3 x w''(x) + [(\frac{-c}{1+\lambda^s})x + 2\beta_3 k + \beta_2]w'(x) + \Big[-(\frac{1}{1+\lambda^s})ck + \frac{1-\gamma}{1+\lambda^s}c\Big]w(x) = 0$$

$$k_i = \frac{(\beta_3 - \beta_2) + \sqrt{(\beta_3 - \beta_2)^2 - 4\beta_3\beta_1}}{2\beta_3} \qquad i = 1, 2$$

Note, this quadratic function has two roots, one is negative and one is positive. To solve for the solution of the transformed differential equation we use transformation, $y = -\frac{c}{\beta_3(1+\lambda^s)}x$, and $w(x) = e^{-y}\Phi(y)$, where Φ is the Kummer's confluent hypergeometric function.

Defining the parameters a, b for the Kummer's confluent hypergeometric function, $\Phi(a, b; y)$ as a particular solution;

$$a_i = k_i - 1 + \gamma, \qquad b_i = \frac{\beta_2}{\beta_3} + 2k_i \quad \Phi(a, b; y) = 1 + \Sigma_{j=1}^{\infty}\frac{(a)_j}{(b)_j}\frac{y^j}{j!}$$

where, $\quad (a)_j = a(a+1)...(a+j-1), \quad (a)_0 = 1.$

If b is not a non-negative integer, k_1 being the positive root and k_2 the negative root then the general solution has the following form:

$$\psi_1(x) = C_{11}\Psi_{11} + C_{12}\Psi_{12}(x) + \psi_{1p}(x) \tag{A.33}$$

$$\Psi_{1i}(x) = x^{k_i}\Phi(a_i, b_i; \frac{c}{\beta_3(1+\lambda^s)}x)$$

$$\psi_{1p}(x) = \Psi_{12}(x)\int_0^x \Psi_{11}(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)} - \Psi_{11}(x)\int_0^x \Psi_{12}(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)}$$

$$W(x) = \Psi_{11}(x)\Psi_{12}'(x) - \Psi_{12}(x)\Psi_{11}'(x)$$

To solve for the homogeneous solution of the differential equation in NT_0, using transformations $x = z^{-1}$, $\psi_2(x) = x^k w(z)$ where k is the root for the quadratic equation $\beta_3 k^2 + (\beta_3 - \beta_2)k + \beta_1 = 0$. This leads to the following equation:

$$\beta_3 z w''(z) - [\frac{q}{1+\lambda^b}z + \beta_2 - 2\beta_3(k+1)]w'(z) - k\frac{q}{1+\lambda^b}w(z) = 0$$

$$k_i = \frac{(\beta_2 - \beta_3) \pm \sqrt{(\beta_2-\beta_3)^2 - 4\beta_3\beta_1}}{2\beta_3} \qquad i = 1, 2$$

Note, this quadratic function has two roots, one is negative and one is positive. To solve for the solution of the transformed differential equation we use transformation $y = -\frac{q}{(1+\lambda^b)\beta_3}z$, and $w(z) = e^{-y}\Phi(y)$, where Φ is the Kummer's confluent hypergeometric function. Defining the parameters a, b for the Kummer's confluent hypergeometric function, $\Phi(a, b; y)$ as a particular solution:

$$a_i = k_i, \qquad b_i = -\frac{\beta_2}{\beta_3} + 2k_i + 2 \quad \Phi(a, b; y) = 1 + \Sigma_{j=1}^{\infty}\frac{(a)_j}{(b)_j}\frac{y^j}{j!}$$

where, $\quad (a)_j = a(a+1)...(a+j-1), \quad (a)_0 = 1.$

If b is not a non-negative integer, k_1 being the positive root and k_2 the negative root then the general solution has the following form:

$$\psi_2(x) = C_{21}\Psi_{21}(x) + C_{22}\Psi_{22}(x) + \psi_{2p}(x) \tag{A.34}$$

$$\Psi_{2i}(x) = x^{-k_i}\Phi(a_i, b_i; \frac{q}{(1+\lambda^b)\beta_3}x^{-1})$$

$$\psi_{2p}(x) = \Psi_{22}(x)\int_0^x \Psi_{21}(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)} - \Psi_{21}(x)\int_0^x \Psi_{22}(z)\frac{\eta(z+1)^{1-\gamma}}{(1-\gamma)\beta_3}\frac{dz}{W(z)}$$

$$W(x) = \Psi_{21}(x)\Psi'_{22}(x) - \Psi_{22}(x)\Psi'_{21}(x)$$

From A.18, A.20, A.26, A.28, A.30, A.31,and A.32 we can find C_{11}, C_{12}, C_{21}, C_{22}, x_0, x_T, and x_e.

Appendix B

Appendix for Chapter 2

Proof of Proposition 2.1

It is straight forward to show that if $\chi \leq \chi_1$, the equilibrium is in the Prime credit region, which entails $\eta = 0$. Now consider $\chi > \chi_1$. It follows that $\eta > 0$. Consider the Poor credit region which requires $a > 0$ and $r = \underline{R}$. By (2.10), $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$. By (2.4)-(2.6), the demand for ℓ is characterized by

$$-c'\left(\frac{\chi + \underline{R} - \bar{R}}{\bar{R} - \chi}\ell\right)\left(\chi + \underline{R} - \bar{R}\right) + \chi\underline{R}v'\left(\frac{\chi\underline{R}}{\bar{R} - \chi}\ell\right) = 0 \tag{B.1}$$

Take derivative wrt χ to get

$$\frac{d\ell}{d\chi} = \frac{c'\left(\bar{R} - \underline{R}\right)\left(\bar{R} - \chi\right)^2 + \ell\underline{R}\chi\left[c''\left(\chi + \underline{R} - \bar{R}\right) - v''\underline{R}\bar{R}\right]}{\chi\left(\bar{R} - \chi\right)\left[\underline{R}^2\chi^2 v'' - \left(\chi + \underline{R} - \bar{R}\right)^2 c''\right]} < 0$$

By (2.11) and since x_1 is constant, $da/d\chi > 0$. Thus, the cutoff of χ_2 between Fair and Poor credit regions is unique. At χ_2, ℓ solving (B.1) satisfies $n\ell = 1 - \lambda\hat{x}_1$. ■

Proof of Proposition 2.2

In the Prime credit region, the equations (2.9), (2.4), (2.20), (2.21) and $c'(k) = \bar{R}u'(x_2)$, which jointly solve (x_1, k, r, τ). By the implicit function theorem, given $\varepsilon' < 0$, $d\chi_1^S/d\phi = |A_2|/|A_1|$, where $|A_1| =$

$$
\begin{aligned}
&\sigma\frac{\varepsilon'}{\varepsilon}u_1'S_E\begin{vmatrix} c'\frac{\bar{R}-\chi}{\underline{R}} & \chi\lambda v' & \chi v' \\ c''\big(\chi+\underline{R}-\bar{R}\big)+\underline{R}\chi^2 v'' & \underline{R}\chi^2\lambda v'' & \underline{R}\chi^2 v'' \\ -\frac{\underline{R}\left(\bar{R}-\chi\right)}{1-\lambda}u_2'' & u_1''+\frac{\lambda\underline{R}\left(\bar{R}-\chi\right)}{1-\lambda}u_2'' & -\frac{\underline{R}\left(\chi+\underline{R}-\bar{R}\right)}{1-\lambda}u_2'' \end{vmatrix} \\
&-(1-\varepsilon)\phi\times \\
&\begin{vmatrix} u_2'c'\left(\bar{R}-\chi\right)+\varepsilon c''S_B & c'u_2'\lambda\big(\chi+\underline{R}-\bar{R}\big)-(1-\varepsilon)u_1''S_E & u_2'c'\big(\chi+\underline{R}-\bar{R}\big) \\ c''\big(\chi+\underline{R}-\bar{R}\big)+\underline{R}\chi^2 v'' & \underline{R}\chi^2\lambda v'' & \underline{R}\chi^2 v'' \\ -\frac{\underline{R}\left(\bar{R}-\chi\right)}{1-\lambda}u_2'' & u_1''+\frac{\lambda\underline{R}\left(\bar{R}-\chi\right)}{1-\lambda}u_2'' & -\frac{\underline{R}\left(\chi+\underline{R}-\bar{R}\right)}{1-\lambda}u_2'' \end{vmatrix}
\end{aligned}
$$

and

$$
|A_2| = \frac{\varepsilon'}{\varepsilon}\tau u_1'S_B\begin{vmatrix} c''\big(\chi+\underline{R}-\bar{R}\big)-\underline{R}\chi^2 v'' & \underline{R}\chi^2\lambda v'' \\ \frac{\underline{R}\left(\bar{R}-\chi\right)}{1-\lambda}u_2'' & -u_1''+\frac{\lambda\underline{R}\left(\bar{R}-\chi\right)}{1-\lambda}u_2'' \end{vmatrix}
$$

Given $\varepsilon' < 0$, $|A_3| < 0$ and $da/d\phi < 0$, which means given χ, if $\phi' > \phi$ and $a > 0$ under ϕ', then $a > 0$ under ϕ. Thus, the cutoff of χ_2^S is higher for higher ϕ. ■

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VITA

Mohammad Nabi Arjmandi was born in Ilam, Iran, where he spent the early years of his life. He embarked on an academic journey that led him to excel in the field of quantitative finance and economics. Nabi holds a Bachelor's degree in Applied Mathematics from Shahid Beheshti University in Tehran, Iran, where his passion for mathematics and quantitative analysis was nurtured.

Driven by a desire to further his knowledge and expertise, Nabi pursued graduate studies in the United States. He earned a Master's degree in Financial Engineering from Claremont Graduate University, a renowned institution for quantitative finance education. Following this, he continued his academic pursuits by obtaining a Master of Economics degree from Washington University in St. Louis.

Nabi's educational background, combined with his dedication to continuous learning, has equipped him with a strong foundation in financial analysis and economics. He is enthusiastic about contributing his insights and skills to the world of finance and macroeconomics, with a commitment to making meaningful contributions to the academic and professional communities